Answer Key for Week 10 Homework

Problem 12.1

Start with the inflow-outflow identity:

(1) \[ I + G + EX \equiv S + (T - TR) + IM \]

Subtract \( IM \) (imports) from both sides to get net exports \( (NX) \) on the left and treat transfer payments \( (TR) \) as negative taxes included in \( T \):

(2) \[ I + G + NX \equiv S + T \]

Substitute in the saving function \( S = -c_0 + (1 - c)YD \) and the tax function \( T = t_0 + tY \) and use the definition of disposable income \( (YD = Y - T) \) to get

(3) \[ I + G + NX = -c_0 + (1 - c)(Y - t_0 - tY) + t_0 + tY = (-c_0 + c_0) + [1 - c(1 - t)]Y \]

Define autonomous expenditure as \( A = I + G + NX + c_0 - ct_0 \), then

(4) \[ A = [1 - c(1 - t)]Y \]

Solving for \( Y \)

(5) \[ Y = \frac{1}{1 - c(1 - t)}A \],

which is identical to equation (2.15) in the textbook.

Taking first differences of each side yields:

(6) \[ \Delta Y = \frac{1}{1 - c(1 - t)} \Delta A \],

which is identical to equation (2.16). Defining the multiplier as \( \mu = \frac{\Delta Y}{\Delta A} \) and applying to

(6) yields

(7) \[ \mu = \frac{\Delta Y}{\Delta A} = \frac{1}{1 - c(1 - t)} \].
which is identical to equation (2.17).

These derivations show that multipliers do not depend on the intercept terms at all. Multipliers depend on marginal and not average behavior – it is the expenditure or taxation out of the last dollar earned (i.e., the marginal propensity to consume and the marginal tax rate) and not expenditure or taxation out of all income (i.e., the average propensity to consume that the average tax rate) that matter.

**Problem 12.6**

(a) Inflow-outflow identity: \( I + G + EX \equiv S + (T - TR) + IM \rightarrow I + G \equiv S + T \)

\[ \rightarrow 300 + 800 = -100 + 0.1(Y - 800) + 800 \]

\[ \rightarrow Y = \frac{300 + 800 + 100 + 0.1(800) - 800}{0.1} = 4,800 \]

\( t = T/Y = 800/4,800 = 1/6 = 16.67 \) percent

(b) \( \mu_A = \frac{1}{1 - c(1 - t)} = \frac{1}{1.9(1 - 1/6)} = 4 \); therefore \( Y \) increases by

\[ \Delta Y = \mu_A \Delta G = 4 \times 100 = 400 \]

(c) The tax multiplier when \( t \) is allowed to vary to keep \( T \) at a targeted level is

\[ \mu_T = \frac{-c}{1 - c} = \frac{-0.9}{1 - 0.9} = -9 \]; therefore \( \Delta Y = \mu_T \Delta T = -9 \times -100 = 900 \)

The new level of \( T \) is 700; the new level of \( Y \) is 5,700; so \( t = T/Y = 700/5,700 = 12.28 \) percent

(d) The increase in government expenditure in (b) sets up a multiplier process. However, at a fixed tax rate \( (t) \) part of the incomes generated are leaked away as increased taxes. This sets up, in effect, a secondary, offsetting multiplier that reins-in the initial stimulus somewhat. In constrast, in (c), the cut in taxes creates a multiplier process as well; but because the tax rate is adjusted downward in order to keep taxes at a target level, no secondary negative multipliers are created. Thus, the effect in (c) is bigger than in (b).

(e) If the budget is always balanced, then the increase \( \Delta T = \Delta G \). In effect, taxes are targeted at a fixed level, allowing \( t \) to vary as necessary. The tax multiplier is given in (c) and the government expenditure multiplier when taxes are targeted is:

\[ \mu_G = \frac{1}{1 - c} = \frac{1}{1 - 0.9} = 10. \] The increased government expenditure sets up an upward stimulus and the matching increase in taxes sets up a downward stimulus. The net effect is the balanced-budget multiplier: \( \mu_{BB} = \frac{1}{1 - c} + \frac{-c}{1 - c} + \frac{1 - c}{1 - c} = 1 \). Therefore, \( \Delta Y = \mu_{BB} \Delta G = 1 \times 100 = 100 \). The new level of \( Y \) is, then, 4,900 and the new level of \( T \) is 900. Thus, \( t = T/Y = 900/4,900 = 18.37 \) percent.
Problem 12.8

(a) Start with the Production-Expenditure Identity: \( Y = C + I + G + NX \) (the Inflow-Outflow Identity as in Problems 12.1 and 12.6 would have done just as well). Then,

\[
Y = 100 + 0.9(Y + 200 - 0.14285Y) + 300 - 20(6) + 400 + 100 \rightarrow \\
[1 - 0.9 + 0.9(0.14285)]Y = 0.228565Y = 960. \text{ Solving } Y = \frac{960}{0.228565} = 4,200. \text{ And the budget is balanced, since } G + TR = 400 + 200 = 0.14285 \times 4200 = tY = T. \\
\]

(b) Proceed as in (a) except set \( TR = 300. \) Therefore,

\[
Y = 100 + 0.9(Y + 300 - 0.14285Y) + 300 - 20(6) + 400 + 100 \rightarrow \\
[1 - 0.9 + 0.9(0.14285)]Y = 0.228565Y = 1060. \text{ Solving } Y = \frac{1050}{0.228565} = 4,594. \text{ Budget deficit is } \Delta Y = G + TR - T = 400 + 300 - tY = 700 - 0.14285 \times 4,638 = 44. \\
\]

(c) To keep the budget balanced \( \Delta T = \Delta G = 100. \) From Problem 12.6 (e) we saw that the balanced-budget multiplier \( \mu_{BB} = 1; \) therefore \( \Delta Y = \mu_{BB} \Delta G = 1 \times -100 = -100. \text{ Changes are relative to the solutions in (a); so to achieve the necessary tax cut: } \\
t = T/Y = 500/4100 = 12.19 \text{ percent}.
\]

Problem 12.9

\( Y = C + I + G + NX \)

(a) \( \rightarrow Y = 100 + 0.9(Y + 200 - 0.14285Y) + 300 - 20(rr) + 400 + 100 \)

Consolidating terms \( \rightarrow [1 - 0.9 + 0.9(0.14285)]Y = 0.228565Y = 1080 - 20(rr) \)

\[ rr = \frac{1080 - 0.228565Y}{20} = 54 - 0.01143Y \]

Thus IS curve is \( rr = 54 - 0.01143Y \)

![The IS Curve](image)

(b) Set \( rr = 6 \) and solve for \( Y: \]

\[
Y = \frac{54 - 6}{0.01143} = 4,200. \text{ Budget balance follows immediately if } Y \text{ is the same as in (a).} 
\]

3
(c) Set $rr = 5$ and solve for $Y$: $Y = \frac{54 - 5}{0.01143} = 4,287$. Budget deficit is $G + TR - T = 400 + 200 - 0.14285(4287) = -12$ – a negative deficit is a surplus!

(d) Change the tax rate in the derivation in (a). Thus, $Y = 100 + 0.9(Y + 200 - 0.12Y) + 300 - 20(rr) + 400 + 100$

Consolidating terms $[1 - 0.9 + 0.9(0.12)]Y = 0.208Y = 1080 - 20(rr)$

$\rightarrow rr = \frac{1080 - 0.208Y}{20} = 54 - 0.0104Y$

Thus IS curve is $rr = 54 - 0.0104Y$

$Y = \frac{54 - 6}{0.0104} = 4,615$

Budget deficit is $G + TR - T = 400 + 200 - 0.12(4615) = 46$ – moves from balance to deficit.

Effect of a Tax Cut on the IS Curve
(a) In general, increases in items that are inflows into the domestic private sector are stimulatory, while increases in outflows are contractionary. Countercyclical inflows thus act as automatic stabilizers, while procyclical outflows act as automatic stabilizers. Consider inflows first.

- **Transfer payments** have risen on trend across the post-World War II period. They do tend to rise somewhat faster in recessions and to slow their growth in expansions. Again, as an inflow, this is stimulatory.

- **Inventory investment** is different. It is highly variable, though very quite small. And it tends to fall in recessions. As an inflow, this is contractionary. However, inventories mainly act as a buffer smoothing the difference between changes in aggregate demand and supply. Inventories tend to rise somewhat before recessions, as firms find sales lagging production. They are unlikely to increase production until inventories fall. Thus, while as an inflow, inventory investment is not in itself stimulatory, it is probably necessary for inventories to shrink before firms feel justified in raising production and ending a recession.

Turning to outflows:

- **Taxes** fall fairly sharply as a percentage of potential GDP in every recession, which is stimulating; and in every expansion they rise, though less sharply and with some variation. As an outflow, they are thus countercyclical in their stimulatory effect.
• *Imports* too rise on trend, but tend to fall more sharply in recessions. Once more a countercyclical stimulus. So, for these three items, their behavior is clearly that of automatic stabilizers.

Summarizing the most important inflows and outflows – transfer payments, taxes, and imports clearly act as automatic stabilizers; inventory investment does not. Though inventory investment has the pattern of automatic destabilizer, it is relatively small and its complex behavior reflects adjustments to disequilibria between aggregate supply and demand that may, in fact, not be destabilizing.

(b) One way to assess the effectiveness of the automatic stabilizers is to consider what would be their multiplied effect on GDP. Consider the largest changes in the data for the three variables that are actually stabilizing during recessions – not counting the most recent recession, for which we do not yet have an NBER trough and, therefore, for which we do not have complete data. Between 1974:3 and 1975:2, taxes fall from 26.8 percent of potential GDP to 21.7 – a fall of 5.1 percentage points. Using a multiplier of 2 (the Obama administration estimates the multiplier at about 1.5), this cut should raise GDP as a share of potential by over 10 percentage points compared to what it would have been without the fall in taxes. This is probably a gross overestimate, both because the multiplier is probably too high and because most recessions have not shown this large a fall in taxes. What is more, in practice multipliers take some time to work themselves through the economy; and, before such a stimulus has its full effect, taxes have typically changed somewhat in the other direction, setting up a downward multiplier that counteracts some of the stimulus. Still, it does point out that automatic stabilizers may act importantly to mitigate the business cycle. The maximum change in transfer payments associated with a recession occurs starting just at the trough of the recession in 1949:4 with a 5 point rise in a single quarter. The stimulus would, therefore, be on the same order as the tax stimulus, but it is reversed more quickly. The maximum change for imports associated with a recession (actually the two recessions in the early 1980s) is the fall from 1980:1 to 1983:1 of 2.8 points, which would have about half the effect of the tax or transfer changes, but still not insignificant. Since these stabilizing actions are the largest in the data, they are likely to indicate the upper bounds on the stabilizing effects that might be expected.

**Problem 12.12**

Generally, an increase in an inflow will shift the IS curve to the right, while an increase in an outflow will shift the IS curve left. Changes in marginal rates may pivot the curve as well. Thus:

(a) A decrease in exports (an inflow) shifts the curve left.
(b) A decrease in imports (an outflow) shifts the curve right.
(c) An increase in the marginal propensity to import acts exactly like an increase in tax rates, since both are outflows. It shifts the curve left, but – as we saw with taxes in the text – steepens it as well.
(d) An increase in investment risk, raises the opportunity cost of investment, thereby decreasing it; and, since investment is an inflow, shifting the curve left.

(e) An increase in the savings rate shifts the curve left (savings are an outflow) and steepens the curve, acting exactly like an increase in tax or import rates.

(f) A decrease in the expected returns to investment, raises the opportunity cost of investment, thereby decreasing it; and, since investment is an inflow, shifting the curve left.

(g) An increase in payments of interest on the government debt shifts the curve right, since interest is a form of transfer payment, which is an inflow.