

integrated variance, the forecasting power of alternative integrated variance measures can be assessed. Although proposing and implementing sensible economic metrics is a difficult task in general, we believe that this is an important hurdle to overcome for the literature on integrated variance estimation.

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# Comment

## Torben G. ANDERSEN

Department of Finance, Kellogg School, Northwestern University, Evanston, IL 60208, and National Bureau of Economic Research ([t-andersen@kellogg.northwestern.edu](mailto:t-andersen@kellogg.northwestern.edu))

## Tim BOLLERSLEV

Department of Economics, Duke University, Durham, NC 27708, and National Bureau of Economic Research ([boller@econ.duke.edu](mailto:boller@econ.duke.edu))

## Per Houmann FREDERIKSEN

Department of Marketing and Statistics, Aarhus School of Business, Fuglesangs Alle 4, 8210 Aarhus V, Denmark ([phf@asb.dk](mailto:phf@asb.dk))

## Morten Ørregaard NIELSEN

Department of Economics, 482 Uris Hall, Cornell University, Ithaca, NY 14853 ([mon2@cornell.edu](mailto:mon2@cornell.edu))

### 1. INTRODUCTION

The article by Hansen and Lunde (henceforth HL) provides an excellent introduction to, overview of, and synthesis of the recent literature on estimating financial return variability from high-frequency data in the presence of market microstructure noise in the observed prices. In addition, importantly, it goes further by extending the existing procedures in important directions, providing abundant empirical evidence in support of the main assertions. The latter is perhaps the most compelling aspect of the work, because it directly pinpoints limitations in currently popular approaches. Among the features stressed are findings suggesting a negative correlation between the underlying "efficient" price and the noise component, time variation in the size of the noise component, and temporal dependence in the noise process. Finally, the magnitude of the noise component has decreased dramatically in recent years and is now quite small relative to the daily return variation for the liquid stocks investigated. The most striking implication of this is that the so-called "volatility signature plot" generally will not diverge to infinity as the sampling frequency increases toward tick-by-tick transaction prices or quotations. In fact, for some

relevant sampling schemes, the signature plot drops off rather than explodes near the origin, implying that the presence of microstructure noise *lowers* the return variation estimated from ultra-high-frequency observations. This is evidently problematic for procedures that rely on an asymptotic theory stipulating that the noise term will be dominant in the limit (for ever-higher sampling frequencies) and thus allows for direct identification and estimation of the variance of the noise term through the use of subsampling schemes. The time series dependence of the noise process further complicates the practical implementation of such strategies. HL provide a foundation for systematic discussions of such issues and further seek to explain when the pertinent theoretical assumptions provide a sensible and reliable guide for real-world applications. This careful balancing of theoretical and empirical perspectives produces a comprehensive exposition of where the literature has originated, as well as insightful suggestions for directions that should be pursued going

forward. In particular, they explain why a move toward accommodating time-varying and dependent noise processes and correlation among the noise and efficient price processes appears to be critical.

The main purpose of this comment is to emphasize an additional important aspect of the underlying “efficient” price process that has been almost entirely ignored in this literature, although both theory and empirical evidence strongly suggest it should be present—namely, the presence of discrete jumps or discontinuities in the price path. From a theoretical perspective, prices should jump when significant new information is made public. Examples of this include regularly scheduled macroeconomic news announcements, as well as firm-specific dividend and earnings announcements. Such behavior is fully consistent with, and arguably even implied by, the standard no-arbitrage semimartingale framework for modeling asset price dynamics (see, e.g., Back 1991). From the empirical side, there is ample recent evidence that jumps are prevalent in asset return series, as has been documented by, among others, Andersen, Benzoni, and Lund (2002), Andersen, Bollerslev, and Diebold (2005), Andersen, Bollerslev, and Dobrev (2005), Andersen, Bollerslev, Frederiksen, and Nielsen (2005), Andersen, Bollerslev, and Huang (2005), Barndorff-Nielsen and Shephard (2004, 2006a), Bates (2000), Eraker (2004), Eraker, Johannes, and Polson (2003), Garcia, Ghysels, and Renault (2004), Huang and Tauchen (2005), Johannes (2004), Pan (2002), Schaumburg (2004), and Todorov (2005). The ramifications of jumps in the price process are manifold. Qualitatively new considerations are introduced into asset and derivatives pricing, and the consequences for portfolio selection and risk management are perhaps even more profound. Moreover, Andersen, Bollerslev, and Diebold (2005) documented that the decomposition of the realized return variation into diffusive and jump components allow for improved forecasts of future return variation. Despite these considerations, the literature on measuring return variation from high-frequency data in the presence of microstructure noise has been, as mentioned earlier, developed almost exclusively within a pure diffusive setting. The only exceptions that we know of are Oomen (2005), who adopted the view of a pure jump framework, and Huang and Tauchen (2005), who studied the impact of iid noise on the high-frequency return-based jump detection techniques proposed by Barndorff-Nielsen and Shephard (2004, 2007). But neither of these articles contended directly with the problems of estimating the diffusive component of the return variation in the presence of market microstructure noise, which is the main theme of HL. It is evident that the focus on the diffusive case has allowed for important progress, but, given the significance of the jump component for practical financial decision making and management, we find the time ripe for this branch of the literature to start developing tools for separately identifying the jump and diffusive components and also assessing the impact of jumps on the proposed estimation procedures for the diffusive return variation. In that spirit, here we propose a few simple diagnostic tools along the lines of the volatility signature plot that may prove helpful for gauging the presence and significance of jumps in the return process even in the presence of microstructure noise.

The remainder of our comment is organized as follows. We first review the concepts of realized power and bipower variation that are critical for the jump detection procedures proposed by Barndorff-Nielsen and Shephard (2004, 2007), hereinafter referred to as BNS. We then generalize the notion of a volatility signature plot to encompass realized power and bipower variation depicted against sampling frequency. As usual, such plots allow us to assess the impact of the microstructure noise on the power variation measures, but, more importantly, we may also use the joint features of the volatility and bipower signature plots to gauge whether there is evidence of a significant jump component in the return process. Finally, we propose minor alterations to these plots that should mitigate the influence of noise and hence provide a more robust indication of the strengths of the different return variation components and the distortions induced by microstructure noise. We apply these new diagnostic tools to the individual Dow Jones components used for illustration throughout by HL, namely Alcoa (AA) and Microsoft (MSFT).

## 2. REALIZED POWER AND BIPOWER VARIATION

Allowing for a more general class of arbitrage-free asset return processes than considered by HL, we stipulate that the efficient log-price evolves according to the following jump diffusive model,

$$dp^*(t) = \mu(t) dt + \sigma(t) dw(t) + \kappa(t) dq(t), \quad 0 \leq t \leq T, \quad (1)$$

where  $\mu(t)$  is a continuous and locally bounded variation process,  $\sigma(t)$  is a strictly positive stochastic volatility process with a right-continuous sample path with well-defined left limits (thus allowing for jumps in volatility that are ruled out by HL),  $w(t)$  is a standard Brownian motion, and  $q(t)$  denotes a counting process with (possibly) time-varying intensity  $\lambda(t)$ . That is,  $P[dq(t) = 1] = \lambda(t) dt$ , where  $\kappa(t) \equiv p^*(t) - p^*(t-)$  refers to the size of the corresponding discrete jumps in the logarithmic price process. The representation in (1) corresponds to the most general asset price dynamics typically contemplated in asset pricing applications, although it does not include Levy processes with infinite jump activity, which have received some attention in the recent mathematical finance literature. In the present setting, the quadratic variation (QV) for the cumulative return process,  $y^*(t) \equiv p^*(t) - p^*(0)$ , equals

$$[y^*, y^*]_t = \int_0^t \sigma^2(s) ds + \sum_{0 < s \leq t} \kappa^2(s), \quad (2)$$

where by definition the summation consists of the  $q(t)$  squared jumps occurring between time 0 and time  $t$ . Of course, in the absence of jumps, or  $q(t) \equiv 0$ , the summation vanishes, and the quadratic variation simply equals the integrated variance of the continuous sample path component, or IV in HL's notation. More generally, however, the return variation is QV, which, in the presence of jumps, is not identical to IV. Note that the drift of the return process in (1) has no impact on these theoretical return variation measures. In fact, the mean drift of the price process will, according to standard arguments, exert a negligible impact on the analysis based on very high-frequency data (see, e.g., the discussion in Andersen, Bollerslev, and Diebold

2004). Thus, without loss of generality and in line with HL, we ignore this term in the sequel, imposing  $\mu(t) = 0$ .

Following HL, let the discretely sampled intraday returns be denoted by  $y_{i,m}^* \equiv p^*(t_{i,m}) - p^*(t_{i-1,m})$ ,  $i = 1, 2, \dots, m$ . The corresponding ideal realized variation (volatility) estimator is then given by

$$RV_*^{(m)} \equiv \sum_{i=1}^m y_{i,m}^{*2}, \quad (3)$$

where we normalize the period  $[t-1, t]$  to be one time unit, referred to as a trading day. Moreover, like HL we have dropped the explicit reference to the particular trading day, and henceforth, unless otherwise noted, we measure all quantities over this  $[t-1, t]$  time interval.

It follows directly from the theory of QV that the realized variation converges uniformly in probability to the increment of the QV process as the sampling frequency of the underlying returns increases, that is,

$$RV_*^{(m)} \rightarrow QV \equiv \int_{t-1}^t \sigma^2(s) ds + \sum_{t-1 < s \leq t} \kappa^2(s) \quad \text{for } m \rightarrow \infty. \quad (4)$$

Hence, even in the ideal case of no noise in the observed prices, the realized volatility estimator will generally converge *not* to the integrated variance but rather to the QV, which includes the impact of the squared jumps that occurred over the course of the trading day.

The BNS jump detection procedure relies on the *realized standardized bipower variation* for disentangling these two components. In particular, define

$$BV_*^{(m)} \equiv \mu_1^{-2} \frac{m}{m-1} \sum_{i=2}^m |y_{i,m}^*| |y_{i-1,m}^*|, \quad (5)$$

where  $\mu_1 \equiv \sqrt{(2/\pi)} = E(|Z|)$  and  $Z$  denotes a standard normally distributed random variable. BNS then showed that, even in the presence of jumps,

$$BV_*^{(m)} \rightarrow IV \equiv \int_{t-1}^t \sigma^2(s) ds \quad \text{for } m \rightarrow \infty. \quad (6)$$

This leads directly to a consistent jump detection procedure, as, evidently,

$$RV_*^{(m)} - BV_*^{(m)} \rightarrow \sum_{t-1 < s \leq t} \kappa^2(s) \quad \text{for } m \rightarrow \infty. \quad (7)$$

Moreover, under the null hypothesis of no jumps, the appropriately scaled and studentized version of the quantity in (7) will asymptotically, for increasingly frequent sampling ( $m \rightarrow \infty$ ), be standard normally distributed,

$$m^{1/2} \frac{RV_*^{(m)} - BV_*^{(m)}}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_{t-1}^t \sigma^4(s) ds]^{1/2}} \Rightarrow N(0, 1). \quad (8)$$

Ignoring the potential complications associated with market microstructure noise, the left side of (8) thus may serve as a test statistic for the presence of jumps, except that the *integrated quarticity* that appears in the denominator is unobserved. Meanwhile, it is possible to show more generally that, in the absence of jumps and under weak auxiliary assumptions, the integrated

power of the volatility coefficient for  $p > 0$  may be consistently estimated by the corresponding standardized *realized power variation*,

$$RPV_*^{(m)}(p) \equiv \mu_p^{-1} m^{(p/2-1)} \sum_{i=1}^m |y_{i,m}^*|^p \rightarrow \int_{t-1}^t \sigma^p(s) ds \quad \text{for } m \rightarrow \infty. \quad (9)$$

where  $\mu_p \equiv E(|Z|^p)$ . Unfortunately, this estimator diverges to infinity for  $p > 2$  in the presence of jumps. Hence the resulting test statistic obtained by replacing the integrated quarticity in the denominator in (8) with the corresponding realized quarticity defined in (9) will have no power (asymptotically) to reject the null hypothesis when in fact there are jumps.

Alternatively, the integrated quarticity may be estimated consistently in a jump robust fashion from a generalization of the realized bipower variation measure by summing products of adjacent absolute returns raised to powers less than two, as in, for example, the standardized *realized tripower quarticity* measure used by Andersen, Bollerslev, and Diebold (2005),

$$TQ_*^{(m)} \equiv \mu_{4/3}^{-3} \frac{m}{m-2} \sum_{i=3}^m |y_{i,m}^*|^{4/3} |y_{i-1,m}^*|^{4/3} |y_{i-2,m}^*|^{4/3} \rightarrow \int_{t-1}^t \sigma^4(s) ds \quad \text{for } m \rightarrow \infty. \quad (10)$$

In the absence of any discrepancies between the observed and the “efficient” prices (i.e., no market microstructure noise), these results thus allow for the construction of a feasible one-sided test for the presence of jumps based on the statistic in (8) coupled with the tripower quarticity measure in (10). Meanwhile, the extensive simulation evidence given by Huang and Tauchen (2005) suggests that a better-behaved finite-sample (finite  $m$ ) test statistic is obtained by invoking a variance-stabilizing logarithmic,  $[\log(RV_*^{(m)}) - \log(BV_*^{(m)})]$  or ratio  $[RV_*^{(m)}/BV_*^{(m)}]$  transformation.

All of these developments are, of course, subject to the criticism that any noise in the observed prices may distort the inference. The question is whether it is feasible to robustify the procedures along lines similar to those proposed by HL. We take an initial look at this issue in the next section.

### 3. BIPOWER VARIATION SIGNATURE PLOTS

We have identified the realized variation and bipower variation measures as the critical elements in the proposed jump detection strategy, whereas formal inference regarding this feature also will involve the integrated quarticity, which in turn may be estimated by the appropriate realized power variation measure or the realized tripower quarticity measure. The volatility signature plot advocated by Andersen, Bollerslev, Diebold, and Labys (2000b) graphs the average sample values measured over a long time span (preferably several years) of realized volatilities (RVs) for various high-frequency return sampling frequencies. These different RV measures should, in the absence of microstructure frictions, provide reasonable estimates of the same average return variability irrespective of the

underlying sampling frequency. Meanwhile, if significant microstructure biases are present for the measures based on the highest sampling frequencies, this should be reflected in systematic deviations of the corresponding average variation measures relative to those computed from returns sampled at lower frequencies. As explained by HL, this diagnostic tool is quite useful for gauging the impact of noise in the various modified RV estimators. Here we propose a similar approach for informally gauging the properties of the bipower variation measure by plotting the sample average values over long calendar periods against the underlying return sampling frequencies. We term the resulting graphical display a *bipower signature plot*.

The comparison of the volatility and bipower signature plots over an identical time period should speak to the impact of microstructure noise on each of these measures, but, perhaps even more importantly, the discrepancy between them should reflect the magnitude of the jump component in the sample return variation as implied by the results in (7) and (8). Toward this end, Figure 1 depicts the standard volatility signature plots and the bipower volatility signature plots for AA and MSFT constructed using returns obtained from the quote midpoints for the period 1998–2002, much in line with the corresponding series of HL. In addition, for simple reference, we have superimposed the average realized variation estimate for the 30-minute frequency as a benchmark indication of an approximately unbiased measure. The use of quote midpoints is motivated by a desire to reduce the noise induced by bid–ask bounce-type effects, whereas the relatively long sample period allows for fairly precise estimates of the average sample values.

The figures are telling. First, as documented by HL, the traditional RV measure drops off at the highest frequencies, suggesting the presence of a significant *negative* correlation between the noise and the underlying efficient price. This effect turns out to be even stronger for the realized bipower variation, where the downward bias is very pronounced. Second, the bipower variation plot is uniformly below the volatility signature plot. The existence of such a gap is precisely what we would expect if jump components exerted a nonnegligible impact on the total return variation. As such, this feature, evident in both plots, is strongly suggestive of the presence of jumps in the underlying efficient price processes. Third, compared with the overall variability of the plots, the discrepancy between the two plots is

remarkably stable over the frequencies corresponding to an intraday return period of 5–30 minutes. One hypothesis is that the microstructure noise tends to impact both measures in a similar direction so that the difference between the two will tend to be more stable than each individual measure. This is obviously of interest for gauging the significance of the jump component in the overall return variability.

Although at a first glance these displays appear to be highly informative, caution is clearly called for, because the strong pattern in the bipower variation plots suggests a rather pronounced impact of the market microstructure noise, and how this affects the general features of the plots is unclear. Consequently, we next introduce new types of measures and displays designed explicitly to mitigate the effects of certain noise components.

#### 4. ROBUSTIFIED SIGNATURE PLOTS

The issue of how microstructure noise may impact the jump detection procedures was explicitly considered by analytical means by Huang and Tauchen (2005), hereinafter HT. Their general conclusion is that the noise is likely to reduce the power of the BNS jump detection tests. In other words, the magnitude of the jump component suggested by Figure 1 may well be biased in that the discrepancy between the two curves is too small. HT also provided the first systematic analysis of a simple variant of the bipower variation, which involves an additional spacing or skipping between the adjacent high-frequency returns used in the computations. This measure will tend to alleviate the impact of iid noise, because it “breaks” the first-order serial correlation induced in the observed returns. Formally, define the generalized (standardized) bipower variation measure computed with  $i$  additional skips between the intraday returns as

$$BV_i^{(m)} \equiv \mu_1^{-2} \frac{m}{m-1-i} \sum_{j=2+i}^m |y_{j,m}| |y_{j-(1+i),m}|. \quad (11)$$

Note that for  $i = 0$  this reduces to the standard bipower variation measure in (5). More generally, however, for  $i \geq 1$ , the staggered nature of the terms serves to reduce the impact of short-lived dependencies in the noise process.

Of course, the RV estimator also may be affected by microstructure noise, the main theme of HL. HL find improved

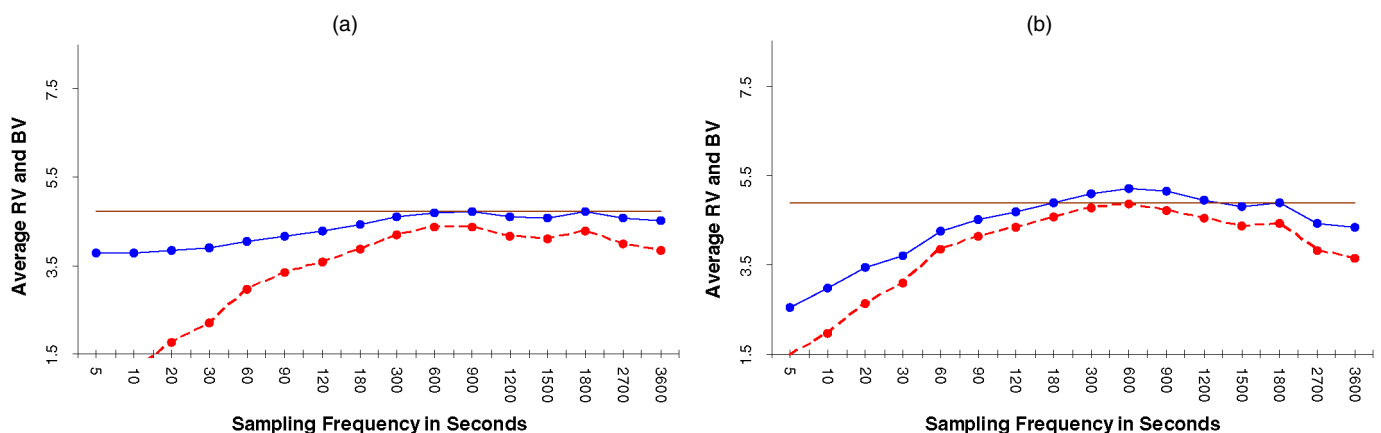


Figure 1. Bipower Variation Signature Plots for (a) AA and (b) MSFT (—●— RV; —●— RV 30 minutes; - - -●- - BV).

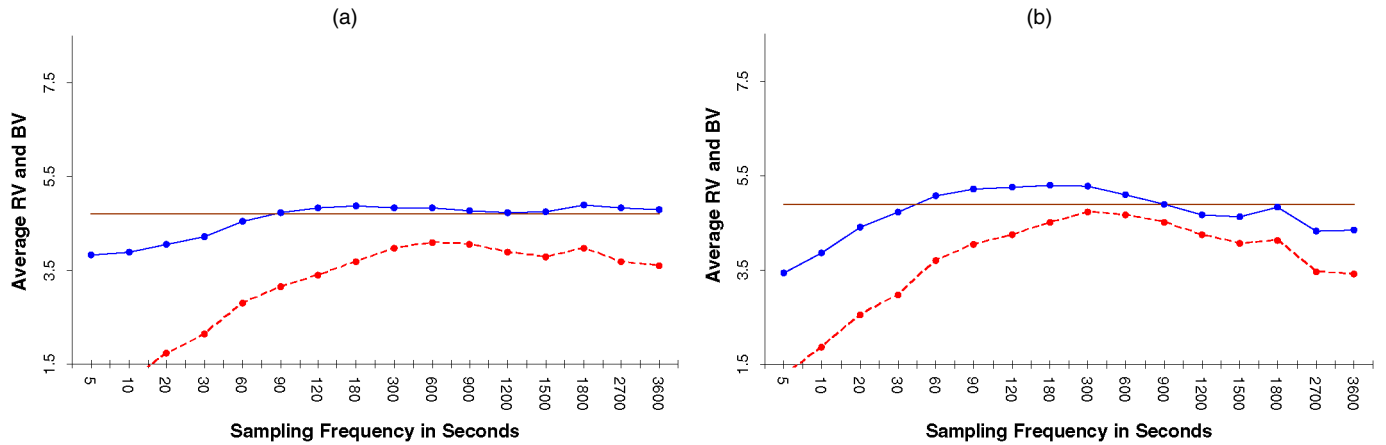


Figure 2. Robustified Signature Plots for (a) AA and (b) MSFT (—●—  $RV_{AC_1}$ ; —  $RV$  30 minutes; - -●-  $BV_1$ ).

empirical performance of the first-order serial correlation-adjusted estimator,  $RV_{AC_1}^{(m)}$ , as used by, for example, French, Schwert, and Stambaugh (1987) in estimating monthly realized volatilities from daily data and first advocated in the context of high-frequency data by Zhou (1996). We present the signature plots for both of these iid “robust” measures in Figure 2, with the generalized bipower variation computed using one skip, that is,  $i = 1$ .

The impacts of the adjustments are readily seen for both curves. In particular, for AA, the RV estimator now appears unbiased from the 60- or 90-second sampling frequency, rather than the 300-second frequency for the uncorrected estimator in Figure 1(a). The impact on the bipower variation measure is even more dramatic, because the modified staggered version delivers an estimate that is lower across the entire range of sampling frequencies. This is consistent with the upward bias in the measure resulting from microstructure noise stressed by HT. Consequently, the gap between the two estimators is now larger, but still quite stable for the frequencies below 5 minutes. Hence these displays suggest an even more important role for the jump component. The results for MSFT are qualitatively very similar, although the increase in the gap between the two curves associated with the jump component appears somewhat smaller.

## 5. POWER VARIATION SIGNATURE PLOTS

We have already alluded to the critical role of the integrated quarticity measure in drawing statistical inference regarding the realizations of the integrated variance for a given trading day. The corresponding realized quarticity measures used in practice are, of course, also affected by market microstructure noise. It is straightforward to define *power variation signature plots* that depict the sample average of the realized fourth-order power variation measure given in (9), say  $RQ_*^{(m)} \equiv RPV_*^{(m)}$  (4), against the underlying sampling frequency. Likewise, we may construct a realized tripower quarticity signature plot based on the jump-robust estimator of the integrated quarticity defined in (10). Moreover, motivated by the desire for robustness against iid noise, a staggered version of the tripower quarticity measure

is naturally defined by

$$TQ_i^{(m)} \equiv \mu_{4/3}^{-3} \frac{m}{m - 2(1 + i)} \times \sum_{j=3+2i}^m |y_{j,m}|^{4/3} |y_{j-(1+i),m}|^{4/3} |y_{j-2(1+i),m}|^{4/3}, \quad (12)$$

which may similarly be displayed in a signature plot format for different values of  $m$ .

Figure 3 presents these different (realized) integrated quarticity signature plots. First, recall that the standardized fourth-order realized power variation measure will diverge at the highest sampling frequencies in the presence of jumps. This feature is readily observed in the plots. The RQ estimator is always well above the corresponding jump robust tripower quarticity,  $TQ$ , and the divergence becomes apparent as we approach the 5-second sampling interval. These features are again much more evident for AA than for MSFT, suggestive of a relatively larger role of jumps for the former stock. Comparing the two TQ measures, we see much better coherence, although the iid noise robust version with  $i = 1$  lies almost universally below the unadjusted ( $i = 0$ ) realized tripower quarticity measure, suggesting an upward bias in the latter measure due to the presence of microstructure noise. This discrepancy is more substantial than a cursory look at the figures may indicate, because the common scale is somewhat distorted by the large values of the corresponding power variation measures. For return intervals ranging between 1 and 30 minutes, the relative difference between the two tripower measures is in fact nontrivial.

These results have important implications for the calculation of reliable standard errors for daily integrated variance estimates. The indication of a severe upward bias in the power variation measure of integrated quarticity implies that the assessment of the precision of daily variation, or volatility, estimates derived from this measure should be interpreted very carefully. Given that the evidence for the presence of both jumps and noise in the high-frequency return series is rather overwhelming at this point, it thus seems critical to apply the jump-robust estimators and also explore the iid noise—or more general noise-robust—integrated quarticity estimators when constructing standard error bands for realized volatility.



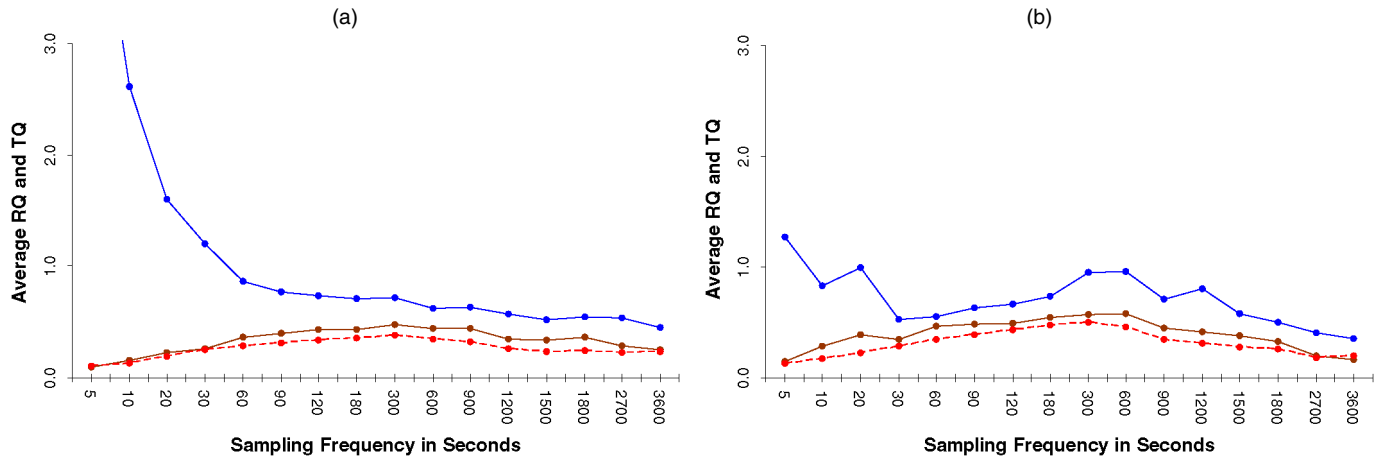


Figure 3. Power Variation Signature Plots for (a) AA and (b) MSFT (—●— RQ; —●— TQ; - - -●- - TQ<sub>1</sub>).

Of course, an additional issue of interpretation crops up as the return variation in the presence of jumps does not equal the integrated variance. Hence the appropriate approach may also involve jump-adjusting the intraday return series before computing the relevant integrated variance measures and confidence bands, along the lines of the procedures recently investigated by Andersen, Bollerslev, and Dobrev (2005) and Andersen, Bollerslev, Frederiksen, and Nielsen (2005).

## 6. CONCLUDING REMARKS

We find the article by HL extremely inspiring. We applaud the constructive elements, as well as the provocative topics raised in the course of their presentation and discussion. Our main purpose in this discussion has been to call attention to another feature of high-frequency return series with direct and important implications for the issues debated by HL—namely, the presence of jumps and their impact on the analysis of high-frequency return-based realized variation measures. We deem the evidence for jumps to be indisputable from the results both presented here and elsewhere, so the issue is mostly to assess their impact on the theory and practical procedures hitherto developed for estimating daily integrated variance series. On that front, we detect a striking impact of jumps in our generalized volatility signature plots. Furthermore, it seems evident that these types of signature plots can be quite informative as to the presence of both jumps and noise in the underlying observed return series. Up to this point, the jump component has been largely ignored in the literature on market microstructure noise-robust volatility estimation. Our simple diagnostics appear to identify systematic biases in these estimators and the associated inference stemming from the neglect of jumps. Hence the larger message is really a call for action to develop a theory for volatility estimation from high-frequency data that are robust simultaneously to noise and jumps. Currently, the only work that we know of dealing with this topic in the jump-diffusion setting is that of Huang and Tauchen (2005), whereas Large (2005) and Oomen (2005) both operated under the assumption of a pure jump process. Our introduction of generalized (and robustified) volatility signature plots presented here, and originally initiated by Andersen, Bollerslev, Frederiksen,

and Nielsen (2005), is directly inspired by this work. We feel these plots may serve as useful tools for assessing the relevance and magnitude of the different effects in the more general context, much along the lines of the currently popular volatility signature plots used extensively throughout by HL. It is worth mentioning that our exposition here has been based on simple “first-generation” RV volatility estimators. It is clearly feasible, and likely advisable, to exploit results on optimal sampling frequency and more exhaustive usage of the underlying high-frequency data through subsampling techniques or kernel-based estimators as has been advocated by, among others, Bandi and Russell (2005), Barndorff-Nielsen, Hansen, Lunde, and Shephard (2004), and Zhang, Mykland, and Ait-Sahalia (2005); see also the recent incisive survey of Barndorff-Nielsen and Shephard (2007).

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# Comment

## Ole E. BARNDORFF-NIELSEN

The T.N. Thiele Centre for Mathematics in Natural Science, Department of Mathematical Sciences, University of Aarhus, Ny Munkegade, DK-8000 Aarhus C, Denmark ([oebn@imf.au.dk](mailto:oebn@imf.au.dk))

## Neil SHEPHARD

Nuffield College, University of Oxford, Oxford OX1 1NF, U.K. ([neil.shephard@nuffield.ox.ac.uk](mailto:neil.shephard@nuffield.ox.ac.uk))

### 1. INTRODUCTION

The article by Hansen and Lunde provides an excellent window through which researchers and new students can view how econometricians have been trying to nonparametrically estimate quadratic variation (QV) in the presence of market frictions. The story that the authors tell us demonstrates an enormous leap forward in recent years, underpinned by the availability of high-frequency data. The matching of continuous-time arbitrage-free price processes with the econometrics has driven this subject and provides a solid basis for further work. Our recent survey (Barndorff-Nielsen and Shephard 2007) provided an overview of recent work that has a rather different emphasis than that provided by Hansen and Lunde.

Fascinatingly, Hansen and Lunde have shown that as theoretical research has progressed, the properties of friction on U.S. equity markets have changed due to the advent of new technology, which has reduced the tick size on many markets, dramatically reducing the variance of frictions and moderately increasing the temporal dependence. The result is that existing no-noise analysis, such as the feasible central limit theorem (CLT) introduced by Barndorff-Nielsen and Shephard (2002), appears to give pretty good predictions of finite-sample behavior when returns are measured through midpoints recorded every 10 or 20 minutes. Given that this was the goal of that research, it is rather pleasing to have it confirmed.

The current research agenda focuses on two issues:

1. Can we exploit even higher-frequency data than 10 minutes to improve the efficiency of our estimator of QV?
2. Do these methods extend to the multivariate case?

We briefly discuss both of these issues. Finally, we make some general observations on the model used for the efficient price, a Brownian stochastic volatility model, and the role of jumps.

### 2. HIGHER-FREQUENCY DATA

Hansen and Lunde suggest that two steps are involved in tackling issue 1. For returns down to the level of 1 minute, a useful approximation is that frictions are uncorrelated with the underlying price. This is a helpful simplifying assumption that appears in much recent econometric theory on this subject, including the subsamplers of Zhou (1996), Zhang, Mykland, and Aït-Sahalia (2005), Zhang (2004), and Aït-Sahalia, Mykland, and Zhang (2005b) and the general kernel approach studied by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2004). The finite-sample behavior of these estimators using 1-minute return data is unclear, however, although we expect this issue to be clarified soon.

The gains from using 1-minute data rather than 10-minute data are less than might be expected. Under the 10-minute return data, we know that the no-noise CLT provides a reasonable approximation, which says that

$$\delta^{-1/2}([P_\delta]_t - [P]_t) \xrightarrow{L} MN\left(0, 2 \int_0^t \sigma_u^4 du\right), \quad (1)$$

where  $[P]$  denotes the QV of  $P$  and  $P_\delta$  is the discretized price process updating the price every  $\delta$  units of time. Hence if the no-noise assumption were true, then moving from 10 minutes to 1 minute would reduce the variance of the estimator by a factor of 10. However, as Hansen and Lunde argue, when there is noise, one should change the estimator to make it more robust. A simple estimator for this is the kernel estimator. Barndorff-Nielsen et al. (2004) extended (1) to general kernels. In particular, in the best-case scenario for the larger data approach, if