Asymmetric Inventory Dynamics and Product Market Search

Linx Chen

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...indeed, to a great extent, business cycles are inventory fluctuations.

Alan Blinder, 1981

1. Economists knew inventory movements
...indeed, to a great extent, business cycles are inventory fluctuations.

Alan Blinder, 1981

Everything that needs to be said has already been said. But since no one was listening, everything must be said again.

Andre Gide

1. Economists knew inventory movements
1. Inventory investment is a component of GDP
2. Clarification: inventory means stock, II means the change (flow)
3. and it’s known as CIPI, change in private inventory
Document and explain two new stylized facts:

1. Inventory investment accounts for much larger share of GDP change in recessions than expansions

2. Inventory-sales ratio lags GDP for four quarters

Standard inventory models (e.g. Wen 2011) fail to account for them

This paper:

- Based on stockout-avoidance motive for inventory (Kahn 1987).
- Augment with product market search
- Matches the two new stylized facts and existing ones.
Fact 1: Asymmetric Share of Inventory Investment

- Inventory investment accounts for 72% of GDP decline in recessions
- This part motivates research on inventory
- But only 8% of GDP increase in expansions
- This part is mostly ignored
- Inventory-investment-to-GDP ratio is negatively skewed
- Stylized facts are determined by behaviors in expansions
Fact 1: Inventory Investment in the 1981 Recession

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Fact 1: Inventory Investment in the 1981 Recession
Fact 1: Inventory investment in the 1982-1990 Expansion
Table: Peak-to-trough Declines in All Postwar Recessions.
Note: Units in billions of 2009 dollar, annualized quarterly

<table>
<thead>
<tr>
<th>Peak</th>
<th>Trough</th>
<th>Inven.Inves.</th>
<th>GDP</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948:4</td>
<td>1949:4</td>
<td>-40.66</td>
<td>-30.68</td>
<td>132%</td>
</tr>
<tr>
<td>1953:2</td>
<td>1954:2</td>
<td>-24.47</td>
<td>-62.77</td>
<td>39%</td>
</tr>
<tr>
<td>1957:3</td>
<td>1958:2</td>
<td>-21.19</td>
<td>-84.98</td>
<td>25%</td>
</tr>
<tr>
<td>1969:4</td>
<td>1970:4</td>
<td>-35.84</td>
<td>-7.19</td>
<td>498%</td>
</tr>
<tr>
<td>1973:4</td>
<td>1975:1</td>
<td>-80.06</td>
<td>-169.95</td>
<td>47%</td>
</tr>
<tr>
<td>1980:1</td>
<td>1980:3</td>
<td>-67.26</td>
<td>-142.02</td>
<td>47%</td>
</tr>
<tr>
<td>1981:3</td>
<td>1982:4</td>
<td>-120.51</td>
<td>-169.73</td>
<td>71%</td>
</tr>
<tr>
<td>1990:3</td>
<td>1991:1</td>
<td>-46.87</td>
<td>-118.38</td>
<td>39%</td>
</tr>
<tr>
<td>2001:1</td>
<td>2001:4</td>
<td>-24.09</td>
<td>-40.20</td>
<td>59%</td>
</tr>
<tr>
<td>2007:4</td>
<td>2009:2</td>
<td>-213.07</td>
<td>-636.23</td>
<td>33%</td>
</tr>
</tbody>
</table>

Avg*: 72%
## Fact 1: Trough-to-peak Increases

<table>
<thead>
<tr>
<th>Trough</th>
<th>Peak</th>
<th>Inven.</th>
<th>Inves.</th>
<th>GDP</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949:4</td>
<td>1953:2</td>
<td>33.28</td>
<td>588.80</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>1954:2</td>
<td>1957:3</td>
<td>20.94</td>
<td>345.24</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>1958:2</td>
<td>1960:2</td>
<td>22.89</td>
<td>320.36</td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>1961:1</td>
<td>1969:4</td>
<td>30.15</td>
<td>1613.21</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>1970:4</td>
<td>1973:4</td>
<td>76.25</td>
<td>754.12</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>1975:1</td>
<td>1980:1</td>
<td>33.01</td>
<td>1232.47</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>1980:3</td>
<td>1981:3</td>
<td>115.93</td>
<td>279.97</td>
<td>41%</td>
<td></td>
</tr>
<tr>
<td>1982:4</td>
<td>1990:3</td>
<td>84.35</td>
<td>2490.81</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>1991:1</td>
<td>2001:2</td>
<td>7.16</td>
<td>3844.74</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>2001:4</td>
<td>2007:4</td>
<td>120.34</td>
<td>2286.52</td>
<td>5%</td>
<td></td>
</tr>
</tbody>
</table>

**Avg:** 8%

**Table:** Trough-to-peak Increases in All Postwar Expansions

**Note:** Units in billions of 2009 dollar, annualized quarterly
Fact 1: Skewness as an Alternative Measure of Asymmetry

Skewness of Inventory Investment Relative to GDP

Skewness = -0.33

Figure: Histogram of Inventory-investment-to-GDP Ratio
Fact 2: Inventory-sales ratio lags GDP

- Inventory-sales ratios lags GDP by four quarters

- Positive cross-correlation is the largest at fourth lag

- The lagging relationship is stable for the entire post-war period
Fact 2: Inventory-sales Ratio Lags GDP by Four Lags

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Figure: HP Filtered (1600)
Inventory-sales ratio being countercyclical is NOT a stylized fact.

It ceases to be countercyclical since the 1990s.

Countercyclicality is crucial for various important results.

Examples: (Bils Kahn 2000, Midrigran Krytsov 2013, Alessandria et al. 2013, Sarte et al. 2015)
Correlation = -0.24 before 1992, Correlation = 0.23 after 1992

Figure: HP Filtered, 40 Quarters Moving Window
Model: Agents

1. Intermediate good producer: produced with labor only.

2. Variety good producer: produced with intermediate goods.

3. Household: “love for variety”
Search and match protocol similar to the labor literature (Pissarides 1994, Diamond 2000).

- Measure 1 of HH matches with measure 1 of varieties.
- Each variety is produced by one monopolistic firm.
- Generate $x$ matches with $d$ aggregate search effort

$$x = M(d, 1)$$

and thus the rate at which HH finds varieties

$$\Psi_D \equiv \frac{x}{d} = M(1, \frac{1}{d})$$
Household: Second Stage Problem

- First stage decides varieties $x$ and consumption level $\tilde{c}$
- Each variety $i$: maximum quantity available for sale $z_i$ and prices $p_i$
- Demand shocks $v_i$ to each variety
- Expenditure minimization problem:
  $$\min_{c_i} x p_i c_i di$$
  s.t. $c_i \leq z_i$  
  $\tilde{c} \leq \left( \int_0^x v_i^{-1/\rho} \left( c_i^{\rho} \right)^{1/\rho} \right)^{-\rho}$
Household: Second Stage Problem

Demand for variety $i$:

\[ c_i = \min \left\{ z_i, v_i \left( \frac{p_i}{\rho} \right)^{\frac{1}{1-\rho}} c \right\} \]

where $c = \frac{\tilde{c}}{x^\rho} = \left( \frac{1}{x} \int_0^x v_i \left( \frac{1}{\rho} \right)^{\frac{1}{1-\rho}} c_i^{\frac{1}{1-\rho}} \right)^{\rho}$

Average price index:

\[ p = \left[ \frac{1}{x} \int_0^x v_i (p_i + \mu_i)^{1-\rho} \right]^{1-\rho} \]

where $\mu_i$ is the Lagrange multiplier associated with $c_i \leq z_i$

Total expenditure:

\[ \int_0^x p_i c_i \, di = px \bar{c} \]
Household: First Stage Problem

Solves the following Bellman’s equation:

\[ H(a) = \max_{c, d, n, a', x} \left( u(x^0 c, d, n) + \beta \mathbb{E} H(a') \right) \]

s.t. \[ a' = wn + a(1 + \Pi) - pcx \]
\[ x = \Psi_D d \]

Consume \( x \) varieties with average level \( c \), search for varieties with effort \( d \), work for wage \( w \), receive profit from all firms \( \Pi \), save with stock purchase \( a' \) (numeraire).

\( \Psi_D \) is variety finding rate, household take as given.
Variety Producer

- Monopolistic competitive, unit measure.
- With probability $x$, the variety producer have access to final good producer’s demand (“matched”).
- Once matched, draw demand shock $v_i$.
- $v_i$ is i.i.d. across time and across varieties
- Decide on pricing and ordering before knowing these shocks.
- Thus generate the incentive to hold inventories.
Solves the following problem:

\[
\mathcal{V}(\epsilon_i) = \max_{y_i, p_i, e'_i} \left\{ -p_M y_i + x \int \left\{ c_i p_i + E m' \mathcal{V}(e'_i) \right\} F(v_i) \right. \\
+ (1 - x) E m' \mathcal{V}(e'_i) \right\} F(v_i) + (1 - x) E m' \mathcal{V}(e'_i)
\]

\[
s.t. \quad c_i = \min \left\{ v_i c \left( \frac{p_i}{p} \right)^{\frac{1}{1 - \delta}}, z_i \right\}
\]

\[
z_i = e_i + y_i
\]

\[
e'_i = \begin{cases} 
(1 - \delta_e) [e_i + y_i - c_i] & \text{"matched"} \\
(1 - \delta_e) [e_i + y_i] & \text{"unmatched"}
\end{cases}
\]

- \(z_i\) is the amount of good \(i\) made available to buyers
- \(p_M\) price of intermediate goods, \(y_i\) the order
Variety Producer

Pricing Decision:

\[ p_i = \frac{\epsilon_i}{\epsilon_i - 1} (1 - \delta_e) E \left( m' P'M \right) \]

where the price elasticity of expected sales is given by:

\[ \epsilon_i = \frac{\rho}{1 - \rho} \frac{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F^v(dv_i)}{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F^v(dv_i) + \left[ 1 - F^v(v_i^*) \right] \left[ e_i + F(n_i) \right]} \]

The cut-off point of stockout \( v_i^* \) is given by:

\[ c_i(p_i, n_i, v_i^*) = z_i \]
Variety Producer

Availability decision:

\[(b - r')x[1 - F'(v_i^*)] = 1 - r'\]

where

\[b = \frac{p_i}{W/p_M}\]

and

\[r' \equiv (1 - \delta_e)E_mP_M/p_M\]

- \(r'\) captures return on holding inventory in the absence of stock-out
- \(x\) is endogenous unlike standard stockout model
- Information structure
  - all varieties choose the same \(z_i\) and \(p_i\)
Perfectly competitive, unit measure

Solves the static problem:

$$\max_n p^M F(n) - wn$$

Production function \(F(n)\)

Allows for exact aggregation despite heterogeneous \(v_i\)
Agents solve their respective optimization problems

Variety goods, intermediate goods and labor markets clear

All variety producers choose the same price

... and same amount of goods available

Solved with 3rd order perturbation.
**Functional Forms**

Production Function:

\[ F(n) = An^{1-\alpha} \]

Utility (Generalized GHH 1988, search behavior):

\[ u(cx^\rho, d, n) = \log \left( cx^\rho - \zeta n^{1+\upsilon_n} - \xi d \right) \]

Distribution of demand shocks \( v_i \) Pareto(\( v_{min}, \sigma_v \)):

\[ F_{v_i}(v) = 1 - \left( \frac{v_{min}}{v_i} \right)^{\sigma_v} \]

Matching function (den Hann et al. 2000):

\[ M(D, 1) = \frac{D}{(D^i + 1^i)^{1/\iota}} \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Rate</td>
<td>0.99</td>
<td>4% Return</td>
<td>Data</td>
</tr>
<tr>
<td>$\nu_n$</td>
<td>Labor Elasticity</td>
<td>0.75</td>
<td>Frish Elas.</td>
<td>Chetty 2011</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor Share</td>
<td>0.67</td>
<td>Data</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Labor Disutility</td>
<td>1.5</td>
<td>1/3 time worked</td>
<td>ATUS</td>
</tr>
<tr>
<td>$v_{min}$</td>
<td>Loc. $v_i$</td>
<td>0.04</td>
<td>Mean 1</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Shape $v_i$</td>
<td>1.05</td>
<td>S.O. Prob=5%</td>
<td>Bils 2004</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elas. of Subs.</td>
<td>1.17</td>
<td>20% markup</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>Deprec. Inven.</td>
<td>0.015</td>
<td>6% annual</td>
<td>Wen 2011</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Match Elasticity</td>
<td>1.18</td>
<td>0.35 elas.</td>
<td>Broda et al 2011</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Search Disutility</td>
<td>0.01</td>
<td>1 hr shopping</td>
<td>ATUS</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>TFP Pers.</td>
<td>0.96</td>
<td></td>
<td>SF-FED TFP</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>TFP Vola.</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table: Calibration
<table>
<thead>
<tr>
<th>Stat.</th>
<th>Data</th>
<th>Model</th>
<th>Wen 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(\frac{\text{Inven.}}{\text{GDP}}, \text{GDP})</td>
<td>0.66</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>AR(1) of \frac{\text{Inven.}}{\text{Sales}}</td>
<td>0.75</td>
<td>0.89</td>
<td>0.77</td>
</tr>
<tr>
<td>corr(\frac{\text{Inven.}}{\text{Sales}}, \text{GDP})</td>
<td>-0.43</td>
<td>-0.30</td>
<td>-0.68</td>
</tr>
<tr>
<td>skewness(\frac{\text{Inven.}}{\text{GDP}})</td>
<td>-0.30</td>
<td>-0.46</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table: Inventory Stylized Facts Performance
Asymmetric Inventory Dynamics and Product Market Search

Performance: Fact 1

- Examine peak-trough shares of inventory investment.
- Dating turning points:
  - Treat the model as generating demeaned growth rates
  - Define recessions to be at least two consecutive periods with GDP contraction rate as large as recessions in the data
  - Matches share of recession periods in data: 20%.
  - Model depress on avg. four quarters and expands 13 quarters
- Recession: inventory investment is 54% of output decline (data 72%)
- Expansion: 25% of output expansion (data 8%)
- Robust to various business cycle dating schemes
Asymmetric Inventory Dynamics and Product Market Search

- Performance: Fact 2

Figure: Inventory-sales Ratio Lags Output by 5 Quarters (Model)
Cross-correlations of HP_in_rISratio and HP_in_rGDP

Figure: IS Ratio Lags Output by 4 Quarters (Data)
Asymmetric Inventory Dynamics and Product Market Search

Mechanism: Fact 1, Role of Product Market Friction

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>ξ = 0.006</th>
<th>ξ = 0.010</th>
<th>ξ = 0.012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady State Varieties, $\bar{x}$</td>
<td>0.91</td>
<td>0.88</td>
<td>0.75</td>
</tr>
<tr>
<td>Peak-to-trough Share</td>
<td>0.40</td>
<td>0.54</td>
<td>0.71</td>
</tr>
<tr>
<td>Trough-to-peak Share</td>
<td>0.29</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>Skewness $(\frac{\text{Inven.}}{\text{Inves.}} / \text{GDP})$</td>
<td>-0.21</td>
<td>-0.46</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

Table: Inventory Performance
Mechanism: Fact 1, Demand Curve

\[ c_i = \min \left\{ z_i, v_i c \left( \frac{p_i}{p} \right)^{\frac{p}{1-p}} \right\} \]

Figure: Demand Curve for Variety \( i \)
Mechanism: Fact 1, Optimal Markup

\[ b_i = \frac{\epsilon_i}{\epsilon_i - 1} r, \quad b_i \downarrow \rho r \quad \text{as} \quad \frac{z_i}{c} \uparrow \infty \]

Figure: Optimal Choices For Markup
Mechanism: Fact 1, Optimal Markup + Optimal Buffer

\[ b_i = \frac{\epsilon}{\epsilon - 1} r_i \]

\[ x(1 - F_v)(b - r') = 1 - r' \]

Figure: Joint Determination of Markup and Safe Buffer
Mechanism: Fact 1, Peak vs Trough

Figure: Asymmetric Responses: Peak vs. Trough
Mechanism: Fact 1, Markup and Safe Stock TS

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Mechanism: Fact 1, Markup and Safe Stock TS
Asymmetric Inventory Dynamics and Product Market Search

Mechanism: Fact 1, Higher $r'$

Figure: Increasing $r'$
Mechanism: Fact 1

- Nonlinearity exists in stock-out model, but unexplored.
- Allowing movement in $x$ enhances the nonlinearity
- Peak and trough are further away along markup decision curve
- Generates quantitatively stronger asymmetry in inventory decision.
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Mechanism: Fact 2, Hump-shape Responses
With product search, expansion of varieties peaks first then the return on holding inventory holding return peak later

Return on holding inventory has no direct impact on sales!

Prolonged impact on buffer stock, thus inventories. But shorter-lived impact on sales.

Generates the lagging relationship of inventory-sales ratio
Two new stylized facts poses challenges to popular DSGE inventory models.

Product market search friction improve an off-the-shelf inventory model’s ability to be consistent with these two facts.

Consistent with household shopping empirics:
- Procyclical search effort
- Expansion of varieties and expenditure.

Consistent with aspects of business cycle asymmetry:
- Positively skewed markup
- Negatively skewed employment

Companion empirical paper documents asymmetric effects of monetary policy shocks, more on asymmetry inventory dynamics
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Figure: Wen 2011
Intuition: Hump-shape

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Intuition: Hump-shape
Broda et al. 2010:

1. UPC level data of HH consumption varieties (nondurable, 60% of CPI basket).
2. Large turnover of varieties HH consumes (75% common good in 4 year period).

Suggests that substantial risk of "out-of-favor" for producers when deciding inventory.