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THE PRICE VARIABILITY-VOLUME RELATIONSHIP ON SPECULATIVE MARKETS

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This paper concerns the relationship between the variability of the daily price change and the daily volume of trading on the speculative markets. Our work extends the theory of speculative markets in two ways. First, we derive from economic theory the joint probability distribution of the price change and the trading volume over any interval of time within the trading day. And second, we determine how this joint distribution changes as more traders enter (or exit from) the market. The model's parameters are estimated by FIML using daily data from the 90-day T-bills futures market. The results of the estimation can reconcile a conflict between the price variability-volume relationship for this market and the relationship obtained by previous investigators for other speculative markets.

1. INTRODUCTION

THIS PAPER CONCERNS the relationship between the variability of the daily price change and the volume of trading on speculative markets. Previous empirical studies [**2**, **3**, **6**, **12**, **14**, **16**] of both futures and equity markets always find a positive association between price variability (as measured by the squared price change ΔP^2) and the trading volume.[2] There are two explanations for the relationship. Clark's [**2**] explanation, which is secondary to his effort to explain why the probability distribution of the daily price change is leptokurtic, emphasizes randomness in the number of within-day transactions. In Clark's model the daily price change is the sum of a random number of within-day price changes. The variance of the daily price change is thus a random variable with a mean proportional to the mean number of daily transactions. Clark argues that the trading volume is related positively to the number of within-day transactions, and so the trading volume is related positively to the variability of the price change.

The second explanation is due to Epps and Epps [**6**]. Their model examines the mechanics of within-day trading. The change in the market price on each within-day transaction or market clearing is the average of the changes in all of the traders' reservation prices. Epps and Epps assume there is a positive relationship between the extent to which traders disagree when they revise their reservation prices and the absolute value of the change in the market price. That is, an increase in the extent to which traders disagree is associated with a larger absolute price change. The price variability-volume relationship arises, then, because the volume of trading is positively related to the extent to which traders disagree when they revise their reservation prices.

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[2] Cornell [**3**] offers considerable empirical documentation on how pervasive the relationship is. His explanation is very similar to that of Epps and Epps [**6**].

The Clark and Epps and Epps models are complementary and they give considerable insight into the behavior of speculative markets. Yet, even when taken together, the two models provide a description of speculative markets that is incomplete and can be extended in two directions. First, both models work with the conditional distribution of the square of the price change over a short interval of time, ΔP^2, given the volume of trading, V, for the same interval of time. Application of either model requires the investigator to specify in advance or discover by nonlinear regression the functional form of the conditional expectation $E[\Delta P^2 \mid V]$. The model we derive below eliminates the need for this. The theory gives an explicit expression for the joint probability distribution of the price change and the trading volume over any interval of time. The joint distribution contains all relevant information about the price variability-volume relationship. Specifically, it determines the conditional distribution of the price change given the volume and the conditional absolute moments of all orders.

Second, neither model considers growth in the size of speculative markets such as that experienced by many of the new financial futures markets. Trading on a new market is initially very thin. If the market is viable, then the trading volume increases secularly as more traders become aware of the market's possibilities. Eventually a steady state is reached. The empirical results of other studies suggest that price variability should increase with the growth in the trading volume. This seems unlikely. In fact, one might conjecture that more traders would tend to stabilize prices.

A case in point is the 90-day T-bills futures market. This was one of the first and most popular of the new financial futures markets. During its growth phase the number of traders increased dramatically. (See the Appendix for more details.) One source of this growth was the gradual diffusion of information about the market;[3] another was the relaxation of regulations restricting financial institutions' access to futures markets. The following display shows the average daily volume ($\bar{V}$) and the variance of the daily price change ($s^2_{\Delta P}$) for four nonoverlapping intervals, each comprised of 219 trading days:

Days	$\bar{V}$	$s^2_{\Delta P}$
1–219	.393	.057
220–438	1.094	.059
439–657	1.811	.018
658–876	5.490	.048

where the volume is measured in thousands of contracts, the price change variance is measured in (\$ thousands)^2, and the sample extends from the first day of trading, January 6, 1976, through June 30, 1979. The volume of trading appears to grow exponentially. A regression of the log of the daily volume of

[3] See Jacobs and Jones [**8**] for details about the opportunities that were available to traders taking positions in both the spot and futures T-bills markets.

trading on a trend term gives

$$\log(V(t)) = -\underset{(.033)}{1.462} + \underset{(.000065)}{.0039t}, \qquad R^2 = .80,$$

where $t = 1, 2, 3, \ldots, 876$ denotes trading days and standard errors are in parentheses. According to the results of previous empirical studies, the variance of the daily price change should share in some of this .39 per cent per day growth in the volume of trading. But the data show that is not the case; if anything, the variance of the daily price change declines over the period. The corresponding regression of the log of the squared daily price change on a trend term gives

$$\log(\Delta P(t)^2) = -\underset{(.186)}{4.632} - \underset{(.00037)}{.00047t}, \qquad R^2 = .002.$$

The theory of speculative trading as currently formulated cannot explain this gentle decline in the variance of the daily price change given the explosive growth of the trading volume.

The purpose of this paper is to derive and estimate a more general model of the price change and the trading volume on speculative markets. Like the Epps and Epps model, our model begins with an equilibrium theory of within-day price determination. A major difference between the two models is the way in which we connect the price change to the trading volume. Epps and Epps's key assumption gives them a nearly exact positive relationship between the absolute value of the change in the market price and the trading volume on each within-day market clearing. We do not invoke their assumption. Instead, we use a variance-components scheme to model the within-day revisions of traders' reservation prices. This allows us to derive the joint probability distribution of the price change and the trading volume for each within-day market clearing. Adding the random number of within-day price changes and volumes gives the daily values of each variable. The result is a bivariate normal mixture model with a likelihood function that depends only on a few easily interpreted parameters.

The model can explain both the results of previous studies and the anomalous data displayed above. If the number of traders is fixed, which is a reasonable assumption for the mature markets studied by others, then the model predicts that the distribution of the daily price change is leptokurtic and that the square of the daily price change is positively related to the daily trading volume. If the number of traders is growing, which is the case in the 90-day T-bills futures market, then the model predicts that the mean trading volume increases linearly with the number of traders. The reason is that the trading volume is one-half of the sum of the absolute changes in the traders' positions; another trader contributes another term to the sum. The model also predicts that the variance of the price change decreases with more traders. The reason for this is that the market price change during a single market clearing is the average of the changes in the traders' reservation prices. More terms in the average tend to wash out the effects of inter-trader differences.

Though the model we derive and estimate can explain many of the stylized facts for new markets and for established markets, there is a natural direction in which further extensions of the model are possible. Specifically, we assume that each trader acts in the same fashion regardless of the total number of traders in the market; e.g., a typical trader acts in the same fashion whether there are 1,000 or 10,000 other active traders in the market. By doing so, we can prove that the average trading volume per trader is independent of the total number of traders and that it depends on only a few simple parameters. This result is particularly helpful in implementing and interpreting the empirical work. There is, however, another class of full-equilibrium rational expectations models [**5**, **7**] which suggest that there is an interaction effect among the traders, and that at a fixed point in time the number of futures contracts per trader may decline as the market expands. This in turn suggests, but still requires formal proof, that the average trading volume per trader (the volume is the number of contracts that change hands per unit time) may decline as the market expands. In Section 5 we show that there is some evidence in favor of this effect, though the effect appears to be small relative to the direct expansion of the market. Indeed, it has to be small, for otherwise the market would implode and trading would cease as more traders enter the market, and this certainly did not happen to the 90-day T-bills futures market.

Section 2 describes the model in more detail. Section 3 applies the model to daily data for the 90-day T-bills futures market. Section 4 contains results about the price variability-volume relationship. Section 5 includes further discussion and suggestions for subsequent research. Section 6 contains the concluding remarks.

2. MODEL

2.1. *Intra-day Trading*

The market consists of J active traders who take long or short positions in a single futures contract. Within the day the market passes through a sequence of distinct Walrasian equilibria. The movement from the $(i-1)$st to the ith within-day equilibrium is initiated by the arrival of new information to the market. The time intervals between successive equilibria are not necessarily of equal length.[4]

At the time of the ith within-day equilibrium the desired position Q_{ij} of the jth trader is given by the linear relation

$$(1) \qquad Q_{ij} = \alpha\left[P_{ij}^* - P_i \right] \qquad (j = 1, 2, \ldots, J)$$

[4]Since buy/sell orders are executed sequentially, many actual transactions at the exchange can comprise what we think of as a single market clearing or transaction.

where $\alpha > 0$ is constant; P_{ij}^* is the jth trader's reservation price, or null price, and P_i is the current market price. Equation (1) abstracts from transaction costs and it assumes that the traders differ only in their reservation prices.[5] A positive value for Q_{ij} represents a desired long position in the contract while a negative value represents a desired short position. These J active traders have reservation prices different from the current market quotation. The inter-trader differences in the P_{ij}^*'s arise from different expectations about the future and from different needs to transfer risk through the market. Nonactive traders use the market quotation as their reservation price. For simplicity, we assume J is fixed within the day. Later we discuss secular growth over days in the number of active traders.

Equilibrium requires $\sum_{j=1}^{J} Q_{ij} = 0$. This implies that the average of the reservation prices

$$P_i = \frac{1}{J} \sum_{j=1}^{J} P_{ij}^*$$

clears the market.

Consider now the movement from the $(i-1)$st to the ith within-day equilibrium. A piece of news arrives to the market and changes the traders' reservation prices. The resulting change in the market price ΔP_i is the average of the increments to the traders' reservation prices; the associated volume of trading V_i is by definition one-half the sum of the absolute values of the changes in the traders' positions. By making use of the equilibrium condition and equation (1), the price change and the trading volume can be written:

$$\Delta P_i = \frac{1}{J} \sum_{j=1}^{J} \Delta P_{ij}^*, \tag{2}$$

$$V_i \equiv \frac{1}{2} \sum_{j=1}^{J} | Q_{ij} - Q_{i-1,j}| = \frac{\alpha}{2} \sum_{j=1}^{J} |\Delta P_{ij}^* - \Delta P_i|, \tag{3}$$

where $\Delta P_{ij}^* \equiv P_{ij}^* - P_{i-1,j}^*$ is the increment to the jth trader's reservation price. In equation (3) the volume is proportional to the mean absolute deviation of the reservation price increments about their mean.

Specification of the joint probability distribution of the increments $\{\Delta P_{ij}^*\}$ induces a joint probability distribution for the change in the market price and the trading volume. Here we do not make Epps and Epps's crucial assumption. (They make no formal distributional assumptions about the increments $\{\Delta P_{ij}^*\}$.

[5]The demand equation (1) in the text is a special case of the demand equation derived by Jacobs and Jones [**8**, equation (4), p. 702]. To get our equation (1), assume all traders have the same coefficient to absolute risk aversion and there is only one contract. Then, everything except the quoted interest rate can be absorbed into a "reservation interest rate," and both rates can be converted to prices using formulas published by the exchange. The reservation prices should be thought of as being *net* of the effect of the market price on the traders' forecasts.

Instead, they assume [**6**, equations (15a) and (15b), p. 309] that there is a functional dependency among the increments. This allows them to write an exact relationship between $|\Delta P_i|$ and V_i to which they append a random error.) We assume a variance-components model

$$\Delta P_{ij}^* = \phi_i + \psi_{ij}, \quad E[\phi_i] = E[\psi_{ij}] = 0, \quad \mathrm{var}[\phi_i] \equiv \sigma_\phi^2, \quad \mathrm{var}[\psi_{ij}] \equiv \sigma_\psi^2, \tag{4}$$

where the ϕ's and the ψ's are mutually independent, both across traders and through time. The component ϕ_i is common to all traders while the component ψ_{ij} is specific to the jth trader. A large realization (in absolute value) for the common component relative to the realizations of the specific components represents a situation in which the traders react nearly unamimously to the new information. In contrast, a small realization for the common component relative to the specific components means that the traders react diffusely to the information. Finally, the assumption that the components are mutually serially independent through time means that there are no delays in the receipt of the new information.[6]

Use the variance-components model (4) to write the ith price change (2) and trading volume (3) as

$$\Delta P_i = \phi_i + \bar{\psi}_i, \qquad \bar{\psi}_i \equiv \frac{1}{J} \sum_{j=1}^{J} \psi_{ij}, \tag{5}$$

$$V_i = \frac{\alpha}{2} \sum_{j=1}^{J} |\psi_{ij} - \bar{\psi}_i|. \tag{6}$$

Interestingly, the common component ϕ_i plays no role in the generation of trading volume. A large realization for ϕ_i relative to the ψ_{ij}'s lead to a price change and little or no trading volume. This is known to occur occasionally on speculative markets.

To make the likelihood calculations in Section 3 feasible we assume the variance components ϕ_i and ψ_{ij} in (4) are normally distributed.[7] Application of the normality gives results about the joint distribution of the price change and the trading volume.

PROPOSITION: (i) *The price change ΔP_i is normally distributed.* (ii) *For large J the volume V_i is approximately normally distributed.* (iii) *ΔP_i and V_i are stochastically independent.* (iv) *Their first two moments (with the dependence on J under-*

[6] The changes in the reservation prices arise from revised expectations about the contract's ultimate price and from revised plans to transfer risk via the futures market. Basically, we assume that these revisions are, in the very short term, dominated by the traders' reactions to new information. Any tendency for the revisions to move in a systematic fashion is not detectable given the very fine level of temporal disaggregation at which we work.

[7] The stable paretian, of course, is another possibility. We choose not to work with it because there is evidence [**2**, **6**, **14**, **15**] that, when compared with the normal mixture model, the stable paretian gives a poorer fit to the marginal distribution of the daily price change.

stood) are

(7a) $$\mu_1 \equiv E[\Delta P_i] = 0,$$

(7b) $$\sigma_1^2 \equiv \mathrm{Var}[\Delta P_i] = \sigma_\phi^2 + \frac{\sigma_\psi^2}{J},$$

(7c) $$\mu_2 \equiv E[V_i] = \left(\frac{\alpha}{2}\right)\sigma_\psi\sqrt{\frac{2}{\pi}}\left(\sqrt{\frac{J-1}{J}}\right)J,$$

(7d) $$\sigma_2^2 \equiv \mathrm{Var}[V_i] = \left(\frac{\alpha}{2}\right)^2\sigma_\psi^2\left(1-\frac{2}{\pi}\right)J + o(J).$$

Item (i) is trivial. Item (ii) is intuitively clear because $V_i = (\alpha/2)\sum_{j=1}^{J}|\psi_{ij} - \bar{\psi}_i|$ is the sum of nearly independent terms. For a normal parent distribution, the asymptotic normality of the (suitably normalized) mean deviation about the mean is a well-known statistical result [**11**, pp. 386–87 and 393–94]. Item (iii) follows from the independence of $\Delta P_i = \phi_i + \bar{\psi}_i$ and the typical deviation from the mean, $\psi_{ij} - \bar{\psi}_i$, whose absolute value enters the sum for the volume. For item (iv), the first two moments (7a) and (7b) for ΔP_i can be computed directly. The expression (7c) for $E[V_i]$ follows from $E|Z| = \sqrt{2/\pi}$ where $Z \sim N(0,1)$, and the expression (7d) for the asymptotic variance of V_i follows from [**11**, p. 394].

Unlike the Epps and Epps model, the price change and the trading volume are stochastically independent on a single transaction or market clearing. Other distributional assumptions can, of course, give dependence but we argue that any dependence will usually be weak. The price change equals the independent common component plus the average over traders of the ψ_{ij}'s, while the trading volume is proportional to the mean absolute deviation of the ψ_{ij}'s about their average. Suppose the ψ_{ij}'s were not normally distributed, but only symmetrically distributed about zero with finite variance. Then the suitably normalized average and MAD about the average will be, for large J, approximately jointly normally distributed with zero covariance.

Although the random variables ΔP_i and V_i are stochastically independent given our assumptions, their common dependence on the ϕ_i's and the ψ_{ij}'s gives rise to functional dependencies among their moments. For example, in (7b) and (7c) both Var[ΔP_i] and $E[V_i]$ are increasing functions of the variance of the specific component, σ_ψ^2.[8]

[8] If σ_ϕ^2 or σ_ψ^2 were generated by random processes, then (7) displays the conditional moments, and unconditionally ΔP_i^2 and V_i would be positively related. For example, suppose σ_ψ^2 is random, say $\sigma_\psi^2(i) = e^{\eta_i}$, where the η_i's are iid random variables. Then by inspection of (7b) and (7c), the unconditional covariance between ΔP_i^2 and V_i is positive as long as the third moment of $\sigma_\psi(i)$ is positive. This mechanism might explain (in the context of our model) why Epps and Epps find a weak but significant relationship between ΔP_i^2 and V_i at the within-day level. We abstract from this kind of randomness, however, in order to make the estimation of the daily model feasible.

The expressions (7b) and (7c) are the key to our resolution of the paradox generated by the data displayed in the introduction. As the number of traders grows secularly over days, $\mathrm{Var}[\Delta P_i]$ declines monotonically to its asymptote σ_ϕ^2. Since ΔP_i is an average the effects of disagreement get washed out; only the common component of the reservation price increments matters in the limit. In contrast, the mean volume of trading grows linearly with J. The absolute values that comprise the sum for V_i in (6) magnify the effects of inter-trader differences in the increments to the reservation prices.

2.2. *The Joint Probability Distribution of the Daily Price Change and Volume of Trading*

The number of traders J is nonrandom and fixed for each day. The number of daily equilibria, I, is random because the number of new pieces of information arriving to the market each day varies significantly. Summing the within-day price changes and trading volumes gives the daily values

$$\Delta P = \sum_{i=1}^{I} \Delta P_i, \qquad \Delta P_i \sim N(0, \sigma_1^2),$$

$$V = \sum_{i=1}^{I} V_i, \qquad V_i \sim N(\mu_2, \sigma_2^2).$$

Both the daily price change and trading volume are mixtures of independent normals with the same mixing variable, I. (We assume that J is always large enough so that the error is negligible in the normal approximation, item (ii) of the proposition, to the distribution of V_i; the likelihood calculations are intractable using the exact distribution.)

Conditional on I the daily price change ΔP is $N(0, \sigma_1^2 I)$ and the daily volume V is $N(\mu_2 I, \sigma_2^2 I)$. Thus another way to write this bivariate normal mixture model is

(8a) $$\Delta P = \sigma_1 \sqrt{I}\, Z_1,$$

(8b) $$V = \mu_2 I + \sigma_2 \sqrt{I}\, Z_2,$$

where Z_1 and Z_2 are $N(0,1)$ random variables and Z_1, Z_2, and I are mutually independent. With ΔP and V written this way the price variability-volume relationship is immediate:

$$\begin{aligned} \mathrm{Cov}(\Delta P^2, V) &= E[\Delta P^2 V] - E[\Delta P^2]E[V] \\ &= \sigma_1^2 \mu_2 E[I^2] - \sigma_1^2 \mu_2 (E[I])^2 \\ &= \sigma_1^2 \mu_2 \mathrm{Var}[I] > 0, \end{aligned}$$

which holds empirically so long as the number of traders is fixed.

This expression for $\mathrm{Cov}(\Delta P^2, V)$ makes clear that the variance-volume relationship arises because ΔP^2 and V are positively related to the unobserved mixing variable, I. If the mixing variable shows no variation, $\mathrm{Var}[I]=0$, then the relationship vanishes. Thus, our explanation for the variance-volume relationship is closer to Clark's than that of Epps and Epps. In fact, our model can be viewed as providing both an economic relationalization and an explicit expression for Clark's conjectured relationship between the trading volume and the mixing variable.

With the model written as (8a) and (8b) it is also clear why Clark and others always report that the trading volume is an imperfect proxy for the mixing variable. In (8b) the volume equals $\mu_2 I$ plus the heteroskedastic "measurement error" $\sigma_2\sqrt{I}\,Z_2$. Only if $\sigma_2 \approx 0$ will Clark's method of dividing the daily price change by a function of the trading volume actually induce normality in the marginal distribution of the price change. From (7d) there is no *a priori* reason to expect σ_2 to be small.

The mixing variable enters the model nonlinearly and there is no way to use functional operations to eliminate I between equations (8a) and (8b). The proper way to eliminate the unobserved mixing variable between the two equations is to integrate it out of the trivariate joint probability density of ΔP, V, and I. This leaves the joint density of the observed variables ΔP and V. Conditional on I the random variables ΔP and V are independent and so their joint conditional density is the product of the marginals

$$f_c(\Delta P, V \mid I; \sigma_1, \mu_2, \sigma_2) = n(\Delta P; 0, \sigma_1^2 I)\, n(V; \mu_2 I, \sigma_2^2 I)$$

where $n(x;\mu,\sigma^2)$ is the normal density. The unconditional joint density is

$$f(\Delta P, V; \sigma_1, \mu_2, \sigma_2, \theta) = \int f_c(\Delta P, V \mid I; \sigma_1, \mu_2, \sigma_2)\, G(dI; \theta) \tag{9}$$

where $G(I;\theta)$ is the marginal distribution function of the mixing variable and θ is a vector containing its parameters. By restricting G to a particular parametric family, either discrete or a continuous approximation, and by using numerical methods to calculate the integral we can estimate the model by maximum likelihood.[9]

As is usually the case in unobserved variables models, the mathematical form of this model is invariant with respect to arbitrary transformations of the scale of the unobserved variable. That is, if a is any positive constant and $I' \equiv I/a$, then the model

$$\Delta P = \left(\sigma_1\sqrt{a}\right)\sqrt{I'}\,Z_1,$$

$$V = (\mu_2 a)I' + \left(\sigma_2\sqrt{a}\right)\sqrt{I'}\,Z_2,$$

[9] This is the random effects model discussed by Chamberlain [**1**, pp. 232–235].

is observationally equivalent to the basic model (8), so long as the parametric family of the mixing variable is closed under scale transformations. To normalize the model we set $E[I'] = 1$. The parameters actually estimated are then

$$\tilde{\sigma}_1 \equiv \sigma_1 \sqrt{E[I]}\,, \tag{10a}$$

$$\tilde{\mu}_2 \equiv \mu_2 E[I], \tag{10b}$$

$$\tilde{\sigma}_2 \equiv \sigma_2 \sqrt{E[I]}\,. \tag{10c}$$

The parameter $\tilde{\sigma}_1$ is the standard deviation of the daily price change and $\tilde{\mu}_2$ is the mean daily trading volume. The parameter $\tilde{\sigma}_2$ is related to the variance of the daily volume by

$$\tilde{\sigma}_2 = \sqrt{\mathrm{Var}[V] - \tilde{\mu}_2^2 (cv)^2}$$

where $cv = \sqrt{\mathrm{Var}[I]}\,/E[I]$ is the coefficient of variation of the mixing variable.

3. ESTIMATION

3.1. *The Data*

The sample is described in more detail in the Appendix. It consists of 876 observations on the daily price change and volume of trading on the 90-day T-bills futures market. In work that is too lengthy to report on in detail here, we were able to verify many of the known facts about the marginal distribution of the daily price changes. Specifically, for the full sample and for various subdivisions of the sample we found that the price changes have mean zero, they are serially uncorrelated, and their frequency distribution is leptokurtic.

3.2. *Preliminary Tests*

For a fixed number of traders the daily volume and the square of the price change are positively related. As the number of traders increases, however, the mean daily volume increases while the variance of the price change decreases towards an asymptote. Daily data on the number of traders are not available, but the series is known to be strongly upward trended (see the Appendix). Thus, introduction of a trend into a variance-volume regression should raise the coefficient on volume. The results, with standard deviations in parentheses, are

$$\log\left(\Delta P(t)^2\right) = -\underset{(.095)}{4.899} + \underset{(.084)}{.246}\log(V(t)), \qquad R^2 = .01,$$

$$\log\left(\Delta P(t)^2\right) = -\underset{(.321)}{2.059} + \underset{(.182)}{1.755}\log(V(t)) - \underset{(.0007)}{.0073t}\,, \qquad R^2 = .10,$$

where $t = 1, 2, 3, \ldots, 876$ indexes trading days. This is very encouraging. In fact, the unitless coefficient of $\log(V)$ in the second of these two regressions is, at the five per cent level, insignificantly different from both of the corresponding estimates that Clark reports for two samples of cotton futures data [**2**, Sample 1: line (b), and Sample 2: line (b), Table II, p. 144]. These regressions, however, are very inefficient because they ignore most of the structure imposed by the theory. In particular, the error terms in these regressions are heteroskedastic.

3.3. *Maximum Likelihood Estimation*

The theory of arrival times and Clark's work suggest that the Poisson and the lognormal distributions are natural candidates for the marginal distribution of the mixing variable. We carried out all estimation using both distributions. The parameter estimates were very similar, though the lognormal usually gave likelihoods five to ten times higher than the Poisson. For this reason, and for comparability with Clark, we report only the lognormal results.

The lognormal mixing variable can be written $I = e^{\theta Z + m}$ where $Z \sim N(0, 1)$ and θ and m are parameters. Its first two moments are $E[I] = e^{\theta^2/2 + m}$ and $\operatorname{Var}[I] = (E[I])^2[e^{\theta^2} - 1]$. It can be normalized to have mean unity by imposing $m = -\theta^2/2$. The remaining parameter θ is related to the unitless coefficient of variation $cv = \sqrt{\operatorname{Var}[I]}\,/E[I]$ through the monotonic transformation $\theta = \sqrt{\ln(cv^2 + 1)}$.

We discuss first the results of applying the model to smaller subsets of data under the assumption that the number of traders is constant within each subset. When the number of traders is constant then the normalized parameters, $\tilde{\sigma}_1$, $\tilde{\mu}_2$, and $\tilde{\sigma}_2$ as defined in (10), are constants and estimable by maximum likelihood. With I written as $e^{\theta Z - \theta^2/2}$, the daily likelihood function (the joint density (9) evaluated at the day t observations $\Delta P(t)$ and $V(t)$) is

$$(11) \qquad L_t(\tilde{\sigma}_1, \tilde{\mu}_2, \tilde{\sigma}_2, \theta) = \int_{-\infty}^{\infty} n\left(\Delta P(t); 0, \tilde{\sigma}_1^2 e^{\theta z - (\theta^2/2)}\right) \cdot n\left(V(t);\ \tilde{\mu}_2 e^{\theta z - (\theta^2/2)}, \tilde{\sigma}_2^2 e^{\theta z - (\theta^2/2)}\right) \cdot (2\pi)^{-1/2} e^{-z^2/2}\, dz.$$

Integrals of this form were computed with an 8-point Gauss-Hermite quadrature rule. This rule integrates exactly against the normal measure any polynomial up to degree 15. Maximum likelihood estimates were computed by using the Goldfeld-Quandt DFP program to maximize the sum of the logs of the integrals with respect to $\tilde{\sigma}_1$, $\tilde{\mu}_2$, $\tilde{\sigma}_2$, and θ. Method of moments estimators provided

TABLE I

SUBPERIOD ESTIMATION[a]

Days	$\tilde{\sigma}_1 = \sigma_1\sqrt{E[I]}$	$\tilde{\mu}_2 = \mu_2 E[I]$	$\tilde{\sigma}_2 = \sigma_2\sqrt{E[I]}$	$\theta = \sqrt{\ln(1 + [\text{coef.var.}(I)]^2)}$
1–200	.233 (.013)	.359 (.014)	.112 (.015)	.420 (.032)
101–300	.228 (.012)	.688 (.026)	.161 (.014)	.518 (.029)
201–400	.235 (.012)	1.001 (.027)	.180 (.019)	.399 (.016)
301–500	.185 (.010)	1.351 (.042)	.257 (.028)	.394 (.020)
401–600	.136 (.007)	1.606 (.006)	.332 (.004)	.412 (.035)
501–700	.134 (.007)	2.280 (.082)	.508 (.053)	.462 (.027)
601–800	.163 (.009)	3.972 (.146)	.880 (.104)	.471 (.024)
701–876	.215 (.012)	6.253 (.245)	1.270 (.125)	.457 (.028)

[a] Standard deviations are in parentheses.

excellent starting values. Convergence to the global maximum always obtained from other starting values as well.[10]

Table I displays the parameter estimates for eight intervals containing 200 days of data each except for the last interval which has 176 days. The intervals overlap by 100 days and so adjacent sets of parameters are correlated. Using overlapping intervals has a smoothing effect similar to that of a moving average.

Several features of the results in Table I merit further description. First, the normalized volume parameters, $\tilde{\mu}_2$ and $\tilde{\sigma}_2$, grow exponentially over the sample. Comparing the estimates in the first and last lines of Table I, we see that $\tilde{\mu}_2$ increases by a factor 17.4 while $\tilde{\sigma}_2$ increases by a factor of 11.3. The model predicts that these two parameters are proportional to $JE[I]$ and $\sqrt{JE[I]}$, respectively, so their growth can be explained directly by expansion in the number of traders, J, and by any indirect effect that more traders have on the mean of the mixing variable. Note that the square root relationship does not hold exactly. If it did then the parameter $\tilde{\sigma}_2$ should increase by a factor of $\sqrt{17.4} = 4.2$, instead of by a factor of 11.3, though it is true that $\tilde{\sigma}_2$ grows more slowly than $\tilde{\mu}_2$, which is a qualitative prediction of the model.

Now contrast the rapid growth in the volume parameters to the behavior of $\tilde{\sigma}_1$, which is the standard deviation of the daily price change. The parameter $\tilde{\sigma}_1$ shows a slight tendency to drift downwards over the sample period. This finding is consistent with the expression (7a) for the variance of the within-day price change, though the last estimate of $\tilde{\sigma}_1$ is well above the downward trend line. Most important, however, is the finding that the price variability does not increase with the growth in the trading volume as the previous price variability-volume studies would suggest.

Examination of the right-most column in Table I indicates that there is no

[10] All calculations were performed in double precision arithmetic and every possible precaution was taken to avoid roundoff errors. The reported results were double-checked for accuracy by repeating roughly half of the estimation with a 16-point Gauss-Hermite quadrature rule. See Davis and Rabinowitz [4] for further details about Gaussian quadrature.

evidence for drift over the sample period in the parameter θ. By itself, however, this finding does not mean that the marginal distribution of the mixing variable is constant over the sample. Because of the need for the normalization (10), any drift in the mixing variable that leaves its coefficient of variation unchanged will not affect the parameter θ. For instance, suppose $I = h(J)I^{\dagger}$ where h is increasing in J and $I^{\dagger}$ is distributed independently of the number of traders. In this setup the coefficient of variation of I is independent of J, and so θ will not change as J trends upwards. This kind of drift in the mixing variable has probably taken place over the sample period, but there is evidence that it cannot be too important relative to the direct expansion in the number of traders. From (7b) and (10a), the standard deviation of the daily price change can be written

$$\tilde{\sigma}_1 \equiv \sigma_1 \sqrt{E[I]} = \sqrt{(\sigma_\phi^2 + \sigma_\psi^2/J)h(J)E[I^{\dagger}]} .$$

If $h(J)$ is rapidly increasing in J, then so is $\tilde{\sigma}_1$ and this does not appear to be the case in Table I.

3.4. *A Parameterized Model*

The subinterval estimation gives encouraging results, but the method generates many parameters to estimate and interpret. This can be remedied by making full use of the theory. The Proposition of Section 2.1 specifies how the parameters of the likelihood function vary with the number of traders. To complete the model we need only to specify how the number of traders evolves as a function of time. Maximum likelihood estimation is then straightforward.

From the Proposition the first three parameters of the likelihood function can be written as the following functions of the number of traders:

$$\text{(12a)} \qquad \tilde{\sigma}_1(J) \equiv \sigma_1(J)\sqrt{E[I]} = \sqrt{\beta_1 + \frac{\beta_2}{J}}, \qquad \beta_1, \beta_2 > 0,$$

$$\text{(12b)} \qquad \tilde{\mu}_2(J) \equiv \mu_2(J)E[I] = \beta_3 J, \qquad \beta_3 > 0,$$

$$\text{(12c)} \qquad \tilde{\sigma}_2(J) \equiv \sigma_2(J)\sqrt{E[I]} = \sqrt{\beta_4 J + \beta_5}, \qquad \beta_4 > 0,$$

where from (7b) the parameters β_1 and β_2 are proportional to σ_ϕ^2 and σ_ψ^2 respectively; from (7c) the parameter β_3 is proportional to $(\alpha/2)\sigma_\psi\sqrt{2/\pi}$ (note that we use $(J-1)/J \approx 1$); finally, from (7d) the parameter β_4 is proportional to $(\alpha/2)^2\sigma_\psi^2(1-(2/\pi))$ while β_5 captures the omitted term. We assume $E[I]$ is constant and independent of J (the results of the subinterval estimation suggests that this is reasonable), and we have absorbed $E[I]$ into the β's.

We use a logistic model for the number of traders as a function of time

$$J(t) = \frac{e^{\omega t}}{1 + \delta e^{\omega t}} \tag{13}$$

where $t = 1, 2, 3, \ldots, 876$ indexes trading days and ω and δ are parameters to be estimated. The logistic model provides more flexibility than a straight exponential and was found to give a marginally better fit to the data. The absence of a multiplicative constant in (13) reflects a normalization to achieve identification. Incidentally, a random error could be incorporated into this expression. This would require, however, another numerical integration. It is not currently feasible to maximize a log-likelihood function that is the sum of the logs of 876 bivariate integrals. We believe the short-term fluctuations in J about its mean are much smaller than the short-term fluctuations in the mixing variable about its mean.

In this version of the model the daily likelihood function is

$$\bar{L}_t(\beta_1, \beta_2, \ldots, \beta_5, \theta, \omega, \delta) = L_t(\tilde{\sigma}_1(J(t)), \tilde{\mu}_2(J(t)), \tilde{\sigma}_2(J(t)), \theta)$$

where L_t is the likelihood function defined in (11). The parameters β_1, $\beta_2, \ldots, \beta_5$, δ, and ω enter the right-hand side of this expression through the functions in (12) evaluated at $J(t)$ from (13). As before, 8-point Gauss-Hermite quadrature was used to evaluate the integrals and the DFP algorithm was used to maximize the sum over the sample of the logs of the integrals. Starting values were obtained from regressions using the subinterval results in Table I. Convergence to the global maximum always obtained from other starting values as well.

Table II reports the parameter estimates for this procedure applied to the entire 876-day sample. As the theory requires, the parameters β_1 through β_4 are all significantly positive. The estimate of δ is statistically significant, though the estimate is so small that the contribution of the logistic term in (13) is minor. The

TABLE II

PARAMETERIZED MODEL

Parameter	Estimate		Functional Form
β_1	.0283	(.0023)	$\tilde{\sigma}_1(J) = \sqrt{\beta_1 + \frac{\beta_2}{J}}$
β_2	.0405	(.0093)	
β_3	.2452	(.0104)	$\tilde{\mu}_2(J) = \beta_3 J$
β_4	.0252	(.0010)	$\tilde{\sigma}_2(J) = \sqrt{\beta_4 J + \beta_5}$
β_5	−.0239	(.0035)	
θ	.4167	(.0097)	$\theta = \sqrt{\ln(1 + [\text{coef.var.}(I)]^2)}$
ω	.0042	(.00014)	
δ	.0063	(.0027)	$J(t) = \frac{e^{\omega t}}{1 + \delta e^{\omega t}}$

expression is dominated by the exponential term, which suggests a .42 per cent per day average growth rate in the number of traders. As will be shown below, this is probably somewhat of an underestimate of the actual growth rate in the number of traders.

3.5. *Model Evaluation and Residual Diagnostics*

The major difference between this model applied to the full sample and the subinterval estimation is that the theory has allowed us to compress all of the structural changes in the market's characteristics into a few elementary functions of time. It is interesting, then, to determine how well the estimated model captures the general trends over the period in price variability and the trading volume. Table III displays the relevant summary statistics for such an appraisal of the model. The first and fourth columns contain, respectively, the actual variance of daily price change, $s^2_{\Delta P}$, and the mean daily trading volume, $\bar{V}$, for eight subintervals of data which overlap by 100 days and for the entire 876-day sample. The second and fifth contain, for the sake of comparison, the averages for each interval of the predicted $\sigma^2_{\Delta P}$'s and $E[V]$'s obtained from simple trend regressions which are displayed at the bottom of the table. The third and sixth columns contain the averages of the predicted values from the estimated model.

TABLE III

SUMMARY STATISTICS ON ACTUAL AND PREDICTED VALUES

Days	$s^2_{\Delta P}$ (\$ thous.)^2	Averaged Predicted $\sigma^2_{\Delta P}$ Regression	Model	$\bar{V}$ Thous. Contracts	Average Predicted $E[V]$ Regression	Model
1–200	.056	.056	.056	.35	.40	.38
101–300	.058	.053	.047	.66	.58	.58
201–400	.066	.050	.040	1.05	.86	.87
301–500	.039	.047	.036	1.35	1.27	1.31
401–600	.022	.044	.034	1.60	1.87	1.95
501–700	.021	.041	.032	2.32	2.76	2.90
601–800	.030	.038	.031	4.05	4.07	4.23
701–876	.053	.035	.030	5.87	5.70	5.81
1–876	.046	.046	.040	2.19	2.23	2.29
R^2's:		.0058	.0055		.685	.691

Regression Equations (876 Obs.)[a]:

$$\Delta P^2 = .059 - (.306 \times 10^{-4})t, \qquad SEE = .101;$$
$$\quad (.007) \quad (.135 \times 10^{-4})$$
$$\log(V) = -1.46 + (.389 \times 10^{-2})t, \qquad SEE = .485.$$
$$\quad (.033) \quad (.648 \times 10^{-4})$$

[a] The predicted values for the volume regression were computed by assuming log-normality, which gave an adequate correction for the bias caused by using a nonlinear function of V on the left-hand side of the regression. Expermentation showed that a similar correction to a $\log(\Delta P^2)$ regression could not remove the bias adequately; hence we use the levels regression for ΔP^2.

The table also shows R^2 statistics which were computed as one minus the residual sum of squares divided by the total sum of squares about the mean of the corresponding dependent variable.

Several conclusions emerge from examination of Table III. First, the predicted values from the estimated model match reasonably closely the general trends in the data that are suggested by the regression equations. There is evidence for a very gentle decline in price variability coupled with a strong upward trend in the trading volume. The R^2's for the estimated model are very close to those for the regression equations, indicating that the model does not miss any of the important trend variation in the data. Note, though, that the R^2's for the explanation of the squared price changes are very small, less than .01, which is an indication of how extremely volatile the daily price changes are and how difficult it is to detect general trends in price variability. Comparison of the sample means of the actual and predicted values shows that the estimated model has a very slight tendency to overpredict the trading volume, while it has a more noticeable tendency to underpredict the price variability. The mean of the residuals from predicting the ΔP^2's is .062 which is roughly 13 per cent of the sample mean of the ΔP^2's. Finally, note that both the regression equation and the estimated model underpredict $\sigma^2_{\Delta P}$ at the very end of the sample. The displayed estimate of $\sigma^2_{\Delta P}$, however, is not conditional on the volume, and near the end of the sample there were a few trading days in which the volume was exceptionally heavy and the price change large in absolute value. These trading days can be thought of as days in which, by chance, the realizations of the mixing variable were exceptionally large. (Note that the model also underpredicts the volume at the end of the sample.) While this finding does cast some doubt on our assumption that the mean of the mixing variable is constant over the entire period, inspection of residual plots indicated the observations for these few days were not so extreme as to be considered outliers, and that they were consistent with the normal stochastic fluctuations in the data.

Although the estimated model provides a reasonably adequate description of the general trends in price variability and the trading volume, further diagnostic work with the model's residuals indicated that there is additional unexplained non-trend variation in the data. Specifically, we conducted two tests for autocorrelation in the model's residuals: the nonparametric runs test and the Durbin–Bartlett test based on the cumulative periodgram. For the ΔP^2 residuals, the test statistics were just under the one per cent critical points, while for the volume residuals test statistics were much further below the one per cent critical points. Inspection of the residuals' spectral densities indicated that the spectral masses are concentrated at the lower frequencies. The source of the autocorrelation is probably nondeterministic low-frequency noise in both the number of traders and the rate at which new information flows to the market (i.e., the mean of the mixing variable). There is no conceptual problem in taking this autocorrelation into account. We could specify low-order ARMA models for these noises and deduce the sample likelihood function. The computational burdens are insur-

TABLE IV

NUMBER OF TRADERS AND THEIR POSITIONS IN THE 90-DAY T-BILLS FUTURES MARKET[a]

November 30, 1977				March 30, 1979			
Number of Contracts	Number of Traders	Position (Contracts) Long	Short	Number of Contracts	Number of Traders	Position (Contracts) Long	Short
1–5	1108	1551	1448	1–4	(6070)[b]	9196	6625
6–10	137	816	877	5–10	549	3406	2927
11–24	114	1561	1574	11–24	262	4083	3339
25–50	83	2224	2656	25–50	189	5926	5097
51–150	51	3492	3650	51–150	115	8433	7922
151 +	19	5457	4902	151 +	63	21506	26668
	1512	15101	15107		7248	52550	52578

Average position[c]: 20.0 contracts per trader — 14.5 contracts per trader

[a] Data were compiled from CFTC market surveys [**9**, Table VII-C] and [**10**, Table IV-A, and the unnumbered table on p. 4]. The long and short open interests do not exactly balance because of minor reporting errors.

[b] The second survey excluded traders in the 1–4 cell. Their positions can be determined as residuals. The figure in parentheses is the authors' estimate based on a minor adjustment for different cell widths and the assumption that in the first cell the average number of contracts per trader did not change between survey dates. The assumption is reasonable for every other cell except 151 +. This average can exceed the cell's upper limit; the reason is that traders spread by holding long and short positions in contracts of different maturity dates.

[c] This is the long plus the short open interest divided by the number of traders.

mountable, however, because the nonlinearity of the model would force us to evaluate many times over an 876-dimensional integral. Nevertheless, even though we cannot correct for it, the autocorrelation is probably a very weak threat to the statistical validity of the results because the sample size is so large. Note that the standard errors for the full sample results in Table II are very small as are the standard errors for the subinterval results in Table I, where the autocorrelation is not a problem.

The model can also be evaluated by using the data in Table IV. According to these data the number of traders increased between November 30, 1977, and March 30, 1979, by the proportion $7248/1512 = 4.79$. The numerator of this ratio is contaminated by an error with an unknown standard deviation; the denominator is exact. According to the estimated logistic model (13), the number of traders increased by the proportion

$$\frac{J(815)}{J(481)} = \underset{(.10)}{3.54}$$

where 481 and 815 correspond to the survey dates and the parentheses contain the estimate's asymptotic standard deviation obtained by the Δ-method. The small standard deviation suggests that sampling error probably cannot explain the entire difference between these two figures, and we shall discuss this further in Section 5. Nonetheless, it is reassuring that the two figures are of the same order of magnitude. The model's estimate is based only on the theory and the daily price and volume data; it is completely independent of the data in Table IV.

4. THE PRICE VARIABILITY-VOLUME RELATIONSHIP

The joint probability density $f(\Delta P, V)$ of the daily price change and the trading volume contains all of the relevant information about the price variability-volume relationship. In particular, there is no need to specify *a priori* the functional form for the conditional expectation $E[\Delta P^2 \mid V]$. It can be obtained directly by numerical integration

$$E[\Delta P^2 \mid V] = \frac{\int (\Delta P)^2 f(\Delta P, V)\, d(\Delta P)}{\int f(\Delta P, V)\, d(\Delta P)}.$$

Since the parameters of the joint distribution, $\tilde{\sigma}_1$, $\tilde{\mu}_2$, and $\tilde{\sigma}_2$, drift over the sample period, the conditional expectation $E[\Delta P^2 \mid V]$ shifts through the period. Figure 1 displays the conditional expectation for five sample points. The integrals were computed with the parameters evaluated at the predicted values from the full-sample model discussed in Sections 3.4 and 3.5. (The general qualitative features of the diagram are unaffected by the previously noted slight tendency to underestimate $\sigma^2_{\Delta P}$; the bias is imperceptible on the scale of the figure.) At a fixed point in time, the figure shows that the function defined by $E[\Delta P^2 \mid V]$ is increasing in the trading volume and it is gently convex, which is in agreement with Clark's previous findings and with simple regression results for this data set. As the market expands the curve flattens out towards a straight line. This resolves the paradox generated by the data for the 90-day T-bills futures market, because the daily observations on ΔP^2 and V correspond to different points along curves like those in Figure 1.

5. FURTHER DISCUSSION AND SUGGESTIONS FOR SUBSEQUENT RESEARCH

The basic result of our theoretical work is that the variance of the daily price change and the mean daily trading volume depend upon three factors: (i) the average daily rate at which new information flows to the market; (ii) the extent

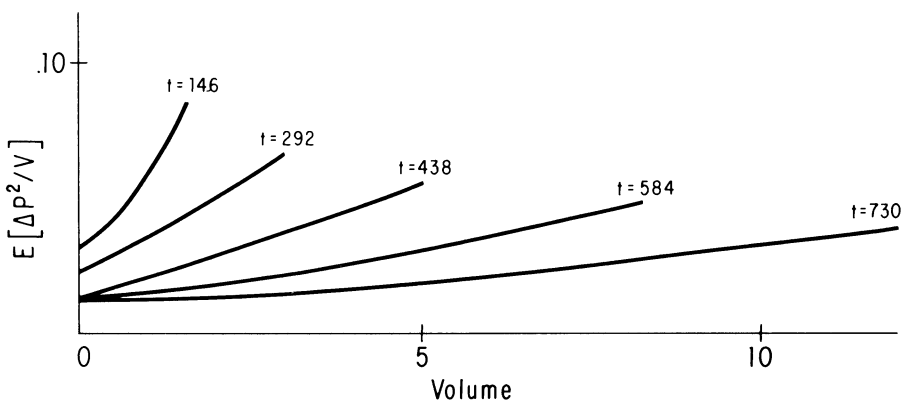

FIGURE 1

to which traders disagree when they respond to new information; and (iii) the number of active traders in the market. In our empirical work we treat, admittedly as a first approximation, factors (i) and (ii) as constant over time and focus directly on the overall expansion of the market. By doing so we can reconcile most of the stylized facts for the new and growing 90-day T-bills futures with the findings of many other studies. Nonetheless, there has to be some error introduced by holding the first two factors constant. Indeed, the autocorrelation in the model's prediction errors can in part be explained by slow-moving forces that determine the rate of flow of new information to the market.

One of the more interesting ways in which our model errs is that when its predictions are compared to the available, albeit limited, survey data (see the last part of Section 3.5), the model underestimates the proportionate increase in the number of traders between the two survey dates. The recent theoretical work by Diamond and Verecchia [5] and Grossman and Stiglitz [7] can help explain why this occurs. Specifically, these models with rational expectations, but noisy private information, suggest that as the market expands the market price becomes a more accurate predictor of the ultimate price of the contract, since the price is averaging the forecasts of more traders. As traders become aware of the increased precision of the forecast implicit in the market price, we might expect the extent to which they disagree in response to new information would decline as the market expands. The relevance of this effect is as follows. Our theoretical work shows that the average daily volume is proportional to three terms:

$$E[V] = (\text{constant}) \cdot \mu_I \sigma_\psi J$$

where μ_I is the average daily rate at which new information flows to the market, σ_ψ measures the extent to which traders disagree in response to the information, and J is the number of active traders in the market. In our empirical work we treat μ_I and σ_ψ as constants, whereas the above-cited work suggests that σ_ψ may decline with more traders. With μ_I and σ_ψ treated as constants our estimate of the proportionate increase in the number of traders is essentially the predicted proportionate increase in the average daily volume between the two survey dates. But this method gives an estimate of the proportionate increase in the number of traders that is about three-fourths of the actual proportionate increase. In other words, the actual number of traders increased proportionately more than $E[V]$.[11] Therefore the product, $\mu_I \sigma_\psi$, must have declined (by about twenty-five per cent). Now, if μ_I changes secularly over the sample period it surely trends upwards, i.e., the rate of flow of new information increases with more traders. Hence, the facts can only be reconciled if the disagreement parameter, σ_ψ, has declined somewhat over the sample period, which is consistent with implications of the other theoretical work.

[11] In particular, the importance of the smallest traders increased between the survey dates. According to Table IV, the positions of traders holding 10 or fewer contracts comprised 31 per cent of the open interest in November, 1977, and their market share rose to 42 per cent of the open interest in March, 1979. For further evidence and discussion see Jaffe and Hobson [9, p. 18].

The theoretical work on speculative trading under rational expectations has not, however, progressed to the point where effects like this can be taken into account explicitly in empirical work. What is needed is the theory of how the stochastic specification of both the rate of flow of new information and traders' reactions to the information change as more traders enter the market. In addition, it is desirable ultimately to make the number of traders endogenous, i.e., to have a theory of the trivariate joint distribution of the price change, the trading volume, and the number of traders. The Grossman and Stiglitz model is a step in this direction, but it is not yet suitable for direct application to data. The techniques we use in proving the Proposition in Section 2 may prove helpful in this endeavor, because our methods show how to derive from economic theory the likelihood function that is required for estimation.

6. CONCLUSION

Three firm conclusions emerge from our work. First, it is possible to derive from a simple economic model of speculative trading the parametric form of the joint probability distribution of the price change and the volume of trading on speculative markets. The joint distribution incorporates all relevant information about the price variability-volume relationship. Once the parameters of the distribution have been estimated by maximum likelihood, then the functional form of the conditional expectation of the squared price change given the volume is known. There is no need for numerous exploratory regressions in order to determine the correct functional form. Second, our applied work suggests that if the volume of trading is strongly trended over the sample period, then the results of a price variability-volume study can be very misleading. A sharp rise in the number of traders in the market can conceal most, and perhaps all of the relationship between the squared price change and the volume of trading. At a minimum, then, any variance-volume study should include preliminary tests for trend in the volume of trading. Finally, the diagnostic work with our model's prediction errors indicates, but by no means confirms, that there is some validity to the predictions about market expansion that are suggested by the newer rational expectations models which incorporate noisy private information. We encourage, then, further development of these models to the point where they can be applied directly to price and volume data.

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APPENDIX

The data for this paper are 876 daily observations on price change and trading volume for the 90-day T-bill futures contracts traded at the Chicago Merchantile Exchange. The contracts call for the delivery of a $1,000,000 face-value U.S. T-bill. Potential problems are caused by the exchange rules that limit the daily price change on a single T-bill contact; however, limit moves occurred only twice in the sample. The sample begins on the first day of trading, January 6, 1976, and ends on June 30, 1979. Weekends were treated like overnight periods. The exchange's formulas were used to convert the quoted interest rates into prices. Clark's [2] method was used to aggregate the prices for different delivery dates into a price for a single composite contract. All price data in this project are for this composite contract and are expressed in thousands of dollars. The trading volume is the total for all contracts and is expressed in thousands of contracts.

Data on the total number of traders in any futures market are not collected at frequent intervals. There are, nonetheless, three sources of information verifying the conclusion that the series for 90-day T-bills was strongly trended upwards during the sample period of this paper. First, when interviewed by us, members of the staffs of the Chicago Mercantile Exchange and the Commodity Futures Trading Comission found the conclusion almost incontestable. Second, since mid-1978 the CFTC [13] has reported monthly the number and aggregate positions of domestic traders holding a position that equals or exceeds 25 contracts. From July, 1978, through June, 1979, the number of these large traders increased by roughly 5 per cent per month. Third, during the sample period the CFTC conducted two market surveys which are summarized in Table IV. Unfortunately, the second survey in March, 1979, excluded traders in the 1-4 contracts cell. In the November, 1977, survey these traders comprised more than half of the market. Our estimate of the number of traders in the omitted cell is subject to error, but the number has to have gone up between the survey dates in order to account for the missing open interest. Note that we assume only that contracts per trader remain constant *within* each cell, not across the various cells.

REFERENCES

[1] CHAMBERLAIN, G.: "Analysis of Covariance with Qualitative Data," *Review of Economic Studies*, 47(1980), 225-238.

[2] CLARK, P. K.: "A Subordinate Stochastic Process Model with Finite Variance for Speculative Prices," *Econometrica*, 41(1973), 135-155.

[3] CORNELL, B.: "The Relationship Between Volume and Price Variability in Futures Markets," *Journal of Futures Markets*, 1(1981), 303-316.

[4] DAVIS, P. J., AND P. RABINOWITZ: *Methods of Numerical Integration*. New York: Academic Press, 1975.

[5] DIAMOND, D. W., AND R. E. VERRECCHIA: "Information Aggregation in a Noisy Rational Expectation Economy," *Journal of Financial Economics*, 36(1981), 221-235.

[6] EPPS, T. W., AND M. L. EPPS: "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distribution Hypothesis," *Econometrica*, 44(1976), 305-321.

[7] GROSSMAN, S. J., AND J. E. STIGLITZ: "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70(1980), 393-408.

[8] JACOBS, R. L., AND R. A. JONES: "The Treasury-Bill Futures Market," *Journal of Political Economy*, 88(1980), 699-721.

[9] JAFFE, N. L., AND R. B. HOBSON: *Survey of Interest-Rate Futures Market*. Washington, D.C.: United States Commodity Futures Trading Commission, 1979.

[10] HOBSON, R. B.: *Futures Trading in Financial Instruments*. Washington, D.C.: United States Commodity Futures Trading Commission, 1978.

[11] MOSTELLER, F.: "On Some Useful 'Inefficient' Statistics," *Annals of Mathematical Statistics*, 17(1946), 377-408.

[12] ROGALSKI, R. J.: "The Dependence of Prices and Volume," *Review of Economics and Statistics*, 60(1978), 268-274.

[13] UNITED STATES COMMODITY FUTURES TRADING COMMISSION: *Commitments of Traders in Commodity Futures with Market Concentration Ratios*; July 31, 1978 through June 30, 1979. Chicago: Commodity Futures Trading Commission.

[14] UPTON, D. E., AND D. S. SHANNON: "The Stable Paretian Distribution, Subordinated Stochastic Processes, and Asymptotic Lognormality: An Empirical Investigation," *Journal of Finance*, 34(1979), 1031-1039.

[15] WESTERFIELD, R.: "The Distribution of Common Stock Price Changes: An Application of Transactions Time and Subordinate Stochastic Models," *Journal of Financial and Quantitative Analysis*, 12(1977), 743-765.

[16] YING, C. C.: "Stock Market Prices and Volumes of Sales," *Econometrica*, 34(1966), 676-685.