Models of Discrimination and Affirmative Action

1 Introduction

- Wage inequality, and in particular, racial income inequality, is an important question in public economics.

- Numerous empirical studies have established that when wages are regressed on a bunch of variables that should matter for productivity (schooling, experience, union membership etc.) and dummy variables for race and sex, the coefficient for the dummies usually turn out to be significantly different from zero.

- While this type of analysis may under- or overstate the extend of racial discrimination, it motivates the theoretical studies on discrimination.

- Recently, disparate treatment and disparate impact received by different racial and/or gender groups in housing, mortgage lending, retailing, policing, judicial system, and even organ transfers have caused tremendous amount of publicity.

- It is important to know the extent of and the reasons for disparate treatments before the best policy to help the minorities can be made.

2 Theoretical Models of Discrimination

1. Taste-based Discrimination: Becker (1959). Common driving force: racist preferences (or racial animus) by some agents in the model, e.g. employers, customers or other employees.

2. Statistical Discrimination: Originated from Phelps (1972) and Arrow (1973).

   - Common ingredient: group identity (which the firms do not directly care about) is used as a proxy for productivity (which the firms do care about).

   - The assumption that the firms do not directly care about the workers' racial (or gender) identity is the distinguishing characteristic from Becker's model of taste-based discrimination.

3. Discrimination due to inter-group interactions:

   - Moro and Norman: discrimination as specialization;

   - Mailath, Samuelson and Shaked: Discrimination due to labor market search friction and wage bargaining. The presence of group B affects the outside option of the firms when bargaining with group W and vice versa.

2.1 A Simple Model of Taste-Based Discrimination:

- Price taking firms that produce output from labor input. The production function is

$$y = f(L),$$

where L is the total labor input;

- Two groups of workers, B and W;

- Each agent supplies labor inelastically at $\bar{L}_B$ and $\bar{L}_W$ respectively for B and W;

- Suppose that the wages for B and W are respectively w_W and w_B;

- Suppose that workers' labor are perfect substitutes. Then the firms' profit function is

$$\pi\left(L_B, L_W\right) = f\left(L_B + L_W\right) - w_B L_B - w_W L_W$$

- Suppose that firm owners have identical, but possibly racist preference

$$u\left(\pi, L_B, L_W\right).$$

- A utility-maximizing firm owner solves

$$\max_{\{L_B, L_W\}} u\left(\pi\left(L_B, L_W\right), L_B, L_W\right).$$

- The first order conditions are

$$\begin{aligned}
\frac{\partial u\left(\pi, L_B, L_W\right)}{\partial \pi}\left[f'\left(L\right) - w_B\right] + \frac{\partial u\left(\pi, L_B, L_W\right)}{\partial L_B} &= 0 \\
\frac{\partial u\left(\pi, L_B, L_W\right)}{\partial \pi}\left[f'\left(L\right) - w_W\right] + \frac{\partial u\left(\pi, L_B, L_W\right)}{\partial L_W} &= 0
\end{aligned}$$

- In equilibrium labor supply must equal labor demand, hence

$$w_W - w_B = \frac{\frac{\partial u(\pi,\bar{L}_B,\bar{L}_W)}{\partial L_W} - \frac{\partial u(\pi,\bar{L}_B,\bar{L}_W)}{\partial L_B}}{\frac{\partial u(\pi,\bar{L}_B,\bar{L}_W)}{\partial \pi}}.$$

- Hence $w_W - w_B > 0$ whenever

$$\frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_W} > \frac{\partial u\left(\pi, \bar{L}_B, \bar{L}_W\right)}{\partial L_B}.$$

2.2 Statistical Discrimination

2.2.1 Phelps (AER, 1972)

- Phelps (1972) is a model of discrimination driven by differences in information technology, i.e., the signal that is used by the firms to infer about a worker's unobserved productivity is less informative for group B members than for group W members.

- Firms are competitive and risk neutral;

- Workers differ both in terms of ability (or productivity) a, which is not observed by the firms. For illustration, assume that in both group B and W, a is distributed according to

$$N\left(\mu_a, \sigma_a^2\right).$$

- Firms, however, observe a noisy signal, θ_i, of worker i's ability, a_i. Specifically,

$$\theta_i = \begin{cases} a_i + \varepsilon_{iB} \text{ if } i \text{ is Black} \\ a_i + \varepsilon_{iW} \text{ if } i \text{ is White.} \end{cases}$$

  where $\varepsilon_{iB} \sim N\left(0, \sigma_B^2\right)$, and $\varepsilon_{iW} \sim N\left(0, \sigma_W^2\right)$. Hence θ_i is an unbiased signal of a_i, but the precision of the signal may depend on the group identity;

- Since firms are risk neutral and competitive, each worker will be offered a wage which equals his expected productivity (ability) conditional on the signal θ_i.

- Using the standard results on Bayesian updating (see for example, page 167 of DeGroot 1970), we obtain

$$w(\theta_i) = \begin{cases} \frac{\sigma_B^2}{\sigma_a^2+\sigma_B^2}\mu_a + \frac{\sigma_a^2}{\sigma_a^2+\sigma_B^2}\theta_i \text{ if } i \text{ is Black} \\ \frac{\sigma_W^2}{\sigma_a^2+\sigma_W^2}\mu_a + \frac{\sigma_a^2}{\sigma_a^2+\sigma_W^2}\theta_i \text{ if } i \text{ is White.} \end{cases}$$

  Hence if, $\sigma_B^2 > \sigma_W^2$, then we have the following implication:

  - Wages are lower for high scoring blacks than for high scoring whites (signals above the prior mean ability μ_a);

  - Wages are higher for low scoring blacks than for low scoring whites (signals below μ_a);

  - Average wages are equal for the two groups (unless if there are difference in the distribution of intrinsic ability a across the groups).

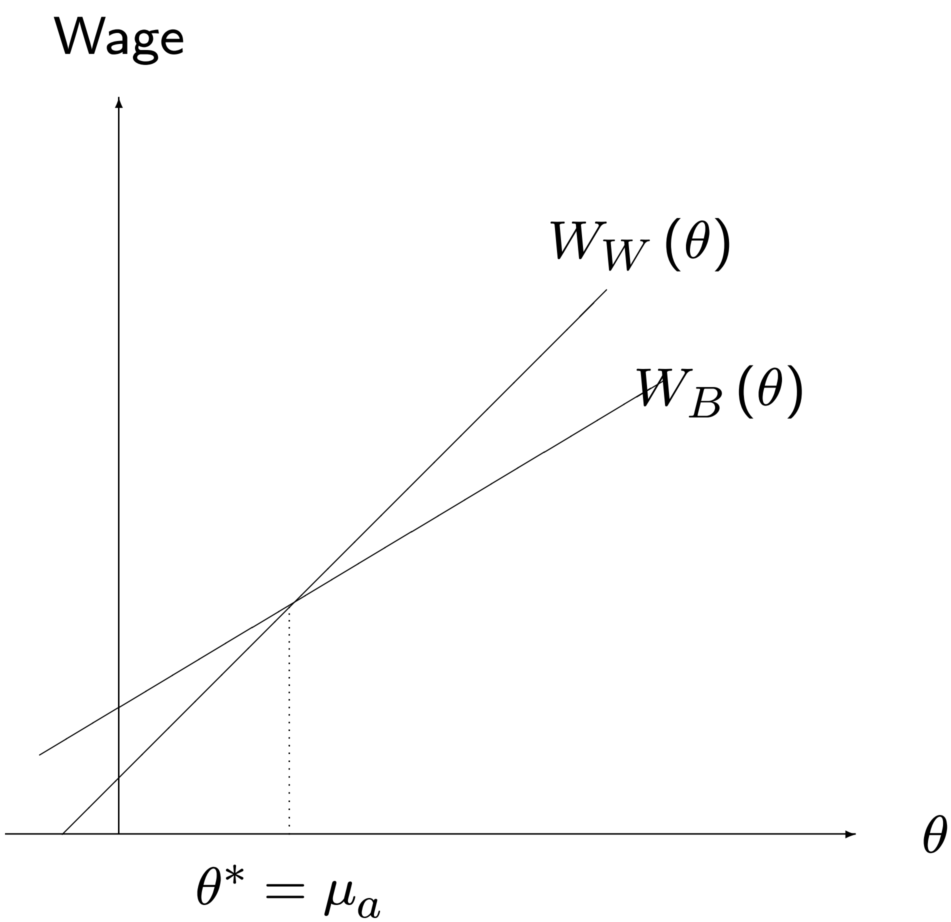

Figure 1: Wage Offers in Phelps Model

2.2.2 Arrow (1973)

- Arrow (1973) is the first to lay out the necessary ingredients of a theory of "self-fulfilling prophecy" with endogenous skill acquisition decisions to interpret discriminatory outcomes.

- The important ingredients of statistical discrimination theory are:

  1. the employers should be able to costlessly observe a worker's race;

  2. the employers must incur some cost before he can determine the employee's true productivity (otherwise, there is no need for surrogate information);

  3. the employers must have some preconception of the distribution of productivity within each of the two groups of workers.

Arrow's Model

- Each firm has two kinds of jobs, skilled and unskilled and they are complementary to each other: let the production function be $f(L_s, L_u)$ where L_s skilled labor and L_u is the unskilled labor, and f is a constant returns to scale production function

- All workers are qualified to perform unskilled jobs; but only a proportion p_w of whites and a proportion of p_b of blacks are skilled.

- The firm must pay a cost r to find out whether the worker is skilled or not, and the firm knows eventually whether a worker is qualified or not.

- Wage determination: his notion of a competitive wage in the skilled job is a contract that pays a worker from group $j = b, w$ a wage $w_j > 0$ if a worker is revealed to be qualified, and 0 otherwise; on the other hand the firm always pays a wage w_u to any worker on the unskilled job.

- Arrow claims that competition among firms will result in a zero profit condition, hence

$$\begin{aligned} r &= p_w \left[f_1 (L_s, L_u) - w_w \right] \\ r &= p_b \left[f_1 (L_s, L_u) - w_b \right]. \end{aligned}$$

  Hence

$$w_w = \frac{p_b}{p_w} w_b + \left(1 - \frac{p_b}{p_w} \right) f_1 (L_s, L_u).$$

  If for some reason $p_b < p_w$, then $w_b < w_w$.

- The wage on the simple task for both groups is

$$w_u = f_2 (L_s, L_u).$$

- I think Arrow had all the ideas right, but his model of wage determination is simply not consistent: since $w_u > 0$ and any unqualified worker who is hired on the skilled job will eventually get a wage 0, why would any unqualified worker agree to be hired on the skilled job in the first place?

However, neglecting the above problems with the model, we have:

- Blacks are paid a lower wage in the skilled task if they are believed to be qualified with a lower probability;

- The explanation of discriminatory behavior is shifted from preferences to beliefs.

Arrow then proceed to provide an explanation for why p_w and p_b differ in equilibrium even though there are no intrinsic differences between groups. Since Arrow's model is problematic in details we will not go through the details, but he assumes that

- A worker becomes qualified as a result of a costly (unobservable) investment;

- Workers invest in skills if the gains of doing so outweighs the costs. Arrow takes the gains to be $w_j - w_u$ (which is obviously inconsistent with the labor market equilibrium condition). Suppose the distribution of skill investment cost is given by $G(\cdot)$. Then the proportion of skilled workers will be $G\left(w_j - w_u\right)$. And equilibrium requires that

$$p_j = G\left(w_j\left(p_j\right) - w_u\right).$$

- Arrow then notes that the system can easily have symmetric as well as asymmetric equilibria. The intuition for the asymmetric equilibria is simple: if very few workers invest in a particular group, the firms will rationally perceive this group as unsuitable for the skilled task and equilibrium wages in the skilled task will be low, which will in turn give little incentive for the workers from this group to invest.

2.2.3 Coate and Loury (AER, 1993)

- CL presents a theoretical analysis of affirmative actions, mainly to understand the incentive effects of affirmative action policies on agents' incentive to invest in skills.

- To do that, they first have to present a theoretical model of why market discrimination against the blacks occurred in the first place. They view it as a statistical discrimination. Their model is an improvement upon Arrow, yet still leaves the wage determination exogenous.

COATE AND LOURY MODEL

- There are more than two competitive firms, and a continuum of workers with unit mass;

- The workers belong to one of two identifiable groups, B or W, with $\lambda \in (0, 1)$ be the fraction of W in the population;

- [Linear Production Function] There are two tasks, a complex task (task one in the paper) and a simple task (task zero in the paper); the complex task can be successful performed only by qualified workers.

- The productivities of (or the firms' net return from) qualified and unqualified workers on the two tasks are summarized by the following table:

		Tasks	
		Complex	Simple
Workers	Qualified	$x_q > 0$	0
	Unqualified	$-x_u < 0$	0

- Workers are born to be unqualified, but they can become qualified through some costly ex ante investment. The skill investment cost c is heterogenous across workers and is distributed according to CDF $G(\cdot)$ which is assumed to be continuous and differentiable.

- Workers' skill investment decisions are unobservable by the firms. Instead, firms observe a noisy signal θ of the worker's qualification: The signal θ is drawn from $[0, 1]$ according to PDF $f_q(\theta)$ if the worker is qualified, and according to $f_u(\theta)$ if he is unqualified. Assume monotone likelihood ratio property on the testing technology:

$$l(\theta) = \frac{f_q(\theta)}{f_u(\theta)}$$

  is strictly increasing and continuous. This MLRP simply says that a worker with a higher θ is more likely to be qualified than one with a lower θ.

- [Part of CL Model that is Not Satisfactory] A worker gets a net benefit ω if he is assigned to the complex task, and 0 if he is assigned to the simple task.

TIMING OF THE GAME:

- STAGE 1: Nature chooses workers' types c;
- STAGE 2: Workers make (unobservable) skill investment decisions;
- STAGE 3: Test results $\theta \in [0, 1]$ observed by firms;
- STAGE 4: Firms decide how to assign the workers to the two tasks.

EQUILIBRIUM:

- We solve the equilibrium of the model from backwards.

- Consider STAGE 4. What is the optimal task assignment rule for the firms?

- Suppose that a firm sees a worker with signal θ from a group where a fraction π has invested in skills.

- The posterior probability that such a worker is qualified is:

$$p(\theta;\pi) = \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1-\pi) f_u(\theta)}.$$

- The expected profit for the firm if it assigns such a worker to the complex task is

$$p(\theta; \pi) x_q - [1 - p(\theta; \pi)] x_u,$$

while the profit is zero if it assigns the worker to the simple task.

- Hence the firm will assign this worker to the complex task if and only if

$$\begin{aligned} & p(\theta; \pi) x_q - [1 - p(\theta; \pi)] x_u \geq 0 \\ \Leftrightarrow \quad & \frac{x_q}{x_u} \geq \frac{1-\pi}{\pi} \frac{f_u(\theta)}{f_q(\theta)}. \end{aligned}$$

Since f_q/f_u is assumed to be monotonically increasing in θ, the above inequality holds if and only if

$$\theta \geq \tilde{\theta}(\pi).$$

- $\tilde{\theta}(\pi)$ is determined as follows:
  - if
    $$\frac{x_q}{x_u} = \frac{1-\pi}{\pi}\frac{f_u(\theta)}{f_q(\theta)} \tag{1}$$
    has a solution , $\tilde{\theta}(\pi)$ is the unique solution;
  - If $x_q/x_u > (1-\pi) f_u(\theta) / \pi f_q(\theta)$ for all $\theta \in [0,1]$, then $\tilde{\theta}(\pi) = 0$;
  - Otherwise, $\tilde{\theta}(\pi) = 1$.

- Whenever $\tilde{\theta}(\pi)$ is interior, we have

$$\frac{d\tilde{\theta}(\pi)}{d\pi} = -l'\left(\tilde{\theta}(\pi)\right)\frac{x_u}{x_q}\frac{1}{\pi^2} < 0.$$

- TO SUMMARIZE:
  - In the task assignment stage, the firm will follow a cutoff rule $\tilde{\theta}(\pi)$: workers with signal θ higher than $\tilde{\theta}$ will be assigned to the complex task and those with signals lower than the cutoff will be assigned to the simple task.
  - Moreover, the cutoff $\tilde{\theta}(\pi)$ is weakly decreasing in π, the fraction of skilled which is weakly decreasing in π, the fraction of skilled workers in that group.

- Now we analyze the workers' optimal skill investment decision at STAGE 2, given the firms' sequentially rational behavior in STAGE 4.

- Suppose that in STAGE 4, the firms follow a cutoff rule at $\tilde{\theta}$. If a worker with cost c decides to invest in skills, his expected payoff will be

$$\left[1 - F_q\left(\tilde{\theta}\right)\right]\omega - c$$

If he does not invest in skills, his expected payoff will be

$$\left[1 - F_u\left(\tilde{\theta}\right)\right]\omega.$$

Hence a worker with cost c will invest if and only if

$$c \leq B\left(\tilde{\theta}\right) \equiv \left[F_u\left(\tilde{\theta}\right) - F_q\left(\tilde{\theta}\right)\right]\omega.$$

This implies that the fraction of workers who rationally invests in skills *given a cutoff* $\tilde{\theta}$ is

$$G\left(B\left(\tilde{\theta}\right)\right) = G\left(\left[F_u\left(\tilde{\theta}\right) - F_q\left(\tilde{\theta}\right)\right]\omega\right). \quad (2)$$

- Note that

$$B'\left(\tilde{\theta}\right) = \omega\left[f_u\left(\tilde{\theta}\right) - f_q\left(\tilde{\theta}\right)\right]$$

is positive if and only if $l\left(\tilde{\theta}\right) < 1$. Hence it is a single peaked function. Moreover, $B(0) = B(1) = 0$.

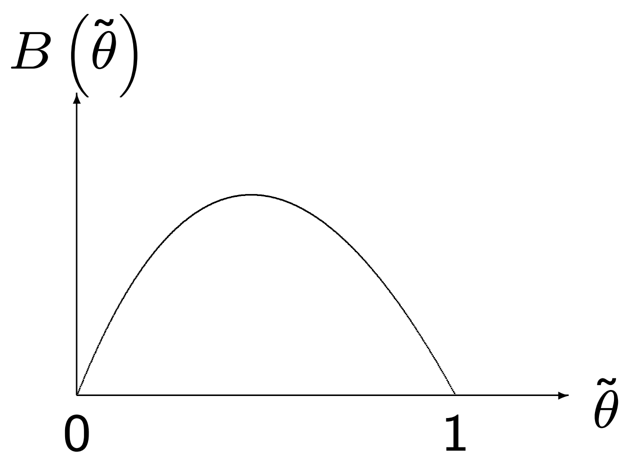

Figure 2: Benefits to Invest in Skills as a Function of the Cutoff $\tilde{\theta}$

- An *equilibrium* of the game is simply $\left(\tilde{\theta}_j^*, \pi_j^*\right), j = B, W$ such that for each j,

$$\begin{aligned} \tilde{\theta}_j^* &= \tilde{\theta}\left(\pi_j^*\right) \\ \pi_j^* &= G\left(B\left(\tilde{\theta}_j^*\right)\right), \end{aligned}$$

where $\tilde{\theta}(\cdot)$ and $G(B(\cdot))$ are defined by (1) and (2) respectively.

- Equivalently, we could redefine the equilibrium of the model as $\pi_j^*, j = B, W$, that satisfy

$$\pi_j^* = G\left(B\left(\tilde{\theta}\left(\pi_j^*\right)\right)\right). \tag{3}$$

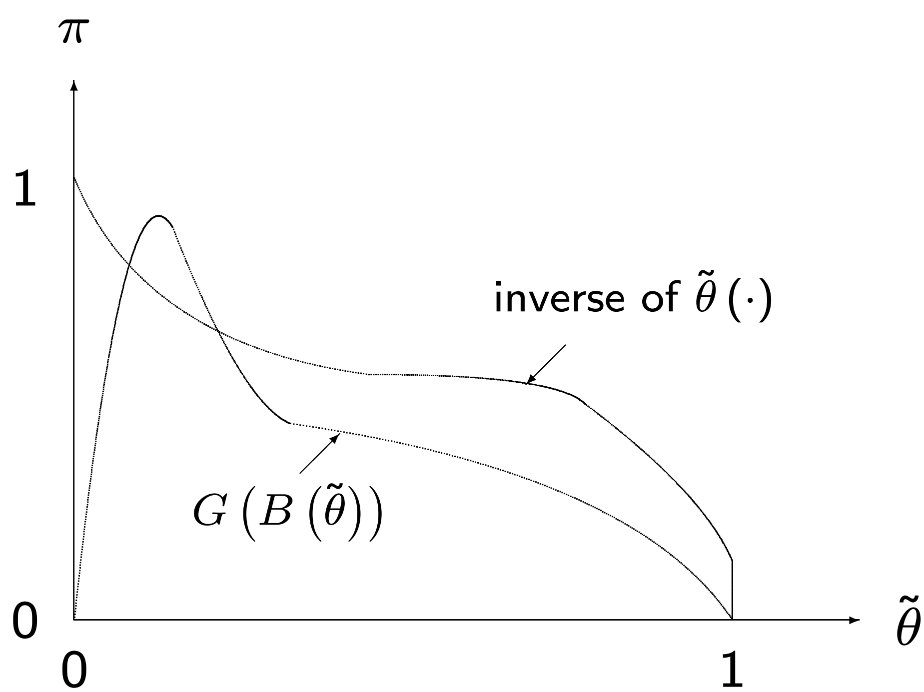

Figure 3: Multiple Equibria

Proposition 1 *If there exist $\tilde{\theta}$ such that*

$$G\left(B\left(\tilde{\theta}\right)\right) > \frac{f_u\left(\tilde{\theta}\right)/f_q\left(\tilde{\theta}\right)}{x_q/x_u + f_u\left(\tilde{\theta}\right)/f_q\left(\tilde{\theta}\right)},$$

then there exist at least two non-zero solutions to Equation (3).

Thus Coate and Loury demonstrate that statistical discrimination is a logically consistent notion in their model.

- Discrimination in this model can be viewed as a coordination failure: Removing discrimination is achieved if somehow blacks and the firms can all be coordinated on the good equilibrium. There is no conflict of interests among the whites and blacks concerning affirmative actions.

- Equilibria are Pareto ranked: The higher the fraction of investors, the higher are the profits to the firms and the higher are the expected net payoff for the workers.