Baumol's (1952) Model of the Transactions Demand for Money

The model based is based on the paper, William J. Baumol, "The Transactions Demand for Money: An Inventory Theoretic Approach," *Quarterly Journal of Economics* 66(4), pp. 545-556.

Key idea: *non-interest-bearing* cash is needed for transactions; leaving cash in a bank account bears interest, so there is an *opportunity cost* to withdrawing cash for spending, measured by the rate of interest (r) that the bank pays; while there is a *direct cost* in terms of time and trouble, travel expense, etc. to withdrawing cash from the bank (these are sometimes called "shoe-leather costs"); so the problem is how optimally to obtain the necessary cash at the lowest cost.

Assumptions:

1. Income is received at regular intervals, say, monthly and are paid into the bank in cash.
2. Expenditures are perfectly steady, the same amount each day.
3. Withdrawals of money from the bank occur at regular intervals, each withdrawal occurring immediately after cash holdings run to zero.

Model:

Variables:

T = transactions (expenditures) per year

W = size of the regular cash withdrawal

T/W = number of withdrawals per year (so, e.g., if there are $100,000 of expenditure in a year and withdrawals are $10,000 each, then there would be 10 withdrawals per year)

b = (actual and shoe-leather) costs per withdrawal

bT/W = total withdrawal costs (= total number of withdrawals $\times$ cost per withdrawal)

r = interest rate

$M^D = ½W$ = *average* cash holdings (at the moment of withdrawal holdings are W, the cash is spent steadily, and just before the next withdrawal, holdings of case are zero; so that on average holdings are halfway between 0 and W)

$rW/2$ = opportunity cost of holding non-interest-bearing cash (i.e., the average amount of interest that would be earned if one left the money in the bank and did not engage in any expenditures)

Solving the Model:

Problem: find the optimal withdrawal size – i.e., the one that minimizes costs. Size of total annual transations (T), withdrawal costs (b), and interest rate (r) are taken as given.. The problem is to determine the optimal withdrawal size, noting that each withdrawal imposes withdrawal costs (b), so that fewer withdrawals reduces costs, while the larger the withdrawal the higher the opportunity cost (i.e., interest foregone). There are thus two effects that have to be balanced – one pushing toward fewer, larger withdrawals and one pushing toward more, smaller withdrawals. Mathematically, the problem is:

(1) $$\min \, Cost = \frac{bT}{W} + r\left(\frac{W}{2}\right)$$

To solve, find the derivatives with respect to *W* and set them to zero (second-order conditions also have to be checked, but we ignore that here).

(2) $$\frac{dCost}{dW} = \frac{-bT}{W^2} + \frac{r}{2} = 0\,.$$

Solving for *W*:

$$W^2 = \frac{2bT}{r}\text{, so that}$$

(3) $$W = \sqrt{\frac{2bT}{r}}\,.$$

And, since $M^D = W/2$,

(4) $$M^D = (1/2)\times\sqrt{\frac{2bT}{r}} = \sqrt{\frac{bT}{2r}}\,.$$

To find elasticities, take natural logarithms of both sides:

(5) $$\log(M^D) = (1/2)[\log(b) + \log(T) - \log(2) - \log(r)]\,.$$

The elascitity of any variable *X* with respect to *Y* is $\varepsilon_{X|Y} = \dfrac{d\log(X)}{d\log(Y)}$.

Therefore, the interest elasticity of the demand for money is

(6) $$\varepsilon_{M^D|r} = \frac{d\log(M^D)}{d\log(r)} = -1/2$$

and the transactions (proportional to income) elascitity of the demand for money is:

(7) $$\varepsilon_{M^D|T} = \frac{d\log(M^D)}{d\log(T)} = 1/2\,.$$