How Does the Life Settlement Affect the Primary Life Insurance Market?*

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Abstract

We study the effect of life settlement market on the structure of the long-term contracts offered by the primary insurance market, as well as on the consumer welfare in a dynamic model of insurance with one-sided commitment as in Hendel and Lizzeri (2003) and Daily, Hendel and Lizzeri (2008). We show that the presence of the settlement market affects the extent as well as the form of dynamic reclassification risk insurance in the equilibrium of the primary insurance market; and that the settlement market generally leads to lower consumer welfare. In the most extreme form, the presence of the settlement market can completely unravel the dynamic contracts to a sequence of spot short-term contracts. We also examine the primary insurers' response to the settlement market when they can offer enriched contracts by specifying optimally chosen cash surrender values (CSVs). We show that the option of specifying CSVs is a useless option when there is no settlement firms; but facing the threat from the settlement firms, contracts with optimally chosen health-contingent CSVs will emerge in the equilibrium of the primary market, and such contracts improve consumer welfare, even though consumer welfare is still lower than that without the secondary market. In contrast, the option to endogenously specify non-health contingent CSVs is useless for the primary insurers.

Keywords: Life insurance, dynamic insurance, secondary market

JEL Classification Codes: G22, L11

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1 Introduction

A life settlement is a financial transaction in which a life insurance policyholder sells the policy to a third party for more than the cash value offered by the life insurance company, thereby the third-party purchaser assumes responsibility for all subsequent premium payments to the life insurance company and becomes the new beneficiary of the life insurance policy at maturation (i.e., the time of the original policyholder's death).[1] The life settlement industry is quite recent, growing from just a few billion dollars in late 1990s to about $12-$15 billion in 2007, and according to some projections, is expected to grow to more than $150 billion in the next decade.[2, 3]

It is useful to provide some background information on the life insurance market. There are two main categories of life insurance products, Term Insurance and Whole Life Insurance.[4] A Term insurance policy covers a person for a specific term at a fixed or variable premium for each year, and if the person dies during the term, the life insurance company pays the face amount of the policy to his/her beneficiaries, provided that the premium payment has never lapsed. The most popular Term Insurance has the premium fixed during the term and is called Level Term Insurance. A Whole Life insurance policy, on the other hand, covers a person's entire life, usually at a fixed premium. Besides the difference in the period of coverage, Term and Whole Life insurances also differ in the amount of cash surrender value (CSV) received if the policyholder surrenders the policy to the insurance company: for Term insurance, the CSV is zero; while for Whole Life insurance, the CSV is typically positive and pre-specified to depend on the length of time that the policyholder has owned the policy, but does *not* specifically depend on the health status of the policyholder when surrendering the policy.[5]

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[1] The legal basis for the life settlement market seems to be the Supreme Court ruling in *Grigsby v. Russell* [U.S. Supreme Court, 1911], which upheld that for life insurance an "insurable interest" only needs to be established at the time the policy becomes effective, but does not have to exist at the time the loss occurs: "... [A] valid policy is not voided by the cessation of the insurable interest, even as against the insurer, unless so provided by the policy itself." The life insurance industry has typically included a two-year contestability period during which transfer of the life insurance policy will void the insurance.

[2] See Chandik (2008, p2).

[3] The life settlement industry actively targets wealthy seniors 65 years of age and older with life expectancies from 2 to up to 12-15 years. This differs from the earlier viatical settlement market developed during the 1980s in response to the AIDS crisis, which targeted persons in the 25-44 age band diagnosed with AIDS with life expectancy of 24 months or less. The viatical market largely evaporated after medical advances dramatically prolonged the life expectancy of an AIDS diagnosis.

[4] There are other variations such as Universal Life Insurance and Variable Life Insurance that combine some features of both Term and Whole Life Insurances. See Gilbert and Schultz (1994) for details.

[5] The CSV for Whole Life insurance policies can depend on holding time of the policy at surrendering. To the

Two main features of the life insurance contracts underline the opportunity for a secondary life insurance market. The first feature is that most life insurance purchased by consumers, either Term and Whole Life, have the feature that the insurance premium stays fixed over the course of the contract. Because policyholders' health typically deteriorate over time, the fixed premium implies that the premium policyholders initially pay exceeds the actuarially fair premium, in exchange for the same level of premium in later years which is typically lower than the spot market premium. This phenomenon is called *front-loading*.[6] Front-loading implies that policyholders of long-term life insurance contracts, especially those with impaired health, often have terms that are much more favorable than what they could obtain in the spot market (i.e., the present actuarial value of the policy). This generates what has been known as the Intrinsic Economic Value (IEV) of the life insurance contract.[7] The second feature is that the cash surrender value for life insurance contracts is either zero for Term insurance, or at a level that does not depend on the health status of the policyholder. Because the IEV of a life insurance policy is much higher for individuals with impaired health, the fact that the CSV does not respond to the health status provides an opening for the gains of trade between policyholders with impaired health and the life settlement companies.[8] Life settlement companies operate by offering policyholders, who are intending to either lapse or surrender their life insurance policies, more cash than the cash surrender value offered by the insurers.[9]

The emerging secondary market for life insurance has triggered controversies between life insurance companies who oppose it, and the life settlement industry who supports it. The views from the two opposing camps are represented by Doherty and Singer (2002) and Singer and Stal-

extent that the holding time of the policy is related to age, it could potentially act as an imperfect proxy for health status. More likely, though, the holding time reflects the time of investment because Whole Life policies are generally considered as both life insurance and savings instrument.

[6] Hendel and Lizzeri (2003) offered both theory and evidence for why front-loading is crucial in providing insurance for reclassification risks. Our theoretical framework borrows from Hendel and Lizzeri (2003) and Daily, Hendel and Lizzeri (2008).

[7] See Deloitte Report (2005, p1).

[8] Deloitte Report (2005, p3) states that the CSV of Whole Life insurance contracts are, by regulation, not allowed to be conditioned on health impairments of the policyholder who surrenders the contract. Doherty and Singer (2002, p18) also argue that regulatory constraints faced by life insurance carriers deter life insurance companies to offer health dependent cash surrender values: "Such an offering of explicit health-dependent surrender values by a life insurance carrier, however, would be fraught with regulatory, actuarial, and administrative difficulties. Life insurance carriers do not offer health-adjusted surrender values, which suggests that these difficulties outweigh the benefits that carriers would obtain by offering health-dependent surrender values to consumers."

[9] The well accepted statistic is that the CSV often represents 10% of the face amount (or the death benefits), while the life settlement value has been about 20% of the death benefits.

lard (2005) on the proponent side and the Deloitte Report (2005, also known as the "Carriers" Report) on the opponent side. Doherty and Singer (2002) argued that a secondary market for life insurance enhances the liquidity to life insurance policyholders by eroding the monopsony power of the carrier. This will increase the surplus of policyholders; moreover it will in the long run lead to a larger primary insurance market. On the other side, life insurance companies, as represented by the Deloitte Report (2005), claim that the life settlement market, by denying them the return on lapsing or surrendered policies, increases the costs of providing policies in the primary market. They allege that these costs will have be passed on to consumers, which would ultimately make the consumers worse off (Deloitte Report 2005).

A key issue in the contention between the opposing sides is the role of lapsing or surrendering in the pricing of life insurance in the primary market.[10] There are a variety of situations in which policyholders may choose to lapse or surrender.[11] First, the beneficiary for whom the policy was originally purchased could be deceased or no longer need the policy;[12] second, the policyholder may experience a negative income shock (or large expense shock from medical expenses etc.) that leads him to favor more cash now than to leave a bequest. In the absence of the life settlement market, when a health-impaired policyholder chooses to lapse or surrender its insurance, the life insurance company pockets the intrinsic economic value of these policies, which potentially allows the life insurance company to offer insurances at lower premium if its pricing is partially supported by lapsation or surrendering. In the presence of the secondary market for life insurance, the life settlement company will purchase these policies as assets, thus the primary insurance company will always have to pay the face amount.

In this paper, we analyze the effect of the secondary market for life insurance on primary market insurance contracts and on consumer welfare in a dynamic model of insurance with symmetric learning and one-sided commitment as in Hendel and Lizzeri (2003, HL henceforth) and Daily, Hendel and Lizzeri (2008, DHL henceforth) where life insurance companies are risk neutral and can commit to contractual terms (including future premia and face amounts), and the consumers are risk averse and cannot commit to remain in the contract they earlier have chosen. The relevant risk is the mortality risk, which is assumed to be symmetrically observed. Because consumers' mortality risks change over time, they face *reclassification risk* for the insurance premium if there are no dynamic contracts. HL showed that in equilibrium life insurance companies will offer dynamic

[10] See a discussion in Daily (2004).

[11] See Doherty and Singer (2002, p.24).

[12] A related situation if the life insurance was purchased to protect a company from the loss of a key executive (the so-called "key-man policy") is that the company may have folded, or the individual could be no longer integral to the business' success.

contracts that provide consumers with reclassfication risk insurance via front-loading of insurance premiums. DHL introduced a new feature into the HL model: they assume that consumers in the second period may lose bequest motive. This, together with the front-loading feature of the equilibrium contract, implies that there are policies in the second period for which the present actuarial value is higher than the surrender value (which is assumed to be zero). Thus, there is scope for life settlement firms to purchase policies from those consumers who no longer have bequest motive.

We extend the DHL analysis in several dimensions. First, we fully characterize the equilibrium contract of the primary insurance market with the presence of the settlement market, assuming that the primary insurers cannot enrich their contract space to set optimally chosen cash surrender values. We show that the life settlement market affects the equilibrium life insurance contracts in a qualitatively important manner: the equilibrium life insurance contract will no longer have flat premium in the second period; instead, consumers with high mortality risks will be offered a second-period premium discount relative to the spot market, but the second-period premium will still increase in the consumers' mortality risk. This may lead to a *smaller* degree of front-loading in the first period. Second, we confirm the DHL result that the presence of the settlement market always leads to a decrease of consumer welfare relative to what could be achieved in the absence of the settlement market; moreover, we provide conditions under which the life settlement market could lead to an extreme form of welfare loss – the complete collapse of reclassification risk insurance as a result of unraveling. Third, we analyze how the possibility that the primary insurers can offer richer contracts, in particular contracts that specify optimally chosen cash surrender values, affect the above results.

Our paper is also related to a large industrial organization literature on secondary markets for durable goods (see, e.g., Hendel and Lizzeri 1999; Stolyarov 2002; House and Leahy 2004). The key difference between the life settlement market and the secondary durable market is as follows. In the durable good case, once the transaction between the primary market seller and the buyer is consummated, the seller's payoff is not directly affected by whether the buyer sells the used durable in the secondary market. In contrast, for the case of life settlements, the primary insurer's payoff is directly impacted by whether the policyholder chooses to lapse, surrender for cash value, or to sell the contract to a settlement firm. This same distinction applies to other secondary markets in financial services, such as the home mortgages resale market and the catastrophic risk reinsurance (see Doherty and Singer 2002 for some description of these markets.)

The remainder of the paper is organized as follows. Section 2 presents a baseline model in which there is no secondary market. Section 3 extends the model to include a secondary market and analyzes its effect on primary market contracts and on consumer welfare. Section 4 presents some numerical results for how contract terms change with respect to various parameters of the model. Section 5 considers how the option of endogenously chosen cash surrender values affect the results; in particular we highlight whether or not allowing allowing health-contingentn CSVs have a crucial effect relative to non-health contingent CSVs. Finally, Section 6 summarizes our findings and discusses directions for both theoretical and empirical research in the future.

2 The Basic Model without Life Settlement Market

In this section, we analyze a model of dynamic insurance that is a slightly modified version of that proposed in Hendel and Lizzeri (2003) and further analyzed in Daily, Hendel and Lizzeri (2008). The two main modifications from these papers are, first, we consider continuos distribution for second-period health states;[13] second, we slightly change the timing of events: we assume that income realization and premium payments occur before the resolution of death uncertainty.[14] Neither modification makes any qualitative difference for this baseline model without the secondary market; when we introduce secondary market, these modifications simplify some of our arguments.

Health, Income and Bequests. There is a perfectly competitive primary market for life insurance that includes individuals (policyholders) and life insurance companies. There are two periods. In the first period, the policyholder has a probability of death $p_1 \in (0, 1)$, which is known to both himself and the insurance companies. We will often refer to p_1 as the policyholder's period 1 health state. In the second period, the policyholder has a new probability of death p_2 which is a random variable drawn from the cumulative distribution $\Phi(\cdot)$ with support $[0, 1]$. We assume that $\Phi(\cdot)$ is continuous and differentiable on the interior of its support with a density $\phi(\cdot)$. We will refer to p_2 as the policyholder's period 2 health state. The realization of a consumer's period 2 health state p_2 is not known in period 1, but p_2 is symmetrically learned (and thus common knowledge) by the insurance company and the consumer at the start of period 2.

In period 1 the policyholder earns an income stream $y-g$ and in period 2 the policyholder earns an income stream $y + g$, where y is interpreted as the mean life-cycle income and $g \in (0, \bar{g}]$ with $\bar{g} < y$ captures the income growth over the periods.[15] Both y and g are assumed to be common knowledge.

When making decisions in period 1, the policyholder cares about two sources of utility: his own consumption should he live, and his dependents' consumption should he die. If the policyholder

[13] HL considered discrete health states.

[14] HL assumed that income and consumption are realized after the death uncertainty is resolved.

[15] See HL for a justification for the assumed income process. We follow their notation.

lives, he has a utility $u(c)$ if he consumes $c \geq 0$; if the policyholder dies, then he has a utility $v(c)$ if his dependents consume $c \geq 0$. $u(\cdot)$ and $v(\cdot)$ are both strictly concave and twice differentiable. However, in the second period, there is a chance that the policyholder no longer has a bequest motive.[16] The probability that the policyholder *retains* his bequest motive is given by $q \in (0,1)$. We assume that the bequest motive is realized at the same time as the health state; moreover, it is private information to the policyholder and thus cannot be contracted upon. If the consumer retains the bequest motive, the policyholder's utility in period 2 is again $u(\cdot)$ in the life state and $v(\cdot)$ in the death state, same as those in period 1; if the consumer no longer has bequest motive, then the policyholder's utility is $u(\cdot)$ in the life state, and some constant which could be normalized to zero in the death state.

It is also assumed that there are no capital markets thus the consumer cannot transfer income from period 1 to period 2. Thus the only way for the consumer to ensure a stream of income for his/her dependents is to purchase some level of life insurance.

Timing, Commitment, and Contracts. Now we provide more details about the timing of events. At the beginning period 1, after learning the period 1 health state p_1, the lifetime income y, and the income growth g, the policyholder may sign a long-term contract with an insurance company. A *long-term contract* specifies a premium and face value for period 1, (Q_1, F_1), and a menu of health-contingent premia and face values for each health state of period 2, $\{(Q_2(p), F_2(p)) : p \in [0,1]\}$. The key assumption is that the insurance companies can commit to these terms in period 2, but that the policyholders cannot. The *one-sided commitment* assumption has two important implications. First, it implies that the period 2 terms of the long-term insurance contract must be as desirable for the consumer as what he could obtain in the spot market; otherwise, the consumer will lapse the long-term contract into a new spot contract. This imposes a constraint on the set of feasible long term contracts that consumers will demand in period 1. Second, if a policyholder suddenly finds himself without a bequest motive, he could lapse his policy by refusing to pay the second period premium.

In period 2, after learning the period 2 health state p_2, the policyholder has three options. He can either continue with the terms of his long-term contract from period 1, or he can let the long-term policy lapse and buy a spot market contract, or he can let the long-term policy lapse and simply remain uninsured. A spot contract is simply a premium and a face value (Q, F) which will have to earn zero expected profit under our assumption that the insurance market is competitive.

The precise timing of the model is outlined below:

[16] As described in the introduction, this could result from divorce, or from changes in the circumstances of the intended beneficiaries of the life insurance.

1. **Period 1**

   (1.1) Policyholder and insurance companies learn p_1, y, g, and q;

   (1.2) The policyholder earns income $y - g$, decides on a long term contract, pays the first period premium Q_1, and consumes the remaining, $y - g - Q_1$;

   (1.3) With probability p_1 the policyholder dies and his dependents receive the face value F_1 of his insurance contract.

2. **Period 2**

   (2.1) If the policyholder survived in period 1, he and insurance companies learn p_2 which is drawn from CDF $\Phi(\cdot)$. Policyholder also learns whether or not he will have a bequest motive.

   (2.2) The policyholder earns income $y + g$ and decides to stay with his long term contract or to lapse. If he lapses he either chooses a spot market contract or decides to remain uninsured. If the consumer retained his contract or repurchased, he pays the premium Q_2 and consumes the remaining, $y + g - Q_2$. If he lapsed and did not repurchase, he consumes his entire income $y + g$.

   (2.3) With probability p_2 the policyholder dies and his dependents receive the face value of his standing insurance contract F_2 (if any).

2.1 Equilibrium Contracts Without Settlement Market

To begin characterizing the equilibrium set of contracts, let us first consider the actions of a policyholder in the second period who no longer has a bequest motive. Given that there is no secondary market for life insurance contracts, and we have not yet allowed the insurance companies to buy back contracts through CSVs, the best course of action for those who no longer have a bequest motive is to simply let the long-term policy lapse and to become uninsured.[17] This is true for any second period health state. Also observe that, in equilibrium, a policyholder with a bequest motive will never strictly prefer to have his long-term policy lapse in the second period. This is so because we will constrain the long-term contracts offered in period 1 to satisfy such "no-lapse" conditions for those who retain bequest motive in the second period.[18]

[17] We introduce "cash surrender value" in Section 5 when we consider the primary market insurers' response to the settlement market.

[18] Formally, the argument is as follows: suppose a policyholder with second period health state p_2 decides to lapse and repurchase on the spot market. The same outcome can be attained if the period 1 contract is modified such that

The competition among primary insurance companies ensures that the equilibrium contract is a long-term contract

$$\left\langle \ (Q_1, F_1), \{(Q_2(p), F_2(p)) : p \in [0,1]\} \ \right\rangle$$

that maximizes

$$u(y - g - Q_1) + p_1 v(F_1) +$$
$$(1 - p_1) \int \left\{ \ q\left[u(y + g - Q_2(p)) + pv(F_2(p))\right] + (1 - q)u(y + g) \ \right\} d\Phi(p) \tag{1}$$

subject to:

$$Q_1 - p_1 F_1 + (1 - p_1)q \int \left\{ \ Q_2(p) - pF_2(p) \ \right\} d\Phi(p) = 0 \tag{2}$$

$$\forall p : Q_2(p) - pF_2(p) \leq 0, \tag{3}$$

where (1) is the expected utility the policyholders receive from the contract; constraint (2) is the *zero-profit* constraint that reflects perfect competition in the primary market; and constraints in (3) guarantee that there will not be lapsation among policyholders with a bequest motive in the second period.[19]

The first order conditions for an optimum with respect to $\langle (Q_1, F_1), \{(Q_2(p), F_2(p)) : p \in [0,1]\}\rangle$ are:

$$[Q_1] : \quad u'(y - g - Q_1) = \mu \tag{4a}$$

$$[F_1] : \quad v'(F_1) = \mu \tag{4b}$$

$$[Q_2(p)] : \quad u'(y + g - Q_2(p)) = \mu + \frac{\lambda(p)}{(1 - p_1)q\phi(p)} \tag{4c}$$

$$[F_2(p)] : \quad v'(F_2(p)) = \mu + \frac{\lambda(p)}{(1 - p_1)q\phi(p)} \tag{4d}$$

where μ and $\lambda(p)$ are respectively the Lagrange multipliers for constraints (2) and (3); $\mu > 0$ and $\lambda(p) \leq 0$ must satisfy the following complementary slackness conditions:

$$\lambda(p)\left[Q_2(p) - pF_2(p)\right] = 0 \tag{5}$$

(1) his policy is the same for all states except p_2; and (2) the terms for state p_2 change to that of the second period spot contract.

[19] To see why constraints in (3) guarantee "no lapsation" for those with bequest motive in the second period, suppose that (3) does not hold for some p. Then, since the spot market is competitive, it is clear that the policyholder could obtain the same face value for a lower premium on the spot market, thus a non-committing policyholder would find it desirable to lapse the long-term contract in exchange for a spot contract second period. Now suppose that (3) holds for all p. Then, to obtain the same face value the policyholder would have to pay at least the premium he is already paying, since insurance companies cannot make negative profits on second period spot market contracts. Hendel and Lizzeri (2003, Appendix) formalize this argument.

The first order conditions (4) imply that in equilibrium,

$$u'(y-g-Q_1) = v'(F_1) \tag{6}$$

$$u'(y+g-Q_2(p)) = v'(F_2(p)). \tag{7}$$

These conditions imply that in equilibrium the consumer obtains *full-event insurance* in every state in both periods. The first order conditions thus provide a one-to-one relationship between the face amounts and premiums policyholders will obtain in equilibrium; moreover, note that the face amount must *decrease* with the premium in every state in both periods.[20] What remains is to characterize the equilibrium set of premiums $\langle Q_1, \{Q_2(p) : p \in [0,1]\}\rangle$.

Let the support of the second-period health states be divided into two subsets: a subset, denoted by $\mathcal{B}$, where the no-lapsation constraint (3) binds; and another, denoted by $\mathcal{NB}$, where (3) does not bind. Lemma 1 shows that the states in the subset $\mathcal{B}$ is healthier than the states in subset $\mathcal{NB}$.

Lemma 1 *If $p \in \mathcal{B}$ and $p' \in \mathcal{NB}$ then $p < p'$ and $Q_2(p) \leq Q_2(p')$.*

Proof. Note that $p \in \mathcal{B}$ and $p' \in \mathcal{NB}$ implies that $\lambda(p) \leq 0$ and $\lambda(p') = 0$. Thus, first order conditions (4c) for p and p' respectively imply that

$$u'(y+g-Q_2(p)) = \mu + \frac{\lambda(p)}{(1-p_1)q\phi(p)} \leq u'\left(y+g-Q_2(p')\right) = \mu.$$

Since u' is decreasing, we have $Q_2(p) \leq Q_2(p')$. Because full event insurance conditions (6) and (7) imply that the face amount decreases with the premium, we must have $F_2(p) \geq F_2(p')$. To prove that $p < p'$, suppose to the contrary that $p \geq p'$. Then first note that $p' \in \mathcal{NB}$ implies that $Q_2(p') < p'F_2(p')$; hence

$$Q_2(p) \leq Q_2(p') < p'F_2(p') \leq pF_2(p)$$

where the last inequality follows from postulated $p \geq p'$ and $F_2(p) \geq F_2(p')$ [Recall that full-event insurance requires that face amount decreases with premium].. Hence $Q_2(p) < pF_2(p)$, contradicting the assumption that $p \in \mathcal{B}$. ■

Lemma 1 implies the existence of a threshold p^* such that $p \in \mathcal{B}$ if $p < p^*$ and $p \in \mathcal{NB}$ if $p > p^*$. Denote by $Q_2^{FI}(p)$ and $F_2^{FI}(p)$ as the *fair premium* and *face amount* for full-event insurance under

[20] This is intuitive because the consumer is equating the marginal utility from his own consumption (which increases with the premium he needs to pay for the insurance) and the marginal utility from his beneficiary consumption (which decreases with the face amount).

health state p in the second period, namely, $\left\{Q_2^{FI}(p), F_2^{FI}(p)\right\}$ uniquely solves:[21]

$$\begin{aligned} Q_2^{FI}(p) - pF_2^{FI}(p) &= 0 && (8) \\ u'\left(y + g - Q_2^{FI}(p)\right) &= v'\left(F_2^{FI}(p)\right). && (9) \end{aligned}$$

Following from Lemma 1, together with the full-event insurance conditions (**??**), we have that:

- If $p < p^*$, then
$$Q_2(p) = Q_2^{FI}(p);$$

- If $p > p^*$, then we know $p \in \mathcal{NB}$, thus $\lambda(p) = 0$. Therefore the first order conditions (4a) and (4c) imply that:
$$u'(y - g - Q_1) = u'(y + g - Q_2(p)),$$
which in turn implies that if $p > p^*$, the premium $Q_2(p)$ is constant and *independent of* p. Moreover, it must satisfy
$$Q_2(p) = 2g + Q_1. \tag{10}$$

We prove a useful lemma for the equilibrium contract premium at $p = p^*$:[22]

Lemma 2 *If $p^* < 1$, then the equilibrium contract satisfies the following at $p = p^*$:*

$$\begin{aligned} Q_2(p^*) &= Q_2^{FI}(p^*) && (11) \\ u'\left(y + g - Q_2^{FI}(p^*)\right) &= u'(y - g - Q_1), && (12) \end{aligned}$$

where $Q_2^{FI}(\cdot)$ is defined as the unique solution to the equation system (8) and (9).

Proof. Suppose $Q_2(p^*) < Q_2^{FI}(p^*)$. Then $p^* \in \mathcal{NB}$, thus

$$u'(y + g - Q_2(p^*)) = u'(y - g - Q_1).$$

But for all $p < p^*$, $Q_2(p) = Q_2^{FI}(p)$, so

$$\lim_{p \to p^*_-} u'(y + g - Q_2(p)) = u'(y + g - Q_2^{FI}(p^*)) \leq u'(y - g - Q_1).$$

This implies that $u'(y + g - Q_2^{FI}(p^*)) \leq u'(y + g - Q_2(p^*))$, a contradiction. Analogously, suppose $u'(y + g - Q_2^{FI}(p^*)) < u'(y - g - Q_1)$. Then there exists $p > p^*$ such that $u'(y + g - Q_2^{FI}(p)) <$

[21] The proof that the equation system (8) and (9) has a unique solution follows from the assumption that u' and v' are both decreasing in their arguments.

[22] That is, $p^* \in \mathcal{B}$, but $\lambda(p^*) = 0$.

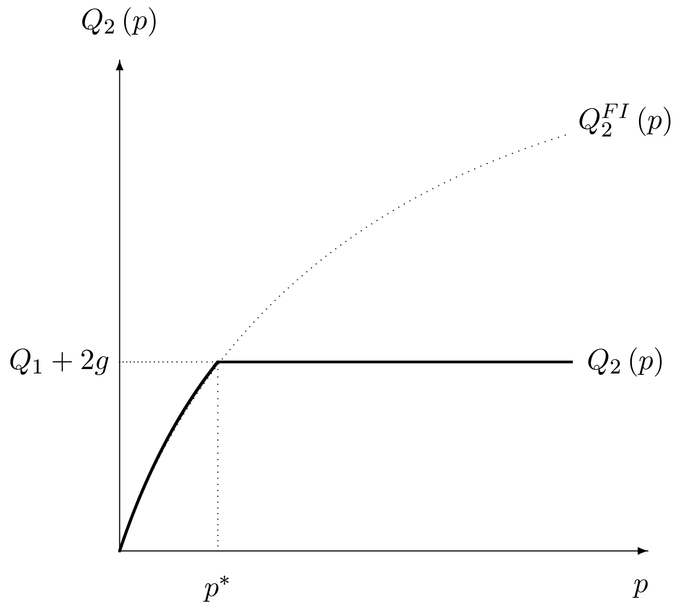

Figure 1: The Equilibrium Second-Period Premium as a Function of Health State p: The Case Without Life Settlement Market.

$u'(y-g-Q_1)$. Since $p \in \mathcal{NB}$, $Q_2(p) < Q_2^{FI}(p)$, and hence $u'(y+g-Q_2(p)) < u'(y-g-Q_1)$, a contradiction. ■

Moreover, (11) and (12) also imply that p^* is uniquely determined by the equality

$$Q_2^{FI}(p^*) = 2g + Q_1.$$

What this implies is that the premium profile over health states is equal to the actuarially fair premium and increasing as p approaches p^*, but becomes flat as p becomes greater than p^*. Figure 1 depicts the equilibrium premia in the second period as a function of the period-2 health state p.[23]

The rest of the equilibrium contract terms, namely the first period premium Q_1 (and thus also F_1) is determined from the zero profit condition (2), which can now be rewritten as:

$$\overbrace{Q_1 - p_1F_1}^{\text{Period 1 Front Loading}} + \overbrace{(1-p_1)q\int_{p^*}^{1}[Q_2(p) - pF_2(p)]\,d\Phi(p)}^{\text{Period 2 Expected Loss}} = 0. \tag{13}$$

Because $Q_2(p) = 2g + Q_1 < Q_2^{FI}(p)$ for $p > p^*$, we know that the insurance company on average expects to suffer a loss in the second period; thus, the zero-profit condition (13) implies that

[23] Hendel and Lizzeri (2003) made the ingeneous observation that excatly the same outcome for the consumers would obtain if the insurance company offers a contract that guarantees the second period premium to be $Q_1 + 2g$ for *all* health states.

$Q_1 - p_1 F_1 > 0$. That is, in the first period, the insurance demands a premium Q_1 that is higher than the actuarially fair premium $p_1 F_1$ for the period-1 coverage alone. This is exactly the phenomenon of *front-loading*. The equilibrium contract requires that consumers may accept a front-loaded premium in the first period in exchange for a flat premium in second period in states $p > p^*$. The flat premium in these states provides *reclassification risk insurance* to the consumers.

We are now ready to make some useful observations. First, one can show that p^* will always be greater than p_1. To see this, suppose to the contrary that $p^* < p_1$, hence $p_1 \in \mathcal{NB}$. From (10), we have $Q_2(p_1) = Q_1 + 2g$. The first order conditions (6) and (7) then imply that if $Q_2(p_1) = Q_1 + 2g$, it must be the case that $F_2(p_1) = F_1$. Since by assumption $p_1 \in \mathcal{NB}$,we have $Q_2(p_1) - p_1 F_2(p_1) = (Q_1 + 2g) - p_1 F_1 < 0$; hence $Q_1 - p_1 F_1 < -2g < 0$. This contradicts the zero-profit constraint, which requires that $Q_1 - p_1 F_1$ is positive whenever $p^* < 1$.

The second observation is the following: there is always a sufficiently small g such that there will be some degree of reclassification risk insurance and front-loading, i.e., $p^* < 1$. To see this, suppose that $p^* = 1$, thus no-lapsation constraint (3) is binding for all p. From the first order conditions, this implies that $u'(y + g - Q_2(p)) \leq u'(y - g - Q_1)$ for all $p \in [0, 1)$. Since u is concave, this in turn implies that $Q_2(p) \leq Q_1 + 2g$ for all p. However, for any $p > p_1$, it must be the case that $Q_2(p) > Q_1$ if $p \in \mathcal{B}$.[24] For sufficiently small g, it is impossible that $Q_2(p) \leq Q_1 + 2g$ for all p at the same time that $Q_2(p) > Q_1$ for all $p > p_1$. Hence, for sufficiently small g, it must be that $p^* < 1$.

The above discussions is summarized below, which replicates Proposition 1 in Hendel and Lizzeri (2003) with some slight modifications.[25]

[24] To see this, note that since the set $\mathcal{NB}$ is empty (i.e., there is no front-loading), it must be the case that (Q_1, F_1) is actuarially fair, i.e.,

$$Q_1 - p_1 F_1 = 0;$$

and we know (Q_1, F_1) has to provide full-event insurance, i.e.,

$$u'(y - g - Q_1) = v'(F_1).$$

We also know that $(Q_2(p), F_2(p))$ must satisfy full-event insurance at p in period 2, i.e.,

$$u'(y + g - Q_2(p)) = v'(F_2(p)).$$

If $Q_2(p) \leq Q_1$, it must be that $F_2(p) > F_1$ for any $g > 0$. But then for all $p > p_1$,

$$Q_2(p) - pF_2(p) < Q_1 - pF_1 \leq Q_1 - p_1 F_1 = 0,$$

contradicting the assumption $p \in \mathcal{B}$. Hence $Q_2(p) > Q_1$ for all $p > p_1$.

[25] We leave out item 4 in Proposition 1 of Hendel and Lizzeri (2003), which claimed that individuals with higher g would choose contracts with lower Q_1 and higher p^* (less front loading). This claim is intuitive because front-loading involves a transfer of wealth from period 1 to the unhealthy states in period 2. The tradeoff between dynamic

Proposition 1 ***(Hendel and Lizzeri 2003)*** *The equilibrium set of contracts satisfies the following:*

1. *All policyholders obtain full event insurance in period 1, and in all health states of period 2. That is, equations (6) and (7) hold.*
2. *There is a second-period threshold health state p^* (which is higher than the period 1 death probability) such that for health states $p \leq p^*$ the second period premiums are actuarially fair and for $p > p^*$ the second period premiums are actuarially unfair. For $p \geq p^*$ the premium is constant and given by $Q_2(p) = Q_1 + 2g$.*
3. *For g small enough, p^* will be strictly less than 1. So reclassification risk insurance is provided for policyholders with low income growth.*

A useful comparative statics is how the bequest motive q in the second period affects the contract profile, front-loading and reclassification risk insurance:

Proposition 2 *Let $q < \hat{q}$. Let unhatted and hatted variables denote equilibrium for q and $\hat{q}$ respectively. Suppose that $\hat{p}^* < 1$. Then in equilibrium $Q_1 < \hat{Q}_1$ and $p^* < \hat{p}^*$.*

Proof. Let $q < \hat{q}$ and suppose $Q_1 \geq \hat{Q}_1$. Then the concavity of u implies that $u'(y - g - Q_1) \geq u'(y - g - \hat{Q}_1)$. Lemma 2 then implies that [notice that $Q_2^{FI}(\cdot)$ as defined by (8) and (9) does not depend on q] :

$$u'(y + g - Q_2^{FI}(p^*)) \geq u'(y + g - Q_2^{FI}(\hat{p}^*)).$$

Since $Q_2^{FI}(\cdot)$ is increasing in its argument, we thus have $p^* \geq \hat{p}^*$. This in turn implies that $\hat{Q}_2(p) \leq Q_2(p)$ and $\hat{F}_2(p) \geq F_2(p)$ for all p. Hence $p\hat{F}_2(p) - \hat{Q}_2(p) \geq pF_2(p) - Q_2(p)$ for all p. Hence

$$\hat{q} \int \left[p\hat{F}_2(p) - \hat{Q}_2(p) \right] d\Phi(p) > q \int \left[pF_2(p) - Q_2(p) \right] d\Phi(p).$$

But the above inequality and the postulated $Q_1 \geq \hat{Q}_1$ contradict the zero profit condition for both q and $\hat{q}$. ■

insurance and consumption smoothing would seem to imply that individuals with low current income and high future income prefer not to front load as much as individuals with relatively higher current income. Their formal argument was that as g increases, more of the no-lapsation constraints bind, increasing p^* and that an increase in p^* leads to a decrease in Q_1. Two mitigating factors, however, may invalidate their argument. First, Q_1 is affected by g as well. Second, the full-insurance fair premium and face amount functions $Q_2^{FI}(\cdot)$ and $F_2^{FI}(\cdot)$ both increase with g.

Proposition 2 states that an increase in the probability of losing bequest motive (i.e. a decrease in q) leads to less front-loading in the first period.[26] At the same time, however, a higher degree of reclassification risk is offered in the second period, i.e., more states are offered actuarially favorable contract terms and premiums are lower across the board. These results are rather intuitive: a higher probability of losing bequest motives (a lower q) makes it easier for the insurance companies to maintain zero profit condition because they employ lapsing-based pricing.

2.2 A Special Case: $u(\cdot) = v(\cdot)$

In this subsection, we explicitly solve for the equilibrium contract profile for the special case where $u(\cdot) = v(\cdot)$. In this special case, the full-event insurance conditions (**??**) immediately imply that

$$y - g - Q_1 = F_1 \tag{14}$$

$$y + g - Q_2(p) = F_2(p). \tag{15}$$

For period-2 health states $p \le p^*$, we know that the no-lapsation constraint (3) binds, thus

$$Q_2(p) = pF_2(p). \tag{16}$$

We immediately know from (15) and (16) that for $p \le p^*$, the equilibrium premium must be:

$$Q_2(p) = \frac{p(y+g)}{1+p} = Q_2^{FI}(p); \tag{17}$$

$$F_2(p) = \frac{y+g}{1+p} = F_2^{FI}(p). \tag{18}$$

For $p > p^*$, we know from (10) that

$$\begin{aligned} Q_2(p) &= Q_1 + 2g; \\ F_2(p) &= y + g - Q_2(p) = y - Q_1 - g. \end{aligned}$$

and p^* is given by (from Lemma 2):

$$p^* = \frac{Q_1 + 2g}{y - g - Q_1}. \tag{19}$$

Finally, the first period premium Q_1 is determined from the zero profit condition (2):

$$Q_1 - p_1(y - g - Q_1) + (1 - p_1)\, q \int_{\frac{Q_1+2g}{y-g-Q_1}}^{1} \left[Q_1 + 2g - p(y - Q_1 - g)\right] d\Phi(p) = 0. \tag{20}$$

[26] Because period-one premium Q_1 is lower and period-one face amount F_1 (from full-event insurance condition) is higher, the amount of front-loading, namely $Q_1 - p_1F_1$, must be lower.

One way to proceed is to rewrite the above equation as a function of p^*. From (19), we know that

$$\begin{aligned} Q_1 &= \frac{p^*(y-g)}{1+p^*} - \frac{2g}{1+p^*} \\ Q_1 + 2g &= \frac{p^*(y+g)}{1+p^*} \\ y - g - Q_1 &= \frac{y+g}{1+p^*} \end{aligned} \tag{21}$$

Plugging the above three equations to (20), we get:

$$\frac{p^*(y-g)}{1+p^*} - \frac{2g}{1+p^*} - \frac{p_1(y+g)}{1+p^*} + (1-p_1)\,q\int_{p^*}^{1}\left[\frac{p^*(y+g)}{1+p^*} - \frac{p(y+g)}{1+p^*}\right]d\Phi(p) = 0,$$

which, after canceling out $1/(1+p^*)$, is:

$$p^*(y-g) - [2g + p_1(y+g)] + (1-p_1)(y+g)\,q\int_{p^*}^{1}(p^*-p)\,d\Phi(p) = 0. \tag{22}$$

Notice that in principle the above is an implicit function that characterizes p^*; once p^* is determined, the rest of equilibrium contract terms $\langle Q_1, F_1, \{(Q_2(p), F_2(p)) : p \in [0,1]\}\rangle$ are all uniquely determined. An important point here is that the characterizing equation for p^* only depends on $\Phi(\cdot)$, but does not depend on any other parameters (e.g., risk aversion) for the utility function. We state this as a proposition because this result provides a stark contrast to the case with a secondary market, where we will show in the next section that preference parameters have an important qualitative effect on the equilibrium contract terms.

Proposition 3 *In the case with no secondary market, if we assume that $u(\cdot) = v(\cdot)$, then the equilibrium contract terms are independent of any other features of the utility functions.*

$\Phi(\cdot)$ is the CDF of Uniform $[0,1]$. We now consider a further special case when $\Phi(\cdot)$ is the CDF of Uniform $[0,1]$, (22) can be further simplified to:

$$\begin{aligned} p^*(y-g) - [2g + p_1(y+g)] + (1-p_1)(y+g)\,q\left[p^*(1-p^*) - \frac{1-p^{*2}}{2}\right] &= 0 \\ \iff p^*(y-g) - [2g + p_1(y+g)] - \frac{(1-p_1)(y+g)\,q}{2}(1-p^*)^2 &= 0 \\ \iff \frac{(1-p_1)(y+g)\,q}{2}(1-p^*)^2 - p^*(y-g) + [2g + p_1(y+g) - y + g] &= 0 \\ \iff \frac{(1-p_1)(y+g)\,q}{2}(1-p^*)^2 + (1-p^*)(y-g) + [p_1(y+g) + 3g - y] &= 0 \end{aligned}$$

Thus, the only admissible solution for $1-p^*$ is

$$1 - p^* = \frac{-(y-g) + \sqrt{(y-g)^2 - 2(1-p_1)(y+g)\,q\,[p_1(y+g) + 3g - y]}}{(1-p_1)(y+g)\,q}$$

Note from the above expression that for $1-p^*$ as given above to be positive, i.e., for $p^* < 1$, it must be the case that g has to be sufficiently small so that $p_1(y+g)+3g-y<0$, i.e.

$$g < \frac{1-p_1}{3+p_1}y.$$

Once p^* is determined, we get Q_1 from (21); $Q_2(p)$ and $F_2(p)$ are also obtained easily once we know p^* and Q_1.

3 Introducing the Life Settlement Market

The Secondary Market. We now introduce the life settlement market. Policyholders, who in the absence of the life settlement market would have let their long term contracts lapse, can now instead sell their contracts to life settlement firms. Recall that in the equilibrium without a secondary market, the long term policies are such that some of the second period contract terms, namely for those states in which the primary insurance contract offers reclassification risk insurance, have positive actuarial present value, denoted by $V_2(p) \equiv pF_2(p) - Q_2(p) > 0$. Policyholders who suddenly find themselves without a bequest motive and holding an actuarially favorable contract will prefer to capture some of that value rather than let the policy lapse. This, as we described in the introduction, is the source of surplus for the life settlement market.

We will assume that the secondary market only operates in period 2. This is an innocuous assumption because the zero-profit condition on the primary market ensures that all period 1 contracts are actuarially fair, and thus there is no surplus to be recovered on a secondary market for period 1 contracts. We will also assume that the decision of whether or not to sell a contract on the secondary market is made at the same time that the lapsation decision is made; that is, after learning the health state and bequest motive but *before* the uncertainty of death is realized. Moreover, we make the natural assumption that the resale price of the contract is paid to the policyholder upon transfer of the policy to the settlement company and *before* the death uncertainty is realized.

3.1 Equilibrium Contracts With Settlement Market

Now we characterize the equilibrium contact in the presence of the secondary market. To start, we will make the following assumption, which will be verified later, that *policyholders sell their contracts on the secondary market if and only if they lose their bequest motive.* We also assume that if the terms of the second-period contract in health state p, $\langle Q_2(p), F_2(p)\rangle$, imply a positive actuarial value $V_2(p) \equiv pF_2(p) - Q_2(p) > 0$, then it can be sold in the settlement market at a price $\beta V_2(p)$ where $\beta \in (0,1)$ is the amount of value the policyholder can recover from the contract;

$\beta \leq 1$ can represent either the degree of competition in the secondary market or the amount of fees/commissions/profits etc. that are spent by settlement firms.[27]

In the second period, a policyholder will sell his contract to the secondary market if and only if he has no bequest motive. A policyholder will never sell and repurchase because the best he can do if he gets full value for his contract is to repurchase the same contract terms. In the equilibrium of the primary market, the insurance companies will choose a long-term contract (we use superscript s to denote the contract terms for the secondary market case):

$$\left\langle \ (Q_1^s, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\} \ \right\rangle$$

to maximize

$$\begin{aligned} & u(y-g-Q_1^s) + p_1 v(F_1^s) + \\ & (1-p_1) \int \left\{ \ q\left[u(y+g-Q_2^s(p)) + pv(F_2^s(p))\right] + (1-q)u(y+g+\boxed{\beta V_2^s(p)}) \ \right\} d\Phi(p), \end{aligned} \tag{23}$$

where $V_2^s(p) \equiv pF_2^s(p) - Q_2(p)$, subject to the following constraints:

$$Q_1^s - p_1 F_1^s + (1-p_1) \int \left\{ \ Q_2^s(p) - pF_2^s(p) \ \right\} d\Phi(p) = 0 \tag{24}$$

$$\forall p : Q_2^s(p) - pF_2^s(p) \leq 0 \tag{25}$$

Note that there are two key difference between the problems with and without the secondary market. First, the consumer's expected utility functions (1) and (23) differ in that in the case with secondary market, the term $\beta V_2^s(p)$enters in (23), reflecting the added amount of consumption in no bequest state of the second period. Second, the zero-profit condition for the insurance company now does not have the q term multiplying $\int \{Q_2^s(p) - pF_2^s(p)\} \, d\Phi(p)$ in (24) [c.f. the expression for the case without the secondary market (2)]. The q term is no longer in the zero profit condition because with a secondary market, no policyholders with a positive actuarial value will let their contracts lapse, and the insurance companies are liable for paying the death benefits for *all* policies in the second period and of course also collect all contracted second-period premiums.

[27] As mentioned in the introduction, currently the life settlement industry typically offers about 20% of the *death benefits* to sellers, after commissions and fees. It may be plausible to think of β in the range of 0.4 to 0.6 in the current market situation.

As in the case without the secondary market, the first order conditions for an optimum are:

$$[Q_1^s]: \qquad u'(y-g-Q_1^s) = \mu \tag{26a}$$

$$[F_1^s]: \qquad v'(F_1^s) = \mu \tag{26b}$$

$$[Q_2^s(p)]: \qquad qu'(y+g-Q_2^s(p)) + \beta(1-q)u'(y+g+\beta V_2^s(p)) = \mu + \frac{\lambda(p)}{(1-p_1)\phi(p)} \tag{26c}$$

$$[F_2^s(p)]: \qquad qv'(F_2^s(p)) + \beta(1-q)u'(y+g+\beta V_2^s(p)) = \mu + \frac{\lambda(p)}{(1-p_1)\phi(p)} \tag{26d}$$

where, again, $\mu \geq 0$ is the Lagrange multipliers for (24) and $\lambda(p) \leq 0$ is the Lagrange multiplier for the no-lapsation constraint at health state p in period 2, with the complementarity slackness condition:

$$\lambda(p)\left[Q_2^s(p) - pF_2^s(p)\right] = 0 \tag{27}$$

From the first order conditions, we immediately have:

$$u'(y-g-Q_1^s) = v'(F_1^s) \tag{28}$$

$$u'(y+g-Q_2^s(p)) = v'(F_2^s(p)), \tag{29}$$

thus again we see that the equilibrium terms of the contract $\langle (Q_1^s, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\}\rangle$ must provide full-event insurance in both periods and in all health states. Lemmas analogous to Lemma 1 and 2 also hold in the secondary market case. Denote the set of states for which the no-lapsation constraints bind and not bind as $\mathcal{B}^s$ and $\mathcal{NB}^s$ respectively.

Lemma 3 *If $p \in \mathcal{B}^s$ and $p' \in \mathcal{NB}^s$, then $p < p'$ and $Q_2^s(p) < Q_2^s(p')$.*

Proof. If $p \in \mathcal{B}^s$ and $p' \in \mathcal{NB}^s$, then $\lambda(p) \leq 0$ and $\lambda(p') = 0$; and $V_2^s(p) = 0$ and $V_2^s(p') > 0$. The first order conditions (26c) for heath states p and p' thus imply that

$$qu'(y+g-Q_2^s(p)) + \beta(1-q)u'(y+g) \leq qu'(y+g-Q_2^s(p')) + \beta(1-q)u'(y+g+\beta V_2^s(p')).$$

Since $u'(y+g) > u'(y+g+\beta V_2^s(p'))$, the above inequality can hold only if $Q_2^s(p) < Q_2^s(p')$. To prove that $p < p'$, suppose to the contrary that $p \geq p'$. Then note that $p' \in \mathcal{NB}^s$ implies that $Q_2^s(p') < p'F_2^s(p')$; hence

$$Q_2^s(p) < Q_2^s(p') < p'F_2^s(p') \leq pF_2^s(p)$$

where the last inequality follows from postulated $p \geq p'$ and $F_2^s(p) \geq F_2^s(p')$ [Recall that full-event insurance requires that face amount decreases with premium]. Hence $Q_2^s(p) < pF_2^s(p)$, contradicting the assumption that $p \in \mathcal{B}^s$. ■

As in the case without a secondary market, Lemma 3 implies that there exists a threshold health state $p^{s*} \in [0,1]$ such that $p \in \mathcal{B}^s$ if $p < p^{s*}$, and $p \in \mathcal{NB}^s$ if $p > p^{s*}$.

Lemma 4 *The equilibrium contract satisfies the following at $p = p^{s*}$:*[28]

$$\begin{aligned} Q_2^s(p^{s*}) &= Q_2^{FI}(p^{s*}) \quad (30)\\ qu'(y+g-Q_2^{FI}(p^{s*})) + \beta(1-q)u'(y+g) &= u'(y-g-Q_1^s). \quad (31) \end{aligned}$$

Proof. Suppose that $Q_2^s(p^*) < Q_2^{FI}(p^{s*})$. Then $p^{s*} \in \mathcal{NB}^s$ and $\lambda(p^*) = 0$. Thus the first order conditions imply:

$$qu'(y+g-Q_2^s(p^{s*})) + \beta(1-q)u'(y+g+\beta V_2^s(p^{s*})) = u'(y-g-Q_1^s). \quad (32)$$

But since $Q_2^s(p) = Q_2^{FI}(p)$ for all $p < p^{s*}$, we have

$$\lim_{p \to p_-^{s*}} qu'(y+g-Q_2^s(p)) + \beta(1-q)u'(y+g) = qu'(y+g-Q_2^{FI}(p^{s*})) + \beta(1-q)u'(y+g) \le u'(y-g-Q_1^s). \quad (33)$$

(32) and (33) together imply:

$$qu'(y+g-Q_2^{FI}(p^{s*})) + \beta(1-q)u'(y+g) \le qu'(y+g-Q_2^s(p^{s*})) + \beta(1-q)u'(y+g+\beta V_2^s(p^{s*})),$$

but this is impossible because we postulated that $Q_2^{FI}(p^{s*}) > Q_2^s(p^{s*})$ and hence $V_2(p^{s*}) > 0$.

To prove (31), we suppose instead that $qu'(y+g-Q_2^{FI}(p^{s*})) + \beta(1-q)u'(y+g) < u'(y-g-Q_1^s)$. Then there must exist $p > p^{s*}$ but sufficiently close to p^{s*} such that:

$$qu'(y+g-Q_2^s(p)) + \beta(1-q)u'(y+g+\beta V_2^s(p)) < u'(y-g-Q_1^s),$$

contradicting that $p \in \mathcal{NB}$ for all $p > p^{s*}$. ■

Lemmas 3 and 4 establish that the equilibrium outcome in the presence of the secondary market has some resemblance to that without the secondary market. We now show that the secondary market also leads to qualitative differences in the way the primary insurance market is able to provide reclassification risk insurance to the consumers.

Proposition 4 ***(Period-2 Premium at Non-binding Health States)*** *In the presence of the life settlement market, for $p > p^{s*}$, the second-period premium $Q_2^s(p)$ is such that:*

1. $Q_2^s(p) < Q_2^{FI}(p)$;
2. *$Q_2^s(p)$ is strictly increasing in p when $q < 1$.*

[28] Thus, as in the previous case, $p^{s*} \in \mathcal{B}^s$, but $\lambda(p^{s*}) = 0$.

Proof. The first statement, of course, just directly follows from the fact that $p \in \mathcal{NB}^s$ if $p > p^{s*}$. To show that $Q_2^s(p)$ increases in p for $p > p^{s*}$, note the equations that need to be satisfied for all $p > p^*$:

$$
\begin{aligned}
& qu'(y+g-Q_2^s(p)) + \beta(1-q)u'(y+g+\beta V_2^s(p)) = u'(y-g-Q_1^s) \\
& V_2^s(p) = pF_2^s(p) - Q_2^s(p) \\
& v'(F_2^s(p)) = u'(y+g-Q_2^s(p)).
\end{aligned}
$$

Taking derivatives with respect to p for each equation we obtain:

$$
\begin{aligned}
& qu''(y+g-Q_2^s(p))\frac{dQ_2^s}{dp} = \beta^2(1-q)u''(y+g+\beta V_2^s(p))\frac{dV_2^s}{dp} \\
& \frac{dV_2^s}{dp} = F_2^s(p) + p\frac{dF_2^s}{dp} - \frac{dQ_2^s}{dp} \\
& v''(F_2^s(p))\frac{dF_2^s}{dp} = -u''(y+g-Q_2^s(p))\frac{dQ_2^s}{dp}.
\end{aligned}
$$

Solving for dQ_2^s/dp, we obtain:

$$
\frac{dQ_2^s}{dp} = \frac{F_2^s(p)}{\frac{qu''(y+g-Q_2^s(p))}{\beta^2(1-q)u''(y+g+\beta V_2^s(p))} + \left[1 + p\frac{u''(y+g-Q_2^s(p))}{v''(F_2^s(p))}\right]}, \tag{34}
$$

which is strictly positive if $q < 1$. It can also be shown that $Q_2^s(\cdot)$ increases at a lower rate than $Q_2^{FI}(\cdot)$ at $p = p^{s*}$. ■

Proposition 4 shows that the life settlement market leads to two differences in how primary insurers provide insurance against reclassification risks. First, reclassification risk insurance will no longer take the form of guaranteed flat premiums in the second period, as is the case when settlement market does not exist; instead the long-term contract now offers partial insurance in the form of premium discounts relative to the spot market premium. Figure 2 depicts the qualitative features of the second-period premium file $Q_2^s(p)$. Second, the health states for which such partial reclassification risk insurance will be provided, i.e., the set $\mathcal{NB}^s$, may be different from the set $\mathcal{NB}$ that is relevant for the case without secondary market. We are unable to prove in general whether $\mathcal{NB}^s$ is larger or smaller than $\mathcal{NB}$.

Comparative Statics with Settlement Market. We now provide some comparative statics results with settlement market.

Proposition 5 *If $q < \hat{q}$ then $Q_1^s \leq \hat{Q}_1^s$ where Q_1^s and $\hat{Q}_1^s$ are respectively the equilibrium period-one premium for q and $\hat{q}$.*

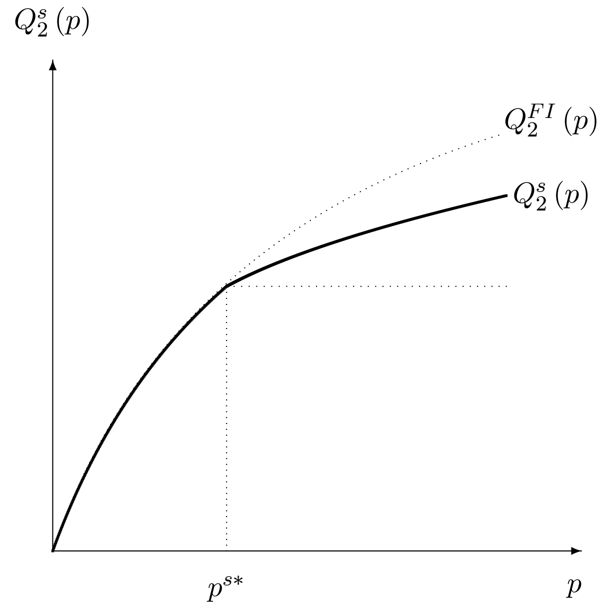

Figure 2: The Equilibrium Second-Period Premium as a Function of Health State p: The Case With Life Settlement Market.

Proof. Suppose to the contrary that $Q_1^s > \hat{Q}_1^s$. Then, $u'(y-g-Q_1^s) > u'(y-g-\hat{Q}_1^s)$. Using Lemma 4, we have:

$$qu'\left(y+g-Q_2^{FI}(p^*)\right)+\beta(1-q)u'(y+g) > \hat{q}u'(y+g-Q_2^{FI}(\hat{p}^*))+\beta(1-\hat{q})u'(y+g).$$

Rearranging the above inequality yields:

$$\beta(\hat{q}-q)u'(y+g) > \hat{q}u'\left(y+g-Q_2^{FI}(\hat{p}^*)\right)-qu'\left(y+g-Q_2^{FI}(p^*)\right). \tag{35}$$

If $\hat{p}^* \geq p^*$, then $Q_2^{FI}(\hat{p}^*) \geq Q_2^{FI}(p^*)$, thus $u'\left(y+g-Q_2^{FI}(\hat{p}^*)\right) \geq u'\left(y+g-Q_2^{FI}(p^*)\right)$. Thus, (35) implies that:

$$\beta(\hat{q}-q)u'(y+g) > (\hat{q}-q)u'(y+g-Q_2^{FI}(p^*)),$$

which is impossible when $\hat{q} > q$.

On the other hand, if $\hat{p}^* < p^*$, then there must exist $\tilde{p} > p^*$ such that $V_2^s(\tilde{p}) = \hat{V}_2^s(\tilde{p})$. Such $\tilde{p}$ must exist for the following reasons. If $\hat{p}^* < p^*$, we know that $\hat{V}_2^s(p) > V_2^s(p)$ for all $\hat{p}^* < p < p^*$. At the same time, the zero-profit conditions together with the postulated $Q_1 > \hat{Q}_1$ imply that $Q_1 - p_1F_1 = \int V_2^s(p)d\Phi(p) > \hat{Q}_1 - p_1\hat{F}_1 = \int \hat{V}_2^s(p)d\Phi(p)$, hence $V_2^s(p)$ must cross $\hat{V}_2^s(p)$ at some point $\tilde{p}$. Note also that $V_2^s(\tilde{p}) = \hat{V}_2^s(\tilde{p})$ implies that $Q_2^s(\tilde{p}) = \hat{Q}_2^s(\tilde{p})$.[29] Because we have postulated that $\tilde{p} > p^* > \hat{p}^*$, we know from the first order conditions that at $p = \tilde{p}$,

$$\begin{aligned}
& qu'(y+g-Q_2^s(\tilde{p})) + \beta(1-q)u'(y+g+\beta V_2^s(\tilde{p})) = u'\left(y-g-Q_1^s\right) \\
> \; & u'\left(y-g-\hat{Q}_1^s\right) \\
= \; & \hat{q}u'(y+g-\hat{Q}_2^s(\tilde{p})) + \beta(1-\hat{q})u'(y+g+\beta\hat{V}_2^s(\tilde{p})) \\
= \; & \hat{q}u'(y+g-Q_2^s(\tilde{p})) + \beta(1-\hat{q})u'(y+g+\beta V_2^s(\tilde{p})),
\end{aligned}$$

which could be true only if $q > \hat{q}$, a contradiction of our initial hypothesis that $q < \hat{q}$. ■

Proposition 5 tells us that the higher the probability that the consumers may lose bequest motives (i.e., the lower is q), the lower is the first-period premium. At a first glance, this may seem to be just an analogous result of Proposition 2 for the case without the settlement market; but this analogy is misleading. When there is no settlement market, a lower q directly increases the probability of lapsation and thus lower the probability that the insurance company has to pay the death benefits. Thus the primary insurance companies under competition will lower first period premiums (lower front-loading) and lower p^* (more dynamic insurance), as shown in Proposition 2. Notice that lower front-loading can be compatible with more dynamic insurance when there is no settlement market because offering more dynamic insurance (i.e. lower p^*) can still be less costly if more of these contracts will lapse (and hence no death benefits payment is necessary) as a result of lower q.

The presence of the settlement market ensures that the primary insurer will not be able avoid paying death benefits even when the consumer does not retain bequest motives. Thus the comparative statics result reported in Proposition 5 does not result from the direct effect that a lower q lowers the probability of paying out death benefits. It instead arises because the *nature* of the dynamic insurance in equilibrium is fundamentally changed: it is now in the form of premium discounts instead of flat premiums. Offering premium discounts is a less costly way of providing dynamic insurance than offering flat premiums, thus allowing the primary insurers to lower the first

[29] To see this, note that both $\langle Q_2^s(\tilde{p}), F_2^s(\tilde{p})\rangle$ and $\left\langle \hat{Q}_2^s(\tilde{p}), \hat{F}_2^s(\tilde{p})\right\rangle$ provide full-event insurance, thus

$$\begin{aligned}
u'\left(y+g-Q_2^s(\tilde{p})\right) &= v'\left(F_2^s(\tilde{p})\right) \\
u'\left(y+g-\hat{Q}_2^s(\tilde{p})\right) &= v'\left(\hat{F}_2^s(\tilde{p})\right).
\end{aligned}$$

If moreover

$$V_2^s(\tilde{p}) = \tilde{p}F_2^s(\tilde{p}) - Q_2^s(\tilde{p}) = \tilde{p}\hat{F}_2^s(\tilde{p}) - \hat{Q}_2^s(\tilde{p}) = \hat{V}_2^s(\tilde{p}),$$

it must be the case that $Q_2^s(\tilde{p}) = \hat{Q}_2^s(\tilde{p})$ and $F_2^s(\tilde{p}) = \hat{F}_2^s(\tilde{p})$.

period premium.[30]

Another important difference in terms of comparative statics results with respect to q between the cases with and without the settlement market is in how q affects p^* and p^{s*}. Proposition 2 tells us that when there is no settlement market, the insurance companies will respond to a lower q by lowering the first period premium *and* increasing dynamic insurance (i.e. a lower p^*). With the settlement market, we already showed that the primary insurers will also respond to a lower q by lowering the first period premium, but it is no longer clear that p^{s*} is also lowered. Indeed, a more plausible conjecture would be that p^{s*} will increase as q decreases. The intuition is that when the period-one premium is lowered, the zero-profit condition would require that the primary insurers offer less insurance (here, again, note importantly that a lower q does not allow the primary insurers to pay less death benefits when there is settlement market). This intuition is complicated, however, by the fact that the shape of $Q_2^s(\cdot)$ itself may be affected by q (as shown in Figure 2, and the first order conditions (26a)-(26d)). At a *global* level, we must expect to see a decline of p^{s*} as q increases from very small values to large values due to Proposition 7 below, since it establishes that with sufficiently small q, p^{s*} must be 1, and we know that when $q = 1$, p^{s*} must be less than 1 (and the same as p^* at $q = 1$). However, at a *local* level, without further assumptions, we are unable to prove that an increase in p^{s*} lowers the second-period loss for the primary insurers.[31] The different reactions of p^* and p^{s*} to q are also illustrated in our numerical results presented in Section 4 (see Figure 4).

Now we state a simple comparative statics result with respect to β, the parameter that measures the competitiveness or the efficiency (i.e. loadings) of the settlement market.

Proposition 6 *An increase in β increases consumer welfare.*

Proof. This result follows from the fact that in the optimization problem given by (23)-(25), the parameter β only shows up in the objective function. Thus, the envelope theorem applies. ■

The effect of β on the structure of equilibrium contracts is much harder to establish. The complication is similar to what we described above related to the effect of q, in that a change in β can potentially affect the shape of $Q_2^s(\cdot)$.

[30] Another useful piece of intuition to explain both Propositions 2 and 5 is that demand for consumption smoothing increases when q gets lower. Specifically, as q gets lower, it becomes more and more likely for the policyholder to be in a high income state with no bequest motive in the second period. As such, they would like to transfer income from this state to the first period, where income is lower, if they could. This transfer occurs indirectly through lower first period premiums when q is lower.

[31] Under additional assumptions which insure that $Q_2^s(\cdot; q)$ does not cross each other as q changes, then indeed one can show that a lower q leads to a higher p^{s*} in the presence of the settlement market.

3.2 Welfare Effects of the Settlement Market

We first consider a limiting result to demonstrate a potentially stark effect of the secondary market on the extent to which reclassification risk insurance can be achieved by the primary insurers.

Proposition 7 ***(Potential for Unraveling)*** *Fix $u(\cdot,), v(\cdot), y$ and $\Phi(\cdot)$. There is a threshold $\hat{q} > 0$ such that if $q < \hat{q}$, then $\mathcal{NB}^s = \emptyset$ for any g, that is, the equilibrium contract is the set of spot market contracts for all period-2 health states.*

Proof. If $\mathcal{NB}^s$ is not empty, then for any $p \in \mathcal{NB}^s$, following the first order conditions the contract terms must satisfy:

$$qu'(y + g - Q_2^s(p)) + \beta(1 - q)u'(y + g + \beta V_2^s(p)) = u'(y - g - Q_1^s), \tag{36}$$

which can be rewritten as:

$$q\left[u'(y + g - Q_2^s(p)) - \beta u'(y + g + \beta V_2^s(p))\right] = u'(y - g - Q_1^s) - \beta u'(y + g + \beta V_2^s(p)) \tag{37}$$

First note that it must be the case that $Q_1^s \geq Q_1^{FI} > 0$, where Q_1^{FI} denotes the actuarially fair premium, and that $V_2^s(p) \geq 0$. Specifically, Q_1^{FI} is implicitly defined by the unique solution to:

$$\begin{aligned} u'(y - g - Q_1^{FI}) &= v'\left(F_1^{FI}\right) \\ Q_1^{FI} &= p_1 F_1^{FI}. \end{aligned}$$

Notice that Q_1^{FI} does not depend on q, but depends on g. Some algebra shows that Q_1^{FI} is decreasing in g. Let $\bar{g}$ be the upperbound of the values that g can take, we know that $Q_1^{FI} \geq \underline{Q_1^{FI}}$ where $\underline{Q_1^{FI}} > 0$ denotes the actuarially fair full-insurance premium if $g = \bar{g}$. Hence the right hand side (RHS) of (37) is bounded below, *for any* $g > 0$, by:

$$RHS > u'(y - \underline{Q_1^{FI}}) - \beta u'(y)$$

Now examine the left hand side (LHS) of (37). We consider two cases. First, suppose that $\lim_{x\to 0} u'(x) \equiv u'(0) < \infty$. Because $Q_2^s(p)$ is always smaller than $y + g$ in equilibrium, we have that

$$u'(y + g - Q_2^s(p)) - \beta u'(y + g + \beta V_2^s(p)) < u'(0).$$

Thus if

$$q < \hat{q} = \frac{u'(y - \underline{Q_1^{FI}}) - \beta u'(y)}{u'(0)},$$

then the LHS of (37) will always be smaller than its RHS; i.e., equation (36) can never be satisfied for any p. Thus, $\mathcal{NB}^s$ must be empty.

Now consider the second case when $\lim_{x\to 0} u'(x) = \infty$. Since $p \in \mathcal{NB}^s$, we have:

$$pF_2^s(p) - Q_2^s(p) > 0 \tag{38}$$

Plugging (29) into inequality (38), we obtain:

$$pv'^{-1}(u'(y+g-Q_2^s(p))) > Q_2^s(p), \tag{39}$$

or equivalently,

$$u'(y+g-Q_2^s(p)) < v'\left(\frac{Q_2^s(p)}{p}\right). \tag{40}$$

Notice that the LHS of (40) is increasing as $Q_2^s(p)$ varies from 0 to $y+g$, and that its RHS is decreasing over the same interval. If $u'(y+g) \geq v'(0)$ then (40) cannot be satisfied for any value of $Q_2^s(p)$ and hence$\mathcal{NB}^s$ must be empty, thus we can without loss of generality consider the case that $u'(y+g) < v'(0)$. Since we are now considering the case in which $u'(0) = \infty$, we know that at $Q_2^s(p) = y+g$, LHS of (40) is $u'(0) > v'((y+g)/p)$ for all p. Because LHS of (40) is continuous and monotonically increasing in Q_2^s, while the RHS of (40) is continuous and monotonically decreasing in Q_2^s, there must exist, for each $p \in \mathcal{NB}^s$ some $x(p;g) < y+g$ such that $u'(y+g-x) = v'(x/p)$, and hence $Q_2^s(p)$ must be bounded above by $x(p;g)$. Moreover, note that, for all g, it can be easily shown that $x(p;g)$ is increasing in p. Thus we can write $\sup_{p\in\mathcal{NB}^s} x(p;g) = x(1;g) \equiv x(g) < y+g$, for all g. Now denote $\bar{u}' \equiv \max_g u'(y+g-x(g)) < \infty$. We hence have

$$\begin{aligned} u'(y+g-Q_2^s(p)) - \beta u'(y+g+\beta V_2^s(p)) &< u'(y+g-Q_2^s(p)) \\ &< u'(y+g-x(p;g)) \\ &\leq u'(y+g-x(g)) \\ &\leq \bar{u}', \end{aligned}$$

where the second inequality follows from $Q_2^s(p) < x(p;g)$; the third inequality follows from $x(p,g) \leq x(g)$, and the last inequality follows from $\bar{u}' \equiv \max_g u'(y+g-x(g))$. Thus, if

$$q < \hat{q} \equiv \frac{u'(y-\underline{Q}_1^{FI}) - \beta u'(y)}{\bar{u}'},$$

then the LHS of (37) will always be smaller than its RHS; i.e., equation (36) can never be satisfied for any p. Thus, $\mathcal{NB}^s$ must be empty. ■

Proposition 7 shows that in the presence of the life settlement market, the primary life insurance market can no longer offer any dynamic reclassification risk insurance for *any* level of g, when q is sufficiently small. This result provides a stark contrast to Claim (3) in Proposition 1 which states that, without life settlement, the equilibrium contract offered by life insurance companies must involve some degree of reclassification risk insurance *when g is sufficiently small.* Proposition 7 thus tells us that settlement market may lead to the unraveling of the capacity of primary life insurance market to offer dynamic reclassification risk insurance.

Note that Proposition 7 provides a clear welfare ranking of the equilibria with and without the settlement market for environments with small g and small q : without settlement market, when g and q are small, the equilibrium contracts must offer dynamic reclassification risk insurance; with settlement market, the equilibrium insurance contracts must be spot contracts. Thus settlement market reduces consumer welfare in these cases.

Our next proposition shows a much stronger result for welfare comparison for the cases with and without settlement market:[32]

Proposition 8 ***(Welfare Effects of The Secondary Market)*** *Consumer welfare is reduced by the presence of a life settlement market.*

Proof. We will show that for any contract that is feasible with a secondary market, there is a contract that is feasible without the secondary market, which makes the consumer weakly better off.

Let $C^s = \langle (Q_1^s, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\}\rangle$ be a feasible contract with a secondary market. Thus:

$$Q_1^s - p_1 F_1^s = (1-p_1)\int V_2^s(p) d\Phi(p),$$

where $V_2^s(p) \equiv pF_2^s(p) - Q_2^s(p)$. Consider the contract $\hat{C} = \left\langle (\hat{Q}_1, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\}\right\rangle$ where $\hat{Q}_1$ is given by:

$$\hat{Q}_1 - p_1 F_1^s = q(1-p_1)\int V_2^s(p) d\Phi(p).$$

Since $q \in (0,1)$, we know that $\hat{Q}_1 < Q_1^s$. That is, $\hat{C}$ is exactly the same contract as C^s except that the first period premium is decreased until the zero profit condition for the no-secondary-market case holds. It is easy to see that $\hat{C}$ is a feasible contract for the case without a secondary market, but infeasible with a secondary market.

We will now show that $\hat{C}$ in a world without secondary market is better than C in a world with secondary market. To see this, let

$$\begin{aligned} W^s(C^s) = & p_1 v(F_1^s) + u(y-g-Q_1^s) + \\ & (1-p_1)\int \left\{q\left[pv(F_2^s(p)) + u(y+g-Q_2^s(p))\right] + (1-q)u(y+g+\beta V_2^s(p))\right\} d\Phi(p) \end{aligned}$$

[32] This is the same result as Proposition 2 (iii) in DHL (2008).

denote the expected consumer welfare associated with contract C^s in a world with a secondary market. Let

$$W\left(\hat{C}\right) = p_1 v(F_1^s) + u(y - g - \hat{Q}_1) + \\ (1-p_1)\int \left\{q\left[pv(F_2^s(p)) + u(y+g-Q_2^s(p))\right] + (1-q)u(y+g)\right\} d\Phi(p)$$

denote the expected consumer welfare associated with contract $\hat{C}$ in a world without a secondary market. Note that

$$\begin{aligned} W\left(\hat{C}\right) - W^s\left(C^s\right) &= u(y-g-\hat{Q}_1) - u(y-g-Q_1^s) \\ &\quad - (1-p_1)(1-q)\int \left[u(y+g+\beta V_2^s(p)) - u(y+g)\right] d\Phi\left(p\right) \\ &\geq u(y-g-\hat{Q}_1) - u(y-g-Q_1^s) \\ &\quad - (1-p_1)(1-q)\left[u\left(y+g+\beta\int V_2^s(p)d\Phi\left(p\right)\right) - u(y+g)\right] \end{aligned}$$

where the inequality is a result of Jensen's inequality due to the concavity of $u(\cdot)$. Further note that:

$$\begin{aligned} &(1-q)\left[u\left(y+g+\beta\int V_2^s(p)d\Phi\left(p\right)\right) - u(y+g)\right] \\ &= (1-q)u\left(y+g+\beta\int V_2^s(p)d\Phi\left(p\right)\right) + qu(y+g) - u(y+g) \\ &\leq u\left(y+g+\beta(1-q)\int V_2^s(p)d\Phi\left(p\right)\right) - u(y+g), \end{aligned}$$

where again the inequality follows from Jensen's inequality. Thus,

$$\begin{aligned} W\left(\hat{C}\right) - W^s\left(C^s\right) &\geq u(y-g-\hat{Q}_1) - u(y-g-Q_1^s) \\ &\quad - (1-p_1)\left[u\left(y+g+\beta(1-q)\int V_2^s(p)d\Phi\left(p\right)\right) - u(y+g)\right] \end{aligned}$$

First note that $Q_1^s - \hat{Q}_1 = (1-p_1)(1-q)\int V_2^s(p)d\Phi\left(p\right)$. By the continuous function theorem, we know that there exists $\delta_1 \in (0,1)$ such that

$$\begin{aligned} & u(y-g-\hat{Q}_1) - u(y-g-Q_1^s) \\ = \quad & u'\left(y-g-Q_1^s + \delta'\left(Q_1^s - \hat{Q}_1\right)\right)\left(Q_1^s - \hat{Q}_1\right). \end{aligned}$$

Similarly, there exists $\delta_2 \in (0,1)$ such that

$$\begin{aligned} & (1-p_1)\left[u\left(y+g+\beta(1-q)\int V_2^s(p)d\Phi(p)\right)-u(y+g)\right] \\ = \; & (1-p_1)\left[u'\left(y+g+\delta_2\beta(1-q)\int V_2^s(p)d\Phi(p)\right)\beta(1-q)\int V_2^s(p)d\Phi(p)\right] \\ = \; & u'\left(y+g+\delta_2\beta(1-q)\int V_2^s(p)d\Phi(p)\right)\beta\left(Q_1^s-\hat{Q}_1\right). \end{aligned}$$

Hence

$$\begin{aligned} & W\left(\hat{C}\right)-W^s\left(C^s\right) \\ \geq \; & \left[u'\left(y-g-Q_1^s+\delta'\left(Q_1^s-\hat{Q}_1\right)\right)-\beta u'\left(y+g+\delta_2\beta(1-q)\int V_2^s(p)d\Phi(p)\right)\right]\left(Q_1^s-\hat{Q}_1\right) \\ \geq \; & 0 \end{aligned}$$

with inequality being strict if $Q_1^s - \hat{Q}_1$ is strictly positive, i.e., if there is dynamic reclassification risk insurance under contract C^s.

Now let C^s be the *equilibrium* contract in the presence of the settlement market. The above argument shows that the contract $\hat{C}$ constructed through a simple reduction of first period premium is feasible in the case without the settlement market; and $\hat{C}$ provides weakly (or strictly, if C^s offers some dynamic insurance) higher expected utility to the consumers for the case without settlement market than C^s would provide for consumers with settlement market. Because $\hat{C}$ is only a candidate contract for the case without settlement market, the equilibrium contract in that case must provide no lower expected consumer welfare than $\hat{C}$. ■

Proposition 8 formalizes an intuitive argument provided in Claim 3 of Proposition 2 in Daily, Hendel, and Lizzeri (2008). They argued that the settlement market effectively transfers resources from period 1 when income is relatively low to period 2 when income is relatively high in events of losing bequest motive. Such transfers, due to concavity of the utility function, is welfare reducing. Their argument seems to hinge on a hypothesis that the first-period equilibrium premium in the primary market with the settlement market is higher than that without. We show below in Section 4 that such a result does not hold in general.[33] Our proof does not rely on such cross-regime comparisons of first-period premiums.

[33] An extreme example of this could be seen as a corollary of Proposition 7: when q is sufficiently small, we know that the primary insurance market could not offer any dynamic insurance, which implies that first period premium is $Q_1^s = Q_1^{FI}$, the actuarially fair premium; in contrast, when g is small, Proposition 1 tells us that, without the settlement market, there will be dynamic insurance, implying that the first period premium $Q_1 > Q_1^{FI}$ because of front-loading.

3.3 The Special Case when $u(\cdot) = v(\cdot)$

It is useful to re-examine the special case when $u(\cdot) = v(\cdot)$ in the case with the secondary market. Recall that in Proposition 3 we showed that in the absence of the secondary market, the terms of the equilibrium life insurance contracts are independent of any other features of these functions (e.g. risk aversion). We now show that in the presence of a secondary market, this invariance property breaks down.

For states $p \leq p^{s*}$, the conditions that characterize $(Q_2^s(p), F_2^s(p))$ are exactly the same as the case without the secondary market:

$$\begin{aligned} Q_2^s(p) &= Q_2^{FI}(p) = \frac{p(y+g)}{1+p}; \\ F_2^s(p) &= \frac{y+g}{1+p}. \end{aligned}$$

However, the conditions chararcterizing $(Q_2^s(p), F_2^s(p))$ for $p > p^*$ differ substantially from the earlier case. This can be easily seen from our proof of Proposition 4 where we showed that for $p > p^{s*}$, $Q_2^s(p)$ is increasing in p. Expression (34) in that proof showed that, for $p > p^{s*}$,

$$\frac{dQ_2^s}{dp} = \frac{F_2^s(p)}{\frac{qu''(y+g-Q_2^s(p))}{\beta^2(1-q)u''(y+g+\beta V_2^s(p))} + \left[1 + p\frac{u''(y+g-Q_2^s(p))}{v''(F_2^s(p))}\right]}.$$

When $u = v$, and noting that full-event insurance ensures that $y + g - Q_2^s(p) = F_2^s(p)$, we know that the term

$$\frac{u''(y+g-Q_2^s(p))}{v''(F_2^s(p))} = 1.$$

But the magnitude of the other term

$$\frac{qu''(y+g-Q_2^s(p))}{\beta^2(1-q)u''(y+g+\beta V_2^s(p))}$$

depends on the shape of u. Thus the extent of reclassification risk insurance in health states $p \in \mathcal{NB}^s$, as measured by the difference between $Q_2^{FI}(p)$ and $Q_2^s(p)$, depends on the shape of u. Moreover, the zero-profit condition will also lead p^{s*} to depend on the shape of u. We summarize this discussion in the following proposition:

Proposition 9 *In the case with secondary market, the equilibrium contract terms depend on the shape of the utility functions when we assume that $u(\cdot) = v(\cdot)$.*

The contrast between Propositions 3 and 9 is illustrated in Figures 10-11 in our numerical examples in the next section. They are also useful as a basis for empirical test of the effect of the

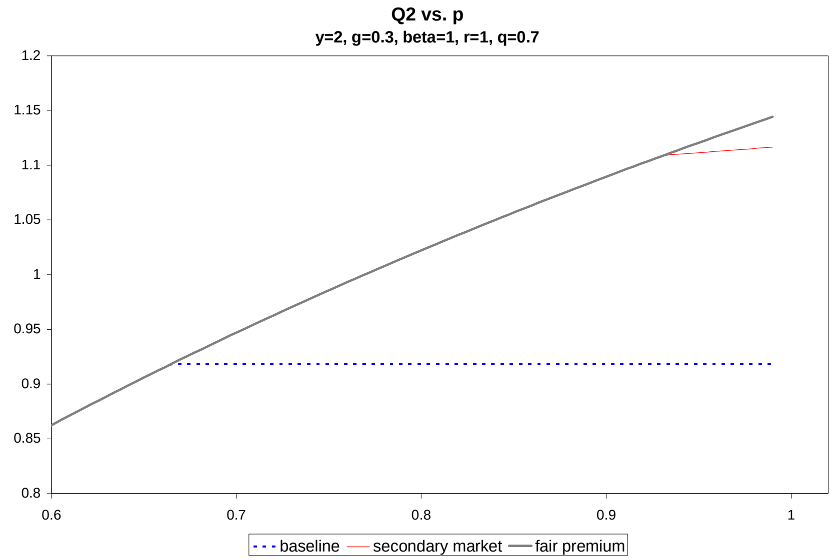

Figure 3: The Relationship Between Q_2 and Q_2^s with p.

secondary market: to the extent that $u(\cdot) = v(\cdot)$ is valid and that consumers' preferences parameters such as risk aversion are measured, we should expect that, in the absence of the secondary market, consumers' choices of life insurance contracts should not be affected by their risk aversion, but in the presence of the secondary market their choices will be affected.

4 Comparing the Equilibrium Outcomes With and Without Secondary Market: Numerical Results

Even though we are able to provide an unambiguous welfare comparison for the equilibrium outcomes with and without the settlement market, we are not able to analytically compare the contractual terms for the equilibria under the two regimes, except for some limiting cases. For example, a corollary of Proposition 7 is that when q is sufficiently small, the first period premium is lower with the settlement market than without, because the settlement market makes reclassification risk insurance impossible.

In this section, we provide more results about the comparisons of the outcomes with and without the secondary market from a parametric example. In this example, we assume that both u and v are constant absolute risk aversion (CARA) utility functions, i.e.,

$$u(x) = v(x) = 1 - \exp(-rx)$$

where $r > 0$ is the absolute risk aversion; and the CDF of the period-two death probabilities Φ is the CDF of a uniform distribution on $[0, 1]$.

Figure 3 depicts the relationship between $Q_2(\cdot)$ and $Q_2^s(\cdot)$ with p in equilibrium. It confirms our analytical characterizations regarding the qualitative properties of Q_2 and Q_2^s (see Proposition 1 and Proposition 4). It shows that in the absence of the secondary market, Q_2 is flat when p exceeds a threshold p^*; while with secondary market, Q_2^s continues to rise with p after p exceeds p^{s*}. In this example, it turns out that $p^{s*} > p^*$; thus the extent of reclassification risk insurance is reduced by the secondary market in two aspects: first, $p^{s*} > p^*$; second, the insurance takes the form of premium discounts in healthy states with secondary market instead of flat premium without the secondary market.

Figure 4 depicts the relationship between the threshold health states p^{s*} and p^*, for the cases with and without secondary market respectively, with the probability of retaining bequest motive q. For the case without the secondary market, it is increasing, thus confirming the predictions of Proposition 2. For the case with the secondary market, it shows that when q is small, p^{s*} is 1 and thus there is no dynamic insurance, which is consistent with our theoretical result presented in Proposition 7. At $q = 1$, p^{s*} indeed coincides with p^* (and both are less than 1) because the secondary market should make no difference when no one intends to lapse their contract. In the intermediate values of q, we see that p^{s*} decreases in q. Thus the settlement market leads to *a qualitatively different relationship* between the threshold states and q; moreover, p^{s*} is always higher than p^* in this numerical example, implying that the extent of dynamic insurance is reduced in the presence of the settlement market. This reduction in the extent of the dynamic insurance occurs both in the sense that the premium is higher (see Figure 3) and smaller coverage of health states (see Figure 4).

Even though it is true that, as DHL (2008) argued in their claim (ii) of Proposition 2, the presence of a settlement market *raises* the amount of front-loading that is required to obtain the *same* amount of reclassification risk insurance, we have seen from Proposition 4 that the presence of the settlement market qualitatively changes how the reclassification risk insurance is delivered (premium discounts in unhealthy states instead of a guaranteed premium) and the extent of the reclassification risk insurance in the sense that p^{s*} differs from p^* (e.g., Figure 4 above shows that p^{s*} is typically higher than p^*, thus implying a smaller extent of insurance). Both of these changes may *reduce* the reclassification risk insurance provided when the settlement market is present. Thus it is possible that the amount of front-loading may be reduced in equilibrium. Figure 5 shows the relationship between Q_1 and Q_1^s as a function of q, the probability that the bequest motive is retained. The figure shows that except for the cases when $q = 0$ or $q = 1$, the first period premium (and thus front-loading) is higher in the case without the secondary market.

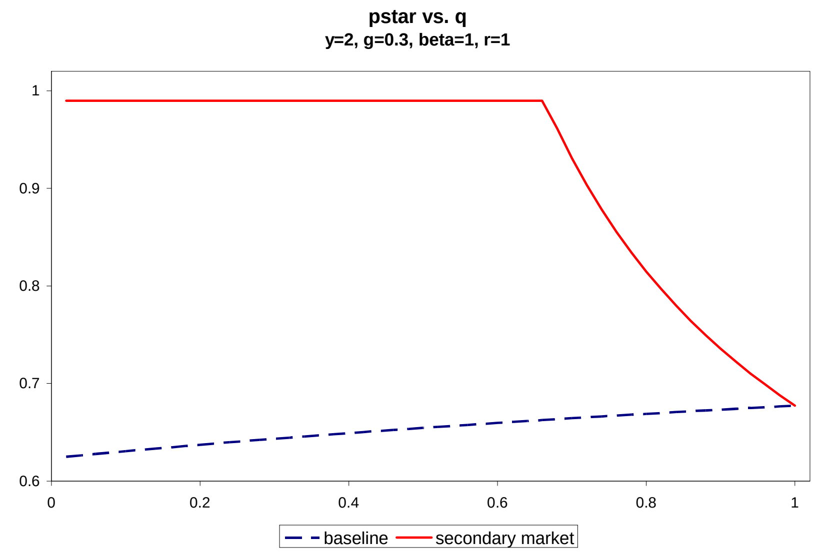

Figure 4: The Relationship Between p^* and p^{s*} with the Probability of Retaining Bequest Motive q.

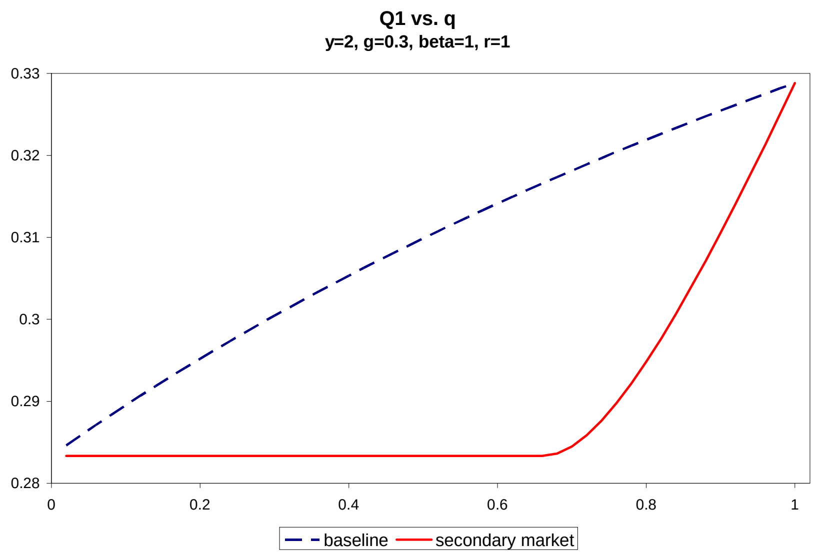

Figure 5: How Does the Secondary Market Affect the First Period Premium, for Varying Values of q?

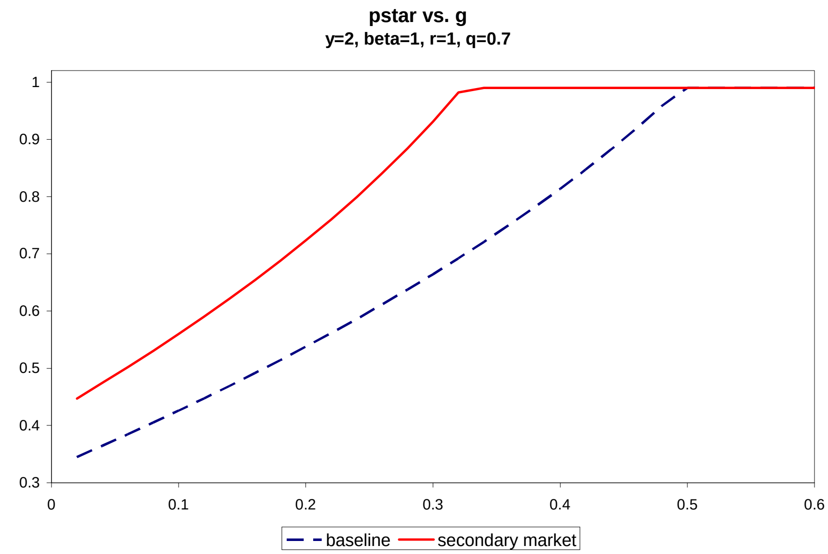

Figure 6: The Relationship Between p^{s*} and p^* and g.

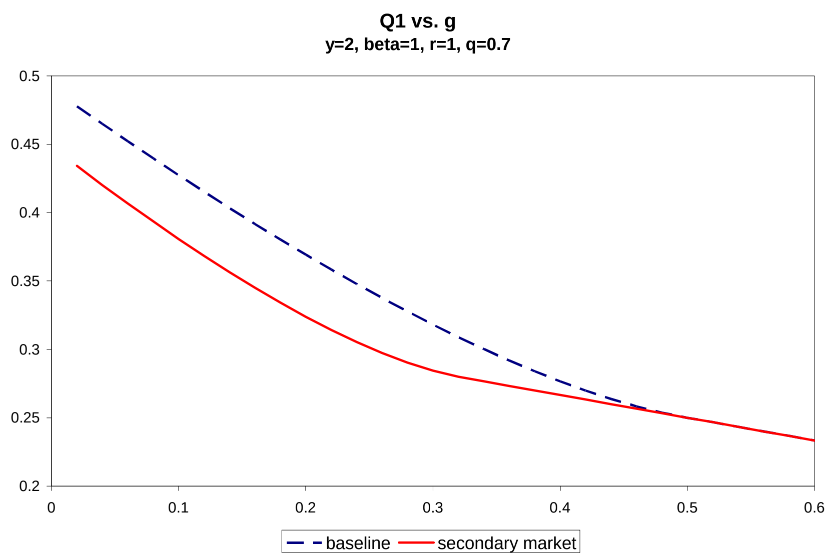

Figure 7: The Relationship Between Q_1 and Q_1^s and g.

Figure 6 shows how the equilibrium threshold states p^* and p^{s*} vary with income growth g. It shows that both p^* and p^{s*} are weakly increasing in g. Figure 7 shows how the equilibrium first period premiums Q_1 and Q_1^s vary with income growth g. We see that when there is no settlement market, consumers with higher income growth choose contracts with lower front loading and less reclassification risk insurance, as predicted by Proposition 1(iv) in Hendel and Lizzeri (2003). The same qualitative relationship holds in the secondary market case. Given our earlier discussion in footnote 25, Figure 7 shows that the force where, *ceteris paribus*, consumers with higher income growth find it less desirable to transfer resources from the first period to the second period dominates the other mitigating forces in this numerical example. In both cases, if g is sufficiently high, consumers will no longer wish to obtain reclassification risk insurance.

Figures 8 depicts the relationship between the equilibrium threshold state p^{s*} and β, the proportion of the contract's actuarial value that the consumer is able to recover from the secondary market. Figure 9 depicts the relationship between equilibrium first period premium and β. From these figures we see that as β increases, so too does the equilibrium level of front-loading and reclassification risk insurance. This is intuitive, because as β becomes higher, reclassification risk insurance becomes more attractive as more values can be recovered in the event that the consumer loses his bequest motive. As such, consumers are willing to front-load a little bit more for the same amount of reclassification insurance. We should be a little careful with the intuition, however, because there is a second effect that works in the opposite direction. This effect is as follows: when β is higher, it becomes cheaper in terms of front-loading to maintain the same level of consumption in the no-bequest state, so we might expect the amount of front-loading to decrease. From the figures, it appears that the first effect dominates the second. Also note that unraveling ($p^{s*} = 1$) is more likely to occur for smaller values of β.

Figure 10 depicts the relationship between threshold states and the risk aversion parameter r, and Figure 11 depicts the relationship between first period premiums and r. In the absence of the secondary market, the results confirms our our prediction from Propositions 3: p^* and Q_1 are independent of r. When there is a secondary market, the results are also intuitive: higher absolute risk aversion decreases p^{s*} and increases Q_1^s, but the degree of front-loading and risk classification insurance is always lower than what is achieved without the secondary market. These figures demonstrate an important qualitative difference between the equilibria with and without the settlement market: the optimal contract is influenced by preference parameters only when there is a settlement market. This could be potentiallyan interesting testable implication.

Finally, Figure 12 plots the difference in consumer welfare between the two regimes with and without a secondary market. Our results in Propositions 8 and 6 are confirmed. The region on the left in which the lines converge is an illustration of unraveling: only when the primary contracts

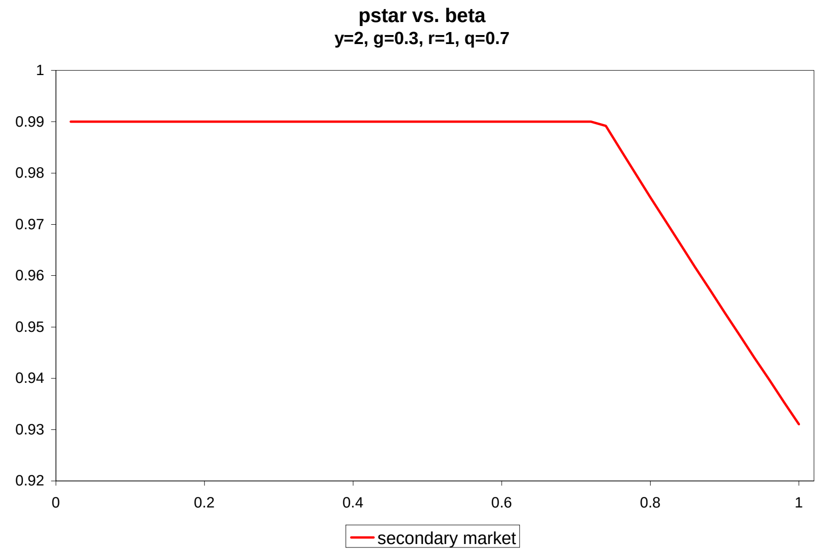

Figure 8: The Relationship Between p^{s*} and β with the Secondary Market.

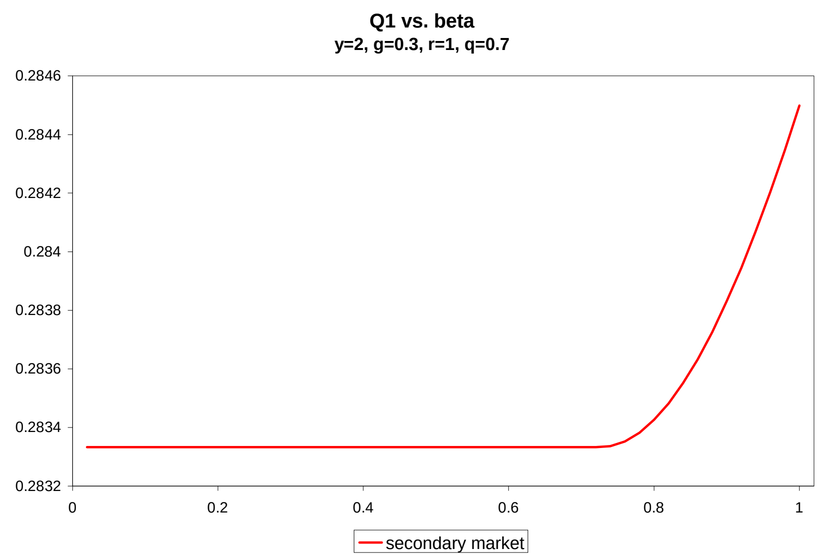

Figure 9: The Relationship Between Q_1^s and β with the Secondary Market.

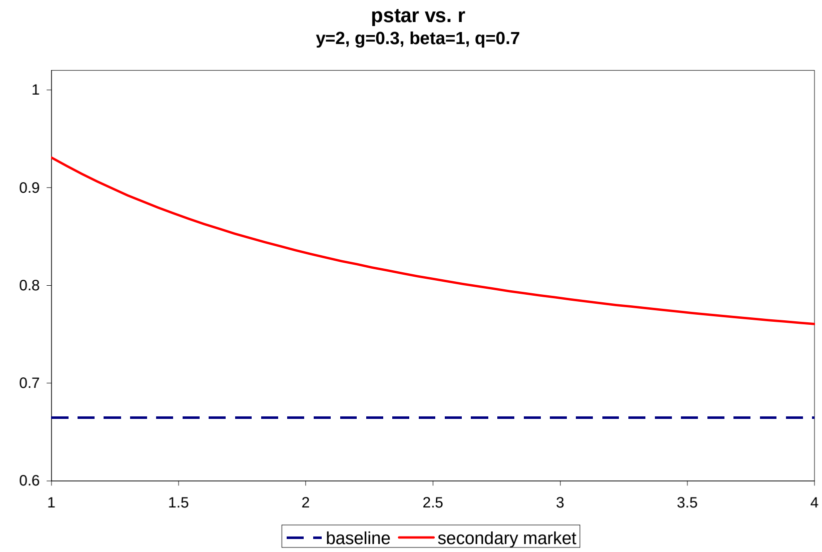

Figure 10: The Relationship Between p^* and p^{s*} with the Absolute Risk Aversion Parameter r.

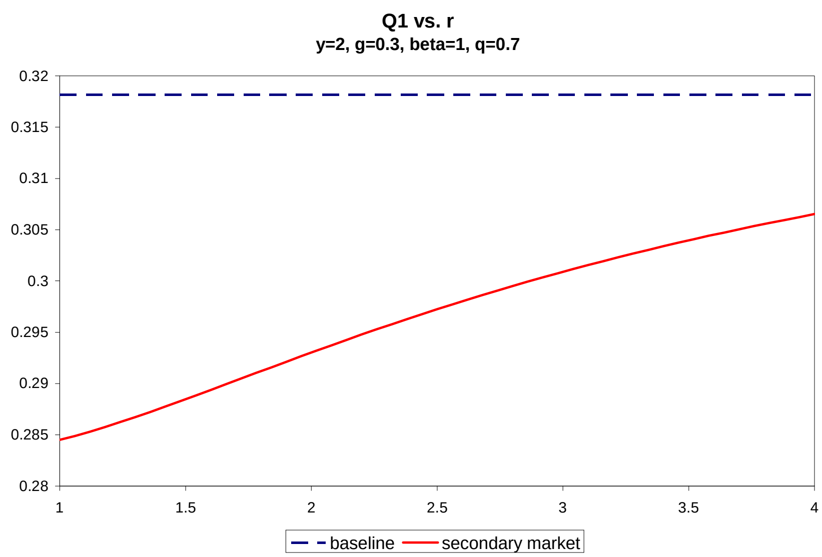

Figure 11: The Relationship Between Q_1 and Q_1^* and the Absolute Risk Aversion Parameter r.

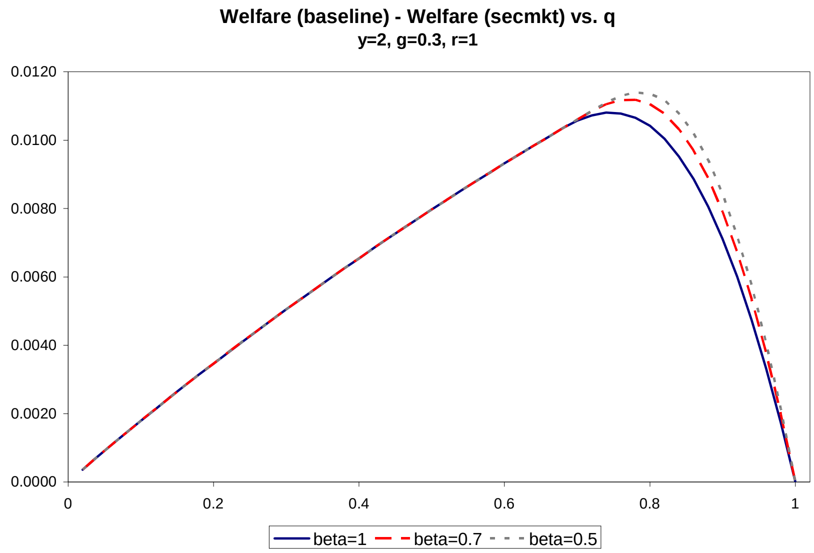

Figure 12: The Welfare Effects of the Life Settlement Market, for Varying Levels of β.

degenerate to spot market contracts can the welfare be the same across different levels of β.

Although we were only able to derive a few of the above results analytically, they appear to be quite robust to different specifications of the parameters.

5 Endogenous Cash Surrender Values

So far, we have limited our discussion of the primary market response to the settlement market subject to the restriction that they can only choose contracts of the form $\langle (Q_1^s, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\}\rangle$. One may imagine that facing the threat from the life settlement market, the primary insurers may start to enrich their contracts by specifying endogenously chosen cash surrender values (CSV).

In this section, we first study the case in which the primary insurers can specify health-contingent CSVs in the life insurance contracts. That is, feasible contracts will now take the form $\left\langle (Q_1, F_1), \left\{(Q_2(p), F_2(p), \boxed{S(p)}) : p \in [0,1]\right\}\right\rangle$ where the additional term $S(p)$ specifies the cash surrender value a policyholder can receive from the primary insurer if he surrenders the contract when his period-two health status is p. Notice that by paying $S(p)$, the primary insurers can *avoid* paying the death benefit, but does not receive the premium payment in the second period; this is significantly different from when the policyholder sells the contract to the settlement market in which case the primary insurer still receives payment of the second-period premium, but remains responsible for paying the death benefits.

5.1 Endogenous Cash Surrender Values Without a Secondary Market

As a benchmark, we here show that, in the absence of a life settlement market, the consumers will optimally choose to set $S(p) = 0$ when the feasible contract space is enriched to include health contingent CSVs. To see this, we will simply characterize the equilibrium contract. The competitive insurance companies will choose to offer the long-term contract $\left\langle (Q_1, F_1), \left\{ (Q_2(p), F_2(p), \boxed{S(p)}) : p \in [0,1] \right\} \right\rangle$ that maximizes:

$$u(y - g - Q_1) + p_1 v(F_1) + \quad (41)$$
$$(1 - p_1) \int \{ q [u(y + g - Q_2(p)) + pv(F_2(p))] + (1 - q) u(y + g + S(p)) \} \, d\Phi(p)$$

subject to the following constraints:

$$Q_1 - p_1 F_1 + (1 - p_1) \int \{ q [Q_2(p) - pF_2(p)] - (1 - q) S(p) \} \, d\Phi(p) = 0 \quad (42)$$

$$\forall p : Q_2(p) - pF_2(p) \leq 0 \quad (43)$$

$$\forall p : S(p) \geq 0 \quad (44)$$

Constraint (44) captures the inability of the consumer to commit to contracts requiring him to pay the insurance company. The first order conditions for an optimum are:

$$u'(y - g - Q_1) = \mu \quad (45a)$$

$$v'(F_1) = \mu \quad (45b)$$

$$u'(y + g - Q_2(p)) = \mu + \frac{\lambda(p)}{(1 - p_1) q \phi(p)} \quad (45c)$$

$$v'(F_2(p)) = \mu + \frac{\lambda(p)}{(1 - p_1) q \phi(p)} \quad (45d)$$

$$u'(y + g + S(p)) = \mu + \frac{\gamma(p)}{(1 - p_1)(1 - q) \phi(p)}, \quad (45e)$$

where $\gamma(p)$ is the Lagrange multiplier for (44) for second-period health state p. It is easy to see that (44) must bind for all p. If it were slack for some p, then $\gamma(p) = 0$ and $u'(y+g+S(p)) = u'(y-g-Q_1)$ which is impossible. From this, we can conclude that without a secondary market, the primary insurance companies will choose in equilibrium not to include a surrender option in their long-term contract. The proposition below summarizes this discussion:

Proposition 10 *In the absence of a settlement market, the option to include surrender values in long-term contracts will not be provided by the life insurance companies; equilibrium contracts and outcomes for the consumers will be the same regardless of whether surrender values are allowed.*

Proposition 10 is consistent with the observation that Term Life insurance products indeed do not carry any cash surrender value. Whole Life insurance policies, as we mentioned in the introduction, often do specify a low amount of Cash Surrender Value if the policyholder cancels the policy prior to death. The industry has typically advertised the cash surrender value option as a redemption of front-loaded premium payments, even though many industry analysts disagree and think that it should be better interpreted as a saving instrument that exploits the tax advantages of life insurance payouts (see, e.g., Gilbert and Schultz 1994, Chapter 6). Proposition 10 suggests the latter option is more appropriate; in the absence of threats from secondary market, it would have been efficient not to specify any cash surrender value in a pure life insurance contract.

5.2 Endogenous Health-Contingent Cash Surrender Values with a Secondary Market

Now we consider equilibrium contracts with health-contingent surrender values in the presence of (or threat of) a secondary market. The key difference from above is that the surrender value can be no less than what the policyholder could obtain from the secondary market. We denote the equilibrium contract in the case of secondary market with surrender values with the superscript ss. The competitive insurance companies offer a long-term contract $\left\langle (Q_1^{ss}, F_1^{ss}), \left\{ (Q_2^{ss}(p), F_2^{ss}(p), \boxed{S^{ss}(p)}) : p \in [0,1] \right\} \right\rangle$ to maximize:[34]

$$u(y - g - Q_1^{ss}) + p_1 v(F_1^{ss}) + \tag{46}$$
$$(1 - p_1) \int \{ q [u(y + g - Q_2^{ss}(p)) + pv(F_2^{ss}(p))] + (1 - q) u(y + g + S^{ss}(p)) \} \, d\Phi(p)$$

subject to:

$$Q_1^{ss} - p_1 F_1^{ss} + (1 - p_1) \int \{ q [Q_2^{ss}(p) - pF_2^{ss}(p)] - (1 - q) S^{ss}(p) \} \, d\Phi(p) = 0 \tag{47}$$

$$\forall p : Q_2^{ss}(p) - pF_2^{ss}(p) \leq 0 \tag{48}$$

$$\forall p : \beta [pF_2^{ss}(p) - Q_2^{ss}(p)] - S^{ss}(p) \leq 0. \tag{49}$$

[34] We omit another constraint for no-lapsation: $S^{ss}(p) \leq V_2^{ss}(p)$ in the formulation of the problem. Lemma **??** below ensures that this constraint never binds and thus the solution to the problem is not affected by the omission.

The first order conditions for an optimum are:

$$u'(y-g-Q_1^{ss}) = \mu \tag{50a}$$

$$v'(F_1^{ss}) = \mu \tag{50b}$$

$$u'(y+g-Q_2^{ss}(p)) = \mu + \frac{\lambda(p)}{(1-p_1)qf(p)} - \frac{\beta\gamma(p)}{(1-p_1)q\phi(p)} \tag{50c}$$

$$v'(F_2^{ss}(p)) = \mu + \frac{\lambda(p)}{(1-p_1)q\phi(p)} - \frac{\beta\gamma(p)}{(1-p_1)q\phi(p)} \tag{50d}$$

$$u'(y+g+S^{ss}(p)) = \mu + \frac{\gamma(p)}{(1-p_1)(1-q)\phi(p)} \tag{50e}$$

where $\gamma(p)$ is the Lagrange multiplier for constraint (49).

From these conditions, we see that constraint (49) must bind for all p because otherwise, $\gamma(p)$ must be equal to 0 and then $u'(y+g+S^{ss}(p)) = u'(y-g-Q_1)$, which cannot hold. Thus,

Lemma 5 *In the presence of a secondary market, health-contingent surrender values with be optimally chosen to be equal to the amount that can be obtained from the secondary market.*

We now state without proof two additional lemmas, which are analogous to Lemmas 3 and 4. The proofs of the following can be adapted without difficulty from the proofs of Lemmas 3 and 4. They follow from the observation that in equilibrium, the following condition must hold for each p:

$$qu'(y+g-Q_2^{ss}(p)) + \beta(1-q)u'(y+g+\beta V_2^{ss}(p)) = (q+\beta(1-q))u'(y-g-Q_1^{ss}) + \frac{\lambda(p)}{(1-p_1)\phi(p)}$$

Lemma 6 *Let $\mathcal{B}^{ss}$ denote the set of states for which (49) binds and $\mathcal{NB}^{ss}$ the set of states for which 49 does not bind. If $p \in \mathcal{B}^{ss}$ and $p' \in \mathcal{NB}^{ss}$ then $p < p'$ and $Q_2^{ss}(p) \leq Q_2^{ss}(p')$.*

Lemma 6 guarantees the existence of a threshold health state p^{ss*} above which second period premiums are actuarially favorable. Lemma 7 below characterizes this threshold state.

Lemma 7 *The equilibrium contract satisfies the following at $p = p^{ss*}$:*

$$Q_2^{ss}(p^{ss*}) = Q_2^{FI}(p^{ss*}) \tag{51}$$

$$qu'(y+g-Q_2^{FI}(p^{ss*})) + \beta(1-q)u'(y+g) = (q+\beta(1-q))u'(y-g-Q_1^{ss}) \tag{52}$$

Lemmas 6 and 7 show that the equilibrium contracts are *qualitatively* similar in the case of secondary market with surrender values as compared to the case of secondary market without surrender values. However, because as we mentioned earlier, even though the primary insurance companies would have to offer cash surrender value that exactly matches what the consumers who lost bequest motives could obtain in the secondary market, their payoffs are fundamentally different depending on whether the primary insurers are offering endogenously chosen CSV. When $S^{ss}(p)$ is endogenously chosen, once a consumer with health state p loses bequest motive, he will surrender his policy to the primary insurance company in exchange for CSV $S^{ss}(p)$, but the insurance company does not receive further premium payment $Q_2^{ss}(p)$ and would not have to pay out death benefits $F^{ss}(p)$. When there is no CSV, the primary insurer will continue to receive premium payments from the settlement firm, but will have to pay the death benefit. It is a priori not clear how the endogenous CSV will affect the quantitative features of the contracts and how it would affect consumer welfare. We examine these issues below.

5.2.1 Cross-Regime Welfare Comparisons

We have already shown that when surrender values are not allowed, the presence of a settlement market is welfare reducing. It will be useful now to compare welfare across the four regimes we have analyzed so far:

Regime A. no secondary market with cash surrender values;

Regime B. no secondary market without cash surrender values;

Regime C. secondary market with endogenous cash surrender values;

Regime D. secondary market without cash surrender values.

Proposition 10 shows that there is no welfare difference between regimes A and B. It is also easy to show that consumer welfare is higher in regime C as compared to regime D:

Lemma 8 *The consumer welfare is higher (strictly higher if $\beta < 1$) in regime C than in regime D.*

Proof. Let the equilibrium in regime D be denoted by $C^s = \langle (Q_1^s, F_1^s), \{(Q_2^s(p), F_2^s(p)) : p \in [0,1]\}\rangle$. Consider a contract $\hat{C}$ that is feasible in regime C constructed as follows:

$$\hat{C} = \left\langle (Q_1^{ss} = \hat{Q}_1, F_1^s), \{(Q_2^s(p), F_2^s(p), S^{ss}(p) = \beta\left[pF_2^s(p) - Q_2^s(p)\right]) : p \in [0,1]\}\right\rangle$$

where $\hat{Q}_1$ is chosen to satisfy the zero-profit condition (47), i.e.,

$$\hat{Q}_1 = p_1 F_1^{ss} - (1 - p_1) \int \{[q + (1 - q)\beta]\left[Q_2^s(p) - pF_2^s(p)\right]\} d\Phi(p).$$

In contrast, Q_1^s in C^s must satisfy the zero-profit condition (24), which implies that:

$$Q_1^s = p_1 F_1^s + (1 - p_1) \int \{Q_2^s(p) - pF_2^s(p)\} d\Phi(p).$$

For any $\beta \in (0,1)$, $\hat{Q}_1 < Q_1^s$. Thus the consumer is strictly better off in regime C with contract $\hat{C}$ than in regime D with the optimal contract C^s. ■

It remains to be shown how welfare compares between regime B and regime C. It turns out the welfare is still higher without a secondary market.

Lemma 9 ***(Welfare Effects of The Secondary Market with Endogenous Cash Surrender Values)*** *Consumer welfare is reduced by the presence of the secondary market, even if contracts are expanded to include optimally chosen surrender values.*

Proof. The proof is similar to that of Proposition 8. Let

$$C^{ss} = \langle (Q_1^{ss}, F_1^{ss}), \{(Q_2^{ss}(p), F_2^{ss}(p), S^{ss}(p)) : p \in [0,1]\}\rangle$$

be the optimal contract with surrender values with a secondary market. As discussed above, $S^{ss}(p) = \beta V_2^{ss}(p)$ for all p. Thus, the zero profit condition implies:

$$Q_1^{ss} - p_1 F_1^{ss} = (1-p_1)[q + \beta(1-q)] \int V_2^{ss}(p) d\Phi(p),$$

where $V_2^{ss}(p) \equiv pF_2^{ss}(p) - Q_2^{ss}(p)$. Consider the contract $\hat{C} = \left\langle (\hat{Q}_1, F_1^{ss}), \{(Q_2^{ss}(p), F_2^{ss}(p)) : p \in [0,1]\}\right\rangle$ where $\hat{Q}_1$ is given by:

$$\hat{Q}_1 - p_1 F_1^{ss} = (1-p_1) q \int V_2^{ss}(p) d\Phi(p).$$

Since $q \in (0,1)$ and $\beta > 0$ we know that $\hat{Q}_1 < Q_1^{ss}$. That is, $\hat{C}$ offers exactly the same face values and second period premiums as C^{ss}, except at a lower first period premium. It is easy to see that $\hat{C}$ is a feasible contract for the case without a secondary market, but infeasible with a secondary market, even if surrender values are allowed.

We will now show that $\hat{C}$ in a world without secondary market is better than C^{ss} in a world with secondary market. To see this, let

$$\begin{aligned} W^{ss}(C^{ss}) =& p_1 v(F_1^{ss}) + u(y - g - Q_1^{ss}) + \\ & (1-p_1) \int \{q[pv(F_2^{ss}(p)) + u(y+g-Q_2^{ss}(p))] + (1-q)u(y+g+S^{ss}(p))\} d\Phi(p) \end{aligned}$$

denote the expected consumer welfare associated with contract C^s in a world with a secondary market. Let

$$\begin{aligned} W\left(\hat{C}\right) =& p_1 v(F_1^{ss}) + u(y - g - \hat{Q}_1) + \\ & (1-p_1) \int \{q[pv(F_2^{ss}(p)) + u(y+g-Q_2^{ss}(p))] + (1-q)u(y+g)\} d\Phi(p) \end{aligned}$$

denote the expected consumer welfare associated with contract $\hat{C}$ in a world without a secondary market. Note that since $S^{ss}(p) = \beta V_2^{ss}(p)$ for all p, all of the equations in the proof of Proposition 8 hold until we reach the inequality:

$$\begin{aligned} W\left(\hat{C}\right) - W^{ss}\left(C^{s}\right) &\geq u(y-g-\hat{Q}_1) - u(y-g-Q_1^{ss}) \\ &\quad - (1-p_1)\left[u\left(y+g+\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right) - u(y+g)\right] \end{aligned}$$

First note that $Q_1^{ss} - \hat{Q}_1 = \beta(1-p_1)(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)$. By the continuous function theorem, we know that there exists $\delta_1 \in (0,1)$ such that

$$\begin{aligned} & u(y-g-\hat{Q}_1) - u(y-g-Q_1^{ss}) \\ = \;& u'\left(y-g-Q_1^{ss} + \delta'\left(Q_1^{ss} - \hat{Q}_1\right)\right)\left(Q_1^{ss} - \hat{Q}_1\right). \end{aligned}$$

Similarly, there exists $\delta_2 \in (0,1)$ such that

$$\begin{aligned} & (1-p_1)\left[u\left(y+g+\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right) - u(y+g)\right] \\ = \;& (1-p_1)\left[u'\left(y+g+\delta_2\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right)\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right] \\ = \;& u'\left(y+g+\delta_2\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right)\left(Q_1^{ss} - \hat{Q}_1\right). \end{aligned}$$

Hence

$$\begin{aligned} & W\left(\hat{C}\right) - W^{ss}\left(C^{ss}\right) \\ \geq \;& \left[u'\left(y-g-Q_1^{ss} + \delta'\left(Q_1^{ss} - \hat{Q}_1\right)\right) - u'\left(y+g+\delta_2\beta(1-q)\int V_2^{ss}(p)d\Phi\left(p\right)\right)\right]\left(Q_1^{ss} - \hat{Q}_1\right) \\ \geq \;& 0 \end{aligned}$$

with inequality being strict if $Q_1^{ss} - \hat{Q}_1$ is strictly positive, i.e., if there is dynamic reclassification risk insurance under contract C^{ss}. Since C^{ss} is the optimal contract with surrender values in the settlement market, and since $\hat{C}$ is a feasible contract when there is no settlement market, the settlement market must be welfare reducing even with optimally chosen surrender values. ■

Gathering results in Proposition 10 and Lemmas 8-9, the following proposition summarizes the ranking of consumer welfare in the four regimes:

Proposition 11 *In terms of consumer welfare, the four regimes can be ranked as follows:*

$$\begin{aligned} & \textit{Regime } A = \textit{Regime } B \\ & \geq \textit{Regime } C \\ & \geq \textit{Regime } D. \end{aligned}$$

5.2.2 Cross-Regime Comparisons of Contract Structure

In this section we compare the structure of the equilibrium contracts across regimes.

Proposition 12 *When a secondary market exists, the p^* is higher when surrender values are not allowed.*

Proof. Let hatted variables denote the equilibrium with surrender values and let unhatted variables denote the equilibrium without surrender values. Suppose $Q_1 \leq \hat{Q}_1$. Then:

$$\int V_2(p)dF(p) \leq \int q\hat{V}_2(p) + \beta(1-q)\hat{V}_2(p)dF(p) \leq \int \hat{V}_2(p)dF(p)$$

So there must exist p such that $V_2(p) \leq \hat{V}_2(p)$ and $Q_2(p) \geq \hat{Q}_2(p)$. At such a p the following must hold:

$$\begin{aligned} &qu'(y+g-Q_2(p)) + \beta(1-q)u'(y+g+\beta V_2(p)) \\ \geq &qu'(y+g-\hat{Q}_2(p)) + \beta(1-q)u'(y+g+\beta\hat{V}_2(p)) \end{aligned}$$

This implies that $u'(y-g-Q_1) \geq (q+\beta(1-q))u'(y-g-\hat{Q}_1)$ and hence:

$$qu'(y+g-Q_2^{FI}(p^*)) + \beta(1-q)u'(y+g) \geq qu'(y+g-Q_2^{FI}(\hat{p}^*)) + \beta(1-q)u'(y+g)$$

So $p^* \geq \hat{p}^*$. Now suppose $Q_1 > \hat{Q}_1$. Then:

$$u'(y-g-Q_1) > u'(y-g-\hat{Q}_1) \geq (q+\beta(1-q))u'(y-g-\hat{Q}_1)$$

And so by the same reasoning above, $p^* > \hat{p}^*$. ∎

5.3 Enriched Contract Space in the Presence of the Secondary Market: The Case of Non-Health Contingent CSV

Health-contingent CSV in the form of $S(p)$ may not be easy to enforce; and in reality, as we mentioned in the introduction, Whole Life insurance often does have CSVs, albeit very low and non-health-contingent. Thus, we also study an enriched contract space in the form of

$$\langle (Q_1, F_1), \{(Q_2(p), F_2(p),) : p \in [0,1]\}, S \rangle$$

where S is a non-health-contingent CSV. The restrictions on CSV to be non-health contingent could result from explicit government regulations or from the difficulties from enforcing contracts with health-contingent CSVs.

An immediate implication from Proposition 10 is that, in the absence of the settlement market, the primary insurance companies will set $S = 0$ when they are restricted to offer only non-heath contingent CSV.

Now we analyze the equilibrium S^* when the primary insurance companies face the threat from the settlement market, but are restricted to react to the threat by offering a non-health contingent CSV. The primary insurance companies will choose a contract $\langle (Q_1, F_1), \{(Q_2(p), F_2(p)) : p \in [0,1]\}, S\rangle$ to maximize:

$$u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)q \int_0^1 \left[u(y+g-Q_2(p)) + pv(F_2(p))\right] d\Phi(p)$$

$$+ (1-p_1)(1-q)\int_{S\geq \beta V_2(p)} u(y+g+S)d\Phi(p) + (1-p_1)(1-q)\int_{S<\beta V_2(p)} u(y+g+\beta V_2(p))d\Phi(p) \tag{53}$$

where $V_2(p) \equiv pF_2(p) - Q_2(p)$, subject to the following constraints:

$$V_2(p) \geq S, \ \forall p \tag{54}$$

$$S \geq 0 \tag{55}$$

$$Q_1 - p_1 F_1 = (1-p_1)q\int_0^1 V_2(p)d\Phi(p) + (1-p_1)(1-q)\int_{S\geq\beta V_2(p)} Sd\Phi(p)$$
$$+(1-p_1)(1-q)\int_{S<\beta V_2(p)} V_2(p)d\Phi(p) \tag{56}$$

To understand the above problem, let us first explain the constraints. Constraint (54) is the analog of no-lapsation constraint in this setting, which requires that the actuarial value of the contract terms for any second period health state must be at least equal to the surrender value. As before, this requirement reflects the consumer's inability to commit: if the actuarial value of the contract was higher than the surrender value, the consumer would simply surrender the contract and repurchase insurance on the spot market. Constraint (55) requires that the surrender value S be non-negative; it reflects the consumer's inability to commit to a negative payout in any state. Constraint (56) is the zero-profit condition reflecting competitiveness of the primary market. The first integral in (56) is the expected loss the insurance company faces from consumers who retain their bequest motive: constraint (54) implies that for these consumers, the insurance company's expected loss is always equal to $V_2(p)$; the second integral in (56) is the expected loss the insurance company faces from consumers who lose their bequest motive and find it optimal to surrender the policy back to the original insurer (for any of such consumers the insurance company loses S); and the third integral is the expected loss the insurer faces from consumers who lose their bequest motive and find it optimal to sell the policy on the secondary market (for a consumer in this category with health state p, the expected loss is $V_2(p)$ for the insurance company).

Now let us explain the objective function. The first integral in (53) is the expected second period utility to consumers with a bequest motive, for whom constraint (54) ensures that they remain with the original contract terms; the second integral is the expected second period utility for consumers who lose their bequest motive, and find it optimal to surrender their contract back to the insurance company for CSV S; the third integral is the expected second period utility for consumers without bequest motive who find it optimal to sell their contract on the settlement market for payment of $\beta V_2(p)$.

The main result for this section is:

Proposition 13 *When the primary insurers are restricted to offer only non-health contingent cash surrender values, they will choose $S^* = 0$ in equilibrium.*

Proof. The Lagrangian for the above maximization problem is:

$$\begin{aligned}
\mathcal{L} &= u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)q \int_0^1 \left[u(y+g-Q_2(p)) + pv(F_2(p))\right] d\Phi(p) \\
&+ (1-p_1)(1-q)\int_{S\geq\beta V_2(p)} u(y+g+S)d\Phi(p) + (1-p_1)(1-q)\int_{S<\beta V_2(p)} u(y+g+\beta V_2(p))d\Phi(p) \\
&+ \int_0^1 \lambda(p)\left[Q_2(p) - pF_2(p) + S\right] dp + \gamma S \\
&+ \mu\left[Q_1 - p_1F_1\right] + \mu(1-p_1)q\int_0^1 \left[Q_2(p) - pF_2(p)\right] d\Phi(p) - \mu(1-p_1)(1-q)\int_{S\geq\beta V_2(p)} S d\Phi(p) \\
&+ \mu(1-p_1)(1-q)\int_{S<\beta V_2(p)} \left[Q_2(p) - pF_2(p)\right] d\Phi(p) \qquad (57)
\end{aligned}$$

where $\{\lambda(p) \leq 0 : p \in [0,1]\}, \gamma \geq 0, \mu \geq 0$ are respectively the Lagrange multiplier for constraints (54), (55), and (56) respectively.

Suppose that the optimal $V_2(\cdot)$ is continuous and monotonically increasing in p (an assumption that we will verify later), we know that for every $S \geq 0$, there exists a p_0 such that $\beta V_2(p) \geq S$ if and only if $p \geq p_0$ where $\beta V_2(p_0) = S$, and thus,

$$\frac{dp_0}{dS} = \frac{1}{\beta V_2'(p_0)} \qquad (58)$$

Using this, the above Lagrangian can be rewritten as:

$$
\begin{aligned}
\mathcal{L} &= u(y-g-Q_1) + p_1 v(F_1) + (1-p_1)q\int_0^1 \left[u(y+g-Q_2(p)) + pv(F_2(p))\right] d\Phi(p) \\
&+ (1-p_1)(1-q)\int_0^{p_0} u(y+g+S)d\Phi(p) + (1-p_1)(1-q)\int_{p_0}^1 u(y+g+\beta V_2(p))d\Phi(p) \\
&+ \int_0^1 \lambda(p)\left[Q_2(p) - pF_2(p) + S\right] dp + \gamma S \\
&+ \mu\left[Q_1 - p_1 F_1\right] + \mu(1-p_1)q\int_0^1 \left[Q_2(p) - pF_2(p)\right] d\Phi(p) - \mu(1-p_1)(1-q)\int_0^{p_0} S d\Phi(p) \\
&+ \mu(1-p_1)(1-q)\int_{p_0}^1 \left[Q_2(p) - pF_2(p)\right] d\Phi(p). \qquad (59)
\end{aligned}
$$

Using the Leibniz integral rule and (58), we have that the derivative of the above Lagrangian with respect to S, evaluated at the optimum (superscripted by $*$), is

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S} &= (1-p_1)(1-q)\int_0^{p_0^*} u'(y+g+S^*)d\Phi(p) + (1-p_1)(1-q)u(y+g+S^*)\frac{\phi(p_0^*)}{\beta V_2^{*\prime}(p_0^*)} \\
&\quad - (1-p_1)(1-q)u(y+g+\beta V_2^*(p_0^*))\frac{\phi(p_0^*)}{\beta V_2^{*\prime}(p_0^*)} + \int_0^1 \lambda(p)dp + \gamma \\
&-\mu(1-p_1)(1-q)\int_0^{p_0^*} d\Phi(p) - \mu(1-p_1)(1-q)S^*\frac{\phi(p_0^*)}{\beta V_2^{*\prime}(p_0^*)} \\
&-\mu(1-p_1)(1-q)\left[Q_2^*(p_0^*) - pF_2^*(p_0^*)\right]\frac{\phi(p_0^*)}{\beta V_2^{*\prime}(p_0^*)} \qquad (60)
\end{aligned}
$$

Note that $\beta V_2^*(p_0^*) = S^*$, so this simplifies to:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial S} &= (1-p_1)(1-q)u'(y+g+S^*)\Phi(p_0^*) + \int_0^1 \lambda(p)dp + \gamma \\
&\quad - \mu\overbrace{(1-p_1)(1-q)\Phi(p_0^*)}^{\text{Term A}} + \mu\overbrace{(1-p_1)(1-q)(1-\beta)V_2^*(p_0^*)\frac{\phi(p_0^*)}{\beta V_2^{*\prime}(p_0^*)}}^{\text{Term B}}. \qquad (61)
\end{aligned}
$$

We now argue that $\frac{\partial \mathcal{L}}{\partial S}$ is strictly negative when S deviates from 0 to a small $\varepsilon > 0$. To see this, note that in the ε-neighborhood of $S = 0$, we have $\gamma = 0$, $\lim_{s\to\varepsilon=0^+} V_2^*(p_0^*) = \varepsilon$, thus

$$
\lim_{s\to\varepsilon=0^+}\frac{\partial \mathcal{L}}{\partial S} = (1-p_1)(1-q)\left[u'(y+g) - \mu\right]\Phi(p_0(0)) + \int_0^1 \lambda(p)dp.
$$

Note that the first order condition with respect to Q_1 implies that

$$
u'(y-g-Q_1^*) = \mu > u'(y+g)
$$

and that $\lambda(p) \le 0$ for all p, we have:

$$
\lim_{s\to\varepsilon=0^+}\frac{\partial \mathcal{L}}{\partial S} < 0.
$$

Thus the optimal $S^* = 0$. At $S^* = 0$, the equilibrium contracts we characterized in Section 3 will be the optimal solution to the problem here. It can be also verified, using variation arguments, that these contracts we characterized in Section 3 also satisfy the necessary optimality conditions with respect to $Q_1, F_1, Q_2(\cdot)$ and $F_2(\cdot)$. ■

To understand the intuition for the result, it is best to focus on the first order derivative (61). Figure 13 depicts the effect of rasing S from 0 to ε on the firm's second period profits. The area labeled A captures the loss in profit from such a change in the sense that the firm will be paying these consumers ε under such a change, whereas they would have received zero before the change. This corresponds to Term A in expression (61). The area labeled B in Figure 13 is the gain for the firm's profit under such a change. The change of S from zero to ε induces some consumers who health state p satisfies $\beta V_2(p) < \varepsilon$ to switch from selling to the settlement market to surrendering to the insurance company for a cash value ε. For these consumers, the insurance company now pays ε instead of $V_2(p)$ that would have to be paid to the settlement company. Area B corresponds to Term B in expression (61). As is clear from the graph, Area A is a first order proportional to ε, while Area B is second order proportional to ε. When ε is small, the firm's second period losses increases. As a result, the insurance company has to increase the first period premium Q_1 to maintain zero profit. The utility cost of increasing the first period premium is exactly $\mu \equiv u'(y - g - Q_1^*)$. The utility gain for the consumer when S increases from 0 to ε is captured by the first term in expression (61), $(1-p_1)(1-q)u'(y+g)\Phi(p_0^*)$. Because the marginal utility in the second period from having ε for those consumers captured in A is $u'(y+g)$, which is smaller than the marginal losses from the increase in the first period premium, μ, such a trade-off is welfare-decreasing. Thus the optimal $S^* = 0$.

Proposition 13 tells us that when primary insurance companies are restricted to respond to the threat of the settlement market by optimally choosing non-health contingent CSVs, such an option is essentially useless. Thus the consumer welfare in this regime is exactly the same as Regime D we described above (secondary market with no CSV).

6 Conclusion

In this paper we have examined in detail the effect of life settlement market on the structure of the long-term contracts offered by the primary insurance market, as well as on the consumer welfare in a dynamic model of insurance with one-sided commitment (Hendel and Lizzeri 2003, Daily, Hendel and Lizzeri 2008). We show that the presence of the settlement market affects the extent as well as the form of dynamic reclassification risk insurance in the equilibrium of the primary insurance market. In the absence of life settlement market the primary insurers provide

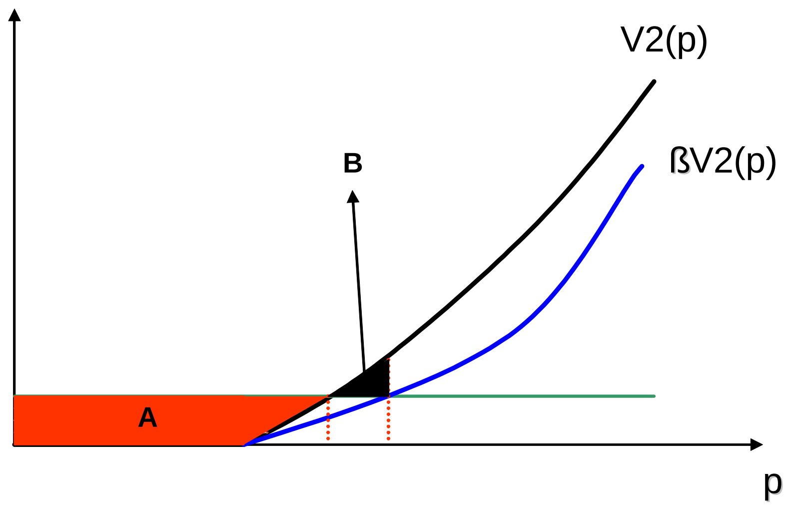

Figure 13: The Effect of Increasing S by $\varepsilon > 0$ on the Firm's Profits in the Second Period.

reclassification risk insurance by demanding a level premium (that is more favorable than the actuarially fair premium) in the second period from individuals with health worse than a threshold p^*. In contrast (see Proposition 4), when there is a secondary market, the primary insurers offer reclassification risk insurance via a premium discount (relative to the actuarially fair premium) in the second period for individuals with health worse than a threshold p^{s*}; moreover, p^{s*} may be different from p^*. We show that in general, settlement market always leads to worse consumer welfare than when there is no secondary market (Proposition 8). In the most extreme form, the presence of the settlement market can completely unravel the dynamic contracts to a sequence of short-term spot contracts with no dynamic risk classification risk insurance at all (Proposition 7).

We also examine the primary insurers' response to the settlement market when they can offer enriched contracts by specifying optimally chosen cash surrender values. We show that when there are no settlement firms, the primary insurers would not exercise the option of specifying CSVs (Proposition 10); but when there is the threat from the settlement firms, we show that if primary firms can specify health-contingent CSVs, then they will indeed optimally choose to include CSV options in the equilibrium contracts, and allowing the optimally chosen CSVs improves the consumer welfare, even though consumer welfare is still lower than that without the secondary market (Proposition 11). We also showed that the option of primary insurers to endogenously choose the CSV is useless if the CSVs are restricted to be non-health contingent, as is practiced now. However, if CSVs can be health contingent, then the primary insurance companies can partially mitigate the welfare losses induced by the emergence of the settlement market.

Our analysis yields a list of testable predictions. First, settlement market leads to the sub-optimality of level premiums. We should expect to see a move away from level-premium insurance as the secondary market expands. Second, we should expect to see that the primary insurers will be more aggressive in specifying health-contingent cash surrender values in the long-term contracts they offer to consumers (or trying to do so). Third, we should expect to see that consumers' preference parameters such as risk aversion play a more important role in their choice of life-insurance contracts (Proposition 3 vs. 9).

While a dynamic contracting model of one-sided commitment is shown to be able to generate predictions of the life-insurance contracts that are broadly consistent with what is observed in the data (Hendel and Lizzeri 2003), the only reason for lapsing in such a framework is the loss of bequest motive. As the viatical market for AIDS patients showed, settlement could be a result of extreme income loss (or equivalently expense increase). In the model we studied here, income risks are assumed away. In a model that explicitly features both income and mortality risks, the demand for life settlement may arise not only because bequest motives may disappear, but also because consumers who experience large negative income shocks may need to re-optimize. In future work, we will explore the interactions between the life insurance market, the life settlement market and potentially self-insurance via savings etc.

There are also several directions for empirical research. For example, focusing on the responses of the primary market insurers on the threat from the settlement market, Proposition 4 showed that level term life insurance policies are no longer optimal. Propositions 9 and 13 showed that primary insurers should have incentives to offer health-contingent CSVs in response to the settlement market, but a non-health contingent CSV is of no use. Do we see these developments in the primary market? Focusing on the life settlement market, the current model predicts that those that sell life insurance to the settlement firms are those with no bequest motives, but with worse health. Does the evidence support this prediction? There are also interesting empirical facts to find out for which the current theoretical model does not have unambiguous predictions. For example, does the settlement market increase or decrease the degree of front-loading in the primary market?

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