Regime-Shifts, Risk Premiums in the Term Structure, and the Business Cycle

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Abstract

Recent evidence indicates that using multiple forward rates sharply predicts future excess returns on U.S. Treasury bonds—the R^2's being around 30%. The projection coefficients in these regressions exhibit a distinct pattern that relates to the maturity of the forward rate. These dimensions of the data in conjunction with the transition dynamics of bond yields offer a serious challenge to term structure models. In this paper we show that a regime-shifts term structure model can empirically account for these challenging data features. Alternative models, such as the affine specification, fail to account for these important features. We find that regimes in the model are intimately related to bond risk premia and real business cycles.

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JEL classification: E43, G12, C51, C52.

1 Introduction

Term structural models with regime-shifts considered in Naik and Lee (1997) and Bansal and Zhou (2002) capture the important feature that the aggregate economy is subject to discrete and persistent changes in the business cycle. The business cycle fluctuations together with the monetary policy response to them have significant impacts on not only the short interest rate, but also the entire term structure. Regime-switching term structure models represent a parsimonious way to introduce interactions between the business cycles, the term structure, and risk premia on bonds. Using the US treasury yield data from 1964 to 1995, Bansal and Zhou (2002) find that the model-implied regime changes usually lead or coincide with economic recessions. Therefore the term structure regimes seem to confirm and complement the real business cycles. This evidence, consequently, also permits the possibility that this class of term structure models may be able to capture the dynamics of risk premia on bonds.

The most common strategy for understanding bond risk premiums is to study deviations from the the Expectations Hypothesis. One form of the violation that the regression of yield changes on yield spreads produces negative slope coefficient instead of unity (Campbell and Shiller, 1991), has been addressed in many recent papers (see Roberds and Whiteman, 1999; Dai and Singleton, 2002; Bansal and Zhou, 2002; Evans, 2003). Another form of the violation of Expectations Hypothesis is that the forward rate can predict the excess bond return (Fama and Bliss, 1987). More recently, Cochrane and Piazzesi (2002) document that using multiple forward rates to predict bond excess returns generates very high predictability of bond excess returns—the adjusted R^2's from the regression being around 30%. Further, they show that the coefficients of multiple forward-rate regressors form a tent shape pattern related to the maturity of the forward rate. The size of the predictability and nature of projection coefficients is quite puzzling and constitutes a challenge to term structure models.

The main contribution of this paper is to account for the predictability evidence from the perspective of latent factor term structure models. When evaluating the plausibility of various term structure models it is important not to focus exclusively on the predictability issue; previous work (see Dai and Singleton, 2000; Bansal and Zhou, 2002; Ahn et al., 2002) highlight the difficulties that many received models have in capturing the transition dynamics of yields (i.e., conditional volatility and conditional cross-correlation across yields). The predictability evidence, in conjunction with the transition dynamics constitutes a sufficiently rich set of data-features to discriminate across alternative term structure models and to evaluate their plausibility. The main empirical finding of this paper is that the regime-shifts term structure models can simultaneously justify the size and nature of bond return predictability and the transition dynamics of yields. More specifically, we find that models with regime-shifts can reproduce the high predictability and the tent-shaped regression coefficients documented in Cochrane and Piazzesi (2002). Additionally, the regime-shifts term structure model reproduces the dynamics of conditional volatility and cross-correlation across yields. On the other hand, commonly used multi-factor CIR and affine models cannot capture these dimensions of the data. Our overall evidence indicates that incorporating regime shifts is important for interpreting key aspects of treasury bond market data.

We use US treasury yield data from 1964 through 2001. The period from 1996 to 2000 poses a tough challenge for standard asset pricing models, with unprecedented long economic growth and bull market run. At the same time this stretch of the data has several economic recessions and periods of economic boom. Using the whole sample we find that the conditional correlation between the long and short yields vary over a range from about 40% to 80%. The conditional volatilities of the long and short yields also reveal very large variations. Despite this, when confronting the U.S. treasury yields data from 1964 to 2001, our regime-shifts model still stands out as the best performing candidate. The regime indicator is related to business cycles in the data; for example, the model-based regime indicator predicts the 2001-2002 recession.

To estimate various models under consideration we use the Efficient Method of Moments (EMM), developed in Bansal et al. (1995) and Gallant and Tauchen (1996). Tests of over-identifying restrictions based on the EMM method provide a way to compare different, potentially non-nested models. This estimation technique forces the model to confront several important aspects of the data, such as the conditional volatility and correlation across different yields. To generate diagnostic evidence to help discrimination across models, we rely on the reprojection methods developed by Gallant and Tauchen (1998). Our empirical evidence suggests that the benchmark CIR and affine model specifications with up to three factors are sharply rejected with p-values of zero. The only model specification that finds support in the data (with p-value of 1%) is our preferred two-factor regime switching model where the market prices of risks depend on regime shifts. Our diagnostics of the various models show that the our preferred regime shifts model specification produces the smallest cross-sectional pricing errors across all the specifications considered in the paper. Using reprojections we compute the conditional correlations and volatility under the null of the various models. Our results show that only the regime-shifts models can capture the large variations in conditional correlations and conditional volatility that are observed in the data.

The remainder of this paper is organized in the following manner. Section 2 reviews the regime shifts term structure model developed in Bansal and Zhou (2002). Section 3 discusses the empirical estimation results, the specifications tests, and an array of diagnostics based on the conditional correlation and volatility. It also examines cross-sectional implications on pricing errors, violations of the expectation hypothesis of forward rate predictability and the link between regime classification and business cycles, especially the recent economic recession. Section 4 contains the concluding remarks.

2 Term Structure Model with Regime-Shifts

In this section, we review the term structure model with regime shifts that is proposed in Bansal and Zhou (2002). The derivation focuses on a single factor specification, the multi-factor extension is straightforward (also see Bansal and Zhou, 2002). To capture the idea that the aggregate economy is subject to regime shifts, we model the regime shifts process as a two state Markov process as in Hamilton (1989). Suppose that the evolution of tomorrow's regime $s_{t+1} = 0, 1$ given today's regime $s_t = 0, 1$ is governed by the transitional probability matrix of a Markov chain

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}, \tag{1}$$

where $\sum_{j=0,1} \pi_{ij} = 1$ and $0 < \pi_{ij} < 1$. In addition to the discrete regime shifts, the economy is also affected by a continuous state variable,

$$X_{t+1} - X_t = \kappa_{s_{t+1}}(\theta_{s_{t+1}} - X_t) + \sigma_{s_{t+1}}\sqrt{X_t}u_{t+1}, \tag{2}$$

where $\kappa_{s_{t+1}}$, $\theta_{s_{t+1}}$, and $\sigma_{s_{t+1}}$, are the regime-dependent mean reversion, long run mean, and volatility parameters respectively. All these parameters are subject to discrete regime shifts. Specifically, $X_{t+1} - X_t = \kappa_0(\theta_0 - X_t) + \sigma_0\sqrt{X_t}u_{t+1}$ if the regime $s_{t+1} = 0$, and $X_{t+1} - X_t = \kappa_1(\theta_1 - X_t) + \sigma_1\sqrt{X_t}u_{t+1}$ if the regime $s_{t+1} = 1$. Note that the innovation in the process (2), u_{t+1}, is conditionally normal given X_t and s_{t+1}. For analytical tractability we assume that the process for regime shifts s_{t+1} is independent of X_{t+1-l}, $l = 0 \cdots \infty$, this is similar to the assumptions made in Hamilton's regime switching models. It is also assumed that the agents in the economy observe the regimes, though the econometrician may possibly not observe the regimes.

The pricing kernel for this economy, is similar to that in standard models, except for incorporating regime shifts

$$M_{t+1} = \exp\{-r_{f,t} - (\frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}})^2 \frac{X_t}{2} - \frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}} \sqrt{X_t} u_{t+1}\}. \tag{3}$$

The above specification of the pricing kernel captures the intuition that these aggregate processes are latent and subject to regime shifts (as in Hamilton, 1989). Note that the λ parameter that affects the risk premia on bonds is also subject to regime shifts and hence depends on s_{t+1}. Bansal and Zhou (2002) present a general equilibrium model that leads to the pricing kernel in (3).

With regime shifts, we conjecture that the bond price with n periods to maturity, at date t depends on the regime $s_t = i$, $i = 0, 1$, and X_t

$$P_i(t, n) = \exp\{-A_i(n) - B_i(n) X_t\}.$$

The one period ahead bond price, analogously depends on s_{t+1} and X_{t+1}

$$P_{s_{t+1}}(t+1, n-1) = \exp\{-A_{s_{t+1}}(n-1) - B_{s_{t+1}}(n-1) X_{t+1}\}.$$

In addition we impose the boundary condition $A_i(0) = B_i(0) = 0$ and the normalization $A_i(1) = 0$, $B_i(1) = 1$, for $i = 0, 1$, that is, $r_{f,t} = X_t$. The key asset pricing condition is,

$$E_t[\mu_{n,s_{t+1},t} + \frac{\sigma^2_{n,s_{t+1},t}}{2} - r_{f,t} | X_t, s_t] = -X_t E_t[B_{s_{t+1}}(n-1)\lambda_{s_{t+1}} | s_t] \tag{4}$$

The conditional mean and volatility of the bond return in regime s_{t+1} is $\mu_{n,s_{t+1},t}$ and $\sigma^2_{n,s_{t+1},t}$ respectively. Equation (4) captures the idea that all risk-premiums and bond prices at date t depend only on s_t and X_t. To further get some intuition regarding this risk premium result, note that $-\sigma_{s_{t+1}} B_{s_{t+1}}(n-1)\sqrt{X_t}$ is the exposure of the bond return to the standardized shock u_{t+1} in regime s_{t+1}. Further, $[\frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}}\sqrt{X_t}]$ is the exposure of the pricing kernel to u_{t+1} in regime s_{t+1}. The covariance between these exposures determine the compensation for risk in regime s_{t+1}. Hence, the risk compensation for regime s_{t+1} is the product

$$-\sigma_{s_{t+1}} B_{s_{t+1}}(n-1)\sqrt{X_t} \times [\frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}}\sqrt{X_t}] = -B_{s_{t+1}}(n-1)\lambda_{s_{t+1}} X_t$$

Given information regarding s_t, X_t, and the regime transition probabilities; agents integrate out the future regime, s_{t+1}, which leads to the risk premium result stated in (4). In the absence of regime shifts, the risk premium in (4), would simply be $-X_t B(n-1)\lambda$. Hence incorporating regime shifts makes the "beta" of the asset, that is the coefficient on X_t, be time-varying and dependent on the current regime. This fashion of making the asset "beta" time varying is potentially important for capturing the behavior of risk premia on bonds. The market price of risk, that is the risk premium for an asset with a unit exposure to u_{t+1}, in this model is $E_t[\frac{\lambda_{s_{t+1}}}{\sigma_{s_{t+1}}}|s_t]\sqrt{X}_t$, and is clearly regime dependent.

Given (4), the solution for the bond prices can be derived, by solving for the unknown coefficients A and B, in particular we show:

$$\begin{bmatrix} B_0(n) \\ B_1(n) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \begin{bmatrix} (1-\kappa_0-\lambda_0)B_0(n-1) - \frac{1}{2}\sigma_0^2 B_0^2(n-1) + 1 \\ (1-\kappa_1-\lambda_1)B_1(n-1) - \frac{1}{2}\sigma_1^2 B_1^2(n-1) + 1 \end{bmatrix} \tag{5}$$

and

$$\begin{bmatrix} A_0(n) \\ A_1(n) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \begin{bmatrix} A_0(n-1) + \kappa_0\theta_0 B_0(n-1) \\ A_1(n-1) + \kappa_1\theta_1 B_1(n-1) \end{bmatrix} \tag{6}$$

with initial conditions $A_0(0) = A_1(0) = B_0(0) = B_1(0) = 0$. Note that bond price coefficients are mutually dependent on both the regimes—current bond prices reflect agent's expectations regarding regime shifts in the future. Finally, the bond yield of a K factor regime-switching model can be derived in an analogous manner,

$$Y_s(t,n) = -\frac{\ln P_s(t,n)}{n} = \frac{A_s(n)}{n} + \sum_{k=1}^{K} \frac{B_{ks}(n)X_{kt}}{n}. \tag{7}$$

The above regime shifts term structure model does not entertain the possibility of separate risk compensation for regime shifts. In other words, the risk premium for a security that pays one dollar contingent on a regime shift at date $t+1$, is zero. The model can be extended to include explicit and separate compensation for regime-shifts risks. Such an extension, however entails additional parameters. We have not discussed or pursued this more embellished version of the model as we found it extremely hard to identify and estimate its parameters. Further, as documented below, the key puzzles in the term structure data, can be accounted for by the more parsimonious model described above.

A recent paper by Dai et al. (2003) incorporates a separate risk premium for regime-shifts, but for analytical tractability, this paper assumes that the within regime volatility is constant. Given the nature of yields data, it would seem that allowing for within regime volatility to be stochastic is quite important. It remains to be seen if the specification which assumes a constant within regime volatility can account for the observed time-varying volatility and conditional cross-correlation of yields. As discussed below in our empirical work, these dimensions of the term structure data are important in discriminating across term structure models.

3 Empirical Estimation and Model Evaluation

3.1 Estimation Methodology

To utilize a consistent approach for evaluation and estimation across the different models we rely on the simulation-based EMM (efficient method of moments) estimator, developed in Bansal et al. (1995) and Gallant and Tauchen (1996). The EMM estimator consists of three steps. The first, called the projection step, entails estimating a reduced form model, termed the auxiliary model, that provides a good statistical description of the data. Multivariate bond yields are difficult data to model as they exhibit extreme persistence in location and scale, time varying correlations, and non-Gaussian innovations. Since we do not have good *a priori* information on the specification of a model to capture all of these features, we utilize a semi-nonparametric (SNP) series expansion. The SNP expansion has a VAR-ARCH Gaussian density as its leading term, and the departures from the leading term are captured by a Hermite polynomial expansion. We elected to use a simpler, ARCH-like leading term, instead of a GARCH-type leading term because of the similar problems with multivariate GARCH-type models of bond yields noted by Ahn et al. (2002).

In the second step, termed the estimation step, the score function from the log-likelihood estimation of the auxiliary model is used to generate moments for a GMM-type criterion function. The score function provides a set of moment conditions that are true by construction and are to be confronted by all term structure models under consideration. In the computations, the score function is averaged over the simulation output from a given term structure model and the criterion function is minimized with respect to the parameters of the term structure model under consideration. By using the scores from the non-parametric SNP density as the moment conditions, each model is forced to match the conditional distribution of the observed 6-month and 5-year yields. Being a GMM-type estimator, EMM provides a chi-squared measure of goodness-of-fit. In particular, the normalized objective function acts as an omnibus specification test, which is distributed as a chi-square (as in GMM) with degrees of freedom equal to the number of scores (moment conditions) less the number of parameters in the particular term structure model. The distance matrix (the weight matrix in GMM) used in constructing the specification test is identical across different model specifications (the null hypotheses). Consequently, the p-values based on this specification test can be directly compared across different structural models to identify the best model specification. For a discussion of the importance of having the same distance matrix, for a consistent comparison across models, see Hansen and Jagannathan (1997). It is well recognized in the literature, that tests for the absence of regime-shifts against a regime shifting alternative require non-standard approaches (see Hansen, 1992; Garcia, 1992). Our approach of comparing all the considered models to a common non-parametric density (the SNP density), allows us to rank order all the considered models according to the p-values implied by the EMM criterion function. The advantage of using the non-parametric SNP, as discussed in Gallant and Tauchen (1999), is that it can asymptotically converge to virtually any smooth distributions, including mixture distributions (as is the case with a model of regime shifts).

The third step is reprojection, or post-estimation analysis of model simulations. Since EMM is a simulation-based estimator, there are available for analysis long simulated realizations from each estimated model. These simulations can be used to compute statistics of interest that can be compared to analogous values computed from the observed data. The reprojected statistics should be thought of as population quantities implied by the model at the estimated parameter values. Among other things, we compute the reprojected Cochrane-Piazzesi forward rate regressions for models with and without regime switching.

3.2 Data Description

The data set is monthly, June 1964 to December 2001, bond yield data obtained from the Center for Research in Security Prices (CRSP). There are total 451 monthly observations, with eight maturities 1, 3, 6, month and 1, 2, 3, 4, 5 year. It is important to recognize that the data period 1964–2001 contains six major recessions and six major expansions, which as stated earlier provides potential economic motivation for incorporating regime shifts. The summary statistics of these monthly yields are displayed in Table 1. On average, the yield curve is upward sloping. The standard deviation, positive skewness, and kurtosis are systematically higher for short maturities than for long ones. To incorporate important time-series and cross-sectional aspects of term structure data we focus on a short term and a long term yield—the yield on the six month bill and the five year note. Time series plots of the basis yields are in Figure 1. It is not unusual for using two or three time series to estimate a model with three or more latent factors, since the identifications are coming from the number of scores (or moment restrictions) generated from the auxiliary model (see, e.g. Chernov et al., 2003).

We very briefly summarize the first step estimation results for the non-parametric SNP specification which was guided by the BIC information criterion; details are available upon request. The leading term of the bivariate SNP density has 1 lag in the VAR based conditional mean ($L_\mu = 1$) and 5 lags in ARCH specification ($L_r = 5$). The preferred specification accommodates departures from conditional normality via a Hermite polynomial of degree 4 ($K_z = 4$). This "semiparametric ARCH" specification is similar to that proposed by Engle and González-Rivera (1991). This specification allows for skewness and kurtosis in the error distribution. The total number of parameters for the specification is $l_a = 28$; hence, there are a total of 28 data-determined moments conditions that each model must confront.

The conditional moments of the estimated SNP density for the observed interest rates are available analytically. It is fairly instructive to focus on some specific aspects of the estimated non-parametric SNP bivariate density. The top panel in Figure 6 gives the estimated conditional volatilities and cross correlations of the 6-month and 5-year yields, which seem to be very persistent and fairly volatile. The short interest rate has a wide range for the conditional volatility which peaks around 1980, while the range for the five year yield volatility is narrow. The range for the conditional correlation is from about 40% to 80%—a wide range indeed. The most volatile period for bond yields, the early 80's sees, is associated with a considerable drop in the conditional correlation. The behavior of the conditional variance and the cross-correlation, as documented above, poses a serious challenge to the various term structure models under consideration.

It is important to note that our estimation of the various term structure models utilizes information in the bivariate SNP density based on the 6-month and the 5-year yields. We do not directly rely on bond excess returns—hence our estimation does not directly utilize information on the predictability of bond returns. We use the estimated model to evaluate via simulation, if model can reproduce the predictability regressions discussed in Cochrane and Piazzesi (2002). These predictability regressions are challenging for two reasons. First, the size of the predictability is fairly high, the R^2's in these projections are quite large. Second, the nature of the predictability—the "tent shape" of the multiple regression coefficients captures the unconditional covariation of future bond returns with current forward rates. A reasonable term structure model should account for both these features of the predictability along with the important data aspects embodied in the bivariate SNP density for 6-month and 5-year yields.

3.3 Model Estimation Results

Table 2 gives the main EMM estimation results for four different models: one-factor regime-switching (1-Factor[RS]), two-factor square-root (2-Factor[CIR]), two-factor regime switching (2-Factor[RS]), and three-factor affine (3-Factor[AF]). Three additional models (not reported here), one-factor square-root, two-factor Naik and Lee (1997), and three-factor square-root, are also estimated with results similar to that in Bansal and Zhou (2002); none of these can replicate the Expectation Hypothesis puzzle and other data features of interest. The results reported here are for simulation size of 50,000. The one factor model with regime-shifts (1-Factor[RS]) is rejected with a p-value less than 0.0000. The two factor square-root model (2-Factor[CIR]) improves things but this specification is still sharply rejected—the model specification test drops to 56.066 with p-value smaller than 0.0003. The best model amongst all specifications is the two-factor regime switching specification with p-value reaching 1%. The estimated regime switching probabilities are both just under 0.95. All the parameters of the model are estimated rather accurately. The transition probabilities reported for the 2-Factor[RS] specification are comparable to those found in other papers (see Gray, 1996; Hamilton, 1988; Cai, 1994).

The 2-Factor[RS] model can be viewed as a three factor model with the regime switching factor being a multiplicative or nonlinear third factor. For a fair comparison of this two factor regime switching model, we also estimate a three-factor affine term structure model, (3-Factor[AF]) preferred by Dai and Singleton (2000)—they find considerable empirical support for this specification using the post 1987 swap yield data. The discrete time counterpart to this affine specification is;

$$\begin{aligned} X_{1t+1} - X_{1t} &= \kappa_1(\theta_1 - X_{1t}) + \sigma_1\sqrt{X_{1t}}u_{1t+1} \\ X_{2t+1} - X_{2t} &= \kappa_2(\theta_2 - X_{2t}) + \sigma_2 u_{2t+1} + \sigma_{23}\sqrt{X_{1t}}u_{3t+1} \\ X_{3t+1} - X_{3t} &= \kappa_3(X_{2t} - X_{3t}) + \sqrt{X_{1t}}u_{3t+1} + \sigma_{31}\sigma_1\sqrt{X_{1t}}u_{1t+1} + \sigma_{32}\sigma_2 u_{2t+1} \end{aligned} \tag{8}$$

Associated with this 3-Factor[AF] specification are three market price of risk parameters, which as before we label as λ_k, $k = 1, 2, 3$. In all there are 13 parameters to estimate. As reported in Table 2, the 3-Factor[AF] specification is sharply rejected with a $\mathcal{X}^2(15) = 42.803$

and a p-value of 0.0017. In a more general semiparametric setting, Ghysels and Ng (1998) reject the affine restrictions on the conditional mean and variance of yields.

Table 3 reports the t-ratio diagnostics for the 28 moment conditions implied by each of the four specifications. These 28 scores (moment conditions) should, for a correctly specified model, be close to zero. If the structural model under consideration matches the particular moment under consideration, then at conventional 5% level of significance the t-ratio should be smaller than 1.96. The reported t-ratios are not adjusted for parameter estimation so these t's are therefore asymptotically slightly downward biased relative to 2.0. They thus must be interpreted with cautious intuition guided by the overall chi-square diagnostics, which are free of such asymptotic bias. For the 1-Factor[RS] model, 17 out of 28 moment tests are rejected, with fitting of conditional volatility especially bad. The 2-Factor[CIR] model only has 9 t-ratios higher than 1.96, and adding one more linear factor dramatically improves the fitting of conditional volatility and conditional mean. It is remarkable that our favored 2-Factor[RS] model matches well all the mean, volatility, and polynomial scores, except for the single ARCH(1) score of the six month yield that is just over 2.0. The 3-Factor[AF] specification is certainly an improvement than the one or two factor models, but it still has 4 out of 13 ARCH scores and 2 out of 9 Hermite scores are not well matched. Overall, our preferred 2-Factor[RS] specification seems to have the greatest advantage in matching the conditional volatility and covariance (i.e., the ARCH scores), and the non-Gaussian polynomials (i.e., the Hermite polynomial parameters), relative to other multifactor CIR or affine specifications.

3.4 Risk Premium Analysis

An important diagnostic is to evaluate if the different model specifications can justify the observed patterns of violations of the Expectations Hypothesis, in particular, as documented in Fama and Bliss (1987), the predictability of forward rates on excess returns. The simple existence of the predictability from forward rate to excess return—R-square significantly higher than zero—is easily explained by any dynamic term structure model with time-varying risk premium. However, the greater challenge, as recently popularized by Cochrane and Piazzesi (2002), is to explain the robust tent-shaped pattern of the slope coefficients when multiple forward rates are used as regressors. Another form of the EH violation (not a focus of this paper) is the negative slope in stead of unity when regressing yield changes on yield spreads (Campbell and Shiller, 1991). Bansal and Zhou (2002) provide evidence that the two-factor regime-shifts model is the only one that can replicate this type of EH violation at the shorter maturities, while all multi-factor models fair well at the longer maturities.

Following the same conventions in Cochrane and Piazzesi (2002), we work with log bond prices, i.e., p_t^k is the log of the price at t of a k *year* bond, and geometric (log) yields and returns, so $y_t^1 = -p_t^1$ is the geometric yield on the 1-year bond. Cochrane and Piazzesi (2002) consider the regression of excess returns of bonds on the yields and the forward rates:

$$ex_{t+12}^k = \beta_{k0} + \beta_{k1} y_t^1 + \sum_{i=2}^{5} \beta_{ki} f_t^i + \epsilon_{t+12}^k, \;\; k = 2, .., 5 \tag{9}$$

where $ex_{t+12}^k = p_{t+12}^{k-1} - p_t^k - y_t^1$ is the excess return on the k year bond and $f_t^k = p_t^{k-1} - p_t^k$ is the forward rate. Note that ex_{t+12}^k is effectively the return on holding a k year bond for one year in excess of the one year yield. This excess return data is collected on a monthly frequency which leads to the usual overlap in return data.

We first check the robustness of the Cochrane and Piazzesi (2002) findings. As shown in the top panel of Table 4, the regression R-square with five forward rates reaches 36%, which confirms their findings. An important note is that the difference between using three, four, or five forward rates is negligible, while reducing to two or one forward rates dramatically decreases the R-square. This seems to suggest the existence of three latent factors, and the use of five regressors creates near perfect co-linearity problem, up to cross-sectional measurement errors that can mask the singularity. We concentrate on the regressions with three forward rates. The estimated coefficients are plotted in the top-left panel of Figure 2 and the tent shape finding of Cochrane and Piazzesi (2002) is quite apparent.

Next, we examine if any of the dynamic term structure models under consideration can meet the challenge of replicating this unique tent-shape phenomenon. Using the estimated parameters of the four models, we simulate 50,000 monthly data and run the same regressions of excess bond returns on forward rates. As seen in the lower panel of Table 4, the 2-Factor[RS] model not only achieves the highest predicting R-squares (20-36%), but also clearly mimics closely the tent-shape regression coefficients. On the other hand, the 2-Factor[CIR] model produces a skewed and inverted tent shape, and the 3-Factor[AF] model produces a inverted tent shape. Both models achieve R-squares around 10-20%. Interestingly, even the 1-Factor[RS] model can replicate to some degree the tent shape, even though its R-square is only about 1%. These patterns are quite apparent in Figure 2. These results suggest that the prediction capability of forward rates for excess returns may be explained by two or three linear factors, while the tent pattern of regression coefficients appears to be due to the regime-switching nature of the yield curve.

The analysis of Duffee (2002) and Dai and Singleton (2002) suggest that allowing more flexible specification of the risk premium parameters for the conditional Gaussian factor model can dramatically improve its ability to match the predictability of excess returns. To explore this argument, we have also estimated the "Preferred Essentially Affine $A_0(3)$ Model" discussed in Duffee (2002) with three Gaussian factors and eight market-price-of-risk parameters (we call it 3-Factor[EA] model). The Chi-square test of overall specification is 29.278 with 9 degrees-of-freedom and a p-value of 0.0006—hence, the model is not supported by the data. The estimation result suggests that the 3-Factor[EA] model overshoots the excess returns predictability, the R-squares range from 26% to 65% vis-a-vis 30% observed in the data. More importantly, it cannot reproduce the tent-shape of the predictability regression coefficients. Further, its performance for cross-sectional pricing error is somewhat worse than the three factor affine model. Our diagnostics for this model specification reveal that the implied conditional volatility and conditional correlations of yields do not match those in the data. Given this result, for brevity, we do not present very detailed evidence on this specification.

3.5 Regime Indicator, Risk Premium, and the Business Cycle

We now explore the cross sectional implications of the term structure models over the maturities that are not used in the model estimation. We also look at the association between the bond market implied regimes and the real business cycle. For the 2-Factor[CIR] and 3-Factor[AF] models, a standard method is used to calculate the pricing errors—since the yield curve solution is linear in the factors, we first invert from two or three basis yields to get the latent factors and then use the linear pricing solution to calculate the non-basis yields. For the 1-Factor[RS] and 2-Factor[RS] models, the presumption that agents in the economy know the current regime implies a strategy to recover the regimes. Specifically, dates are classified into regimes according to which of the two yield curves produces the smallest pricing error. Under the null of correct specification, the pricing error should be zero given the true regime and the population parameter values. For more details see Bansal and Zhou (2002).

Table 5 reports the time-series average of pricing errors $1/T\sum_{t=1}^{T}\mathrm{PE}_s(t)$ or other statistics from the cross-sectional average $\mathrm{PE}_s(t) = 1/N\sum_{n=1}^{N}|\hat{Y}_s(t,n) - Y_s(t,n)|$, where $\hat{Y}_s(t,n)$ is the calculated yield and $Y_s(t,n)$ is the observed yield for maturity n at time t (where the current state s is inferred from minimizing the pricing errors of the two yield curves, as mentioned above). It is clear from the sample statistics that the 2-Factor[RS] model has the smallest average pricing error and also the smallest standard deviation in the pricing error. The maximal pricing error associated with the 2-Factor[RS] specification is also the smallest. Further, on average the pricing error is only about 27 basis points for the annualized percentage yields. The 3-Factor[AF] specifications have average pricing errors of 31 basis points, which in an absolute sense is also quite small. The 1-Factor[RS] and 2-Factor[CIR] models achieve similar pricing result as 44-45 basis points.

It has been well recognized that the slope of the yield curve (i.e., spread) has the ability to predict future real GDP growth—in particular, negative spreads tend to predict a recession e.g., see Harvey (1988) and Estrella and Hardouvelis (1991). Figure 3 recreates this linkage between the monthly yield spread, our regime indicator for regime 0 (our low regime), and the NBER business cycles recession indicator. Most of the time, it seems that the economy is in regime 1. The total number of regime switches recovered from the sample period is 44. The regime relates to the NBER business cycles. Our low regime (regime 0) obtains during or before recessions in the economy. In the data, the correlation between NBER business cycle indicator and the yield spread (5 year yield minus 6 month yield) is 15%. In general, the yield curve becomes inverted (or flat) several months before the economic growth becomes negative (or depressed). Our regime indicator is mostly zero, as Figure 3 shows, when the yield curve becomes inverted (or flat). The correlation between the model based regime indicator and the yield spread (5 year yield minus 6 month yield) is 24%—that is, our high regime (regime 1) coincides with high yield spread and our low regime (regime 0) largely coincides with low yield spread. Therefore, as in Bansal and Zhou (2002), the regime indicator has power to predict recessions. The correlation between the NBER business cycle (NBER recession as regime 0 and NBER boom as regime 1) and our regime indicator is 0.1117. In the context of modeling the short interest rate, Ang and Bekaert (2002) also document the links between regime shifts and business cycles.

Fama and Bliss (1987) attribute the time-varying risk premium in bonds to the business cycle. In particular, their argument is that the bond excess return is high when the economy is in recessions and low in expansions. The top panel of Figure 4 shows that our regime zero and negative ex-post excess returns bear close relation—the correlation between our regime indicator and ex-post bond excess returns is 21%. That is our high regime (regime 1) tends to coincide with high ex-post returns. We also explore how the expected excess returns relate to the regimes. The middle panel of Figure 4 plots the fitted expected return in the data based on the excess return forward rate projection discussed above. The correlation in the data between our regime indicator and the expected excess return is 32%, i.e., high risk premia and the high regime (regime 1) tend to go together. In this sense our regimes can also be thought of as ranking on high and low risk premium on bonds. In the last panel of Figure 4 we plot the reprojected expected excess returns for bonds from our preferred 2-Factor[RS] model. The reprojected expected excess return for this model duplicates the expected excess return patterns observed in the data. Further, the reprojected expected excess return has a correlation of 37% with our regime indicator. The overall evidence indicates that our regime indicator tracks the time-varying risk premium on the bond market. As discussed above none of the other models can replicate the Cochrane and Piazzesi (2002) predictability regressions; consequently they also cannot account for the expected risk premium dynamics plotted in the middle panel Figure 4.

3.6 The Reprojected Conditional Volatility and Correlation

As a final diagnostic we assess the various models' capabilities to match the shape and track the conditional distribution and covariance characteristics of the data. Following Gallant and Tauchen (1998) we compute the reprojected conditional density of the two basis yields. Given the estimated null model and the simulated output for yields, the reprojected conditional density is obtained by re-estimating the parameters of the SNP density. Moments of interest, such as the conditional variances and correlations implied by the model specification can then be computed. These conditional moments are simply functions of the conditioning information used to estimate the reprojected density. Given the conditioning information, the implications of a given null model for any conditional moment of interest can be evaluated on the observed data and compared to the conditional moment implied by the unrestricted SNP density.

Figure 5 plots the reprojected conditional density (evaluated at the sample mean), for the different models under consideration. The unrestricted 6 month yield SNP density has high peak and narrow shoulders and the unrestricted density for the 5 year yield is skewed to the left and moderately peaked. The reprojected densities for the 3-Factor[AF] model do capture the peakedness of the 5 year yield but miss the peak of the 6 month yield and the skew of 5 year yield. On the other hand, the reprojected densities for the 1-Factor[RS] and 2-Factor[CIR] models capture somewhat the skewness of the 5 year yield but largely miss the peak of both yields. The 2-Factor[RS] regime shift model has greater success in capturing the left skew of the five year yield and the peak of both yields.

Figure 6 displays the conditional volatility and cross correlation for the various model specifications as implied by the reprojected densities. Note that, in the data, the dynamics of the conditional variance of the 6-month yield is quite different from that of the 5-year yield. The range for the conditional volatility for the 6-month yield rate is much larger than for the 5-year yield—the high end being almost three times the lowest for the 6-month, and two times for the 5-year yield. The short yield volatility is more persistent, while the long yield volatility seems more choppy. The 1-Factor[RS] model does not reflect any time-variations of short and long rate volatilities, although the levels of volatility are matched. The 2-Factor[CIR] model has difficulty in matching the short rate volatility and does somewhat better in matching the volatility of the 5-year yield. The 2-Factor[RS] model is capable of duplicating the projected volatility of the short rate extremely well, and that of the long yield volatility almost completely. The 3-Factor[AF] model seem to capture the volatility of the short rate much better than the 2-Factor[CIR] model, however, its capability to mimic the long rate volatility is diminished relative to the 2-Factor[CIR] model.

The rightmost subplots of Figure 6 provide evidence regarding the conditional correlation between the 6-month and 5-year yields. The 2-Factor[RS] model succeeds in capturing the wide range of the correlation observed across these yields. The correlation varies from 40% to 80%. Note that although the conditional volatility increases during the volatile period of the early 80's, the conditional correlation decreases—suggesting that the volatilities of the two yields rise more rapidly relative to the conditional covariance. The 1-Factor[RS] model, with only one linear factor, not surprisingly presents a nearly constant correlation very close to unity. The 2-Factor[CIR] and the 3-Factor[AF] specifications have difficulty in capturing the conditional covariance. However, the 3-Factor[AF] specification seems doing a considerably better job of capturing the conditional covariance relative to the 2-Factor[CIR] specification. The 2-Factor[RS] model comes quite close to capturing virtually all the observed dynamics of the conditional correlation between these yields. The main message of this evidence, is that our preferred regime shifts term structure model, is quite successful in capturing the conditional volatility and cross-correlation dynamics of yields. In addition, it captures the size and nature of the predictability of bond returns.

4 Concluding Remarks

Business cycle movements between economic expansions and recessions affect macroeconomic variables, financial markets, and in particular, the term structure of interest rates. In this paper, we incorporate the well documented feature of regime-shifts as in Hamilton (1988) into the standard term structure model such as Cox et al. (1985). We uncover additional important new evidence on the empirical success of regime switching models beyond that reported in Bansal and Zhou (2002).

The empirical work is conducted on nominal U.S. treasury bill and bond yields from 1964 to 2001. For estimation and specification tests of the various models, we use the Efficient Method of Moments estimation technique developed in Bansal et al. (1995) and Gallant and Tauchen (1996). A two factor regime shifting model is the only specification that fits the data according to the usual chi-square test of the restrictions; other models, including the multi-factor CIR and affine, are rejected. Furthermore, the preferred two-factor regime switching model matches the semiparametric moments with acceptable t-ratio diagnostics. In terms of cross-sectional implications, the preferred model achieves the smallest pricing error among all the specifications considered.

Regime switching and the risk premium for holding bonds appear closely connected—we show that the main channel that the regime shifts model accommodates is a time-varying "beta" with respect to risk factors. Our empirical evidence indicates that of the considered models—only the regime switching model can account for the size of the predictability (i.e., high R^2's) and the tent-shape structure of regression coefficients in the generalized Expectations Hypothesis regressions of excess bond returns on forward rates (Cochrane and Piazzesi, 2002). It is also able to account for the conditional volatility and conditional cross-correlation across yields. We find that there is an intimate link between business cycles, the slope of the yield curve, expected excess return of bond, and the regimes extracted from our term structure model.

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Table 1: Summary Statistics

There are 451 monthly observations of the yields with eight maturities. The data are obtained from CRSP (Center for Research in Security Prices) Treasury bill and bond files, ranging from June 1964 to December 2001.

	1 Month	3 Month	6 Month	1 Year	2 Year	3 Year	4 Year	5 Year
Mean	5.9424	6.3765	6.5971	6.8106	7.0156	7.1711	7.2909	7.3545
Stdv	2.4499	2.5767	2.6038	2.5239	2.4559	2.3814	2.3491	2.3240
Skew	1.4278	1.3717	1.3041	1.1737	1.1288	1.1283	1.1003	1.0565
Kurt	5.4659	5.1336	4.8778	4.4157	4.1226	4.0313	3.9196	3.7344

Table 2: Model Estimation by Efficient Method of Moments

The four term structure models are laid out in Section II. The 1-Factor[RS] or 2-Factor[RS] model refers to the regime shifts specification. The 2-Factor[CIR] model is the Cox-Ingersoll-Ross model with two factors. The 3-Factor[AF] model is the affine specification mentioned in the main text. The simulation size of EMM (efficient method of moments) is chosen to be 50,000 for all the four models.

	1-Factor[RS]	2-Factor[CIR]	2-Factor[RS]	3-Factor[AF]
Factor 1 Regime 0				
θ_{10}	.00566 (.00021)	.00548(.00051)	.00501(.00069)	.14e-6(.01e-6)
κ_{10}	.01678 (.00201)	.03515(.00304)	.01109(.00285)	.03530(.00247)
σ_{10}	.00652 (.00034)	.00508(.00032)	.00504(.00039)	.00006(.00000)
λ_{10}	-.00721 (.00165)	.02624(.00178)	.01877(.00273)	-.04136(.00223)
Factor 1 Regime 1				
θ_{11}	.00218(.00031)		.00629(.00060)	
κ_{11}	.01498(.00243)		.04655(.00971)	
σ_{11}	.00194(.00018)		.00075(.00021)	
λ_{11}	-.00324(.00276)		-.00673(.00310)	
Factor 2 Regime 0				
θ_{20}		.00091(.00008)	.00039(.00310)	.00340(.00024)
κ_{20}		.02666(.00305)	.01817(.00004)	.02487(.00660)
σ_{20}		.00545(.00011)	.00305(.00502)	-.00005(.00001)
λ_{20}		-.04212(.00389)	-.04938(.00024)	.00097(.00012)
σ_{23}				-.27376(.05107)
Factor 2 Regime 1				
θ_{21}			.00031(.00003)	
κ_{21}			.02982(.00603)	
σ_{21}			.00476(.00020)	
λ_{21}			-.05977(.00576)	
Factor 3				
κ_3				.01925(.00074)
σ_{31}				-344.37(43.686)
σ_{32}				-.45467(.00257)
λ_3				336.76(2.9700)
Transitional Probability $Pr\{s_{t+1}\|s_t\}$				
π_{00}	.97564(.00565)		.94007(.00008)	
π_{11}	.94489(.00001)		.93005(.00005)	
Specification Test				
$\mathcal{X}^2$	94.523	56.066	23.211	42.803
p-Value	.00000	.00003	0.0100	.00017
d.o.f.	18	20	10	15

Table 3: Diagnostic t-Ratios

The SNP score generator is explained in Section 3.2. The t-ratios are testing whether the fitted sample moments are equal to zero, as predicted by population moments of the SNP density.

Parameter	Description	1-Factor[RS]	2-Factor[CIR]	2-Factor[RS]	3-Factor[AF]
Hermite $A(1)$	00 00				
$A(2)$	01 00	0.30	-1.038	-0.752	0.528
$A(3)$	10 00	2.13	0.240	-0.646	0.898
$A(4)$	02 00	1.47	1.874	1.809	2.215
$A(5)$	11 00	-3.13	-2.258	1.251	-1.402
$A(6)$	20 00	2.36	-2.752	1.921	-1.538
$A(7)$	03 00	0.08	-0.072	-0.152	1.431
$A(8)$	30 00	0.40	-1.093	-0.442	-0.582
$A(9)$	04 00	1.05	2.018	1.634	2.384
$A(10)$	40 00	2.20	-1.230	1.423	-0.389
Mean $\psi(1)$	u(1)	2.61	0.263	-1.022	1.100
$\psi(2)$	u(2)	-0.69	-0.716	-0.299	-0.487
$\psi(3)$	u(1), y(1), lag 1	-1.75	0.859	0.963	0.568
$\psi(4)$	u(2), y(1), lag 1	-0.11	-0.407	-0.342	-0.213
$\psi(5)$	u(1), y(2), lag 1	-2.31	0.534	1.312	0.017
$\psi(6)$	u(2), y(2), lag 1	0.29	-0.047	-0.219	0.085
ARCH $\tau(1)$	R(1)	1.85	-3.402	1.264	-2.140
$\tau(2)$	R(2)	-4.27	-2.924	0.155	-2.692
$\tau(3)$	R(3)	3.98	3.579	1.369	2.962
$\tau(4)$	R(1), z(1), lag 5	2.56	-1.606	1.576	-0.640
$\tau(9)$	R(3), z(2), lag 5	2.76	2.063	0.104	1.641
$\tau(10)$	R(1), z(1), lag 4	2.57	-1.307	1.858	-0.467
$\tau(15)$	R(3), z(2), lag 4	2.80	1.916	0.933	1.891
$\tau(16)$	R(1), z(1), lag 3	1.68	-2.097	1.008	-1.621
$\tau(21)$	R(3), z(2), lag 3	4.41	3.474	1.963	3.198
$\tau(22)$	R(1), z(1), lag 2	2.99	-0.212	1.644	-0.003
$\tau(27)$	R(3), z(2), lag 2	2.25	1.846	0.879	1.597
$\tau(28)$	R(1), z(1), lag 1	3.46	-0.529	2.061	0.325
$\tau(33)$	R(3), z(2), lag 1	2.62	1.893	1.294	1.811

Table 4: Predictability of Bond Excess Returns Using Multiple Forward Rates

The dependent variable in all the regressions below is the one year return from holding a bond with n years to maturity less the yield on a bond with one year to maturity. This annual excess return is tracked monthly. All R^2s are adjusted for degrees of freedom. The sample size in the data is 451 observations. In the top panel the predictability regression is run using 1, 2, 3, 4, and 5 year forward rates as regressors. As the R^2 using 1yr, 3yr, 5yr forward rates is almost the same as using additional forward rates (see 1yr, 3-5yr, and 1-5yr) we focus on the 1yr, 3yr, 5yr projection. Newey-West robust standard errors are reported in parenthesis in the panel "Regression Coefficients and R^2 in Data" for this projection. The results reported in 1-Factor[RS], 2-Factor[CIR], 2-Factor[RS], 2-Factor[AF] panels are based on simulating 50,000 observations from the estimated term structure model and running the same regression as reported in the "Regression Coefficients and R^2 in Data" panel.

R-Square	4yr	1yr, 3yr	1yr, 3yr, 5yr	1yr, 3-5yr	1-5yr
			R^2's in the Data		
2 Year Bond	0.1744	0.2619	0.3088	0.3187	0.3280
3 Year Bond	0.1322	0.2538	0.3326	0.3357	0.3373
4 Year Bond	0.1368	0.2634	0.3406	0.3617	0.3639
5 Year Bond	0.1297	0.2640	0.3163	0.3308	0.3336
Coefficient	Intercept	1yr	3yr	5yr	R-Square
			Regression Coefficients and R^2 in Data		
2 Year Bond	-2.2222 (0.5747)	-0.6753 (0.1743)	1.7041 (0.2527)	-0.7245 (0.2109)	0.3088
3 Year Bond	-3.5737 (1.0078)	-1.4040 (0.3207)	3.5688 (0.4704)	-1.6963 (0.3657)	0.3326
4 Year Bond	-4.9032 (1.4403)	-2.0580 (0.4597)	5.0008 (0.6552)	-2.3245 (0.4864)	0.3406
5 Year Bond	-6.2848 (1.7667)	-2.5018 (0.5674)	5.6134 (0.8329)	-2.3573 (0.6004)	0.3163
Coefficient	Intercept	1yr	3yr	5yr	R-Square
			1-Factor[RS]		
2 Year Bond	8.3712	-0.4714	4.4622	-4.9444	0.0164
3 Year Bond	15.0127	-0.8423	7.9971	-8.8619	0.0149
4 Year Bond	20.1520	-1.1259	10.7298	-11.8906	0.0138
5 Year Bond	24.0829	-1.3394	12.8183	-14.2055	0.0129
Coefficient	Intercept	1yr	3yr	5yr	R-Square
			2-Factor[CIR]		
2 Year Bond	-1.8475	-0.2066	-0.0302	0.3613	0.1741
3 Year Bond	-3.6219	-0.3211	-0.0105	0.6765	0.2209
4 Year Bond	-5.5087	-0.3954	0.0380	0.9938	0.2538
5 Year Bond	-7.6055	-0.4542	0.1060	1.3377	0.2718
Coefficient	Intercept	1yr	3yr	5yr	R-Square
			2-Factor[RS]		
2 Year Bond	-3.3175	-0.8523	1.9875	-0.6116	0.1914
3 Year Bond	-6.1451	-1.4279	3.2531	-0.8669	0.2308
4 Year Bond	-8.9064	-1.8229	4.0214	-0.8262	0.2936
5 Year Bond	-11.9532	-2.1004	4.4245	-0.5051	0.3621
Coefficient	Intercept	1yr	3yr	5yr	R-Square
			3-Factor[AF]		
2 Year Bond	9.3180	0.6074	-1.8067	1.3361	0.1256
3 Year Bond	16.4960	1.2536	-3.7574	2.8143	0.1745
4 Year Bond	22.6622	1.9470	-5.8732	4.4451	0.2206
5 Year Bond	28.6284	2.6990	-8.1863	6.2503	0.2579

Table 5: Average Absolute Pricing Error (Basis Points)

There are eight maturities (1, 3, 6 month; 1, 2, 3, 4, 5 year) for each of 451 dates. The absolute pricing error of 1-Factor[RS] model is over 7 points; 2-Factor[CIR] over 6 points; 2-Factor[RS] over 6 points; and 3-Factor[AF] over 5 points. The summary statistics of the absolute pricing errors are calculated over the 451 dates for each of the four models.

	1-Factor[RS]	2-Factor[CIR]	2-Factor[RS]	3-Factor[AF]
Mean	45	44	27	31
Median	34	40	19	23
Std.	33	24	22	28
Min.	5	5	3	1
Max.	223	156	154	188

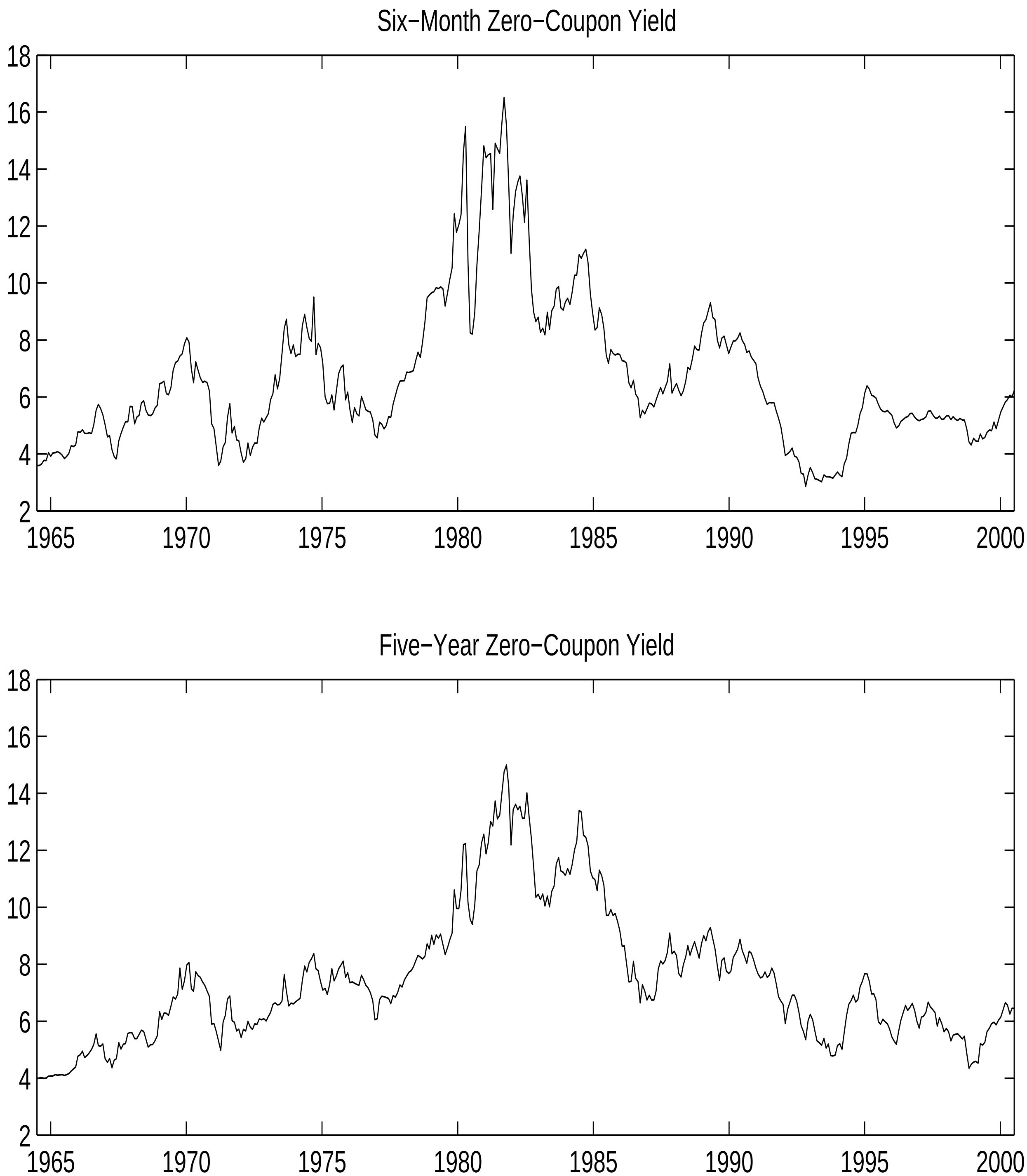

Figure 1: Observed Short Rate and Long Rate

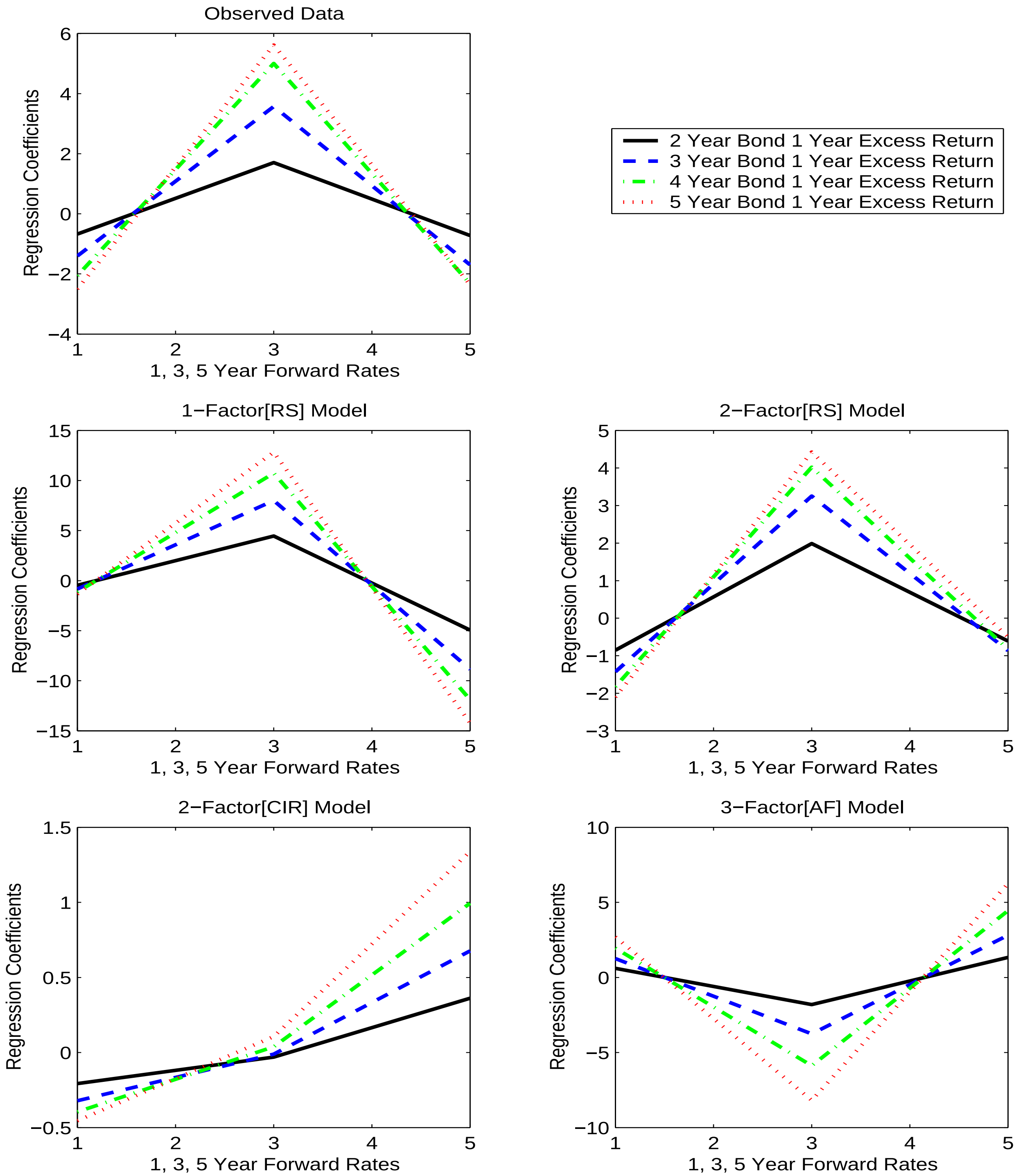

Figure 2: Predictability Regression Coefficients

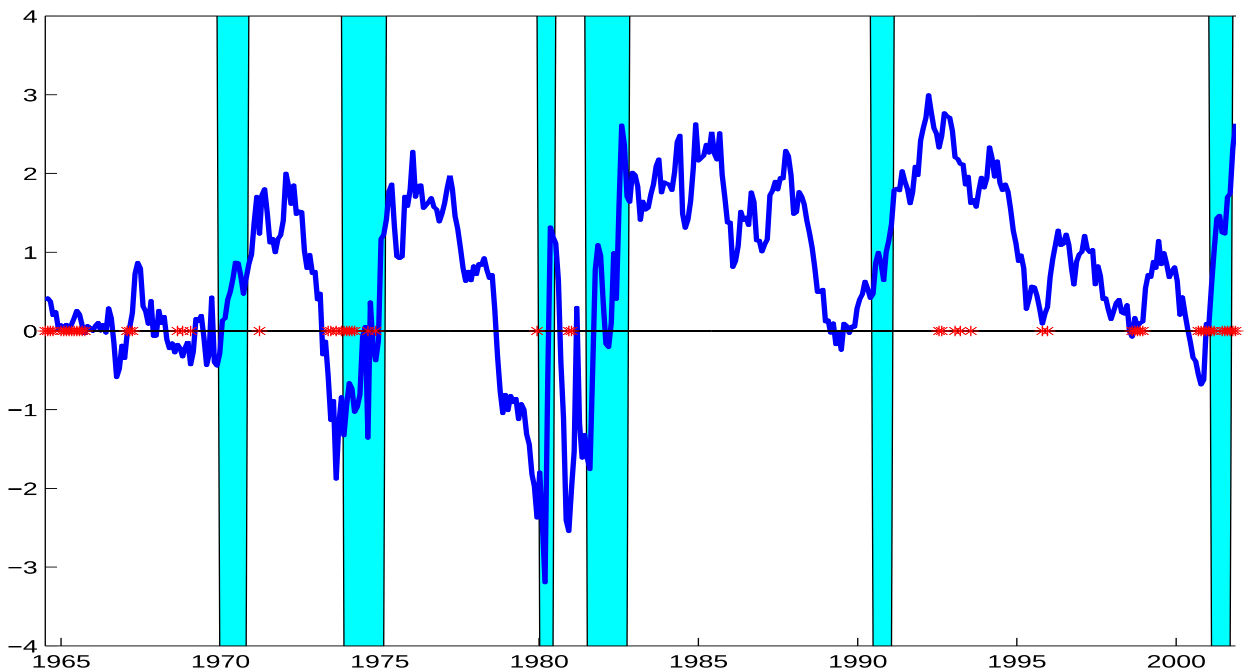

Figure 3: Yield Spread, Regime Indicator, and Business Cycle
The thick line is the five year yield minus the six month yield (yield spread), the shaded area is the NBER recession period, and the star is the indicator of our low regime (regime 0) from our preferred 2-Factor[RS] model. The high regime (regime 1) corresponds to all dates without the star.

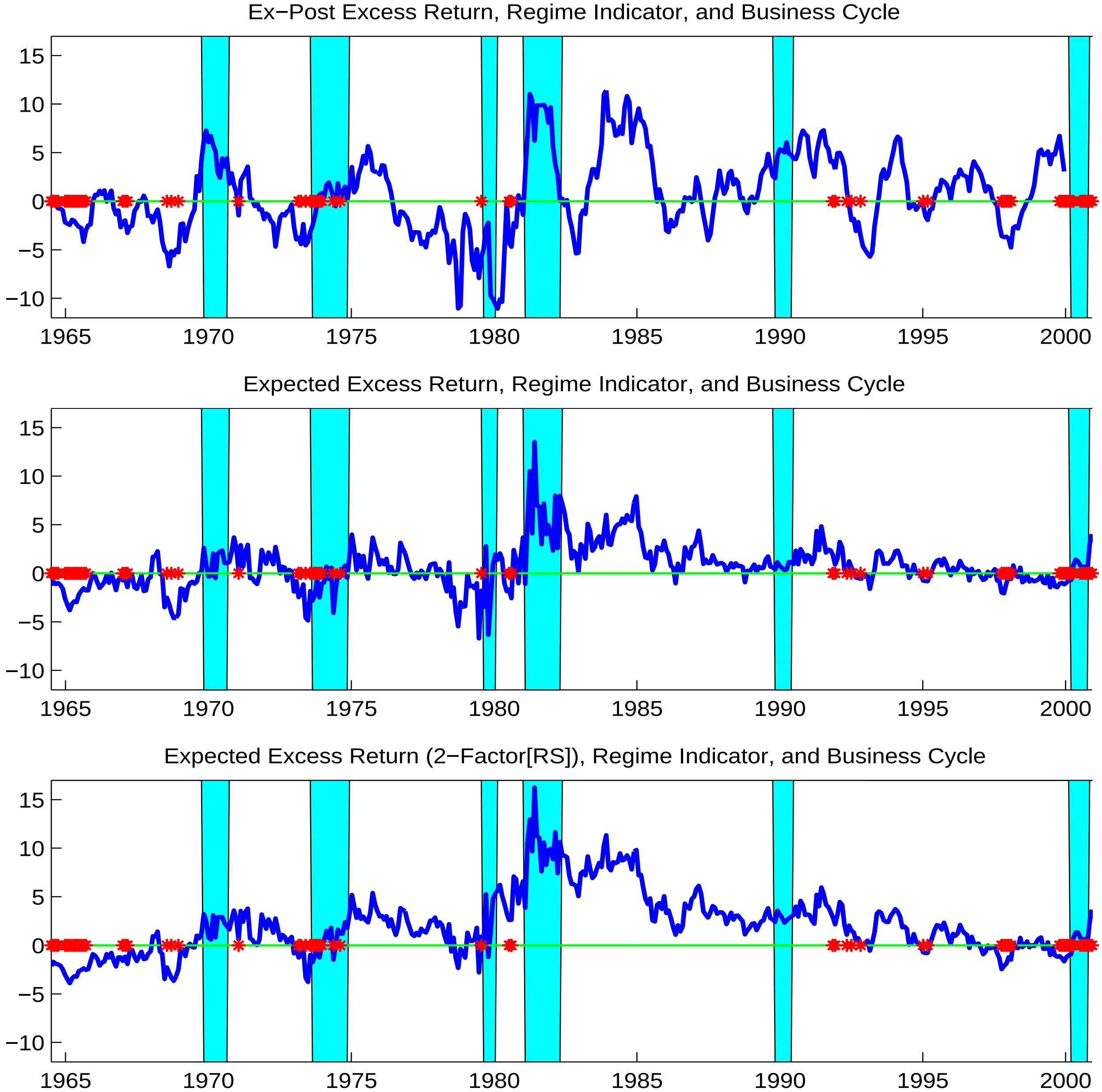

Figure 4: Excess Return, Regime Indicator, and Business Cycle
The shaded area is the NBER recession period, and the star is the indicator of the low regime (regime 0) from our preferred regime-shifts term structure model. The thick line is, respectively, the annual ex-post excess return (top panel), the expected excess return based on projecting future ex-post excess returns on three forward rates (middle panel), and the reprojected expected excess return from our 2-Factor[RS] model (bottom panel). All ex-post and expected excess returns are averages (across bonds) using the 2-5 year bonds.

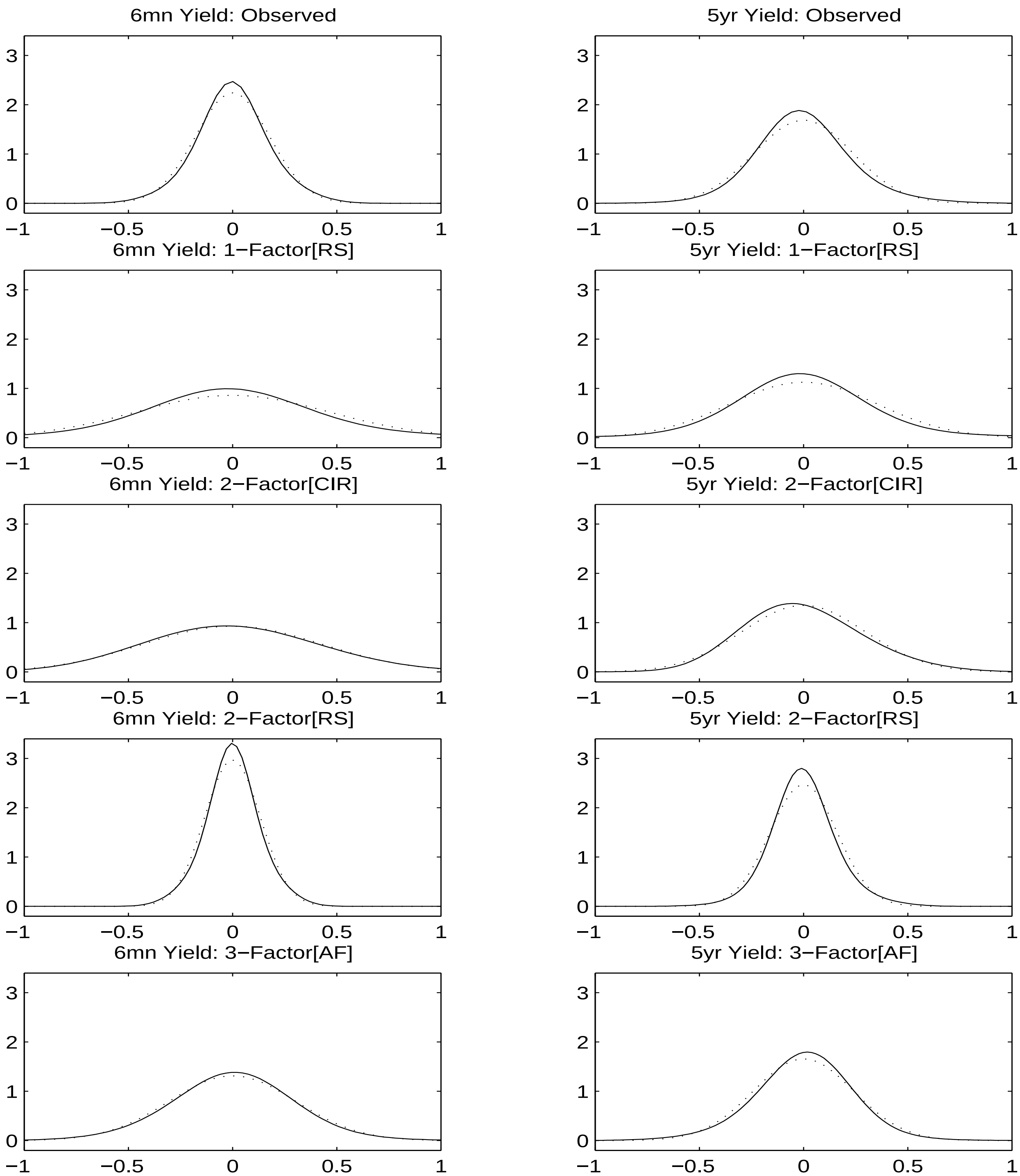

Figure 5: Reprojected Densities

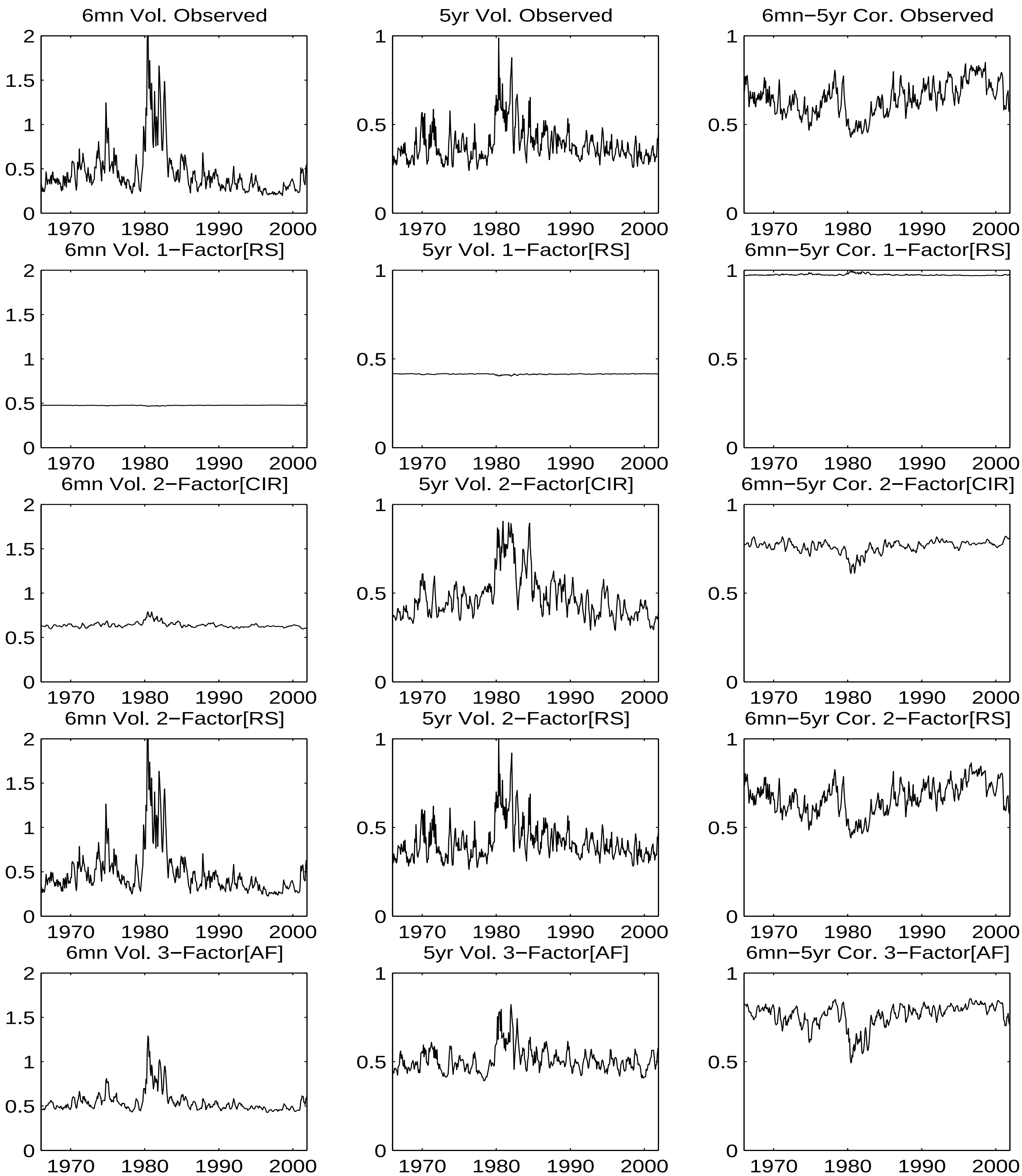

Figure 6: Reprojected Volatilities and Correlations