

Midterm Examination
Three Questions, 100 points

Note: For all questions, full credit **requires** clear explanations or derivations.

1 [30] Consider the random effects panel data model

$$y_{it} = x'_{it}\beta + u_i + \epsilon_{it}$$

where x'_{it} are strictly exogenous, ϵ_{it} are *iid* $(0, \sigma_\epsilon^2)$ for $t = 1, 2, \dots, T$, $i = 1, 2, \dots, N$, and the u_i satisfy the assumptions as given in class with $\text{Var}(u_i) = \sigma_u^2$.

1-A Briefly describe the feasible random effects estimator.

1-B Suppose we modify the problem so that the u_i are heteroskastic with variances $\text{Var}(u_i) = \sigma_{u_i}^2$. Describe the GLS estimator of β under the assumption that the $\sigma_{u_i}^2$ and σ_ϵ^2 are known.

1-C Describe a feasible GLS procedure for Part **1-B**.

2 [30] Consider the seemingly unrelated regression model

$$y_i = X_i\beta_i + \epsilon_i, \quad \text{Cov}(\epsilon_i, \epsilon_j) = \sigma_{ij}I,$$

where y_i is $T \times 1$, X_i is $T \times K$, $i = 1, 2, \dots, N$.

2-A Outline a feasible F-GLS procedure for estimation of

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{pmatrix}$$

2-B Can you do a Hausman test that the F-GLS estimator in **2-A** is statistically significantly different than the estimator obtained via equation-by-equation OLS? How would you interpret the outcome of the test?

2-C Suppose you were concerned that some of the variables in the X_i might be endogenous. Suppose also that you have additional exogenous variables so that, for each i , there is a matrix of exogenous variables Z_i , $T \times L$, $L \geq K$. Explain how to estimate β via GMM under the maintained assumptions about the error covariance structure. Hint: Start by assuming the σ_{ij} are known, derive the objective function, then proceed to a feasible version.

3 [40] Consider the equation

$$y_{1t} = \gamma_{12}y_{2t} + \gamma_{14}y_{4t} + \beta_{11}x_{1t} + \beta_{13}x_{3t} + \epsilon_{1t}$$

which is embedded in a four equation system ($M = 4$) with $K = 5$ exogenous variables.

3-A Discuss the rank and order conditions as they pertain to this equation. Be as specific as you can.

3-B How could you estimate this equation by 2SLS?

3-C How would you estimate this equation by Limited Information Maximum Likelihood (LIML)?

3-D In reference to **3-C**, how would you conduct an LM test of the null hypothesis that x_2 does not belong in the equation?