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**Long-run Causal Order: A Preliminary Investigation**

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Abstract

of

Long-run Causal Order: A Preliminary Investigation

The concept of long-run causal order and its relationship to nonstationarity in time series data is explored using some ideas drawn from the literature on graphical causal modeling. Ordinary variables, which are nonstationary only because they are caused by a distinct nonstationary variable, are distinguished from fundamental trends, which are nonstationary owing to their own-dynamics. Although systems of equations in which the own dynamics appears stationary may, in principle, generate nonstationary behavior through their interaction, it is argued that such behavior is nongeneric and that typically ordinary variables trend because an (often latent) fundamental trends is among their causes. The possibility of inferring the long-run causal structure among a set of time-series variables from an exhaustive examination of weak exogeneity in irreducibly cointegrated subsets of variables is explored and illustrated.

Keywords: graphical causal modeling, causal search, cointegrated vector autoregression (CVAR), weak exogeneity, irreducible cointegrating relations

JEL Classification: C32, C51, C18
In the long run, we are all dead.

JOHN MAYNARD KEYNES

In the long run, we are simply in another short run.

VARIOUSLY ATTRIBUTED

Contrary to Keynes’ famous dictum in the long run we are all dead,
the long run is with us every day of our lives

WALT ROSTOW

1. The Problem of Causal Order in the CVAR

Katarina Juselius’s and Søren Johansen’s most famous contributions to econometrics, studied in detail and applied in his monograph (Johansen 1995) and in her textbook (Juselius 2006) and, jointly and singly, in a large number of journal articles, concern the cointegrated vector autoregression (CVAR). The CVAR focuses special attention on the nonstationary components and the long-run properties of the time series. The questions we shall address in this paper are how the long-run properties of the CVAR can be given a structural interpretation and how that interpretation might support inference of the long-run causal structure from the observable characteristics of the nonstationary data.

There are two significant traditions in time-series econometrics.\(^1\) The Cowles Commission in the 1940s and ’50s pioneered structural econometrics that conceived of the econometric problem as one of articulating and measuring economic mechanisms (Koopmans 1950; Hood and Koopmans 1953; see Morgan 1990 for a history). The articulation of mechanisms was generally referred to as the identification problem. The major resource for securing identification was a priori economic theory. Early on, structural and causal articulation were regarded as synonymous, although subsequently causal language fell from favor (Hoover 2004). In his contribution to the 1953 Cowles volume, Herbert Simon (1953) drew on the language of experiments (actual or metaphorical) to suggest that an identified system of dynamic equations provided a map of the space of interventions in the economy.\(^2\) Simon demonstrated an isomorphism between a structurally identified model and a causally well-ordered model.

A second econometric tradition, grounded more in time-series statistics, focused on process rather than structure (e.g., see Wold 1960; Granger 1969). Granger defined causality in terms of incremental predictability. Sims (1972) introduced Granger-causality into empirical macroeconomics; and, frequently, thereafter, an equivocation between Granger’s notion of causality and structural notions became common place. But Granger himself was aware that Granger-causality did not

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\(^1\) For discussions of various approaches to causality in macroeconomics and macroeconometrics, see Hoover (2001; 2008; 2012).

address the questions of control and counterfactual policy analysis that motivated structural understandings of causality, such as those of Simon and the Cowles Commission (Granger 1969; 1995; also White and Lu 2010, p. 194). While both the structural and the process approaches to econometrics have a concept of causation, those concepts are distinct. They may, nonetheless, be mutually informative. White and Lu (2010) and White and Pettenuzzo (2014), for instance, analyze the conditions under which Granger-causality can be provide information relevant to assessing structural causality (see also Hoover 2001, pp. 150-155).

The vector autoregression (VAR) arises out of the process tradition. Building on earlier criticisms of Liu (1960) and others, Christopher Sims (1980) introduced the VAR into macroeconometrics as part of a critical response to the Cowles Commission approach. Christopher Sims (1980, p. 1), attacked the structural interpretation of econometric models for using “incredible” identifying restrictions. Initially, he offered the VAR – a system of reduced-form equations in which all variables are endogenous – as a workable alternative to identified structural models.

There is a tendency to treat process accounts of causality as essentially atheoretical and data driven and to treat structural accounts as necessarily relying on a priori theory. These connections are more of accident of the history of econometrics than essential. In the case of the VAR, it rapidly became clear that reduced-form VARs were inadequate to the needs of counterfactual policy analysis – perhaps the most important use of macroeconometric models (Cooley and LeRoy 1985, Sims 1982, 1986). The structural VAR (SVAR), which imposes a causal order on the contemporaneous relationships among the endogenous variables was seen to provide the minimum restrictions needed to identify independent shocks, which were taken to be the drivers of a dynamic system, and policy analysis was largely reduced to working out the impulse responses to those shocks (see Duarte and Hoover 2012 and Hoover and Jordá 2001).

While the problem that had motivated Sims in the first place, the incredibility of the identifying restrictions, had been minimized in the SVAR, it was not eliminated; and the question, how we are to know the correct contemporaneous causal order, remains an open one. In truth, economic theory rarely provides a clear or decisive answer. In practice in most, though not all cases, SVARs were identified by assuming certain triangular causal orderings of the contemporaneous variables. Since all such causal orders are just-identified, they have the same likelihood function, and, thus, there is no empirical basis for choosing among them, so long as “empirical” is restricted to likelihood information. At this point SVAR practitioners typically claim that it is necessary to invoke prior information from economic theory or practical institutional knowledge or common sense to pick among the equivalent causal orders. But, in fact, empirical evidence can be brought to bear on the choice. When the underlying data-generating processes (DGPs) are over-identified, information about conditional dependence and independence among the variables provides information that can be used, in some cases, to distinguish among possible causal orders. This approach has been developed with great sophistication – mainly for non-time-series data – in the so-called graphical-
causality or Bayes-net literature (Spirtes, Glymour, and Scheines 2000; Pearl 2009). Swanson and Granger (1997) first applied a simple graphical causal search algorithm to the problem of determining the contemporaneous causal structure of an SVAR. Subsequently, more sophisticated algorithms have been applied and shown to be effective in a wide range of circumstances (Demiralp and Hoover 2003; Demiralp, Hoover, and Perez 2008; and references therein).

Meanwhile, time-series econometrics discovered the importance of nonstationary processes and the concept of cointegration (Engle and Granger 1987). In light of these developments, the SVAR was reformulated into the CVAR. Throughout the paper, we will consider cases in which we, in fact, know the true DGP, but observe only some part of it. To be clear, our operating assumption is that a complex data-generating process governs the behavior of the economy; and the aim of structural causal modeling is to uncover a (partial) representation of the true DGP that is adequate to pragmatically required levels of detail and precision to support inter alia prediction and counterfactual analysis. A key question will be how much information about the DGP can be recovered from the observables.

The issues that concern us, can be explicated in a standard CVAR with one lag and no deterministic components and variables integrated of degree one (notated I(1)) and taken to be a reduced form of a part of the economy’s unobserved DGP:

$$\Delta x_t = \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \varepsilon_t,$$

(1)

where \(x = [x_1, x_2, \ldots, x_r]'\), \(\Gamma\) and \(\Pi\) are \(p \times p\) matrices of parameters, \(\varepsilon_t \sim \text{IN}(0, \Omega)\) is \(p\)-element vector of normal residuals, and \(t\) subscripts indicate time. The residuals contain both unobserved causes, which we shall call shocks, and various sorts of error. In general, the individual elements of \(\varepsilon\) are not independent, so that \(\Omega\) is not a diagonal matrix.

What Johansen and Juselius call a structural CVAR is related to equation (1):

$$A_0 \Delta x_t = A_1 \Delta x_{t-1} + a \beta' x_{t-1} + u_t,$$

(2)

where \(u_t = A_0 \varepsilon_t \sim \text{IN}(0, \Sigma)\); \(A_0\) is a \(p \times p\) matrix with ones on the main diagonal such that \(\Sigma = A_0 \Omega A_0'\) and \(\Sigma\) is diagonal; and \(A_1 = A_0 \Gamma\) (Juselius 2006, equation (12.2), p. 208; cf. equation (15.5), p. 276; Johansen 1995, pp. 78-79). If the variables in \(x\) are cointegrated (i.e., if a linear combination of nonstationary variables is itself stationary), then \(\Pi\) has reduced rank \((r)\) and may be written as \(\Pi = a \beta'\), where \(a\) and \(\beta\) are \(p \times r\) matrices, and \(a = A_0 \alpha\).

3 "Graphical" (or "graph-theoretic") causal search to "Bayes-net," should be the preferred term, as the search methods do not require a Bayesian approach to statistics. For compact treatments of the approach and the basic algorithms, see Cooper (1999) and Demiralp and Hoover (2003).

4 On the general methodology of modeling in relation to the CVAR see Hoover, Johansen, and Juselius (2008) and Hoover and Juselius (2015).
Should equation (2) be regarded as structural? Johansen and Juselius call it “structural” because

1. since different models may share the same reduced form, the matrix $A_0$ is not in general unique and the choice of a particular $A_0$ provided that it has $p \times (p - 1)/2$ zero restrictions, identifies the causal ordering of the contemporaneous variables in the sense of Simon (1953; see also Hoover 2001, ch. 3);\(^5\)
2. the long-run parameters of $\alpha$ and $\beta$ can be recovered, provided that at least $r \times (r - 1)$ restrictions are imposed on $\beta$ (note, however, that in general $\alpha$ and $\beta$ are not unique, but may take different values such that $\Pi = \alpha \beta'$; so that it is necessary to choose among admissible sets of restrictions);
3. the elements of $u$ (in contrast to those of $\epsilon$) are independent of each other and may, therefore, be taken to be the very shocks that are entangled in $\epsilon$, which is reflected in the fact that $\Sigma$ is diagonal, provided of course that the chosen $A_0$ corresponds to the data-generating process (DGP) for the economy.\(^6\)

Equation (1) is meant to characterize a limited number of observable variables, and implicitly other unobserved variables, possibly important to the behavior of the observables, that it does not capture. To the degree that influential variables are unobserved, the relationship of the observables, even when transformed as in (2), is not likely to be structural. But even if equation (1) is adequate, equation (2) is structural in, at best, a limited sense.

One aspect of this claim can be grasped by comparing the practical importance of contemporaneous identification to the practical irrelevance of long-run identification. As noted $A_0$ is not unique. Each admissible choice of $A_0$ defines a distinct causal order and, in general, though all just-identified orders imply the same likelihood function, each will define a different set of shocks and different impulse-response functions. The matrices $\alpha$ and $\beta$ are also in general not unique, but their product must equal $\Pi$, and it is only $\Pi$ that matters to the impulse-response functions. Thus, there is a narrower empirical basis for choosing among the admissible just-identified $\alpha$s and $\beta$s.

There is a second reason to question the structural status of equation (2). If the rank of $\Pi$ is $r < p$, then the variables in $x$ contain $q = p - r$ common stochastic trends. It is possible that some of the common trends are endogenously generated by the observable variables; but, as we will argue, they will more often result from latent exogenous trends driving the observable variables (see Section 3 below). When the system is driven by latent trends essential elements of the underlying structure are not represented in the econometric model, so that (2) can, at best be a partial structural model.

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\(^5\) The problem of which admissible $A_0$ corresponds to the DGP is typically resolved by some a priori assumption about causal order. However, starting with Swanson and Granger (1997), graph-theoretic causal search algorithms have been used to make the choice of $A_0$ based on facts about conditional dependence and independence in the data (see Section 2 below and Demiralp and Hoover (2003) and Demiralp, Hoover, and Perez (2008)).

\(^6\) The elements of $u$ include both shock and error. Our focus here is on the shocks; in practice, we cannot ignore the error.
These two issues – the invariance of the observed behavior under different choices of long-run identification and the latency of the principal causal drivers – are related. Together they form the major hurdle to applying graph-theoretic search algorithms to discover long-run causal structure. The goal of this paper is to provide a coherent account of the causal order of a CVAR and to make some preliminary suggestions about how the methods of graphical causal search in conjunction with cointegration analysis might can aid in the empirical discovery of its long-run, as they have already aided in the discovery of the short-run, causal structure.

2. Graph-Theoretic Causal Order

Several econometricians have given structural accounts of long-run behavior in the CVAR, they have focused mainly on the use of theory to provide the necessary identification (Davidson and Hall 1991; Pesaran and Shin 2002; Pesaran and Smith 1998; Pagan and Pesaran 2007). In contrast to economists’ frequent reliance on a priori theory, in the case of stationary data, considerable headway has been made (mostly, but not entirely outside of economics) in developing graphical causal search algorithms that can narrow the class of admissible identifications (sometimes to a unique scheme) (Spirtes, Glymour, Scheines 2000; Pearl 2009). As a preliminary to examining how some of these ideas might be extended to the nonstationary case, it will be helpful to review selectively some aspects of graphical causal analysis.

2.1 Graphs and Causal Structure

In Simon’s (1953) account, a structural model is a system of equations representing mechanisms in the world. Although the account can be generalized considerably (see Hoover 1990; 2001, chapter 3), it will do for our purposes to restrict our attention to linear equations and to treat each equation as the representation of the causal mechanism determining its left-hand-side variable (the effect) in terms of right-hand-side variables (the direct causes). The coefficients on the right-hand-side variables are assumed to be variation-free in the sense of Engle, Hendry, and Richard (1983); that is, any coefficient taking a particular value does not restrict the others from taking any value within their ranges (Hendry 1995, p. 163; see also Davidson and Hall 1991, p. 244; and Hoover 2001, section 7.3).

We analyze a restricted version of the structural approach to causality, in that it does deal with nonlinearities, such as cross-equation restrictions, that might arise in economic optimization problems or from systemic restrictions, such as may be generated under rational expectations. In part this is a pragmatic choice to deal with the easier case first; in part, it is to maintain tighter contact with the existing graph-theoretic causal search literature; and, in part, it arises from a yet-untested conjecture that considerable empirical progress can be made with respect to long-run cause in a

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7 Hoover (1990; 2001, chs. 2 and 3) provides a detailed account of Simon’s approach and of it generalization to nonlinear systems, including ones with cross-equation restrictions among the parameters.
simple framework. The structural approach can nonetheless be further generalized; see for example Hoover (1990, appendix; 2001, esp. ch. 3) and White and Chalak (2009).

Graph-theoretic causal analysis represents structural systems of equations as a directed graph. The variables form the nodes or vertices of the graph, and edges connects a pair of vertices. Edges come in several forms, but we will use only one – the single-headed arrow “\(\rightarrow\)”, which means “directly causes”. Direct causes are also referred to as the parents of the effect or child. We restrict ourselves to directed acyclic graphs (DAGs), which are adequate to the typical CVARs found in the macroeconomics literature. Graphical causal modeling is not, however, restricted to DAGs: the literature has also addressed cyclical graphs (for example, graphs in which \(A\) causes \(B\), \(B\) causes \(C\), and \(C\) causes \(A\)) and simultaneous graphs (a particularly tight form of cyclicity in which \(A\) causes \(B\) and \(B\) causes \(A\)) (see Richardson 1998, Phiromswad and Hoover 2013, and the references therein).

2.2 Graphs and Conditional Independence

The key idea in graph-theoretic accounts of causal structure is the mapping between the causal graph and the probability distribution described of the true DGP and its reduced form. The mapping is based on Reichenbach’s (1956, p. 156) Principle of the Common Cause: if any two variables, \(A\) and \(B\), are probabilistically dependent, then either \(A\) causes \(B\) (\(A \rightarrow B\)) or \(B\) causes \(A\) (\(A \leftarrow B\)) or they have a common cause (\(A \leftarrow C \rightarrow B\)). Essentially, the idea behind the principle is that correlations may not be causation, but correlations nevertheless must have a causal explanation. The Principle of the Common Cause is generalized as the causal Markov condition (Spirtes et al. 2000, p. 29; see also Pearl 2009, p. 30).

Without going into detail, the graph encodes certain facts of (conditional) probabilistic dependence and independence among the variables. If the data were, in fact, generated by a system of equations corresponding to the graph – as they would be, for example, in a simulation – then the joint probability distribution for those variables would embody the encoded probabilistic relations.

A key idea is that, when probabilistic dependence between variables is induced by their having a common cause, then conditioning on the common causes renders the variables conditionally independent. The translation of equations into graphs also generates another characteristic pattern of causal graphs. When two or more variables are causes of another variable, then several arrows will point into the effect variable – e.g. \(A \rightarrow C \leftarrow B\) graphs an equation in which \(A\) and \(B\) are the causes of \(C\), and \(C\) is said to be a collider on the directed path between \(A\) and \(B\). If \(A\) and \(B\) are unconditionally probabilistically independent and collide at \(B\), they will be probabilistically dependent conditional on \(C\). With stationary data, the presence of colliders helps to orient the arrows in a graph. In the nonstationary case, as we shall see presently, colliders also represent points at which new local trends are generated.

A final useful concept from graphical causality is causal sufficiency:

**Definition 1.** A set of variables is causally sufficient if, and only if, any variable that is excluded from the set directly causes at most one variable within the set (Sprites et al. 2000, p. 22).
The point of invoking causal sufficiency is that the actual DGP of the economy is more complicated than any model of observable variables that an economist might analyze. When a set of variables is causally sufficient, the excluded variables are not common causes and do not induce probabilistic dependence among the observables, so that it is possible to analyze the subset of variables without loss of causal information. Clearly, causal sufficiency is a very special case that will rarely be strictly true for our models, but that sometimes might be approximately true. When it fails, then we necessarily face a latent-variable problem.

Graph-theoretic search algorithms work backwards from the data by systematically evaluating conditional dependence and independence relations for subsets of variables statistically and then deducing logically what graph or class of graphs or, equivalently, what econometric specifications could have generated those facts. We investigate the possibility of employing a strategy that developed for stationary data to infer long-run causal structure using facts about cointegration and weak exogeneity, rather than facts of causal dependence and independence.

3. Causal Relations Among Stochastic Trends

3.1 Where Do Stochastic Trends Come From?

The nonstationarity of the variables in a system of equations such as equation (1) may arise in two ways. Consider two distinct DGPs or structural models. The first corresponding to the graph Figure 1:

\[
\Delta X_t = \Delta \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \Phi_{XX} X_{t-1} + \Phi_{XT} T_{t-1} + \varepsilon_t,
\]

\[
= \begin{bmatrix} -0.5 & 0.0 & 0.0 & 0.0 \\ 1.0 & -0.5 & 0.0 & 0.0 \\ 1.0 & 1.0 & -0.5 & 0.0 \\ 1.0 & 1.0 & 1.0 & -0.5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}_{t-1} + \begin{bmatrix} 1.0 & 1.0 \\ 1.0 & 0.5 \\ 0.5 & 1.0 \\ 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_{2,1} \end{bmatrix}_{t-1} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}_t,
\]

where the Ts are exogenous I(1) trends:

\[
\Delta T_t = \Delta \begin{bmatrix} T_1 \\ T_{2,j} \end{bmatrix} = H_t = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}_t.
\]

---

8 See Cooper (1999), Spirtes et al. (2000, chs. 5 and 6), and Pearl (2009, ch. 2). The Tetrad software package implements Spirtes et al.’s (2000) algorithms, as well as additional algorithms, and can be downloaded from Carnegie Mellon University’s Tetrad Project website: http://www.phil.cmu.edu/tetrad/.
and the $\epsilon$'s and the $\eta$'s are identically, independently distributed (i.i.d.) random shocks.

The second distinct structural model is:

**Model 2**

$$\Delta X_t = \Delta \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \Pi X_{t-1} + Z_t$$

where the $\zeta$'s are i.i.d. random shocks.

Model 1 shows the first of the two ways that variables may display stochastically trending behavior: the $T$'s trend stochastically independently of the other variables in the system because of their fundamental random-walk structure and transmit that behavior to the $X$s (i.e., were $\Phi_{XT} = 0$, $\Phi_{XX}$ which does not contain an autoregressive root of unity, would not generate a trend). Model 2 shows the second way: here the $X$'s trend, not because of an exogenous cause, but because of the fine-tuning of their structural parameters (cf. Davidson and Hall 1991, p. 239). The rank of $\Pi$ in Model 2 is two, indicating that there are cointegrating relation among the $X$'s, and, therefore, two trends. The cointegrating relations of Model 2 are characterized by its

$$\beta' = \begin{bmatrix} 1.00000 & 1.00000 & 1.00000 & -0.50000 \\ 0.84887 & 0.24254 & -0.24254 & 0.00000 \\ 1.34890 & 0.74254 & 0.25746 & -0.25000 \\ 0.13023 & 0.24149 & 0.34851 & -0.15000 \end{bmatrix}.$$  

Now consider the situation in which Model 1 is the data-generating process (DGP) and the $T$'s are not observable, but latent variables. As it turns out the parameters of the two models were chosen so that the $X$ variables in Model 1 display exactly these same cointegrating relationships. From the point of view of trends and cointegration, the two models are observationally equivalent.

The two models are nonetheless structurally distinct. In Model 1, the I(1) behavior arises because of distinct, though unobserved, variables in the DGP; while in Model 2, the I(1) behavior is a systemic characteristic, not attributable to specific variables, and crucially dependent on the exact values of the elements of $\Pi$. We can call the I(1) behavior and the cointegrating relations among the $X$'s in Model 1 generic in the sense that it is robust to changes in the values of the structural parameters (i.e., to changes that do not alter the causal graph (Figure 1)), whereas in Model 2 those properties are fragile.

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9 Here $\beta' = \Phi_{XT} \perp \Phi_{XX}$, where the subscript $\perp$ indicates the orthogonal complement.
To illustrate, suppose that the parameters of Model 2 are altered, such that the values of $\Pi$ in equation (5) are now

$$
\Pi = \begin{bmatrix}
1.00000 & 1.00000 & 1.00000 & -0.50000 \\
0.84887 & 0.24254 & -0.24254 & 0.00000 \\
0.50000 & 0.74254 & 0.25746 & -0.25000 \\
0.13023 & 2.00000 & 0.34851 & -0.15000
\end{bmatrix},
$$

where the bold parameters are the ones that have been changed. Now the rank of $\Pi$ is four, and there is no cointegration among the variables. In contrast, consider the same sort of changes to the structural parameters of Model 1 in (3), so that $\Phi_{XX} = \begin{bmatrix}
-0.5 & 0.0 & 0.0 & 0.0 \\
1.0 & -0.5 & 0.0 & 0.0 \\
0.5 & 1.0 & -0.5 & 0.0 \\
1.0 & 2.0 & 1.0 & -0.5
\end{bmatrix}$. There are still only the two trends, $T_1$ and $T_2$, in the system. If these trends are treated as latent, then the values (but not the number) of the cointegrating relationships are changed:

$$
\beta' = \begin{bmatrix}
1.00000 & 2.00000 & 1.00000 & -0.50000 \\
0.60634 & 0.24254 & -0.24254 & 0.00000
\end{bmatrix}.
$$

Thus, a small change in the structural parameters in (5) resulted in a complete loss of cointegration; while a similar change in (3) left the cointegration among the $X$'s unaffected. The generic nature of the cointegration properties of systems like (3) is the result of the trend behavior of the $X$'s having a an independent cause based in exogenous variables that are fundamentally I(1); while the fragility of cointegration in a DGP like (5) is the result of it arising only from the fine-tuning of the structural parameters. Such fine-tuning could arise in specific cases for good economic reasons; but, in the spirit of Reichenbach’s Principle of the Common Cause, we should assume that it would not be the the general case, unless we can point to an economic explanation of why trending behavior arises in a particular case.\(^{10}\)

In Model 1, we can point to specific variables that are the source of the trends. In this case, we will say that the variables are driven by genuine (or real) fundamental trends, whether those trends are themselves observed or are latent. It is conventional in the CVAR literature to say that any system of I(1) variables with reduced-rank contains trends equal to the number of variables in the system less the number of cointegrating relations (the rank). These trends may generally be represented as the cumulation of the permanent shocks to the CVAR, which are backed out of the shocks to the $X$s by imposing identifying assumptions (see Juselius 2006, ch. 15, esp. section 15). These representations are generally not uniquely identified, even when there are latent fundamental trends in the DGP, and may not correspond to any structural variable in the world at all, as in Model

\(^{10}\) An analogous case arises in the graph-theoretic search literature in the guise of fragile failures of faithfulness – i.e., failures of the estimated probability distributions to reflect all of the independence relationships implied by the graph of the DGP (Sprites et al. 2000, p. 41; Pearl 2009, pp. 62-63; Hoover 2001, pp. 45-49, 151-153, 168-169).
2, in which case, we might call them virtual trends, since they do not correspond to a particular variable—observable or latent. Our working hypothesis is that the trending behavior typically observed in macroeconomic data arises from the existence of (possibly latent) fundamental trends, and our goal is to explore some of the implications of that assumption for the long-run causal structure of the world and for the possibilities of uncovering that structure (or, as least, parts of it) empirically.\footnote{The robustness of trend behavior in CVARs driven by exogenous, latent trends would explain why the trends estimated in CVARs are often robust to widening the data set and recommends Juselius’s specific-to-general approach: once the trends can be characterized, then any new variable is either redundant or carries information with respect to a new trend (Juselius 2006, ch. 22; Johansen and Juselius 2014).}

3.2 Causal Order in the Long Run

What exactly is meant by long-run causal order? Consider a simple structural dynamic system:

\[
x_t = \mu + \alpha x_{t-1} + \beta (y_{t-1} - y_{t-2}) + \varepsilon_t^x
\]

\[
y_t = \gamma + \delta y_{t-1} + \lambda x_{t-1} + \varepsilon_t^y.
\]

This system can be represented in the causal graph in Figure 2.a.

The long run at time \( t \) can be defined to be the counterfactual situation towards which the system would be heading if it were subjected to no further shocks (i.e., when \( \varepsilon_{t+n} = e_{t+n}^x = 0 \), for all \( n \geq 1 \)) and the dynamics are allowed to fully work themselves out, so that the variables are no longer changing (i.e., \( x_{t+n} = x_{t+n-1} \) and \( y_{t+n} = y_{t+n-1} \) as \( n \to \infty \)). For parameter values that generate convergence, we will call these long-run values of the variables \( x_t^\infty \) and \( y_t^\infty \). They remain indexed by \( t \), since where the variables will end up depends on the time at which we stop shocking the system.

Applying these ideas, each of the equations (7) and (8) can be transformed to its long-run version by setting all shocks and all differenced variables to zero, and all level variables to their common values, and simplifying equation by equation (in order not to disturb the structural nature of the equations). Consider first the case in which \( |\alpha| < 1 \) and \( |\delta| < 1 \), so that the variables are stationary:

\[
x_t^\infty = \frac{\mu}{1-\alpha}
\]

\[
y_t^\infty = \frac{\gamma + \lambda x_t^\infty}{1-\delta}.
\]
The causal graph of this long-run structure is given in Figure 2.B, which is much less complex than that of Figure 2.A. Also, note that, while \( x \) and \( y \) (at different lags) cause each other in the system (7) and (8) and Figure 2.B, \( x \) is a one-way cause of \( y \) in the long-run.

In the stationary case, the system settles down to fixed values, regardless of initial conditions. In contrast consider a nonstationary case in which \( \mu = \gamma = 0 \) and \( \alpha = 1 \). Under this parameterization, the long-run system is then

\[
x_i^\infty = x_i^\infty
\]

\[
y_i^\infty = \frac{\lambda}{1 - \delta} x_i^\infty.
\]

Although equation (11) shows that \( x_i^\infty \) is not caused in the long-run by \( y_i^\infty \), unlike in the stationary case, it has its own dynamic, and the value of \( x_i^\infty \) changes as time advances, so that we can write (11) as

\[
x_i^\infty = x_i.\]

The long-term causal relation in (12) is still appropriately represented in Figure 2.B. It is important to note that the equations (11) or (11)' and (12) (and, equally, equations (9) and (10)) do not represent the reduced-forms or explicitly the values of the variables in the long run any more than equations (7) and (8) represent explicitly the value of the variables along the dynamic paths. Rather they are the long-run causal structural equations (causes on the right, effects on the left), which, of course, can be solved to find the long-run values. Figure 2.A shows that if we intervened in the process governing \( y \) (e.g., by changing one of the parameters \( \lambda \) or \( \delta \)), it would have various dynamic implications for \( x \), as well as for \( y \). But Figure 2.B and its associated equations (11)' and (12) show that, in the long-run, \( x \) is ordered recursively ahead of \( y \), so that the intervention would not transmit from \( y \) to \( x \), once all the dynamics had been worked out.

We would not, therefore, in this structural form substitute (10)' into (11), to give the long-run solution for \( y_i^\infty \), as such a substitution and, in general, taking linear combinations of distinct structural equations, as one does in solving for a reduced-form, destroys the correspondence between the systems of equations and the causal structure, represented in a causal graph. We now turn to characterizing the sources of long-run trends, such as \( x_i^\infty \) and of their causal relationships to each other and to non-trend variables.

---

12 The reduction of the dynamic behavior of the variables of (7) and (8) in the nonstationary case illustrate temporal aggregation characteristic of nearly-decomposable systems analyzed by Simon (1996, ch. 8) and Ando and Simon (1961); for a fuller discussion, see Hoover (2015).
## 3.3 Fundamental Trends and Ordinary Variables

Consider a structural DGP that generates a simple I(1) process:

\[
x_t = x_{t-1} + \varepsilon_t,
\]

(13)

where time \( t = 0, 1, 2, \ldots \). The process can be expressed in moving-average form:

\[
x_t = x_0 + \sum_{j=1}^{t} \varepsilon_j,
\]

(14)

where \( x_0 \) is an initial value. Similarly, a simple I(2) process

\[
\Delta x_t = \Delta x_{t-1} + \varepsilon_t,
\]

(15)

can be expressed as

\[
x_t = x_1 + \Delta x_1 (t-1) + \sum_{i=2}^{t} \sum_{j=2}^{i} \varepsilon_{ij},
\]

(16)

where \( \Delta x_1 \) and \( x_1 \) are initial values.

Now, consider a variable \( y \), which is I(1) but has an independent I(1) cause \( z \). Thus, \( z \) is described by analogues to (13) and (14):

\[
z_t = z_{t-1} + \varepsilon^z_t,
\]

(17)

and

\[
z_t = z_0 + \sum_{j=1}^{t} \varepsilon^z_j.
\]

(18)

And \( y \) can be expressed as

\[
y_t = y_{t-1} + \alpha z_{t-1} + \varepsilon^y_t,
\]

(19)

where \( \alpha \) is a parameter measuring the strength of the causal connection between \( z_t \) and \( y_t \). The shock terms \( \varepsilon^z_t \) and \( \varepsilon^y_t \) are serially and mutually uncorrelated. This is a requirement of a structural causal model consistent with Reichenbach’s Principle that probabilistic dependencies must have a causal explanation. Substituting (17) into (18), yields

\[
y_t = y_{t-1} + \alpha \left( z_0 + \sum_{j=1}^{t} \varepsilon^z_j \right) + \varepsilon^y_t.
\]

(20)
The moving-average form of (19) is

\[ y_t = y_1 + \alpha z_0 (t-1) + \sum_{j=2}^{t} e_j + \alpha \sum_{i=2}^{j} \sum_{j=1}^{i} e_{j-1} \]

where \( y_0 \) and \( z_0 \) are initial values.

Equation (21) shows that the variable \( y \) is I(2). But consider the thought-experiment in which \( e_t = 0 \) for every \( t \), which is the same as the causal connection between \( z \) and \( y \) having been severed. In that case, \( y \) – now driven entirely by its internal dynamic – would be I(1) and not I(2). Looked at as a “dynamic processor,” the structural equation (19) for \( y \) is fundamentally an I(1) generator. The variable \( y \) becomes I(2) only because the I(1) series \( z \) is run through this I(1) generator, which raises its natural order of integration by one degree.

A key feature is captured in a distinction between ordinary and own orders of integration:

**Definition 2.** The ordinary order of integration of a variable \( x \) is the number of times it must be differenced to render it stationary – that is, it is the property than we have hitherto indicated by saying \( x \) is I(\( n \)), which says that \( \Delta^n x \) is stationary and \( \Delta^{n-1} x \) is not.

Indicate the ordinary order of integration for a variable \( x \) as I(\( x \)), so that “\( x \) is I(\( n \))” can also be written as I(\( x \)) = \( n \). When we use “order of integration” without qualification, we mean the ordinary order of integration.

**Definition 3.** The own order of integration of a variable \( x \) (indicated by the operator \( \Xi(x) \)) is the order of integration that would result from the structural equation for \( x \), considered independently from all other structural equations and setting all variables, except for \( x \), its lagged values, and its own shock, to zero.

Thus, for equations (13) and (17), \( \Xi(x) = \Xi(z) = 1 \) and I(\( x \)) = I(\( z \)) = 1; and for equation (14), \( \Xi(x) = 2 \) and I(\( x \)) = 2; whereas for (19), the own and ordinary orders of integration diverge, so that \( \Xi(y) = 1 \) and I(\( y \)) = 2.

It is easy to show that if a variable \( y \) has causes \( z_1, z_2, \ldots, z_n \), the relationship of the ordinary to the own order of integration is given by

\[ I(y) = \Xi(y) + \max[I(z_1), I(z_2), \ldots, I(z_n)]. \tag{22} \]

We can now define explicitly the notions that we used implicitly earlier:

**Definition 4.** An ordinary variable \( (x) \) is a variable for which \( \Xi(x) = 0 \).

**Definition 5.** A nonstationary processor \( (y) \) is a variable for which \( \Xi(y) \geq 1 \).

**Definition 6.** A fundamental trend of order \( n \) is the cumulation of the shocks to a nonstationary processor \( (y) \), such that \( \Xi(y) = I(y) = n \).

The order of integration \( (n) \) corresponds to the number of unit roots in the own dynamics of \( y \). The fundamental trend and the variable that generates it (i.e., the nonstationary processor) are not identical, since the own dynamics of the nonstationary processor may involve roots less than one in
absolute value – i.e., it may involve stationary movements around the fundamental trend (Juselius 2006, section 14.3). The fundamental trend is, then, the long-run component of the nonstationary processor. The requirement that $\Xi(y) = I(y)$ ensures that the order of integration is determined exclusively by the own dynamics, which what make the trend fundamental.

We can demonstrate:

**Proposition 1.** Fundamental trends are probabilistically independent of each other.

**Proof.** Consider two nonstationary processors that generate distinct trends, $x$, for which $\Xi(x) = I(x) = m$ and $y$ for which $\Xi(y) = I(y) = n$. Suppose that they are probabilistically dependent. By Reichenbach’s Principle of the Common Cause, variables are probabilistically dependent if, and only if, one is the cause of the other or both are the effects of a common cause. Consider the case in which one is the direct cause of the other: $x \rightarrow y$. Since $y$ is a nonstationary processor, its order of integration should be $I(y) = \Xi(y) + I(x) = m + n$; but that contradicts the assumption that $\Xi(y) = I(y) = n$ and, therefore that $y$ generates a fundamental trend. It follows, then, that not $(x \rightarrow y)$, which in turn implies that probabilistic dependence cannot be induced by a direct causal connection when the effect generates a fundamental trend. Consider the second case in which $x$ and $y$ have a common cause $z$: $x \leftarrow z \rightarrow y$. If $\Xi(z) > 0$, a similar argument shows that neither $x$ nor $y$ could generate a fundamental trend. And if $\Xi(z) = 0$, then the probabilistic dependence induced by $z$ between $x$ and $y$ is confined to the stationary components of those variables. Thus, either we must reject that $z$ is a common causes of $x$ and $y$ and, therefore, that $x$ and $y$ are probabilistically dependent or that the probabilistic dependence involves the the fundamental trends. We can conclude, therefore, that $x$ and $y$ cannot generate fundamental trends.

When one variable causes another, the fundamental trends of the cause are passed on to the effect. An ordinary variable can be I(1), so long as the largest order of integration of one of its causes is $I(z) = 1$. Equation (22) shows that that there could be more than one I(1) cause, since the order of integration of the effect depends on the maximum order of integration of the set of causes. Thus, an ordinary variable $y$ can be caused by any number of I(1) variables and yet remains I(1). On the other hand, if $\Xi(y) = 1$ and the maximum ordinary order of integration among its causes were $I(z) = 1$, then $I(y) = 2$.

We can define the element that accounts for the cointegration among variables:

**Definition 7.** A local trend ($T$) is a linear combination of fundamental trends resulting from colliders in the causal graph.

Although generally local trends arise from colliders, it is convenient to treat fundamental trends as a degenerate form of local trend in which there is a zero weight on all but one constituent in the linear combination of fundamental trends, so that fundamental trends are a subset of local trends. If two variables are cointegrated, then they share a common local trend.

The notion of own order of integration and the distinction between ordinary variables and fundamental trends combined with some commonplace empirical observations allows us to say some relatively important things about long-term causal structure.
Consider the causal relations among variables for which the largest ordinary order of integration is I(1). These variables are typically analyzed as being driven by stochastic trends fewer in number than the total number of variables. In such a system, the following propositions are true:

**Proposition 2.** The fundamental trends must necessarily be I(1).

**Proposition 3.** Any variable \(x\) in the system that is not identical with one of the fundamental trends has \(\Xi(x) = 0\).

**Proposition 4.** Fundamental trends can cause only ordinary variables and not other fundamental trends.

*Proof.* Any fundamental trend \(T_j\) in the system has the property of being both I\((T_j) = 1\) and \(\Xi(T_j) = 1\); but if any such trend had a cause that were I\((1)\), then it would have to be that I\((T_j) = 2\), which is a contradiction.

Turn now to systems that contain both I\((1)\) and I\((2)\) variables. In practice, econometricians rarely find more than one I\((2)\) variables in a data set. Restricting ourselves to the case of a single I\((2)\) variable:

**Proposition 5.** The I\((2)\) variable can cause only ordinary variables and not another fundamental trend.

*Proof.* Analogous to the argument for proposition 4: if it did, it would generate an I\((3)\) variable, which *ex hypothesi* is not in the system.

**Proposition 6.** The own order of integration of the I\((2)\) variable \((x)\) is either \(\Xi(x) = 2\), in which case it generates a fundamental trend, or \(\Xi(x) = 1\) with one or more I\((1)\) trends causing it, in which case all the fundamental trends \((T_j)\) in the system are \(\Xi(T_j) = 1\).

Although these six propositions very likely do not exhaust what can be learned about the causal structure of long-run relations involving fundamental trends from this simple analysis, two points are striking and, perhaps, somewhat surprising: first, unlike the case of short-run relationships, substantial conclusions about causal order may be inferred from facts about integration without any need to appeal to an conditioning relationship; second, the analysis combined with some commonplace empirical observations known to most practitioners of CVAR analysis, suggest that the causal structure of the long-run is simpler – there are fewer and less dense causal connections among trends than among ordinary variables.\(^{13}\)

4. Graphical Analysis of the CVAR

4.1. The Canonical CVAR of a Causally Sufficient, Acyclical Graph

Consider first the long-term structure of a causally sufficient CVAR with an acyclical causal structure in which the fundamental trends are represented explicitly. In the remainder of the paper, we consider only cases for a strong form acyclicity in which we do not permit any feedback from one

\(^{13}\) Note, however, that we have continued to restrict consideration to recursive orderings and not allowed for simultaneity or cyclicity. We conjecture, but do not provide the analysis here, that if both \(\Xi(T_b) = 1\) and \(\Xi(T_i) = 1\) and the two trends are simultaneous \((T_b \Leftrightarrow T_i)\) both series will prove to be I\((2)\).
variable to another, even with a time delay. Thus, we rule out cases such as $X_t \rightarrow Y_{t+1} \rightarrow X_{t+2}$. Consider only the case in which all variables are I(1) and in which the I(0) dynamics have been concentrated out and in which the contemporaneous causal order has been imposed (Juselius 2006, section 7.1):

$$\Delta \xi_j = \Psi_{\xi j}^\prime + H_j,$$  \hspace{1cm} (23)

where $\xi = [X', T']'$; $T$ is a $q \times 1$ vector of fundamental trends; $X$ is a $p \times 1$ vector of ordinary variables, which may be trending (i.e., I(1)), but are not fundamental trends; $H' = [\varepsilon_{1t}, \ldots, \varepsilon_{p,t}, \eta_{1t}, \ldots, \eta_{q,t}]'$ is a $(p+q) \times 1$ vector of shocks to ordinary variables ($\omega_t, t = 1, 2, \ldots, p$) and to fundamental trends ($\eta_j, j = 1, 2, \ldots, q$), each of the elements of which is an identically independently distributed random variable and $H_j \sim IN(0, \Omega)$, where $\Omega$ is diagonal.

The system can be partitioned as

$$\Delta \xi_j = \begin{bmatrix} \Delta X \\ \Delta T_{j-1} \end{bmatrix} = \begin{bmatrix} \Psi_{XX} & \Psi_{XT} \\ \Psi_{TX} & \Psi_{TT} \end{bmatrix} \begin{bmatrix} X \\ T_{j-1} \end{bmatrix} + \begin{bmatrix} H_X \\ H_T \end{bmatrix} = \Psi_{\xi j}^\prime + H_j,$$  \hspace{1cm} (24)

where the submatrix of parameters $\Psi_{XX}$ is a full rank $p \times p$, while $\Psi_{XT}$ is $p \times q$, $\Psi_{TX}$ is $q \times p$, and $\Psi_{TT}$ is $q \times q$.

Because $X$ is the vector of ordinary variables, $\Psi_{XX}$ is full rank and the eigenvalues of $L + \Psi_{XX}$ must be less one in absolute value.\textsuperscript{14} If the variables in $T$ are the actual I(1) fundamental trends, as opposed to ordinary variables that serve as the conduits of the fundamental trends into the observable system, they must, as shown in Section 3.4, be mutually causally independent, requiring $\Psi_{TT} = 0_{qq}$ and strongly exogenous, requiring $\Psi_{TX} = 0_{q,p}$ (Johansen 1995, p. 77; Juselius 2006, p. 263).

By analogy with the example in Section 3.2, the long-run causal structure of the ordinary variables can be defined as follows: Let $D$ be the $p \times p$ matrix with the values of the main diagonal of $\Psi_{XX}$ on its main diagonal and zeroes elsewhere. Then,

$$X_i^\infty = -(D^{-1} \Psi_{XX} - I)X_i^\infty - D^{-1} \Psi_{XT} T_i^\infty.$$  \hspace{1cm} (25)

\textsuperscript{14} $\Psi_{xx}$ is assumed to be full rank, because all variables are assumed to have an ordinary order of integration I(1) and an own order $\Theta(1)$; thus a reduced rank then could occur only if, contrary to our assumption, the specific value of the elements of $\Psi_{xx}$ were fine-tuned to induce linear dependency; but, in Section 3.1 we set such nongeneric cases aside as outside the scope of our current interest.

\textsuperscript{15} Again, as noted in Section 3.3 with respect to equations (8) and (9) or (10) and (11), equation (24) is not a reduced form or long-run solution; it is the long-run causal structure. The matrix $D^{-1}$ is simply a matrix of normalizing factors, enforcing the convention that long-run effects are placed on the left-hand side and causes on the right-hand side of the equation.
The $\Psi$ matrix in (23) can be decomposed analogously to the $\Pi$ matrix in (2) such that $\Psi = \alpha \beta'$, where $\alpha$ is $(p + q) \times r$ and $\beta'$ is $r \times (p + q)$. The transitional causal structure embedded in $\Psi$ that governs the transmission of shocks and ultimately determines the long-run causal structure reflected in (25) can be represented in this $\alpha \beta'$-decomposition in the following **canonical** way: variables that are both cointegrated and directly causally connected are represented by the individual cointegrating relations expressed in $\beta$ and the effects of causes are indicated by non-zero coefficients in $\alpha$. To take a concrete example, consider a specific causal structure embedded in a CVAR like (2) and represented graphically in Figure 3. (With causal time-series graphs, we suppose henceforth that the arrows correspond to a one-period lag between a direct cause and its effect.)

Thus, the causally canonical representation of Figure 3 would be given as

\[
\Delta \xi_t = \Psi \xi_{t-1} + H_t = \begin{bmatrix}
p_{11} & 0 & 0 & 0 & 0 & p_{16} & 0 \\
0 & p_{22} & 0 & 0 & 0 & p_{26} & p_{17} \\
0 & 0 & p_{33} & 0 & 0 & 0 & p_{37} \\
0 & p_{42} & 0 & p_{44} & 0 & 0 & 0 \\
0 & p_{52} & p_{53} & 0 & p_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
D \\
E \\
FT_1 \\
FT_2 \\
\end{bmatrix} + H_t
\]  

(26)

\[
= \alpha \beta' \xi_{t-1} + H_t = \begin{bmatrix}
\alpha_{11} & 0 & 0 & 0 & 0 \\
0 & \alpha_{22} & 0 & 0 & 0 \\
0 & 0 & \alpha_{33} & 0 & 0 \\
0 & 0 & 0 & \alpha_{44} & 0 \\
0 & 0 & 0 & 0 & \alpha_{55} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
D \\
E \\
FT_1 \\
FT_2 \\
\end{bmatrix} + H_t
\]

The rules governing the translation of the Figure 3 or any graph into the CVAR are straightforward:

i. each single-variable direct causal pair or each collider is represented by a cointegrating relationship corresponding to a unique row of the $\beta'$ matrix where the value of the parameter for the effect is normalized to unity;

ii. there are as many adjustment parameters in $\alpha$ as there are rows in $\beta'$ (at most one per row) with the column of each non-zero parameter in $\alpha$ corresponding to the row of one of the effects (i.e., corresponding to the row in which that variable is normalized to unity) in $\beta'$;

iii. if any variable is a cause, but not an effect with respect to all the other variables, it corresponds to a zero row in $\alpha$ (and, thus, is weakly exogenous).
The $\beta$ matrix thus tells us which variables are related causally and, therefore, connected by edges, and the $\alpha$ matrix (equivalently the normalization of $\beta'$) tells us which way the arrows point for those edges.

Except for trivial reorderings of the variables and rescalings, the CVAR (26) uniquely represents the causal graph in Figure 3. Algebraically, however, the matrices $\alpha$ and $\beta$ are not unique. They can rotated to form other pairs ($\alpha^*$ and $\beta^*$) such that $\Psi = \alpha^*\beta^*$. The $\alpha\beta'$-representation and the $\alpha^*\beta^*$-representation yield the same value of the likelihood function. The problem of causal search is to find empirical information, other than the value of the likelihood function, that would allow us to select the canonical representation as in CVAR (26) that corresponds to the graph of the data-generating process.

4.2. Trends, Cointegration, and Weak Exogeneity

4.2.1. Formation and Sharing of Local Trends

We can think of the causal graph of a system of I(1) variables as representing the channels of transmission of these trends. Each collider corresponds to the creation of a local trend, and the causal variables involved in the collider are cointegrated with the effect variable. The transmission of a local trend from one variable to a single other variable also implies the cointegration of the cause and the effect.

Although causal connections produce cointegration, cointegration itself is not essentially a causal notion. Instead, cointegration results either a) when a local trend is shared by two variables or b) whenever the number of variables sharing the same fundamental trends, whether or not they share the same local trends (i.e., whether or not they share the fundamental trends in the same proportions), exceeds the number of fundamental trends. Thus, in case b), if there is a set of variables each of which is driven by the same $q$ fundamental trends, then any $q+1$ of them will be cointegrated. A causal connection is, thus, sufficient for the cointegration of the complete set of causes with their effect, but it is not necessary.

Proposition 7. Causal Cointegration: If each member of the set of parents of a variable $C$ in a causal graph is I(1), then the set of variables consisting of $C$ and its parents, is cointegrated.

It is convenient to write the fact that a set of variables is cointegrated as $\text{CI}(Z)$, where $Z$ is a set of variables with two or more members. Thus, if the variables $A$ and $B$ are cointegrated, we can write this as $\text{CI}([A, B])$. Two terms will prove useful:

Definition 8. A cointegrating group is a set of variables in which every pair of variables shares the same common local trend – i.e., every pair is cointegrated.

Definition 9. A collider group is a set of variables that are cointegrated because they form a collider.

The variables in a cointegration group share a single common local trend; while the variables in a collider group generate a new local trend. The same variable may be part of both a cointegration group and a collider group. Other sets of cointegrating variables may be in neither type of group.
Davidson (1998, p. 91) introduces a useful concept, which we define here slightly differently that he does:

**Definition 10.** A set of variables is irreducibly cointegrating (notated IC(·)) if, and only if, it does not contain a subset that is itself cointegrated.

### 4.2.2. A State-space Analysis of the CVAR

It will prove useful to examine the relationship between weak exogeneity and the causal graph. Weak exogeneity is not in itself a causal property; rather it is a property related to the manner in which a likelihood function can be decomposed into a conditional and marginal probability distribution under a given parameterization (Engle, Hendry, and Richard 1983). Weak exogeneity is essentially the condition that guarantees that the parameters of interest can be estimated efficiently from the conditional likelihood function.

Given a DGP, the weak exogeneity status of its variables will depend on the model we estimate. So, for example, if (26) were the DGP and we estimated a model with precisely the form of the DGP, then $FT_1$ and $FT_2$ would be weakly exogenous in the conditional model

$$\{A, B, C, D, E\} \mid \{FT_1, FT_2\}, \{A, B, C, D, E\} \mid i$$

for the coefficients $\psi_i$ or $(\alpha_i, \beta_i), i = 1, 2 \ldots , 5, j = 1, 2 \ldots , 7$. Our main interest, however, will be in the case in which only a subset of the data of the DGP is observed – leaving other variables in the DGP latent. So, for example, we might consider data generated by (26), but observe only $B, C,$ and $E$. These variables can be modeled in a CVAR form, but the coefficients of the model will not in general be the same as those of (26), though we could compute them if we knew the DGP. Still, we can ask the question whether we can decompose the likelihood function of this model, with some unobserved variables, in a manner that renders some of the observed variables weakly exogenous with respect to the coefficients of a conditional model for the remaining observable variables.

We can note this weak exogeneity using a new symbol “$\rightarrow^\alpha$”, which means “is weakly exogenous for” and is to be distinguished from “$\rightarrow$”, which means “directly causes.” Thus, $X \rightarrow^\alpha Y$ can be read as “the variables in the set $X$ are weakly exogenous for the coefficients of a CVAR model of $Y$ conditional on $X$” or, leaving the relativity to a particular set of parameters implicit, “$X$ is weakly exogenous for $Y$.” If we know the causal graph of the DGP, then we can read the various weak exogeneity relationships for models of different subsets of variables from information in the causal graph. As a result, if we can identify weak exogeneity relationships for different subsets, we may be able to work backwards to determine which causal graphs could have generated them.\(^{16}\)

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\(^{16}\) Connecting weak exogeneity and, therefore, efficient estimation to causal order is reminiscent of LeRoy’s (1995) approach to causality (cf. Hoover 2001, pp. 170-174). An importance difference, however, is that LeRoy defines causal orderings in terms of efficient estimation, while we seek only the implications of causal orderings defined in terms of structural parameterizations for weak exogeneity and tests of weak exogeneity as evidence of what the underlying structural relations might be.
The object of the analysis is to use tests of long-run weak exogeneity in CVARs of the form of equation (1) for observable variables only to discover restrictions on allowable causal ordering of the underlying DGP (24). Long-run weak exogeneity corresponds to a zero row in the $\alpha$ matrix of the CVAR, so a critical goal is, given a particular DGP, to determine what it implies for the $\alpha$ matrix of a CVAR of the subset of observable variables.

Fundamental trends are assumed to be latent. In order to analyze cases in which some subsets of ordinary variables could also be latent, partition $X_t = [X_l, X_u]$, where the $X_l$ are observed and the $X_u$ (are treated as) unobserved. Then, rather than partitioning $\Psi$ as in (24), partition it as

$$
\Psi = \begin{bmatrix}
M & C \\
0 & 0 \\
\end{bmatrix},
$$

depending on whether any ordinary variables are latent. The $m \times p$ null element in the lower left-hand corner of the $\Psi$ matrix corresponds to the assumption that the fundamental trends are strongly exogenous, and the $m \times m$ null element in the lower right-hand corner indicate that fundamental trends do not cause one another.

The submatrix $M = \begin{bmatrix} M_{11} & M_{12} \\
M_{21} & M_{22} \\
\end{bmatrix}$ contains the parameters of the ordinary variables. Only the parameters in $M_{11}$ relate exclusively to the $p_1$ observed ordinary variables, while the other $M_{ij}$ contain parameters that relate partly or exclusively to the $p_2$ latent ordinary variables. The submatrix

$$
C = \begin{bmatrix}
C_1' & C_2' \\
\end{bmatrix},
$$

contains the parameters in $C_1'$ that relate to the effects of the latent fundamental trends on the observed ordinary variables and those in $C_2'$ that relate to the their effects on the unobserved ordinary variables.

A state-space representation of CVAR (24) can then be given:

1. $\Delta X_{1t+1} = M_{11} X_{1t} + M_{12} X_{2t} + C_1 T_t + \varepsilon_{1t+1}'$  \hspace{1cm} (27)
2. $\Delta X_{2t+1} = M_{21} X_{1t} + M_{22} X_{2t} + C_2 T_t + \varepsilon_{2t+1}'$  \hspace{1cm} (28)
3. $\Delta T_{t+1} = \eta_{t+1}'$  \hspace{1cm} (29)

where $t = 0, 1, \ldots, n - 1$, and $T_0 = 0$ and $X_0 = 0$. The shocks are partitioned into those affecting ordinary variables ($\varepsilon$) and those affecting the latent variables ($\eta$), with $(\varepsilon, \eta) \sim \text{i.i.d.} \ N_{p+m}(0, \Omega)$,
\[ \Omega = \begin{pmatrix} \Omega_\varepsilon & 0 \\ 0 & \Omega_\eta \end{pmatrix}, \] and \( \Omega_\varepsilon \) diagonal. In keeping with the distinction between ordinary variables and fundamental trends, we assume that the eigenvalues of \( I_p + M_1 \) and \( I_1 + M_{11} \) and \( I_1 + M_{12} \) are less than one in absolute value, so that the source of the nonstationarity of \( X_t \) is the fundamental trends, rather than its own dynamics.

The matrix \( C \) represents the proportions of fundamental trends present in observable variables but transmitted to them through latent causal connections and not via causal relationships among the observable variables. Thus, while the non-zero entries of \( M \) correspond to the edges in a causal graph, \( C \) is not given a direct graphical interpretation. The fundamental trends are embedded in \( T \), but the variables included in \( T \) should be regarded as local trends, which may either be latent fundamental trends directly causing the observed variables or ordinary variables that carry some linear combination of fundamental trends and cause the observable variables. Therefore, \( \Omega_\eta > 0 \) need not be diagonal.

Suppose that the DGP graph is described as in system (27)-(29), and we wish to know whether any of the observed variables \( (X_{i1}) \) are weakly exogenous in a CVAR of the observed variables only. This comes down to the question of whether \( \alpha \) in that CVAR has any zero rows. Johansen’s (2018) Theorem 2 proves that the \( \alpha \) of such a CVAR can be written as:

\[ \alpha = \Sigma (M_{12}V_{2T} + C_1V_{TT})_{\perp}, \quad (30) \]

where \( V_{2T} \) is the covariance of the unobserved \( X_t \)'s with the fundamental trends and \( V_{TT} \) is the variance of (also unobserved) fundamental trends – both conditional on the observed \( X_t \)'s (see Johansen 2018, Section 2 and equation (A8)); \( \Sigma \) is a \( p_1 \times p_1 \) matrix of the variance of the prediction errors \( X_{it} - E_{it-1}(X_{it}) \); and the subscript \( \perp \) indicates the orthogonal complement. Since the orthogonal complement is defined only up to multiplication from the right by a full-rank \( p_1 \times p_1 \) matrix and the zero rows are invariant within class, we can safely ignore \( \Sigma \), so that, if there is a zero row in \( (M_{12}V_{2T} + C_1V_{TT})_{\perp}, \) there is also a zero row in \( \alpha \).

4.2.3. Weak Exogeneity and Causal Order

The state-space representation and Johansen’s Theorem 2 offer a tool for analyzing weak exogeneity for subsets of variables in the DGP. These, in turn, correspond in systematic ways to facts about the causal structure of the DGP itself. Consider some illustrative cases:

**Case 1.** Consider the causal graph in Figure 4, in which all ordinary variables are observed and only the fundamental trends are unobserved, then the key components of equation (30) are given by
\[ X_t = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} ; \quad M_{11} = \begin{bmatrix} \psi_{11} & 0 & 0 \\ 0 & \psi_{22} & 0 \\ \psi_{31} & \psi_{32} & 0 \end{bmatrix} , \text{ and the remaining elements of } M \text{ are zero;} \]

\[ C_1 = \begin{bmatrix} \psi_{14} & \psi_{15} \\ \psi_{24} & \psi_{25} \\ 0 & 0 \end{bmatrix} \text{ and } V_T \text{ is full rank. Thus equation (30) becomes } \alpha = \Sigma(C_1V_T)_{\perp} \text{ and } \]

\[ (C_1V_T)_{\perp} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} , \text{ implying that } A \text{ and } B \text{ are weakly exogenous for } C \text{ (i.e., } [A, B] \mapsto C).^{17} \]

Notice that it does not matter, in this specific case, what the causal relations are among the observables, since they are encoded in the \( M_{11} \) matrix, which plays no part in the determination of \( \alpha \) in equation (30). What matters is which variables convey the fundamental trends to the observables.

**Case 2.** Unfortunately, the simple mapping between weak exogeneity and causal connection suggested by Case 1 does not hold up. Consider Figure 5, which adds the variable \( D \) and edges connecting to other variables in Figure 4. Here the two fundamental trends are the parents in a collider at \( C \). The analysis proceeds just as in Case 1 with the relevant matrices of the state-space formulation given by

\[ X_t = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \text{ and } C_1 = \begin{bmatrix} \psi_{15} & \psi_{16} \\ \psi_{25} & \psi_{26} \\ 0 & 0 \\ 0 & \psi_{46} \end{bmatrix}. \]

These imply that \( (C_1V_T)_{\perp} = \begin{bmatrix} 0 \\ * \\ 0 \\ * \\ 1 \end{bmatrix} ; \) so that there are no zero rows in \( \alpha \), and, therefore, none of the variables are weakly exogenous. \(^{18} \) The variables \( A, B, C, D \) are cointegrated (\( \text{CI}([A, B, C, D]) \)); but, so is each three-member subset of these variables, implying not \( \text{IC}([A, B, C, D]) \). This is a robust finding: the parents in a collider are weakly exogenous only when the colliding set is irreducibly cointegrated.

**Case 3.** It is tempting to think that we might consider an irreducible subset of the variables in Figure 5, such as \( [A, B, C] \) and find the same weak exogeneity relations as we did in Figure 4. That,

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\(^{17}\) The orthogonal complement for any matrix is not, in general, unique; but each admissible complement spans the same space and places zeroes in the same positions.

\(^{18}\) This is, as in similar cases, a generic claim and does not rule out that zero rows in \( \alpha \) might occur for carefully chosen parameter values.
however, does not work. In analyzing the subset, we are effectively treating $D$ as an unobservable variable. The state-space representation for this reduced system gives:

$$X_{tt} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}; \quad X_{2t} = [D]_{tt}; \quad M_{12} = \begin{bmatrix} 0 \\ 0 \\ \psi_{34} \end{bmatrix}; \quad C_i = \begin{bmatrix} \psi_{15} & \psi_{16} \\ \psi_{25} & \psi_{25} \\ 0 & 0 \end{bmatrix}; \quad \text{and } V_{2t} = \begin{bmatrix} 0 & \ast \end{bmatrix}, \text{ so that}

$$(M_{12}V_{2t} + C_iV_{TT})_{\perp} = \begin{bmatrix} 0 \\ \ast \\ \ast \end{bmatrix} \perp \begin{bmatrix} 0 & \ast \\ \ast & \ast \\ \ast & \ast \end{bmatrix} = \begin{bmatrix} \ast \\ \ast \\ \ast \end{bmatrix}; \quad \text{and, in general, } \alpha$$

no zero rows, so that none of the variables is weakly exogenous.

Although $D$ as unobservable, in the DGP that actually determines the value of the observable variables, it provides a conduit from the fundamental trends to $C$ that is distinct from the observable conduits, $A$ and $B$. It is as if the graph of Figure 5 has been transformed into Figure 6, where the dashed arrow indicates a causal connection between $FT$ and $C$, mediated by $D$ in the DGP but not observable in the CVAR of the subset $\{A, B, C\}$. Unobserved mediating causes, like $D$, can make an indirect causal connection appear to be direct.

**Case 4.** In Case 3, weak exogeneity failed to obtain, even though the causal connections were genuine. It can also happen that weak exogeneity does obtain, even when causal connections are missing. Consider Figure 7. The graph shows not $(A \rightarrow C)$ and not $(B \rightarrow D)$ and not $(B \rightarrow E)$, although $B$ does indirectly cause $E$. Using the same state-space methods, but omitting the details here, we can show that $\{A, B\} \rightarrow \{C, D, E\}$. And, looking at subsets of variables $\{A, B\} \rightarrow D$. Thus, $\{A, B, D\}$ have the same apparent pattern of weak exogeneity as found for $\{A, B, C\}$ in Case 1 (Figure 4); yet these variables do not form a collider group in Figure 7. But notice CI$(\{A, B, D\})$, but also CI$(\{A, D\})$. The set $\{A, B, D\}$ is not irreducibly cointegrated. It appears that a mapping between weak exogeneity and causal connections can be established only in irreducibly cointegrated sets.

**Case 5.** Weak exogeneity may fail to track direct cause. Consider a causal chain:

$$FT \rightarrow A \rightarrow B \rightarrow C \rightarrow D.$$

All four observable variables form a single cointegration group, sharing the single fundamental trend. Note that $B \rightarrow C$ and that $\{B, C\}$ form a cointegration group. We might be tempted to conclude that these facts would warrant inferring what is, in fact, true that $B \rightarrow C$. A similar case shows the problem: $A \rightarrow C$ and CI$(\{A, C\})$; but, in fact, it is not true that $A \rightarrow C$. Equally, $B \rightarrow D$ and CI$(\{B, D\})$; but, in fact, it is not true that $B \rightarrow D$. The example shows that we have to be careful in making such inferences, but not that they are hopeless. Note that $A \rightarrow \{B, C, D\}$; $B \rightarrow \{C, D\}$; and $C \rightarrow D$; so that the variables form a nested hierarchy with $A$ at the top. This hierarchy can be reinterpreted as a chain: $A \rightarrow B$ and all variables lower in the hierarchy; $B \rightarrow C$ and all variables lower in hierarchy; $C \rightarrow D$; and $D$ is not weakly exogenous for any variable. Such as chain recapitulates the causal graph. The lesson is that when a variable is weakly exogenous for another variable in a cointegration group, it is a direct cause only if it is adjacent in the sense of sitting at the immediately higher step of the hierarchy.
These cases show us how to read weak exogeneity off a causal graph. There are four criteria:

A. Within a set of variables that form a cointegration group, a particular variable is weakly exogenous for the group if, and only if, it the sole source of the local trend that cointegrates the group.

B. The parents in any set of variables that form a collider group in which two or more local trends are combined are weakly exogenous for the child in the collider group, provided that the number of variables in the group is fewer than one plus the number of fundamental trends carried by those variables.

C. If a collider fulfills criterion B, then in any set that replaces one or more weakly exogenous parents with a variable in the same cointegration group as that parent, provided the variable is itself weakly exogenous for the parent, will also be weakly exogenous for the child. (Thus, in Figure 7, in the collider \{A, C, E\}, \{A, C\} → E; but in the set in which B replaces C (both in the same collider group), \{A, B\} → E.)

D. If a collider fulfills criterion B, then any variable that is weakly exogenous for the child, either as a parent or as a member of the same cointegration group that replaces the parent, will be weakly exogenous for a variable that replaces the child from a cointegration group that includes the child and for which it is weakly exogenous. (Thus, in Figure 3, \{FT1, FT2\} → B, but in the set that replaces B with D, which are both in the same cointegration group, \{FT1, FT2\} → D.)

The inferential lessons of Cases 1-5, can be summarized in three rules, consistent with visual reading of the graph:

**Rule 1.** If A → B, then not B → A.

Rule 1 simply says that causation cannot run against the direction of weak exogeneity.

**Rule 2.** In a cointegration group, if A → B and there is no C such that A → C and C → B, then A → B.

Rule 2 says that bivariate weak exogeneity coincides with direct causation, provided that the variables are adjacent.\(^{19}\)

**Rule 3.** A triple of variables forms a collider A → C ← B, if i) IC(\{A, B, C\}); ii) \{A, B\} → C; iii) it is not the case that A is a member of a cointegration group Z such that, for any member D ∈ Z (excluding A), A → D and \{B, D\} → C, and mutatis mutandis for B; and iv) it is not the case that C is a member of a cointegration group Z such that for any member D ∈ Z (excluding C) that D → C.

Rules 3 says that if two variables are weakly exogenous for a third, they form a triple, provided that each of the weakly exogenous variables is adjacent to the third variable (established by conditions iii) and iv)).

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\(^{19}\) The rule refers to the DGP, so that an unobserved intermediate cause would appear to warrant the inference of a direct causal connection when only an indirect connection existed in the DGP. This implies that widening the data set might, in effect, open the “black box” and provide more refined information about causal mechanisms.
4.3. Inferring Causal Structure from Empirical Observation

The DGP that adequately represents the causal structure in the economy is not directly observable. But might it be inferred on the basis of data and not simply imposed as a priori restrictions on the CVAR? Based on our analysis of long-run causal structure, can we recover reliable information about the underlying DGP from the facts of cointegration and weak exogeneity analogously to the way in which graphical causal search algorithms infer causal structure for stationary data from empirical evidence about probabilistic dependence and independence among subsets of variables? We analyze two cases — with and without causal sufficiency.

4.3.1. Long-run Causal Search in a Causally Sufficient Graph

Davidson (1998, section 3) proposes a search algorithm that identifies every irreducible cointegrating set of variables within a CVAR. He then uses that information where possible to identify the cointegrating relations in the $\beta'$ matrix. This strategy is successful in some cases and not others. There is an analogy with causal search for stationary variables. Despite the slogan, “correlation is not causation,” it is sometimes possible to infer causal direction from tests of unconditional dependence. For example, for a causally sufficient set of three stationary variables with an acyclical data-generating process, if $A$ and $C$ are not correlated, but $A$ and $B$ and $B$ and $C$ are correlated, then $A \rightarrow B \leftarrow C$ is the only consistent causal graph. In most cases, however, unconditional independence is not enough. Relations of conditional dependence and independence provides a richer source of information for inferring the direction, as well as the existence of causal edges (see Section 2.2 above).

Davidson’s schema places cointegration in something like the role of unconditional independence (or correlation) in the stationary case. Davidson’s inferential scheme can be further developed by explicitly recognizing, first, that the ultimate source of nonstationarity in any set of variables is often found in latent trends and, second, that assessment of weak exogeneity may provide evidence of causal asymmetry. Within irreducibly cointegrated subsets of the variables, weak exogeneity can function in something like the role of conditional independence, when processed according to the three rules in the last section and may provide richer, empirically grounded information about the identification of the CVAR. As with causal search in the stationary case, the application of these rules will not identify every possible causal graph, but will sometimes be able to partially or completely uncover the underlying causal structure.

Consider the DGP in Figure 3 and assume that its variables are causally sufficient and all (including the fundamental trends) are observed. We are interested in the logic of causal inference rather than the statistical problem of inference, so we also assume that we know the correct facts with respect to the cointegration rank and cointegration and weak exogeneity among any subset of variables. (In the language of the causal search literature, we assume that we have an oracle.) Can we use this information to recover the graph of the DGP?

The inference problem can be viewed as how to place the zero and non-zero coefficients the $\alpha$ and $\beta'$ matrices in equation (26).
Given that we know that the cointegration rank is 5, we know that there are two fundamental trends. This implies that $\alpha$ is $7 \times 5$ and $\beta' \times 5 \times 7$. Since $FT_1$ and $FT_2$ are weakly exogenous with respect to all other variables in the system, we may conclude that, even if they are not identical with the fundamental trends (which in this case, of course, they are), they are at least the unique sources introducing those trends into the system. And we are entitled to enter zeroes in the entire rows of $\alpha$ corresponding to $FT_1$ and $FT_2$. Without loss of generality, we may enter non-zero $\alpha$s along the main diagonal of the submatrix excluding the $FT_1$- and $FT_2$-rows of $\alpha$ and zeroes everywhere else. Similarly, we may enter ones on the main diagonal of the submatrix of $\beta'$ that excludes the last two columns.

With two fundamental trends, no irreducible cointegrating relation can involve more than three variables. Exhaustive consideration along Davidson’s lines would produce 21 possible cointegrating pairs and 35 possible cointegrating triples. Similarly, we need to consider possible weak exogeneity of variables within each irreducibly cointegrating subset. Most of subsets are not irreducibly cointegrating or do not contain weakly exogenous variables, so rather than tediously listing the weak-exogeneity status of all 56 subsets systematically, we just note the salient ones.

From the facts that $CI\{A, FT_1\}$ and that there are no other variables in this cointegration group and that $FT_1 \rightarrow A$, Rule 2 implies $FT_1 \rightarrow A$, which justifies the placement of $\beta_5$ in row 1 of $\beta'$ and zeroes in the remaining unassigned places in that row. Analogous reasoning with respect to $\{C, FT_2\}$ implies $FT_2 \rightarrow C$ and justifies the placement of $\beta_6$ and the zeroes in row 3. And again with respect to $\{B, D\}$, analogous reasoning justifies the placement of $\beta_8$ and the zeroes in row 4. In addition in this case, Rule 1 and the fact that $B \rightarrow D$ imply that not $(D \rightarrow B)$ and justify the zero in row 2, column 4.

Rule 3 and the facts that $IC\{FT_1, FT_2, B\}$, that $B$ is not part of a cointegration group with either $FT_1$ or $FT_2$, and that $\{FT_1, FT_2\} \rightarrow B$ allows us to identify the collider $FT_1 \rightarrow B \leftarrow FT_2$ and justifies the placement of $\beta_5$ and $\beta_6$ and the remaining zeroes in row 2 of $\beta'$.

Rules 3 and the facts that $IC\{B, C, E\}$, $\{B, C\} \rightarrow E$, and not $(C \rightarrow FT_2)$, with which it forms a cointegration group, allows us to identify the collider $B \rightarrow E \leftarrow C$ and justifies the placement of $\beta_8$ and $\beta_7$ and the zeroes in row 5 of $\beta'$. And with that we were able to recover the entire DGP graph using only the facts of cointegration and weak exogeneity.

4.3.2. Long-run Causal Search in the Presence of Latent Trends

The CVARs typically estimated in practice most often do not contain variables that are weakly exogenous for the whole system, which could, therefore, be identified as the conduit of the fundamental trends to the other variables in the system. It is, therefore, worth considering how the principles of search might operate when fundamental trends are latent variables. It is possible to apply the rules of Section 4.2 to the variables generated according to equation (26) when only the ordinary variables ($A, B, C, D, E$), but not the fundamental trends ($FT_1$ and $FT_2$), are observed.
For some of the causal edges, the reasoning of Section 4.3.1 is still applicable, and we would be able to infer the edges shown in Figure 8: $B \rightarrow D$ and $B \rightarrow E \leftarrow C$. The remainder of Figure 8 requires further comment.

We are unable to infer the edges between $FT_1, FT_2$ and $A, B,$ and $C$ for the simple reason that the two fundamental trends are not observed and the inference of the edges in which they are involved requires their observability. However, we do know from the fact that the cointegration rank is 3 that there are two fundamental trends. What we cannot say, however, precisely how those two trends enter directly into the observable system. They may, in fact, be transmitted through ordinary variables that are also latent. We do know that they must enter through $A, B,$ or $C$. If that were not the case and a fundamental trend entered through $D$ or $E$, we would not have found that CI($\{B, D\}$) or $\{B, C\} \rightarrow E$. This is indicated in Figure 8 by the oval enclosing the ordinary variables and the circles (indicating their latency) around the fundamental trends. The arrows running from the latent fundamental trends to the oval, stopping short of the particular variables indicates that we know these variables are caused by these trends, albeit we do not know exactly what the connections are.

Thus, instead of (26), we can fill in the causally ordered CVAR equation (31) with the ambiguous information depicted in Figure 8, where the question marks indicate parameters that correspond to possible, but yet-to-be-determined causal edges:

$$\Delta \xi_t = \Psi \xi_{t-1} + H_t$$

$$= a \beta \xi_{t-1} + H_t = \begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ 0 & \alpha_{22} & 0 & 0 & 0 \\ 0 & 0 & \alpha_{33} & 0 & 0 \\ 0 & 0 & 0 & \alpha_{44} & 0 \\ 0 & 0 & 0 & 0 & \alpha_{55} \end{bmatrix} \begin{bmatrix} 1 & ? & ? & 0 & 0 \\ ? & 1 & ? & 0 & 0 \\ 0 & \beta_{42} & 0 & 1 & 0 \end{bmatrix} + H_t$$

Here the two variables, $L_1$ and $L_2$ are latent variable that may either be the underlying fundamental trends or ordinary variables that transmit distinct linear combinations of the fundamental trends to the system. In the later case, the covariance between their error terms in $\Omega$ may be nonzero.

Neither the graph nor (31) conveys all the information that we have. We know, for instance, that there are two fundamental trends and that at least one of the fundamental trends must be a causal influence on each of $A, B,$ and $C$. If that were not so, then the only way that all three variables could carry the trends and be irreducibly cointegrated would be for them to form a collider group in which one pair is weakly exogenous for the remaining variable. Given the DGP, we know that the weak exogeneity search would not have found that. Furthermore, we know that no two of $A, B,$ and $C$ can have a common latent cause. If that were not true, that pair would form a cointegration group, which, given the DGP, the search for cointegrating pairs would not have, in fact, discovered. These
two conclusions imply that each of the three observed variables carries the fundamental trends in distinct proportions. These facts place restrictions on how the last two columns of the $\beta'$ in (31) can be filled in to be consistent with the DGP.

5. Conclusion

In the history of econometrics, the problem of identification and the notion of causal order have long been connected – both in the work of Simon and the early Cowles Commission program and in the literature on SVARs. Typically, economists have relied heavily on the idea that a priori restrictions derived somehow from economic theory would provide the needed identification. Recent work on graphical causal modeling, however, has shown that there is often unexploited information that could provide a firmer, empirical basis for identification. In the case of cross-sectional data or the contemporaneous causal orderings of SVARs, the graphical causal modelers have stressed the information contained in conditional independence relationship encoded in the probability distribution of the data. Conditional independence may also be a resource in the case of the long-run dynamics of the CVAR, although the fact that nonstationary data involves non-standard distributions poses some challenges. We have suggest here that nonstationary data also present the opportunity to take a different approach.

Where do the trends we observe among macroeconomic variables come from? We showed that it is possible for the structure of the DGP to be such that a set of observable variables trends without any fundamental trends acting as drivers. Yet, we have argued that these cases rely on particular parameterizations that are likely not to be robust to small changes in parameter values and that call out for an economic explanation of why they arise at all. Once a distinction is drawn between ordinary variables (stationary processors) and fundamental trends (nonstationary processors), it is clear that a more robust account for nonstationary behavior is that it is transmitted from its fundamental sources to variables that without these fundamental trends as direct or indirect causes would not naturally be nonstationary. In typical CVAR analysis, econometricians mostly do not find variables that themselves can be identified as the source of fundamental trends. This suggests that, in most cases, fundamental trends are latent variables, and any sort of structural or causal analysis of CVARs must account for their latency.

We suggested – somewhat informally – that combining Davidson’s suggestion of a comprehensive search for sets of irreducible cointegrating relations with a similar comprehensive search of weak exogeneity among those sets could provide a non-a priori empirical basis for discovering identifying restrictions on cointegrating relations, as well as information on causal direction. We showed that in a simple example, the complete causal graph of the CVAR could be recovered. But, in most cases in the face of latent variables, these restrictions are unlikely to provide complete identification. Nevertheless, as in our illustration, some of the cointegrating relations may be identified, even when there are latent trends. It is also possible that, in some cases, it would be possible to recover estimates of the trends using state-space methods (see e.g., Johansen and Tabor 2017). Finally, viewing the CVAR through the lens of latent fundamental trends reinforces Juselius’s advocacy of simple-to-general modeling in the CVAR context (Juselius 2006, ch. 22, esp. sections
22.2.3 and 22.3). Cointegrating relations are robust to widening the data set to include more variables. The aim of such widening can be seen as an effort to discover the observable variables that are the counterpart of the latent trends in narrower data sets.

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Figures

Figure 1. Causal structure of the Model 1

Figure 2. (A) The dynamical causal structure of system (7) and (8)

(B) The long-run causal structure of system (7) and (8)

Figure 3. The causal structure of the DGP of CVAR (25).
Figure 4. Causal structure of the DGP of Case 1

Figure 5. Causal structure of the DGP of Case 2

Figure 6. Virtual causal structure of Case 2 when $D$ is unobserved

Figure 7. Causal structure of the DGP of Case 4
Figure 8. Recoverable structure of Figure 3 graph when fundamental trends are latent.