Comment

on

Peter R. Hansen and Asger Lunde:

Realized Variance and Market Microstructure Noise

by

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1. Introduction

The paper by Hansen and Lunde (henceforth HL) provides an excellent introduction, overview and synthesis of the recent literature on estimating financial return variability from high-frequency data in the presence of market microstructure noise in the observed prices. But, importantly, it also goes further by extending the existing procedures in important directions, providing abundant empirical evidence in support of the main assertions. The latter is perhaps the most compelling aspect of the work, as it directly pinpoints limitations in currently popular approaches. Among the features stressed are findings suggesting a negative correlation between the underlying “efficient” price and the noise component, time-variation in the size of the noise component and temporal dependence in the noise process. Finally, the magnitude of the noise component has decreased dramatically in recent years, and it is now quite small relative to the daily return variation for the liquid stocks investigated. The most striking implication is that the so-called volatility signature plot generally will not diverge to infinity as the sampling frequency increases towards tick-by-tick transaction prices or quotations. In fact, for some relevant sampling schemes the signature plot drops off rather than explode near the origin, implying that the presence of the microstructure noise lowers the return variation estimated from ultra high-frequency observations. This is evidently problematic for procedures which rely on an asymptotic theory stipulating that the noise term will be dominant in the limit (for ever higher sampling frequencies) and thus allows for direct identification and estimation of the variance of the noise term through the use of sub-sampling schemes. The time series dependence of the noise process further complicates the practical implementation of such strategies. HL provide a foundation for systematic discussions of such issues and further seek to explain when the pertinent theoretical assumptions provide a sensible and reliable guide for real-world applications. This careful balancing of theoretical and empirical perspectives produces a comprehensive exposition of where the literature has originated as well as insightful suggestions for directions that should be pursued going forward. In particular, they explain why a move towards accommodating time-varying and dependent noise processes and correlation among the noise and efficient price processes appears critical.

The main purpose of this comment is to emphasize an additional important aspect of the underlying “efficient” price process that has been almost entirely ignored in this literature, although both theory and empirical evidence strongly suggest that is should be present, namely
the presence of discrete jumps or discontinuities in the price path. From a theoretical perspective, prices should jump when significant new information is made public. Examples include regularly scheduled macroeconomic news announcements as well as firm specific dividend and earnings announcements. Such behavior is fully consistent with - arguably even implied by - the standard no-arbitrage semi-martingale framework for modeling asset price dynamics, see, e.g., Back (1991). From the empirical side, there is ample recent evidence from many sources that jumps are prevalent in asset return series, as documented by, e.g., Andersen, Benzoni and Lund (2002), Andersen, Bollerslev and Diebold (2005), Andersen, Bollerslev and Dobrev (2005), Andersen, Bollerslev, Frederiksen and Nielsen (2005), Andersen, Bollerslev and Huang (2005), Barndorff-Nielsen and Shephard (2004, 2006), Bates (2000), Eraker (2004), Eraker, Johannes and Polson (2003), Garcia, Ghysels and Renault (2004), Huang and Tauchen (2005), Johannes (2004), Pan (2002), Schaumburg (2004), and Todorov (2005). The ramifications of jumps in the price process are manifold. Qualitatively new considerations are introduced into asset and derivatives pricing, and the consequences for portfolio selection and risk management are perhaps even more profound. Moreover, Andersen, Bollerslev and Diebold (2005) document that the decomposition of the realized return variation into diffusive and jump components allow for improved forecasts of future return variation. In spite of these considerations the literature on measuring return variation from high-frequency data in the presence of microstructure noise has, as mentioned, been developed almost exclusively within a pure diffusive setting. The only exceptions we know of are Oomen (2005) who adopts the view of a pure jump framework, and Huang and Tauchen (2005) who studies the impact of \textit{i.i.d.} noise on the high-frequency return based jump detection techniques proposed by Barndorff-Nielsen and Shephard (2004, 2006). Neither of these papers, however, contend directly with the problems of estimating the diffusive component of the return variation in the presence of market microstructure noise, which is the main theme of HL. It is evident that the focus on the diffusive case has allowed for important progress but given the significance of the jump component for practical financial decision making and management, we find the time ripe for this branch of the literature to start developing tools for both separately identifying the jump and diffusive components and assessing the impact of jumps on the proposed estimation procedures for the diffusive return variation. In that spirit, we shall propose a few simple diagnostic tools along the lines of the volatility signature plot which may prove
helpful for gauging the presence and significance of jumps in the return process even in the presence of microstructure noise.

The remainder of our comment is organized as follows. We first review the concepts of realized power and bipower variation which are critical for the jump detection procedures proposed by Barndorff-Nielsen and Shephard (2004, 2006), hereafter BNS. We then generalize the notion of a volatility signature plot to encompass realized power and bipower variation depicted against sampling frequency. As usual, such plots allow us to assess the impact of the microstructure noise on the power variation measures but, more importantly, we may also use the joint features of the volatility and bipower signature plots to gauge whether there is evidence of a significant jump component in the return process. Finally, we propose minor alterations to these plots that should mitigate the influence of noise and hence provide a more robust indication of the strength of the different return variation components and the distortions induced by microstructure noise. We apply these new diagnostic tools to the individual Dow-Jones components employed for illustration throughout in HL, namely Alcoa Inc. (AA) and Microsoft Corp. (MSFT).

2. Realized Power and Bipower Variation

Allowing for a more general class of arbitrage-free asset return processes than considered in HL, we stipulate that the efficient log-price evolves according to the following jump diffusive model,

\[ dp^*(t) = \mu(t) dt + \sigma(t) dw(t) + \kappa(t) dq(t), \quad 0 \leq t \leq T, \quad (1) \]

where \( \mu(t) \) is a continuous and locally bounded variation process, \( \sigma(t) \) is a strictly positive stochastic volatility process with a sample path that is right continuous and has well-defined left limits (thus allowing for jumps in volatility which are ruled out by HL), \( w(t) \) is a standard Brownian motion, and \( q(t) \) denotes a counting process with (possibly) time-varying intensity \( \lambda(t) \). That is, \( P[dq(t)=1] = \lambda(t)dt \), where \( \kappa(t) = p^*(t) - p^*(t-) \) refers to the size of the corresponding discrete jumps in the logarithmic price process. The representation in equation (1) corresponds to the most general asset price dynamics typically contemplated in asset pricing applications,
although it does not include Levy processes with infinite jump activity that have received some
attention in the recent mathematical finance literature. In the present setting, the quadratic
variation ($QV$) for the cumulative return process, $y^*(t) = p^*(t) - p^*(0)$, equals
\[
[y^*, y^*]_t = \int_0^t \sigma^2(s) \, ds + \sum_{0 < \delta \leq t} \kappa^2(s),
\]
where by definition the summation consists of the $q(t)$ squared jumps that occurred between time
0 and time $t$. Of course, in the absence of jumps, or $q(t) = 0$, the summation vanishes and the
quadratic variation simply equals the integrated variance of the continuous sample path
component, or $IV$ in the HL notation. More generally, however, the return variation is $QV$ and in
the presence of jumps this is not identical to $IV$. Notice, that the drift of the return process in
equation (1) has no impact on these theoretical return variation measures. In fact, the mean drift
of the price process will, according to standard arguments, exert a negligible impact on the
analysis based on very high-frequency data; see, e.g., the discussion in Andersen, Bollerslev and
Diebold (2004). Thus, without loss of generality and in line with HL, we ignore this term in the
sequel, imposing $\mu(t) = 0$.

Following HL, let the discretely sampled intraday returns be denoted by $y_{i,m}^* = p^*(t_{i,m}) - p^*(t_{i-1,m})$, $i=1,2,\ldots,m$. The corresponding ideal realized variation (volatility) estimator is then
given by,
\[
RV^{(\infty)}_\sigma = \sum_{i=1}^m y_{i,m}^* \sigma^2,
\]
where we normalize the period $[t-1,t]$ to be one time unit, referred to as a trading day. Moreover,
as in HL we have dropped the explicit reference to the particular trading day, and henceforth
unless otherwise noted we measure all quantities over this $[t-1,t]$ time interval.

It follows directly from the theory of quadratic variation that the realized variation
converges uniformly in probability to the increment of the quadratic variation process as the
sampling frequency of the underlying returns increases. That is,
\[
RV^{(\infty)}_\sigma \to QV = \int_{t-1}^t \sigma^2(s) \, ds + \sum_{t-1 < \delta \leq t} \kappa^2(s), \quad \text{for } m \to \infty.
\]
Hence, even in the ideal case of no noise in the observed prices, the realized volatility estimator will generally not converge to the integrated variance but to the quadratic variation, which includes the impact of the squared jumps that occurred over the course of the trading day.

The BNS jump detection procedure relies on the realized standardized bipower variation for disentangling these two components. In particular, define

\[
BV_{\tau}^{(m)} = \mu_1^{-2} \left( m(m-1) \right) \sum_{t=2}^{m} |y_{t,m}| \cdot |y_{t-1,m}|,
\]

where \( \mu_1 = \sqrt{2/\pi} = E(|Z|) \), and \( Z \) denotes a standard normally distributed random variable. BNS then show that, even in the presence of jumps,

\[
BV_{\tau}^{(m)} \rightarrow IV = \int_{t-1}^{t} \sigma^2(s) ds, \quad \text{for } m \rightarrow \infty.
\]

This leads directly to a consistent jump detection procedure, as evidently,

\[
RV_{\tau}^{(m)} - BV_{\tau}^{(m)} = \sum_{t-\Delta \tau} \chi^2(s), \quad \text{for } m \rightarrow \infty.
\]

Moreover, under the null hypothesis of no jumps the appropriately scaled and studentized version of the quantity in equation (7) will asymptotically, for increasingly frequent sampling \((m \rightarrow \infty)\), be standard normally distributed,

\[
\frac{m^{1/2} \left( RV_{\tau}^{(m)} - BV_{\tau}^{(m)} \right)}{\left[ (\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_{t-1}^{t} \sigma^4(s) ds \right]^{1/2}} \rightarrow N(0,1).
\]

Ignoring the potential complications associated with market microstructure noise, the left-hand-side of equation (8) may therefore serve as a test statistic for the presence of jumps, except that the integrated quarticity, which appears in the denominator is unobserved. Meanwhile, it is possible to show more generally, in the absence of jumps and under weak auxiliary assumptions, that the integrated power of the volatility coefficient, for \( p>0 \), may be consistently estimated by...
the corresponding standardized realized power variation,

\[
RPV_{t}^{(m)}(p) = \sqrt{\mu_p^{-1} m (\nu^2 - 1)} \sum_{i=1}^{\nu} | \gamma_{i,t^*}^{(i)} |^p + \int_{t-1}^{t} \sigma^p(s) ds, \quad \text{for } m \to \infty.
\]

where \( \mu_p \equiv E(|Z|^p) \). Unfortunately, this estimator diverges to infinity for \( p > 2 \) in the presence of jumps. Hence, the resulting test statistic obtained by replacing the integrated quarticity in the denominator in equation (8) with the corresponding realized quarticity defined in (9) will have no power (asymptotically) to reject the null hypothesis when in fact there are jumps.

Alternatively, the integrated quarticity may be estimated consistently in a jump robust fashion from a generalization of the realized bipower variation measure by summing products of adjacent absolute returns raised to powers less than two, as in, e.g., the standardized realized tripower quarticity measure employed in Andersen, Bollerslev and Diebold (2005),

\[
TQ^{(m)}_{t^*} = m \mu_{4/3}^{-3} \left( m/(m-2) \right) \sum_{i=3}^{\nu} | \gamma_{i-1,t^*}^{(i-1)} |^{4/3} | \gamma_{i-2,t^*}^{(i-2)} |^{4/3} \rightarrow \int_{t-1}^{t} \sigma^4(s) ds, \quad \text{for } m \to \infty.
\]

In the absence of any discrepancies between the observed and the “efficient” prices, i.e., no market microstructure noise, these results therefore allow for the construction of a feasible one-sided test for the presence of jumps based on the statistic in equation (8) coupled with the tripower quarticity measure in equation (10). Meanwhile, the extensive simulation evidence in Huang and Tauchen (2005) suggests that a better behaved finite sample (finite \( m \)) test statistic is obtained by invoking a variance-stabilizing logarithmic, \( \log(RV_{t}^{(m)}) - \log(BV_{t}^{(m)}) \), or ratio,

\[
RV_{t}^{(m)} / BV_{t}^{(m)},
\]

transformation.

All of these developments are, of course, subject to the criticism that any noise in the observed prices may distort the inference. The question is whether it is feasible to robustify the
procedures along lines similar to those proposed by HL. We turn towards an initial look at this issue next.

3. Bipower Variation Signature Plots

We have identified the realized variation and bipower variation measures as the critical elements in the proposed jump detection strategy, while formal inference regarding this feature also will involve the integrated quarticity, which in turn may be estimated by the appropriate realized power variation measure or the realized tripower quarticity measure. The volatility signature plot advocated by Andersen, Bollerslev, Diebold and Labys (2000) graphs the average sample values measured over a long time span - preferably several years - of realized volatilities, $RV$, for different high-frequency return sampling frequencies. These different realized volatility measures should, in the absence of microstructure frictions, provide reasonable estimates of the same average return variability irrespective of the underlying sampling frequency. Meanwhile, if there are significant microstructure biases present for the measures based on the highest sampling frequencies, this should be reflected in systematic deviations of the corresponding average variation measures relative to those computed from returns sampled at lower frequencies. As explained by HL, this diagnostic tool is quite useful for gauging the impact of noise in the various modified realized volatility estimators. Here, we propose a similar approach for informally gauging the properties of the bipower variation measure by plotting the sample average values over long calendar periods against the underlying return sampling frequencies. We term the resulting graphical display a bipower signature plot.

The comparison of the volatility and bipower signature plots over an identical time period should speak to the impact of microstructure noise on each of these measures, but perhaps even more importantly the discrepancy between them should reflect the magnitude of the jump component in the sample return variation as implied by the results in equations (7) and (8). To this end, Figures 1A and 1B depict the standard volatility signature plots as well as the bipower volatility signature plots for AA and MSFT constructed using returns obtained from the quote midpoints for the period 1998-2002, much in line with the corresponding series in HL. In addition, we have for simple reference superimposed the average realized variation estimate for the 30-minute frequency as a benchmark indication of an approximately unbiased measure. The
use of quote midpoints is motivated by a desire to reduce the noise induced by bid-ask bounce type effects, while the relatively long sample period allows for fairly precise estimates of the average sample values.

The figures are telling. First, as documented in HL, the traditional RV measure drops off at the highest frequencies, suggesting the presence of a significant negative correlation between the noise and the underlying efficient price. This effect turns out to be even stronger for the realized bipower variation, where the downward bias is very pronounced. Second, the bipower variation plot is uniformly below the volatility signature plot. The existence of such a gap is precisely what we should expect if jump components exert a non-negligible impact on the total return variation. As such, this feature evident in both of the plots is strongly suggestive of the presence of jumps in the underlying efficient price processes. Third, compared to the overall variability of the plots, the discrepancy between the two plots is remarkably stable over the frequencies corresponding to an intraday return period of between 5 and 30 minutes. One hypothesis is that the microstructure noise tends to impact both measures in a similar direction so that the difference between the two will tend to be more stable than each of the individual measures. This is obviously of interest for gauging the significance of the jump component in the overall return variability.

Although at a first glance these displays appear highly informative, caution is clearly called for as the strong pattern in the bipower variation plots suggests a rather pronounced impact of the market microstructure noise, and it is unclear how this affects the general features of the plots. Consequently, we will next introduce new types of measures and displays explicitly designed to mitigate the effects of certain noise components.

4. Robustified Signature Plots

The issue of how microstructure noise may impact the jump detection procedures is explicitly considered by analytical means in Huang and Tauchen (2005), hereafter HT. Their general conclusion is that the noise is likely to reduce the power of the BNS jump detection tests. In other words, the magnitude of the jump component suggested by Figure 1 may well be biased in that the discrepancy between the two curves is too small. HT also provides the first systematic analysis of a simple variant of the bipower variation which involves an additional spacing or
skipping between the adjacent high-frequency returns used in the computations. This measure will tend to alleviate the impact of \textit{i.i.d.} noise, as it “breaks” the first order serial correlation induced in the observed returns. Formally, define the generalized (standardized) bipower variation measure computed with \(i\) additional skips between the intraday returns as

\[
BV_i^{(m)} = \mu_i^{-2} \left( \frac{m}{m - 1 - 2i} \right) \sum_{j=2i}^{m} |y_{j,m}| |y_{j-2i,0,m}|.
\]

Notice that for \(i=0\) this reduces to the standard bipower variation measure in equation (5). More generally however, for \(i \geq 1\) the staggered nature of the terms serves to reduce the impact of short lived dependencies in the noise process.

Of course, the realized volatility estimator may also be affected by microstructure noise, the main theme of the HL paper. HL find improved empirical performance of the first-order serial correlation adjusted estimator, \(RV_{AC}^{(6)}\), as used by, e.g., French, Schwert and Stambaugh (1987) in estimating monthly realized volatilities from daily data and first advocated in the context of high-frequency data in a paper in this journal by Zhou (1996). We present the signature plots for both of these \textit{i.i.d.} “robust” measures in Figures 2A and 2B, with the generalized bipower variation computed using one skip, i.e., \(i=1\).

The impacts of the adjustments are readily seen for both curves. In particular, for AA the RV estimator now appears unbiased from the 60 or 90 second sampling frequency, rather than the 300 second frequency for the uncorrected estimator in Figure 1A. The impact on the bipower variation measure is even more dramatic, as the modified staggered version delivers an estimate that is lower across the entire range of sampling frequencies. This is consistent with the upward bias in the measure resulting from microstructure noise stressed by HT. As a consequence, the gap between the two estimators is now larger, while it is still quite stable for the frequencies below 5 minutes. Hence, these displays suggest an even more important role for the jump component. The results for MSFT are qualitatively very similar although the increase in the gap between the two curves associated with the jump component appears somewhat smaller.

5. Power Variation Signature Plots

We have already alluded to the critical role of the integrated quarticity measure for
drawing statistical inference regarding the realizations of the integrated variance for a given
trading day. The corresponding realized quarticity measures used in practice are, of course, also
affected by market microstructure noise. It is straightforward to define power variation signature
plots that depict the sample average of the realized fourth order power variation measure given in
equation (9), say \( RQ_i^{(m)} = RPP_i^{(m)}(4) \), against the underlying sampling frequency. Likewise, we
may construct a realized tripower quarticity signature plot based on the jump-robust estimator of
the integrated quarticity defined in equation (10). Moreover, motivated by the desire for
robustness against i.i.d. noise, a staggered version of the tripower quarticity measure is naturally
defined by,

\[
TQ_i^{(m)} = m^{\frac{-3}{4}} \left[ \frac{m}{m-2(1+i)} \right] \sum_{j=3+2i}^m \left| y_{j,m} \right|^{4/3} \left| y_{j-(1+i),m} \right|^{4/3} \left| y_{j-2(1+i),m} \right|^{4/3},
\]

which may similarly be displayed in a signature plot format for different values of \( m \).

Figures 3A and 3B present these different (realized) integrated quarticity signature plots.
First, recall that the standardized fourth order realized power variation measure will diverge at
the highest sampling frequencies in the presence of jumps. This feature is readily observed in the
plots. The \( RQ \) estimator is always well above the corresponding jump robust tripower quarticity,
\( TQ \), and the divergence becomes apparent as we approach the five second sampling interval.
These features are again much more evident for AA than MSFT, suggestive of a relatively larger
role of jumps for the former stock. Comparing the two \( TQ \) measures, we see much better
coherence, although the i.i.d. noise robust version with \( i=1 \) almost universally lies below the
unadjusted \( i=0 \) realized tripower quarticity measure, suggesting an upward bias in the latter
measure due to the presence of microstructure noise. This discrepancy is more substantial than it
may seem from a cursory look at the figures, as the common scale is somewhat distorted by the
large values of the corresponding power variation measures. For return intervals ranging between
one and thirty minutes the relative difference between the two tripower measures is in fact non-
trivial.

These results have important implications for the calculation of reliable standard errors
for daily integrated variance estimates. The indication of a severe upward bias in the power
variation measure of integrated quarticity implies that the assessment of the precision of daily
variation, or volatility, estimates derived from this measure should be very carefully interpreted. Given that the evidence for the presence of both jumps and noise in the high frequency return series is rather overwhelming at this point, it thus seems critical to apply the jump-robust estimators and also explore the \textit{i.i.d.} noise - or more general noise robust - integrated quarticity estimators when constructing standard error bands for realized volatility. Of course, an additional issue of interpretation crops up as the return variation in the presence of jumps does not equal the integrated variance. Hence, the appropriate approach may also involve jump-adjusting the intraday return series before computing the relevant integrated variance measures and confidence bands, along the lines of the procedures recently investigated by Andersen, Bollerslev and Dobrev (2005) and Andersen, Bollerslev, Frederiksen and Nielsen (2005).

6. Concluding Remarks

We find the paper by HL extremely inspiring. We applaud the constructive elements as well as the provocative topics raised in the course of their presentation and discussion. Our main purpose with this discussion has been to call attention to another feature of high-frequency return series with direct and important implications for the issues debated by HL, namely the presence of jumps and their impact on the analysis of high-frequency return based realized variation measures. We deem the evidence for jumps to be indisputable - both from the results presented here and elsewhere - so the issue is mostly to assess their impact on the theory and practical procedures hitherto developed for estimating daily integrated variance series. On that front, we detect a striking impact of jumps in our generalized volatility signature plots. Further, it seems evident that these types of signature plots can be quite informative about the presence of both jumps and noise in the underlying observed return series. Up until this point, the jump component has largely been ignored in the literature on market microstructure noise robust volatility estimation. Our simple diagnostics appear to identify systematic biases in these estimators and the associated inference stemming from the neglect of jumps. Hence, the larger message is really a call for action on developing a theory for volatility estimation from high-frequency data that is robust simultaneously to noise and jumps. Currently, the only work we know of dealing with this topic in the jump-diffusion setting is Huang and Tauchen (2005), while Large (2005) and Oomen (2005) both operate under the assumption of a pure jump process. Our
introduction of generalized (and robustified) volatility signature plots presented here, and originally initiated in Andersen, Bollerslev, Frederiksen and Nielsen (2005), is directly inspired by this work. We feel these plots may serve as useful tools for assessing the relevance and magnitude of the different effects in the more general context, much along the lines of the currently popular volatility signature plots used extensively throughout the HL paper. It is worth mentioning that our exposition here has been based on simple “first-generation” realized volatility estimators. It is clearly feasible, and likely advisable, to exploit results on optimal sampling frequency and more exhaustive usage of the underlying high-frequency data through sub-sampling techniques or kernel-based estimators as advocated by, e.g., Bandi and Russell (2005), Barndorff-Nielsen, Hansen, Lunde and Shephard (2005) and Zhang, Mykland and Aït-Sahalia (2005); see also the recent incisive survey in Barndorff-Nielsen and Shephard (2005).
Figures 1A and 1B: Bipower Variation Signature Plots
Figures 2A and 2B: Robustified Signature Plots
Figures 3A and 3B: Power Variation Signature Plots
References


