THE ROLE OF COMMITMENT IN DYNAMIC CONTRACTS: EVIDENCE FROM LIFE INSURANCE*

IGAL HENDEL AND ALESSANDRO LIZZERI

We use data on life insurance contracts to study the properties of long-term contracts in a world where buyers cannot commit to a contract. The data are especially suited to test a theory of dynamic contracting since they include information on the entire profile of future premiums. All the patterns in the data fit the theoretical predictions of a model with symmetric learning and one-sided commitment à la Harris and Holmstrom. The lack of commitment by consumers shapes contracts in the way predicted by the theory: all contracts involve front-loading (prepayment) of premiums. Front-loading generates a partial lock-in of consumers; more front-loading is associated with lower lapse. Moreover, contracts that are more front-loaded have a lower present value of premiums over the period of coverage. This is consistent with the idea that front-loading enhances consumer commitment and that more front-loaded contracts retain better risk pools.

I. INTRODUCTION

The effect of agents’ inability to commit to long-term plans has been shown to have far-reaching implications for economic arrangements. The goal of this paper is to examine empirically the role of imperfect commitment in shaping contractual relations using data from the life insurance market.

Lack of consumer commitment can generate inefficiencies in insurance markets because short-term contracts do not offer insurance against reclassification risk: bad news about the health status of a consumer resulting in increased premiums. Lack of long-term insurance is considered an important market failure in health and life insurance markets, and it has underscored much of the policy debate concerning health reform (see Diamond [1992]).

Several factors make life insurance suitable for studying commitment to long-term contracts. Learning (about health) is a quantitatively important phenomenon. Life insurance contracts are simple and explicit, and issues such as asymmetric information and misreporting of accidents are not important in the case of life insurance (see Section V). In contrast, other contractual

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relations such as labor contracts are plagued by unobservables and by the presence of implicit agreements. Finally, very rich data on contracts are available.\footnote{Previous empirical work on dynamic contracts includes Chiappori, Salanie, and Valentin [1999] who test a model of wage dynamics using data from one large French firm. They find support for theories of learning with downward rigidity of wages. Murphy [1986] investigates a longitudinal sample of CEOs and compares learning with incentives; his evidence is mixed. Dionne and Doherty [1994] present a dynamic model of adverse selection and with evidence from auto insurance. Crocker and Moran [1998] studies employee lock-in as a way to increase commitment to long-term health insurance.}

We first present a theoretical framework to understand how we should expect lack of commitment to be reflected in observed contracts. The model adapts to the life insurance market the Harris-Holmstrom [1982] model of symmetric learning that was first developed in a labor market context. The main predictions of the model are then tested using data on the United States life insurance industry.

We assume that the initial health status of the insured is known to all, and that new information about her health type is revealed over time symmetrically to all market participants. We also assume that there is one-sided commitment; i.e., insurance companies can commit to long-term contracts but buyers cannot.

Because of learning, short-term (spot) contracts involve reclassification risk: future premiums will reflect the future health status of consumers. Long-term contracts would solve the problem if consumers could commit: insurance companies would offer future premiums reflecting the average future health of the pool. Lack of commitment makes such a contract infeasible: healthy consumers would drop out to obtain cheaper coverage from competing companies. Hence, to obtain insurance against reclassification risk, consumers must prepay some of the premiums. The prepayment, or front-loading, locks the consumer into the contract. To be completely successful, the front-loaded amount must guarantee that no consumers drop coverage in the future: the future premium must be no higher than the fair premium for the healthiest type. Such a contract would implement the same allocation that would be achieved under consumer commitment: it provides full insurance against reclassification risk.

All contracts we observe in the life insurance market in the United States (see subsection IV.A) are front-loaded, and some of the contracts come close to achieving full insurance. However, most of these contracts leave consumers subject to some reclas-
sification risk. The model provides an explanation for the variety of contracts, linking them to consumers' heterogeneity in willingness to front-load. More importantly, given the variety of observed contracts, the model will generate testable predictions about the cross section of contracts.

Front-loading can be considered as the ex ante price that consumers pay for future caps on premiums, a result of their inability to commit. Good risks can still reenter the market in the future while bad risks are protected against steeper premium increases if they remain in the front-loaded policy pool. The key comparative statics generated by the model are that contracts that are more front-loaded lock in consumers to a greater extent. Thus, more front-loaded contracts retain a better risk pool and have a lower present value of premiums.

As predicted by the model, life insurance contracts are offered in several varieties, involving different time profiles of premiums which differ in how steeply premiums increase over time. Different premium profiles represent different degrees of front-loading (commitment). We use the contract diversity to test the implications of the model. In accordance with the contract theoretic predictions, we find that virtually every contract is front-loaded and that there is a negative correlation between front-loading and the present value premiums. Remarkably, most of the (large) dispersion in the present value of premiums can be accounted for by the extent of front-loading. We confirm the role of front-loading as a commitment device by looking at the relation between lapsation (a term commonly used in the actuarial literature that means voluntary termination of coverage) and front-loading. As predicted, more front-loaded contracts have lower lapsation. Thus, all the patterns in the data suggest that contracts are designed in a way that fits a model of symmetric learning with one-sided commitment. Alternative commitment assumptions are rejected by the data. Standard asymmetric information models deliver predictions that are inconsistent with the data.

Finally, understanding and quantifying the inefficiency from the lack of bilateral commitment in life insurance is of interest for the policy debate on health insurance. Lack of renewability and the absence of insurance of reclassification risk are some of the concerns raised about health insurance (e.g., in Clinton's reform proposal, new state regulation during the 1990s). However, the health care market suffers from a variety of other problems (see Cutler [1993]). Isolating the problems due to the lack of bilateral
commitment should be useful for understanding the source of inefficiency in health insurance (see Concluding Remarks).

II. THE CONTRACTS AND THE DATA

II. A. The Contracts

The life insurance market is segmented in Term and Cash Value contracts. We focus on the market for Term insurance for two reasons. (1) term is simpler since it involves pure insurance: a fixed sum is paid upon death of the insured if death should occur within the period of coverage. In contrast, Cash Value offers a combination of insurance and savings. (2) we do not have data on cash values. Term contracts represented over 37 percent of all ordinary life insurance in 1997 according to LIMRA [1997].

All contracts are unilateral (see McGill [1967]): the insurance companies must respect the terms of the contract for the duration, but the buyer can look for better deals at any time. The contract becomes void once the buyer stops paying premiums, and there are no cash flows upon termination. These features fit a model of one-sided commitment.

The main distinction among term contracts is the premium profile. Annual Renewable Term (ART) contracts charge premiums that increase yearly. Level Term contracts (LTn) offer premiums that increase only every $n$ years, ranging from 2 to 30 years. Term contracts are renewable, without medical examination, up to a prespecified age (typically 70 or older). Future terms for renewal are specified at the moment of underwriting.

Column 2 of Table I illustrates an aggregate ART. This contract has premiums that vary exclusively by age. Column 3 illustrates a ten-year Level Term contract (LT10). We discuss the

2. We focus on the individual market. In employment-related insurance commitment is enhanced by job lock-in.

3. Severance payment would enhance buyers’ ability to commit to stay in the contract. However, for legal reasons they are not used in any life insurance contract. Severance payments would be considered a stipulated “penalty,” and would not be enforced by a Court. “A Common Law tradition prevents Courts from enforcing terms stipulating damages that exceed the actual harm caused by breach” [Cooter and Ulen 1999]. Insurance companies do not suffer any ex post damage from the loss of a consumer.

4. Premiums are guaranteed for a prespecified period of time, ranging from a year to twenty years. Since no medical examination is needed to renew, price changes are not insured specific. Furthermore, insurers rarely change the announced premiums: Consumer Reports (July 1993) found that only one increased term premiums between 1989 and 1993. Hence, preannounced premiums are the future premiums.
difference between these contracts in detail later. For now, note that contracts differ substantially in the time profile of premiums; the premiums for the ART start below those for the LT10 but increase much more rapidly.

The right side of Table I illustrates a Select and Ultimate (S&U) ART. These contracts offer state contingent prices: low premiums are contingent on the insured showing that he is still in good health. Thus, premiums vary by issue age and duration since underwriting (last date that good health was proved). Rows show the age at latest underwriting; columns the years since underwriting (captured by Policy Year). For instance, at age 41 the insured would pay $475 (first row, second column of matrix) for renewing his one-year-old policy if he does not requalify (pass the medical). If he had requalified, he would have paid $385 (second row, first column). The incentive to requalify increases with age; at 59 the insured who does not requalify pays $1750 instead of $1340. The big difference in premiums in the alternative scenarios of a S&U ART shows that learning about the health status of an individual is an important phenomenon, and that consumers may be left to suffer significant reclassification risk. A 59 year old could be charged between $1340 and $6375.

The market shares of these contracts are ARTs (aggregate

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TABLE I

TYPES OF TERM CONTRACTS

<table>
<thead>
<tr>
<th>Age</th>
<th>ART</th>
<th>LT10</th>
<th>S&amp;U ART</th>
<th>Policy year</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>459</td>
<td>574</td>
<td>370</td>
<td>475</td>
</tr>
<tr>
<td>41</td>
<td>499</td>
<td>574</td>
<td>385</td>
<td>490</td>
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<tr>
<td>42</td>
<td>539</td>
<td>574</td>
<td>400</td>
<td>530</td>
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<tr>
<td>49</td>
<td>909</td>
<td>574</td>
<td>630</td>
<td>890</td>
</tr>
<tr>
<td>50</td>
<td>974</td>
<td>1064</td>
<td>690</td>
<td>945</td>
</tr>
<tr>
<td>51</td>
<td>1044</td>
<td>1064</td>
<td>735</td>
<td>1050</td>
</tr>
<tr>
<td>58</td>
<td>2009</td>
<td>1064</td>
<td>1245</td>
<td>1750</td>
</tr>
<tr>
<td>59</td>
<td>2289</td>
<td>1064</td>
<td>1340</td>
<td>1785</td>
</tr>
</tbody>
</table>

These are contracts offered in 7/1997 to a preferred nonsmoker, male, by Northwestern Mutual (ART and LT10) and Jackson National (S&U ART) for $500,000 of coverage.

ART = annual renewable term policy.
LT10 = term policy with level premiums for ten years.
S&U ART = annual contract that allows for reclassification, by showing good health.
and S&U 22.4 percent, LT5 34.8 percent, LT10 5.23 percent, and LT20 11.0 percent (LIMRA [1997]).

This initial look at contracts highlights some patterns that we will investigate more systematically in what follows. First, there are both contingent (e.g., S&U ART) and noncontingent (e.g., aggregate ART) contracts. Second, the time profile of premiums differs across contracts: some have a flatter profile (are more front-loaded). Furthermore, contracts that have a low initial premium, have high premiums later on.

II. B. The Data

The source of information on contracts is the Compulife Quotation System (July 1997), an information system compiled for the use of insurance agents which contains quotes by the main United States life insurers (240 firms). Premiums are reported by consumer characteristics such as age, smoking status, gender, etc. It is an accurate source of pricing data since agents are forbidden to offer discounts by the antirebating laws. For the purpose of testing the predictions of the model, the unique feature of Compulife, is that it reports the whole profile of future premiums faced by a buyer at the moment of underwriting. Thus, we observe the entire contract time profile, not just the starting premium. The virtue of these data is that they are generated by using the pricing rule of dozens of insurance companies. We punch in the demographics and face value into the Compulife pricing software, and we obtain the premiums for all future dates and contingencies. This is ideal for testing the model since, when we compare contracts, we literally fix all demographic characteristics and only vary the contract profile.

In our sample we included insurers that satisfy a dual criterion of size and solvency. They must be in categories IX and above of A.M. Best for size (in terms of sales of term policies) and rating A or better in terms of solvency (according to A.M. Best). This guarantees product homogeneity and avoids nonrepresentative policies. Fifty-five insurers satisfy the criteria.\(^5\)

Table II presents descriptive statistics of the contracts in our sample. The table presents the mean premium and standard deviation by type of contract. The third and fourth columns con-

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5. Twenty-one of the firms in our sample held 46.8 percent of the 2937 billion dollars of in-force face amount of term insurance in 1996 [Best's Review July 1996]. We do not have sales information for the other companies in our sample, but clearly a large share of the market is covered.
cern the first-year premium for a 40 year old male nonsmoker who just passed the medical screening. The fifth and sixth columns report the present value of premiums for twenty years of coverage. When the contract is state contingent, because premiums depend on whether the consumer does or does not requalify, we need to specify what contingency we look at. The present value is computed under the assumption that the buyer never requalifies. As will be shown later, this is a useful benchmark because it allows a theoretically meaningful comparison of contracts. We use an 8 percent interest rate in computing present values.\footnote{We chose a high interest rate for two reasons: to account for both time preference and for the probability of surviving to a specific date, and because results are reinforced if the interest rate is lower. We experimented with other interest rates and our qualitative results remain unchanged.}

Table II shows substantial variability in the present value of premiums, in particular, across contract types. This variability will be used to test the model.\footnote{Price dispersion may seem inconsistent with competition. However, Winter [1981] found that, after controlling for contract characteristics, only a small fraction of the dispersion remains unexplained. He concluded that observed premiums are consistent with the hypothesis of competition. We will control for these policy characteristics.}

\section{Theory}

Before presenting the formal analysis, we informally discuss the main ideas of the model. The key ingredients of the model are
symmetric learning (information about the health type of the insured is revealed over time), one-sided commitment (insurers commit, buyers do not), and buyer heterogeneity in the cost of front-loading.

Consider a two-period model where information about risk types (health states) is revealed at the beginning of the second period. A contract that guarantees full insurance against reclassification risk has second-period premiums that are independent of the health state. If buyers cannot commit to stay with a contract, this constant premium should be smaller than the actuarially fair premium for the best health state. Otherwise, the healthier buyers would find better rates on the spot market and drop coverage. However, this contract (that retains even the healthiest buyers) suffers second-period losses amounting to the difference between the second-period premium (calculated to retain the healthiest buyer) and the cost of covering the entire pool. Therefore, this contract must front-load premiums: a surcharge in the first period to cover future expected losses.

It is clear then that front-loading is beneficial: paying premiums up front reduces reclassification risk. One of our empirical findings is that almost all contracts display front-loading. However, many contracts leave buyers to suffer considerable reclassification risk, suggesting that many buyers are not willing to front-load enough to fully insure. There are several reasons why consumers may find front-loading costly. For instance, with capital market imperfections, the tighter the credit constraints the more costly it is to front-load premiums.

If it is costly for consumers to front-load, equilibrium contracts offer imperfect insurance against reclassification risk: in return for the front-loading, policies offer a cap on the premium required in the second period. To capture the variety of degrees of front-loading offered in the life insurance market, we assume that consumers are heterogeneous in the cost of front-loading. Consumers with more resources in the first period (those that face looser credit constraints) will front-load more and face a lower cap on the premium in the second period, which translates into lower reclassification risk. Front-loading and the comparative static with respect to the premium profiles will provide the main testable implications of the model.
III. A. Model

Consider a market with insurance buyers and sellers. Buyers wish to insure a stream of income for their dependents. When considering a future period, a buyer has two distinct sources of utility. If he will be alive, and he consumes $c \geq 0$, his future utility is given by $u(c)$. However, if he will be dead, his future utility is given by the consumption of his dependents. In this case, if his dependents consume $c \geq 0$, his future utility is given by $v(c)$. The functions $u$ and $v$ are both assumed to be strictly concave and twice differentiable.

There are two periods. In period 1 all agents have identical death probabilities $p$. We can think of $p$ as representing a specific health status in period 1. The model can be readily extended to allow several risk categories. In period 1 there are 3 stages: in stage 1 insurance companies offer contracts. At stage 2 buyers choose a contract. At stage 3 uncertainty about death is resolved, and consumption takes place. In period 2 there are four stages. In stage 0 uncertainty about the health status of each buyer is realized. A consumer with health status $i$ dies with probability $p_i$ in period 2. We order the health status so that $p_1 < p_2 \ldots < p_N$. We assume that $p_1 \geq p$ (health worsens over time). The probability of being in state $i$ is denoted by $\pi_i$. Uncertainty (learning) about the second-period risk category creates reclassification risk, since it potentially leaves the buyer facing uncertain premiums in the second period. We assume that information is symmetric: at the end of stage 0 all insurance companies observe the health status of all buyers.\(^8\) Stages 1 through 3 are the same as in the first period.

We assume that there is perfect competition between insurance companies.\(^9\) We also assume one-sided commitment: insurance companies can commit to future premiums, whereas consumers freely choose between staying with their period 1 contract and switching to one of the spot contracts offered in period 2. Therefore, the set of feasible first-period contracts is the set of unilateral contracts, i.e., those contracts that terminate the mo-

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8. We discuss this assumption in detail in Section V.
9. The market for term life insurance is quite competitive: there are 400 competing companies [Life Insurance Factbook], and term insurance is increasingly viewed as a commodity (Record 1997). There is a regulatory presence which mainly pertains to financial solvency, not pricing. “Life insurance rates for individual insurance are regulated only in a most indirect sense. It is felt that competition is an adequate regulator over any tendencies toward rate excessive-ness” [Black and Skipper 1994].
ment the buyer stops paying the premiums; there are no cash flows upon termination.\textsuperscript{10}

A first-period contract consists of first-period premium $Q_1$ and face amount $F_1$, and a vector of premiums and face amounts $(Q_2^1, F_2^1), \ldots, (Q_N^N, F_N^N)$ indexed by the second-period health status of an individual. Contract terms can depend on the information revealed in the second period. A second-period contract consists of a premium and face amount $(Q_2^i, F_2^i)$ indexed by the second-period health status: in the second period, firms can offer different contracts to buyers in different risk categories. A first-period contract is a long-term contract to which the company unilaterally commits, whereas a second-period contract is a short-term (spot) contract.

Lack of consumer commitment per se need not preclude the possibility of achieving full insurance. In a world with no other friction, all consumers would front-load sufficiently to guarantee that they have no incentive to drop out of the contract in the future. An implication of such a solution is that all consumers would be fully insured against reclassification risk. Furthermore, there would be no contract variety: all consumers would choose the same contract.

Explaining the existing variety of contracts, and obtaining predictions about them, requires understanding why most consumers are reluctant to fully front-load. For simplicity, and to be closer to the formulation of Harris and Holmstrom, we assume that capital markets are completely absent.\textsuperscript{11} The results would be unchanged if we assumed a borrowing rate that is higher than the lending rate. This kind of capital market imperfection seems to be quite reasonable given the interest rates charged on unsecured debt (e.g., median interest rate charged on credit card balances is 18 percent). Also, note that to explain the variety of behavior, we do not need all buyers to face such imperfections. We capture consumer heterogeneity by assuming differences in the income process.\textsuperscript{12} If alive, the consumer receives an income of $y - g$ in the first period and $y + g$ in the second period, with $g \geq 0$.

\textsuperscript{10} These are exactly the kind of contracts offered in the U. S. life insurance market; see McGill [1967] or Daily [1989].

\textsuperscript{11} Note that this assumption does not mean that consumers are constrained in their front-loading. Consumers have enough resources to sufficiently front-load their premiums, but, absent capital markets, front-loading requires giving up current consumption, which introduces the trade-off between consumption smoothing and insurance against reclassification risk.

\textsuperscript{12} Others sources of heterogeneity in front-loading are discussed in the working paper [Hendel and Lizzieri 2002]. These include heterogeneous and uncertain needs for life insurance and heterogeneous beliefs probability of requalification.
Thus, all consumers have the same permanent income, but different consumers have different income growth, as represented by the parameter $g$. If the buyer is dead, then his dependents receive no income except for the face amount of the insurance. Because of the absence of credit markets, individuals with higher $g$ are more tightly constrained. Heterogeneity in $g$ is what drives the different choice of contracts in our model.

III.B. Solving the Model

The approach we follow to obtain the equilibrium contracts is the following. In a competitive equilibrium, allocations must maximize consumers’ expected utility subject to a zero profit constraint and a set of no-lapse constraints that capture consumers’ inability to commit. Solving this constrained maximization problem delivers the set of premiums and face amounts that must be available to a consumer in a competitive market. However, there are several ways in which this set of premiums and face amounts can be made available to consumers. The constrained maximization problem below is constructed via fully contingent contracts that involve no lapse. We later describe how to obtain the same allocation with noncontingent contracts that are quite common in the life insurance market.

In a competitive equilibrium, premiums and face amounts that are fully contingent on the health state $Q_1, F_1$, $(Q_2^1, F_2^1)$, \ldots, $(Q_N^N, F_N^N)$ must maximize consumers’ expected utility:

\begin{align*}
(1) \quad & p_i v(F_1) + (1 - p)u(y_1 - g - Q_1) \\
& + (1 - p) \sum_{i=1}^N \pi_i [p_i v(F_2^i) + (1 - p_i)u(y_2 + g - Q_2^i)],
\end{align*}

subject to the zero profit condition,

\begin{align*}
(2) \quad & (1 - p)Q_1 - pF_1 + (1 - p) \sum_{i=1}^N \pi_i [(1 - p_i)Q_2^i - p_iF_2^i] = 0
\end{align*}

and the no lapse constraints: for $i = 1, \ldots, N$, and for all $Q_2^i, F_2^i$ such that $(1 - p_i)Q_2^i - p_iF_2^i > 0$,

\begin{align*}
(3) \quad & p_i v(F_2^i) + (1 - p_i)u(y_2 + g - Q_2^i) \\
& \geq p_i v(F_2^i) + (1 - p_i)u(y_2 + g - Q_2^i).
\end{align*}
The constraint in equation (2) requires that insurance companies break even on average. This must hold under competition. The no-lapse constraints defined by equation (3) are the additional constraints imposed by lack of consumer commitment. They require that, at every state in the second period, the buyer prefers staying in the long-term contract rather than switching to a competing insurance company. In other words, an equilibrium contract must be such that there does not exist another contract that is profitable and that offers the buyer higher utility in any state of period 2.

We say that a consumer gets full event insurance in state $i$ of date 2 (the definition for date 1 is analogous) if

$$v'(F^i_2) = u'(y_2 + g - Q_2^i).$$

We call $Q_2^i(FI)$ the fair premium for full insurance in state $i$, namely, the premium that guarantees zero profits.

The following proposition, which is proved in the Appendix, provides a characterization of equilibrium contracts.

**Proposition 1.** In the equilibrium set of contracts:

(i) All consumers obtain full event insurance in period 1 and in all states of period 2.

(ii) For every $g$ there is an $s$ such that $Q_2^i = Q_2^i(FI)$ for $i = 1, \ldots, s - 1$, and $Q_2^i = Q_2^s$ for $i = s, \ldots, N$, where $Q_2^i < Q_2^i(FI)$ for $i = s + 1, \ldots, N$.

(iii) There is a $\hat{g}$ such that, if $g < \hat{g}$, then there is front-loading.

(iv) More front-loaded contracts provide more insurance against reclassification risk: contracts with higher $Q_1$ involve a lower $s$. Buyers with higher $g$ choose contracts with less front-loading.

Part (i) follows from the fact that, under competition, event insurance is offered at a fair rate—hence, buyers will choose to fully insure event risk.\textsuperscript{13}

Part (ii) says that there is a premium cap: at health state $s$ or

\textsuperscript{13} Parts (ii) and (iii) of Proposition 1 are adaptations of Harris and Holmstrom [1982] to our environment. Part (i) was also shown by De Garidel [1999]. Cochrane [1995] studies a similar problem. He shows that lack of commitment does not preclude full reclassification risk insurance. His result is seemingly at odds with ours, as well as with Harris and Holmstrom [1982]. However, there is no such inconsistency; under his assumptions consumers are eager to front-load sufficiently to obtain full insurance. In our context this means that $s = 1$, which obtains when $p_1$ is much larger than $p$ and $g$ is small. He only looks at a subcase...
worse, the premiums are capped at a common price which is cheaper than the spot rate for each of those states. In the optimal contract, consumers transfer income from the first period to the bad states in the second period. The reason why premiums in the bad states are all the same is the following: given that the no-lapsation constraints are nonbinding in those states, transferring resources across these states is costless, leading to an elimination of all variability in premiums at the optimum. If, in contrast, the consumer is healthier than in state \( s \), the premiums are actuarially fair. These are the states where the no-lapsation constraints are binding. In those states, buyers must be offered spot market rates.

Part (iii) says that contracts are front-loaded so long as income does not increase too rapidly.

Part (iv) says that insurance companies should offer a menu of contracts differing in the trade-off between front-loading and reclassification risk. Less front-loaded contracts appeal to buyers with lower first-period income, i.e., consumers who find front-loading more costly; \( g \) captures the marginal rate of substitution between a dollar today and a dollar in the future. Different contracts are offered to cater to consumers with different willingnesses to front-load in exchange for different degrees of reclassification risk insurance.

It can be shown that for \( g \leq 0 \), the model predicts that premiums are upwardly rigid. This is a direct counterpart of the downwardly rigid wages in Harris and Holmstrom. Thus, the model predicts that the most front-loaded contracts have flat premiums, which is exactly what we observe in the data.

III. C. Contract Equivalence and Empirical Implementation of the Model

Proposition 1 obtains equilibrium allocations via fully contingent contracts that involve no lapsation. However, as we saw in Section II, both contingent and noncontingent contracts are common in the life insurance market. We now show that noncontingent contracts can also be optimal. We then show how we account for lapsation, which is quite common in life insurance,
and we discuss how we compute the present value of premiums for contingent contracts.

To gain an idea of how equivalence is possible, note that a buyer who chooses an LT20 or an aggregate ART (which are noncontingent) is free to drop out of the contract at any date to (for instance) purchase an S&U ART.

More formally, fix an equilibrium contingent contract $C$. A noncontingent contract $NC$ is also an equilibrium contract if, in each state, the consumer for whom the $C$ contract is optimal can obtain the same utility with the $NC$ contract and the insurance companies offering the contracts obtain the same profits with either contract. Clearly, this requires that the consumer drops out of $NC$ in the appropriate states.

From part (ii) of Proposition 1, the terms of a $C$ contract are constant: $Q^s_2 = Q^s_i$ and $F^s_2 = F^s_i$ for $i = s, \ldots, N$. In states $i = 1, \ldots, s - 1$, the premiums and face amounts equal those offered in the spot market: the buyer pays the actuarially fair premium. Fix the terms of the noncontingent contract $NC$ to be the same as for contract $C$ in the first period, and $(Q^s_2, F^s_2)$ in the second period. Thus, the two contracts are equivalent by construction in the first period and in the bad states of the second period. However, in states $i = 1, \ldots, s - 1$ contract $C$ has better terms. But in those states the consumer can obtain the same terms by dropping out of contract $NC$ and purchasing spot contracts. Because, by Proposition 1, in each state $i = 1, \ldots, s - 1$, contract $C$ has the same terms as a spot contract, the consumer obtains the same utility via $NC$ and $C$. Insurers are indifferent between selling the two kinds of contracts as well: in state $s$ through $N$ consumers stay in the contract under both contracts, and the terms are exactly the same. In states 1 to $s - 1$, the two alternative contracts induce different behavior by the consumer, but equal (zero) profits for the firms: in those states premiums in contract $C$ are actuarially fair. Thus, in those states, firms are indifferent between retaining and not retaining the consumers. This shows that the two ways of achieving the equilibrium allocation are equivalent.

This equivalence result will help us in two ways: first, by explaining within our model the existence of noncontingent contracts in the life insurance market. Second, the result helps us compare contingent and noncontingent contracts. All we have to do is transform each contingent contract into its equivalent noncontingent contract, along the lines of the previous discussion.
The importance of having comparable noncontingent contracts comes from the next proposition, which is at the core of our empirical analysis. It presents the main comparative statics across noncontingent contracts. Due to the equivalence result we will be able to apply it to all contracts.

**Proposition 2.** Consider two contracts offered in the first period that are not contingent on the health state in period 2. In equilibrium, the contract with the higher first-period premium has a lower present value of premiums, it is chosen by consumers with lower income growth, and has lower lapsation (retains a healthier pool of consumers).

*Proof.* The contract with the higher first-period premium must have a lower second-period premium since otherwise no consumer would choose it. Thus, in the second period this contract will retain a healthier pool of consumers. This implies that the average cost of this contract (payments to dependents of deceased policy holders) is lower. Under competition this implies that the present value of the premiums must also be lower. The fact that the contract with the higher first-period premium is chosen by consumers with lower income growth follows from Proposition 1.

The importance of Proposition 2 is that it provides a way to test the theory by comparing noncontingent contracts that display different degrees of front-loading. The equivalence result discussed above provides a way to extend the comparison to the entire set of contracts by telling us how to evaluate contingent contracts. Whenever we compute the present value of premiums for a contingent contract, we do so by looking only at the nonrequalifying states: in the terminology of the model we consider the portion of the contract \((Q_1,F_1), (Q_2^s,F_2^s)\), where \(s\) is the state obtained in Proposition 1. The resulting contract is exactly the noncontingent contract that implements the same allocation (as discussed before in Proposition 2). An example: for the S&U ART contract described in Table I, this means computing the present value of the first row of the matrix. Note that, absent the equivalence result, we would not be able to compute the present value of premiums since we do not have data on requalifying probabilities; these probabilities depend on the underwriting stringency (the state \(s\) in terms of our model) for individual contracts. The
model allows us to compare contracts even in the absence of such information.

To be consistent with the theory, in the empirical analysis of contingent contracts, we should consider requalification to be a lapsation, i.e., when a consumer requalifies, we consider him to have started a new contract. This is consistent with industry practice, which treats requalification as a new contract, and it is the theoretically correct way to compare contracts. Of course, in our empirical analysis we make sure that our findings are not an artifact of our definitions: we also test the predictions by looking only across contracts that are directly comparable.

**Summary of predictions.** In the next section we look at the time profile of premiums of the various policies to test the implications of the model. The model predicts the following. 1. Optimal contracts involve front-loading. 2. Contracts with higher front-loading retain higher proportions of insureds in the long run, i.e., have lower lapsation. 3. Since better risk types have higher incentive to lapse any given contract, higher lapsation implies worse pools. 4. A worse pool (lower front-loading) translates into higher present value of premiums for a fixed period of coverage.

**IV. Testing the Implications of the Model**

To test the model, we use the variety of offered contracts. The different types of contracts are characterized by different premium profiles, or slopes (namely, steepness of premium increases over time). The slope of the premiums determines the extent of front-loading. The steeper the premiums, the less front-loaded the contract. We want to establish first the extent of front-loading of contracts offered, and then the relation between front-loading and lapsation, as well as with the quality of the insured pool.

**IV. A. Front-loading**

Table III shows the yearly premiums paid, over twenty years, by a 40 year old purchasing half a million dollars of coverage under three different contracts. The numbers are averages over all contracts of each type in our sample. Column 1 is for aggregate ARTs. Column 3 is for twenty-year level term contracts (LT20). Columns 5 and 7 describe two extreme scenarios for S&U ARTs (recall from Table I that the complete description of this type of contract is a matrix): in column 5 the buyer never satisfied the
criterion for requalifying; in column 7 he was lucky and requalified year after year.

Observe the substantial difference in payment profiles across contracts. By definition, premiums in LT20s are constant in nominal terms. Due to discounting and aging, the level term contract presents a high degree of front-loading. The first payment in the twenty-year policies is 83 percent higher than in the yearly select contracts; this gap represents a lower bound on the magnitude of front-loading.

The initial overpayment creates consumer lock-in or commitment to the contract. When the insured reaches age 50, he cannot be lured by a S&U ART, since the remaining premiums are lower in LT20 contracts than the premiums he would pay in S&U ARTs (even if he were to keep requalifying as a select customer).

We now argue that even ARTs are front-loaded. To check this, in Table III we report the ratio of the yearly premium over the probability of death; the ratio was normalized to equal 100 at

<table>
<thead>
<tr>
<th>Age</th>
<th>S&amp;U ART Aggregate ART</th>
<th>LT20 NoRequal</th>
<th>Requal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AvgPm</td>
<td>Prm/Death</td>
<td>AvgPm</td>
</tr>
<tr>
<td>40</td>
<td>645</td>
<td>100</td>
<td>866</td>
</tr>
<tr>
<td>42</td>
<td>739</td>
<td>60</td>
<td>866</td>
</tr>
<tr>
<td>44</td>
<td>852</td>
<td>50</td>
<td>866</td>
</tr>
<tr>
<td>46</td>
<td>1,000</td>
<td>44</td>
<td>866</td>
</tr>
<tr>
<td>48</td>
<td>1,184</td>
<td>39</td>
<td>866</td>
</tr>
<tr>
<td>50</td>
<td>1,395</td>
<td>37</td>
<td>866</td>
</tr>
<tr>
<td>52</td>
<td>1,611</td>
<td>34</td>
<td>866</td>
</tr>
<tr>
<td>54</td>
<td>1,877</td>
<td>30</td>
<td>866</td>
</tr>
<tr>
<td>56</td>
<td>2,223</td>
<td>27</td>
<td>866</td>
</tr>
<tr>
<td>58</td>
<td>2,746</td>
<td>27</td>
<td>866</td>
</tr>
</tbody>
</table>

AvgPm = average premium of all policies of a specific type in our sample paid at different ages by a consumer who buys (qualifies for) insurance at age 40.

Pram/Death = average premiums divided by probability of death at each age. We use the 1985/90 select and ultimate actuarial table by the Society of Actuaries. Select and Ultimate tables reflect mortality experience for insured population who passed a medical. We are using death probabilities of a person passing the medical at age 40.

NoRequal = premium profile of an S&U ART buyer who failed to requalify (show he is in good health) as good risk in later years.

Requal = premium profile of a buyer who requalified (passing a medical) year after year.
age 40.\textsuperscript{14} This ratio should be constant absent front-loading. In contrast, the ratio declines over time. For example, for a 40 year old and an aggregate ART policy the ratio halves within four years, and it keeps declining to below a third of its initial level. Hence, contracts do not break even period by period.

More surprisingly, even S\&U ARTs, obvious candidates for involving no long-term insurance, involve some degree of front-loading. Column 6 confirms this. Absent front-loading, the ratio (column 6) should increase over time, since the denominator reflects the entire population while the numerator reflects the worsening pool that fails to requalify year after year. In contrast, the ratio mildly declines.

Note that on average the first year premium of aggregate ARTs is 36 percent higher than that of select contracts despite the fact that both are yearly contracts. This reflects the fact that in an aggregate contract the insurance company is offering a lower cap on future premiums in the event that the consumer is in poor health. The aggregate contract offers more insurance of reclassification risk, at the cost of higher initial payments.

To summarize, all the available contracts involve front-loading, as suggested by the theory. Let us contrast this finding with the alternative assumptions on commitment. If consumers could commit, front-loading would not be necessary to achieve the efficient allocation. If front-loading is costly, we would not observe it. At the other extreme, suppose that insurance companies cannot commit to future terms of the contract. Then front-loading would not provide any insurance against reclassification risk. Thus, in neither of these scenarios would we observe front-loading.

\textbf{IV. B. The Negative Relation between Front-Loading and the Present Value of Premiums}

We saw in the previous subsection that contracts differ in the extent of front-loading. We now show that there is a systematic

\textsuperscript{14} The death probabilities used for Table III are taken from select and ultimate tables compiled by the Individual Life Insurance Experience Committee [1999]. Select and ultimate tables reflect the mortality experience of insured individuals, namely, those who passed a medical examination, as a function of both age and time elapsed since they passed the medical. Reported numbers understate the extent of front-loading since most contracts do not retain the best draws. One caveat concerns the trend of decreased mortality. The mortality tables refer to past populations, the contracts may thus reflect an expected decline in mortality. However, the declines in the ratios that we observe in Table III are too big to be explained by this.
The relation between present value of premiums and the extent of front-loading along the lines of Proposition 2.

We look at the relation between the slope of the premiums and the present value of premiums over twenty years of coverage. To do this, we proxy the slope of the premium profile by the ratio of the first over the eleventh premium, \( Q(1st)/Q(11th) \). Table IV shows basic statistics for contracts offered to 40 year old male nonsmokers who qualify as preferred risks. Note the wide range of premium slopes and present values.

A higher \( Q(1st)/Q(11th) \) ratio means that more is paid up-front (there is more front-loading). The prediction of the model is that contracts with a higher \( Q(1st)/Q(11th) \) should have lower present value of premiums. We now investigate whether the relation between slopes and present values is indeed negative in the data. We also ask how much of the variability in present values can be explained by contract slopes.

The first column of Table V presents the regression of the log of the present value of the premiums on the \( Q(1st)/Q(11th) \) ratio and other contract characteristics. The front-loading variable is highly significant; and it explains most of the variability in the present value of premiums. Excluding this variable from the regression drops the \( R^2 \) from 74.4 percent to 16.6 percent (column

<table>
<thead>
<tr>
<th>( Q(1st)/Q(11th) )</th>
<th>Mean</th>
<th>Std dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>16,055</td>
<td>5,245</td>
<td>6,871</td>
<td>28,754</td>
</tr>
<tr>
<td>ln(PV)</td>
<td>9.62</td>
<td>0.36</td>
<td>8.83</td>
<td>10.27</td>
</tr>
</tbody>
</table>

\( Q(1st)/Q(11th) \) is the ratio of the first to the eleventh premium; we use it to capture the slope of premiums.

\( PV \) = present value of the cost of twenty years of coverage at \( r = 0.08 \).

15. Recall that the present value is calculated conditional on the consumer not requalifying. As argued in subsection III.C, this is the theoretically meaningful measure.

16. Recall from subsection II.B that a contract with a lower present value of premiums does not dominate a contract with a higher one. Consumers who choose a contract with a low \( Q(1st)/Q(11th) \) ratio (hence with a lower present value of premiums) will drop coverage in the future if they remain healthy, since in this case they obtain lower rates by initiating a new contract. The theory says that the pool of consumers who stays with a contract in the long run is not the same for contracts with different slopes. The evidence presented in subsection IV.C is consistent with this.
2). The effect is economically significant as well, one standard deviation increase in \( Q(1st)/Q(11th) \) translates into a 28 percent decline in the cost of coverage.

The third column presents a similar regression after omitting the LT20s from the sample. We do so to check whether these contracts, which have no variability in \( Q(1st)/Q(11th) \), were responsible for the negative relation between premiums and slope. The negative relation is still strong among the rest of the contracts as well. Columns 4 repeats the exercise for a sample of five- and ten-year contracts.

Column 5 includes only the noncontingent contracts in the sample. The purpose of this column is to test the robustness of the results within noncontingent contracts. Recall that to compare contingent contracts we have to appeal to the equivalence result in subsection III.C. One might thus wonder about the robustness of these results, once we only compare contracts that do not require such a theoretical interpretation. By comparing noncontingent contracts, we test Proposition 2 directly, without relying on the equivalence result. The negative relation is confirmed.
Moreover, to assess the economic magnitude of the estimated effect of front-loading, we compare the predicted gap in cost between the average LT20 and the average aggregate ART. The former, by definition has a slope of 1, while the latter has a slope of 0.44 (see Table III). By multiplying the differences in slopes by the coefficient in the regression (-0.73), we find that, on average, by decreasing premium slopes from the average aggregate ARTs to a flat contract, the present value of premiums declines by 37.7 percent.

The same picture arises when we look at different ages. In the working paper [Hendel and Lizzeti 2002] we compare contracts across ages. We also included fixed firm effects to make sure the cross-contract differences are not due to firm differences.

**IV. C. Front-Loading and Lapsation**

The role of front-loading as a device to provide more insurance depends on locking in consumers. We now attempt to provide further corroboration for the theoretical prediction that less front-loaded contracts suffer from more health-related lapse.

Table VI presents lapsation data, from LIMRA [1996] for the period 1993–1994. The sample includes all the contracts in force at the beginning of the 1993–1994 policy year, for 33 top United States insurers (namely, all LIMRA members). In total, the sample contains over 7 million policies that were in force in 1993 (approximately one-third of all policies in the United States in 1993). Lapse rates are percentages of face amount (or number of

<table>
<thead>
<tr>
<th>Contract year</th>
<th>% of face amount</th>
<th>% of policies</th>
<th>% of face amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ART</td>
<td>Term</td>
<td>ART</td>
</tr>
<tr>
<td>1</td>
<td>11.8</td>
<td>14.2</td>
<td>15.0</td>
</tr>
<tr>
<td>2</td>
<td>14.1</td>
<td>11.4</td>
<td>14.8</td>
</tr>
<tr>
<td>3–5</td>
<td>13.4</td>
<td>8.0</td>
<td>12.4</td>
</tr>
<tr>
<td>6–10</td>
<td>10.1</td>
<td>5.0</td>
<td>9.4</td>
</tr>
<tr>
<td>11+</td>
<td>7.0</td>
<td>3.9</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Data Source: LIMRA’s Long-Term Ordinary Lapse Survey—United States [1996].

Term includes all term level contracts, from 2 to 30 years of length.

Term = term contracts other than ARTs.
policies) in force at their 1993 policy anniversaries, that lapse on or before their 1994 anniversary.

These data are not ideal since they do not provide a complete breakdown of lapsation by type of contract. It only allows us to contrast ARTs with term contracts of longer length (2 to 30 year level contracts). The model predicts that longer term contracts, which involve more front-loading, should suffer lower rates of lapsation. Moreover, we expect the difference in lapsing to be more pronounced when there is more health-related learning (namely, for older age groups).

The first two columns represent lapsation measured as the percent of the face amounts. “Contract Year” measures the age of the contract. For example, “Contract Year” = 1 means contracts issued during 1993, while “Contract Year” = 2 means contracts issued in 1992.

The numbers are in line with Proposition 2. Aside from contracts just issued, on which not much lock-in has occurred yet, Term (longer) contracts have lower lapsation, and a steeper decline in lapsation over time. A similar picture appears in the next two columns that measure lapsation as a percent of the number of policies.

The second part of the table separates lapsation by age groups. We expect more health-related information to be revealed in the 40–59 age-group than in the 20–39. Thus, we expect the effect of front-loading in reducing lapsation to be more pronounced for the older group. As expected, we find that the difference in lapsation between ARTs and Term is bigger for the older group.

IV. D. Accidental Death Insurance: A Test Case

Accidental death insurance, which is a special type of life insurance contract, provides an ideal way to further test whether the main forces behind the model are responsible for the shape of the available contracts in the industry.

An accidental death insurance policy is essentially the same product as a term policy, with the exception that it only pays if death is accidental; it does not pay if death is due to illness. Accidental death rates are quite flat between the ages of 25 to 60, and firms are unlikely to learn much about the characteristics of the consumers. Since learning is a key force behind our results, we expect the predictions of the model to have less bite in the Accidental insurance market. In other words, there is no reason to predict that front-loading will be used to improve long-run
insurance. We found that both the probability of an accident and premiums are flat between ages 25 to 60; namely, there is no front-loading (see Jaffe [1998] and quotes available on the web).

Thus, a very similar product which involves no learning about risk characteristics displays no front-loading and no relation between the present value of premiums and front-loading; this can be viewed as an experiment whereby removing the key assumption about learning makes the main theoretical predictions disappear.

V. ALTERNATIVE EXPLANATIONS

In this section we explore and attempt to rule out alternative explanations for the empirical findings.

Adverse selection. One possible concern is the presence of asymmetric information. Cawley and Philipson [1999] found no evidence of adverse selection in term life insurance. This can be explained by the fact that buyers have to pass a medical examination and answer a detailed questionnaire. (See Society of Actuaries [1995] for a detailed description of underwriting practices and the stringency of medical screening.) Misrepresentation or concealment of material information would render the policy void. Furthermore, insurance companies have an information clearing system where they share information about buyers.

Even absent asymmetric information about the current risk category, in the context of long-term contracts, there is another potential source of adverse selection. Buyers may have superior information about their living habits which influence future death probabilities. This is plausible. However, if this asymmetric information was an important force, more front-loaded contracts would attract buyers with worse future health who are seeking a lower cap on their future premiums. But then more front-loading would be associated with higher cost of coverage. We find the opposite: our numbers suggest that the prevailing force shaping contracts is not asymmetric information, otherwise front-loaded contracts would be more costly.

Our discussion of adverse selection is only valid for term life insurance. There is evidence that annuity markets suffer from adverse selection [Brugiavini 1990; Friedman and Warshawsky 1990]. Note that there is no medical exam for annuities.

Fixed underwriting costs. Underwriting involves fixed costs. Because more front-loaded contracts have lower lapsation,
these fixed costs are incurred less frequently. This would explain the lower present value of premiums of more front-loaded contracts.

If this hypothesis were correct, then, as a fraction of the face amount, the savings from lower lapsation would decrease as the face amount increases, since the fixed underwriting costs would become relatively less important. Thus, fixed costs cannot account for the observed relation between front-loading and the present value of premiums.

**Cross-firm differences in underwriting standards.** If firms specialize in specific types of contracts, with those firms offering more front-loaded contracts performing stricter underwriting, then we would obtain a negative relation between the present value of premiums and front-loading. We have no reason to believe that this specialization takes place. However, to rule out this explanation, we resort to within-firms contract comparisons performed in two ways. First, we ran the regression in Table V with firm fixed effects, and we found that results are unchanged from the previous analysis. Second, we confirmed that the relation between front-loading and cost of coverage holds for every company individually that offers several contract types (see footnote 17).

**Correlation mortality-income.** Since mortality and income are negatively correlated, the lower cost of more front-loaded contracts could be caused by higher income consumers purchasing those contracts. This explanation can be evaluated by looking at larger face amounts. If income levels (through the correlation with mortality) were creating the link between front-loading and present value, then that relation should disappear as the face amount becomes large, since only the very rich will buy a $10 million policy. This does not happen (see footnote 17).

**VI. Concluding Remarks**

We have shown that term life insurance contracts fit the theoretical predictions of a model with symmetric learning and one-sided commitment. They are not in line with alternative

17. We compared present values for different levels of coverage, from $100,000 to $10 million dollars, for all the insurers in our sample that offer comparable ARTs and LT20s. For every company the same pattern arises. The ratio of present values of the ART over the LT20 is larger than 1; moreover, the ratio does not decline in face amount (see details in the working paper).
assumptions on commitment. They also cannot be explained by standard asymmetric information models. The life insurance industry reacts to the absence of consumer commitment to long-term contracts by front-loading premiums. Front-loading creates consumer lock-in, which in turn reduces reclassification risk.

Our analysis suggests that in the life insurance industry the extent of the distortions due to dynamic information revelation are not large. According to our numbers, some of the commonly purchased term contracts come very close to capturing all the gains from long-term insurance. However, this achievement comes at a cost because it requires consumers to pay more money up front than would be optimal from the point of view of intertemporal consumption smoothing. It is worth noting that the industry achieved this partial solution to the problem of reclassification risk without need of regulation (e.g., no imposition of guaranteed renewability).

Understanding and quantifying the inefficiency from the lack of bilateral commitment in life insurance is also of interest in light of the policy debate over health insurance. In the health insurance market, front-loaded contracts are not offered, and reclassification risk is a major concern. The health care market suffers from a variety of other problems. What makes the performance of these two industries so different? There are several differences between these markets: first, while life insurance is about insuring an income stream, health insurance is about health care treatment. Thus, the amount needed to be front-loaded to generate long-term insurance is proportional to income in the life insurance market while independent of income in the health insurance market. Hence, health insurance front-loading is likely to be unaffordable to low income households. Second, treatment cost risk is nondiversifiable, since it affects all patients (for more on this argument see Cutler [1993]). Finally, health insurance involves a quality of service that is susceptible to opportunism on the part of insurance companies (in particular, under managed care). Hence, consumers are likely to be more reluctant to lock in to a health insurer, who can later opportunistically reduce quality.

An interesting connection exists between life insurance and some credit contracts (e.g., mortgages). Borrowers cannot easily commit not to prepay if interest rates drop. However, there is an important difference: interest rate risk is aggregate risk that banks offering mortgages cannot easily diversify. In contrast,
reclassification risk is an idiosyncratic risk that is fully diversifiable. Thus, we expect more insurance against reclassification risk than of interest rate risk. Indeed, no more than a couple of “points” are front-loaded in common mortgage contracts, only moderately reducing the present value of long-run payments. This is in contrast to the life insurance case where there is a wide range of front-loading with a large impact on long-run payments.

Most life insurance contracts are in nominal terms. It is puzzling that contracts in real terms are not more common. A possible explanation is that buyers prefer decreasing levels of coverage in real terms: the present value of the income to be insured declines as the buyer ages. Furthermore, inflation-adjusted contracts would need more front-loading, simply because the future stakes, subject to adverse retention, would be bigger than in a nominal contract. This may also explain consumers’ preference for nominal contracts. Finally, this puzzling feature is common to a wide variety of securities: inflation-indexed bonds are not very common.

APPENDIX

Proof of Proposition 1

First note that we can replace the set of constraints (3) with the following, simpler, set of constraints:

\[(1 - p_i)Q^i_2 - p_iF^i_2 \leq 0 \text{ for } i = 1, \ldots, N.\]

To see this, note that if \((Q_1,F_1), (Q^1_2,F^1_2), \ldots, (Q^N_2,F^N_2)\) maximize (1) subject to (2) and (5), then there is no state \(i\), and no \((Q^i_2,F^i_2)\) that makes positive profits and gives buyers a higher utility in that state. Thus, (3) is satisfied. Conversely, suppose that \((Q^i_2,F^i_2)\) are such that (5) is violated. Then it is clear that (3) is violated as well since a competing insurance company can offer terms that are slightly better for buyers than \((Q^i_2,F^i_2)\) and that still make positive profits.

Let \(\mu\) be the Lagrange multiplier for the constraint in (2) and \(\lambda_i\) the Lagrange multiplier for the \(i\)th constraint in (5).

The first-order conditions for an optimum are

\[u'(y - g - Q_1) = \mu,\]
\[v'(F_1) = \mu,\]
(8) \[-(1-p)\pi_i u'(y+g-Q_2^i) + (1-p)\pi_i \mu + \lambda_i = 0 \forall i,
\]
(9) \[ (1-p)\pi_i u'(F_2^i) - (1-p)\pi_i \mu - \lambda_i = 0 \forall i,
\]
(10) \[ (1-p_i)Q_2^i - p_i F_2^i \lambda_i = 0 \forall i. \]

Part (i) follows from first combining equations (6) and (7) and then combining equations (8) and (9).

To prove part (ii), note first that equations (8) and (9) imply that

\[ F_2^i = (v')^{-1}(u'(y+g-Q_2^i)). \]

If constraint \(i\) in equation (5) is binding, \((1-p_i)Q_2^i = p_i F_2^i\). Substituting from equation (11), we obtain

\[ (1-p_i)Q_2^i = p_i (v')^{-1}(u'(y+g-Q_2^i)). \]

From equation (12), we see that, since \(u'\) and \(v'\) are decreasing functions, if \(i\) and \(j\) are two binding constraints with \(i > j\) (so that \(p_i > p_j\)), then \(Q_2^i > Q_2^j\).

Suppose now that constraint \(k\) in equation (5) is nonbinding. Then equation (8) simplifies to

\[ u'(y+g-Q_2^k) = \mu. \]

Thus, if constraints \(k\) and \(l\) are nonbinding, \(Q_2^k = Q_2^l\).

If in contrast, constraint \(i\) in equation (5) is binding,

\[ u'(y+g-Q_2^i) = \mu + \lambda_i (1-p)\pi_i < \mu, \]

where the inequality holds because, if constraint \(i\) is binding, \(\lambda_i < 0\).

Thus, if constraint \(i\) is binding and \(k\) is not, \(Q_2^i < Q_2^k\). We now want to show that if \(i\) is binding and \(k\) is not, then \(i < k\). Since \(k\) is not binding, substituting from equation (11), we obtain

\[ (1-p_k)Q_2^k < p_k (v')^{-1}(u'(y+g-Q_2^k)). \]

Thus, \(Q_2^i < Q_2^k\) and \(p_i > p_k\) render equations (12) and (15) incompatible proving that indeed \(p_i < p_k\).

To prove part (iii), we need to show that the first-period premium \(Q_1\) must be larger than actuarially fair premium \(Q_1(\text{FI})\). If any of the no-lapsation constraints (5) is not binding, then \(Q_1 > Q_1(\text{FI})\) is immediate from the zero profit condition for the insurance company (equation (2)).
Suppose instead that all the no-lapsation constraints are binding. Substituting from equation (6) into equation (8), we obtain
\[ u'(y + g - Q_2^i) = u'(y - g - Q_1) + \lambda / (1 - p) \pi_i. \]

Thus, \( Q_1 > Q_2^N(FI) - 2g \). This inequality clearly requires that \( Q_1 > Q_1(FI) \) if \( g \) is small enough since \( p_N > p \).

Part (iv) is obvious: as \( g \) increases, more and more of the no-lapsation constraints become binding. This means that, as \( g \) grows, \( s \) becomes larger. When \( s \) is larger, \( Q_1 \) declines.

REFERENCES

Life Insurance Fact Book, American Council of Life Insurance, several issues.