The Great Moderation of the Term Structure of U.K. Interest Rates*

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Abstract

This paper studies the joint evolution of monetary policy, the term structure of interest rates and the U.K. economy across policy regimes. The interaction between the macroeconomy and the term structure is modelled using a time-varying VAR augmented with the factors from the yield curve. We document a great moderation in the dynamics of the yield curve, with the factors being persistent and volatile before the introduction of inflation targeting in 1992 but becoming stable afterwards. The introduction of time-variation in the Factor Augmented VAR improves the fit of the model and makes the expectation hypothesis consistent yields close to actual yields, even at long maturities. Monetary policy shocks had a significant impact on the volatility of inflation, output and the policy rate over the pre-inflation targeting era, but their contribution has been negligible in the current regime. Shocks to the level of the yield curve accounted for a large fraction of inflation variability only before 1992.

JEL classification: E44, E52, C15.
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1 Introduction

The term structure of interest rates and the rest of the economy are intimately related. On the one hand, the expectations of future inflation and real activity can be important determinants of the yield curve. On the other hand, the yield curve can have strong predictive power for real activity and, to a lesser extent, inflation.

The strong link between the yield curve and the macroeconomy has been the common theme of a growing empirical literature. Recent examples include, Diebold, Rudebusch and Aruoba (2006) and Diebold and Li (2006), who use a generalised version of the Nelson Siegel model to show that this link is bi-directional.

Another strand of research has provided strong evidence in favor of instability in the dynamics of the US yield curve. Prominent examples include Rudebusch and Wu (2006), Diebold, Li and Yue (2006) and Cogley (2004). This evidence is compatible with a growing literature on the macroeconomic side that has shown that the dynamics of inflation and real activity were characterised by significant time variations in several industrialized economies. McConnell and Perez-Quiros (2000), Kim and Nelson (1999b), Cogley and Sargent (2005), and Sims and Zha (2006) for the US, Benati (2004) for the UK and Mumtaz and Surico (2006) for a panel of G7 countries find evidence of both variance and parameter instability. Moreover, Clarida *et al.* (2000) for the US and Nelson (2001) for the UK provide empirical support for the notion that the appointment of Paul Volcker as Fed Chairman in October 1979 and the adoption of an explicit inflation target by the British monetary authorities in December 1992 represented unprecedented monetary policy breaks.

The purpose of this paper is twofold. First, we wish to assess the extent of time variation in the dynamics of some main macroeconomic variables *and* the yield curve in the UK. Second, we are interested to investigate whether the analysis of the term structure of interest rates can shed new lights on the UK macroeconomic performance of the last fifteen years or so. It is worth emphasizing that most existing studies have analysed the dynamics of UK macroeconomy and the term structure in isolation and (to our knowledge) a systematic investigation into the *evolution* of the link between the economy and the yield curve *over time* had not yet been carried our.

We specify the link between macro and finance as in the Nelson-Siegel generalization of Diebold, Rudebusch and Aruoba (2006). To capture potential instabilities in monetary policy, the yield curve and the structure of the economy, we extend their framework using a VAR model with time-varying coefficients and stochastic volatilities, augmented with the factors from the yield curve.

The United Kingdom is a particularly interesting case as its economy has undergone significant statutory changes in the recent past: in 1992 a regime of inflation targeting was established in the aftermath of the speculative attacks that forced the sterling out of the Exchange Rate Mechanism (ERM); in 1997 the Bank of England acquired operational independence and sole responsibility for setting interest rates to meet the inflation target. The introduction of the inflation targeting framework in the U.K. coincided with a structural break in the dynamics of real GDP and inflation; during the post-1992 period, the volatilities of the business-cycle components of several macroeconomic indicators have reached the lowest values since the collapse of Bretton Woods (see Benati (2004)). These facts have been know as the Great Stability (see Bean (2005)) and identifying the source(s) of the events represent a crucial challenge for our understanding of the transmission mechanism of monetary policy through the economy and the term structure of interest rates.

Our results suggest three interesting conclusions. First, there is strong evidence that the dynamics of the yield curve have changed significantly over time. In particular, there is a large decline in the persistence and the volatility associated with the yield curve factors, with the 1990s characterised by the lowest historical values. The largest decline in the dynamics of the yield curve factors coincides with the remarkable fall in the estimated volatility and persistence of the macroeconomic variables included in our model. Second, allowing for time-variation improves the fit of the model: the expectation hypothesis consistent yields based on our time-varying model are very close to actual yields, even at long maturities. This is in contrast to the results from a fixed coefficient FAVAR, also reported in the paper, and suggests that the rejection of the expectation hypothesis documented in earlier contributions may be due, at least in part, to *omitted* parameter instability. Third, the impact of monetary policy shocks and the shocks to the level of the yield curve has varied over time. Monetary policy shocks contributed significantly to inflation, output and policy rate volatility only before the introduction of inflation targeting in 1992. In addition, there is a significant shift around 1992 in the direction of the impact of monetary policy on the level of the yield curve. Level factor shocks contributed significantly to output and inflation volatility before 1992, but modestly during the inflation targeting period. In contrast, the contribution of this shock to variation in the policy rate appears significantly higher during the current regime. Furthermore, the level factor shocks led to a significant and persistent increase in inflation in the 1970s, but to a short-lived response over the post-1992 period. To the extent that the level of the yield curve captures the bond market's perception of future inflation, these results are consistent with the notion of improved monetary policy in the UK.

The paper is organised as follows. Section 2 presents a generalization of the Nelson-Siegel model using a FAVAR with time-varying coefficients and stochastic volatilities. Section 3 describes estimates of persistence and volatility. The evidence on the Expectations Hypothesis and the fit of the time-varying model is revisited in Section 4. Section 5 presents time-varying impulse response and variance decomposition analysis. Details on the estimation procedure are provided in the appendix.

2 Modelling yield curve and macro dynamics

Earlier empirical contributions based on UK data have shown that the dynamics of the yield curve and key macroeconomic variables have evolved significantly over time. This is particularly true if the selected sample covers most of the post-WWII period. While the recent macro-finance literature has convincingly advocated the case for the existence of a bi-directional link between the term structure and the rest of the economy, to the best of our knowledge this paper is among the first attempt to model time variation in the yield curve and the economy simultaneously. To this end, we design a generalization of Nelson-Siegel interpolation in the context of a FAVAR model featuring time-varying coefficients and stochastic volatilities. It is worth emphasizing that we also allow for time variation in the cross correlations between UK macro and financial factors.[1]

[1] For applications on US data, see Mumtaz and Surico (2008), and Koopman et. al. (2007), who study the forecasting ability of an alternative time-varying specification of the Nelson-Siegel model.

2.1 A generalization of Nelson-Siegel model

Our model is a generalisation of the latent dynamic factor model used in Diebold, Rudebusch and Aruoba (2006). Following Nelson and Siegel (1987), Diebold, Rudebusch and Aruoba (2006) assume that information about the term structure of interest rates can be summarised by three factors that represent the 'level', 'slope' and 'curvature' of the yield curve. They include these yield curve factors and measures of real activity, inflation and the central bank rate in a VAR model which is used to model the interaction between these variables. We generalise this approach by allowing the parameters of the VAR model to be time-varying.

An intuitive way to represent our model is to cast it into state-space form. The observation equation of the state space system is based on the yield curve model developed by Nelson and Siegel (1987):

$$y_t(\tau) = L_t + \frac{1 - e^{-\tau\lambda}}{\tau\lambda} S_t + \left(\frac{1 - e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda} \right) C_t + e_t(\tau) \tag{1}$$

where $y_t(\tau)$ denotes yields at maturity τ and L_t, S_t and C_t denote the (unobserved) level, slope and curvature factors. Equation 1 relates the yield data to the unobserved factors.

The dynamics of these factors are described by the following *time-varying* VAR

$$Z_t = \gamma_t + \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} + v_t \tag{2}$$

where $Z_t = \{L_t, S_t, C_t, \pi_t, \tilde{y}_t R_t\}$ denotes the data matrix and $VAR(v_t) = \Omega_t$. Note that along with the unobserved factors, Z_t contains three macroeconomic variables: de-trended output $(\tilde{y}_t)$, annualized monthly inflation (π_t) and the policy interest rate (R_t).

Following Cogley and Sargent (2005) and Primiceri (2005) among others, we postulate a random walk for the evolution of the VAR coefficients:

$$\Phi_t = \Phi_{t-1} + \eta_t \tag{3}$$

where $\Phi_t = \left[\alpha_t, \beta_{t,p}\right]$. In addition, the covariance matrix of the VAR innovations Ω_t is assumed to be time-varying. In particular, we allow both the volatility of v_t and the contemporaneous relationship amongst the elements of v_t to evolve as random walks. Finally, we allow the yield specific shocks $e_t(\tau)$ to be serially correlated and heteroscedastic.

This set-up of the dynamic factor model has three potential advantages over the specification employed in Diebold, Rudebusch and Aruoba (2006). First, the dynamics of each variable in our model are allowed to change over time. This is particularly important for variables such as inflation, with number of recent studies (see for e.g. Cogley *et al.* (2005)) highlighting large changes in its trend and persistence. Second, the model allows the contemporaneous and dynamic relationship amongst the yield curve and macroeconomic variables to change over time. This extension is particularly important given the numerous structural changes in the UK economy over the last 30 years. Third, our specification incorporates heteroscedastic shocks and thus accounts for the change in volatility documented in studies such as Benati (2004). It should be noted, however, that althought our time-varying FAVAR model offers a flexible modelling framework, it contains a very simple characterisation of the yield curve. Future work could, therefore, focus on incorporating more structure in equation 1 possibly along the lines suggested in Ang and Piazzesi (2003).

2.2 Estimation

The model in equations (1) to (18) is estimated using the Bayesian methods described by Kim and Nelson (1999a). In particular, we employ a Gibbs sampling algorithm that approximates the posterior distribution. The algorithm exploits the fact that given observations on Z_t the model is a standard time-varying parameter model.

A detailed description of the prior distributions and the sampling method used in the estimation is given in the appendix. Here, we summarise the basic algorithm. Note that as in Diebold and Li (2006), we set $\lambda = 0.0609$, which is the value that maximizes the loading on the curvature factor at 30 months.

1. Given initial values for the factors, simulate VAR parameters and hyperparameters

    - The VAR coefficients ϕ_t and the off-diagonal elements of the covariance matrix α_t are simulated using the methods described by Carter and Kohn (2004).
    - The volatilities of the reduced form shocks H_t are drawn using the date by date blocking scheme introduced by Jacquier *et al.* (2004).

- The hyperparameters Q and S are drawn from an inverse wishart distribution while the elements of G are simulated from an inverse gamma distribution.

2. Given initial values for the factors, draw the stochastic volatilities $\psi_t(\tau)$ and the serial correlation coefficient $\rho(\tau)$.

   - Given data on Z_t and $y(\tau)$ and a value for λ, $\psi_t(\tau)$ are drawn using the methods described in Jacquier *et al.* (2004).
   - Conditional on a draw for $\psi_t(\tau)$, $\rho(\tau)$ are drawn using the method described in Kim and Nelson (1999a) for a heteroscedastic AR model.

3. Simulate the factors conditional on all the other parameters.

   - This is done by employing the methods described by Bernanke *et al.* (2005) and Kim and Nelson (1999a).

4. Go to step 1.

We use 60000 Gibbs sampling replications and discard the first 55000 as burn-in. The posterior moments vary little over the retained draws providing evidence of convergence (See appendix B).

2.3 Data

We consider U.K. Treasury yields with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months. The yields are derived from bid/ask average price quotes, from January 1970 through March 2006, using the methods developed by Svensson (1995). To initialize the factors and calibrate priors for the VAR, we use a pre-sample of three years starting in January 1970. Therefore the results presented in the following section refer to the period January 1973 - March 2006. Inflation is measured as monthly annualised change in the consumer price index, the policy instrument is the Bank Rate and real activity is measured by de-trended industrial production. Following Cogley *et al.* (2005) we construct an approximate measure of trend real activity y_t^* using the recursion $y_t^* = y_{t-1}^* + 0.075\left(y_t - y_{t-1}^*\right)$ where y_t denotes industrial production and $\tilde{y}_t = y_t - y_t^*$. As discussed in Cogley *et al.* (2005), the time-varying dynamics in the VAR refine this initial guess of the trend in real activity.

3 Reduced-form evidence

This section describes the empirical results of the generalized Nelson-Siegel model developed in Section 2.

The estimates of the factors are displayed in Figure 1 as black lines. The red lines represent the central 68% posterior bands while the light blue refer to the 'empirical counterparts'. The empirical counterparts of the factors can be thought of as crude proxies for the level, slope and curvature of the yield curve and following Diebold and Li (2006) are calculated as:

$$\begin{array}{lc} Level: & y_t(3) + y_t(24) + y_t(120)/3 \\ Slope: & y_t(3) - y_t(120) \\ Curvature: & 2y_t(24) - y_t(3) - y_t(120) \end{array}$$

The factors are estimated very precisely, with tight confidence bands. The estimates track the empirical counterparts well. The correlation between the level factor and its counterpart, for instance, is 0.91, which is 14% higher than the number obtained by Diebold, Rudebusch and Aruoba (2006) using U.S. data and a time-invariant yield-macro model.

The bottom left panel reports the inflation forecasts produced by the National Institute of Economic and Social Research (NIESR) and a financial market measure of inflation expectations derived from 10 year index linked gilts. The correlation between our estimated level factor and the forecasts of the NIESR, which is available at quarterly frequency between March 1973 and December 2003, is remarkably high: 0.84.[2]

The high correlation between the level of the yield curve and inflation forecasts is consistent with the view that movements in the level factor may capture movements in inflation expectations. According to this view, the sharp decline in the 1990s and the remarkable stability of the level factor since the end of the 1990s can be seen as a great moderation in inflation expectations.

Figure 2 considers implications for the persistence of the yield curve and inflation implied by our time-varying FAVAR. The figure presents the estimated (3 month ahead)

[2] The estimated correlation is significant at the 1% level. The correlation between the first difference of the two series is 0.44 which is, again, significant at 1%.

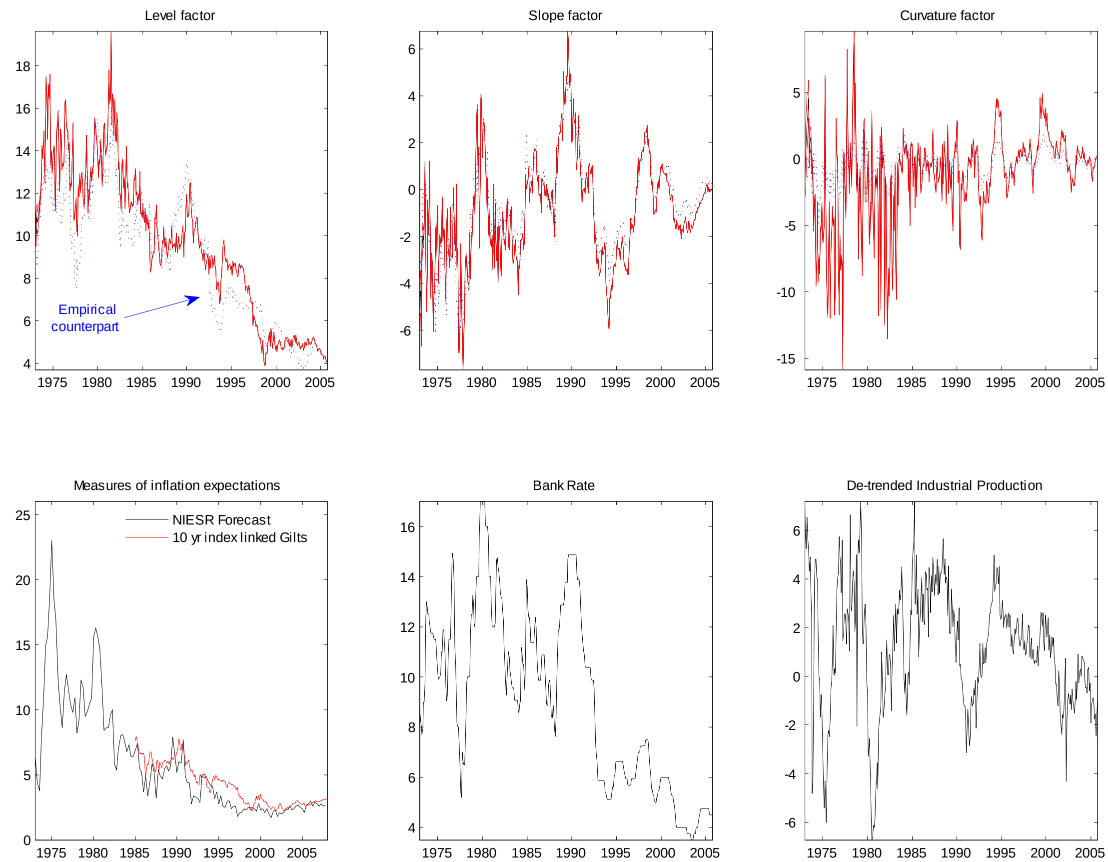

Figure 1: Factors and their empirical counterparts: The top three panels of the figure plot the estimated yield curve factors (black and red lines) and compares them with proxies constructed directly from the data. The bottom left panel shows some measures of inflation expectations available for the UK.

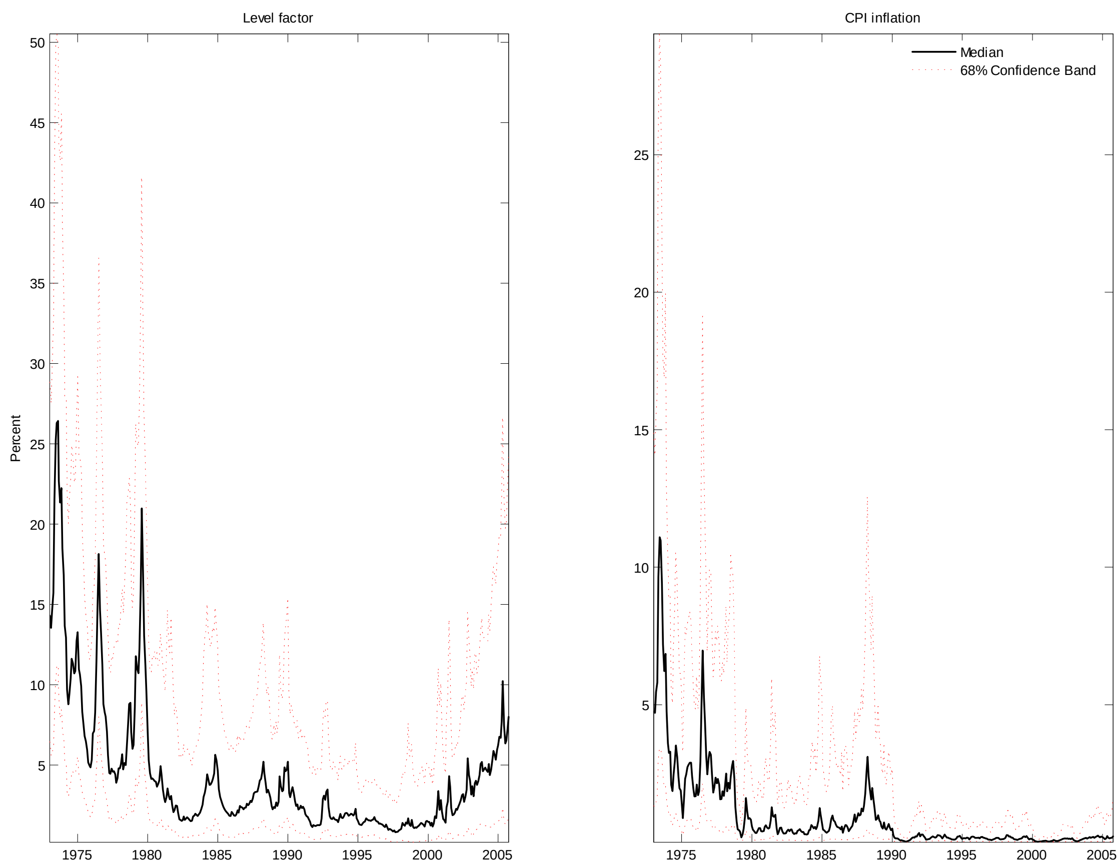

Figure 2: Persistence of the level factor and CPI inflation: This figure plots an estimate of the time-varying multivariate R^2.

time-varying R^2, a measure of persistence recently used by Cogley *et al.* (2008). As discussed in Cogley *et al.* (2008) this measure provides information about the contribution of past shocks to current and future variation in the variable of interest. A high value of the R^2 statistic, therefore, implies higher persistence.

The estimated R^2 confirms recent evidence on inflation persistence presented in Benati (2004). The 1970s and the 1980s were characterised by high persistence, with most of the variation in inflation concentrated at the long run frequencies. There was a large fall in inflation persistence in the post-1992 period, however, with the R^2 substantially lower. It is interesting to note that the persistence of the level factor follows a similar pattern. The R^2 has its peak in the mid and the late 1970s with the largest decline in the early 1980s coinciding with the onset of Margaret Thatcher's premiership[3].

Homoskedasticity is a recurrent assumption in the macro-finance literature. Our results suggest, however, that significant time variation also characterises the evolution of the volatilities of the observed and unobserved factors. Figure 3 plots the estimated unconditional volatility of each variable, where this is computed as the sum of the spectral density over the frequencies. Figure 4 plots the estimated diagonal elements of the time varying covariance matrix (see equation 13). The innovation variances of the level, slope and curvature factors and their estimated unconditional variance displays a stable declining path, with sporadic and short lived increases, the last of which occurred around the ERM crisis of 1990. The volatility of shocks to inflation and inflation variance has also declined over time from the peaks in the mid 1970s and the early 1980s; the last significant increase occurred during the ERM crisis of the early 1990s. Output variance and the volatility of its shock follow a similar pattern, with the largest increases materialising before 1992. Interestingly, the volatility of the innovations to the Bank rate has virtually disappeared since the adoption of the inflation targeting framework in 1992[4].

[3] Note that we get very similar results using the estimate (normalised) spectral density for each variable. The peaks of the density occur at the low frequency and are higher for pre-1980 sample period. These results are available on request from the author.

[4] Our estimates for $\psi_t^{1/2}$ display a similar pattern. That is, the estimated volatility (for all horizons) is at its highest during the 1970s and the 1980s, with the inflation targeting period associated with the greatest stability. These results are available on request.

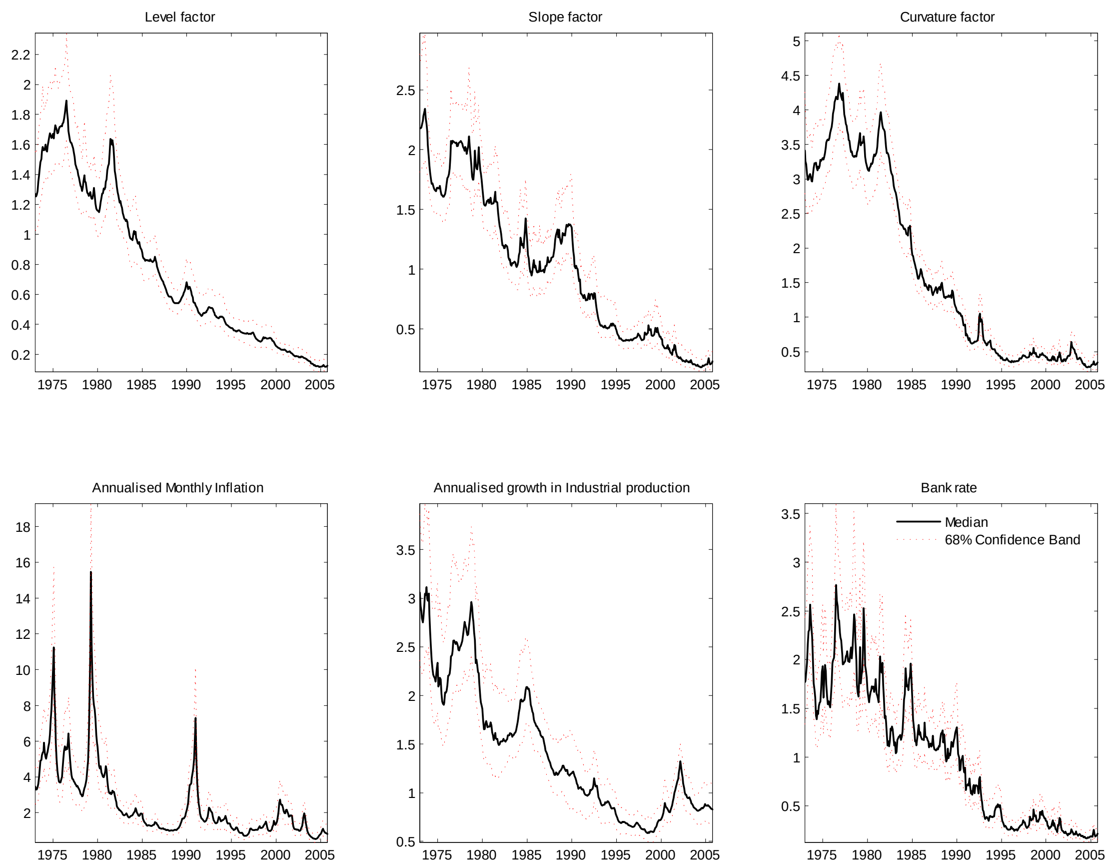

Figure 3: Estimates of unconditional volatility of each variable calculated as the sum (over frequencies) of the estimated spectrum.

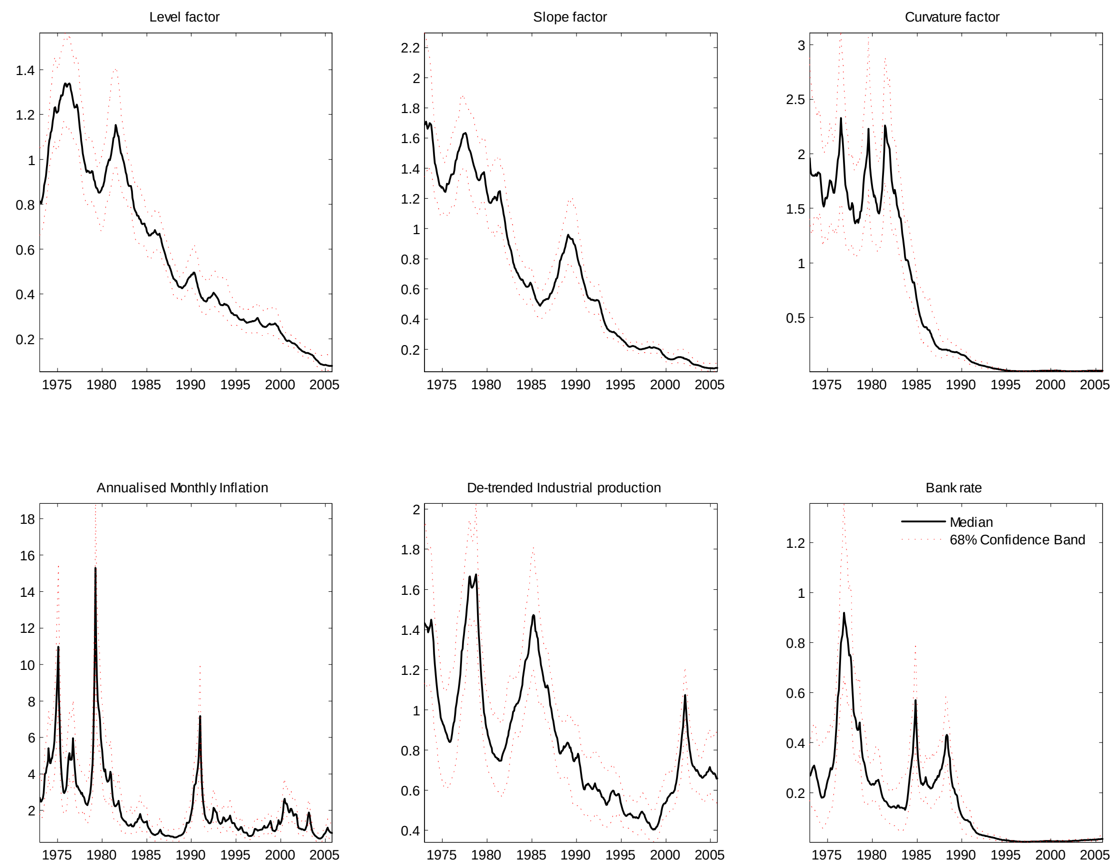

Figure 4: Estimated standard deviation of the FAVAR residuals

4 The evidence on the expectations hypothesis revisited

The expectations theory of the term structure predicts that movements in long rates are due to movements in expected future short rates. Any differences between actual long rates and expected short rates reflect a term premium, which is typically assumed to be vary across maturities.

A long standing empirical literature has shown that the expectations hypothesis is rejected across a number of countries. In a recent contribution, Ravenna and Seppälä (2007) find that the systematic component of monetary policy as well as habit formation are crucial, in a sticky price model, to account for the empirical rejections of the expectations hypothesis. As argued by Fuhrer (1996), a change in the conduct of monetary policy and/or the structure of the economy is likely to have significant implications for the ability of expected short rates to track actual long rates. Furthermore, Assenmacher-Wesche and Gerlach (2008) show, using US data, that there is less evidence against the expectations hypothesis at business cycle frequencies, where monetary policy can reasonably be expected to exert the maximum impact.

Our framework allows us to re-visit the evidence on the expectations hypothesis for the UK based on the time-varying generalization of Nelson-Siegel model. We are interested to assess whether shifts in the systematic component of monetary policy, the structure of the economy and the term structure of interest rates can account, at least partially, for the finding in earlier empirical contributions that the expectations hypothesis consistent yields track poorly actual yields: apparent deviations from the expectations theory may reflect *omitted* parameter instability.

4.1 Computing expectations hypothesis consistent yields

The Expectations Hypothesis (EH) consistent (pure discount) bond yield is:

$$y_t(\tau)^{EH} \equiv \left(\frac{1}{\tau}\right)\sum_{i=0}^{\tau-1} E_t y_{t+i}(1) + c_t(\tau) \tag{4}$$

where τ and c_τ represent the maturity and the term premium.

Our goal is to compare the predictions of the EH using two different models based on *fixed* and *time-varying* coefficients, respectively. In a fixed coefficient model, the computation of the RHS of (4) involves the implicit assumption that agents form their expectations using a model that is fixed over time. The assumption of time-invariance in monetary policy and the structure of the UK economy, however, is very strong as both narrative and empirical evidence have provided large support for the existence of significant structural changes over the sample we consider (see Nelson (2001)).

In our proposed time varying FAVAR, in contrast, changes in monetary policy and the structure of the economy are allowed, but not required, through the drifting coefficients. The important implication of our flexible modeling choice is that agents (implicitly) update their beliefs about the economy and monetary policy at each point in time, and use such updated beliefs to forecast the long term interest rate.

Note that the law of motion for the VAR coefficients in equation (3) implies that future evolutions in agents' beliefs are random variables. When computing the forecast of the long term interest rate, we take this uncertainty into account through Montecarlo integration (see for e.g. Koop *et al.* (1996)).[5] The Bayesian approach taken in this paper provides us, then, with a very natural way of accounting for parameter uncertainty when constructing bands around the central predictions of the expectations hypothesis.[6]

In Figure 5, we compare actual yields with the theoretical yields constructed via (4) on the basis of the time-varying model (1)-(3), (17)-(18) conditional on the information available at time $t-1$. The theoretical yields track actual yields very well and the predictions of the model are accurate, especially at short maturities where the actual yields rarely fall outside the 68% confidence interval. At the 5 and 10 year maturities, the estimated theoretical yields still fit the data well. There are, however, sporadic but noticeable deviations, which occurred during the mid-1970s, and the early 1990s.

Figure 6 presents the same estimates from a restricted version of the FAVAR, where time-variation in the VAR coefficients and stochastic volatility in the observation and the transition equation is removed. Note that this model is similar to the one estimated by Diebold, Rudebusch and Aruoba (2006). Figure 6 suggests that the time-invariant model

[5] Our approach is therefore different from the 'anticipated utility' version of the expectations hypothesis used in Cogley (2004) where agents update their beliefs each period but then keep them fixed over the forecast horizon.

[6] In a classical framework, a time-varying parameter model imposes such a heavy computational burden that considering parameter uncertainty becomes unfeasible (see Carriero *et al.* (2006) for an alternative procedure based on recursive estimations).

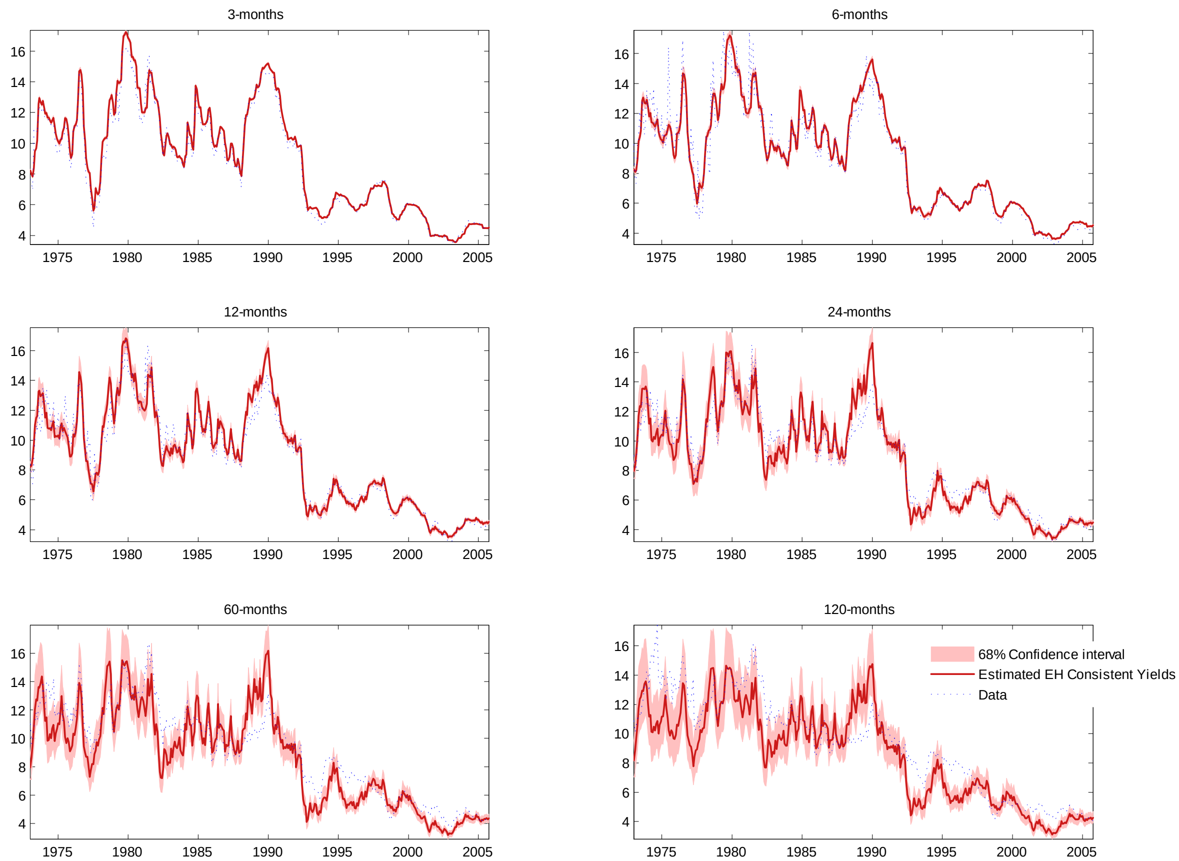

Figure 5: Posterior bands for the Expectation Hypothesis consistent yields from the time-varying FAVAR. These are computed as the discounted sum of the FAVAR forecast of the interest rate at each point in time.

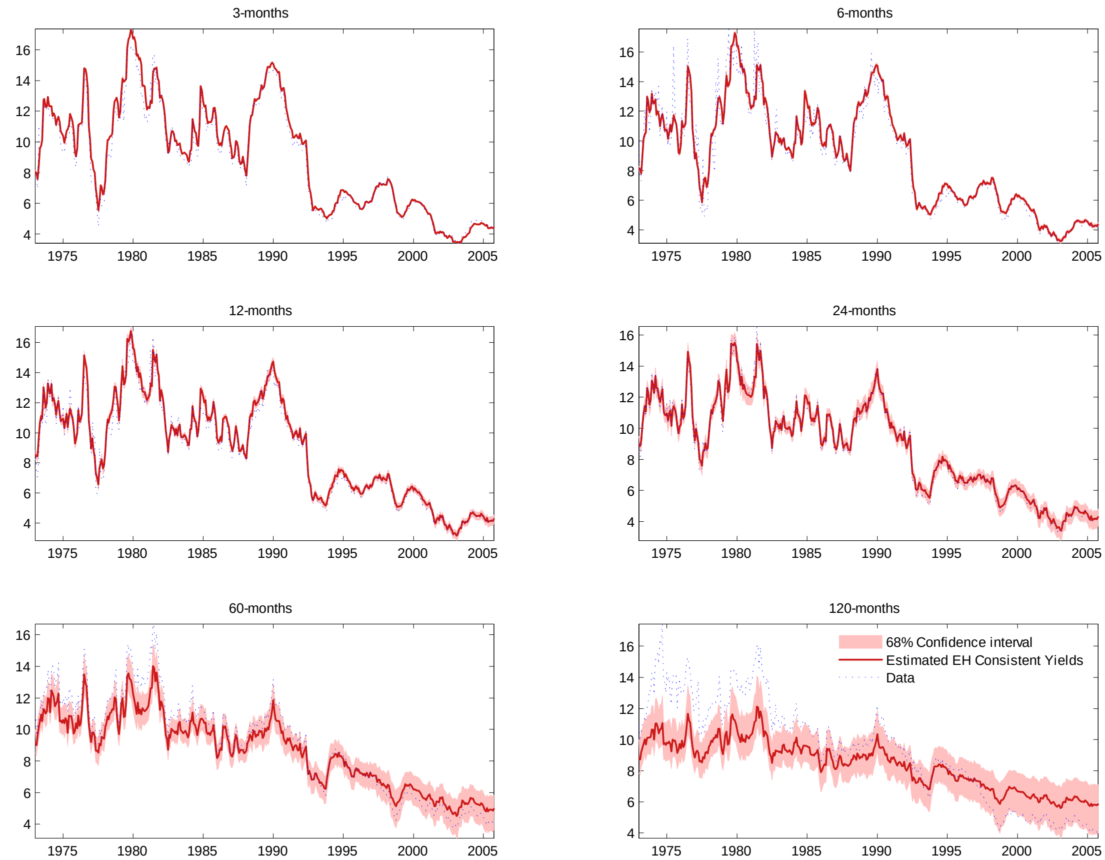

Figure 6: Posterior bands for the EH consistent yields from a fixed coefficient FAVAR. These are computed as the discounted sum of the FAVAR forecast of the interest rate.

fits the shorter term yields very well. However at long horizons, the fit worsens. For example the theoretical 10 year yield diverges systematically from the actual yields throughout the sample period.

4.2 Estimating the term premium: fixed vs. time-varying coefficients

The difference between actual and theoretical yields can be interpreted as a measure of the term premium. A simple way of comparing the performances of the fixed and time-varying coefficient models is to plot the term premia associated with the estimates of the two models. For the sake of exposition, in Figure 7 we report results for the 10 year maturity. The figures for the other maturities are available upon request.

The term premium implied by the fixed coefficient model is depicted in the bottom panel and it is sizable and statistically significant over most of the sample. On the basis of such evidence, one may conclude that deviations from the EH are large and persistent, and that the failure of the EH is the rule rather than the exception. The term premium implied by the time-varying coefficient model, however, offers a quite different picture.

The top panel of Figure 7 shows that the term premium associated with the time-varying specification fluctuates around zero over most of the sample. The light blue areas represent 68% posterior bands. In two notable episodes, however, the EH consistent yield significantly under-predicts the actual yield: the oil crises of the 1970s and the ERM crisis of 1992. Our finding corroborates the view that around those times investors were asking a positive premium to hold long term bonds in an effort to hedge themselves against further increases in inflation.

Note that the positive premium in the couple of years following the introduction of inflation targeting in December 1992 coincides with the credibility building of the new policy regime. Since the Bank of England was granted independence in May 1997, deviations from the expectations theory have been modest.

The time-varying specification includes more sources of variation than the fixed coefficients specification. In the time-invariant model, the shocks to the VAR equations are the only source of variation. In the time-varying specification, there are also shocks attached to each VAR coefficient, in addition to the disturbance covariance matrix. In theory, the time-varying model is more suited to tracking shifts in agents' beliefs through the evolution

Figure 7: The estimated term premium for the 10 year government bond. This is computed as the difference between the 10 year government bond yield and the estimated Expectation Hypothesis consistent 10 year yield from the respective FAVAR models.

of the VAR coefficients. In practice, Figure 7 reveals that the term premia can be largely overestimated when the instability of the VAR relationships is not accounted for.

5 Structural evidence

In this section we adopt a more structural approach and consider the role played by the monetary policy shock and the shock to the level of the yield curve. There are a number of reasons why we focus on this aspect of the yield curve. Firstly, the level of the yield curve is related to the implicit target of the central bank and variation around this target may reflect agents' beliefs about the credibility of monetary policy. Secondly, as figure 1 shows, the level factor moves closely with inflation expectations. One interpretation of this comovement is that the shocks to this variable possibly capture shifts in inflation expectations which may be induced by an unanticipated shift in the central bank implicit inflation target. Of course our model does not contain the theoretical structure to pin these variables down precisely and the level factor remains only an indicator of their possible movements.

We use a simple recursive identification scheme with the following ordering $L_t, S_t, C_t, , \pi_t, \tilde{y}_t, R$. Therefore, the identified monetary policy shock embodies the assumption, which is common in the VAR literature, that the policy rate affects macroeconomic variables (output and inflation) and the yield curve with at least one month lag. As noted by Diebold, Rudebusch and Aruoba (2006), the intuition behind this ordering is the fact that the yield curve observations are dated at the beginning of the month. Under this identification scheme, the shock to the level factor is assumed to be contemporaneously unaffected by the other variables in the system. We interpret this as an exogenous shock to the 'market's perception of long-run inflation' (see Diebold, Rudebusch and Aruoba (2006) pp 324).

We assess the role played by these shocks by computing time-varying impulse response functions and variance decomposition. As our model features time-variation, the impulse response functions may change over time: we therefore report the responses for selected years covering different policy regimes.

5.1 Impulse response analysis

Following Diebold, Rudebusch and Aruoba (2006) we consider the dynamic relationships between the macro variables and the yield curve variables through impulse response analysis. The time-varying nature of our model implies that unlike Diebold, Rudebusch and Aruoba (2006) we can explore how these dynamic relationships have changed over time. Note, however, that as the coefficients change over time this feature has to be taken into account when estimating the impulse response functions. Following Koop *et al.* (1996) we define the impulse response functions as:

$$IRF = E\left(Z_{t+k}|\Psi_{t+k},\mu\right) - E\left(Z_{t+k}|\Psi_{t+k}\right) \tag{5}$$

where Ψ denotes all the parameters and hyperparameters of the VAR and k is the horizon of interest. Equation (5) states that the impulse response functions are calculated as the difference between two conditional expectations. The first term denotes a forecast of the endogenous variables conditioned on a shock μ. The second term is the baseline forecast, which is conditioned on the scenario where the shock equals zero.

Figure 8 shows the impulse response of key variables in the model to a monetary policy shock over the 1970s, the 1980s and over the current inflation targeting regime[7].The impulse responses for these sub-samples are computed as the average response over this period–in particular, for a given Gibbs draw of the FAVAR parameters, we compute the impulse response functions for each month and then compute the average for the appropriate sub-sample. The shock is normalised so that it increases the central bank interest rate by 100 basis points at all dates.

Consider the impact of the policy shock on the two macroeconomic variables. The response of inflation suggests strong evidence of a price puzzle in the 1970s and the 1980s. However, the magnitude of the positive response is much smaller over the inflation targeting period, with the response insignificantly different from zero at all horizons. These results are similar to those reported in Castelnuovo and Surico (2005) (see also Benati and Surico (2007)) who argue, using a sticky price model, that the price puzzle is most evident when estimation is carried out over periods where the monetary authority is less responsive to inflation (i.e. the Taylor principle is violated). According to this argument, the pattern of

[7] The full set of (time-varying) impulse responses are available on request from the authors.

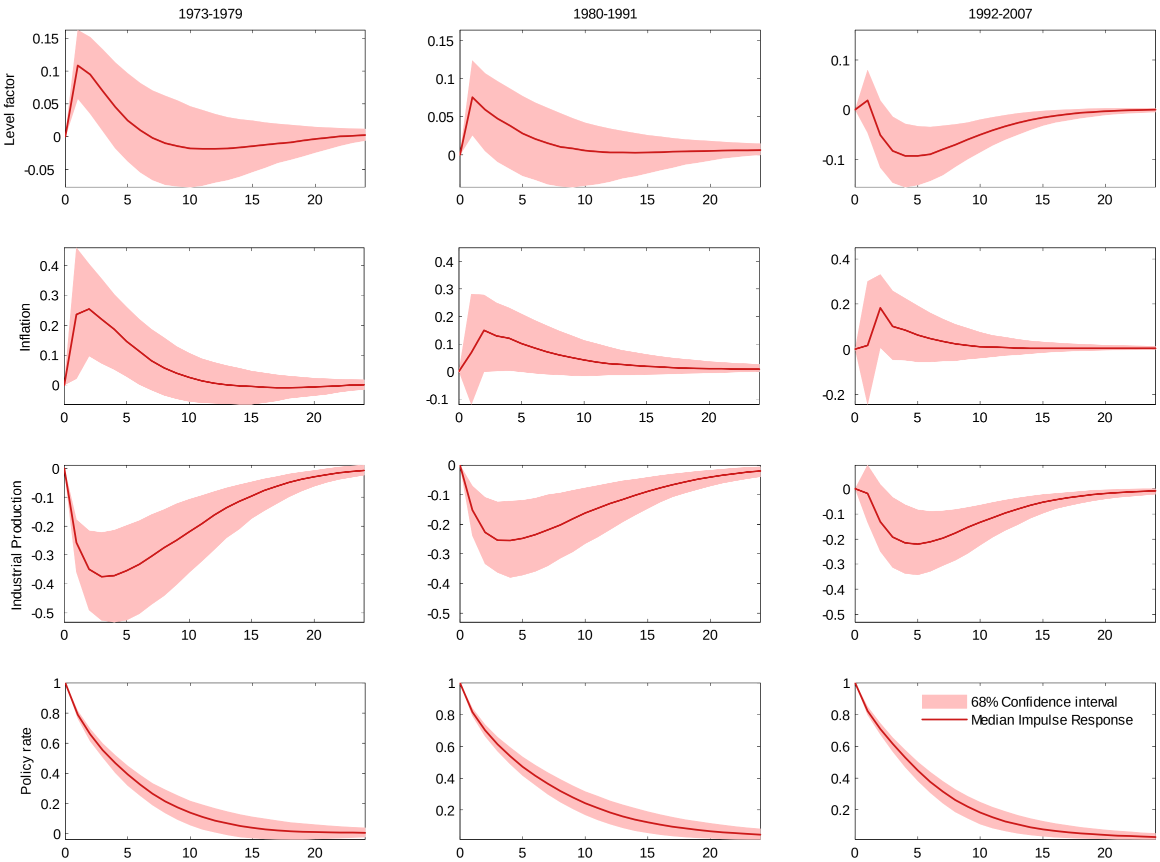

Figure 8: Time-varying impulse response to a monetary policy shock that raises the bank rate by 100 basis points. The impulse responses in each sub-sample are averages of the impulse response in each month in that sample.

inflation responses displayed in Figure 8 would be consistent with an increase in activism by the central bank over the post-1992 period. The response of output to a monetary policy shock is largest over the 1970s, with output falling by around 0.4%, 6 months after the shock. In the 1980s and the 1990s, the magnitude of the response is around 50% smaller. These results mirror those reported by Boivin and Giannoni (2006) for the US.

The response of the level factor to the monetary policy shock suggests some interesting conclusions. As discussed in Diebold, Rudebusch and Aruoba (2006), the direction of the response of the level of the yield curve to a monetary contraction may depend on the credibility of monetary policy. For example, if monetary policy is credible, then the level factor may fall in response to higher policy rates because expectations of future inflation decline. The estimated response of the level factor in the 1970s and the 1980s suggests that this was *not* the case.

The level factor increases significantly in response to a policy contraction. This possibly suggests that the central bank was unable to curb inflation expectations in the 1970s. In contrast, the response during the inflation targeting regime is significantly negative. The shift in the response of the level factor to a monetary policy shock is consistent with the notion of a marked improvement in the credibility of monetary policy.

Figure 9 displays the impulse response to a shock to the level factor. In the pre-inflation targeting period, inflation displays a strong persistent positive response to the level factor, with the inflation response taking 20 months to become insignificant. The persistence is significantly smaller over the inflation targeting period, with the inflation response being not statistically different from zero already in the second month.

Following Diebold, Rudebusch and Aruoba (2006), one interpretation of these estimates is that they represent the impact of an increase in inflation expectations. Under this interpretation of the shock, our results are consistent with the idea that during the pre-inflation targeting period, the response of the monetary authority to an increase in inflation expectations was more muted, leading to a significant increase in actual inflation. The pattern of the policy rate response also supports this view, with the Bank rate responding by less over the 1970s. These results are similar to those reported for the US by Leduc *et al.* (2002), who use survey based measures of inflation expectations in their estimated VAR model.

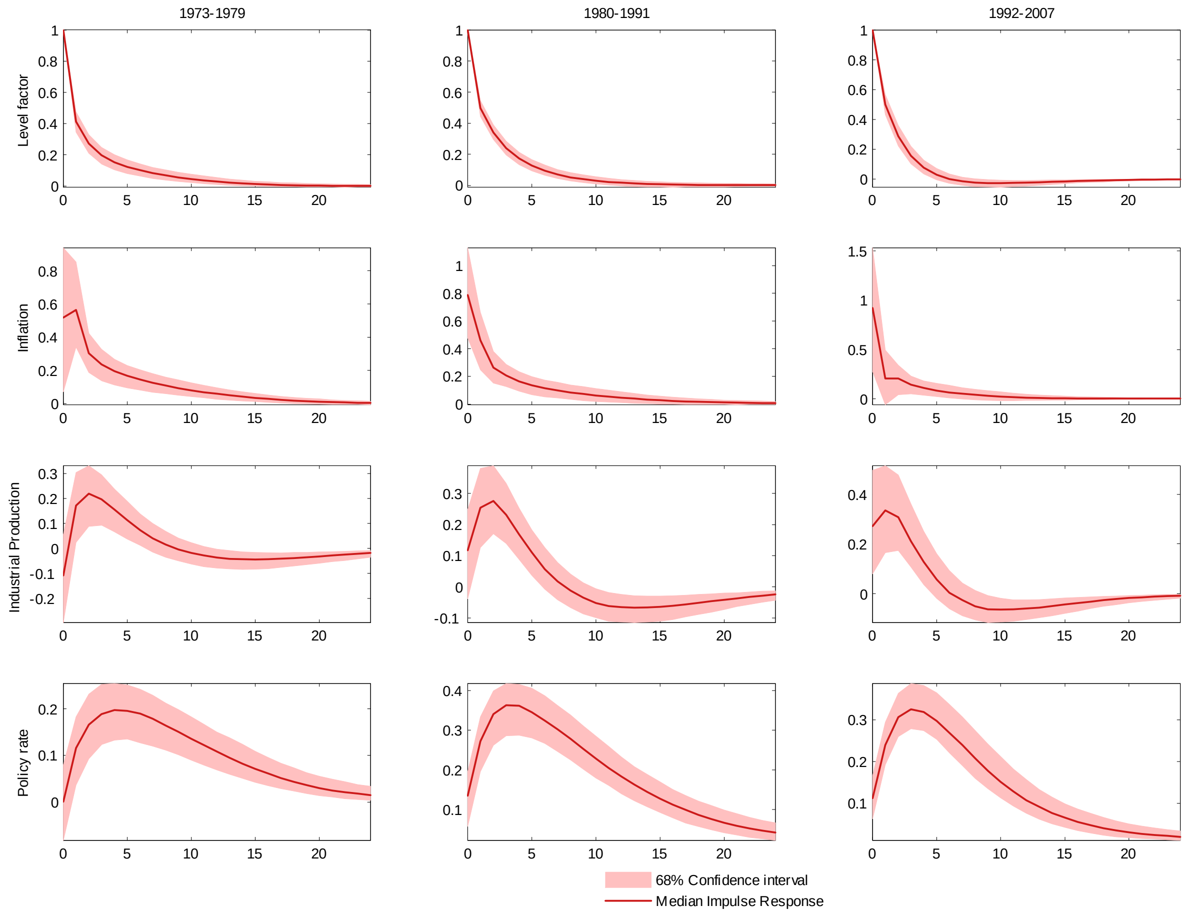

Figure 9: Time-varying response to a level factor shock which raises the level factor by 100 basis points. The impulse responses in each sub-sample are averages of the impulse response in each month in that sample

5.2 Variance Decomposition

In this section, we decompose, at each point in time and at different frequencies, the unconditional variance of each endogenous variable in the FAVAR into contributions from the monetary policy shock and the shock to the level of the yield curve. A number of interesting questions can be assessed with this exercise. First, we can trace how the link between the macroeconomy and the yield curve has evolved over time. Second, it allows us to assess the role of economic events and changes in policy regimes.

Figures 10 and 11 display the proportion of the unconditional variance of each variable accounted for by the monetary policy shock and the shock to the level factor. As in Canova and Gambetti (2006) , these are calculated as the ratio of the unconditional variance due to the shock of interest and the 'total' unconditional variance.[8] That is, the figures depict the following ratio:

$$\frac{\int_{\varpi} f^*_{t|T}(\omega)_i}{\int_{\varpi} f_{t|T}(\omega)_i} \tag{6}$$

$f_{t|T}$ denotes the spectral density:

$$f_{t|T}(\omega)_i = s(I - \tilde{\beta}_{t|T}e^{-i\omega})^{-1}\frac{\Omega_{t|T}}{2\pi}\left[(I - \tilde{\beta}_{t|T}e^{-i\omega})^{-1}\right]' s' \tag{7}$$

where s is a selection vector. $f^*_{t|T}(\omega)_i$ denotes the estimate of the spectrum under the assumption that only the shock of interest is operating. This is calculated as:

$$f^*_{t|T}(\omega)_i = s(I - \tilde{\beta}_{t|T}e^{-i\omega})^{-1}\frac{\Omega^*_{t|T}}{2\pi}\left[(I - \tilde{\beta}_{t|T}e^{-i\omega})^{-1}\right]' s' \tag{8}$$

where the covariance matrix $\Omega^*_{t|T}$ is built (using equation 12) imposing that the only non-zero element in H_t is the variance of the shock under consideration. For example, when considering the contribution of the monetary policy shock, all elements of H_t except the last one are set to zero. This implies that $f^*_{t|T}(\omega)_i$ represents the spectral density of variable i due to the monetary policy shock.

Figure 10 shows that the monetary policy shock makes the largest contribution to the variance of the level factor in the 1970s and the 1980s. A similar pattern emerges

[8]These figures should be interpreted as follows: The Z axis in each figure represents the percentage contribution of each shock to the variation in each variable of the VAR at different frequencies at each point in the sample. The frequencies are depicted on the Y axis, while the X axis represents time.

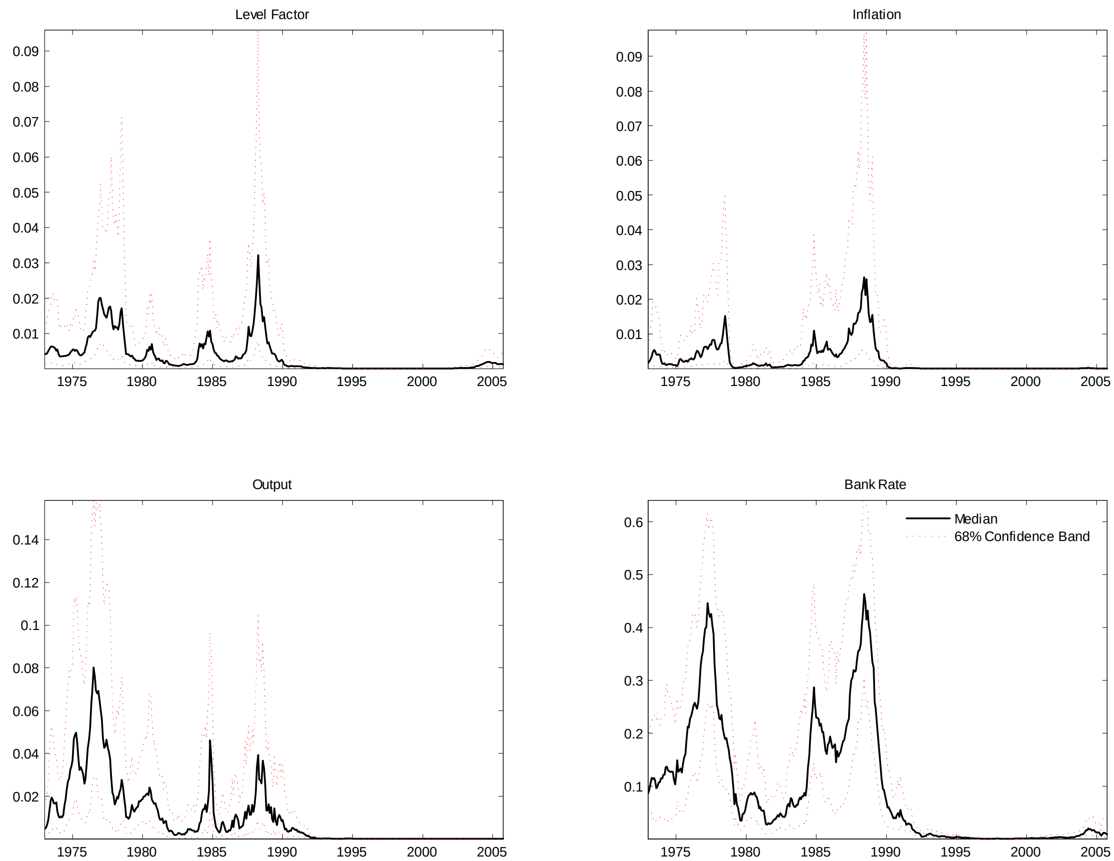

Figure 10: Unconditional variance due to the monetary policy shock.The panels show the percentage of the variance of each of the variables that is explained by the monetary policy shock.

in its contribution to the variation in inflation and output, with the policy shock being important in the mid 1970s, the early and late 1980s. In the post-1992 period, however, the policy shock plays little role in yield curve and macroeconomic fluctuations, indicating that deviations from systematic monetary policy are less important over the inflation targeting period.

The contribution of the monetary policy shock to the policy interest rate shows the most striking variation. In the pre-1992 period, the policy shock accounts for most of the variance of the policy rate. This suggests that the policy rate before 1992 was mainly driven by concerns other than inflation and unemployment (and the information contained in the yield curve). After 1992, however, the policy shock explains only a very small fraction of short-term interest rate variability. This is suggestive of a systematic change in monetary policy since the adoption of the inflation targeting framework, with (i) policy rates moving in response to inflation and output, and (ii) deviations from this policy 'rule' being substantially smaller in magnitude and significance.

Figure 11 shows how the shock to the level factor equation contributes to the movement in each of the endogenous variables. The figure shows that the shock to the level factor explains (at most) about 40% of the variation in inflation over the 1980s. The 'peaks' in the total contribution occur at the start of the Thatcher era, around the time of the 'sterling crisis' in the mid-1980s, during the ERM crisis in 1992 and around the time of central bank independence in 1997. It is likely that these periods were characterised by considerable uncertainty about the level at which rates would have settled eventually. This pattern also suggests that shocks to the central bank's target were more important in driving inflation in the period before the Bank of England was granted independence. In other words, agents' beliefs about the central bank's actions have become less uncertain since 1997.

The contribution of the level factor to output remained at around 20% until the late 1990s and has been negligible afterwards. The contribution of this shock to the policy rate is relatively small in the 1970s but becomes increasingly important in the post-1980 period. This pattern is matched by the contribution of the shock to the slope factor. The contribution is higher in the 1980s and especially during the inflation targeting period. These results suggest that the contributions of the level factor to the policy rate and to the policy stance (as reflected by the slope factor) were at their highest levels in the 1980s

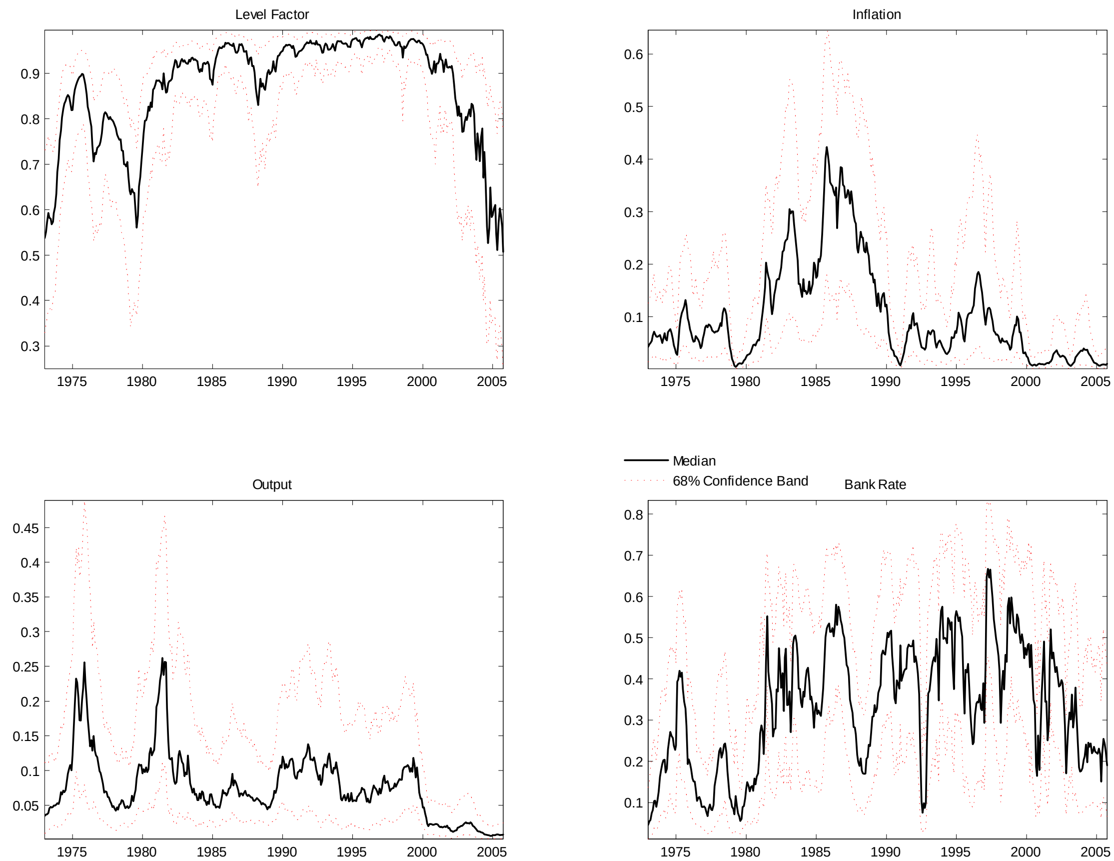

Figure 11: Unconditional variance due to the level factor shock. The panels show the percentage of the variance of each of the variables that is explained by the level factor shock.

and the 1990s, with the contribution of this shock being significantly smaller in the 1970s.

6 Conclusions

This paper has studied the evolution of the yield curve and the macroeconomy in the U.K. over the last forty years. A contribution of the paper is to introduce time-varying parameters and stochastic volatilities in a FAVAR macro-finance framework.

The dynamics of the yield curve have changed significantly over time with a great moderation in their volatility and persistence. In addition, there has been a shift in the relationship between the yield curve and macroeconomic variables. In the pre-inflation targeting period, an unanticipated rise in the level of the yield curve had a large persistent impact on inflation, and it contributed significantly to long run fluctuations in inflation. Since the adoption of the inflation targeting framework, in contrast, shocks to the level factor have become less important for inflation. After 1992, the volatility of the monetary policy shock has virtually disappeared and, unlike in the pre-1992 sample, the variance of the policy rate has been mainly accounted for by movements in the level factor.

Our model fits the data well: the theoretical yields computed as average projections of the time-varying FAVAR are very close to actual yields, and deviations from the expectations hypothesis are rare. The findings of this paper suggests that large estimates of the term premium computed on the basis of fixed-coefficient models may reflect omitted parameter instability.

References

Ang, Andrew and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* **50**(4), 745–787. available at http://ideas.repec.org/a/eee/moneco/v50y2003i4p745-787.html.

Assenmacher-Wesche, Katrin and Stefan Gerlach, 2008, Ensuring Financial Stability: Financial Structure and the Impact of Monetary Policy on Asset Prices, *CEPR Discussion Papers 6773*, C.E.P.R. Discussion Papers.
URL: *http://ideas.repec.org/p/cpr/ceprdp/6773.html*

Bean, Charles, 2005, Monetary Policy in an Uncertain World, Bank of England Speech 238.

Benati, L., 2004, Evolving post-World War II U.K. Economic Performance, *Journal of Money, Credit, and Banking* **36**, 691–718.

Benati, Luca and Haroon Mumtaz, 2007, U.S. evolving macroeconomic dynamics - a structural investigation, *Working Paper Series 746*, European Central Bank.

Benati, Luca and Paolo Surico, 2007, VAR Analysis and the Great Moderation, *Mimeo*, Bank of England.

Bernanke, B. S., J. Boivin and P. Eliasz, 2005, Measuring the Effects of Monetary Policy: A Factor Augmented Vector Autoregressive (FAVAR) Approach, *Quarterly Journal of Economics* **120**, 387–422.

Boivin, Jean and Marc P Giannoni, 2006, Has Monetary Policy Become More Effective?, *The Review of Economics and Statistics* **88**(3), 445–462.
URL: *http://ideas.repec.org/a/tpr/restat/v88y2006i3p445-462.html*

Canova, Fabio and Luca Gambetti, 2006, Structural Changes in the US Economy: Bad Luck or Bad Policy?, *CEPR Discussion Papers 5457*, C.E.P.R. Discussion Papers.

Carriero, Andrea, Carlo A. Favero and Iryna Kaminska, 2006, Financial factors, macroeconomic information and the Expectations Theory of the term structure of interest rates, *Journal of Econometrics* **127**(1-2), 339–358.

Carter, C and P Kohn, 2004, On Gibbs sampling for state space models, *Biometrika* **81**, 541–553.

Castelnuovo, E and P Surico, 2005, The Price Puzzle and Indeterminancy, Bank of England Working Paper no 288.

Clarida, R, J Gali and M Gertler, 2000, Monetary policy rules and macroeconomic stability: evidence and some theory, *Quarterly Journal of Economics* **115**, 147–180.

Cogley, T. and A. M. Sbordone, 2005, A Search for a Structural Phillips Curve, *Staff Report 203*, Federal Reserve Bank of New York.

Cogley, T. and T. J. Sargent, 2005, Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S., *Review of Economic Dynamics* **8**, 262–302.

Cogley, Timothy, 2004, Changing beliefs and the term structure of interest rates: Cross-equation restrictions with drifting parameters, *Review of Economic Dynamics* **8**(2), 420–451.

Cogley, Timothy, Giorgio E. Primiceri and Thomas J. Sargent, 2008, Inflation-Gap Persistence in the U.S, *NBER Working Papers 13749*, National Bureau of Economic Research, Inc.
URL: *http://ideas.repec.org/p/nbr/nberwo/13749.html*

Cogley, Timothy, Sergei Morozov and Thomas J. Sargent, 2005, Bayesian fan charts for U.K. inflation: Forecasting and sources of uncertainty in an evolving monetary system, *Journal of Economic Dynamics and Control* **29**(11), 1893–1925.
URL: *http://ideas.repec.org/a/eee/dyncon/v29y2005i11p1893-1925.html*

Diebold, F.X. and C Li, 2006, Forecasting the Term Structure of Government Bond Yields, *Journal of Econometrics* **130**, 337–364.

Diebold, F.X., C Li and V Yue, 2006, Global Yield Curve Dynamics and Interactions: A Generalized Nelson-Siegel Approach, Mimeo, University of Pennsylvania.

Diebold, F.X., G.D. Rudebusch and S.B. Aruoba, 2006, The Macroeconomy and the Yield curve: a dynamic latent factor approach, *Journal of Econometrics* **131**, 309–338.

Fuhrer, Jeffrey C, 1996, Monetary Policy Shifts and Long-Term Interest Rates, *The Quarterly Journal of Economics* **111**(4), 1183–1209.
URL: *http://ideas.repec.org/a/tpr/qjecon/v111y1996i4p1183-1209.html*

Jacquier, E, N Polson and P Rossi, 2004, Bayesian analysis of stochastic volatility models, *Journal of Business and Economic Statistics* **12**, 371–418.

Kim, C-J. and C. R. Nelson, 1999a, *State-Space Models with Regime Switching*, MIT Press, Cambridge, Massachusetts.

Kim, Chang-Jin and Charles R. Nelson, 1999b, Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle, *The Review of Economics and Statistics* **81**(4), 608–616.

Koop, G, H Pesaran and S Potter, 1996, Impulse response analysis in nonlinear multivariate models, *Journal of Econometrics* **74**(1), 119–147.

Leduc, Sylvain, Keith Sill and Tom Stark, 2002, Self-fulfilling expectations and the inflation of the 1970s: evidence from the Livingston Survey, *Technical report*.

McConnell, M and G Perez-Quiros, 2000, Output Fluctuations in the United States: What has Changed Since the Early 1980's, *American Economic Review* **90**, 1464 – 1476.

Mumtaz, H and P Surico, 2006, Evolving International Inflation Dynamics. World and country specific factors, Bank of England working paper forthcoming.

Negro, Marco Del, 2003, Discussion of Cogley and Sargent's "Drifts and volatilities: Monetary policies and outcomes in the post WWII U.S.", *Technical report*.

Nelson, Charles R and Andrew F Siegel, 1987, Parsimonious Modeling of Yield Curves, *Journal of Business* **60**(4), 473–89.

Nelson, Edward, 2001, UK monetary policy 1972-97: a guide using Taylor rules, *Bank of england working papers*, Bank of England.

Primiceri, G, 2005, Time varying structural vector autoregressions and monetary policy, *The Review of Economic Studies* **72**(3), 821–852.

Ravenna, Federico and Juha Seppälä, 2007, Monetary policy, expected inflation and inflation risk premia, *Research Discussion Papers 18/2007*, Bank of Finland.
URL: *http://ideas.repec.org/p/hhs/bofrdp/2007*$_0$18*.html*

Rudebusch, G.D. and T Wu, 2006, Accounting for a Shift in Term Structure Behavior with No-Arbitrage and Macro-Finance Models, *Journal of Money, Credit and Banking* **forthcoming**.

Sims, C. A. and T. Zha, 2006, Were there Regime Switches in US Monetary Policy?, *American Economic Review* . forthcoming.

Svensson, Lars E O, 1995, Estimating forward rates with the extended Nelson and Siegel method, *Sveriges Riksbank Quarterly Review* **3**, 13–26.

Appendix A: Priors and Estimation

Our time-varying FAVAR model consists of the following observation equation

$$y_t(\tau) = L_t + \frac{1-e^{-\tau\lambda}}{\tau\lambda} S_t + \left(\frac{1-e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda}\right) C_t + e_t(\tau) \tag{9}$$

where $y_t(\tau)$ denotes yields at maturity τ and L_t, S_t and C_t denote the (unobserved) level, slope and curvature factors.

The dynamics of these factors are described by the following transition equation

$$Z_t = \gamma_t + \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} + v_t \tag{10}$$

where $Z_t = \{L_t, S_t, C_t, \pi_t, \tilde{y}_t R_t\}$ denotes the data matrix and $v_t = \omega_t \Omega_t^{1/2}$ with $\omega_t \sim N(0, I)$.

The VAR coefficients $\Phi_t = \left[\alpha_t, \beta_{t,p}\right]$ evolve as a random walk

$$\Phi_t = \Phi_{t-1} + \eta_t \tag{11}$$

The covariance matrix Ω_t is factored in the following way:

$$\Omega_t = A_t^{-1} H_t (A_t^{-1})' \tag{12}$$

The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & h_{4,t} & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{5,t} & 0 \\ 0 & 0 & 0 & 0 & 0 & h_{6,t} \end{bmatrix} \tag{13}$$

$$A_t \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 & 0 \\ \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 & 0 \\ \alpha_{61,t} & \alpha_{62,t} & \alpha_{63,t} & \alpha_{64,t} & \alpha_{65,t} & 1 \end{bmatrix} \tag{14}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + u_t$$

Following Primiceri (2005), we postulate that the non-zero and non-one elements of the matrix A_t evolve as driftless random walks.

$$\alpha_t = \alpha_{t-1} + \varepsilon_t \ , \tag{15}$$

We assume that that the idiosyncratic component $e_t(\tau)$ is serially correlated and heteroscedastic but uncorrelated across maturities $E\left(e_t(i)' e_t(j)\right) = 0$ for $i \neq j$. In particular:

$$e_t(\tau) = \rho(\tau) e_{t-1}(\tau) + \psi_t^{1/2}(\tau) \epsilon_t \tag{16}$$

where the volatility $\psi_t(\tau)$ follows a geometric random walk

$$\ln(\psi_t) = \ln\left(\psi_{t-1}(\tau)\right) + \omega_t$$

Finally the vector $[\ v_t', \eta_t', \tau_t', \nu_t', \omega_t]'$ is distributed as

$$\begin{bmatrix} v_t \\ \eta_t \\ \epsilon_t \\ u_t \\ \omega_t \end{bmatrix} \sim N(0, V), \tag{17}$$

$$V = \begin{bmatrix} \Omega_t & 0 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 & 0 \\ 0 & 0 & S & 0 & 0 \\ 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & R \end{bmatrix} \text{ and } G = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_6^2 \end{bmatrix}, R = \begin{bmatrix} \kappa_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & . \end{bmatrix} \tag{18}$$

Prior Distributions and starting values

Factors

We center our prior on the factors (and obtain starting values) by using the least squares estimator employed by Diebold and Li (2006). The prior covariance of the states $(P_{0/0})$ is set equal to an identity matrix.

Elements of $\psi(\tau)$

Let $\tilde{e}_t(\tau)$ denote the idiosyncratic components obtained using the initial least squares estimates of the factors. The prior for the elements of $\psi(\tau)$ is as follows:

$$\ln \psi_0(\tau) \sim N(\ln \tilde{\mu}_0(\tau), I_6 \times 10)$$

where $\tilde{\mu}_0(\tau)$ is the variance of $\tilde{e}_t(\tau)$.

Elements of $\rho(\tau)$

We use a flat prior for elements of $\rho(\tau)$.

VAR coefficients

The prior for the VAR coefficients is obtained via a fixed coefficients VAR model estimated over the sample 1973:01 to 1973:12. Φ_0 is therefore set equal to

$$\Phi_0 \sim N(\hat{\phi}^{OLS}, V^{OLS})$$

Elements of H_t

Let $\hat{v}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data described above. The prior for the diagonal elements of the VAR covariance matrix (13) is as follows:

$$\ln h_0 \sim N(\ln \mu_0, I_6 \times 10)$$

where μ_0 are the diagonal elements of $\hat{v}^{ols}$. This follows Cogley and Sargent (2005).

Elements of A_t

Following Benati and Mumtaz (2007), the prior for the off diagonal elements A_t is

$$A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$$

where $\hat{a}^{ols}$ are the off diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$. The choice of this prior is arbitrary but reflects our attempt to scale each element of A_0 in such a way as to take its magnitude into account.

Hyperparameters

The prior on Q is assumed to be inverse Wishart

$$Q_0 \sim IW\left(\bar{Q}_0, T_0\right)$$

where $\bar{Q}_0$ is assumed to be $var(\hat{\phi}^{OLS}) \times 10^{-5}$ and T_0 is the length of the sample used for calibration. Note that we use a more conservative prior than Cogley and Sargent (2005) and Primiceri (2005). Our choice is primarily based on computational reasons. In particular, with six endogenous variables and monthly data, draws of VAR coefficients that satisfy the stability criteria are difficult to obtain. Therefore, following Negro (2003), we tighten the prior on Q_0.

The prior distribution for the blocks of S is inverse Wishart:

$$S_{i,0} \sim IW(\bar{S}_i, K_i)$$

where $i = 1..6$ indexes the blocks of S. $\bar{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\bar{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3}. In specifying this prior, we again follow Benati and Mumtaz (2007).

Following Cogley and Sargent (2005), we postulate an inverse-Gamma distribution for the elements of G and R,

$$\begin{aligned} \sigma_i^2 &\sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right) \\ \kappa_i^2 &\sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right) \end{aligned}$$

Simulating the Posterior Distributions

Factors and Factor Loadings

This closely follows Bernanke *et al.* (2005). Details can also be found in Kim and Nelson (1999a).

Factors We fix $\lambda = 0.0609$. Conditional on a value for $\lambda, \rho(\tau)$ and $\psi(\tau)$, the factors are drawn using the methods of Carter and Kohn (2004). In particular we re-write equation 1 as follows:[9]

$$
\begin{aligned}
y_t(\tau) - \rho(\tau)\, y_{t-1}(\tau) &= b_1 \left(L_t - \rho(\tau)\, L_{t-1}\right) + b_2 \left(S_t - \rho(\tau)\, S_{t-1}\right) + \\
&\quad + b_3 \left(C_t - \rho(\tau)\, C_{t-1}\right) + \left(e_t(\tau) - \rho(\tau)\, e_{t-1}(\tau)\right)
\end{aligned}
$$

where:

$$
\begin{aligned}
b_1 &= 1 \\
b_2 &= \frac{1 - e^{-\tau\lambda}}{\tau\lambda} \\
b_3 &= \frac{1 - e^{-\tau\lambda}}{\tau\lambda} - e^{-\tau\lambda}
\end{aligned}
$$

and $e_t(\tau) - \rho(\tau)\, e_{t-1}(\tau) = \epsilon_t$ represents a serially uncorrelated (but heteroscedastic) error term. Using this transformed observation equation, and collecting the unobserved factors L_t, S_t and C_t into the matrix F_t, the conditional distribution is linear and Gaussian:

$$
\begin{aligned}
&F_T \backslash \Xi \tilde{\ } N\left(F_{T\backslash T}, P_{T\backslash T}\right) \\
&F_t \backslash F_{t+1}, \Xi \tilde{\ } N\left(F_{t\backslash t+1, F_{t+1}}, P_{t\backslash t+1, F_{t+1}}\right)
\end{aligned}
$$

where $t = T-1, ..1$, Ξ denotes a vector that holds all the other FAVAR parameters and:

$$
\begin{aligned}
F_{T\backslash T} &= E\left(F_T \backslash X_{i,t}, R_t, \Xi\right) \\
P_{T\backslash T} &= Cov\left(F_T \backslash X_{i,t}, R_t, \Xi\right) \\
F_{t\backslash t+1, F_{t+1}} &= E\left(F_t \backslash X_{i,t}, R_t, \Xi, F_{t+1}\right) \\
P_{t\backslash t+1, F_{t+1}} &= Cov\left(F_t \backslash X_{i,t}, R_t, \Xi, F_{t+1}\right)
\end{aligned}
$$

As shown by Carter and Kohn (2004), the simulation proceeds as follows. First we use the Kalman filter to draw $F_{T\backslash T}$ and $P_{T\backslash T}$ and then proceed backwards in time using:

$$
F_{t|t+1} = F_{t|t} + P_{t|t} P_{t+1|t}^{-1} \left(F_{t+1} - F_t\right)
$$

$$
P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}
$$

[9] See also Kim and Nelson (1999a) for a detailed description of how to deal with dynamic factor models with serially correlated idiosyncratic components.

Elements of $\rho(\tau)$ Given a draw for F_T and the yields $y(\tau)$ we compute $e_t(\tau)$. The draw for $\rho(\tau)$ involves dealing with an AR(1) model with a heteroscedastic error term. We proceed by removing the heteroskedasticity by computing the following normalised idiosyncratic components $\tilde{e}_t(\tau) = \frac{e_t(\tau)}{\psi^{1/2}(\tau)}$ where $\psi(\tau)$ is the draw for stochastic volatility obtained from the previous iteration. Following this transformation, the draw for $\rho(\tau)$ is standard and can be carried out via the methods described in Kim and Nelson (1999a) (Chapter 7).

Elements of $\psi(\tau)$ Given $e_t(\tau)$ and $\rho(\tau)$, we apply the Metropolis Hastings algorithm in Jacquier *et al.* (2004) for each maturity τ to draw the stochastic volatilities $\psi(\tau)$.

Time Varying VAR

Given an estimate for the factors, the model becomes a VAR model with drifting coefficients and covariances. This model has become fairly standard in the literature and details on the posterior distributions can be found in a number of papers including Cogley and Sbordone (2005) and Primiceri (2005). Here, we describe the algorithm briefly.

VAR coefficients Φ_t As in the case of the unobserved factors, the time-varying VAR coefficients are drawn using the methods described by Kim and Nelson (1999a). We only retain draws that are stable at each point in time.

Elements of H_t Following Cogley and Sbordone (2005), the diagonal elements of the VAR covariance matrix are sampled using the methods described by Jacquier *et al.* (2004).

Element of A_t Given a draw for Φ_t the VAR model can be written as

$$A_t'\left(\tilde{Z}_t\right) = u_t$$

where $\tilde{Z}_t = Z_t - \gamma_t - \sum_{p=1}^{P} \beta_{t,p} Z_{t-p} = v_t$ and $VAR(u_t) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state space models described by Kim and Nelson (1999a) can be applied.

VAR hyperparameters Conditional on Z_t, $\phi_{l,t}$, H_t, and A_t, the innovations to $\Phi_{l,t}$, H_t, and A_t are observable, which allows us to draw the hyperparameters—the elements of Q, S, and the σ_i^2—from their respective distributions.

Appendix B: Convergence

In Figure below, we plot the posterior means of key model parameters. These means are computed for every 20^{th} draw of the first 2000 retained draws of the Gibbs sampler. The Figure shows that there is little evidence of large fluctuations in the posterior means. This provides evidence in favour of convergence.

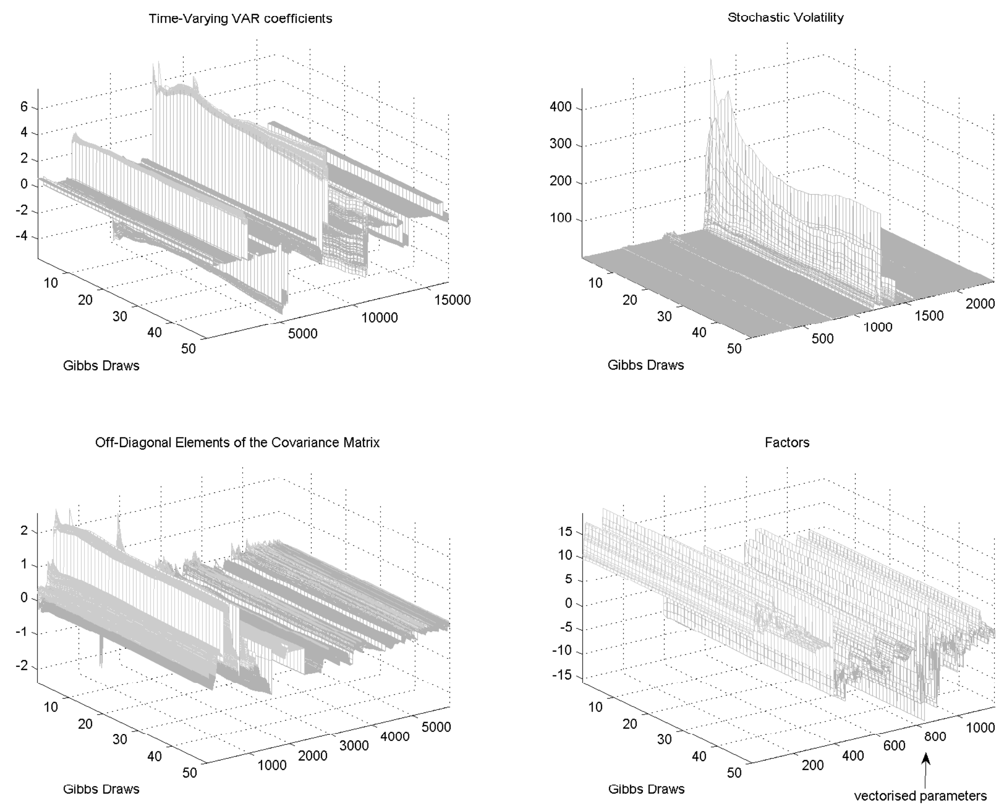

Figure 12: Posterior means of key parameters