A fall in K (arrow 1 on graph) is initially a movement along the capital curve of the production function, to point \( Y_1 \) and \( K_1 \). Since \( Y \) must be the same in the Labor panel, the labor curve must shift down (arrow 2) to intersect at \( L_0 \) and the new \( Y_1 \). As it becomes less steep the mpL falls, so that profit-maximizing firms must pay a lower real wage at each level of \( L \), so \( L^D \) curve shifts to \( L^D' \) (arrow 3), implying that \( L \) will fall. This, in turn, implies that \( Y \) will be even lower and that the capital curve must shift down to pass to reflect the now lower \( L \) down (arrow 4)). When all the adjustments are done, the final capital and labor curves reach a mutual equilibrium is reached at \( L_1, K_1, \) and \( Y_2 \). At this point \( L \) has fallen and \( (w/p) \) has fallen and \( Y \) has fallen.

2. Cobb-Douglas Production Function

Convert the production function to growth rates:

\[ Y = AL^\alpha K^{1-\alpha} \]

\[ \dot{Y} = \dot{A}L^\alpha + \alpha \dot{L} + (1-\alpha)K \]

or, rewrite to show the shares of growth accounted for by \( A, L \) and \( K \):

\[ 1 = \frac{\dot{A}}{\dot{Y}} + \alpha \frac{\dot{L}}{\dot{Y}} + (1-\alpha) \frac{\dot{K}}{\dot{Y}} \]

To calculate the shares, substitute the average growth rates of labor, capital, and output in for the growth rates in the formula.

Time from peak to peak: 9.75 years
\[
\bar{Y} = \left( \frac{3392.6}{2279.2} \right)^{1 \over 0.75} - 1 = 4.16\%
\]

\[
\bar{K} = \left( \frac{9436.2}{6580.6} \right)^{1 \over 0.75} - 1 = 3.77\%
\]

\[
\bar{L} = \left( \frac{144,400.1}{116,731.6} \right)^{1 \over 0.75} - 1 = 2.21\%
\]

Shares of Growth: Technical progress (A), Labor (L), and Capital (K)

\[L\text{ share } = \alpha \frac{\bar{L}}{\bar{Y}} = \frac{0.69(2.21)}{4.16} = 37\%
\]

\[K\text{ share } = (1 - \alpha) \frac{\bar{K}}{\bar{Y}} = \frac{0.31(3.77)}{4.16} = 28\%
\]

\[Technical\ Progress\ share = \frac{\hat{A}}{\bar{Y}} = 1 - \alpha \frac{\hat{L}}{\bar{Y}} - (1 - \alpha) \frac{\hat{K}}{\bar{Y}} = 100 - 37 - 28 = 35\%
\]

3. An open market sale of T-bills will increase the public holdings of T-bills (right shift of government supply in diagram below), driving the price down. This implies the interest rate on T-bills will increase.

![Diagram of government supplies, public demands, and open market sale](image)

The sale also leads to a decrease in banks’ holdings of reserves, as funds must be transferred to the Fed (shown as left shift in diagram below). This implies that \(r_{FF}\) increases.
The higher interest rates in the Fed funds market may cause substitution towards T-bills causing a left shift of the public’s demand curve in the first diagram. There may also be substitution because of the higher yield in the T-bill market (but for simplicity this is omitted from the diagram, as it is likely to be small.)

Since the expectations theory of the term structure implies because of arbitrage longer rates are averages of current and expected short rates, longer rates will rise at all maturities, though not necessarily by as much as short rates (that will depend on people’s expectations about the path of future short rates). Higher interest rates will reduce investment as the opportunity cost of investment has increased. This will have a multiplier effect, reducing GDP as well. The implied fall in the growth rate of GDP will, according to Okun’s law, increase $\Delta U$ and thus either slow the fall in the unemployment rate or actually raise the unemployment rate. This change in $U$ will reduce the gap between actual unemployment and NAIRU in the Phillips curve and, therefore, reduce the acceleration of inflation, either lowering the rate increase in inflation or actually lowering the inflation rate.

**Summary:**
- T-bill rate rises
- Federal funds rate rises
- Long-term bond rate rises
- Investment falls
- Unemployment either falls more slowly or rises
- Inflation rate either rises more slowly or falls
4.
a) The modified balance growth path is the rate of growth at which GDP grows at the same rate as capital adjusted or capital productivity and labor adjusted for labor productivity and the growth of the participation rate. It is the rate of growth growth (\( \hat{Y}^* \)) such that the rate of unemployment remains constant. \( \text{NAIRU} \) (the nonaccelerating inflation rate of unemployment) is the rate of unemployment (\( U^* \)) at which prices do not accelerate. NAIRU can also be defined as the nonaccelerating inflation rate of resource utilization and is the rate of resource utilization (e.g., as measured by \( \hat{Y}^* \)) such that prices do not accelerate.

b) Okun’s Law: \[
\Delta U = -0.43\left(\hat{Y} - \frac{1.16}{0.43}\right) = -0.43(\hat{Y} - 2.70)
\]

Phillips Curve: \[
\Delta \hat{p} = -0.53\left(U - \frac{4.58}{-0.53}\right) = -0.53(U - 8.64)
\]

\[
\hat{Y}^* = 2.70 \quad U^* = 8.64
\]

c) \[\Delta U = -0.43(4.4 - 2.70) = -0.73\]

Therefore, \( U_{51} = U_{50} + \Delta U = 6.8 - 0.73 = 6.07 \)

d) \[\Delta \hat{p} = -0.53(6.8 - 8.64) = 0.97\]
\[\hat{P}_{51} = \hat{P}_{50} + \Delta \hat{p} = 2 + 0.97 = 2.97\]

e) Since theoretically \( \hat{Y}^* = \hat{\theta} + n + \hat{P} \hat{\theta} \) and \( \hat{\theta} \) has risen by 1%, \( \hat{Y}^* \) has also risen. Now \( \hat{Y} < \hat{Y}^* \rightarrow \Delta U > 0 \) therefore \( U \uparrow \). Higher \( U \rightarrow \uparrow U - U^* \rightarrow \) lower acceleration and perhaps deceleration of inflation.

Authorities would want to increase \( \hat{Y} \) by 1% point to keep the economy on a balanced growth path. In the long run, an increase in \( \theta \) is as good as an increase in real wages (as \( w/p = a \theta \) for the Cobb-Douglas production function). But in the short run, an increase in \( \theta \) leads to an increase in \( U \) unless the growth rate of output increases enough—a possible bad situation if authorities do not meet the challenge.
5. a) See curve marked “section a)” in diagram below.

![Yield Curves Diagram](image)

b) 

2009: \( r_{1,09} = 0.27 \)

2010: \( r_{2,09} = 0.67 = \frac{0.27 + r_{1,10}^c}{2} \Rightarrow r_{1,10}^c = 2(0.27) - 0.67 = 1.07 \)

2011: \( r_{3,09} = 1.12 = \frac{0.27 + 1.07 + r_{1,11}^c}{3} \Rightarrow r_{1,11}^c = 3(1.12) - 0.27 - 1.07 = 2.02 \)

2012: \( r_{4,09} = 1.57 = \frac{0.27 + 1.07 + 2.02 + r_{1,12}^c}{4} \Rightarrow r_{1,12}^c = 4(1.57) - 0.27 - 1.07 - 1.12 = 2.92. \)

c) 

\( r_{1,09} = 0.27 + 1 = 1.27 \)

\( r_{1,10}^c = 1.07 + 2 = 3.07 \)

\( r_{1,11}^c = 2.02 + 3 = 5.02 \)

\( r_{1,12}^c = 2.92 + 4 = 6.92 \)

So,
\[
\begin{align*}
    r_{1,09} &= 1.27 \\
    r_{2,09} &= \frac{1.07 + 3.07}{2} = 2.07 \\
    r_{3,09} &= \frac{1.07 + 3.07 + 5.02}{3} = 3.05 \\
    r_{4,09} &= \frac{1.07 + 3.07 + 5.02 + 6.92}{4} = 4.02
\end{align*}
\]

For graph, see curve marked “section c) in diagram above.

6. a) 6. **Extra Credit** (low value; do this question only if finished with the others).
   a) A second stimulus ($50 billion).
   b) Unemployment rate fell by 0.2 to 10%; employment also fell a small amount by 11,000?
   c) Ben Bernanke.