

**Midterm Exam: Overnight Take Home
Three Questions
Allocated as 35, 30, 35 Points, 100 Points Total**

Due Midnight, Wednesday, October 5, 2016

Exam Rules

This exam is totally open book, including all texts, lecture notes, and previous Matlab scripts including starter scripts.

However, you must not talk or communicate with each other or anyone else about the exam or course material. This restriction includes e-mail, voice, phone, text, or any other means of communication. The answers (text and code) that you turn in must be your own work but can be based, of course, on the allowable materials described immediately above. You must also turn in your Matlab codes.

There will be a pledge/text file in your course dropbox folder that you must sign (electronically) confirming that you have not communicated with anyone or had assistance from anyone with any part of the exam. If this file is not signed then we will not grade the exam and record a zero.

Exam Notes for Students: The questions below generally start with asking you to do things you already did in the projects, so you may just reuse your code for those parts. Then the questions go on to ask about additional material not already on the projects. I suggest you first do the easier parts based on previous projects and then go back and answer the remaining questions.

Exam questions follow

1 [35 pts] Suppose it is 9:00 the morning of September 16, 2008, in the middle of the financial crisis. You work for a mutual fund manager who has a \$75 million position in a particular stock, and the manager is worried about a very large loss over the upcoming trading day starting in the morning and ending at 16:00. Specifically, the manager asks you for the probability that the value of the position (ignoring rare jumps) will decline by 2 percent or worse over the day and for the probability that the value of the position will decline by 4 percent or worse over the day. The position size of \$75 million is very realistic and the potential losses of 2 or 4 percent translate to \$1.5 and \$3.0 million, respectively. The manager is asking you for

$$\Pr(r_t^c \leq -q), \quad q = 2 \text{ or } q = 4$$

where $r_t^c = \sum_{i=1}^n r_{t,i}^c$, and $r_t^c \sim N(0, IV_t)$. September 16, 2008, corresponds to $t = 426$, but since you do not know IV_t the only choice is to use TV_{t-1} as an estimate of IV_t .

1-A [20] For your two stocks, estimate the probabilities described above and calculate confidence intervals around TV_{t-1} to compute 95 percent Lower and Upper bounds for the estimated probabilities. Set $\alpha = 4.5$ and $k_n = 11$ in the computations. Display a filled-in version of this table:

Estimate of $\Pr(r_t \leq -q)$				
Stock	q	Estimate	Lower bound	Upper bound
1	2			
1	4			
2	2			
2	4			

Comment on the estimation accuracy: Think, if the width of the confidence interval is in the range 0.01–0.06, then that level of accuracy would likely suffice for most applications. If the width is something like 0.20–0.30, then the estimates would be too imprecise for any practical application.

QUESTION 1 CONTINUED ONTO NEXT PAGE

1-B [15] (Value at Risk) Above, we fixed the loss and determined the probability of the loss. In risk management, it is common to turn the problem around by fixing the probability at some $\alpha \in [0, 1]$ and determine the associated \$ loss that could occur with the probability fixed at α .¹ Keep in mind that since r_t^c is expressed as a percent, eg, $r_t^c \pm 1$ percent or $r_t^c \pm 2$ percent, the actual \$ gain or loss on an investment of $V = \$75$ million is $\pm \frac{r_t^c}{100} V$. So we seek a number Q such that

$$\Pr\left(\frac{r_t^c}{100} \times V \leq Q\right) = \alpha.$$

Note that when $\alpha = 0.01$ then

$$\Pr(Z \leq z_{0.01}) = 0.01, \text{ and } z_{0.01} = -2.326.$$

To get the Value at Risk (VaR), we only need to proceed as in **1-A** to put the first probability above in Z terms, and then we solve for Q with $V = 75$. We would commonly denote this $\alpha = 0.01$ -level VaR on $V = 75$ as $Q_{0.01}(75)$.

Complete a table like the one below for each of your two stocks:

Estimated VaR $Q_{0.01}(75)$ at $\alpha = 0.01$ (1 percent)				
Stock	α	VaR	Lower bound VaR	Upper bound VaR
1				
2				

Comment briefly on the accuracy of the VaR estimates; would your boss find them helpful or uselessly imprecise?

¹Not to be confused with the truncation constant, e.g., $\alpha = 4.5$, that you used before to separate continuous and jump returns.

2 [30 pts] A long-short hedge fund follows a trading strategy defined by a long position in one stock and a short position in the other. The idea is that the fund has identified positive future prospects for the stock held long and negative future prospects for the stock that it short.² The rules of short trading are set up so that the return on the short stock is just the negative of the return if it were held long. Thus if $r_{t,i}^A$ is the return on stock A and $r_{t,i}^B$ the return on stock B, the return on the long-short portfolio is just

$$r_{t,i}^{LS} = r_{t,i}^A - r_{t,i}^B$$

Usually, hedge funds charge investors rather large fees, and investors thereby demand that the fund demonstrate that its strategy is market neutral, i.e., that the return has a zero beta with the market. The reason is that an investor can easily, and cheaply, earn the average market return by using a mutual fund or ETF that matches the market return, so there is no point in paying the hedge fund the fees for the market return.

2-A [15] Compute the long-short 5-min return (long stock 1, short stock 2) for the two stocks in your folder. Use your earlier script to estimate the realized beta and 95 percent confidence intervals day-by-day for the long-short return. That is the beta between the long-short return and the market return. Set $k_n = 11$ and use 100 repetitions to keep the computations of the confidence intervals doable. Make a nice plot of the realized beta and the intervals that would be suitable to show to an investor/client. Comment on whether at least visually the long-short position appears market neutral.

2-B [15] Most investors demand more than just a plot and insist on formal statistical tests. We can test the null hypothesis of zero beta at the 5 percent level by checking whether a 95 percent confidence interval includes zero or not. That is, we reject the null hypothesis if zero does not lie in the 95 percent confidence interval. Run the test day-by-day for your data set and compute the average number of rejections over all the days in your data set. A common rule is to declare a hedge fund **not** market neutral if there are 10 percent or more rejections. (Five percent would be expected by simple statistical fluctuations so the 10 percent rule adds in a small additional allowance.) Would the hedge fund holding long stock 1 and short stock 2 be considered market neutral?

²The hedge fund Greenlight Capital has generated some very high returns for investors on large long-short portfolios.

3 [35 pts] Some critics point out (correctly) that the standard jump detection method as used in the this class properly identifies valid jumps, but it can occasionally mislabels continuous returns as jumps. This mislabeling can occur during episodes of very low overall volatility, such as 2014–2015, and especially between 11:30–13:30, the low part of the volatility U-shape during the lunch hour. Generally, a return of 0.10 or less in magnitude, $\frac{1}{10}$ of one percent, that gets labeled as a jump would be considered an invalid jump. The purpose of this problem is to label the set of jumps in your two stocks as valid or not. We should not simply compare the magnitude of the observed jump return to 0.10 because that fails to account for the statistical error in estimating the jump return itself. Proceed as follows.

3-A [15] Do this for each of your two stocks: The first step is to get the confidence bounds for the jumps and keep track of the days and times. Run your jump detector (with jump threshold parameter $\alpha = 4.5$) over the data set, but exclude jumps before 10:30 and after 15:00 to avoid nuisance end-effects. Let NJ denote the number jumps for the stock, and create a matrix with NJ rows given by

$$(|r_{t,i}^d| \quad r_{t,i}^d \quad C_{L,t,i} \quad C_{U,t,i} \quad \text{YMD}_t \quad \text{TIME}_i)$$

where $C_{L,t,i}$, $C_{U,t,i}$ are the lower and upper confidence bands about the jump as determined in Project 4 with $k_n = 11$, using 100 bootstrap repetitions, and where YMD_t and TIME_i are the year-month-day and time variables available in the raw data set. Report these matrices for each of your stocks.

3-B [20] We should evaluate the statistical validity of a declared jump by checking whether its 95 percent confidence interval excludes -0.10 and 0.10 . In other words, a jump is judged valid if

$$\text{Valid Jump: } C_{U,t,i} < -0.10 \text{ or } C_{L,t,i} > 0.10.$$

Answer the following:

- i) What proportion of the jumps of each stock are valid by the above criterion?
- ii) Do the invalid jumps tend to occur in the later part of the data sets, 2014–2015, and also around the lunch time 11:30–13:30?