Preference Revelation and Implementation

- If agents' preferences are known to "Public Good Board", Lindahl mechanism can implement the first best outcome;

- If agents' preferences are private information, however, Lindahl mechanism is not incentive compatible;

- Clarke (1971) - Groves (1973) - Ledyard mechanisms:

  – is incentive compatible;

  – can implement the first best outcome when agents' preferences (quasi-linear) are private information;

  – but it does not satisfy voluntary participation and balanced budget.

- Hurwicz (1975) showed that there is no incentive compatible, voluntary participation, balanced budget mechanism that implements the first best outcome;

- What is the best outcome that can be implemented then? Lots of papers on that. Mailath-Postlewaite (1991) proved an asymptotic impossibility result: as the number of agents goes to infinity, the probability of public good provision in any IC, IR, BB mechanism goes to zero.

1 A Quasi-Linear Model

- n agents: $i \in \{1, ..., n\}$ with wealth w_i;

- A private good and a public good;

- Agents have quasi-linear utility functions:

$$u_i(x_i, G) = v_i(G) + x_i;$$

  – Assumption: while the quasi-linear form of $u_i(x_i, G)$ is known to the government - or the "Public Good Board (PGB)", the function $v_i(G)$ is agent i's private information

- Production function of public good: $f(z) = z$.

- PGB wants to design a mechanism as follows:

– Ask each agent to report their own value function $\tilde{v}_i(\cdot)$. Agents could potentially misreport;

– For any *reported* profile $(\tilde{v}_1(\cdot), ..., \tilde{v}_n(\cdot))$, the mechanism specifies the level of the public good to be provided G, and the taxes from each of the n agents, $(T_1, ..., T_n)$, as a function of the reported profile.

- Desirable Properties of the mechanism:

1. *Incentive compatibility*: truth telling is an equilibrium (preferrably weakly dominant strategy equilibrium) of the game induced by the mechanism;

2. *Voluntary Participation (no coersion):* No agents are made worse off by participating in the mechanism than being in autarky.

3. *Efficiency.*

4. *Balance Budget.*

2 Mechanism A

- Ask each individual to report $\tilde{v}_i(\cdot)$;
- For any reported profile $\tilde{\mathbf{V}} = (\tilde{v}_1(\cdot), ..., \tilde{v}_n(\cdot))$, the level of public good $\hat{G}$ will be

$$\hat{G}\left(\tilde{\mathbf{V}}\right) = \max_{\{G\}} \left\{ \sum_{j=1}^{n} \tilde{v}_j(G) - G \right\}$$

- Agent i's total tax will be given by

$$\hat{T}_i\left(\tilde{\mathbf{V}}\right) = \hat{G}\left(\tilde{\mathbf{V}}\right) - \sum_{j \neq i} \tilde{v}_j\left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right).$$

  That is, agent i's tax equals the entire cost of the public good less the aggregate utility accruing to other people from the public good.

- An important observation of the tax rule: in determining $\hat{T}_i$, agent $i's$ reported v_i does not directly appear in the definition of $\hat{T}_i$. His expressed demand for the public good does have an indirect effect on $\hat{T}_i$ since it entered into PGB's determination of $\hat{G}$. But if i were to understate his demand, say by declaring that $v_i(G)$ were zero for all G, he would still have to pay a tax, since $\hat{T}_i$ depends on the chosen quantity of the public good and everyone else's expressed valuation of that chosen quantity, not i's.

- Truth-telling is an equilibrium in weakly dominant stratigies:

  Consider agent i. Suppose that other agents report $\tilde{\mathbf{V}}_{-i} = (\tilde{v}_1, ..., \tilde{v}_{i-1}, \tilde{v}_{i+1}, ..., \tilde{v}_n)$. Agent i knows the mechanism, so she knows that if she reports $\tilde{v}_i$, PGB will choose $\hat{G}$ to maximize

$$\sum_{j=1}^{n} \tilde{v}_j(G) - G$$

and sets a tax $\hat{T}_i$ to be

$$\hat{G} - \sum_{j \neq i} \tilde{v}_j \left(\hat{G}\right).$$

Agent i, however, want to maximize

$$\begin{aligned} & \max_{\{\tilde{v}_i\}} v_i \left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) + \left(w_i - \hat{T}_i\left(\tilde{\mathbf{V}}\right)\right) \\ = \;& v_i \left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) + \left(w_i - \hat{G}\left(\tilde{\mathbf{V}}\right) + \sum_{j \neq i} \tilde{v}_j \left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right)\right) \\ = \;& w_i + \left[\sum_{j \neq i} \tilde{v}_j \left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) + v_i \left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) - \hat{G}\left(\tilde{\mathbf{V}}\right)\right] \end{aligned}$$

By the definition of $\hat{G}\left(\tilde{\mathbf{V}}\right)$, we know that if agent were to report $\tilde{v}_i = v_i$, the above objective function will always be maximized.

An Example:

- Suppose that $\hat{G}$ is a \$1000 bridge, and suppose that there are five users who assign the following values to it:

$$\begin{aligned} v_1\left(\hat{G}\right) &= 0, v_2\left(\hat{G}\right) = 500, v_3\left(\hat{G}\right) = 100 \\ v_4\left(\hat{G}\right) &= 200, v_5\left(\hat{G}\right) = 300. \end{aligned}$$

Under mechanism A,

$$\begin{aligned} \hat{T}_1(\mathbf{V}) &= -100, \hat{T}_2(\mathbf{V}) = 400, \hat{T}_3(\mathbf{V}) = 0, \\ \hat{T}_4 &= 100, \hat{T}_5 = 200. \end{aligned}$$

Note that:

  - agent 1 is getting a grant rather than paying a tax;
  - the total tax is \$600, short of the \$1000 cost of the bridge - mechanism A is not self-financing.

3 Mechanism B: Clarke-Groves Mechanism

- Is there a way to fix the tax rule so that we can always be sure that total tax receipts will cover the cost of the public good, i.e.,

$$\sum_{i=1}^{n} \hat{T}_i \geq \hat{G},$$

while the incentives to tell the truth is maintained?

- The answer is given by Clarke-Groves mechanism, which is a fix based on mechanism A above. First note:
  - if we add a term to i's tax $\hat{T}_i$ above that does not depend on v_i or $\hat{G}$, i's incentive to tell the truth will be preserved.

Clarke-Groves mechanism:

- Ask each individual to report $\tilde{v}_i(\cdot)$;

- For any reported profile $\tilde{\mathbf{V}} = (\tilde{v}_1(\cdot), ..., \tilde{v}_n(\cdot))$, the level of public good $\hat{G}$ will be

$$\hat{G}\left(\tilde{\mathbf{V}}\right) = \max_{\{G\}} \left\{ \sum_{j=1}^{n} \tilde{v}_j(G) - G \right\}$$

- Set $\hat{\hat{T}}_i\left(\tilde{\mathbf{V}}\right)$ as follows:

$$\begin{aligned} \hat{\hat{T}}_i\left(\tilde{\mathbf{V}}\right) &= \hat{T}_i\left(\tilde{\mathbf{V}}\right) + S_i\left(\tilde{\mathbf{V}}_{-i}\right) \\ &= \hat{G}\left(\tilde{\mathbf{V}}\right) - \sum_{j \neq i} \tilde{v}_j\left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) + S_i\left(\tilde{\mathbf{V}}_{-i}\right). \end{aligned}$$

where

$$S_i\left(\tilde{\mathbf{V}}_{-i}\right) = \max_{\{G\}} \sum_{j \neq i} \left[\tilde{v}_j(G) - \frac{G}{n} \right].$$

Interpretation:

- Since $S_i\left(\tilde{\mathbf{V}}_{-i}\right)$ is independent of what i reports, we know from previous analysis that the above mechanism is still incentive compatible.

- What is S_i? Imagine that everyone is first assigned an equal share $1/n$ of the cost of G. $S_i\left(\tilde{\mathbf{V}}_{-i}\right)$ is the maximum surplus of all agents other that i if i does not participate at all while paying his own share $1/n$ of the cost;

- We can rewrite $\hat{\hat{\hat{T}}}_i\left(\tilde{\mathbf{V}}\right)$ as

$$\hat{\hat{\hat{T}}}_i\left(\tilde{\mathbf{V}}\right) = \frac{\overbrace{\hat{G}\left(\tilde{\mathbf{V}}\right)}^{\text{pre-assigned cost share}}}{n} \overbrace{+S_i\left(\tilde{\mathbf{V}}_{-i}\right) - \sum_{j\neq i}\left[\tilde{v}_j\left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) - \frac{\hat{G}\left(\tilde{\mathbf{V}}\right)}{n}\right]}^{\text{externality } i \text{ imposes on all other agents}}$$

- Remark: By definition,

$$S_i\left(\tilde{\mathbf{V}}_{-i}\right) - \sum_{j \neq i}\left[\tilde{v}_j\left(\hat{G}\left(\tilde{\mathbf{V}}\right)\right) - \frac{\hat{G}\left(\tilde{\mathbf{V}}\right)}{n}\right] \geq 0,$$

we know that

$$\begin{aligned} \hat{\hat{T}}_i\left(\tilde{\mathbf{V}}\right) &\geq \frac{\hat{G}\left(\tilde{\mathbf{V}}\right)}{n} \\ \Rightarrow \sum_{i=1}^{n} \hat{\hat{T}}_i\left(\tilde{\mathbf{V}}\right) &\geq \hat{G}\left(\tilde{\mathbf{V}}\right). \end{aligned}$$

Thus self-financing is achieved. The problem is, we may have budget surplus.

- How do we deal with the budget surplus?
  - If we throw away the budget surplus, then Pareto optimality is not achieved;
  - If we return the budget surplus to the agents, then the incentive compatibility of the CG mechanism may be defeated unless the refund rule is chosen wisely. Unfortunately, Lurwicz showed that there exists no tax scheme that always achieves efficiency, incentive compatibility, and exact budget balance.
- CG mechanism is the basis of the recent literature of the efficiency auction mechanisms.