Overconfidence, Morale and Wage-Setting Policies*

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Abstract

Psychologists have consistently documented people’s tendency to be overconfident about their own ability. We interpret workers’ confidence in their own skills as their morale. We investigate the implication of worker overconfidence on the firm’s optimal wage-setting policies. In our model, a wage contract both provides incentives and conveys to the worker the firm’s opinion about her ability, hence affects her morale. We provide conditions for the non-differentiation wage policy to be superior to the differentiation wage policy. In numerical examples, we show that worker overconfidence is a necessary condition for the firm to prefer no wage differentiation so as to preserve some workers’ morale; and a non-differentiation wage policy will itself breed more worker overconfidence, thus “overconfidence begets overconfidence.” Moreover, wage compression is more likely when aggregate productivity is low.

Keywords: Overconfidence, Worker Morale, Wage-setting Policies

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1 Introduction

Most of corporate America abstain from wage differentiation among workers assigned to similar tasks. Baker, Gibbs and Holmstrom (1994) studied the wage policy of a large firm and found that individual workers’ wages are largely determined by their cohorts (the year of entry into the firm) and their job levels. Wage differentiation among workers assigned to similar tasks are especially avoided when selective wage cuts are involved. Bewley’s (1999) pioneering survey of wage-setting practices of over three hundred business executives and personnel managers showed that an overwhelming number of firms believed that selective wage cuts would hurt worker morale, so they would lay workers off rather than offer them a lower wage.¹ It is worth adding that wage differentiation boosts the morale of some workers while hurting the morale of others. So these executives must mean that the benefit from morale boost to some workers is outweighed by the cost from morale loss to others.

Worker morale is a buzz word in the business world.² But what exactly is worker morale? How is morale affected by wage-setting practices, and vice versa? How does morale affect worker incentives? What determines whether a firm should adopt a differentiation or a non-differentiation wage policy? How do aggregate productivity fluctuations affect the benefits and costs of wage differentiation?

In Bewley’s (1999) survey, respondents view morale in many different aspects. A small manufacturing company owner responded that “Morale is having employees feel good about working for the company and respecting it. The employee with good morale likes his work ...”; a general manager of a large company said that “Moral equals motivation.” Bewley (1999) summarizes that morale meant “emotional attitudes toward work, co-workers, and the organization. Good morale meant a sense of common purpose consistent with company goals and meant cooperativeness, happiness or tolerance of unpleasantness, and zest for the job.” In this paper, we interpret a worker’s morale as her confidence in her own ability. This

¹Not all firms oppose to wage differentiation in the workplace. Former GE Chairman and CEO Jack Welch, for example, believed that strong workforces are built by treating individuals differently: “Some contend that differentiation is nuts – bad for morale. ... Not in my world.” (Welch, 2001)

²A search using keyword “morale” in Lexis-Nexis Academic Business News Database retrieves more than 1000 documents in the previous six months.
is consistent with some of the above responses but a little narrower, and we believe that it is a relevant view: in our model, a worker with high confidence of her own ability believes that she can “make a difference” (in increasing output and obtaining a high bonus) by exerting effort; thus, in our model, a worker’s morale is an intrinsic motivation. A worker has “high morale” when she thinks that her effort has a large impact on output; and conversely, a worker is demoralized when she believes that her costly effort is basically useless.

Because wage differentiation among retained employees hurts the morale of some workers, a firm may find it too costly to keep such demoralized workers and would rather lay them off altogether. An interesting puzzle arises because in many cases, laid-off workers believe that their relationship with the firm could still generate positive joint surplus, relative to the sum of their outside options. Why, then, would the firm choose not to keep the workers by lowering their wages? The standard answer rests on some form of non-transferable utility. Our answer is different: the workers simply over-estimate the surplus from the relationship because of their overconfidence.

In our model, a principal — the firm — hires many agents — the workers — to produce output. Each worker’s output depends on her own effort and ability, not on those of other workers. As is standard, effort is not contractible. But, ability is uncertain. Moreover, the principal privately observes a performance evaluation of each worker, which is informative about her ability, while workers observe each other’s contract offers. Hence, incentive contracts play a signaling besides their traditional allocative role, and affect worker incentives through both channels. The firm can either condition its wage offers on performance evaluations (differentiation policy), or conceal its opinion about workers’ abilities by offering the same contract to all employees (non-differentiation policy).

Our main innovation is to allow firm and workers to hold different initial beliefs regarding the workers’ ability. This assumption is motivated by the findings in psychological research (reviewed in the next section) that people tend to be overconfident about their own possession of any desirable traits, in particular, one’s own ability. We show that whether the differentiation or non-differentiation wage policy is optimal for the firm crucially depends on the divergence between the firm and workers in their initial beliefs. In simulated examples, wage differentiation dominates no differentiation when the firm and workers share identical
initial beliefs; but as the workers become more overconfident, the non-differentiation policy eventually dominates. The intuition is that, any wage differentiation will have two effects. The first is a *sorting effect*, which is beneficial to the firm since it allows it to tailor incentive contracts to each worker’s ability. The second is a *morale effect*, which is a double-edged sword: on the one hand, wage differentiation breaks bad news to some workers, depresses their morale, thus lowers their incentives to exert effort and hurts the firm’s profits (negative morale effect); on the other hand, it also breaks good news to other workers and boost their morale (positive morale effect). When workers are initially sufficiently more confident about their ability than the firm, the negative morale effect dominates the positive morale effect, and the difference can more than compensate the sorting effect, thus making wage differentiation undesirable.

In our model, the effect of wage differentiation on worker morale arises because of relative wage comparisons. Thus the non-differentiation wage policy is related to the concept of “fairness.” Indeed, unfairness by the employer is likely to impact on production through, if any, the resulting loss in worker morale. These ideas are appealing and intuitive from simple introspection, and form the basis for a prominent efficiency wage theory. Solow (1979), Akerlof (1982) and Akerlof and Yellen (1990) have pioneered the theoretical work on the effects of fairness considerations in wage-setting. Inspired by equity theory in social psychology and social exchange theory in sociology, and supported by ample field evidence, they postulated that worker effort depends not on the offered wage *per se*, but on its divergence from a “fair” reference wage, which usually depends on what other comparable workers earn. For example, Akerlof and Yellen (1990, p. 256) state that “[..] when people do not get what they deserve, they try to get even.”

Instead of assuming it, our model rationalizes the “fair-wage/effort” behavioral hypothesis by showing that, when workers suffer from the often observed human judgement bias, namely self overconfidence, firms may in fact find it optimal to treat workers equally despite different performance measurements. Related to our paper, Rotemberg (2002) studies a model in which employers possess some signals about the workers’ productivity, and in-

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3Bewley (1999) documented that, though it is practiced rarely, cross-the-board cuts in compensations, beginning with the management’s, would credibly signal the company’s trouble and entail no loss in morale.
vestigates the effect of the employee’s perception of the employer’s precision in evaluating individual abilities on income distributions. In his model, “fair” evaluation is akin to “accurate” evaluation. In contrast, in our model, it is the outcome, rather than the precision, of the firm’s performance evaluation that is unknown to the worker. Wage compression relative to expected productivities arises in our model because the wage-setting employer finds it optimal to strategically hide its information about the employees’ abilities in order to preserve morale.

Our paper is also related to an emerging economics literature that explores the connection between intrinsic and extrinsic motivations.4 Benabou and Tirole (2002, Section 2) analyze a mechanism design problem of a principal who is privately informed about the cost of a task that an agent is to perform. They show that, when both base wage and bonus are feasible, the only perfect Bayesian equilibrium satisfying the intuitive criterion of Cho and Kreps (1987) is separating, hence the firm’s incentive scheme always reveal its private information to the worker. In our model, the firm simultaneously employs many (in fact, a continuum of) workers. Thus, the common candidate deviation to break a pooling equilibrium is not valid in our context since any such proposed deviation contract will create an informational externality in that it also affects the morale of other workers who are not offered the same contract.

Prendergast (1991) also examines the issue of strategic information revelation by an informed principal to its workers, in the context of a training-promotion problem similar to our model, but with common prior beliefs. He did not show whether and under which conditions the firm will prefer a pooling contract. In our model with common priors, it never does.

Bewley (1999, Chapter 21) presents a model in which a worker’s realized pace of work is directly affected by her mood (Bewley’s conceptualization of morale) possibly through some unmodelled physiological process.5 In contrast, in our model morale is not a direct input of either the production or the utility function of the workers; instead, a worker’s morale affects her incentives by affecting her perception of the effects of her effort.

4See Kreps (1997) for a review of the related social and industrial psychology and economics literature.
5Also see Athreya (2002) for a model in which worker morale directly affects her cost of effort.
A similar dilemma of information revelation by a principal is analyzed by Feess, Schieble and Walzl (2001). In their model, an agent spends effort to forecast the quality of a project; and the principal could privately observe an additional informative signal about the project and decides, *ex ante*, whether to disclose his signal to the agent before she exerts effort. In our model, the principal can condition its disclosure decision on the private information realization. Our paper differs from both Benabou and Tirole (2002) and Feess et al. (2001) in that we consider a firm with many workers who observe each other’s received offers. Lizzeri, Meyer and Persico (2002) analyze a two-period principal agent or tournament model, where each agent can observe neither her first-period output nor her own ability. They focus on the desirability of performing interim performance evaluation and revealing interim output to the agents. In their setting, worker ability is only a parameter in the principal’s objective function, which affects neither the agent’s marginal productivity of effort (as in our model) nor its cost (as in Benabou and Tirole, 2002). Therefore, revelation of information on ability does not have the direct impact on the incentive of the agents that we call the “morale effect.”

The remainder of the paper is structured as follows. Section 2 provides a brief review of the large psychological literature underlying our behavioral assumption that workers are overconfident of their own ability or skills; Section 3 presents the model; Section 4 characterizes the optimal wage contract for any belief pair of the worker and the firm; Section 5 provides examples to illustrate when the non-differentiation wage-setting policy is optimal for the firm; Section 6 presents some discussion of the model and results; and finally, Section 7 concludes.

### 2 Psychological Evidence of Overconfidence

Psychological evidence of overconfidence is first and foremost reflected in the “*above average*” effect, whereby well over half of survey respondents typically judge themselves in

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6 Other authors have also provided similar reviews. See Babcock and Loewenstein (1997), Camerer and Lovallo (1999), Malmendier and Tate (2001), Compte and Postlewaite (2001), among others.
possession of more desirable attributes than fifty percent of other individuals. In Svenson (1981), 81 American and 80 Swedish students were asked to judge their skill in driving and how safe they were as drivers. The results showed an astonishing level of overconfidence. 82.5% of American and 51.4% of Swedish subjects rated themselves as safer than 70% of other driver. 92.8% of American and 68.7% of Swedish subjects rated themselves as above average in their driving skills. In particular, 46.3% of Americans regard themselves among the most skillful 20%. In Larwood and Whittaker (1977), 72 undergraduate management students and 48 presidents of New York state manufacturing firms are asked to rate themselves relative to their classmates or fellow presidents in IQ, likelihood of success, predicted growth in a hypothetical marketing problem, etc. The results indicate an overwhelming dose of overconfidence: of the 72 students, only 10 felt that they were merely of average intelligence relative to their own classmates and only 2 thought themselves below average; and only 18 of the 72 subjects predicted that their hypothetical firm’s sales would be below the industry average. The executive sample also predicted inordinate success, even though more moderate than the students. In Meyer (1975), less than 5% of employees rated themselves below the median.

Psychological evidence of overconfidence is also reflected in the “fundamental attribution error” (Aronson 1994), that is, people tend to attribute their successes to ability and skill, but their failure to bad luck or factors out of their control. Such self-serving biases are bound to lead to overconfidence. Psychologists have gathered a great deal of evidence for the observation that we take credit for the good and deny the bad. For example: (a) Students who do well on an exam attribute their performance to ability and effort, whereas those who do poorly attribute it to a poor exam or bad luck (Arkin and Maruyama 1979); (b) Gamblers perceive their success as based on skill and their failure as a fluke (Gilovich 1983); (c) When married couples estimate how much of the housework each routinely did, their combined total of housework performed amounts to more than 100 percent - in other words, each person thinks he or she did more work than their partner think he or she did (Ross and Sicoly 1979); (d) Two-person teams performing a skilled task accept credit for the good scores but assign most of the blame for the poor scores to their partner (Johnston 1967); (e) When

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7It seems that “above median” effect is a better term.
asked to explain why someone else dislikes them, college students take little responsibility for themselves (i.e., there must be something wrong with this other person), but when told that someone else likes them, the students attributed it to their own personality (Cunningham, Starr and Kanouse 1979).8

3 The Model

Initial Beliefs. Consider a firm who employs for one period a continuum of workers with unit measure. Workers differ in their ability (or interchangeably, productivity, talent) denoted by \( a \). To simplify the analysis, we assume that a worker’s ability \( a \in \{a_l, a_h\} \), where \( a_h > a_l > 0 \). We refer to \( a_l \) and \( a_h \) respectively as low and high ability. For each worker, neither she nor the firm knows the true value of her ability. The firm has an objective prior belief \( q_0 \) that each worker has high ability. We assume that the workers’ ability types are independent, hence by Large Numbers \( q_0 \) is also the proportion of high ability workers in the firm. In contrast, each worker has a prior belief \( p_0 \) that she has high ability. Importantly, we allow the worker and the firm to have heterogeneous beliefs about \( a \), that is, \( p_0 \) and \( q_0 \) do not have to be equal.9 In particular, as motivated by the psychological evidence reviewed in Section 2, we will be interested in the case when the workers are overconfident about their ability relative to the firm’s objective assessment, that is, when \( p_0 \geq q_0 \). The initial beliefs \( p_0 \) and \( q_0 \) are common knowledge, and when they are not equal, we assume that each party believes the other to be wrong in its assessment of ability.

Production Technology. For simplicity, we assume that each employed worker can pro-

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8Written reports given by drivers involved in automobile accidents provides less systematic, but equally telling evidence of self-serving bias: (1) The telephone pole was approaching fast; I attempted to swerve out of its way, when it struck the front of my car; (2) An invisible car came out of nowhere, struck my vehicle, and vanished; (3) My car was legally parked as it backed into the other vehicle; (4) As I reached an intersection, a hedge sprang up, obscuring my vision. I did not see the other car; (5) A pedestrian hit me and went under my car. (San Francisco Sunday Examiner and Chronicle, April 22, 1979, p. 35).

9We are aware that heterogenous priors violate Harsanyi’s doctrine. See Morris (1995) for a persuasive argument that neither Bayesian decision theory nor standard theories of rationality requires agents to have the same priors. See Morris (1994) and Allen and Gale (1999) for models in which agents have heterogenous prior beliefs.
duce two levels of output, which are, without loss of generality, normalized to 0 (low output) or $y$ (high output) where $y > 0$. The production technology is stochastic as follows: if a worker with ability $a_j, j \in \{h, l\}$ exerts effort $n \geq 0$, then the probability of high output is

$$\Pr (Y = y | a_j, n) = \pi_j (n), j \in \{h, l\}$$

where $\pi_j' > 0, \pi_j'' < 0$ and satisfy $\pi_j (0) = 0, \lim_{n \to \infty} \pi_j (n) = 1$ and $\pi_h (n) > \pi_l (n)$ for all $n > 0$. In words, we assume that a worker produces low output unless she exerts positive effort. We assume that output realizations conditional on individual values of $a$ and $n$ are independent across workers. An important and interesting case is that ability and efforts are complements in production, but most of our results can be obtained without such an assumption (see Section 4.1 for more details).

The cost of effort is independent of the worker’s ability and is given by $C (n)$ with $C (0) = 0, C' > 0$ and $C'' \geq 0$. Following the standard assumptions in principal-agent models, we assume that a worker’s effort is not observable to the firm, while her output is observable and verifiable by all parties (including possibly the court).

**Performance Evaluation.** The firm, before beginning (or continuing) the relationship, receives a signal $\theta \in \{\theta_h, \theta_l\}$ of each individual’s ability. There are a couple of possible interpretations of the signal $\theta$: it could be the test results after training period if the relationship is new; or it could be non-output performance evaluation of the worker’s ability from the previous periods if the relationship is continuing. The signals are independently and identically drawn as follows:

$$\alpha = \Pr \{\theta_h | a = a_h\} = \Pr \{\theta_l | a = a_l\},$$

where $\alpha > 1/2$ measures the signal’s informativeness of a worker’s ability. Importantly, it is assumed that the realization of each worker’s signal $\theta$ is privately observed by the firm. A worker does not directly observe her performance evaluation $\theta$, but she may possibly infer $\theta$ from the wage contract offered by the firm.

**Belief Updating and Morale.** The firm uses each worker’s signal $\theta$ to update its belief about her ability. We write $q_\theta$ as the firm’s posterior belief from Bayes’ rule that a worker
with signal $\theta$ has high ability, that is,

$$q_h = \frac{q_0 \alpha}{q_0 \alpha + (1 - q_0) (1 - \alpha)},$$

$$q_l = \frac{q_0 (1 - \alpha)}{q_0 (1 - \alpha) + (1 - q_0) \alpha}.$$  

A worker does not directly observe her performance evaluation $\theta$, but if she ever infers $\theta$ from contract offers (that is, when the firm offers different wage contracts to workers with different performance evaluations), she will form a posterior belief $p_\theta$ regarding her ability by Bayes’ rule:

$$p_h = \frac{p_0 \alpha}{p_0 \alpha + (1 - p_0) (1 - \alpha)},$$

$$p_l = \frac{p_0 (1 - \alpha)}{p_0 (1 - \alpha) + (1 - p_0) \alpha}.$$  

Note that we allow the firm and the workers to agree to disagree on their priors and their subsequent interpretations of the performance evaluations. A worker’s confidence about her ability is what we mean by worker morale in this paper: $p_0$ is the worker’s initial morale; and $p_\theta$ is her morale if she learns her performance evaluation from the wage offers. It is clear that knowing that she receives a low evaluation hurts, while a high evaluation boosts, a worker’s morale since $p_l < p_0 < p_h$.

Since the firm employs a continuum of workers, it knows beforehand, by Large Numbers, that a measure $q_0$ of workers has ability $a_h$, a measure $\alpha q_0 + (1 - \alpha) (1 - q_0)$ of workers will obtain signal realizations $\theta_h$, and so on. In other words, the firm does not encounter any aggregate surprises. The role of the performance evaluations for the firm is to identify probabilistically (“sort”) workers of ability $a_h$. After observing the signals, the firm knows again by Large Numbers that a fraction $q_h$ of those workers with high evaluations is indeed of ability $a_h$; while for the remaining $1 - q_h$ fraction of the high evaluations was inaccurate. The firm reasons similarly for workers for whom it has given a poor performance evaluation $\theta_l$.

Preferences. The firm and the workers are risk-neutral. To make the problem interesting, we assume that the workers have limited liability, i.e., a worker’s total compensation cannot be negative. The outside option of the firm, namely the value of a vacancy, is $V_0 \geq 0$;
and the outside option of a worker, namely the value of unemployment, is $U_0 \geq 0$ at the beginning of this relationship. Recall that the low output realization was normalized to zero. We assume that both $U_0$ and $V_0$ are constant. In particular, the worker’s outside option $U_0$ is independent of her morale, which seems reasonable only when ability is firm-specific. It will become clear that pooling is more likely to dominate separating contracts if $U_0$ is instead increasing and convex in the worker’s morale.

**Wage Contract.** Because the output level is assumed to be either high or low, a contingent wage contract is simply a *two-tier contract* $\{w, \Delta\}$, where $w$ is the *base wage* to be received regardless of the level of output, and $\Delta$ is the *bonus* to be paid only when output is high. The firm decides, for each worker, whether to offer her a contract, and if so, the terms of contract. It is assumed that the firm can credibly commit to any contract. We assume that each worker can observe all wage contracts, or lack thereof, offered by the firm to her colleagues. A worker compares her wage contract with others’, makes inference about her performance evaluation privately observed by the firm, and adjust her morale accordingly.

The firm can adopt two possible wage-setting policies. The first is a *non-differentiation policy* (or interchangeably, pooling contract policy) under which the firm offers the same wage contract to all workers irrespective of their individual performance evaluations. A worker does not induce any inference on her own ability, thus her morale is maintained at the initial level $p_0$. The second is a *differentiation policy* (or interchangeably, separating contracts policy) under which different wage contracts are offered according to the individual performance evaluations. A worker infers in equilibrium her individual performance evaluation and updates her morale to $p_h$ or $p_l$ accordingly. We are interested in when the firm will

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10 In principle, the firm could condition payments to a worker on the entire distribution of outputs, as in a tournament. However, by Large Numbers, for any set of contract offers the resulting distribution of outputs is known beforehand by each worker, and does not affect her incentives. Hence, our two-tier contract covers this case too.

11 This assumption might appear unrealistic in many contexts, as it is well-known that many employers consider wage information disclosure taboo, occasionally prohibiting it formally in their labor contracts. However, this kind of secrecy policy is hard to implement in practice. In addition, for those cases where information-sharing is effectively prevented, our analysis maybe useful as a counterfactual, to understand *why* firms have an interest in such confidentiality rules and workers in breaking them.
find differentiation or non-differentiation wage-setting policies optimal.

4 Analyzing the Model

In this section, we characterize the firm’s optimal wage contract for a generic belief pair \( \{p, q\} \) regarding the worker’s ability. The relevant values of \( p \) and \( q \) may depend on whether differentiation or non-differentiation wage-setting policies are in consideration.

4.1 Worker’s Problem

Suppose that a worker with morale \( p \) is offered a wage contract \( \{w, \Delta\} \), she chooses whether to accept the employment, and if so, her optimal effort level \( n^* \) by solving problem (1) below:

\[
\max \left\{ \max_{n \geq 0} \left\{ w + \Delta [p \pi_h(n) + (1 - p) \pi_i(n)] - C(n) \right\} ; \ U_0 \right\},
\]

where the inner maximizing problem yields her expected utility when she accepts the contract and optimally chooses effort. If we temporarily ignore the non-negativity constraint on \( n \), the inner maximization problem is concave with the necessary and sufficient first order condition:

\[
\Delta [p \pi_h'(n) + (1 - p) \pi_i'(n)] = C''(n),
\]

which yields a unique optimal level of effort \( n^*(\Delta, p) \).

For \( n^*(\Delta, p) \) to be positive, the bonus must be large enough. Simple algebra shows that \( n^*(\Delta, p) \geq 0 \) if and only if

\[
\Delta \geq \frac{C'(0)}{p \pi_h'(0) + (1 - p) \pi_i'(0)} \equiv \underline{\Delta}
\]

If a firm does not find it optimal to provide such a high enough bonus, then such workers with morale \( p \) will be laid off. A firm is willing to offer a bonus at least \( \underline{\Delta} \) only if \( y > \underline{\Delta} \); otherwise, due to our limited liability assumption on the worker, the firm will for sure lose money. To make things interesting, we take this as an assumption:

**Assumption 1** \( y > \underline{\Delta} \).
Not surprisingly, the worker’s effort level does not depend on the base wage $w$ and is increasing in the level of bonus $\Delta$. It is not definite whether $n^* (\Delta, p)$ increases in a worker’s morale $p$. However, it follows from implicit function theorem that $n^* (\Delta, p)$ increase in $p$ if ability and effort satisfy the following local complements condition.

**Definition 1 (Local Complements)** Ability and effort are local complements at effort level $n^*$ if $\pi_h'(n^*) > \pi_i'(n^*)$.

Now we discuss under what restrictions on the primitives of the model will the local complements condition will be satisfied. Since both $\int_0^\infty \pi_h'(n) \, dn = \int_0^\infty \pi_i'(n) \, dn = 1$, ability and effort can not be complements for every level of $n$. That is, in our setup, if ability and effort are complements in some regions, they must be local substitutes in some other regions. However, we can provide a sufficient condition for ability and effort to be local complements at the possible optimal effort level $n^*$. Suppose that $\pi_h'' < \pi_i'' < 0$, then it can be shown that there exists an effort level $\hat{n}$ such that $\pi_h'(\cdot)$ crosses $\pi_i'(\cdot)$ from above at $\hat{n}$. In other words, $\pi_h'(n) > \pi_i'(n)$ if and only if $n < \hat{n}$ and $\pi_h(\hat{n}) = \pi_i(\hat{n})$. To induce an effort level $\hat{n}$ from the worker with morale $p$, the corresponding bonus level $\hat{\Delta}$ must satisfy the first order condition (2) evaluated at $n = \hat{n}$, i.e.,

$$\hat{\Delta} \left[ p \pi_h'(\hat{n}) + (1 - p) \pi_i'(\hat{n}) \right] = C'(\hat{n}).$$

By the definition of $\hat{n}$, we have $\pi_h'(\hat{n}) = \pi_i'(\hat{n})$. Thus, $\hat{\Delta} = C'(\hat{n}) / \pi_h'(\hat{n})$. If $\hat{\Delta} \geq y$, then the firm will never optimally choose to offer a bonus as high as $\hat{\Delta}$; which in turn guarantees that the effort level induced by the firm under the optimal contract will always be less than $\hat{n}$. Hence the local complements condition will be satisfied.

In our model, when the local complements condition is satisfied at $n^*$, morale serves an intrinsic motivation; and the strength of the intrinsic motivation is affected by the extrinsic motivation, i.e., the firm’s wage policy. The following lemma provides sufficient conditions under which the optimal effort $n^*$ is concave in $\Delta$, a property that will be useful later. Its proof is relegated in the appendix.

**Lemma 1** Sufficient conditions for $n^*(\cdot, p)$ to be concave in $\Delta$ are:
1. $C''' \geq 0$, and

2. $p\pi'_h + (1 - p)\pi'_l$ is log-concave.

4.2 Firm’s Problem

Now we analyze the firm’s choice of optimal contract for a single worker, given posterior beliefs $q$ and $p$ on ability. The posteriors are generated by prior beliefs $q_0, p_0$, performance evaluation outcome $\theta$, and information on $\theta$ revealed to the worker by the contract itself (if any).

Given a wage contract $\{w, \Delta\}$ and the worker’s subsequent optimal effort choice $n^*$, the perceived expected utility of a worker with morale $p$ from accepting the wage contract $\{w, \Delta\}$, is

$$U(w, \Delta; p) = w + [p\pi_h(n^*(\Delta, p)) + (1 - p)\pi_l(n^*(\Delta, p))] \Delta - C(n^*(\Delta, p)); \quad (4)$$

which must exceed the outside option for the contract to be acceptable:

$$U(w, \Delta; p) \geq U_0. \quad \quad (5)$$

The perceived expected profit to the firm with belief $q$ from offering a contract $\{w, \Delta\}$ to a worker with morale $p$ is

$$V(w, \Delta; p, q) = [q\pi_h(n^*(\Delta, p)) + (1 - q)\pi_l(n^*(\Delta, p))] (y - \Delta) - w \quad \quad (6)$$

which must exceed the outside option for the contract to be offered

$$V(w, \Delta; p, q) \geq V_0. \quad \quad (7)$$

**Definition 2** A contract $\{w, \Delta\}$ is $(p, q)$-feasible if it satisfies (5) and (7).

The firm’s problem is

$$\max_{\{w \geq 0, \Delta \geq \Delta\}} V(w, \Delta; p, q)$$

s.t. (5), (7). \quad \quad (8)

**Lemma 2** The solution to the firm’s problem (8) exists.
Proof. Note that the firm’s expected output is bounded above by \( qy \) (when \( n = \infty \)). Since the firm can guarantee itself \( V_0 \geq 0 \), the firm will for sure choose \( w \leq qy \) and \( \Delta \leq y \). We can without loss of generality restrict the firm’s feasible contract set to

\[
\{(w, \Delta) : w \in [0, qy], \quad \Delta \in [\Delta, y], \quad U(w, \Delta; p) \geq U_0\},
\]

Since \( U \) is continuous in \((w, \Delta)\), this is a compact set. Since \( V(w, \Delta; p, q) \) is continuous, by Weierstrass Theorem it has a maximum, which we can then compare with \( V_0 \). Thus (8) always has a solution.

We note a simple lemma:

**Lemma 3** In any optimal contract \( \{w, \Delta\} \), the worker’s participation constraint (5) must bind if \( w > 0 \).

**Proof.** If \( w > 0 \) but (5) does not bind, then the firm could slightly reduce \( w \) without changing the incentives to exert effort, since \( n^* \) depends only on the bonus \( \Delta \); and without violating the participation constraint of the worker. This increases the firm’s profits, a contradiction to the firm’s optimality.

Workers’ participation constraint in general will be slack because of limited liability assumption.

We now establish that when the worker is at least as confident as the firm, a case we focus on, then the firm can without loss of generality only choose a level of bonus and no base wage:

**Proposition 1** If \( p \geq q \), then any optimal contract has a zero base wage.

**Proof.** The proof is by contradiction. If there exists an optimal contract with \( w > 0 \), then by Lemma 3 the worker’s participation constraint must bind. Hence we can solve for \( w \) from (5) holding as an equality to obtain

\[
w = U_0 - [p\pi_h(n^*(\Delta, p)) + (1 - p)\pi_l(n^*(\Delta, p))]\Delta + C(n^*(\Delta, p)).
\]
Replacing it into the objective function of the firm in the inner maximization problem of (8), we obtain after some simplification,

\[
\max_{\Delta \in [\Delta, y]} \left\{ \Delta (p - q) \left[ \pi_h (n^* (\Delta, p)) - \pi_l (n^* (\Delta, p)) \right] \\
+ \left[ q \pi_h (n^* (\Delta, p)) + (1 - q) \pi_l (n^* (\Delta, p)) \right] (y - \Delta) \right\}.
\]

The first order derivative with respect to \( \Delta \) is

\[
(p - q) \left[ \pi_h (n^* (\Delta, p)) - \pi_l (n^* (\Delta, p)) \right] + [q \pi_h' (n^* (\Delta, p)) + (1 - q) \pi_l' (n^* (\Delta, p))] (y - \Delta) \frac{\partial n^* (\Delta, p)}{\partial \Delta}
\]

which is strictly positive whenever \( \Delta < y \) and \( p \geq q \). Hence the firm maximizes profits by loading maximum incentives and offering \( \Delta = y \). But then the firm never obtains any output and pays a positive base wage \( w > 0 \). Thus the firm’s expected profit is negative by offering the optimal wage contract with \( w > 0 \), violating the firm’s participation constraint. A contradiction.

Note, by continuity, Proposition 1 should also hold for any \( p < q \) as long as it is sufficiently close to \( q \); but, when the worker is sufficiently under-confident, the optimal base wage \( w \) may be strictly positive. The reason is as follows: when \( q \) is sufficiently larger than \( p \), the firm does not want to offer too high a bonus because it expects the worker to receive it very often; however, a low bonus alone may not satisfy the worker’s participation constraint, thus the firm has to make it up with a strictly positive base wage.

When \( p \geq q \), Proposition 1 tells us that the optimal contract will have a zero base wage. Thus we can re-write the firm’s problem (8) as

\[
\max_{\Delta \in [\Delta, y]} \left\{ [q \pi_h (n^* (\Delta, p)) + (1 - q) \pi_l (n^* (\Delta, p))] (y - \Delta) \right\};
\]

s.t. (5), (7).

(9)

To characterize the solution to problem (9), we first ignore the participation constraints (5), (7) and check them \textit{ex post}.

\textbf{Lemma 4} If \( n^* (\cdot, p) \) is concave in \( \Delta \), then the inner maximization problem in (9) has a
unique solution $\Delta^* (p, q) \in (\Delta, y)$ which implicitly solves the following equation:

$$0 = \frac{\partial V (0, \Delta; p, q)}{\partial \Delta}$$

$$= (y - \Delta) [q \pi_h (n^* (\Delta, p)) + (1 - q) \pi_l (n^* (\Delta, p))] \frac{\partial n^* (\Delta, p)}{\partial \Delta}$$

$$- [q \pi_h (n^* (\Delta, p)) + (1 - q) \pi_l (n^* (\Delta, p))] ;$$

(10)

**Proof.** Eq. (10) provides the first order condition for the inner problem in (9). Clearly, the second derivative of the value is negative if $n^* (\cdot, p)$ is concave in $\Delta$. Evaluated at $\Delta$, the second term of Eq. (10) is zero and thus smaller than the first term; and evaluated at $y$, the first term is zero and thus larger than the first term. Therefore, there is a unique solution $\Delta^* (p, q) \in (\Delta, y)$ to Eq. (10).

The candidate optimal bonus $\Delta^* (p, q)$ is increasing in output in case of success $y$ which follows directly from applying the Implicit Function Theorem on Eq. (10). Our model does not yield unambiguous relationships between the optimal bonus $\Delta^* (p, q)$ and worker morale $p$ or firm belief $q$. One can show, however, that $\Delta^* (p, q)$ will be strictly decreasing in $p$ if $\partial^2 n^* (\Delta, p) / \partial \Delta \partial p \leq 0$, which is not a general property of the optimal effort function determined in (2).

Finally, applying the Envelope Theorem on Problem (9), we immediately have:

**Proposition 2** If ability and effort are local complements at $n^* (\Delta^* (p, q), p)$, then the firm’s maximum expected profit from hiring a worker with morale $p$, $V (0, \Delta^* (p, q); p, q)$, is strictly increasing in $p$ for any fixed $q$.

Thus, when ability and effort are local complements, it is desirable for the firm to hire overconfident workers because worker overconfidence serves as the intrinsic motivation for higher effort, offsetting the moral hazard inefficiency.

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12It is interesting to contrast with Benabou and Tirole (2002), which make a stark prediction that, in equilibrium, a higher bonus is necessarily associated with bad news in the sense that a principle will offer a higher bonus only when she knows the task to be more difficult. The main reason is that in Benabou and Tirole’s model, the agent makes a discrete choice of whether or not to exert effort, which is affected in a multiplicatively separable fashion by the principal’s private information (if revealed) and the bonus.
So far, we have neglected firm’s and worker’s participation constraint in problem (9). The solution $\Delta^* (p, q)$ characterized in Lemma 4 will be the actual solution to problem (8) if \{0, $\Delta^* (p, q)$\} is $(p, q)$-feasible. Otherwise, we consider two cases: (1). If \{0, $\Delta^* (p, q)$\} violates the firm’s participation constraint (7), then the firm will offer no contract to a worker with morale $p$, and lay her off. The reason is simple: by definition $V (0, \Delta^* (p, q); p, q)$ is the maximum the firm can achieve by choosing $\Delta$ when $w = 0$ (recall that we established in Proposition 1 that no optimal contract with $w > 0$ exists). (2). If the firm’s participation constraint (7) holds but the worker’s (5) fails at \{0, $\Delta^* (p, q)$\}, then the firm will have to consider a new problem by offering a higher bonus level. Specifically, the firm will optimally pick a bonus that makes the worker just willing to participate. Such a bonus level, denoted by $\bar{\Delta} (p)$, solves

$$U (0, \bar{\Delta}; p) = U_0. \quad (11)$$

It is easy to see that $\bar{\Delta} (p) > \Delta^* (p, q)$ because the left-hand side of Eq. (11) is increasing in $\Delta$. The firm is willing to offer a contract \{0, $\bar{\Delta} (p)$\} if and only if its expected surplus from offering such a contract yields more than its outside option $V_0$, i.e.,

$$V (0, \bar{\Delta} (p); p, q) \geq V_0. \quad (12)$$

We summarize the above discussions in the following:

**Proposition 3** Fix any generic belief pair by a worker and the firm $(p, q)$, with $p \geq q$. Consider $\Delta^* (p, q)$ and $\bar{\Delta} (p)$ respectively defined by (10) and (11).

1. If \{0, $\Delta^* (p, q)$\} is $(p, q)$-feasible, then it is the optimal contract;

2. If \{0, $\Delta^* (p, q)$\} is not $(p, q)$-feasible, but \{0, $\bar{\Delta} (p)$\} satisfies (12), then \{0, $\bar{\Delta} (p)$\} is the optimal contract;

3. In any other cases, the firm offers no contract to the worker.

We introduce the following notation to ease exposition below:

$$\bar{\Delta} (p, q) = \begin{cases} 
\Delta^* (p, q) & \text{if } \{0, \Delta^* (p, q)\} \text{ is } (p, q)\text{-feasible} \\
\bar{\Delta} (p) & \text{if } \begin{cases} 
\{0, \Delta^* (p, q)\} \text{ is not } (p, q)\text{-feasible, but} \\
\{0, \bar{\Delta} (p)\} \text{ satisfies (12)} 
\end{cases} \\
\emptyset & \text{otherwise.} 
\end{cases} \quad (13)$$
where $\emptyset$ stands for "no contract". Also define $\hat{V}(p, q)$ as the firm’s maximal profit from offering the optimal contract (including the null contract), i.e.,

$$\hat{V}(p, q) = V\left(0, \hat{\Delta}(p, q); p, q\right)$$

where $V(0, \emptyset; p, q) \equiv V_0$.

### 4.3 The Optimal Contract

Proposition 3 characterizes the firm’s optimal contract (or no contract) to a single worker for a generic belief pair $(p, q)$ with $p \geq q$. We now characterize the optimal set of contracts offered to all workers, taking into account the effect on the firm’s beliefs of observing performance evaluations and on the workers’ beliefs of observing the contract offers.

**Non-Differentiation Wage Policy.** If the firm adopts a non-differentiation wage-setting policy, every worker will maintain her morale at the original level $p_0$. However, the firm still possesses private information regarding each individual worker. That is, the firm still knows precisely who obtains good performance evaluation $\theta_h$ and who obtains bad performance evaluation $\theta_l$. The firm’s optimal bonus under a non-differentiation policy is the solution to the following problem:

$$\max \left\{ \max_{\Delta \in [\Delta, \hat{\Delta}]} \left\{ \left[ \alpha q_0 + (1 - \alpha) (1 - q_0) \right] V(0, \Delta; p_0, q_h) + \left[ \alpha (1 - q_0) + (1 - \alpha) q_0 \right] V(0, \Delta; p_0, q_l) \right\} ; \quad V_0 \right\}$$

s.t. (5).

(14)

where the first term in the objective function of the inner maximization problem is the firm’s expected profit from all the workers who have received $\theta_h$; and the second term is the firm’s expected profit from all the worker who have received $\theta_l$. Recall that from law of large numbers, a measure $\alpha q_0 + (1 - \alpha) (1 - q_0)$ of workers receive a good performance evaluation, and the remaining $\alpha (1 - q_0) + (1 - \alpha) q_0$ measure receive a bad evaluation.

Using the linearity of the function $V(w, \Delta; p, q)$ in $q$ [see (6)] and the martingale property
of the firm’s belief, we immediately have

\[
[\alpha q_0 + (1 - \alpha) (1 - q_0)] V (0, \Delta; p_0, q_h) + [\alpha (1 - q_0) + (1 - \alpha) q_0] V (0, \Delta; p_0, q_l)
\]

\[
= [q_0 \pi_h (n^* (\Delta, p_0)) + (1 - q_0) \pi_l (n^* (\Delta, p_0))] (y - \Delta).
\]

Thus, problem (14) is identical to problem (9). This implies that the optimal bonus under a non-differentiation policy is exactly given by \( \hat{\Delta} (p_0, q_0) \). Thus we reach an important conclusion: under a non-differentiation policy, the firm’s private performance evaluation is irrelevant, both strategically (by definition of pooling) and statistically because of the law of large numbers and expected utility. The firm will offer a contract \( \hat{\Delta} (p_0, q_0) \) as described by (13) to all workers and its expected surplus under a non-differentiation policy is \( \hat{V} (p_0, q_0) \).

Differentiation Wage Policy. If the firm adopts a differentiation wage-setting policy, then the contracts will reveal to each worker the performance evaluation privately observed by the firm. Thus, the firm offers the contract \( \{0, \hat{\Delta} (p_\theta, q_\theta)\} \) to a worker with evaluation \( \theta \in \{\theta_h, \theta_l\} \). Thus overall the firm’s expected surplus under a differentiation policy is

\[
[\alpha q_0 + (1 - \alpha) (1 - q_0)] \hat{V} (p_h, q_h) + [\alpha (1 - q_0) + (1 - \alpha) q_0] \hat{V} (p_l, q_l).
\]

Comparing Wage Policies. The difference between the firm’s expected profits from the differentiation and non-differentiation policies can be usefully decomposed into three components as follows:

\[
[\alpha q_0 + (1 - \alpha) (1 - q_0)] \hat{V} (p_h, q_h) + [\alpha (1 - q_0) + (1 - \alpha) q_0] \hat{V} (p_l, q_l) - \hat{V} (p_0, q_0)
\]

\[
= \left[ [\alpha q_0 + (1 - \alpha) (1 - q_0)] \left[ \hat{V} (p_h, q_h) - \hat{V} (p_0, q_0) \right] 
+ [\alpha (1 - q_0) + (1 - \alpha) q_0] \left[ \hat{V} (p_l, q_l) - \hat{V} (p_0, q_0) \right] \right]
\]

\[
+ [\alpha q_0 + (1 - \alpha) (1 - q_0)] \left[ \hat{V} (p_h, q_h) - \hat{V} (p_0, q_h) \right]
\]

\[
+ [\alpha (1 - q_0) + (1 - \alpha) q_0] \left[ \hat{V} (p_l, q_l) - \hat{V} (p_0, q_l) \right].
\]

(15)
The first component is the sorting effect. It captures the gain under a differentiation policy if the firm can tailor workers’ contracts according to their individual performance evaluations without, hypothetically, altering the worker’s morale. Standard revealed-profit-maximization arguments (see Lemma 5 below) show that the sorting effect is non-negative, thus in favor of differentiation policy. The second component captures the gain due to higher morale from informing workers with high performance evaluation of the good news. When ability and effort are local complements, this effect is non-negative since \( \tilde{V}(\cdot, q) \) is increasing in the worker’s morale \( p \) under the local complements condition. Hence we call this effect as the positive morale effect. The last term captures the loss due to lower morale from informing workers with low performance evaluation of the bad news. Again under the local complements condition, it is non-positive due to the monotonicity of \( \tilde{V}(\cdot, q) \) in the worker’s morale \( p \). Thus we call this effect as the negative morale effect.

**Lemma 5** If \( \Delta(p_0, q_j) = \Delta^*(p_0, q_j), j \in \{ 0, h, l \}, \) then the sorting effect is strictly positive for almost all \( q_0 \in (0, 1) \), and \( p_0 \geq q_0 \).

**Proof.** Under the assumption,

\[
\tilde{V}(p_0, q_0) = [q_0 \pi_h (n^*(\Delta^*(p_0, q_0), p_0)) + (1 - q_0) \pi_l (n^*(\Delta^*(p_0, q_0), p_0))] [y - \Delta^*(p_0, q_0)]
\]

\[
= [\alpha q_0 + (1 - \alpha) (1 - q_0)] \begin{bmatrix}
q_h \pi_h (n^*(\Delta^*(p_0, q_0), p_0)) \\
+ (1 - q_h) \pi_l (n^*(\Delta^*(p_0, q_0), p_0))
\end{bmatrix} [y - \Delta^*(p_0, q_0)]
\]

\[
+ [\alpha (1 - q_0) + (1 - \alpha) q_0] \begin{bmatrix}
q_l \pi_h (n^*(\Delta^*(p_0, q_h), p_0)) \\
+ (1 - q_l) \pi_l (n^*(\Delta^*(p_0, q_h), p_0))
\end{bmatrix} [y - \Delta^*(p_0, q_h)]
\]

\[
< [\alpha q_0 + (1 - \alpha) (1 - q_0)] \begin{bmatrix}
q_h \pi_h (n^*(\Delta^*(p_0, q_l), p_0)) \\
+ (1 - q_l) \pi_l (n^*(\Delta^*(p_0, q_l), p_0))
\end{bmatrix} [y - \Delta^*(p_0, q_l)]
\]

\[
+ [\alpha (1 - q_0) + (1 - \alpha) q_0] \begin{bmatrix}
q_l \pi_h (n^*(\Delta^*(p_0, q_l), p_0)) \\
+ (1 - q_l) \pi_l (n^*(\Delta^*(p_0, q_l), p_0))
\end{bmatrix} [y - \Delta^*(p_0, q_l)]
\]

\[
= [\alpha q_0 + (1 - \alpha) (1 - q_0)] \tilde{V}(p_0, q_h) + [\alpha (1 - q_0) + (1 - \alpha) q_0] \tilde{V}(p_0, q_l),
\]

where the second equality follows from the martingale property of Bayes’ updating; and the inequality follows from revealed profit maximization of the firm and the fact that the firm’s objective function in problem (9) is nonlinear.
The only instances that the sorting effect is zero are either (1) \( q_0 \in \{0, 1\} \); or (2) optimal contracts under all belief pairs \( \{p_0, q_j\}; j \in \{0, h, l\} \), are all \( \bar{\Delta}(p_0) \); or (3) no contracts are offered under all belief pairs \( \{p_0, q_j\}; j \in \{0, h, l\} \).

In general, it is difficult to determine the net trade-offs of differentiation and non-differentiation policies for any initial pair of beliefs \((p_0, q_0)\). Indeed, we rely on numerical examples in Section 5 to illustrate these trade-offs. However, we have the following two general characterizations for some special cases of initial belief pairs:\(^\text{13}\)

**Proposition 4** For any \( q_0 \in (0, 1) \),

1. the firm’s expected profit under a differentiation policy is higher than that under a non-differentiation policy as \( p_0 \to 1 \);

2. the firm’s expected profit is higher under a differentiation policy for any \( p_0 = q_0 \) if and only if \( \bar{V}(p, p) \) is convex in \( p \).

**Proof.** To prove the first statement, note that from Lemma 5, we know that the sorting effect is strictly positive for any \( q_0 \in (0, 1) \), but the morale effects are equal to zero when \( p_0 \) is equal to 1. The conclusion then follows from continuity.

To prove the “if” part of the second statement, note that if \( p_0 = q_0 \), we have \( p_h = q_h, p_l = q_l \); moreover, \( p_0 \) and \( q_0 \) are convex combinations of \( p_h \) and \( p_l \) with weights exactly given by \( \alpha q_0 + (1 - \alpha)(1 - q_0) \) and \( \alpha (1 - q_0) + (1 - \alpha)q_0 \) respectively. The “only if” part obviously follows from the definition of convexity.

In Bayesian decision theory the value function is convex in beliefs by a simple revealed preference argument and the martingale properties of beliefs. Simply put, an informative signal is always beneficial ex-ante. This force is at play here too, and is behind the sorting effect of private information. The firm wants to allocate efficiently workers that it perceives having different abilities to different carefully tailored incentive contracts. As indicated above, this effect emerges formally if we fix worker’s beliefs and shut down the signaling

\(^{13}\)The first statement of Proposition 4 provides a rationale of Jack Welch’s management philosophy mentioned in footnote 1.
role of wage offers, thus preserving only the decision-theoretic part of the mechanism design problem.

However, here contract offers have also a signaling role, and exert a \textit{morale effect} by altering the worker’s beliefs. Due to this strategic interaction, in general one is unable to establish the convexity of $\hat{V}(p, p)$, and Proposition 4 is not generally applicable. What Proposition 4 provides is a simple characterization of the superiority of the differentiation policy in terms of convexity of $\hat{V}(p, p)$ when the firm and workers share identical initial beliefs $p_0 = q_0$. If instead, the workers are overconfident, i.e. $p_0 > q_0$, then convexity no longer suffices. In either case, we show below by parametric examples that a non-differentiation wage-setting policy may indeed dominate a differentiation policy.

The main insight of this paper is that the initial divergence of the levels of confidence between the firm and workers will naturally lead to the changes in magnitudes of the sorting and morale effects. In particular, moderate level of worker overconfidence are likely to enlarge the negative morale effects, and overwhelm the sorting and positive morale effects, thus making non-differentiation policy superior as demonstrated in numerical examples in Section 5.

5 \textbf{Numerical Illustrations}

In this section, we present a parametric example of the above general model, by specifying the functions $\pi_h(\cdot), \pi_l(\cdot)$ and $C(\cdot)$, and show that worker overconfidence is an important determinant of whether the firm will favor a non-differentiation wage-setting policy.

We specify that both the probability functions of high output and effort cost functions are exponential. More specifically, let

\[
\pi_j(n) = 1 - \exp\{-a_j n\}, \quad j \in \{h, l\} \\
C(n) = \exp\{\lambda n\} - 1
\]

where \(\lambda > 0\). Note that $C(0) = 0$. In this formulation, ability and effort are strategic complements in production. Notice that $C''' > 0$ and $p \pi_h' + (1 - p) \pi_l'$ is log-concave.

When offered a wage bonus $\Delta$, the optimal effort level $n^*(p, \Delta)$ of a worker with morale
p is the implicit solution to the following equation (which is simply Eq. 2 for this example):

\[
[pa_h \exp \{-a_h n\} + (1 - p) a_l \exp \{-a_l n\}] \Delta = \lambda \exp \{\lambda n\}.
\]

The optimal bonus of the firm \(\Delta^* (p, q)\) is obtained by numerically solving Eq. 10.

### 5.1 Understanding the Trade-offs

First, we set the outside options of the firm and the worker to zero (i.e., \(U_0 = V_0 = 0\)) so that the optimal contract in Proposition 3 is given by \(\Delta^* (p, q)\) for every pair \((p, q)\).

We first show that the non-differentiation policy can dominate the differentiation policy when the worker’s initial morale \(p_0\) is sufficiently higher than the firm’s initial belief \(q_0\) and sufficiently smaller than 1. Figure 1 depicts the firm’s expected profits under the two policies as a function of the worker’s initial morale \(p_0\) where other parameter values are set at \(q_0 = 1/2, a_l = 1, a_h = 1.5, \lambda = 1, \alpha = 0.9, y = 2\). In this parameter configuration, as well as in those we will explore later in comparative statics exercises, it can be verified that effort and ability are local complements for all feasible effort levels. It is shown in the figure that the non-differential policy provides a higher expected profit for the firm when worker’s initial morale \(p_0\) is in an interval \(\left[p^*_0, \bar{p}^*_0\right]\) where \(p^*_0\) and \(\bar{p}^*_0\) are respectively called the lower and upper threshold.\(^{14}\) For any \(\{q_0, \alpha, a_l, a_h, y\}\), they are the two solutions to the following equation:

\[
\tilde{V}(p_0, q_0) = [\alpha q_0 + (1 - \alpha) (1 - q_0)] \tilde{V}(p_h, q_h) + [\alpha (1 - q_0) + (1 - \alpha) q_0] \tilde{V}(p_l, q_l).
\]

(16)

Importantly, the lower threshold initial worker morale \(p^*_0\) exceeds the firm’s initial belief \(q_0\), and the upper threshold \(\bar{p}^*_0\) is less than 1. That is, for the non-differentiation wage policy to dominate the differentiation policy, workers must be sufficiently but not excessively overconfident. Panel A of Figure 2 depicts the lower threshold \(p^*_0\) as a function of \(q_0\). Note that the lower threshold \(p^*_0\) increases in the firm’s initial belief \(q_0\) and is always higher than \(q_0\) (note that it lies below the dashed 45 degree line). Thus overconfidence of the worker relative to the firm’s initial belief is a necessary condition for the firm to adopt non-differentiation wage policy.

\(^{14}\)In Figure 1, \(\bar{p}^*_0\) is indistinguishable from 1 because of precision level. The actual value of \(\bar{p}^*_0\) for the figure is 0.999914.
wage policy. Indeed, Panel A also suggests that overconfidence “begets” overconfidence. To see this, suppose that the true proportion of high ability workers, \( q_0 \), is low. Then Panel A indicates that the firm will be very likely to adopt a non-differentiation policy because the threshold \( p^*_0 \) is also low. This implies that for a given distribution of initial worker beliefs, a firm facing a low quality labor force is more likely to engage in no wage differentiation. Ex post, the majority of the workers receive a poor performance evaluation but never learn it, hence they become even more overconfident in their own ability relative to the firm. So the larger the proportion of low ability overconfident workers to begin with, the larger the average reinforcement of overconfidence in equilibrium.

Panel B of Figure 2 plots the level of worker overconfidence (relative to the firm’s) necessary for the optimality of non-differentiation wage policy. Note that the necessary level of overconfidence is modestly positive for all levels of \( q_0 \) and it is maximized when \( q_0 = 1/2 \). Panel B also shows that, even allowing workers’ initial confidence to partially reflect the truth, the non-differentiation policy is more likely for low \( q_0 \). Again, overconfidence begets overconfidence.

Finally, Panel C depicts the upper threshold \( \bar{p}^*_0 \) as a function of \( q_0 \). The main message is that it is very close to 1 (above 0.9999) for the whole domain of \( q_0 \). To summarize, in this exponential example, the non-differentiation wage policy is superior if and only if the worker’s initial morale \( p_0 \) lies above the lower threshold \( \underline{p}^*_0 \) and below the upper threshold \( \bar{p}^*_0 \). The most important fact is that the lower threshold \( \underline{p}^*_0 \) is higher than \( q_0 \), which, together with subsequent graphs, implies that worker overconfidence (but not extreme overconfidence), is a necessary condition for the firm to adopt a non-differentiation policy.

Why does moderate level of worker overconfidence cause the firm to favor the non-differentiation over the differentiation policy? The trade-offs between the two policies can be better understood via the decomposition in expression (15).

Figure 3 shows how the sorting and morale effects change as the worker’s initial morale increases. Panel A shows that the sorting effect is strictly positive and increases in \( p_0 \). Panel B shows that the positive morale effect decreases in \( p_0 \) and approaches zero as \( p_0 \) approaches 1. The reason is simple: the morale boost from knowing of a good performance evaluation gets smaller when the worker’s initial confidence gets higher (provided that it is higher than
$q_0 = 0.5$). Panel C shows that the negative morale effect initially declines and then reverts to zero. The reason is that the morale loss from knowing of a bad performance evaluation is non-monotonic (U-shaped) in $p_0$. Panel D shows the total effects, which implies that the non-differentiation policy dominates the differentiation policy if and only if $p_0 \in \left(p_0^*, \bar{p}_0^*\right)$.

Why would the negative morale effect start to dominate the other two effects as $p_0$ increases? From expression (15), we know that there are two forces in play that affect the relative strength of the negative and positive morale effects as the worker’s initial morale $p_0$ varies. The first force is the statistical force from Bayesian updating; and the second force is due to the curvature of $\tilde{V}$. The second force is a relatively unimportant one because $\tilde{V}$ is quite smooth in $p$ and $q$. The statistical force is depicted in Figure 4, which shows the ratio of morale boost from knowing of a good signal over the morale loss from knowing a bad signal. This ratio declines to $(1 - \alpha) / \alpha = 1/9$ as the worker’s initial morale approaches 1. In other words, the morale loss from a bad signal will dominate the morale boost from a good signal as $p_0$ increases. This explains why the negative morale effect will eventually dominate the positive morale effect as $p_0$ is large enough.\footnote{Note, however, as $p_0$ goes to 1, both morale effects go to zero, even though the negative morale effect dominates the positive morale effect. Since the sorting effect is always positive, and in fact increases in $p_0$ for a fixed $q_0$, the differentiation policy wins as $p_0$ approaches 1.}

### 5.2 The Effects of Outside Options

We have shown that some worker overconfidence is necessary for the firm to choose a non-differentiation policy when we set $U_0 = V_0 = 0$. We now discuss the effects of positive outside options.

First, our conclusion still holds when the firm’s outside option $V_0$ is positive and large enough to bind at some optimal contract $\{0, \Delta^* (p_j, q_j)\}, j = 0, h, l$, while $U_0 = 0$. To see why, suppose that the firm prefers to differentiate when $p_0 = q_0$ and $U_0 = 0$, and when the firm’s participation constraint is not binding. This implies from our previous calculations
that

\[ [\alpha p_0 + (1 - \alpha) (1 - p_0)] V (0, \Delta^* (p_h, p_h); p_h, p_h) + [\alpha (1 - p_0) + (1 - \alpha) p_0] V (0, \Delta^* (p_l, p_l); p_l, p_l) > V (0, \Delta^* (p_0, p_0); p_0, p_0) , \]

because the optimal bonus is given by \( \Delta^* (p_j, p_j) \) for \( j \in \{0, h, l\} \) when \( U_0 = 0 \). Now increase the firm’s outside option \( V_0 \) and suppose that the firm’s participation constraint binds at some belief level. The above inequality, together with the fact that \( V (0, \Delta^* (p, p); p, p) \) increases in \( p \) (by local complementarity of effort and ability), implies that

\[
[\alpha p_0 + (1 - \alpha) (1 - p_0)] \max \{ V (0, \Delta^* (p_h, p_h); p_h, p_h), V_0 \} \\
+ [\alpha (1 - p_0) + (1 - \alpha) p_0] \max \{ V (0, \Delta^* (p_l, p_l); p_l, p_l), V_0 \} \\
\geq \max \{ V (0, \Delta^* (p_0, p_0); p_0, p_0), V_0 \} .
\]

That is, differentiation also strictly dominates in these circumstances, except for the uninteresting case where the firm lays off even high ability workers, after revealing the good news to them. Hence, some worker overconfidence is needed for the firm to adopt a non-differentiation wage policy.

Second, we consider the other extreme case where \( V_0 = 0 \) but \( U_0 \) is positive and large enough to bind at some optimal contract \( \{0, \Delta^* (p_j, q_j)\} \) where \( j \in \{0, h, l\} \). Suppose that the outside option binds only for the most pessimistic workers who receive bad news in a differentiation contract. Suppose that the firm and the workers share identical initial beliefs, i.e., \( p_0 = q_0 \). In these circumstances the differentiated contract must offer \( \Delta (p_l) > \Delta^* (p_l, p_l) \) in order for these workers to be just willing to work. Figure 5 depicts the firm’s expected profits under the differentiation and non-differentiation wage policies as a function of the worker’s outside option \( U_0 \) where \( p_0 = q_0 = 0.5 \) and where \( U_0 \) is in the range that will make the participation constraints bind for those workers who are revealed a bad performance evaluation. It can be seen that the firm’s expected profit is higher under the differentiation wage policy than that under the non-differentiation wage policy. Thus some worker overconfidence is necessary for the non-differentiation wage policy to be superior.
5.3 Comparative Statics

The main theoretical and empirical prediction of the model is the range of initial levels of worker confidence in which the firm will prefer a non-differentiation policy to a differentiation policy. In the context of this example, it is the interval \((\bar{p}_0^*, \tilde{p}_0^*)\). Since the upper threshold \(\bar{p}_0^*\) is extremely close to 1 (above 0.9999), we will conduct comparative statics of the lower threshold \(\underline{p}_0^*\) with respect to parameters of the model.

5.3.1 Aggregate Productivity

Aggregate productivity shocks are proxied by \(y\) in our model. How does \(y\) affect the benefits and costs of wage differentiation? Fix the level of initial worker morale, are firms more willing to engage in non-differentiation wage policy when \(y\) is higher (i.e., in a boom)? Since the upper threshold \(\bar{p}_0^*\) is almost flat with respect to \(y\), we will focus on the lower threshold \(\underline{p}_0^*\). Figure 7 depicts the non-monotonic relationship between the necessary level of worker overconfidence \(\underline{p}_0^*\) and the aggregate productivity shock \(y\).

The reason for the non-monotonic relationship between \(\underline{p}_0^*\) and \(y\) is quite subtle. It can be numerically verified that, for any fixed belief pair \((p, q)\), the sorting effect is increasing and convex in \(y\), the negative morale effect is U-shaped in \(y\), and the positive morale effect is inverted U-shaped in \(y\). The latter two relationships arise because of the curvature of \(\tilde{V}\). Overall, the total effects have a U-shaped relationship with \(y\). When the productivity \(y\) is small, for any \((p, q)\), the negative morale effect is small because the discouraged workers are not able to produce too much in any case. Thus, in order for the firm to prefer a non-differentiation policy, the workers must be quite overconfident, thus a higher \(\underline{p}_0^*\) is necessary. When the productivity \(y\) is high, the negative morale effect starts to fall again (because the firm offers a higher bonus), and the sorting effect starts to pick up fast (since the sorting effect is convex in \(y\)). Thus again a higher worker overconfidence is needed for the firm to adopt a non-differentiation policy.

The macro implication of this comparative statics is straightforward. If the aggregate productivity shocks are in the increasing region of Figure 7, then the model predicts that the firm is more likely to adopt a non-differentiation wage policy when \(y\) is low (in a recession) than when \(y\) is high (in a boom). That is, there is more wage compression in a recession. If the
firm prefers wage differentiation, then layoffs of workers with poor performance evaluations will be more likely when \( y \) is low. As Bewley (1999) found, firms must choose between layoffs and wage rigidity in recessions because selective wage cuts would trigger a loss in morale among those workers, making them no longer employable.

5.3.2 Precision of Performance Evaluation

The level of overconfidence necessary for the non-differentiation policy to dominate is also affected by the precision of performance evaluation. The relationship is quite intuitive: the higher \( \alpha \) is, the stronger the sorting effect in favor of differentiation policy and the more worker overconfidence is required for non-differentiation policy to be superior. Figure 6 depicts the relationship between \( p_0^* \) and \( \alpha \) in this exponential example.

5.3.3 Heterogeneity of Worker Ability

Finally, Figure 8 depicts how worker heterogeneity affect the firm’s optimal wage-setting policy. The graph is constructed as follows. We keep constant the mean level of worker ability perceived by the firm is \( q_0 a_h + (1 - q_0) a_l = 5/4 \) and create mean-preserving increases in worker heterogeneity by varying \( a_h \) in interval \([1.25, 2.5]\) while setting \( a_l = 2.5 - a_h \). Thus, the larger \( a_h \), the higher heterogeneity in worker ability. The necessary level of worker overconfidence required for the non-differentiation policy to be optimal increases as the worker ability becomes more heterogeneous because the sorting effect becomes stronger. Note that in the extreme case in which \( a_l = 0 \), that is, low ability worker never produces high output even with effort, it never pays the firm to adopt the non-differentiation policy since \( p_0^* = 1 \).
5.4 A Second Example

The same set of numerical analysis is also conducted in another parametric family of examples in which the following functional forms are adopted:

\[ \pi_j(n) = \frac{a_j n}{1 + a_j n}, j \in \{h, l\}, \]

\[ C(n) = \lambda n, \text{ where } \lambda > 0. \]

Note that here \( \pi_j \) is not log-concave. Nonetheless, the optimal effort function \( n^* (\cdot, p) \) is still concave in \( \Delta \). The qualitative results from this example are identical to the exponential example. To summarize:

- For every \( q_0 \in (0, 1) \), there exists \( p_0^* > q_0 \) and \( \bar{p}_0 \approx 1 \) such that non-differentiation policy dominates differentiation wage policy if and only if \( p_0 \in \left( p_0^*, \bar{p}_0 \right) \);
- The lower threshold level of worker confidence \( p_0^* \) is non-monotonic (U-shaped) in \( y \), increases in the precision of performance evaluation \( \alpha \) and the worker heterogeneity.

6 Discussion

1. In this paper, a worker’s morale does not directly affect the marginal productivity or the marginal cost of effort, a channel emphasized by Bewley (1999). Instead, we emphasized an indirect channel: a worker’s morale affects her incentives to exert effort through affecting the worker’s perceived marginal productivity of effort. However, we believe that our major insight - workers will react asymmetrically to good and bad news when they are moderately overconfident - will lead a firm to prefer a non-differentiation wage policy even if the morale affects marginal productivity or marginal cost of effort directly.

2. We assumed that all workers have identical initial beliefs \( p_0 \), and the firm also holds identical initial beliefs \( q_0 \) about all the workers. Suppose instead, that there are \( k \) different levels of initial belief pairs, \( (p_0^1, q_0^1), \ldots, (p_0^k, q_0^k) \), and a large number of workers in each cell. Then as long as these belief pairs are commonly known by all the workers, the optimal wage setting policy derived in the paper can be simply interpreted as the optimal wage policy.
conditional on a belief pair \((p_j^i, q_j^i), j = 1, \ldots, k\). Thus in such a firm with many different belief pairs, workers do observe wage differentiation, but a worker’s morale is only affected by the wage contract offers received by her co-workers in the same belief pair type.

3. Our model makes the stark prediction that there is complete wage compression when a firm finds the non-differentiation wage policy to be superior. This strong prediction is due to the simplifying assumptions of the model. First, as we mentioned above, when the firm has different types of workers in terms of initial belief pairs \((p_j^i, q_j^i), j = 1, \ldots, k\), the economic forces we highlight in this paper will be consistent with within-firm wage differentiation even when the firm favors the non-differentiation wage policy for any belief pair \((p_j^i, q_j^i), j = 1, \ldots, k\). Second, we assumed in the paper a very restricted space for the performance evaluation signals with only \(\theta_h\) and \(\theta_l\). As we enrich the signal space to include more performance evaluation outcomes, it is conceivable that the morale concerns emphasized in our paper will lead a firm to favor a semi-pooling wage policy as follows: the firm reveals extremely good and extremely bad news, but conceal all mediocre news.\(^{16}\) Under such a wage policy, we will then observe wage compression, but not complete wage equalization, within the firm.

4. A staple of our analysis is that workers are overconfident about their own ability. We then investigate the implication of such overconfidence on the firm’s wage setting policy under the assumption that workers process information revealed by the firm’s wage contracts in a rational Bayesian fashion. In other words, the workers’ bias in our model lies in the prior, not in the information processing. A different approach would be to assume that workers have the correct initial belief, but are biased in their information processing, for example, that they suffer from attribution bias as mentioned in Section 2. When workers suffer from severe attribution bias, the firm will undoubtedly be in favor of a differentiation policy. The reason is obvious: workers who receive bad news will simply attribute it to bad luck and hence will not lower their morale; but workers who receive good news will boost their

\(^{16}\)This seems to be the policy of GE under the leadership of Jack Welch. In Chapter 11 of Welch (2001), he said: “Differentiation comes down to sorting out the A, B, and C players.” The top 20 percent of the players are identified as the A players and highly rewarded; the bottom 10 percent are identified as the C players and fired; and the middle 70 percent are identified as B players.
morale. Of course, any worker who suffers from attribution bias will mostly likely have been overconfident by the time they join the work force. Thus, it seems that worker overconfidence is a natural assumption. When the workers are biased in both their initial beliefs and their information processing, the economic forces emphasized in our model will survive, albeit reduced in strength.

7 Conclusion

In this paper, we investigate the implication of worker overconfidence, which is supported by a large body of psychological evidence, on the optimal wage-setting policies of the firms. More specifically, we examine the optimal contract design problem of a principal facing many agents. The principal receives individualized performance evaluations of the agents and decides if it is in its interest to offer differential wage contracts to workers depending on, thus revealing, her performance evaluation.

We decompose the trade-offs between a differentiation and non-differentiation policy into three components: first, a sorting effect, which allows the firm to tailor individual contracts to her perceived ability, in favor of the differentiation policy; second, a positive morale effect, which means that the morale of the workers with high performance evaluation will be boosted under a differentiation policy, also in favor of differentiation policy; and third, a negative morale effect, which means that the morale of the workers with low performance evaluation will be hurt, in favor of non-differentiation policy. We show in numerical examples, and conjecture in general that, the differentiation policy dominates the non-differentiation policy when the firm and workers share identical initial beliefs. However, worker overconfidence can effectively tilt the balance in favor of the non-differentiation policy. We robustly show in examples that when workers are sufficiently overconfident than the firm, the non-differentiation policy can be the optimal policy. By providing a theoretical link between a behavioral assumption of worker overconfidence and wage-setting practices of the firm, we help explain why firms emphasize against wage disclosure, and abstain from wage differentiation among their workers, as documented by Bewley (1999).

The most interesting extension of the model is to introduce dynamics. As firms accumu-
late more (private) evidence of the worker, while engaging in non-differentiation wage policy, it is possible that at a certain point, the trade-off may be in favor of differentiation policy. At that time, workers who have accumulated a string of bad performance evaluations, but never told so earlier, will receive the full string of news in one dosage, even being laid off.

**Appendix: Proof of Lemma 1.**

**Proof.** Applying the implicit function theorem to the first order condition (2), we obtain

\[
\frac{\partial n^*(\Delta, p)}{\partial \Delta} = \frac{p\pi_h'(n^*) + (1 - p)\pi_i'(n^*)}{C''(n^*) - \Delta[p\pi_h''(n^*) + (1 - p)\pi_i''(n^*)]} > 0.
\]

Thus,

\[
\frac{\partial^2 n^*(\Delta, p)}{\partial \Delta^2} = \frac{[p\pi_h'' + (1 - p)\pi_i''] n''}{C''(n^*) - \Delta[p\pi_h'' + (1 - p)\pi_i'']} - \frac{[p\pi_h' + (1 - p)\pi_i'] \left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\} n'' - [p\pi_h'' + (1 - p)\pi_i'']} \frac{\left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\}^2}{\left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\}^2},
\]

where, with some abuse of notation, we write \( n'' = \frac{\partial n^*(\Delta, p)}{\partial \Delta}. \) After imposing the common denominator as \( \left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\}^2, \) the numerator is

\[
[p\pi_h'' + (1 - p)\pi_i''] n'' \left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\} - [p\pi_h' + (1 - p)\pi_i'] \left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\} n'' - [p\pi_h'' + (1 - p)\pi_i''] \frac{\left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\}^2}{\left\{ C'' - \Delta[p\pi_h'' + (1 - p)\pi_i''] \right\}^2}.
\]

which is clearly negative when \( C''' \geq 0 \) and \( p\pi_h' + (1 - p)\pi_i' \) is log-concave. \( \square \)
References


Figure 1: Firm’s Expected Profits under Differentiation (Solid Curve) and Non-Differentiation (Dashed Curve) Policies as a Function of the Worker’s Initial Morale $p_0$. Other Parameter Values are set at $q_0 = 0.5$, $a_l = 1$, $a_h = 1.5$, $\lambda = 1$, $\alpha = 0.9$, $y = 2$, $U_0 = V_0 = 0$. 
Figure 2: The Lower and Upper Thresholds $p_0^*$ and $\bar{p}_0^*$ as a Function of $q_0$ $\alpha = 0.9, a_l = 1, a_h = 1.5, y = 2, U_0 = V_0 = 0.$
Figure 3: Sorting and Morale Effects as Functions of Worker Morale $p_0$ : $q_0 = 0.5, a_l = 1, a_h = 1.5, y = 2, \alpha = 0.9, U_0 = V_0 = 0$. 

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Figure 4: The Ratio of Morale Boost over Morale Loss as a Function of $p_0$ : $\alpha = 0.9$.

Figure 5: Firm’s Expected Profits under Differentiation (Solid Line) and Non-Differentiation (Dashed Line) Wage Policies as a Function of the Workers’ Outside Option $U_0$ : $p_0 = q_0 = 0.5$, $\alpha = 0.9$, $a_t = 1$, $a_h = 1.5$, $y = 2$, $V_0 = 0$.  

Figure 6: The Relationship Between $p^*_0$ and $\alpha$ : $q_0 = 0.5, a_l = 1, a_h = 1.5, y = 2, U_0 = V_0 = 0$.

Figure 7: The Relationship Between $p^*_0$ and $y$ : $q_0 = 0.5, a_l = 1, a_h = 1.5, \alpha = 0.9, U_0 = V_0 = 0$.  

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Figure 8: The Relationship Between $p_0^*$ and Worker Ability Heterogeneity as Measured by $a_h$: $q_0 = 0.5, y = 2, a_t + a_h = 2.5, U_0 = V_0 = 0.$