Spurious Jump Detection and Intraday Changes in Volatility*

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*The Duke Community Standard was upheld in the completion of this assignment.
1 Introduction

Recent literature suggests that discontinuities, or “jumps,” are an essential part of financial price changes. For instance, an efficient market will incorporate unanticipated news instantaneously, leading to a sudden change in the price of affected assets. Jumps introduce a new dimension to risk management, complicate the derivatives pricing problem, and make general equilibrium pricing models less tractable.

Given the theoretical relevance of jumps, it is important to be able to detect them in data. Several authors have proposed nonparametric statistical tests that determine whether a particular time interval contains a jump, or whether an individual price movement is likely to reflect a jump. Barndorff-Nielsen and Shephard (2004, 2006) distinguish between two measures of integrated volatility, one jump-robust and one not, that together offer a way to test whether a sample contains jumps. Jiang and Oomen (2008) exploit the higher-order sample moments of returns to identify periods that contain jumps, and Ait-Sahalia and Jacod (2007) examine the difference between higher-order moments computed at two different sampling frequencies. Attempting to identify whether individual price changes are jumps, Lee and Mykland (2008) compare the magnitude of each change with a sliding-window measure of local volatility.

Although these tests are all designed to distinguish jumps from the diffusive component of volatility, some recent work suggests that they produce incoherent results. Schwert (2008) finds that tests proposed by different authors identify different days that contain jumps. Even more alarming, he also finds that tests are not even consistent with themselves, detecting different jumps when the sampling frequency is adjusted. For instance, one measure derived from Barndorff-Nielsen and Shephard detects jumps on 6.9% of days at 10-minute sampling and 6.4% of days at 15-minute sampling, but only 1.21% of days at both frequencies simultaneously.

We provide a simple explanation that accounts for many of these contradictory results: dramatic intraday changes in volatility, combined with a coarse sampling frequency that distorts the asymptotic properties of our estimators, cause the jump statistic to detect jumps even in a completely diffusive price sequence. Half or more of the jumps detected in actual data appear to be artifacts of this behavior, and existing estimates of the empirical significance of jumps may be dramatically overstated.

First, in Section 2, this paper discusses standard stochastic models of stock price evolution. In Sections 3.1 and 3.2, it describes how Barndorff-Nielsen Shephard (BNS) and Jiang Oomen jump statistics are calculated. Next,
in Section 3.3 it calls upon existing literature to establish the necessity of using staggered returns to compensate for the effects of microstructure noise. Continuing the theoretical discussion of jump test statistics, it shows in Section 3.4 that many of the components of these statistics are biased in samples with large changes in volatility, possibly leading to overdetection of jumps.

Prior to examining the importance of this effect in an empirical context, Section 4 describes the details of the high-frequency pricing data that we will use. Section 5 discusses the strong intraday pattern in volatility that is evident in this data, and relates this finding to the earlier discussion about the effects of changing volatility on our test statistics. Section 6 outlines some simulations and empirical work that will test the susceptibility of the jump statistics to intraday swings in volatility, and Section 7 provides the results. Finally, Section 8 draws some general conclusions from this work.

2 Stochastic Models of Returns

Consider a standard stochastic model of stock price evolution, given by a stochastic differential equation for log-prices \( p(t) \):

\[
dp(t) = \mu(t)dt + \sigma(t)dW(t)
\]  

(1)

Here, \( \mu(t)dt \) represents the time-varying drift component of prices, while \( \sigma(t)dW(t) \) represents the time-varying volatility component, where \( W(t) \) is standard Brownian motion and \( \sigma(t) \) is the volatility level. To incorporate jumps in price into our model, we add an additional term:

\[
dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t) dq(t)
\]  

(2)

where \( q(t) \) is a counting process that increments by one with each jump and \( \kappa(t) \) gives the magnitude of each jump.

While examining high-frequency data over relatively short time intervals, the drift process is generally insignificant enough to ignore. Unfortunately, it is often difficult to distinguish the other two terms: the jump process and continuous variation. We discuss two families of statistical tests that attempt to isolate significant jumps.
3 Jump Tests

3.1 Barndorff-Nielsen Shephard Tests

Barndorff-Nielsen and Shephard (2004) propose a test that compares two measures of variance to determine whether there is a statistically significant jump component during the sample period. The first measure, Realized Variance, converges as the sample frequency approaches infinity to the integrated variance plus a jump component, while the second measure, Bipower Variation\(^1\), converges to the integrated variance alone. Formally:

\[
RV = \sum_{i=2}^{n} r_i^2 \longrightarrow \int_{0}^{T} \sigma^2(s)ds + \sum_{i=1}^{n} \kappa^2(t_i)q(t_i) \quad (3)
\]

\[
BV = \mu_{1}^{-2} \left( \frac{n-1}{n-2} \right) \sum_{i=3}^{n} |r_i||r_{i-1}| \longrightarrow \int_{0}^{T} \sigma^2(s)ds \quad (4)
\]

where \(r_i = p(t_i) - p(t_{i-1})\) is the geometric return from time \(t_{i-1}\) to time \(t_i\), and \(\mu_{1} = E(|Z|^\alpha)\) for \(Z \sim N(0,1)\).

Clearly, the asymptotic difference between \(RV\) and \(BV\) will the jump component of variation. To test the significance of the detected jump component, however, we need to find the conditional standard deviation, which requires the Integrated Quarticity \(\int_{0}^{T} \sigma^4(s)ds\). Anderson, Bollerslev, and Diebold (2007) propose using the realized Tripower Quarticity statistic to estimated integrated quarticity, while Barndorff-Nielsen and Shephard (2004) suggest the Quadpower Quarticity estimator:

\[
TP = n\mu_{4/3}^{-3} \frac{n-1}{n-3} \sum_{i=4}^{n} |r_i|^{4/3}|r_{i-1}|^{4/3}|r_{i-2}|^{4/3} \longrightarrow \int_{0}^{T} \sigma^4(s)ds \quad (5)
\]

\[
QP = n\mu_{4}^{-4} \frac{n-1}{n-4} \sum_{i=5}^{n} |r_i||r_{i-1}||r_{i-2}||r_{i-3}| \longrightarrow \int_{0}^{T} \sigma^4(s)ds \quad (6)
\]

Combining our estimates of integrated variance and integrated quarticity, we can make several possible test statistics. According to simulations by Huang and Tauchen, however, the following max-adjusted statistics

\[^{1}\text{The formula we give here is slightly different from the typical formula for Bipower Variation, with } n \text{ and } n-1 \text{ having been replaced by } n-1 \text{ and } n-2, \text{ respectively. The two expressions are asymptotically equivalent, but we choose the latter because it is more natural in the finite-sample context and is unbiased in the important limiting case of constant volatility. We make similar modifications to several other statistics.}\]
(which are asymptotically standard normal) perform best:

$$Z_{RJ,TP} = \frac{RJ}{\sqrt{\left(\frac{\pi}{2}\right)^2 + \pi - 5} \left(\frac{1}{n}\right) \max \left(1, \frac{TP}{BV}\right)}$$  \hspace{1cm} (7)$$

$$Z_{RJ,QP} = \frac{RJ}{\sqrt{\left(\frac{\pi}{2}\right)^2 + \pi - 5} \left(\frac{1}{n}\right) \max \left(1, \frac{QP}{BV}\right)}$$  \hspace{1cm} (8)$$

$$Z_{log,TP} = \frac{\log(RV) - \log(BV)}{\sqrt{\left(\frac{\pi}{2}\right)^2 + \pi - 5} \left(\frac{1}{n}\right) \max \left(1, \frac{TP}{BV}\right)}$$  \hspace{1cm} (9)$$

$$Z_{log,QP} = \frac{\log(RV) - \log(BV)}{\sqrt{\left(\frac{\pi}{2}\right)^2 + \pi - 5} \left(\frac{1}{n}\right) \max \left(1, \frac{QP}{BV}\right)}$$  \hspace{1cm} (10)$$

where the relative jump statistic $RJ$ is defined as $\frac{RV - BV}{RV}$. Among these, Huang and Tauchen identify $Z_{RJ,TP}$ as the statistic with the best finite-sample properties. We test the null hypothesis that a sample period contains no jumps using a standard $z$-test.

### 3.2 Jiang-Oomen Tests

Jiang and Oomen (2008) propose a jump detection scheme motivated by the literature on variance swaps. The test relies on a statistic they called *Swap Variance*, given by:

$$SwV = 2 \sum_{i=2}^{n} (R_i - r_i)$$  \hspace{1cm} (11)$$

where $r_i$ is the geometric return defined before and $R_i$ is the arithmetic return $\frac{P(t_i) - P(t_{i-1})}{P(t_{i-1})}$. Jiang and Oomen then use the difference between Swap Variance and Realized Variance as the basis of their test statistic. Effectively, this exploits the higher-order moments of returns to identify discontinuous movements in the price series:

$$SwV - RV = \frac{1}{3} \sum_{i=2}^{n} r_i^3 + \frac{1}{12} \sum_{i=2}^{n} r_i^4 + \ldots$$  \hspace{1cm} (12)$$
With a fine enough sampling frequency, jumps will cause a detectable increase in the value of this statistic, because high \( r_i \) values caused by discontinuities in the price process will be amplified by the larger exponents in the expansion.

To achieve a test statistic with an asymptotically standard normal distribution, we need to compute a scaling factor that depends on the Integrated Sexticity:

\[
\Omega_{SwV} = n^2 \frac{\mu_6}{9} \frac{(n-1)\mu_4^2}{n-4} \sum_{i=5}^{n} |r_i|^{3/2} |r_{i-1}|^{3/2} |r_{i-2}|^{3/2} |r_{i-3}|^{3/2}
\]  \hspace{1cm} (13)

From here, we can formulate several \( z \)-statistics that test the null hypothesis of no jumps in a sample period:

\[
JO_{\text{Diff}} = \frac{n}{\sqrt{\Omega_{SwV}}} (SwV - RV)
\]
\[
JO_{\text{Log}} = \frac{nBV}{\sqrt{\Omega_{SwV}}} (\log(SwV) - \log(RV))
\]
\[
JO_{\text{Ratio}} = \frac{nBV}{\sqrt{\Omega_{SwV}}} \left( 1 - \frac{RV}{SwV} \right)
\]

Jiang and Oomen provide Monte Carlo finite sample experiments that suggest that the \( JO_{\text{Ratio}} \) statistic is best. Again, we test the null hypothesis of no jumps using a standard \( z \) test.

### 3.3 Accounting for Market Microstructure Noise

Our data on stock prices is imperfect. It does not exactly reflect the fundamental values given by the models in Section 2, and instead contains an element of microstructure noise. Mathematically, the observed log-price \( p^*(t) \) is given by:

\[
p^*(t) = p(t) + \epsilon_t
\]  \hspace{1cm} (17)

where \( \epsilon_t \) represents a short-term deviation from the fundamental price \( p(t) \).

This noise can distort the results of jump tests. In particular, if it is i.i.d, we will see negative serial correlation of returns at very high frequencies. If an unusually high \( \epsilon_t \) causes the observed price to display positive returns in one period, the price is more likely to decrease in the next period, as \( \epsilon_{t+1} \) will probably be lower than \( \epsilon_t \).

Anderson, Bollerslev, and Diebold (2004) suggest breaking this correlation by staggering returns: using only
price data from every $N$ minutes, even when observed prices are available at intervals of 1 minute or less. Haung and Tauchen (2005) confirm that this procedure makes jump tests more robust to microstructure noise. As a general approach to market microstructure noise, Anderson, Bollerslev, Diebold, and Labys (2000) recommend volatility signature plots, which display how the average realized variance corresponds to the sampling frequency. At small intervals, realized variance will be high due to microstructure noise, inflated by changes caused by the $\epsilon_t$ term. (In fact, if noise is i.i.d, realized variance will go to infinity as sampling becomes arbitrarily fine.) This effect will diminish as the sampling interval increases, and we can balance the objectives of robustness to microstructure noise and preserving the asymptotic properties of our estimators by choosing the interval where variance appears to stabilize. Using this technique, we will find that an interval of 15 minutes appears to be optimal for the stocks in our sample, and use it in our subsequent analysis.

3.4 The Effects of Dynamic Volatility

Consider the expression for bipower variation over finitely spaced price data:

$$BV = \mu_1^{-2} \left( \frac{n-1}{n-2} \right) \sum_{i=3}^{n} |r_i||r_{i-1}|$$ (18)

Say that the sequence of geometric returns is generated by a diffusive process with deterministic volatility $\sigma(t)$. Then we can write:

$$BV = \mu_1^{-2} \left( \frac{n-1}{n-2} \right) \sum_{i=3}^{n} |Z_i||Z_{i-1}|\sigma_i\sigma_{i-1}$$ (19)

where $Z_i$ and $Z_{i-1}$ are i.i.d standard normal random variables, and $\sigma_i = \int_{t_{i-1}}^{t_i} \sigma(s)ds$. Recalling that $E[|Z_i|] = \mu_1$ by definition, taking expectations we find:

$$E[BV] = \mu_1^{-2} \left( \frac{n-1}{n-2} \right) \sum_{i=3}^{n} E[|Z_i||Z_{i-1}|]\sigma_i\sigma_{i-1}$$ (20)

$$= \frac{n-1}{n-2} \sum_{i=3}^{n} \sigma_i\sigma_{i-1}$$ (21)
Now, applying the arithmetic-geometric mean inequality, we find:

\[ E[BV] = \frac{n-1}{n-2} \sum_{i=3}^{n} \sigma_i \sigma_{i-1} \quad (22) \]

\[ \leq \frac{n-1}{n-2} \left( \frac{\sigma_2^2 + \sigma_n^2}{2} + \sum_{i=3}^{n-1} \sigma_i^2 \right) \quad (23) \]

\[ = \frac{n-1}{n-2} \left( E[RV] - \frac{\sigma_2^2 + \sigma_n^2}{2} \right) \quad (24) \]

\[ = E[RV] + \frac{n-1}{n-2} \left( \frac{E[RV]}{n-1} - \frac{\sigma_2^2 + \sigma_n^2}{2} \right) \quad (25) \]

Equality will only hold when all \( \sigma_i \) are the same, and the inequality will be more pronounced as variation in \( \sigma_i \) increases. From the last line, we see that as long as \( \sigma_i \) are not all the same, the expected value of \( BV \) is less than the expected value of \( RV \) plus a boundary term. The boundary term will be positive if the average of square volatility in the first and last periods is less than the average of square volatility by period throughout the sample, and negative otherwise. Thus, if volatility tends to be higher at the beginning and end of a day, the gap between expected \( BV \) and \( RV \) over that day will be even larger than the first inequality suggests.

Recall that \( RV \) and \( BV \) are both estimators of integrated variance, and in a diffusive price process, they will asymptotically give the same quantity. Still, as long as volatility is not constant and the boundary term is not too positive, in finite samples the expected value of \( BV \) will be less than the expected value of \( RV \), skewing the distribution of the BNS test statistics and biasing the test in favor of finding jumps where none actually exist.

Using the same inequality argument, we can conclude that our jump-robust estimators of quarticity and sexticity, which rely on multiplying consecutive geometric returns to dampen the effect of a jump in one period, will also be biased downward and affected by a similar boundary term. This may further damage the BNS test statistic, since either tripower quarticity or quadpower quarticity is used in the denominator. The effect on the denominator, however, is not clear, because in the forms we examine it also contains a \( BV^2 \) term, which will also be downwardly biased and may cancel out some or all of the downward bias from \( TP \) or \( QP \).

Similarly, we may expect problems with the Jiang-Oomen statistics, although the exact effect is again unclear. In all cases, the \( \Omega_{SwV} \) statistic in the denominator, which relies on a sexticity estimate, is likely to be biased downward. With the log and ratio statistics, however, downward bias of the \( BV \) term in the numerator may
counteract this effect, leaving the overall change unclear. Explicit computation is necessary to determine the comparative magnitudes of these biases and their cumulative effect.

4 Data

The stock whose results we examine in most detail will be JP Morgan (JPM), which displays intraday volatility behavior representative of high-volume securities in general. We also perform our tests on simulations using data from a sample of stocks chosen for their breadth, high volume, and high market capitalization: Coca-Cola (KO), Exxon Mobil (XOM), Intel (INTC), Microsoft (MSFT), and Wal-Mart (WMT).

The price data were obtained from price-data.com, a commercial data vendor, and include every minute from 9:35 AM to 4:00 PM on trading days from 1997 to early 2009. The number of trading days actually included in the sample for each stock ranges from 2264 to 2924 days; the smallest value comes from Exxon Mobil, which is only recorded after its 1999 merger.

5 Intraday Pattern in Volatility

Although volatility is generally considered to be a long-memory process, spot volatility undergoes a dramatic pattern during most days: it starts high, moves down by a factor of two or more to a midday trough, and then moderately increases before the end of the day. Figure 1 displays the average absolute geometric return for each minute during the trading day in the JPM data, which is representative of the pattern in most high-volume securities.

As mentioned in the previous section, rapid changes in volatility during the sample period will bias several statistics ($BV$, $TP$, $QP$, and $\Omega_{SwV}$) downward, particularly if boundary terms like the one in Equation 25 are negative. Figure 1 makes clear that these terms will indeed be negative: volatility is higher at the beginning and end of each day than in the sample as a whole.
6 Measuring the Effect of Dynamic Volatility

To investigate the effect of these large intraday swings in volatility, we carry out simple simulations of a diffusive price process. In one simulation, volatility is assumed to be constant throughout the day, and in another, it is determined by the minute-by-minute volatility pattern of the stock in question. Neither simulation incorporates any other dynamics—leverage effects, feedbacks, or jumps—into the volatility process. The geometric returns at each minute are simply generated as random normal variables, with standard deviation scaled in the second case by the intraday volatility pattern.

We generate 100,000 “trading days,” with 385 minutes each, of simulated data for both types of volatility process, and apply all seven tests listed earlier—four variants of the BNS test and three Jiang-Oomen tests—to examine each day for the presence of a jump. We also apply the tests directly to the actual data, and compare the fraction of jumps detected with the fraction in the simulated data. In the process, we maintain data on the components of the jump tests: the average values of RV, BV, and other estimators, along with the test statistics themselves. To see how our results change with different sampling frequencies, we repeat the jump tests at all sampling intervals from length 1 minute to 30 minutes. We keep the probability space the same: we are simply performing different tests on the same simulated (and real) pricing data.

7 Results

In Table 1, we see the results from running several jump tests on diffusively simulated data using 15-minute intervals, and how these jump frequencies compare to the fraction of jumps detected in the actual price data. We find that the relative jump BNS test using tripower quarticity is the least likely to produce Type I error, which is consistent with the Monte Carlo results of Huang and Tauchen. As predicted by our earlier theoretical discussion, including the intraday volatility pattern in our simulation significantly increases the likelihood of Type I error. Under most tests, the fraction of spurious jumps detected in the simulated data is half or more of the fraction detected in the actual data. In particular, with the $Z_{RJ,TP,0.99}$ statistic, the number of “jumps” in the simulated data is approximately 55% of the number of jumps detected in the real data, suggesting that many of the jumps identified in empirical studies that use this test are artifacts of the data.
Table 10 illustrates how the different components of the BNS and Jiang Oomen jump tests are affected by the introduction of dynamic volatility. We compare the mean estimates from our simulated data to the true expected values of these components, which are easily estimated from the volatility process used in our simulation. As predicted, since our simulation consists of diffusive price movements, the realized variance statistic is nearly unbiased for integrated volatility in both cases, with a very small mean percentage error. Bipower variation, however, is strikingly affected by the introduction of intraday volatility patterns: while it is unbiased with static volatility, after the pattern is added it displays a bias of nearly negative 4 percent. For comparison, in the actual data, total BV is 6.61% less than total RV. Meanwhile, the quarticity estimates are even more significantly affected, as TP and QP are biased downward by 19% and 26%, respectively. We can see how this biased estimation affects the sample distribution of z-scores in Figure 1, which shows superimposed kernel density plots of z-scores both with and without the intraday pattern included in the simulation. As is apparent, the intraday volatility dynamics cause the z-score distribution to widen, as negatively biased quarticity estimates make the denominator of the statistic smaller, and move to the right, as negatively biased bipower variation leads the numerator to have a mean greater than zero. Figure 3 illustrates the increasing negative bias of BV as the length of the sampling interval increases.

Returning to table 1, we see that an even higher fraction of jumps detected by the Jiang Oomen tests can be explained by a diffusive price process following an intraday volatility pattern. Consistent with the results of Jiang and Oomen (2008), we find that the JOratio test is least likely to produce Type I error. The fraction of jump days detected by this test on the scaled, simulated data is almost two-thirds of that detected in the actual data, implying that only a minority of “jump” days detected in the real data are likely to contain actual, statistically significant discontinuous movements in price.

In table 10, we observe results for several more stocks. The table displays the BNS and Jiang Oomen test variants least likely to produce Type I error, and identified in the literature for the best finite sample properties, ZRJ,TP and JOratio. The tests are performed on simulated data scaled by the volatility pattern of each respective stock, and on the real data from each stock itself. Our results are broadly consistent for all stocks: again, for the BNS test, the fraction of jumps identified in the simulated data is roughly half the fraction in the actual data, and for the Jiang Oomen test the ratio is roughly two-thirds.
8 Conclusion

Motivated by recent research that identifies incoherent results from common jump tests, we demonstrate that the finite sample properties of these tests are not robust to large intraday swings in volatility. At a sampling interval (15 minutes) chosen to limit the effects of market microstructure noise in practical applications, Monte Carlo simulations reveal that a pure diffusion process scaled by the intraday volatility pattern produces data with one-half to two-thirds the fraction of jump days—depending on the test—obtained from the actual data. The full-sample difference between realized variance and bipower variation in the simulation, often interpreted as an indicator of the size of the jump component of volatility, is also more than half its value in the real data. Existing estimates of jumps that rely on these tests are therefore likely to be significantly overstated.

It is important to note that the weaknesses of our jump tests are not the consequence of intraday volatility patterns per se, but rather of a volatility process that displays intraday swings of sufficient magnitude. Any additional changes in volatility will cause further distortion to the size of our test statistics; as a result, our results likely represent a lower bound on the extent to which currently detected jumps are spurious. This poses a serious problem for any attempt to determine whether outliers in price movement data arise from complicated volatility dynamics or jumps in the price process itself. As we have seen, a higher rate of change in the volatility process directly inflates the fraction of spurious jumps detected in the data, and our current set of tools is therefore liable to confuse the two phenomena.

Overall, these results emphasize the importance of looking beyond the asymptotic properties of statistical tests to determine the reality of finite sample application. Despite the impressive high-frequency data at our disposal, financial markets are not yet liquid enough for asymptotic results to carry much practical meaning, as microstructure noise forces to limit our sampling frequency to less impressive levels.

References


9 Figures

Figure 1: Intraday Volatility Pattern for JPM

![Intraday Volatility Pattern for JPM](image)

Figure 2: Kernel Density Estimates for $Z_{RJ,TP}$ Test Statistic With and Without Intraday Pattern

![Kernel Density Estimates for $Z_{RJ,TP}$ Test Statistic](image)

Figure 3: Negative Bias of Bipower Variation as an Estimator of Integrated Variance: Simulated Data with Intraday Pattern

![Negative Bias of Bipower Variation as an Estimator of Integrated Variance](image)
## Tables

All tables display statistics that use 15-minute sampling intervals.

### Table 1: Fraction of Days Flagged as Jumps: JPM

<table>
<thead>
<tr>
<th>Test</th>
<th>Simulated, No Pattern</th>
<th>Simulated With Pattern</th>
<th>Actual Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{RJ,TP,0.99}$</td>
<td>0.0144</td>
<td>0.0328</td>
<td>0.0592</td>
</tr>
<tr>
<td>$Z_{RJ,TP,0.999}$</td>
<td>0.0018</td>
<td>0.0073</td>
<td>0.0151</td>
</tr>
<tr>
<td>$Z_{RJ,QP,0.99}$</td>
<td>0.0149</td>
<td>0.0347</td>
<td>0.0626</td>
</tr>
<tr>
<td>$Z_{RJ,QP,0.999}$</td>
<td>0.0021</td>
<td>0.0073</td>
<td>0.0188</td>
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<tr>
<td>$Z_{log,TP,0.99}$</td>
<td>0.0329</td>
<td>0.0621</td>
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<td>0.0110</td>
<td>0.0270</td>
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<td>0.0334</td>
<td>0.0639</td>
<td>0.1023</td>
</tr>
<tr>
<td>$Z_{log,QP,0.999}$</td>
<td>0.0114</td>
<td>0.0284</td>
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</tr>
<tr>
<td>$JO_{diff,0.99}$</td>
<td>0.0616</td>
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</tr>
<tr>
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<td>$JO_{ratio,0.999}$</td>
<td>0.0174</td>
<td>0.0400</td>
<td>0.0626</td>
</tr>
</tbody>
</table>

### Table 2: Percentage Difference Between Average Daily Sample Statistics, from Simulated Data, and True Values of Estimated Quantities: JPM

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Without Intraday Pattern</th>
<th>With Intraday Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV</td>
<td>0.11%</td>
<td>0.09%</td>
</tr>
<tr>
<td>BV</td>
<td>0.30%</td>
<td>-3.77%</td>
</tr>
<tr>
<td>TP</td>
<td>0.45%</td>
<td>-18.82%</td>
</tr>
<tr>
<td>QP</td>
<td>0.62%</td>
<td>-25.51%</td>
</tr>
</tbody>
</table>
Table 3: Fraction of Jump Days Detected in Simulated Data with Intraday Pattern and in Real Data, for Various Stocks

<table>
<thead>
<tr>
<th>Stock</th>
<th>$Z_{RJ,TP,0.99}$</th>
<th>$J_{O_{ratio,0.99}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTC (simulated)</td>
<td>0.0302</td>
<td>0.0669</td>
</tr>
<tr>
<td>INTC (actual)</td>
<td>0.0510</td>
<td>0.0970</td>
</tr>
<tr>
<td>JPM (simulated)</td>
<td>0.0328</td>
<td>0.0761</td>
</tr>
<tr>
<td>JPM (actual)</td>
<td>0.0592</td>
<td>0.1098</td>
</tr>
<tr>
<td>KO (simulated)</td>
<td>0.0267</td>
<td>0.0568</td>
</tr>
<tr>
<td>KO (actual)</td>
<td>0.0578</td>
<td>0.0971</td>
</tr>
<tr>
<td>MSFT (simulated)</td>
<td>0.0301</td>
<td>0.0634</td>
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<tr>
<td>MSFT (actual)</td>
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<td>0.0979</td>
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<td>WMT (simulated)</td>
<td>0.0262</td>
<td>0.0600</td>
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<tr>
<td>WMT (actual)</td>
<td>0.0592</td>
<td>0.0904</td>
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<tr>
<td>XOM (simulated)</td>
<td>0.0258</td>
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<tr>
<td>XOM (actual)</td>
<td>0.0477</td>
<td>0.0888</td>
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