Abstract

In this paper we propose an endogenous growth model of commodity-rich economies in which: (i) long-run (steady-state) growth is endogenous and yet independent of commodity prices; (ii) commodity prices affect short-run growth through transitional dynamics; and (iii) the status of net commodity importer/exporter is endogenous. We argue that these predictions are consistent with historical evidence from the 19th to the 21st century.

J.E.L. Codes: O3; O4; Q4
Keywords: Economic growth; Commodity prices; Net commodity importer/exporter
1 Introduction

Historical evidence from the 19th to the 21st century provides three stylized facts for commodity-rich countries. (1) Commodity prices are generally un-correlated with long-run growth and (2) commodity prices are instead correlated with growth in the short-run; i.e., movements in commodity prices have “level effects” on income, but no “growth effects.” These two facts, which we discuss in Section 2, raise an important question: what is the economic mechanism that drives the short-run co-movement between commodity prices and growth to vanish in the long-run? Moreover, (3) the status of commodity importer/exporter changes over time. In fact, commodity-rich economies switch from being net importers to net commodity exporters and vice-versa. For instance, Canada became a net oil exporter in the mid 1980’s (see Issa et al., 2008) and China was a net oil exporter until the early 1990’s and according to the U.S. Energy Information Administration (EIA) it became the world’s second-largest net importer of crude oil in 2009. Furthermore, recent developments in the world oil market have reignited the long-standing debate about the macroeconomic impact of oil price shocks on oil-importing and oil-exporting economies.¹

These considerations motivate our work. We study the relationship between commodity prices and growth using a model of endogenous growth that draws a marked distinction between the steady-state (long-run) and the transitional dynamics (short-run) relationship between commodity prices and growth.² Moreover, the model provides conditions on the level of the commodity price and the country’s commodity endowment that jointly determine whether an economy is a net importer or exporter of the commodity. Therefore, it answers the question of when and how commodity price shocks have harmful effects on the level of economic activity, total factor productivity (TFP), and welfare.

In Section 3 we propose a small open economy (SOE) model of endogenous growth. Specifically, we assume that the endowment of the commodity is exogenous and constant, and that commodity prices are taken parametrically by the agents inside the model.³ Thus, we abstract from the determination of world commodity prices and focus on their effects

¹Oil prices have plummeted in recent months; the decline in the price of the West Texas Intermediate (WTI)—from approximately 105 dollars per barrel in June 2014 to 47 in January 2015—has been large from an historical perspective in both relative and absolute terms.

²We recognize that variants of the neoclassical growth model would be consistent with the first two facts. In that type of models, long-run (steady-state) growth is determined by the pace of exogenous technical progress which is independent of commodity prices by assumption.

³We view price-taking as a convenient assumption since it affords analytical tractability. We acknowledge that, for certain commodities and time periods, countries may have some degree of market power (e.g., New Zealand supplies close to half of the total world exports of lamb and mutton).
on aggregate variables such as consumer expenditures on home and foreign goods, value of manufacturing production, and TFP.\textsuperscript{4}

To explain the economic mechanism that drives the results it is useful to describe the structure of the model. As in Peretto (1998, 1999), the model combines horizontal (expanding-variety) and vertical (cost-reducing) innovation. Manufacturing is the engine of long-run growth. In this sector, incumbent firms engage in two activities: (1) they use labor and materials to produce intermediate goods supplied to the downstream consumption sector (materials are purchased from an upstream sector which uses labor and the commodity as inputs); and (2) they allocate labor to reduce unit production costs. Market structure is endogenous in that both firm size and the mass of firms are jointly determined in free-entry equilibrium. In fact, firm size, which is proportional to the rate of gross profitability, is the key variable regulating the incentives to reduce costs.

Movements in commodity prices affect the economy via two channels: (1) they change the value of the endowment thus inducing income/wealth effects—“commodity wealth channel”—and (2) they affect the demand for the commodity in the materials sector and, through the demand of materials in manufacturing and inter-sectoral labor reallocation, have cascade effects through all the vertical cost structure of production—“cost channel.”

In Section 4 we derive a “long-run commodity price super-neutrality” result: the steady-state growth rate of TFP is independent of commodity prices. The mechanism that drives this result is the sterilization of market-size effects: given the number of firms, movements in commodity prices change the size of the manufacturing sector, firm size (market share), and so incentives to vertical innovation. Ceteris paribus, this would have steady-state growth effects. However, as the size and so the profitability of incumbent firms change, the mass of firms endogenously adjusts to bring the economy back to the initial steady-state level of firm size, thereby sterilizing the long-run growth effects of commodity price changes.

We argue that the neutrality of commodity prices for long-run growth is critical for the model to be consistent with two basic time-series observations: commodity prices exhibit large and persistent long-run movements (see Jacks, 2013) whereas trend growth in several commodity-rich economies (e.g., Western offshoots) exhibits no such large persistent changes (see Section 2). Put differently, if long-run (steady-state) growth depended on commodity

\textsuperscript{4}Kilian (2008b, 2009) argues for the need to account for the endogeneity of energy prices when studying their effects on the economy. We acknowledge that studying the joint dynamics of commodity prices and growth, and their interdependence, is of first-order importance but it goes beyond the scope of this paper. See Peretto and Valente (2011) and Peretto (2012) for papers that endogenize the price of the commodity within the same class of models we use in this paper.
prices then we would observe correlated swings in growth rates of real GDP per capita, but this is at odd with the data. This argument, which parallels that in Jones (1995), draws an analogy between the effects of commodity price on growth and the literature on the (lack of) growth effects of taxation (see Easterly and Rebelo, 1993; Easterly et al., 1993; Stokey and Rebelo, 1995; Mendoza et al., 1997; Peretto, 2003; Jaimovich and Rebelo, 2012).

We also point out that the long-run commodity price super-neutrality result has stark implications for the long-standing discussion on the Prebisch-Singer hypothesis.\(^5\) Note that we make no attempt to explain why commodity prices would fall relative to the prices of imported goods. However, we show that a downward trend in the commodity/imports relative price has no steady-state growth effects.

In Section 5 we derive conditions for which a commodity price boom increases, decreases, or leaves unchanged the value of manufacturing production and so short-run (transitional) growth; the sign of the effect depends upon the substitution possibilities between labor and materials in manufacturing, and between labor and the commodity in materials. Thus, we identify four cases: after a commodity price boom, (1) the value of manufacturing production raises if the demand for the commodity is overall inelastic—“global complementarity”—(2) it falls if the demand for the commodity is overall elastic—“global substitution”—(3) it does not change if manufacturing and materials sectors have Cobb-Douglas production functions—“Cobb-Douglas-like economy”—and (4) it raises or falls depending on the initial level of the price if materials and manufacturing sectors display opposite substitution/complementarity properties—“complementarity/substitution switch.”\(^6\)

These model’s predictions are related to the literature on the “curse of natural resources” and the “Dutch Disease.”\(^7\) How a commodity price boom affects manufacturing production is ultimately an empirical matter. Yet the empirical literature provides a spectrum of findings ranging from (i) little/no effect (see Gelb, 1988; Sala-i-Martin and Subramanian, 2003; Black et al., 2005; Caselli and Michaels, 2013), (ii) positive (see Allcott and Keniston, 2014; Smith, 2014), to (iii) negative effects (see Ismail, 2010; Rajan and Subramanian, 2011; Harding and Venables, 2013; Charnavoki and Dolado, 2014). In this regard, our model identifies in

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\(^5\)The Prebisch-Singer hypothesis (see Prebisch, 1950; Singer, 1950) posits that in the long-run commodity prices fall relative to the prices of the manufactured goods that the commodity-exporting country imports from abroad. See Harvey et al. (2010) for recent empirical evidence.

\(^6\)Note that the substitution/complementarity effects in this paper are reversed compared to those in Peretto and Valente (2011); this is because we focus on commodity price changes in a SOE, instead of commodity endowment changes in a world equilibrium as in Peretto and Valente’s work.

\(^7\)The “Dutch Disease” hypothesis posits that a boom in the natural resource sector shrinks manufacturing production through crowding out and an appreciation of the real exchange rate.
the overall substitution possibilities between labor and the commodity a key conditioning variable that the empirical literature has so far abstracted from. In the model, the overall substitutability between labor and the commodity is subsumed in (i) the price elasticity of the demand for materials in manufacturing, (ii) the price elasticity of the demand for the commodity in materials, and (iii) the commodity share in materials production costs, which can all be mapped into observable variables and/or estimated.\(^8\)

In Section 5 we further show that the deep technological parameters determining the overall substitutability between labor and the commodity, the level of the commodity price, and the country’s own commodity endowment (relative to population size) jointly determine the status of commodity importer/exporter; such specialization result formalizes the notion of comparative advantage in commodity trade embedded in the model.\(^9\) The equilibrium of the model features a trade-off between the rate at which the economy transforms the commodity endowment into home consumption goods—“internal transformation rate” (ITR)—and the rate at which it transforms the commodity endowment into foreign consumption goods—“external transformation rate” (ETR). Thus, if the ITR dominates the ETR, the economy is a commodity importer; otherwise, it is a commodity exporter.

In Section 6 we discuss the effects of commodity price changes on welfare. The equilibrium of the model suggests that a commodity-rich economy can gain \textit{in terms of welfare} from a permanent increase in the commodity price \textit{even though} it is a commodity importer. This happens because the revenues from the sales of the commodity endowment go up one-for-one with the commodity price whereas the cost of commodity consumption does not. Specifically, commodity consumption responds negatively to the commodity price increase; such effect is strong when the domestic demand for the commodity is elastic, i.e., under global substitution.

In Section 7 we provide a simple numerical exercise that further illustrates the dynamic response of the model economy to an unexpected commodity price shock. We offer some concluding remarks in Section 8.

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\(^8\)Note that in U.S. data the degree of substitutability largely varies across types of resources. For instance, Jin and Jorgenson (2010) document evidence of complementarity for several products of mining/harvesting activities (metal mining, oil and gas, coal mining, primary metals, non-metallic mining, tobacco products) and of substitutability for others (lumber and wood, stone and clay, non-tobacco agricultural products), but they generally reject the Cobb-Douglas unit elasticity specification.

\(^9\)Note that this specialization result is absent in Peretto and Valente (2011) where the status of net commodity importer/exporter is \textit{exogenously} given.
2 Motivating Facts

In this section we detail the main empirical observations that motivate our work. Taken in isolation, each of these observations are well-known in the respective literature. Yet, we argue that a unitary view of these otherwise stylized facts provides new insights into the economics of commodity prices and growth.

Figure 1: U.S. Real GDP per Capita and Energy Prices

Notes: Data for the U.S. real GDP per capita are from the Angus Maddison’s dataset which is publicly available at http://www.ggdc.net/maddison/maddison-project/home.htm. Real energy prices are available from David Jacks’s website at http://www.sfu.ca/~djacks/data/boombust/index.html. Trend (red line) is the long-run trend (LR) component of the series as in Definition 1.

Empirical work on long-run trends in commodity prices and growth has been for long time hindered by the shortness of the time period for which reliable data are available. However, Angus Maddison (see Bolt and van Zanden, 2013) for real GDP per capita and Jacks (2013) for commodity prices have provided data that span the 19th, 20th, and 21st century. The increased time span, approximately 150 years of data, allows us to relate the long-run trend components in commodity prices and growth for several commodity-rich
countries. This is especially important for the current paper since we aim at drawing a marked distinction between the steady-state (long-run) and transitional dynamics (short-run) link between commodity prices and growth.

![Graphs of Real GDP per Capita](image)

**Figure 2: Real GDP per Capita in the Western Offshoots**

*Notes:* Data for real GDP per capita are from the Angus Maddison’s dataset which is publicly available at [http://www.ggdc.net/maddison/maddison-project/home.htm](http://www.ggdc.net/maddison/maddison-project/home.htm). Trend (red line) is the long-run trend (LR) component of the series as in Definition 1.

Consistently with the literature on commodity price super-cycles (see Cuddington and Jerrett, 2008; Jerrett and Cuddington, 2008; Erten and Ocampo, 2013; Jacks, 2013), we adopt the following definition of Long-Run (LR).

**Definition 1 (Long-run trend).** *Given a time series $x_t$, the Long-Run trend (LR) component, $x_t^{LR}$, corresponds to the component of $x_t$ with periodicity larger than 70 years.*

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10We use a band-pass filter, as implemented by Christiano and Fitzgerald (2003), to isolate the Short-Run component (SR), $x_t^{SR}$, which corresponds to the component of $x_t$ with periodicity between 2 and 70 years. The Long-Run (LR) trend component is then $x_t^{LR} = x_t - x_t^{SR}$. The choice of the band-pass filter is dictated by our aim at contrasting the long-run (low-frequency) with the short-run (high-frequency) properties of the data. An Hodrick-Prescott (HP) filter could in principle serve the same purpose, but we would need
The first fact follows directly from Definition 1 above.

**Fact 1.** Commodity prices exhibit large and persistent long-run movements whereas growth rates of real GDP per capita exhibit no such large persistent changes.

Fact 1, which we take as one of the key empirical observations of our analysis, posits an important disconnect between the long-run properties of commodity prices and growth. As a result, we argue it is a litmus test for endogenous growth models along the lines of Jones (1995): if long-run (steady-state) growth depended on commodity prices then we would observe correlated swings in growth rates which is at odd with the data. This type of argument is analogous to the one made by Stokey and Rebelo (1995) in the context of taxation and growth.

Figure 1 illustrates Fact 1 for the U.S. and energy prices. On the one hand, the LR component in real GDP per capita is almost a straight line implying that trend growth has been approximately constant for the last 150 years. Figure 2 shows that a similar pattern emerges for all Western offshoots.\(^\text{11}\) On the other hand, commodity prices exhibit large and persistent movements in the LR component. This observation is not specific to energy prices but it is a robust finding across several commodities (e.g., animal products, grains, metals, minerals, precious metals, softs).\(^\text{12}\)

The second fact that we highlight is the following.

**Fact 2.** Commodity prices and growth rates of real GDP per capita co-move in the short-run.

Evidence for Fact 2 comes from a variety of sources. Despite mixed evidence on the sign of the relationship, overall the empirical literature strongly supports the view that commodity prices are generally correlated with growth in the short-run.

The first source of evidence is the empirical literature on the “curse of natural resources.” Sachs and Warner (1995, 1999, 2001) find a statistically significant negative relationship

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\(^{11}\)We refer the reader to the Online Appendix to the paper for time-series plots of real GDP per capita in several other commodity-rich countries, i.e., Argentina, Brazil, Chile, and Colombia. Due to the lack of long time series for real GDP per capita, we are unable to extend our sample to other developing commodity-rich economies. Yet, the countries we consider are all resource/commodity rich economies.

\(^{12}\)See Jacks (2013) for an extensive treatment of long-run trends, medium-term cycles, and short-run boom/bust episodes in commodity prices. We also refer the reader to the Online Appendix to the paper for time-series plots of several other commodity prices, i.e., animal products, grains, metals, minerals, precious metals, and softs.
between natural resource intensity (e.g., exports of natural resources in percent of GDP) and average growth over a twenty-year period. However, the existence of a resource curse has been called into question by several papers (see Deaton and Miller, 1995; Brunnschweiler and Bulte, 2008; Alexeev and Conrad, 2009; Smith, 2013). The common theme of these papers is that a resource boom is indeed associated with positive instead of negative growth effects as the resource curse hypothesis would predict.

We share with the resource curse papers their focus on the low-frequency relationship between commodity prices and growth. However, we differ from them in that we draw a sharp distinction between what we consider to be a long-run (steady-state) as opposed to a short-run (transitional dynamics) commodity price/growth relationship.

The second source of evidence is the literature on oil prices and the business cycle (see Hamilton, 1996, 2003, 2009; Kilian, 2008a,c, 2009). This strand of literature focuses instead on the high-frequency relationship between oil prices and growth, as such it abstracts from the possibility of growth effects of oil prices in the long-run.

3  A Model Economy

Time is continuous and indexed by \( t \geq 0 \). Throughout, we omit time subscripts unless needed for clarity.

3.1  Overview

We consider a small open economy (SOE) populated by a representative household that supplies labor services inelastically in a competitive labor market. The household faces a standard expenditure/saving decision problem: it chooses the path of expenditures (home and foreign goods) and savings by freely borrowing and lending in a competitive market for financial assets at the prevailing interest rate.\(^\text{13}\) Household’s income consists of returns on asset holdings, labor income, profits, and commodity income which is the (constant) commodity endowment valued at the world commodity price.

The production side of the economy consists of four sectors: (1) consumption goods, (2) intermediate goods or manufacturing, (3) materials, and (4) extraction. The consumption

\(^{13}\text{It is possible to think of our model economy as taking the world interest rate parametrically. Since the model has the property that the domestic interest rate jumps to its steady-state level, which is pinned down by the domestic discount rate, as long as the SOE has the same discount rate as the rest of the world, the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.}\)
sector consists of a representative competitive firm which combines differentiated intermediate goods to produce an homogeneous final good. Upon entry (horizontal innovation), manufacturing firms combine labor services and materials to produce differentiated intermediate goods. They also engage in activities aimed at reducing unit production costs (vertical innovation). Entry requires the payment of a sunk cost. Materials are supplied by an upstream competitive sector which uses labor services and the commodity as inputs. Finally, the extraction sector sells the commodity endowment to the materials sector and potentially abroad.

Manufacturing is the engine of endogenous growth. Specifically, the economy starts out with a given range of intermediate goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms devote labor to cost-reducing (or, equivalently, productivity enhancing) activities. As each firm strives to figure out how to improve efficiency, it contributes to the pool of public knowledge that benefits the future cost-reducing activities of all firms. This allows the economy to grow at a constant rate in steady state, which is reached when entry stops and the economy settles into a stable industrial structure.

### 3.2 Households

The representative household solves the following maximization problem:

\[
\max_{\{Y_H, Y_F\}} U(t) = \int_t^\infty e^{-\rho(s-t)} \log u(s) ds, \quad \rho > 0
\]  

(1)

where

\[
\log u = \varphi \log \left( \frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left( \frac{Y_F}{P_F L} \right), \quad 0 < \varphi < 1
\]  

(2)

subject to the budget constraint,

\[
\dot{A} = rA + WL + \Pi_H + \Pi_M + p\Omega - Y_H - Y_F,
\]  

(3)

where \( \rho \) is the discount rate, \( \varphi \) controls the degree of home bias in preferences, \( A \) is assets holding, \( r \) is the rate of return on financial assets, \( W \) is the wage, \( L \) is population size which equals labor supply since there is no preference for leisure, \( Y_H \) is expenditure on
home consumption goods whose price is $P_H$, and $Y_F$ is expenditure on foreign consumption goods whose price is $P_F$. In addition to asset and labor income, the household receives the dividends paid out by the producers of the home consumption goods, $\Pi_H$, the dividends paid out by firms in the materials sector, $\Pi_M$, and the revenues from sales of the domestic commodity endowment, $\Omega > 0$, at the price $p$. The solution to this problem consists of the optimal consumption/expenditure allocation rule,

$$\varphi Y_F = (1 - \varphi) Y_H,$$  \hspace{1cm} (4) 

and the Euler equation governing saving behavior,

$$r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \hspace{1cm} (5)$$

### 3.3 Trade Structure

The economy can be either an importer or exporter of the commodity. In the first case (commodity importer), it sells the home consumption good to buy the commodity in the world market. As in the SOE tradition, we assume the world commodity market accommodates any demand/supply at the exogenous constant price $p$. In the second case (commodity exporter), the economy accepts the foreign consumption good as payment for its commodity exports. The foreign good is imported at the constant exogenous price $P_F$. Only final goods and the commodity are tradable. The balanced trade condition, which is also the market clearing condition for the consumption good market, is $Y_H + Y_F + p (O - \Omega) = Y$, where $Y$ is the aggregate value of production of the home consumption good. Using the consumption expenditure allocation rule (4), we can rewrite the balance trade condition as

$$\frac{1}{\varphi} Y_H + p (O - \Omega) = Y, \hspace{1cm} (6)$$

where $O$ denotes the home use of the commodity. From (6) it is easily established that: (1) $O > \Omega$ (commodity importer) implies $Y > (1/\varphi) Y_H$, i.e., the model economy exchanges home consumption goods for the commodity; and conversely, (2) $O < \Omega$ (commodity exporter) implies $Y < (1/\varphi) Y_H$, i.e., it exchanges the commodity for foreign consumption goods.
3.4 Consumption Goods

The home (homogeneous) consumption good is produced by a representative competitive firm with the following technology:

\[ C_H = N^\chi \left[ \frac{1}{N} \int_0^N X_i^{\frac{\epsilon-1}{\epsilon}} \, di \right]^{\frac{1}{\epsilon}}, \quad \chi > 0, \, \epsilon > 1 \]  

(7)

where \( \epsilon \) is the elasticity of product substitution, \( X_i \) is the quantity of the non-durable intermediate good \( i \), and \( N \) is the mass of goods. Based on Ethier (1982) we separate the elasticity of substitution between intermediate goods from the degree of increasing returns to variety, \( \chi \). The final good producer solves the following maximization problem:

\[ \max_{\{X_i\}} \Pi_H = P_HC_H - \int_0^N P_iX_i \, di \]

subject to (7). This structure yields the demand curve for each intermediate good,

\[ X_i = Y \cdot \frac{P_i^{-\epsilon}}{\int_0^N \frac{P_i^{1-\epsilon}}{di}}, \]

(8)

where \( Y = P_HC_H \). Since the sector is perfectly competitive, \( \Pi_H = 0 \).

3.5 Manufacturing

The typical firm produces one differentiated good with the following technology:

\[ X_i = Z_i^\theta \cdot F(L_{X_i} - \phi, M_i), \quad 0 < \theta < 1, \, \phi > 0 \]

(9)

where \( X_i \) is output, \( L_{X_i} \) is production employment, \( \phi \) is a fixed labor cost, \( M_i \) is use of materials, and \( Z_i^\theta \) is the firm’s total factor productivity (TFP) which is a function of the stock of firm-specific knowledge, \( Z_i \). \( F(\cdot) \) is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are

\[ W\phi + C_X(W, P_M)Z_i^{-\theta} \cdot X_i, \]

(10)

where \( C_X(\cdot) \) is the associated unit-cost function which is homogeneous of degree one in its arguments; Hicks-neutral technological change internal to the firm shifts this function downward. The firm accumulates knowledge according to the technology
\[ \dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \] (11)

where \( \dot{Z}_i \) is the flow of firm-specific knowledge generated by productivity-enhancing activities employing \( L_{Z_i} \) units of labor (for an interval of time \( dt \)) and \( \alpha K \) is labor productivity in such activities, which depends on the stock of public knowledge \( K \); public knowledge accumulates as a result of spillovers:

\[ K = \sigma(N) \cdot \int_0^N Z_i di, \quad 0 < \sigma(N) < 1 \]

which posits that the stock of public knowledge \( K \) is the weighted sum of firm-specific stocks of knowledge \( Z_i \). The weight \( \sigma(N) \) is a function of the number of existing varieties \( N \) and captures in reduced form the extent of spillovers effects; Peretto and Smulders (2002) provide the micro-foundations for this class of spillovers function.

Specifically, we use \( \sigma(N) = 1/N \) which represents the average technological distance between differentiated products: when a firm \( i \) adopts a more efficient process to produce its own differentiated good \( X_i \), it also generates not-excludable knowledge which spills over into the public domain. However, the extent at which this new knowledge can be used by another firm, say \( j \neq i \), arguably depends on how far in the technological space the differentiated products \( X_i \) and \( X_j \) are; such notion of technological distance is captured in reduced form by the term \( \sigma(N) = 1/N \), which formalizes the idea that as the number of varieties increases the average technological distance between existing products increases as well. This in turn translates into lesser spillovers effects from any given stock of firm-specific knowledge.

### 3.6 Materials

A competitive firm uses labor services, \( L_M \), and the commodity, \( O \), as inputs to produce materials, \( M \), which are purchased by the manufacturing sector at the price \( P_M \). The production technology is \( M = G(L_M, O) \), where \( G(\cdot) \) is a standard production function, which is homogeneous of degree one in its arguments. Total production costs are

\[ C_M(W, p) M, \] (12)

where \( C_M(\cdot) \) is the associated unit-cost function which is homogeneous of degree one in the wage, \( W \), and the commodity price, \( p \).
3.7 Taking Stock: Vertical Cost Structure

Let us assess what we have so far. Given the vertical structure of production, a commodity price change has cascade effects: (1) it directly affects production costs and so pricing, i.e., $P_M$, in the upstream materials sector through the unit-cost function $C_M(W, p)$; (2) the change in $P_M$ in turn affects production costs and so pricing, i.e., $P_i$, in manufacturing through the unit-cost function $C_X(W, P_M)$; (3) the change in $P_i$ finally affects production costs and so pricing, i.e., $P_H$, in the consumption goods sector through the demand for intermediate goods. Thus, the initial change in the price of the commodity affects the home Consumer Price Index (CPI).

Note also that the materials sector competes for labor services with the manufacturing sector. This captures the inter-sectoral allocation problem faced by the economy.

4 Firms’ Behavior and General Equilibrium

In this section we first construct the equilibrium in manufacturing and materials sectors. We then impose general equilibrium conditions to study the aggregate dynamics of the economy.

4.1 Firms’ Behavior in Manufacturing

The typical intermediate firm maximizes the present discounted value of net cash flows:

$$\max_{\{L_X, L_Z, M_i\}} V_i(t) = \int_t^\infty e^{-\int_t^s [r(v) + \delta]dv} \Pi_i(s) ds, \quad \delta > 0$$

where $\delta$ is a “death shock.”*14 Using the cost function (10), instantaneous profits are

$$\Pi_i = \left[ P_i - C_X(W, P_M)Z_i^{-\theta} \right] X_i - W\phi - WLZ_i,$$

where $LZ_i$ are labor services allocated to cost reduction.*15 Each firm $i$ maximizes $V_i(t)$, which is the value of the firm, subject to the cost-reduction technology (11), the demand schedule (8), taking as given $Z_i(t) > 0$ (initial stock of knowledge), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (rivals’ knowledge accumulation paths), and $Z_j(t') \geq 0$ for $t' \geq t$ (knowledge irreversibility

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14 $\delta > 0$ is required for the model to have symmetric dynamics in the neighborhood of the steady-state.  
15 If $\phi = 0$, then horizontal innovation becomes a source of steady-state growth as in first-generation models of endogenous growth à la Romer (1990). In this case, however, the model jumps to the steady state and displays scale effects. Hence, it would be inconsistent with our motivating facts.
constraint). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms, \( N(t) \).

To characterize entry, we assume that upon payment of a sunk cost, \( \beta WY/N \), an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.\(^\text{16}\) Once in the market, the new firm solves a problem identical to the one outlined above for the incumbent firm. Therefore, a free-entry equilibrium requires \( V_i(t) = \beta W(t)Y(t)/N(t) \) for all \( t \).

Appendix A.1 shows that the equilibrium thus defined is symmetric and it is characterized by the following factor demands:

\[
WL_X = Y\frac{\epsilon - 1}{\epsilon}S^L_X + W\phi N, \tag{13}
\]

and

\[
PM_X = Y\frac{\epsilon - 1}{\epsilon}S^M_X, \tag{14}
\]

where the shares of the firm’s variable costs due to labor and materials are respectively,

\[
S^L_X \equiv \frac{WL_{X_i}}{C_X(W, P_M)Z^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log W},
\]

and

\[
S^M_X \equiv \frac{PM_{X_i}}{C_X(W, P_M)Z^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.
\]

Note that \( S^L_X + S^M_X = 1 \). Associated to these factor demands are the rates of return to cost reduction, \( r_Z \), and entry, \( r_N \):

\[
r = r_Z \equiv \frac{\alpha}{W} \left[ \frac{Y}{\epsilon N}\theta(\epsilon - 1) - W\frac{L_Z}{N} \right] + \frac{\dot{W}}{W} - \delta, \tag{15}
\]

and

\[
r = r_N \equiv \frac{N}{\beta Y} \left[ \frac{Y}{\epsilon N} - W\phi - W\frac{L_Z}{N} \right] + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N} + \frac{\dot{W}}{W} - \delta. \tag{16}
\]

Neither the return to cost reduction in (15) nor the return to entry in (16) directly depend on factors related to the commodity market. Why is this the case? The technology

\(^\text{16}\) See Peretto and Connolly (2007) for an interpretation of this assumption and alternative formulations that yield the same results.
(9) yields a unit-cost function that depends only on input prices and it is independent of the quantity produced and thus of inputs use. Since the optimal pricing rule features a constant markup over unit cost, the firm’s gross-profit flow (revenues minus variable costs), $Y/\epsilon N$, is independent of input prices. Equations (15) and (16), then, capture the idea that investment decisions by incumbents and entrants do not directly respond to conditions in the commodity market because they are guided by the gross-profit flow. Conditions in the commodity market have instead an indirect effect through aggregate spending on intermediate goods, $Y$, which are nonetheless sterilized by net entry/exit of firms.

### 4.2 Firms’ Behavior in Materials Sector

Given the unit-cost function (12), competitive producers of materials operate along the infinitely elastic supply curve:

$$P_M = C_M (W, p).$$

(17)

In equilibrium then materials production is given by (14) evaluated at the price $P_M$. Defining the commodity share in material costs as

$$S_M^O \equiv \frac{pO}{C_M (W, p) M} = \frac{\partial \log C_M (W, p)}{\partial \log p},$$

we can write the associated demand for labor and the commodity:

$$WL_M = WM \frac{\partial C_M (W, p)}{\partial W} = Y \frac{\epsilon - 1}{\epsilon} S_M^M (1 - S_M^O),$$

(18)

and

$$pO = pM \frac{\partial C_M (W, p)}{\partial p} = Y \frac{\epsilon - 1}{\epsilon} S_M^M S_M^O.$$  

(19)

### 4.3 General Equilibrium

The main equilibrium conditions of the model are: the rate of return to saving (5), to cost reduction (15), and to entry (16); labor demand in manufacturing (13) and materials sector (18); and the household’s budget constraint (3).\textsuperscript{17} Asset market equilibrium requires return

\textsuperscript{17}The households’ budget constraint (3) and balanced trade (6) imply clearing in the labor market; i.e., $L = L_N + L_X + L_Z + L_M$, where $L_N$ are labor services to enter manufacturing, $L_X$ and $L_Z$ are employment in production and cost reduction of incumbents, respectively, and $L_M$ is employment in the materials sector.
equalization, i.e., \( r = r_{A} = r_{Z} = r_{N} \), and that the value of the household’s portfolio equal the total value of the securities issued by firms, i.e., \( A = NV = \beta Y \).\(^{18}\) We choose labor as the numeraire, i.e., \( W = 1 \), which is a convenient normalization since it implies that all expenditures are constant.

The following proposition characterizes the equilibrium value of home manufacturing production, balanced trade, and expenditures on home and foreign consumption goods.

**Proposition 1.** At any point in time, the value of home manufacturing production and the balanced trade condition are, respectively:

\[
Y(p) = \frac{L}{1 - \xi(p) - \rho \beta} \quad \text{with} \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S^M_X(p) S^O_M(p),
\]

and

\[
\frac{1}{\varphi} Y_H(p) - p \Omega = Y(p) (1 - \xi(p)).
\]

The associated expenditures on home and foreign consumption goods are, respectively:

\[
Y_H(p) = \varphi \left[ \frac{L (1 - \xi(p))}{1 - \xi(p) - \rho \beta} + p \Omega \right],
\]

and

\[
Y_F(p) = (1 - \varphi) \left[ \frac{L (1 - \xi(p))}{1 - \xi(p) - \rho \beta} + p \Omega \right].
\]

Because \( Y_H(p) \) and \( Y_F(p) \) are constant, the interest rate is \( r = \rho \) at all times.

**Proof.** See Appendix A.2.

The following proposition characterizes the equilibrium dynamics of the model.

**Proposition 2.** Let \( x = Y/\epsilon N \) denote the gross profit rate. The general equilibrium of the model reduces to the following system of piece-wise linear differential equations in the gross profit flow, \( x \):

\(^{18}\)The first equality derives from the symmetry of the equilibrium: \( A = \int_{0}^{N} V_i di = NV_i = NV \); the second equality derives from imposing the free-entry equilibrium condition: \( V_i = \beta Y/N \).
\[ x = \begin{cases} 
\frac{(\phi \alpha - \rho - \delta)}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)} - \frac{\rho + \delta}{\alpha \theta (\epsilon - 1)} & \text{if } \phi \leq x \leq x_N \\
\phi - \frac{\rho + \delta}{\beta \epsilon} - \left[ \frac{1 - \theta (\epsilon - 1)}{\beta \epsilon} - (\rho + \delta) \right] x & \text{if } x_N < x \leq x_Z \\
\phi - \frac{\rho + \delta}{\beta \epsilon} - \left[ \frac{1 - \theta (\epsilon - 1)}{\beta \epsilon} - (\rho + \delta) \right] x & \text{if } x > x_Z,
\end{cases} \] (24)

where \( x_N \equiv \frac{\phi}{1 - \epsilon \rho \beta} \) and \( x_Z \equiv \frac{\rho + \delta}{\alpha \theta (\epsilon - 1)} \). Assuming that

\[ \frac{\phi - (\rho + \delta) / \alpha}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)} > \frac{\rho + \delta}{\alpha \theta (\epsilon - 1)}, \]
the economy asymptotically converges to the steady-state value of \( x \),

\[ x^* = \frac{\phi - (\rho + \delta) / \alpha}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)} > x_Z \] (25)

The associated steady-state growth rate of cost-reduction is

\[ \hat{z}^* = \frac{(\phi \alpha - \rho - \delta) \theta (\epsilon - 1)}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)} - (\rho + \delta) > 0. \] (26)

**Proof.** See Appendix A.3.

Let \( T_N \) and \( T_Z \) denote the time when \( x \) crosses the thresholds \( x_N \) and \( x_Z \), respectively. The closed-form solution for the global equilibrium dynamics implied by the system (24) in Proposition 2 is

\[ x(t) = \begin{cases} 
0 \leq t \leq T_N & x_0 + \nu_0 \left( e^{\delta t} - 1 \right) \\
T_N < t \leq T_Z & x_N + (\bar{x}^* - x_N) (1 - e^{-\nu_N t}) \\
t > T_Z & x_Z + (x^* - x_Z) (1 - e^{-\nu_Z t})
\end{cases} \] (27)

where

\[ \nu_0 = \frac{L/\epsilon N_0}{1 - \xi (p) - \frac{T}{\epsilon}}, \quad \nu_N = \frac{1 - \beta \epsilon (\rho + \delta)}{\beta \epsilon}, \quad \nu_Z = \frac{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)}{\beta \epsilon}, \]

and \( \bar{x}^* \) is the steady state that the system would reach if it did not cross the threshold.
and so stopped in the region $x_N < x \leq x_Z$.\(^{19}\) The expressions for the dates $T_N$ and $T_Z$ are

$$T_N = \frac{1}{\delta} \log \left( 1 + \frac{x_N - x_0}{\nu_0} \right) \quad \text{and} \quad T_Z = \frac{1}{\nu_N} \log \left( \frac{\bar{x}^* - x_N}{\bar{x}^* - x_Z} \right).$$

Proposition 2 states a “long-run commodity price super-neutrality” result: the steady-state growth rate of cost reduction, $\dot{Z}^*$, which is the only source of steady-state growth in the model, is independent of the commodity price, $p$.

The mechanism that drives this super-neutrality result is the sterilization of the market-size effect. To see this, (1) fix the number of firms at $\bar{N}$, then a change in the commodity price affects the size of the manufacturing sector $Y(p)$ (see Proposition 1), firm’s gross profitability $x \equiv Y(p)/(\epsilon \bar{N})$ (see Proposition 2), and thereby incentives to vertical innovation. Ceteris paribus, this would have steady-state growth effects. (2) Now let the mass of firms vary as in the free-entry equilibrium; as the profitability of incumbent firms varies, the mass of firms endogenously adjusts (net entry/exit) to bring the economy back to the initial steady-state value of firm size. As a result, the entry process fully sterilizes the long-run growth effects of the initial price change.\(^{20}\)

### 4.4 Total Factor Productivity and Growth

In the region $x(t) > x_Z$ of the system (24), aggregate total factor productivity (TFP) is

$$T = N^\chi Z^\theta. \quad (28)$$

As a result, $\dot{T}(t) = \chi \dot{N}(t) + \theta \dot{Z}(t)$, where $\dot{T}(t) \equiv \dot{T}(t)/T(t)$. Using (26), $\dot{T}(t)$ in steady state is

\(^{19}\)The system in (27) refers to the region of the parameter space in which $x_N < x_Z$ and $x^* > x_Z$ (case A); in this case, horizontal innovation comes first and vertical innovation follows guaranteeing positive steady-state growth. The model also features the case $x_N < x_Z$ and $x_N < x^* < x_Z$ (case B); in this case, vertical innovation never arises in equilibrium and, as a result, the economy features no steady-state growth. The global dynamics of the model are well defined also in the case in which the ranking of the thresholds is inverted, i.e., $x_N > x_Z$ (case C). To streamline the presentation of the results, for the rest of the paper we focus on case A; this parameter restriction is arguably the most relevant one since positive long-run growth is exhibited by both developing and developed commodity countries.

\(^{20}\)Note that the mechanism that yields sterilization of commodity price changes in the long run is also responsible for the sterilization of the so-called “scale effect,” i.e., steady-state growth is independent of population size. See Peretto (1998) and Peretto and Connolly (2007) for a detailed analysis of the mechanism driving the sterilization of the scale effect in this class of models.
\[
\dot{T}^* = \theta \dot{Z}^* = \theta \left[ \frac{(\phi \alpha - \rho - \delta) \theta (\epsilon - 1)}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)} - (\rho + \delta) \right] \equiv g. \tag{29}
\]

In the neighborhood of the steady-state \(x^* > x_Z\), the dynamics of the gross-profit rate, \(x\), are governed by the following differential equation:

\[
\dot{x} = \nu (x^* - x),
\]

where

\[
\nu \equiv \frac{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)}{\beta \epsilon}, \quad \text{and} \quad x^* \equiv \frac{\phi - (\rho + \delta) / \alpha}{1 - \theta (\epsilon - 1) - \beta \epsilon (\rho + \delta)}.
\]

We thus work with the solution

\[
x(t) = x_0 e^{-\nu t} + x^* (1 - e^{-\nu t}), \tag{30}
\]

where \(x_0 \equiv x(0)\) is the initial condition for \(x(t)\). The following proposition characterizes the time path for aggregate TFP.

**Proposition 3.** Consider an economy starting at time \(t = 0\) with initial condition \(x_0\). At any time \(t > 0\) the log of TFP is

\[
\log T(t) = \log (Z_0^0 N_0^\chi) + gt + \left(\frac{\gamma}{\nu} + \chi\right) \Delta (1 - e^{-\nu t}), \tag{31}
\]

where

\[
\Delta \equiv \frac{x_0}{x^*} - 1.
\]

**Proof.** See Appendix A.4.

Equation (31) shows that commodity prices affect the time path of aggregate TFP only through the displacement term, \(\Delta\). Steady-state growth, \(g\), and the speed of reversion to the steady state, \(\nu\), are both independent of the commodity price, \(p\).

### 4.5 The Prebisch-Singer Hypothesis Revisited

The Prebisch-Singer hypothesis posits that in the long-run commodity prices fall relative to the prices of the manufactured goods that the commodity-exporting country imports from abroad (see Prebisch, 1959; Singer, 1950).
In this section we study the long-run (steady-state) growth effects of the Prebisch-Singer hypothesis. Specifically, we make no attempt to explain why commodity prices would fall relative to the prices of imported goods. In contrast, we take the downward trend in the commodity/imports relative price as given, and derive the implications for steady-state growth.

Consider the case of a commodity-exporting economy. The balanced trade condition in (6) suggests that an economy exporting part of its commodity endowment, i.e., \( O < \Omega \), is, in fact, exchanging the commodity for the foreign consumption good. As a result, the relative price \( p/P_F \) is the one relevant for the Prebisch-Singer hypothesis. In the model, the price for the foreign consumption good, \( P_F \), is an exogenous constant. As such, a downward trend in the commodity price, \( p \), results in the same trend in the relevant price ratio, \( p/P_F \).

The following corollary characterizes the main result of the section.

**Corollary 1.** Let

\[
p(t) = p_0 e^{-g_p t},
\]

where \( p_0 \equiv p(0) \) is the initial price at \( t = 0 \), and \( g_p > 0 \) is the downward trend in the commodity price, \( p(t) \). The steady-state growth rate of aggregate total factor productivity (TFP), \( g \), is independent of the downward trend, \( g_p \), i.e.,

\[
\lim_{t \to \infty} g(t) = g \quad \text{for all} \quad g_p \geq 0.
\]

**Proof.** The result follows directly from the super-neutrality result in Proposition 2.

As the commodity price, \( p(t) \), decreases, the value of manufacturing production, \( Y(t) \), changes as well (the sign of the change depends on overall substitution/complementarity, see Proposition 4). In the limit as the price of the commodity approaches zero, the cost function \( \xi(p) \) and so the value of manufacturing production \( Y(t) \) approach a constant. Over the transition towards the steady state, the mass of firms endogenously changes via net entry/exit such that, in the limit, steady-state firms’ market size is independent of the commodity price. A similar argument applies in the case of an upward trend in the commodity price.
5 Manufacturing Production, Specialization, Prices

In this section we study (i) how a permanent change in the commodity price affects the value of manufacturing production; and (ii) how the status of commodity importer/exporter is endogenously determined within the model as a function of the commodity endowment and price, and deep technological parameters.

The following lemma derives a set of elasticities that are the key determinants of the comparative statics.

Lemma 1. Let

$$
\epsilon^M_X \equiv - \frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S^M_X}{\partial \log P_M} = 1 - \frac{\partial S^M_X}{\partial P_M} \cdot \frac{P_M}{S^M_X},
$$

$$
\epsilon^O_M \equiv - \frac{\partial \log O}{\partial \log p} = 1 - \frac{\partial \log S^O_M}{\partial \log p} = 1 - \frac{\partial S^O_M}{\partial p} \cdot \frac{p}{S^O_M}.
$$

Then,

$$
\xi' (p) = \left( \frac{\epsilon - 1}{\epsilon} \right) \cdot \frac{\partial (S^O_M (p) \cdot S^M_X (p))}{\partial p} = \frac{\xi (p)}{p} \cdot \Gamma (p),
$$

where

$$
\Gamma (p) \equiv (1 - \epsilon^M_X (p)) \cdot S^O_M (p) + 1 - \epsilon^O_M (p).
$$

Proof. See Appendix A.6.

Commodity price effects.—The key object in Lemma 1 is \( \Gamma (p) \), which is the elasticity of \( \xi (p) \equiv \left( \frac{\epsilon - 1}{\epsilon} \right) S^O_M (p) \cdot S^M_X (p) \) with respect to the commodity price, \( p \). According to (19), \( \Gamma (p) \) is the elasticity of the home demand for the commodity with respect to the commodity price, holding constant manufacturing expenditure. It thus captures the partial equilibrium effects of price changes in the commodity and materials markets for given market size.

Differentiating (20), rearranging terms, and using (32) yields

$$
\frac{d \log Y (p)}{dp} = \frac{\xi' (p)}{1 - \xi (p) - \beta \rho} = \frac{\xi (p)}{p \left[ 1 - \xi (p) - \beta \rho \right]} \cdot \Gamma (p),
$$

21 The following comparative statics effects are related to the literature on the “Dutch Disease” hypothesis, which posits that a boom in the natural resource sector crowds out manufacturing production.
which says that the effect of a commodity price change on the value of manufacturing production depends on the overall pattern of substitutability/complementarity subsumed in the price elasticities of materials, $\epsilon_X^M$, and commodity demand, $\epsilon_M^O$, and in the commodity share of materials production costs, $S_M^O$.

The following proposition states the results formally.

**Proposition 4.** Depending on the properties of the function $\Gamma(p)$, there are four cases:

1. **Global complementarity.** Suppose that $\Gamma(p) > 0$ for all $p$. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically increasing function of $p$.

2. **Cobb-Douglas-like economy.** Suppose that $\Gamma(p) = 0$ for all $p$. This occurs when $S_M^O$ and $S_X^M$ are exogenous constants. Then, manufacturing expenditure $Y(p)$ in (20) is independent of $p$.

3. **Global substitution.** Suppose that $\Gamma(p) < 0$ for all $p$. Then, manufacturing expenditure $Y(p)$ in (20) is a monotonically decreasing function of $p$.

4. **Endogenous switching from complementarity to substitution.** Suppose there exists a price $p^v$ at which $\Gamma(p)$ changes sign, from positive to negative. Then, the value of manufacturing production $Y(p)$ in (20) is a hump-shaped function of $p$ with a maximum at $p^v$.

**Proof.** See Appendix A.7.

The Cobb-Douglas-like case in Proposition 4 occurs when the production technologies in the materials and manufacturing sectors are both Cobb-Douglas, i.e., $\epsilon_X^M = \epsilon_M^O = 1$; we do not discuss this case further since it is a knife-edge specification in which commodity price changes have no effect on manufacturing production. The main insight derived from Proposition 4 is that the sign of the comparative statics effect depends on the substitution possibilities between labor and materials in manufacturing, and between labor and the commodity in materials. Arguably, the most interesting case is when the function $\Gamma(p)$ switches sign as the model generates an endogenous switch from overall complementarity to substitution. This happens if production in manufacturing and materials sectors displays opposite substitution/complementarity properties; e.g., materials production exhibits labor-commodity complementarity while manufacturing exhibits labor-materials substitution. In this latter case, there exists a threshold price $p^v$ such that $\Gamma(p) < 0$ for $p < p^v$ and $\Gamma(p) > 0$ for $p > p^v$. 


for $p > p^*$: when $p$ is low, the cost share $S_M^O (p)$ is relatively small and the function $\Gamma (p)$ is then dominated by the term $1 - \epsilon_M^O (p)$, which is positive since complementarity implies $\epsilon_M^O (p) < 1$ (i.e., inelastic commodity demand); conversely, when $p$ is high, the cost share $S_M^O (p)$ is relatively large and $\Gamma (p)$ is dominated by the term $1 - \epsilon_X^M (p)$, which is negative since substitution implies $\epsilon_X^M (p) > 1$ (i.e., elastic materials demand).

Overall, the equilibrium of the model suggests that a commodity price boom induces a decline in manufacturing activity (i.e., “Dutch Disease”) when the economy exhibits overall substitution. The reason is that when demand is overall elastic, the commodity price change at the top of our vertical production chain causes a large change in the quantity used; such change reflects the entire set of adjustments, forward and backward, that take place in the economy. A commodity price boom instead raises manufacturing activity when the economy exhibits overall complementarity between labor and the commodity.

Note also that changes in the commodity endowment $\Omega$ have no effect on the value of manufacturing production $Y$, but they positively affect expenditures on home $Y_H$ and foreign $Y_F$ consumption goods (see Proposition 1). 22

The determination of the commodity importer/exporter status.—An important building block of the model economy we study is that the commodity is used as input into the domestic production of materials. As a result, the status of commodity importer/exporter is endogenously determined within the model as a function of the endowment, $\Omega$, price, $p$, technological properties subsumed in the term $\xi(p)$, and other relevant parameters.

The following proposition characterizes the commodity exporting/importing region.

**Proposition 5.** The economy is an exporter of the commodity when

$$\frac{\Omega}{L} > \frac{\xi(p)}{p[1 - \xi(p) - \beta p]}.$$ 

**Proof.** See Appendix A.8.

Proposition 5 provides a formal notion of “commodity supply dependence.” For a given commodity price, $p$, there exists a threshold for the commodity-population endowment ratio $\Omega/L$ such that: (i) if $\Omega/L$ lies below the threshold, the economy is a commodity importer, i.e., $O > \Omega$, and conversely (ii) if $\Omega/L$ is above the threshold, the economy is a commodity exporter, i.e., $O < \Omega$. This is a specialization result: the equilibrium features a trade-off between the rate at which the economy transforms the commodity endowment into home

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22 See Arezki et al. (2015) for evidence on the effects of giant oil discoveries on the current account and other macroeconomic aggregates.
consumption goods—internal transformation rate (ITR)—and the rate at which it transforms the commodity endowment into foreign consumption goods—external transformation rate (ETR); such trade-off depends on the country’s own commodity endowment, the level of the commodity price, and all the deep technological parameters of the domestic vertical structure of production. Therefore, if the ITR dominates the ETR, the economy is a commodity importer; otherwise, it is a commodity exporter.

Figure 3 illustrates the determination of the commodity importer/exporter region.

An alternative way to interpret commodity trade is to note that, for a given relative endowment $\Omega/L$, there exists a commodity price threshold $p^d$ such that for $p < p^d$ the economy is a commodity importer whereas for $p > p^d$ the economy is a commodity exporter. On the one hand, economies with a larger commodity endowment are commodity exporters for a larger range of prices. On the other hand, economies with no commodity endowment, $\Omega = 0$, must be commodity importers for all $p$.

6 Welfare

In this section we derive the closed-form solution for the welfare in the region $x(t) > x_Z$ of the system (24). The following proposition characterizes the time path for welfare.

**Proposition 6.** Consider an economy starting at time $t = 0$ with initial condition $x_0$. At any time $t > 0$ the instantaneous utility flow is
\[
\log u(t) = \log \varphi \left( \frac{p\Omega}{L} + \frac{1 - \xi(p)}{1 - \xi(p) - \rho \beta} \right) - \varphi \log c(p) + \varphi g t + \varphi \left( \frac{\gamma}{\nu} + \chi \right) \Delta (1 - e^{-\nu t}) ,
\]

where

\[
\Delta \equiv \frac{x_0}{x^*} - 1.
\]

The resulting level of welfare is

\[
U(0) = \frac{1}{\rho} \left[ \log \varphi \left( \frac{p\Omega}{L} + \frac{1 - \xi(p)}{1 - \xi(p) - \rho \beta} \right) - \varphi \log c(p) + \frac{\varphi g}{\rho} + \frac{\varphi \left( \frac{\gamma}{\nu} + \chi \right)}{\rho + \nu} \Delta \right].
\]

**Proof.** See Appendix A.6.

Equation (34) identifies four channels through which commodity prices affect welfare: (1) the so-called “windfall effect” through the term \( p\Omega \); (2) the commodity-labor substitutability effect through the term \( (1 - \xi(p))/(1 - \xi(p) - \rho \beta) \); (3) the “cost of living/CPI effect” through the term \( c(p) \equiv C_X(W, C_M(W, p)) \); and (4) the “curse or blessing effect” through transitional dynamics associated with the term \( \Delta \) (i.e., initial displacement from the steady state), and steady-state growth, \( g \).

Specifically, (1) captures static forces that the literature on the curse of natural resources has discussed at length. That is, an economy with a commodity endowment experiences a windfall when the price of the commodity raises. However, in our model economy, this is not analogous to a lump-sum transfer from abroad in that the commodity is used for home production of materials, as such the value of manufacturing production endogenously adjust to the commodity price change; this adjustment is captured by the substitutability effect (2). Note that in our environment the analogous of a pure lump-sum transfer corresponds to an increase in the commodity endowment, \( \Omega \). The cost of living/CPI effect (3) is due to the fact that the economy uses the commodity for the domestic production of materials; thus, an increase in the commodity price works its way through the domestic vertical structure of production—from upstream materials production to downstream manufacturing—and it manifests itself as a higher price of the home consumption good (i.e., higher CPI). The curse/blessing effect (4) captures instead dynamic forces that are critical for our analysis. The steady-state growth rate of aggregate TFP is independent of the commodity price,
As argued above, this is due to the sterilization of market-size effects (see Section 4.3). However, there are effects due to transitional dynamics of TFP: (i) cumulated gain/loss from the acceleration/deceleration of the rate of cost reduction, and (ii) cumulated gain/loss from the acceleration/deceleration of product variety expansion; these two transitional effects amplify the change in the value of manufacturing production induced by the change in the commodity price.

Commodity dependence, commodity price boom, and welfare.—Overall, the equilibrium of the model suggests that an economy with a positive commodity endowment can gain in terms of welfare from a commodity price boom even though it is a commodity importer. Why is this the case? The reason is that revenues from sales of the endowment, $p\Omega$, go up one-for-one with $p$ while commodity demand, $pO$, does not. Specifically, commodity consumption, $O$, responds negatively to an increase in $p$; this effect is strong if home commodity demand is elastic, i.e., under global substitution.

The key insight derived from the equilibrium of the model is that what matters for welfare is not the commodity trade balance, but how manufacturing activity reacts to commodity price changes. Under global substitution, the contraction of the commodity demand after a price boom mirrors the contraction of manufacturing activity, which is the manifestation of the specialization effect discussed above. The Schumpeterian mechanism at the heart of the model amplifies such a contraction—the instantaneous fall in $Y$—into a deceleration of the rate of TFP growth. The economy eventually reverts to the initial steady-state growth rate $g$, but the temporary deceleration contributes negatively to welfare.

With these considerations in mind, let us now consider a permanent increase in the commodity price: for $p' > p$ we write

$$
\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1.
$$

The term $\Delta$ is the percentage displacement of the state variable $x$ from its steady state that occurs at time $t = t_0$, when the commodity price jumps up from $p$ to $p'$. The numerator is the value of profitability holding constant the mass of firms; the denominator is instead the value of profitability at the end of the transition, that is, when the mass of firms has fully adjusted to the new market size.

Let us consider a commodity-importing economy under overall substitution, $\Gamma(p) < 0$. Figure 4 illustrates three possible paths of $\log u(t;p')$ as the economy transits to the new steady state with a permanently higher commodity price, $p' > p$.  

[26]
Since aggregate TFP is predetermined at $t = t_0$, the impact response in $\log u(t_0)$ is driven by the jump in the home CPI index, the windfall effect, and the commodity-labor substitutability effect. However, these forces work in opposite directions such that the initial jump in utility has an ambiguous sign. After the initial impact response, the transition path of $\log u(t)$, for $t > t_0$, is governed by the transitional dynamics of aggregate TFP: the permanent fall in manufacturing activity—from $Y(p)$ to $Y(p') < Y(p)$—produces a slowdown in TFP growth, which in turn is due to a slowdown of net entry and a reduction in cost-reducing activity.\(^{23}\)

As a result, a commodity price boom is welfare improving if and only if the windfall effect through $p\Omega$ is large enough to compensate for the commodity-labor substitutability effect, the cost of living effect, and the curse effect through $\Delta < 0$. The closed-form solution for welfare (34) in Proposition 6 shows how model’s parameters determine the relative weights of these effects.

7 A Numerical Example

To further illustrate the dynamic properties of the model, we conduct a simple numerical exercise. We calibrate the model economy in the region $x(t) > x_Z$ of the system (24).

\(^{23}\)Note that as $t \to \infty$, the slope of the three transition paths depicted in Figure 4 converge to the same constant, $\varphi g$, see equation (33) in Proposition 3. This happens because, as discussed in Section 4.4, the steady-state growth rate of aggregate TFP, $g$, is independent of the commodity price.
7.1 Parameterization

One period is one year. Table 1 contains the baseline parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon/(\epsilon - 1)$</td>
<td>Mfg price markup</td>
<td>1.3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Mfg prod. function: $X_i = Z_i^\theta F(L_{X_i} - \phi, M_i)$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>0.02</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Death rate</td>
<td>0.035</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Mfg entry cost: $V_i = \beta \cdot \frac{Y}{N}$</td>
<td>1</td>
</tr>
</tbody>
</table>

We set $\epsilon = 4.33$ to match a price markup of 30 percent. Overall, the available evidence for the U.S. provides estimates of markups in value added data that range from 1.2 to 1.4.\textsuperscript{24} Hence, we target a markup in the manufacturing sector of $\mu = \epsilon/(\epsilon - 1) = 1.3$ that is at the middle of the available range of estimates. The condition for a symmetric equilibrium, $\theta(\epsilon - 1) < 1$, imposes a restriction on the calibration of $\theta$, i.e., $\theta \in [0, 1/(\epsilon - 1)]$. As a result, the calibrated value of $\epsilon = 4.33$ provides an upper bound on $\theta$, i.e., $\theta \in (0, 0.3)$. Since we have no reference value guiding our choice, we set $\theta = 0.15$ at the middle of the possible range. The death rate is set to $\delta = 0.035$ to match the average closing rate of establishments in the U.S. manufacturing sector for 1992-2012. Data for closing establishments are from the Business Employment Dynamics (BED) survey of the Bureau of Labor Statistics (BLS).\textsuperscript{25}

The requirement of positive eigenvalues over all the state space provides a restriction on the calibration of the entry cost’s parameter, $\beta$. Specifically, $\nu > 0$ implies $\beta \in \left[0, \frac{1-\theta(\epsilon-1)}{\epsilon(\rho+\delta)}\right]$.\textsuperscript{26} We set $\beta = 1$, which is within the set identified by the restriction above. Finally, we set the time discount rate to $\rho = 2\%$, which implies a 2 percent interest rate.

\textsuperscript{25}Survey homepage: \url{http://www.bls.gov/bdm/}.
\textsuperscript{26}Let $\nu_N$ and $\nu_Z$ denote the eigenvalues of the dynamical system in the region $x_N < x(t) \leq x_Z$ and $x(t) > x_Z$ in (24), respectively. The two eigenvalues are in the following relationship: $\nu_Z = \nu_N - \theta(\epsilon - 1)/\beta \epsilon < \nu_N$. 

28
7.2 Dynamic Response to a Commodity Price Shock

In this section we compute the dynamic response of the gross profit rate $x \equiv Y/\epsilon N$ to a “shock” that temporarily displaces $x$ from its steady-state value $x^* > x_Z$; we force the model to be in the neighborhood of the steady state (i.e., in transition dynamics) and illustrate how the model economy reverts back to the original steady state $x^*$.

Figure 5 plots the time path of

$$x(t) = x_0 e^{-\nu t} + x^* (1 - e^{-\nu t})$$

where $x(t) = x_0 e^{-\nu t} + x^* (1 - e^{-\nu t})$, the eigenvalue of the differential equation for $x$ is

$$\nu = \frac{1 - \theta (\epsilon - 1)}{\beta \epsilon} - (\rho + \delta) \text{ for } x(t) > x_Z,$$

and the initial percentage displacement from the steady state is $\Delta = \left( \frac{x_0}{x^*} - 1 \right)$—“profit rate shock.” In Figure 5, we consider a profit rate shock of $\Delta = 10\%$. The parameter values in Table 1 result in an eigenvalue of $\nu = 0.06$, which implies an half-life of $t_{1/2} \approx 11.5$ years.

**Commodity price shock.**—To explain the mapping between a commodity price shock and what we named “profit rate shock,” we now consider the scenario of a permanent fall in the commodity price—from $p$ to $p'$ with $p' < p$—and an economy operating under global substitution, i.e., $\Delta Y \equiv Y(p') - Y(p) > 0$ for all $p' < p$. The long-run commodity price super-neutrality result in Proposition 2 implies that $x^*(p') = x(p)$ for all price pairs $(p', p)$. So, after an unexpected (permanent) fall in the commodity price, the value of manufacturing production jumps from $Y(p)$ to the new steady-state level $Y(p')$. In contrast, the number of firms, $N$, is a predetermined variable, thus it does not respond on impact. The initial impact response in $x(t)$ (i.e., $x(0) = \Delta$) is followed by transitional dynamics driven by net firm entry, $\dot{N} > 0$. Eventually the mass of firms endogenously adjusts—from $N(p)$ to $N(p')$ with $N(p') > N(p)$—such that in steady state the initial jump in $Y(p')$ is fully neutralized.

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27Recall that the gross profit rate $x$ is the key variable regulating the incentives to innovate and hence driving the relevant equilibrium dynamics of the model. Without loss of generality, we consider a shock to the variable $x$ since this allows us to circumvent calibration of the function $\zeta(p)$ in (20). Since there is a one-to-one mapping between the function $\zeta(p)$ and the commodity price $p$, a shock to $x$ can be interpreted as a transformation of the shock to the commodity price, $p$.

28In the region $x(t) > x_Z$ of the system (24), the equilibrium gross profit flow $x$ follows a linear differential equation, hence the speed of reversion to the steady state is fully determined by the magnitude of the eigenvalue, $\nu$. Note that this is not a property of the global equilibrium dynamics; if a shock takes the economy in the region $x(t) < x_N$, the reversion to the steady state $x^*$ is highly non-linear. That is, the speed of reversion depends on the current state of the economy, $x(t)$.
Figure 5: Dynamic Response to a “Profit Rate Shock”

Notes: The figure plots the time path of the gross profit rate $x$ as percent deviation from the steady state $x^* > x_Z$ in the region $x(t) > x_Z$ in (24): $x(t)/x^* - 1 = \Delta e^{-\nu t}$, where $x(t) = x_0 e^{-\nu t} + x^*(1 - e^{-\nu t})$, $\Delta = 10\%$, and $\nu = \frac{1 - \theta (e^{-\nu t})}{\beta} - (\rho + \delta) = 0.0604$. See Table 1 for parameter values.
i.e., $x^*(p') = x^*(p)$.

The insight of the analysis is that firm size is the key driver of the economy’s dynamic response to a commodity price shock. The impact response is exclusively driven by the response of the value of manufacturing production, which instantaneously adjusts to the new equilibrium level. This model’s property allows us to focus on manufacturing production circumventing the calibration of the co-shares function $\xi(p)$, which determines how shocks to the commodity price, $p$, map into changes in the value of manufacturing production, $Y(p)$. Thus, changes in $Y(p)$ are a one-to-one transformation of shocks to the price of the commodity, $p$. After the initial impact response, dynamics is driven by the adjustment of the number of firms via net entry/exit.

8 Conclusions

We study the relationship between commodity prices, commodity trade, and growth within an endogenous growth model of commodity-rich economies. In the model, long-run (steady-state) growth is endogenous and yet independent of commodity prices. However, commodity prices affect short-run growth through transitional dynamics in aggregate TFP. We argue that these predictions are consistent with historical data from the 19th to the 21st century: commodity prices exhibit large and persistent long-run movements whereas growth rates of real GDP per capita in the Western Offshoots (i.e., U.S., Australia, Canada, New Zealand) exhibit no such large persistent changes. This argument, which parallels that in Jones (1995), draws an analogy between the effects of commodity prices on growth and the literature on the (lack of) long-run growth effects of taxation (see Easterly and Rebelo, 1993; Easterly et al., 1993; Stokey and Rebelo, 1995; Mendoza et al., 1997). We show that the overall substitutability between labor and the commodity is key to the understanding of how movements in commodity prices affect commodity-importing or commodity-exporting economies. Importantly, the commodity-labor substitutability properties of our economy are subsumed in observables, such as (i) the price elasticity of the demand for materials in manufacturing, (ii) the price elasticity of the demand for the commodity in materials, and (iii) the commodity share in materials production costs. Finally, we further argue that the overall substitutability between labor and the commodity, the country’s own commodity endowment, and the level of the commodity price, jointly determine whether a commodity-rich economy is a net commodity importer or exporter.
Appendix

A.1 Firm’s Behavior and Free-Entry Equilibrium

To characterize the typical firm’s behavior, consider the Current Value Hamiltonian (CVH, henceforth):

\[
CVH_i = \left[ P_i - C_X(W, P_M)Z_i^{-\theta} \right] X_i - W\phi - WLZ_i + z_i\alpha KLZ_i,
\]

where the co-state variable, \( z_i \), is the value of the marginal unit of knowledge. The firm’s knowledge stock, \( Z_i \), is the state variable of the problem whose law of motion is equation (11); labor services allocated to cost reduction, \( LZ_i \), and the product’s price, \( P_i \), are control variables. Firms take the public knowledge stock, \( K \), as given. Since the Hamiltonian is linear in \( LZ_i \), there are three cases: (1) \( W > z_i\alpha K \) implies that the value of the marginal unit of knowledge is lower than its cost. As result, the firm does not allocate labor to cost-reducing activities; (2) \( W < z_i\alpha K \) implies that the value of the marginal unit of knowledge is higher than its cost. This case violates general equilibrium conditions and, as such, it is ruled out since the firm would demand an infinite amount of labor to employ in cost reduction; and (3) \( W = z_i\alpha K \), which is the first order condition for an interior solution given by the equality between marginal revenue and marginal cost of knowledge accumulation.

The problem of the firm also consists of the terminal condition,

\[
\lim_{s \to \infty} e^{-s \int_{s}^{\infty} \left[ r(u) + \delta \right] du}Z_i(s) = 0,
\]

and a differential equation for the co-state variable,

\[
r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(W, P_M)Z_i^{-\theta - 1} \left( \frac{X_i}{z_i} \right),
\]

that defines the rate of return to cost reduction as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies.

The optimal pricing rule is

\[
P_i = \left( \frac{\epsilon}{\epsilon - 1} \right) C_X(W, P_M)Z_i^{-\theta}. \tag{A.1}
\]
Peretto (1998, Proposition 1) shows that under the restriction $1 > \theta (\varepsilon - 1)$ the firm is always at the interior solution, where $W = z_i \alpha K$ holds, and the equilibrium is symmetric.

The cost function (10) produces the following conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

The price strategy (A.1), symmetry and aggregation across firms yield (13) and (14). In the symmetric equilibrium, $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where $L_Z$ is aggregate labor in cost reduction. By taking logs and time-derivative of $W = z_i \alpha K$, using the demand curve (8), the cost-reduction technology (11), and the price strategy (A.1), one reduces the first-order conditions to (15).

Taking logs and time-derivative of $V_i$ yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i} - \delta.$$

The sunk entry cost is $\beta Y/N$. Labor allocated to entry is $L_N$. The case $V > \beta Y/N$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and, as such, it is ruled out since it would violate general equilibrium conditions. The case $V < \beta Y/N$ yields $L_N = -\infty$, which means that the non-negativity constraint on $L_N$ binds as such $L_N = 0$. A free-entry equilibrium requires $V = \beta Y/N$. Using the price strategy (A.1), the rate of return to entry becomes (16).

### A.2 Proof of Proposition 1

Since consumption goods and materials sectors are competitive, $\Pi_H = \Pi_M = 0$. The consumption expenditure allocation rule (4) and the choice of numeraire yield

$$\dot{A} = r A + L + p \Omega - \frac{1}{\varphi} Y_H.$$

By rewriting the domestic commodity demand (19) as

$$p O = Y \cdot \xi (p), \quad \xi (p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M (p) S_M^O (p),$$

allows us to rewrite the balanced trade condition as
Substituting the expressions for financial wealth, \( A = \beta Y \), and the balanced trade condition in the household’s budget constraint (3), and using the rate of return to saving in (5), yields

\[
\frac{1}{\varphi} Y_H - p\Omega = Y (1 - \xi (p)) .
\]

Differentiating the balanced trade condition yields

\[
\frac{1}{\varphi} \dot{Y}_H = \dot{Y} (1 - \xi (p)) \Rightarrow \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y} Y}{Y Y_H} \varphi (1 - \xi (p)) = \frac{\dot{Y}}{Y} Y_H (1 - \xi (p)) + p .
\]

Substituting back in the budget constraint and rearranging terms yields

\[
\frac{\dot{Y}}{Y} = \frac{Y (1 - \xi (p)) + p\Omega}{p\Omega} \left[ \rho + \frac{L - Y (1 - \xi (p))}{\beta Y} \right] .
\]

This differential equation has a unique positive steady-state value of manufacturing production:

\[
Y (p) = \frac{L}{1 - \xi (p) - \rho\beta} .
\]

We ignore, for simplicity the issue of potential indeterminacy, assuming that \( Y \) jumps to this steady-state value. The associated expenditures on the home and foreign goods, respectively, are

\[
Y_H (p) = \varphi \left[ \frac{L (1 - \xi (p))}{1 - \xi (p) - \rho\beta} + p\Omega \right] ;
\]

\[
Y_F (p) = (1 - \varphi) \left[ \frac{L (1 - \xi (p))}{1 - \xi (p) - \rho\beta} + p\Omega \right] .
\]

Since \( Y_H (p) \) and \( Y_F (p) \) are constant, the saving rule (5) yields that the interest rate is \( r = \rho \) at all times.
A.3 Proof of Proposition 2

The return to entry (16) and the entry technology \( \hat{N} = (N/\beta Y) \cdot L_N - \delta N \) yield

\[
L_N = \frac{Y}{\epsilon x} \left[ x - \left( \phi + \frac{L_Z}{N} \right) \right] - \rho \beta Y.
\]

Taking into account the non-negativity constraint on \( L_Z \), we solve (11) and (15) for

\[
\frac{L_Z}{N} = \begin{cases} 
\theta (\epsilon - 1) x - (\rho + \delta) / \alpha & x > x_Z \equiv \frac{\rho + \delta}{\alpha \theta (\epsilon - 1)} \\
0 & x \leq x_Z.
\end{cases}
\]

(A.2)

Therefore,

\[
L_N = \begin{cases} 
\frac{Y}{\epsilon} \left[ 1 - \theta (\epsilon - 1) - \frac{\phi - (\rho + \delta)}{x} \right] - \rho \beta Y & x > x_Z \\
\frac{Y}{\epsilon} \left( 1 - \frac{\phi}{x} \right) - \rho \beta Y & x \leq x_Z.
\end{cases}
\]

So we have

\[
L_N > 0 \text{ for } \begin{cases} 
x > \frac{\phi - (\rho + \delta)}{1 - \theta (\epsilon - 1) - \epsilon \rho \beta} & x > x_Z \\
x > \frac{\phi}{1 - \epsilon \rho \beta} & x \leq x_Z.
\end{cases}
\]

We look at the case

\[
\frac{\phi}{1 - \epsilon \rho \beta} \equiv x_N < \frac{\rho + \delta}{\alpha \theta (\epsilon - 1)} \equiv x_Z,
\]

which yields that the threshold for gross entry, \( x_N \), is smaller than the threshold for cost reduction, \( x_Z \).\(^{29}\)

To obtain the value of \( Y \) when \( L_N = 0 \), first note that

\[
L_N = 0 \text{ for } \frac{1}{\epsilon} \left( 1 - \frac{\phi}{x} \right) \leq \rho \beta.
\]

The household budget constraint yields

\[
0 = N \left( \frac{Y}{\epsilon N} - \phi \right) + L + \Omega p - \frac{1}{\phi} Y_H.
\]

Using the balanced trade condition and rearranging terms yields

\(^{29}\)The global dynamics are well defined also when this condition fails and \( x_N > x_Z \). We consider only the case \( x_N < x_Z \) to streamline the presentation since the qualitative results and the insight about the role of the commodity price remain essentially the same.
\[ Y = \frac{L - \phi N}{1 - \xi (p) - \frac{1}{\epsilon}}. \]

This equation holds for

\[ x \leq x_N \equiv \frac{\phi}{1 - \epsilon \rho \beta} \Leftrightarrow N \geq N_N \equiv \frac{\phi}{1 - \epsilon \rho \beta} \epsilon. \]

The interpretation is that with no labor allocated to entry, there is net exit and thus saving of fixed costs. This manifests itself as aggregate efficiency gains as intermediate firms move down their average cost curves. Note that in this region,

\[ Y(t) = \frac{L - \phi N_0 e^{-\delta t}}{1 - \xi (p) - \frac{1}{\epsilon}}, \]

which shows that the value of intermediate production grows as a result of net exit. The consolidation of the market results in growing profitability, that is,

\[ x = \frac{(L/\epsilon N_0) e^{\delta t} - (\phi/\epsilon)}{1 - \xi (p) - \frac{1}{\epsilon}} \Rightarrow \dot{x} = \frac{(\delta L/\epsilon N_0) e^{\delta t}}{1 - \xi (p) - \frac{1}{\epsilon}}; \]

the expression for \( \dot{x} \) suggests that the economy must enter the region with positive net entry. Therefore, the only condition needed to ensure convergence to the steady state with active cost reduction is \( x^* > x_Z \).

### A.4 Proof of Proposition 3

Taking logs of (28) yields

\[ \log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s) \, ds + \chi \log N_0 + \chi \log \left( \frac{N(t)}{N_0} \right). \]

Using the expression for \( g \) in (29), and adding and subtracting \( \hat{Z}^* \) from \( \hat{Z}(t) \), we obtain

\[ \log T(t) = \log (Z_0^\theta N_0^\chi) + gt + \theta \int_0^t \left[ \hat{Z}(s) - \hat{Z}^* \right] \, ds + \chi \log \left( \frac{N(t)}{N_0} \right). \]
Using (A.2) and (30) we rewrite the third term as

\[ \theta \int_0^t \left( \dot{Z}(s) - \ddot{Z}^* \right) ds = \alpha \theta^2 (\epsilon - 1) \int_0^t (x(s) - x^*) ds \]

\[ = \gamma \left( \frac{x_0}{x^*} - 1 \right) \int_0^t e^{-\nu s} ds \]

\[ = \frac{\gamma}{\nu} \left( \frac{x_0}{x^*} - 1 \right) \left( 1 - e^{-\nu t} \right), \]

where

\[ \gamma \equiv \alpha \theta^2 (\epsilon - 1) x^*. \]

Observing that \( N(t) = Y(p)/\epsilon x(t) \) yields \( \dot{N}/N = -\dot{x}/x \), we use (30) to obtain

\[ \frac{N(t)}{N_0} = \frac{1 + \left( \frac{N^*}{N_0} - 1 \right)}{1 + \left( \frac{N^*}{N_0} - 1 \right) e^{-\nu t}}. \]

We then rewrite the last term as

\[ \chi \log \left( \frac{N(t)}{N_0} \right) = \chi \log \frac{1 + \left( \frac{N^*}{N_0} - 1 \right)}{1 + \left( \frac{N^*}{N_0} - 1 \right) e^{-\nu t}} \]

\[ = \chi \log \left( 1 + \left( \frac{N^*}{N_0} - 1 \right) \right) - \chi \log \left( 1 + \left( \frac{N^*}{N_0} - 1 \right) e^{-\nu t} \right). \]

Approximating the log terms, we can write

\[ \chi \log \left( \frac{N(t)}{N_0} \right) = \chi \left( \frac{N^*}{N_0} - 1 \right) - \chi \left( \frac{N^*}{N_0} - 1 \right) e^{-\nu t} \]

\[ = \chi \left( \frac{N^*}{N_0} - 1 \right) \left( 1 - e^{-\nu t} \right). \]

Observing that

\[ \frac{N^*}{N_0} - 1 = \frac{x_0}{x^*} - 1, \]

these results yield (31).
A.5 Proof of Proposition 6

Consider

\[
\log u = \varphi \log \left( \frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left( \frac{Y_F}{P_F L} \right)
\]

\[
= \varphi \log \left( \frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left( \frac{1 - \varphi}{\varphi} \frac{Y_H}{P_F L} \right)
\]

\[
= \log \left( \frac{Y_H}{L} \right) - \varphi \log P_H + (1 - \varphi) \log \left( \frac{1 - \varphi}{\varphi} P_F \right)
\]

\[
= \log \left( \frac{Y_H}{L} \right) - \varphi \log c(p) + \varphi \log T - \varphi \log \left( \frac{\epsilon}{\epsilon - 1} \right) + (1 - \varphi) \log \left( \frac{1 - \varphi}{\varphi} P_F \right).
\]

To simplify the notation, and without loss of generality, we set

\[
(1 - \varphi) \log \left( \frac{1 - \varphi}{\varphi} P_F \right) + \varphi \log (N_0^X \sigma_0^\theta) - \varphi \log \left( \frac{\epsilon}{\epsilon - 1} \right) \equiv 0.
\]

This is just a normalization that does not affect the results. We then substitute the expression derived above into (1) and write

\[
U(p) = \int_0^\infty e^{-\rho t} \left[ \log \varphi \left( \frac{1 - \xi(p)}{1 - \xi(p) - \rho \beta} + \frac{p \Omega}{L} \right) - \varphi \log (c(p)) + \varphi \gamma t \right] dt
\]

\[
+ \varphi \left( \frac{\gamma}{\nu} + \chi \right) \Delta \int_0^\infty e^{-\rho t} \left( 1 - e^{-\nu t} \right) dt.
\]

Integrating, we obtain (34).

A.6 Proof of Lemma 1

Observe that

\[
\epsilon_X^M \equiv - \frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M}
\]

so that \( \epsilon_X^M \leq 1 \) if

\[
\frac{\partial S_X^M}{\partial P_M} = \frac{\partial}{\partial P_M} \left( \frac{P_M M}{P_M M + L_X} \right) \geq 0.
\]

This in turn is true if

\[
(1 - S_X^M) \frac{\partial (P_M M)}{\partial P_M} - S_X^M \frac{\partial L_X}{\partial P_M} \geq 0.
\]
Recall now that total cost is increasing in $P_M$ so that

$$\frac{\partial (P_MM)}{\partial P_M} + \frac{\partial L_X}{\partial P_M} > 0 \Rightarrow \frac{\partial (P_MM)}{\partial P_M} > -\frac{\partial L_X}{\partial P_M}.$$  

It follows that

$$\frac{\partial L_X}{\partial P_M} \leq 0$$

is a sufficient condition for $\epsilon_X^M \leq 1$ since it implies that both terms in the inequality above are positive. The proof for $\epsilon_M^O \leq 1$ is analogous.

### A.7 Proof of Proposition 4

Differentiating (20) yields

$$\frac{d \log Y(p)}{dp} = -\frac{d \log (1 - \xi(p) - \beta \rho)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta \rho}.$$  

It is useful to write $\xi'(p)$ as

$$\xi'(p) = \frac{\xi(p)}{p} \left[ (1 - \epsilon_X^M(p)) S_M^O(p) + 1 - \epsilon_M^O(p) \right],$$

which shows that the sign of $\xi'(p)$ depends on the upstream and downstream price elasticities of demand, and on the overall contribution of the commodity to manufacturing costs. Assume for example that $1 - \epsilon_X^M(p) < 0$ (i.e., labor-materials substitution) and $1 - \epsilon_M^O(p) > 0$ (i.e., labor-commodity complementarity), then there exists a threshold price $p^v$ such that:

$$\xi'(p^v) = \frac{\xi(p^v)}{p^v} \left[ (1 - \epsilon_X^M(p^v)) S_M^O(p^v) + 1 - \epsilon_M^O(p^v) \right] = 0,$$

i.e.,

$$(\epsilon_X^M(p^v) - 1) S_M^O(p^v) = 1 - \epsilon_M^O(p^v).$$

### A.8 Proof of Proposition 5

Equations (19) and (20) yield

$$\Omega \geq O \Leftrightarrow \frac{\Omega}{L} \geq \frac{\xi(p)}{p [1 - \xi(p) - \beta \rho]}.$$
References


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OA.1 Commodity Prices and Real GDP per Capita

This section contains time-series plots of several commodity prices (i.e., animal products, grains, metals, minerals, precious metals, and softs) and real GDP per capita in few Latin American countries (i.e., Argentina, Brazil, Chile, and Colombia) that are not included in the main body of the paper. Data for commodity prices are downloaded from David Jacks’s website at http://www.sfu.ca/~djacks/data/boombust/index.html. Data for real GDP per capita are downloaded from the Angus Maddison’s dataset which is publicly available at http://www.ggdc.net/maddison/maddison-project/home.htm. According to Definition 1 in the paper, the trend (red line) is the long-run trend (LR) component of the series which corresponds to the component with periodicity larger than 70 years.¹

¹We use a band-pass filter, as implemented by Christiano and Fitzgerald (2003), to isolate the Short-Run component (SR), $x_t^{SR}$, which corresponds to the component of $x_t$ with periodicity between 2 and 70 years. The Long-Run trend (LR) component is then $x_t^{LR} = x_t - x_t^{SR}$.

Figure OA.1: Real Animal Products Prices
Figure OA.2: Real Grains Prices

Figure OA.3: Real Metals Prices
Figure OA.4: Real Metals Prices II

Figure OA.5: Real Precious Metals Prices
Figure OA.6: Real Minerals Prices

Figure OA.7: Real Softs Prices
Figure OA.8: Real GDP per Capita in Latin America