Industry and the Family: Two Engines of Growth*

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Abstract

We generalize the class of endogenous growth models in which the scale of the economy has level rather than growth effects, and study the implications of different demographic and technological factors when both fertility choice and research effort are endogenous. The model incorporates two dimensions of technological progress: vertical (quality of goods) and horizontal (variety of goods). Both dimensions contribute to productivity growth but are driven by different processes and hence respond differently to changes in fundamentals. Specifically, while unbounded vertical progress is feasible, the scale of the economy limits the variety of goods. Incorporating a linearity in reproduction generates steady-state population growth and variety expansion. We thus have two engines of growth generating dynamics that we compare with observed changes in demographics, market structure, and patterns of growth. Numerical solutions yield the important insight that, while endogenous, fertility responds very little to industrial policies. Demographic shocks, in contrast, have substantial effects on growth.

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1 Introduction

This paper develops a model that captures the basic qualitative aspects of economic growth from 1870 to the present day. For the United States the facts are as follows:

1. The in-house corporate R&D lab is the dominant research method for bringing about innovations. Accordingly, most innovations are brought to market by existing firms.[1]

2. Most genuinely new products, in particular those that start new industries, are brought to market by new firms.

3. R&D effort is increasing in firm size and firm size is generally increasing over time.

4. With the exception of the major shock caused by the two World Wars, fertility is decreasing.

5. Growth appears to fluctuate around a constant mean. However, several shocks during this period create visible transitions, most notably after the Great Depression and World War II (WWII).

Our model features endogenous growth and endogenous fertility. It embodies the facts about market structure indicated in 1-3 and it is consistent with the stylized facts about growth and fertility in 4-5.

Recent formulations of the theory of endogenous technological change acknowledge that there exist two dimensions of technological advance: vertical (quality of goods) and horizontal (variety of goods). We present a model in which research by incumbent firms increases the quality of goods, while entry of new firms expands the variety of goods. Both types of technological progress contribute to productivity growth, although they are driven by different processes and respond differently to policies and exogenous shocks. In particular, in steady state, vertical progress is unaffected by population size while the scale of the economy limits the number of firms that can operate and, therefore, limits the variety of goods. We incorporate a linearity in reproduction that generates steady-state population growth. The resulting steady-state variety expansion, combined with vertical innovation, yields two interdependent engines of growth.

[1] This has been true since approximately 1870, when the first in-house R&D labs were established in the chemical and electrical engineering industries in the U.S. and Germany. See Freeman and Soete (1997), p. 19, 20, 69, 89, Scherer and Ross (1990) and Mowery and Rosenberg (1989).

The key distinction between the two dimensions of technological progress is the manner in which existing knowledge affects research productivity. The notion that accumulation of knowledge by firms on the intensive margin expands public knowledge and generates spillovers is familiar from the standard literature on growth (Aghion and Howitt 1990, Romer 1990, Grossman and Helpman 1991). Less familiar is the notion that in the horizontal dimension knowledge accumulation leads to greater specialization of R&D, implying a dilution of spillovers (Peretto and Smulders 2002). Incorporation of these concepts yields a model wherein constant returns to scale to knowledge in the vertical dimension support endogenous growth independently of population size and growth, while the productivity of horizontal research is not linear in the number of firms so that the expansion of the number of products is not self-sustaining.[2] Along the horizontal dimension, therefore, positive population growth is necessary to allow for positive firm entry in steady state. The *qualitative* distinction between the two dimensions of technological change assigns to endogenous fertility a crucial role in our understanding of the fundamentals that drive modern growth.

This setup allows us to draw a distinction between *intensive* and *extensive* growth. The former is characterized by slow population growth in economies where technological progress takes the form of innovations developed and implemented within existing firms, with relatively little firm entry. Extensive growth, in contrast, is characterized by fast population growth in economies where the rapid expansion of the resource base allows high entry rates of new firms. We study the evolution of five variables:

- the economy's growth rate;
- the relative contribution of intensive and extensive growth;
- the population growth rate;
- the number of firms per capita;
- the average number of employees per firm.

These variables provide a rich description of the growth path of the economy in terms of the *rate* and *direction* of technological progress and in terms of *industrial density* and *firm size*.

[2] While horizontal innovation also benefits from spillovers from existing knowledge, the costs of entry are also increasing with this stock since the entrant joins the industry at the average quality level. These effects cancel each other, leaving horizontal R&D to be independent of the stock of knowledge.

The model's structure is sufficiently simple to reduce to a two-dimensional system of differential equations whose behavior we can study analytically. The transitional dynamics of the model can be compared with observed changes in market structure and patterns of economic and demographic growth. To complement our qualitative results, we carry out a quantitative analysis of the dynamics in response to exogenous shocks.

In our exercises we distinguish between shocks favoring R&D by incumbent firms versus shocks favoring entry of new firms. A decrease in R&D costs for incumbents (vertical innovation) leads to faster quality growth and slower population growth, which in turn slows down variety expansion. As a result, the composition of the economy's growth rate changes, with intensive growth becoming relatively more important. Since both forms of technological progress contribute to productivity growth, the effect of this shock on the growth rate of income per capita is theoretically ambiguous. Our numerical exercise, however, shows that per capita income growth increases since the fall in population growth is very small. In contrast, a decrease in R&D costs for entrants (horizontal innovation) reduces industrial density and lowers intensive growth. Extensive growth increases, but primarily in the short run since in the long run it is determined by the only marginally higher population growth. As a result, the shock lowers per capita income growth.

We also consider demographic shocks, such as changes in mortality and changes in the cost of childrearing. These shocks have theoretically ambiguous effects on economic growth due to the *interdependence* of demographic and industrial factors. In our numerical work, however, we resolve these ambiguities and provide insights on how these shocks affect economic and demographic growth.

What do we learn from our approach and these results? A natural benchmark for comparison is Peretto (1998b), which considers exogenous population growth within a similar industrial setting. The value added of endogenous population growth is the inclusion of an additional margin of choice. Specifically, our model has an industrial block and a demographic block that interact in capital and labor markets. In the capital market the return to investing in innovation is compared to the return to investing in children. In the labor market, industry competes for labor with reproduction. In Peretto (1998b) population growth is not a choice variable. It neither competes for resources with the industrial sector nor earns a return that households compare to the returns to financial assets. What does this additional margin of choice give us? First, we find theoretically ambiguous results concerning parameters that have unambiguous effects in the case of exogenous population growth. Second, while in Peretto (1998b) it is only possible to consider exogenous increases or decreases in population growth, in our model we can consider richer demographic shocks. For example, we can distinguish between the effects of a reduction in the cost of childrearing and those of a fall in mortality. Both shocks yield higher net fertility and could be thought as equivalent ways to generate (rationalize) the shocks to exogenous population growth analyzed in Peretto (1998b). Our model shows that with endogenous fertility the two shocks are fundamentally different. Finally, in our model the compositional effects of changes in the fundamentals reflect society's adjustment along two interdependent dimensions, an adjustment that takes into account that more effort in one dimension involves an opportunity cost in terms of the other. These compositional effects are trivial in Peretto (1998b) in the sense that they are driven exclusively by the change in the rate of vertical innovation (since the rate of horizontal innovation is pinned down by the exogenous growth rate of population). In summary, in this paper we are able to 1) consider richer and more explicit demographic shocks, 2) show that demographic shocks play a quantitatively important role in the economy, and 3) show that fertility is quantitatively insensitive to industrial shocks. Moreover, we do a much better job of matching the facts since Peretto (1998b), by construction, cannot match the falling rate of fertility that characterizes the period under consideration.

It is also useful to relate our work to the recent literature that has introduced demographic change in models of growth (Jones 2001, Hansen and Prescott 2002, Galor and Weil 2000, Lucas 2002).[3] All these papers consider the Industrial Revolution and argue that endogenous demographic change is necessary to understand the historical evidence. They thus incorporate fertility choice in models where growth is driven by technological change and/or human capital accumulation. Unlike these contributions, we make no attempt to explain why and when the Industrial Revolution occurred. Our model is rich enough that we could use it to generate an endogenous regime switch (in a spirit similar to the Galor-Weil approach). We do not pursue this avenue, however, in order to focus on the modern growth period and on the analysis of the effects of changes in fundamentals, some of which have a policy interpretation. Thus, the goal of our paper is to think more generally about the implications of the interdependence of industry and demography in an environment where they compete for resources.

[3]Other recent related papers include Dalgaard and Kreiner (2001), Kelly (2001) and Kögel and Prskawetz (2001).

While our focus is different, our work is related to the literature on the Industrial Revolution in that it incorporates fertility choice and technological change as interdependent determinants of economic growth. The main difference is that while we agree that demographic factors must be included in a good theory of growth, we believe that those contributions do not provide a good description of industry and its evolution. Thus, a key distinction between those papers and our work is that we treat technological progress as a costly activity undertaken by firms that operate within a specific market structure. We thus follow the more traditional endogenous growth literature that has focused on models where private incentives motivate technological progress. Admittedly, this literature has focused on modeling the driving force behind technological progress and ignored population growth which, historical evidence suggests, play a very important role. In line with this criticism, our model allows for endogenous fertility without sacrificing the principle that private incentives shaped by the structure of the market motivate technological progress. It thus provides a richer description of the interaction between demographic and industrial factors.[4] In addition, and more importantly, it brings back to the forefront the very force that drives growth over recorded human history.

The paper is organized as follows. In the next section we discuss the key features of the modern growth period, emphasizing changes in market structure and demographics. In Section 3, we set up the model and discuss its preliminary properties. In Section 4, we solve the model and characterize the economy's equilibrium dynamics and its steady state. In Sections 5 and 6, we study qualitatively and quantitatively the economy's response to exogenous shocks to key parameters. We conclude in Section 7.

2 Illustrative Data

Figures 1 through 5 present raw data for the United States from 1870 to 1996 on net population growth, per capita gross domestic product (GDP) growth, average firm size in the manufacturing industry, manufacturing establishments per capita and per manufacturing employee, and the population level and number of manufacturing establishments.

[4] Specifically, because of its focus on in-house research, this model is more relevant to the period starting with the late nineteenth century when the corporate R&D lab began to impose itself as the dominant way of undertaking innovation. There is ample evidence of fundamental changes in the mode of conducting industrial R&D during the 20th century. For a more detailed discussion of these issues in the context of a growth model of this class, see Peretto (1998a).

Figure 1 suggests there might be as many as five distinct phases for population growth from 1870 to 1996: the late 19th century, the turn of the century up to World War I (WWI), the Interwar period, the post-WWII period, and the late 20th century. Similarly, looking at Figure 2 we see many shocks to per capita GDP growth, including but not limited to WWI, the Great Depression, and WWII. Maddison (1995) dates the beginning of modern growth at 1820 and suggests that since then there have been five distinct "phases of capitalist development" delineated by time periods during which the growth process clearly "...differed from those which preceded and followed"(p. 59). Moreover, he argues that the transition from one phase to another was not endogenous, but rather the result of exogenous historical accidents.[5] Despite this variability, however, we can identify the following general trends.

1. With the exception of shocks caused by the two World Wars, fertility decreases over time (Figure 1).

2. The growth rate is not constant and appears to fluctuate around a constant mean (Figure 2). Several major shocks, however, create transitions that are clearly visible. The most obvious are the Great Depression and WWII.

3. Firm size generally increases over time (Figure 3).

4. Manufacturing establishments and population are highly correlated, supporting the setup of our model in which population growth bounds the rate of firm entry in steady state (Figure 4).

5. Manufacturing establishments per employee and manufacturing establishments per capita follow one another closely (Figure 5). Hence, establishments per capita are a reasonably good proxy for establishments per employee since the two time-series exhibit a similar qualitative behavior. (This allows us to simplify the model as we explain below.)

Overall the data suggest that the modern growth period is not a single unified period but a sequence of periods during which demographic and industrial structure interact and respond to different exogenous shocks. Yet, one can identify some general trends. Our model generates dynamics that

[5] According to Maddison's dating, the five phases cover the following periods: 1820-1870, 1870-1913, 1913-1950, 1950-1973, 1973-1992. Each coincides with intervals between major historic events.

are consistent with these trends. According to Madison (1995), moreover, these trends are common across all advanced industrial economies. Hence, we provide data for the United States – the best documented example – but our claims above apply to Western Europe as well.

3 A model of endogenous fertility and growth

We build on previous work by Becker and Barro (1988), Barro and Sala-i-Martin (1995), and Peretto (1998a, 1998b). We consider a closed economy populated by a representative family whose members supply labor services and consumption loans in competitive labor and capital markets. Each member is endowed with one unit of labor.

Our model considers technological progress in a single country in isolation. Over the period that we consider, however, spillovers have made the frontier of knowledge accessible to all leading countries. Hence, the technological growth that we model is more appropriately seen as applying to the group of countries that industrialized at roughly the same time – the U.S. and Western Europe.

3.1 Consumption, Saving and Fertility

We follow Barro and Sala-i-Martin (1995, Ch. 9), who study a continuous time version of the model developed by Becker and Barro (1988) where parents care about consumption, (adult) family size and the number of children. At time t the family maximizes

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \left[\log C(\tau) + \eta \log N(\tau) + \psi \log \dot{N}(\tau)\right] d\tau, \qquad (1)$$

where C is consumption per capita, N is family size, which is also the economy's population, $\dot{N}$ is the number of children. $\eta > 0$ is the elasticity of instantaneous utility with respect to family size; $\psi > 0$ is the elasticity with respect to children. $\rho > 0$ is the discount rate.

Let l be the fraction of its time endowment that each family member allocates to rearing children. Since each individual has one unit of labor, we have $0 \leq l \leq 1$. Births per family member are given by $b = \frac{l}{\beta}$, where β, the cost of reproduction, is independent of population size. Assuming an exogenous mortality rate, $\delta > 0$, population growth is

$$n \equiv \frac{\dot{N}}{N} = b - \delta = \frac{l}{\beta} - \delta. \qquad (2)$$

Hence, there is constant population growth if people allocate a constant fraction of their time to reproduction.

The family maximizes utility (1) subject to (2) and the flow budget constraint

$$\dot{A} = (r - n) A + (1 - l) W - E, \tag{3}$$

where A is asset holdings per capita, r is the rate of return on assets, W is the wage rate, and E is consumption expenditure per capita. Notice that population growth appears as a depreciation rate of financial assets per capita; this property plays an important part in interpreting the results that follow.

To derive first-order conditions, we form the Hamiltonian

$$\begin{aligned} H &= \log E - \log P_C + (\eta + \psi) \log N + \psi \log \left(\frac{l}{\beta} - \delta \right) + \\ & \nu \left[\left(r - \frac{l}{\beta} + \delta \right) A + (1 - l) W - E \right] + \mu \left(\frac{l}{\beta} - \delta \right) N, \end{aligned}$$

where ν and μ are, respectively, the shadow values of wealth and people. Notice that we used the fact that $E = CP_C$, where P_C is the price of consumption that the family takes as given. Taking derivatives with respect to the control variables E and l and the state variables N and A, we obtain:

$$\frac{1}{E} = \nu;$$

$$\frac{\psi}{\frac{l}{\beta} - \delta} + \mu N = \nu (A + \beta W);$$

$$\frac{\eta + \psi}{N \mu} + \left(\frac{l}{\beta} - \delta \right) + \frac{\dot{\mu}}{\mu} = \rho;$$

$$r - \left(\frac{l}{\beta} - \delta \right) + \frac{\dot{\nu}}{\nu} = \rho.$$

The first two equations say, respectively, that the household equates the marginal utility of consumption expenditure to the marginal utility of financial wealth and the marginal utility of a child to his marginal cost. The marginal utility of the child consists of his direct contribution to utility, plus his contribution to future fertility; his marginal cost consists of his contribution to diluting financial wealth per capita plus the value of the lost wages due to the time spent rearing him. The third and fourth equations are asset-pricing equations stating, respectively, that the rate of return from adding a family member and from adding one unit of financial wealth must equal the intertemporal discount rate. The return to a family member consists of his direct contribution to utility plus his contribution to future net fertility plus the change in his shadow value. The return to financial wealth consists of the market interest rate net of the rate of population growth[6] plus the change in its shadow value.

The first and fourth equations yield the Euler equation

$$\frac{\dot{E}}{E} = r - \rho - n \tag{4}$$

The third equation reduces to the unstable differential equation,

$$\frac{\eta + \psi}{N\mu} = \rho - \frac{\dot{\mu}}{\mu} - \frac{\dot{N}}{N},$$

which implies that $N\mu = \frac{\eta+\psi}{\rho}$ at all times. Using this result we can rewrite the second first-order condition as

$$n = \frac{l}{\beta} - \delta = \frac{\psi}{\frac{1}{E}(A + \beta W) - \frac{\eta+\psi}{\rho}}, \tag{5}$$

which states that population growth is increasing in consumption expenditure per capita, E, and decreasing in financial wealth per capita, A, and the wage rate, W.[7] The feasibility constraint $1 \geq l \Rightarrow 1 \geq \beta(n + \delta)$ yields

$$E \leq \frac{A + \beta W}{\frac{\psi\beta}{1-\beta\delta} + \frac{\eta+\psi}{\rho}}.$$

[6] This is a depreciation effect similar to the one that obtains in the standard neoclassical growth model (see Barro and Sala-i-martin 1995, Ch. 2).

[7] The intuition for the negative effect of financial wealth is that an increase in A raises the marginal cost of children through the depreciation effect and requires an increase in their marginal utility; this is brought about by a fall in fertility. It is important that we be clear that this relation is *not* the policy function expressing fertility choice as a function of the state of the system – in fact, it contains the current wage and current expenditure in addition to current assets per capita. As a consequence, it cannot answer questions like: What is the effect on fertility of a windfall increase in wealth? In other words, equation (5) is *not* the solution of the model that one should compute in order to determine the effects on fertility choice of an increase in initial wealth or in the lifetime wage (eg. the consumption function that one computes for the neoclassical growth model; see Barro and Sala-i-Martin 1995, Ch. 2).

This inequality implies that the denominator of (5) is positive so that whenever the economy is on a feasible path with $l \leq 1$ population growth is positive. This is intuitive. Since the marginal utility of children goes to infinity as the number of children goes to zero, the household never chooses $n = 0$ in equilibrium.

3.2 Production, Innovation and Entry

A representative competitive firm produces a final consumption good, Y, according to

$$Y = F^{\chi - \frac{\epsilon}{\epsilon - 1}} \left[\int_0^F \left(Z_i^\theta X_i \right)^{\frac{\epsilon - 1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon - 1}}, \tag{6}$$

where X_i is the quantity of the non-durable intermediate good i.[8] At a point in time, the state of technology is represented by the quality, Z_i, and variety, F, of intermediate goods. We follow Ethier (1982) and separate the elasticity of substitution between intermediate goods, $\epsilon > 1$, from the degree of increasing returns to the variety of intermediate goods, $\chi > 0$. Moreover, we allow quality to enter with elasticity less than one, $0 < \theta < 1$.

The final good producer maximizes

$$\Pi_Y = P_Y Y - \int_0^F P_i X_i di$$

subject to (6). This yields

$$X_i = NEP_i^{-\epsilon} Z_i^{\theta(\epsilon - 1)} \left[\int_0^F \left(\frac{P_j}{Z_j^\theta} \right)^{1-\epsilon} dj \right]^{-1}, \tag{7}$$

where we have used the fact that because the consumption good has no other use, market clearing requires $P_Y Y = NE$. (This condition can also be written $Y = NC$ since $E = P_C C$ and $P_C = P_Y$.)

[8] Introducing labor in (6) complicates the model without adding insight. The advantage of doing so would be that we could distinguish between employment in manufacturing (intermediate goods) and employment in, say, services (the final good). This, in turn, would allow us to discuss the rallocation of employment between macrosectors of the economy. For simplicity, we ignore this dimension of change. As we emphasized, the data discussed in the previous section, in particular Figure 3, suggest that the qualitative behavior of our variable measuring industrial density does not depend on the reallocation of labor into and out of manufacturing.

Each intermediate good in (7) is supplied by a specialized firm. With a continuum of firms, each firm is atomistic and takes the term in brackets as given. Monopolistic competition, therefore, prevails and firms face isoelastic demand curves.

We distinguish between incumbents who undertake vertical research to improve the quality of the intermediate good they produce, and entrants who pay an entry cost to establish themselves as the producers of a new variety of intermediate goods. Intermediate goods production follows

$$X_i = L_{X_i}, \tag{8}$$

where X_i is output and L_{X_i} is labor employment. Incumbent firms improve the quality, Z_i, of their differentiated product according to

$$\dot{Z}_i = \frac{L_{Z_i}}{\alpha} K, \tag{9}$$

where L_{Z_i} units of labor are employed for an interval of time dt. The productivity of vertical research depends on the exogenous parameter $\alpha > 0$ and on the stock of public knowledge, K, defined as the average quality level of intermediate firms,

$$K \equiv \int_0^F \frac{1}{F} Z_i di.$$

It is worth emphasizing the reasoning for considering the average quality level. In the horizontal dimension knowledge accumulation leads to greater specialization of R&D. An innovation in an industry that is far removed from one's own will be of little benefit to one's own research. This implies that as an economy becomes larger, supporting more and more varieties, the scale effect due to knowledge spillovers vanishes asymptotically as a result of increasing specialization. The microfoundations for this idea are developed in Peretto and Smulders (1999). Here we posit a spillovers function that immediately yields a zero scale effect because this simplifies the analysis of the dynamics. This is reasonable since we study an economy with positive steady-state population growth, which eventually grows to be large.

This specification implies that there are constant returns to scale to knowledge and, furthermore, that the scale of the economy has no effect on the stock of knowledge. Hence, the model supports a steady state with constant economic growth in the presence of constant population growth.

We now turn to entrants into the intermediate goods sector. An entrepreneur can create a new firm by running an R&D project that uses labor and public knowledge to develop a new differentiated product and its manufacturing process. Since this operation requires labor, it entails a sunk, entry cost. The entrepreneur who incurs this cost becomes an entrant and joins the industry with initial quality equal to average quality, Z. We assume that the cost of entry is proportional to the ratio of the entry level of quality, Z, to the public knowledge stock, K. In other words, the cost of entry is increasing in the initial quality of the intermediate good which will be produced and decreasing in public knowledge because the positive intertemporal spillover from current research to future research is enjoyed by both incumbents and entrants. In this specification, these terms offset each other so that the entry cost ends up depending solely on exogenous factors, like entry subsidies, and we can write it as $\xi > 0$. We can thus posit that new varieties are developed (new intermediate firms are established) according to

$$\dot{F} = \frac{L_F}{\xi}, \tag{10}$$

where L_F is aggregate labor devoted to entry over the time interval dt. The associated rate of horizontal innovation, $f \equiv \frac{\dot{F}}{F}$, is decreasing in F. Hence, in the absence of population growth the rate of horizontal innovation goes to zero as the number of firms becomes large. This means that the population *level* bounds the number of firms that operate in steady state. Only with positive population growth is steady-state horizontal innovation possible.

Our setup describes a world where technology advances on two margins: the intensive margin, where incumbents improve the quality of their products, and the extensive margin, where entrepreneurs expand the variety of products. To close this section, we now discuss firms' behavior on these two margins and the associated returns to vertical and horizontal innovation. An intuitive derivation of these rates of return is provided in the Appendix; here we focus on the key features of the framework and on intuition.

On the intensive margin, the typical firm chooses a time-path of vertical R&D spending that maximizes the present discounted value of profits,

$$V_i(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \left[P_i(\tau) X_i(\tau) - L_{X_i}(\tau) - L_{Z_i}(\tau)\right] d\tau,$$

subject to (7), (8), (9), $Z_i(t) > 0$ (initial quality is given), $Z_j(\tau)$ for $\tau \geq t$ and $j \neq i$ (the firm takes as given the innovation paths of its rivals), and $\dot{Z}_j(\tau) \geq 0$ for $\tau \geq t$ (innovation is irreversible). Proceeding as in Peretto (1998a, Proposition 1), we can show that under the restriction $1 > \theta(\epsilon - 1)$

the Nash equilibrium of this R&D game is symmetric[9] and generates the rate of return to vertical innovation

$$r_Z = \pi \frac{\theta(\epsilon - 1)}{\alpha} - z, \tag{11}$$

where

$$z \equiv \frac{\dot{Z}}{Z} = \frac{L_Z K}{Z\alpha} = \frac{L_Z}{\alpha}$$

is the rate of growth of quality and $\pi \equiv \frac{NE}{\epsilon F}$ is the firm's cash flow (revenues minus variable labor costs). We adopt the convention that firm-level variables without the i subscript denote industry averages.

Expression V_i evaluated along the optimal R&D path is the stock market value of the firm. A Nash equilibrium with entry exists whenever the value of the firm equals the entry cost, $V_i = \xi$. This follows from two considerations. First, we must rule out situations with $V_i > \xi$ because they imply infinite demand for labor in entry and thus violate the economy's resources constraint. Second, whenever $V_i < \xi$ setting up a new firm generates a loss that no entrepreneur is willing to incur. For the time being, we point out that when entrants are active, the rate of return to entry is

$$r_F = \frac{1}{\xi} \left[\pi - \alpha z \right]. \tag{12}$$

Notice that both r_Z and r_F depend positively on the firm's cash flow and negatively on the rate of vertical innovation, z. In the case of the return to vertical innovation, r_Z, the negative dependence on z reflects the intertemporal spillover of public knowledge to the other firms. In the case of the return to horizontal innovation, r_F, the negative dependence on z reflects fixed incumbency costs. To see this, observe that (12) states that the firm breaks even in the sense that its cash flow, π, covers flow R&D expenditure, αz, plus the flow cost of servicing the initial sunk cost of entry, $r\xi$, where r is the market interest rate that in equilibrium must be equal to the return to entry r_F.

[9] The gist of the proof is to show that there are *economic* decreasing returns to R&D so that no one firm can take over the whole market by exploiting *physical* increasing returns to knowledge accumulation. Intuitively, this requires a low elasticity of cost reduction and a low elasticity of substitution (high taste for variety) so that the increase in market share from an increase in the knowledge stock, $\theta(\epsilon - 1)$, is less than 1. This is an Inada-like condition ensuring that the individual firm's rate of return to R&D is everywhere decreasing in the firm's knowledge stock.

3.3 Economic Growth

We now define economic growth formally. Substituting (7) into (6), imposing symmetry, dividing through by population size, and taking logs and time derivatives, we obtain the growth rate of income per capita

$$g \equiv \frac{\dot{Y}}{Y} - n = (\chi - 1) f + \theta z + \frac{\dot{E}}{E}. \tag{13}$$

This expression breaks the growth rate of income per capita into its three components: the rate of entry of new firms, f,which raises specialization and thus productivity if $\chi > 1$; improvement of the quality of existing intermediate goods; and the growth rate of consumption expenditure per capita, a proxy for the rate of market growth due to expanding demand. (Notice that population growth drops out because of its contribution to aggregate consumption expenditure.)

In steady state, the rate of entry is pinned down by population growth, $f = n$, and expenditure per capita is constant. We thus obtain

$$g^* = (\chi - 1) n^* + \theta z^*. \tag{14}$$

The effect of population growth on income per capita growth depends on the degree of increasing returns to variety. This is the important property that static increasing returns due to specialization give rise to a *scale effect in levels* that translates population growth into income per capita growth. We have sufficient flexibility in the model to obtain that population growth negatively affects per capita income growth if the scale effect in levels is weak, if $\chi < 1$, without this necessarily implying negative growth of income per capita.[10] This is intuitive: if static increasing returns to specialization are not sufficiently strong, the dominant effect of population growth is the standard one of spreading aggregate income over more heads, thereby producing falling income per capita.

4 General Equilibrium

Our model has two main blocks: an industry block, where we study the determination of the rates of vertical and horizontal innovation, and a de-

[10] In the semi-endogenous growth models of Jones (1995) and Segerstrom (1998), long-run growth is *proportional* to the exogenous rate of population growth. In the endogenous growth model of Jones (1998), long-run growth is again proportional to population growth but the latter is endogenous.

mography block, where we study the determination of population growth. The interaction of the two blocks in the capital and labor markets generates industrial and demographic change. In the capital market, the rate of return to investing in innovation (quality-improving and variety-expanding) is compared to the rate of return to investing in children. In the labor market, industrial activity (production, innovation and entry) competes with reproduction for labor. In this section, we study the equilibrium dynamics generated by this interaction.

4.1 General Equilibrium Conditions

Recall that $\pi \equiv \frac{NE}{\epsilon F}$ is the firm's cash flow and let $x \equiv \frac{F}{N}$ be the number of firms per capita, our measure of industrial density. This is not the inverse of average firm size since the number of employees per firm is given by $s \equiv \frac{(1-l)N}{F}$.

In equilibrium, total assets are given by the aggregate stock market value of firms. Hence, $A = \frac{VF}{N}$. In the following analysis we normalize the wage, $W \equiv 1$. With this notation, the household's optimal consumption plan must satisfy the budget constraint

$$\frac{\dot{V}}{V} = r - f + \frac{1 - \beta (n + \delta)}{xV} - \frac{\epsilon \pi}{V}, \tag{15}$$

the Euler equation

$$\frac{\dot{\pi}}{\pi} = r - \rho - f \tag{16}$$

and the transversality condition

$$\lim_{T \to \infty} e^{-\int_t^T [r(v) - n(v)] dv} V(T) x(T) = 0.$$

Differentiating the value of the firm, we obtain the familiar expression for the rate of return to equity

$$r = \frac{\pi - \alpha z}{V} + \frac{\dot{V}}{V}. \tag{17}$$

Substituting (17) into the household's budget constraint, we obtain

$$[Vf + \pi (\epsilon - 1) + \alpha z] x = 1 - \beta (n + \delta), \tag{18}$$

where

$$n = \frac{\psi}{\frac{1}{\epsilon\pi}\left(\frac{\beta}{x} + V\right) - \frac{\eta+\psi}{\rho}}.$$

Notice that $V = \xi$ whenever $f > 0$. On the right-hand-side of (18) we have net labor supply, which is given by the aggregate labor endowment net of the time spent raising children. On the left-hand side we have the total demand of labor, which is given by employment in horizontal R&D, production, and vertical R&D. This, of course, is the economy's resources constraint.[11]

The equilibrium path of the economy must satisfy the Euler equation (16), the resources constraint (18), the relation

$$\frac{\dot{x}}{x} = f - n,$$

the feasibility constraint

$$\pi \leq \frac{1}{\epsilon} \frac{V + \frac{\beta}{x}}{\frac{\psi\beta}{1-\beta\delta} + \frac{\eta+\psi}{\rho}},$$

and the transversality condition. In addition, it has to satisfy the equilibrium conditions for the industry side of the model, where we determine the rate of return to investment, r, and the rate of vertical innovation, z. The rate of horizontal innovation, f, is determined residually by the resources constraint.

4.2 Industry Equilibrium and Dynamics

To analyze industry equilibrium, plot the rates of return to vertical and horizontal R&D, equations (11) and (12), in (z, r) space. Under the restriction $\theta(\epsilon - 1) < 1$ one of two growth regimes arises, according to the following conditions:

- Entry with no vertical R&D when $\frac{\alpha}{\xi} \geq 1 > \theta(\epsilon - 1)$;
- Vertical R&D and entry when $1 > \theta(\epsilon - 1) > \frac{\alpha}{\xi}$.

To see this, observe that the condition $\theta(\epsilon - 1) < 1$ implies that the r_Z curve intersects the horizontal axis to the left of the point where the r_F curve does. Accordingly, two configurations are possible. If the r_Z curve is flatter

[11] This expression *is* the labor market clearing condition $(L_X + L_Z) F + L_F = N(1 - l)$.

or has a lower intercept than the r_F curve, if $\frac{\alpha}{\xi} \geq 1 > \theta(\epsilon - 1)$, the returns to horizontal innovation dominate the returns to vertical innovation for all possible values of z. Financial investors then deny resources to incumbent firms and put them into start-up projects. In this case, equilibrium is at the point where the r_F curve intersects the vertical axis. If $1 > \theta(\epsilon - 1) > \frac{\alpha}{\xi}$, in contrast, the r_Z curve has a higher intercept than the r_F curve and an intersection exists. The restriction $\xi > \alpha$ on the slopes of (11) and (12) ensures stability of this equilibrium in the sense that firms have no incentive to deviate from it.

We are interested in the interior equilibrium with both vertical and horizontal innovation. Associated to this equilibrium are the rate of vertical innovation

$$z = \pi \frac{\xi\theta(\epsilon - 1) - \alpha}{\alpha\left(\xi - \alpha\right)} \tag{19}$$

and the rate of return to investment (vertical *and* horizontal R&D)

$$r = \pi \frac{1 - \theta(\epsilon - 1)}{\xi - \alpha}. \tag{20}$$

The intersection of (11) and (12) expresses the arbitrage condition that the returns to the two types of investment must be equalized for agents to be willing to finance both at the same time.

(19) and (20) describe the rate of vertical innovation, z, and the rate of return to investment, r, as two linear functions of the cash flow of firms, π. Substituting (19) and (20) into, respectively, (15) and (16), and using the relation $\frac{\dot{x}}{x} = f - n$, we obtain the system:[12]

$$\frac{\dot{\pi}}{\pi} = \frac{\epsilon\pi}{\xi} - \rho - \frac{1 - \beta\left(n + \delta\right)}{\xi x}; \tag{21}$$

$$\frac{\dot{x}}{x} = \frac{1 - \beta\left(n + \delta\right)}{\xi x} - n + \pi\left[\frac{1 - \theta(\epsilon - 1)}{\xi - \alpha} - \frac{\epsilon}{\xi}\right], \tag{22}$$

where n is given by

$$n = \frac{\psi}{\frac{1}{\pi}\frac{\beta + \xi x}{\epsilon x} - \frac{\eta + \psi}{\rho}}. \tag{23}$$

[12]The fact that the value of the firm is pinned down by the cost of entry eliminates this variable from the system.

Observe that our change of variables yields that population growth depends positively on gross profits per firm and firms per capita. The former property is straightforward since π is proportional to consumption per capita and population growth is increasing in consumption per capita. The latter property is more subtle. Due to our change of variables, firms per capita affect fertility through two channels: the positive expenditure effect and the negative financial wealth effect. The expenditure effect dominates, leading n to depend positively on firms per capita.

We illustrate the dynamics of the system in the phase diagram in Figure 7. A technical appendix available on request provides the complete characterization of the transitional dynamics, along with arguments that allow us to rule out unstable trajectories and establish that the equilibrium path is unique: given initial conditions, the economy jumps on the saddle-path and converges to the steady state that we characterize below.[13]

4.3 Steady-State Analysis

The $\dot{\pi}=0$ and $\dot{x}=0$ loci are:

$$\frac{1-\beta\left(n+\delta\right)}{\xi x}=\frac{\epsilon\pi}{\xi}-\rho; \tag{24}$$

$$\frac{1-\beta\left(n+\delta\right)}{\xi x}=\pi\left[\frac{\epsilon}{\xi}-\frac{1-\theta(\epsilon-1)}{\xi-\alpha}\right]+n. \tag{25}$$

Equations (19), (23) and (24)-(25) describe the steady state in (z,π,x,n) space. From here it is useful to proceed in steps.

First notice that (24) and (25) yield

$$\pi=\frac{\xi-\alpha}{1-\theta(\epsilon-1)}\left(n+\rho\right). \tag{26}$$

This equation describes the capital market equilibrium condition that the rate of return to saving equal the rate of return to investment. We therefore label this curve the capital market locus, CC. Next solve (23) for industrial density to obtain

$$x=\frac{\beta}{\epsilon\pi\left(\frac{\eta+\psi}{\rho}+\frac{\psi}{n}\right)-\xi}. \tag{27}$$

[13] We need to emphasize the following. Figure 7 depicts the projection of the three-dimensional dynamics of the model onto the two-dimensional plane (x,π). As we discuss in detail in the technical appendix, the dynamics of this economy take place at different levels of V according to whether there is entry or no action on the extensive margin.

Substituting this expression into (25) yields

$$\pi = \frac{\frac{1-\beta\delta}{\beta}}{\frac{\epsilon}{\xi}\left(\frac{\eta+\psi}{\rho}+\frac{\psi}{n}\right)\frac{1-\beta(n+\delta)}{\beta}-\frac{\epsilon}{\xi}+\frac{1-\theta(\epsilon-1)}{\xi-\alpha}}. \tag{28}$$

This equation describes the equilibrium of the labor market. We label it the resources locus, RR.[14]

In Figure 7 we determine profits per firm, π^*, and population growth, n^*, as the intersection of the resources locus RR and the capital market locus CC. There are two possible cases, depending on whether $-\frac{\epsilon}{\xi}+\frac{1-\theta(\epsilon-1)}{\xi-\alpha} \lesseqgtr 0$. These are represented in, respectively, panels (a) and (b). In either case the steady state is unique. In the former case it always exists, in the latter it does if the RR locus evaluated at $n = \frac{l}{\beta} - \delta$ is higher than the CC locus evaluated at that point. This requires

$$\rho < \frac{\frac{\xi-\alpha}{1-\theta(\epsilon-1)}}{\frac{\xi}{\epsilon}-\frac{\xi-\alpha}{1-\theta(\epsilon-1)}}\frac{1-\beta\delta}{\beta}.$$

We then use (19) to determine the rate of growth of quality, z^*, and (27) to determine industrial density, x^*. From (14) we finally obtain the rate of growth of income per capita.

For our purposes, it is useful to use (19) and (26) to rewrite (14) as

$$g^* = \pi^*\left[(\chi-1)\frac{1-\theta(\epsilon-1)}{\xi-\alpha}+\theta\frac{\xi\theta(\epsilon-1)-\alpha}{\alpha(\xi-\alpha)}\right]-(\chi-1)\rho. \tag{29}$$

This equation does not contain parameters from the fertility block of the model. Hence, demographic factors affect growth exclusively through their effects on firms' cash flow, π. The response of the growth rate to changes in cash flow, in turn, depends on the degree of increasing returns to variety, χ, in the production function (6). If increasing returns to variety are sufficiently large, the term is positive and the firm's cash flow affects steady-state growth positively.

[14] If we consider the birth rate of the economy, $b = n + \delta$, we can construct the CC and RR loci in (π, b) space. The interesting property is that in this case the death rate, δ, enters the CC locus but not the RR locus. Thus, while the effects of all the other parameters on the birth rate, b, are the same as on net population growth, n, the effects of δ differ. We specify how below.

5 The Transition Path and History

How does the model's transition to the steady state compare to the data? The answer depends on the initial condition, the initial number of firms per capita. Freeman and Soete (1997) present the historical context in which various Schumpetarian waves of innovation took place. They argue that the creation of in-house R&D labs in the 1870s came about in a period of increasing economies of scale and increasing firm size. Quoting Chandler (1977), they suggest that the development of the railway and telegraph system in the U.S. in the mid 1800s "...enabled the USA to exploit its great natural endowments and its vast size to reap scale economies unequalled elsewhere in the world. The railways also made a major contribution by the development of management techniques for large enterprises" (p. 60). They also argue that the availability of cheap steel and electrical power provided the opportunity for a multitude of new investment opportunities. These opportunities further required organizational and managerial innovations to take advantage of these technical innovations: "The managerial and organizational innovations which accompanied, followed, or preceded the technical innovations mostly facilitated a larger scale of operation by the leading firms" (p. 80). Hence there were strong forces pushing for increased market concentration in key industries both in the U.S. and abroad. This also made economies of scale in R&D equally important. For example, in describing the German chemical industry, Freeman and Soete explain that "although individual inventor-entrepreneurs made the major process innovations of the nineteenth century, by the end of that century the scale of flow process experimentation was putting it beyond the reach of the individual ingenious chemist..." (p. 89).

The period starting in 1870 is thus one of increasing market concentration and increasing firm size. In terms of our model, therefore, the relevant initial condition is one with more firms per capita than in steady state. In other words, we consider an economy approaching the steady state from the right in our phase diagram in Figure 7. For illustrative purposes, and to better tie the model to data, we linearize and solve the model numerically in the neighborhood of the steady state. To better quantify the implications of various policies and exogenous shocks, we choose parameter values that produce reasonable steady-state values relative to contemporary U.S. data. Table 1 presents the parameter values that we use.

Figure 8 presents the transition path for the relevant variables. The path is consistent with the stylized facts. We see decreasing population growth, increasing firm size, increasing rates of innovation and income per capita growth. We also observe a higher rate of vertical innovation than horizontal innovation, consistent with the fact that most innovations are brought to market by established firms. Moreover, since this is a model in which the scale of the economy has level rather than growth effects, we see positive steady-state population growth without yielding the counterfactual prediction of explosive income per capita growth. The transition is remarkably consistent with the general trends we observe in the U.S. from 1870 to today.

6 Dynamic Effects of Changes in Fundamentals

In this section, we consider two sets of experiments. The first focuses on the industry block by considering reductions in R&D costs. The second set of experiments focuses on the fertility block by considering demographic shocks. As for the case of R&D costs, in the case of changes in child-rearing costs there is a corresponding policy that accomplishes similar effects via a subsidy financed with non-distortionary taxes or a tax whose revenue is rebated in a lump-sum fashion to households.

6.1 Experiment 1: Lower Research Costs

An important advantage of our model is that it allows us to distinguish between shocks that favor one technological dimension over the other. Most R&D-based growth models are either of the quality-ladder or of the variety-expansion type. Hence, any discussion of changes in the cost of research is by definition limited to the one dimension of technological progress incorporated in the model. To see the importance of this distinction, let us consider a decrease in research costs faced only by incumbents, then only by entrants, and then by both.

Lower vertical R&D costs. Consider the effects of a five percent reduction in α. The three graphs in Figure 9 illustrate, respectively, changes in market structure, rates of innovation, and economic and demographic growth over the transition to the new steady state. As one would expect, the immediate impact is to push up vertical innovation, while decreasing horizontal innovation. Increased profitability raises the rate of return to both horizontal and vertical innovation; see equations (11) and (12). Hence, firm entry returns to a level only slightly below its original one (since in the long run it is constrained by population growth, which decreases slightly). From equation (23), we see that population growth depends positively on both profits and firms per capita. Firms per capita can not jump. Hence, the initial jump up in n is caused by the immediate increase in profitability, π. The combination of temporarily faster population growth and slower firm entry leads to a permanent fall in the number of firms per capita, x. As x declines over the transition, population growth slows to its new slightly lower steady-state level.

From (13), we know that per capita income growth depends positively on vertical innovation, but depends positively or negatively on horizontal innovation depending on the degree of increasing returns to variety. In the parameterization illustrated here, $\chi > 1$ implies that growth depends positively on horizontal innovation (and hence in steady-state, on population growth). The rise in growth caused by the jump in z is partially offset by the initial fall in firm entry. However, as firm entry recovers partially and population growth diminishes, per capita income growth continues to rise towards its new higher steady-state level. Overall, the increase in steady-state growth is entirely attributable to the increase in vertical R&D since horizontal innovation is now lower. Hence, a shock (or policy) that stimulates vertical R&D is quite successful at improving growth.[15]

Lower entry costs. Figure 10 shows the graphs for a five percent reduction in the cost of entry, ξ. This reduction in entry costs has drastically different implications from the reduction in vertical research costs just discussed. As expected with lower entry costs, firm entry shoots up, pushing down profits per firm. Decreased profits lower vertical innovation and population growth. However, the initial jump in firm entry combined with the fall in population growth leads to a gradual increase in firms per capita. This allows population growth to recover and even rise slightly relative to its initial steady-state level. The initial boom in horizontal innovation is large enough to initially pull up the growth of income per capita, despite the fall in vertical innovation. Still, as firm entry converges to an only marginally higher steady-state, bounded by the only marginally increased steady-state population growth, per capita income growth falls to its new lower steady-state level. This long run decrease in the growth rate of income per capita is therefore entirely attributable to the fall in vertical innovation.[16] So a shock

[15] Note that if $\chi < 1$ our results would be similar except for the shape of the transition path for growth. There would be an overshooting of growth since the initial fall in firm entry would boost g. Per capita income growth would then have gradually fallen reaching a slightly higher steady state (relative to the case with $\chi > 1$) as firm entry rose in transition to its lower steady-state level.

[16] Once again it is worth noting that if there were smaller increasing returns to variety, this temporary boom in g would not occur. Instead there would be an initial negative overshooting of g.

(or policy) beneficial to horizontal innovation (and presumably growth) only marginally increases horizontal innovation while strongly harming vertical innovation, and hence ends up damaging economic growth.

A key thing to note when considering these types of industrial shocks (or policies) is that effects on vertical R&D are permanent, whereas effects on horizontal R&D are concentrated in the short run, leaving only minor long run effects. This is due to the very nature of the market structure that we consider whereby in the long run horizontal innovation is bounded by population growth. Consequently, we observe that a decrease in vertical R&D costs leads to a permanent increase in the rate of vertical innovation. Horizontal innovation decreases primarily in the short run and in the long run is only slightly lower. In contrast, decreased firm entry costs permanently lower vertical innovation, while only temporarily giving a strong boost to horizontal innovation since in the long run it is pinned down by population growth, which increases only marginally.

Lower vertical R&D and entry costs. Consider a five percent reduction in both α and ξ. As with most models with horizontal and vertical innovation there is a neutrality result that a proportional and equal cost reductions in both types of R&D do not have long-run growth effects (see, e.g., the analysis of R&D subsidies in Peretto 1998b and Young 1998). The reason is that both rates of return are proportional to the cost of R&D. Hence, equal proportional reductions in R&D costs on the two margins cancel out in the arbitrage condition that defines the equilibrium of the industry.

Figure 11 shows the transition to the new steady state with reduced industrial density and lower profits per firm. With the subsidies to in house R&D and firm entry, there is an immediate jump in both vertical and horizontal innovation. The economy's growth rate initially shoots up due to the initial increase in both types of innovation. Decreased profits push population growth down in the short run. In the long run, population growth is unchanged (since increased firms per capita exactly offset the decrease in profits). This result is analytical, not just numerical.[17] Firm entry and vertical innovation, with their relative costs unchanged return to their original steady-state levels. So the effect of equal and proportionate cost reductions in both types of research is only a short run boom and reduced long run market concentration.

Overall these experiments show that industrial shocks (and equivalently industrial policy) affect economic growth almost entirely through its effects

[17] Substitution of (26) into (28) yields an implicit equation in n with the property that identical proportional changes in α and ξ leave n unchanged.

on vertical innovation. Why is horizontal innovation less important? Because in the long run it is bounded by population growth, which while endogenous, is not very responsive to industrial policies. Hence, the key insight we gain from these numerical exercises is that when one considers plausible parameter values for fertility choice, we find that households make only marginal changes in their fertility decisions in response to industrial policies. Since these choices affect the resource constraint of the economy and limit the number of firms that can exist at any moment in time, there is relatively little long run play that can occur through firm entry. The intensive margin, instead, is not bounded and is thus the one dimension able to respond strongly in the long run.

6.2 Experiment 2: Demographic Shocks.

We now consider demographic shocks that directly affect (and work through) population growth. We consider a fall in mortality and a rise in the cost of rearing children.

Fall in mortality. Figure 12 illustrates the effects of a 5 percent fall in the mortality rate, δ. The immediate consequence in the fall in mortality is to push up net population growth, while immediately lowering the fraction of time, l, devoted to childrearing. Higher n increases firm profits thus encouraging increases in both vertical and horizontal innovation. After the initial fall in l, the resources people devote to childrearing partially recover since profits are higher and firms per capita are increasing over the transition. Despite the lowered steady-state birth rate, net population growth remains permanently higher as a direct consequence of decreased mortality. Economic growth, g, increases permanently both because of the increase in vertical and in horizontal innovation. The temporary boom when g initially overshoots is attributable to the initial overshooting of firm entry.

Increase in the cost of reproduction. Figure 13 considers the effects of a five percent increase in β. As expected with increased costs of reproduction, there is an immediate drop in the birth rate. Population growth continues to fall over the transition (due to decreasing firms per capita). Profits per firm drop immediately to their new lower steady-state level. Consequently, vertical innovation and firm entry both drop. Despite lower birth rates, resources devoted to childrearing overshoot initially as a direct consequence of increased reproduction costs, and then fall back partially as families decide to have fewer children. Since resources devoted to childrearing jump initially and then partially return, the huge initial pulling of resources away from horizontal innovation, combined with lower profits, causes it to overshoot downwards and then gradually approach its new lower steady-state from below. Growth of income per capita falls immediately and recovers partially (as horizontal innovation partially recovers), gradually reaching its new lower steady-state. Here we see that an increase in the cost of reproduction has effects similar to those we might expect if our natural resource, which is fixed at any one point in time, were decreased, leaving fewer resources for all types of R&D.

The main mechanism in these demographic experiments is the reallocation of labor between reproduction and innovation. In the case of decreased mortality, fewer resources are needed to achieve the household's planned rate of population growth. This frees up labor to be used in R&D, leading to greater growth of income per capita. On the other hand, increased reproduction costs, force more resources to be devoted to childrearing, drawing them away from innovation, thus harming economic growth. Hence, demographic shocks/policies have positive or negative effects on economic growth depending on whether they require less or more resources to be devoted to childrearing.

7 Conclusion

This paper has presented a model with endogenous fertility and technological progress that occurs in two dimensions: vertical (quality of goods) and horizontal (variety of goods). While both types of technological progress contribute to productivity growth, they are driven by different processes and respond differently to policies and exogenous shocks. In particular, in steady state vertical progress is unaffected by population size while the scale of the economy limits the number of firms that can operate and, therefore, limits the variety of goods. It follows that positive steady-state population growth allows for positive steady-state variety expansion. The model, therefore, has two interdependent engines of growth. This allows us to draw a distinction between *intensive* and *extensive* growth. The former is characterized by slow population growth in economies where technological progress takes the form of innovations developed and implemented within existing firms, with relatively little formation of new firms. Extensive growth, in contrast, is characterized by fast population growth in economies where the rapid expansion of the resource base allows high entry rates of new firms.

R&D cost reductions for incumbents (vertical innovation) lead to faster intensive growth, higher industrial density, and slightly slower population growth (extensive growth). Overall, income per capita grows more quickly.

In contrast, R&D cost reductions for entrants (horizontal innovation) reduce industrial density and intensive growth. Extensive growth increases, but only temporarily since in the long run it is pinned down by the only slightly higher rate of population growth. As a result, growth of income per capita is lower.

Our simulations show that while endogenous, demographic growth responds very little to industrial shocks. This implies that firm entry has strong effects on per capita income growth only in the short run since in steady state it is bounded by population growth which changes very little. Almost all of the play from industrial policies like research subsidies will therefore come from their direct effects on vertical R&D. In contrast, demographic policies affect economic growth positively or negatively depending on whether they release or pull resources away from industrial activity, by requiring fewer or greater resources to be devoted to childrearing. These results suggest that when studying the effects of industrial policy it may be reasonable as a first approximation to take population growth as exogenous. When considering demographic shocks, however, this is clearly inappropriate given their strong effects on economic growth.

These experiments highlight the importance of carefully modeling both industry and the family when attempting to describe the modern growth period. We take the recent literature which incorporates demographic factors into models of economic growth to heart, but emphasize that the role of technological progress must not be downplayed in such endeavors. We have, therefore, created a model that allows for endogenous fertility without sacrificing the principle that private incentives shaped by the structure of the market motivate technological progress. This allows us to study industry and the family as two interdependent forces driving a country's process of economic transformation. The model thus provides a richer description of the interaction between demographic and industrial factors. In addition, and more importantly, it brings back to the forefront the very force that drives growth over recorded human history.

8 Appendix: Derivation of the Rates of Return to Vertical and Horizontal R&D

Suppose that firms finance R&D by issuing ownership claims on the flow of profits generated by quality-improving innovations. Let the market value of such financial assets be q_i. A firm is willing to undertake R&D if the value of the innovation is at least as large as its cost. Using (9), we can write

$$q_i = \frac{\alpha}{K} \Leftrightarrow L_{Zi} > 0. \tag{30}$$

Since the innovation is implemented in-house, its benefits are determined by the marginal profit it generates. Thus, the return to the innovation must satisfy the arbitrage condition

$$r = \frac{\partial \Pi_i}{\partial Z_i}\frac{1}{q_i} + \frac{\dot{q}_i}{q_i}. \tag{31}$$

To calculate the marginal profit, observe that the firm's problem is separable in the pricing and R&D decisions. The optimal pricing strategy is the traditional mark-up rule $P_i = \frac{\epsilon}{\epsilon - 1}$. The firm's profit can be written

$$\Pi_i = \frac{NE}{\epsilon} Z_i^{\theta(\epsilon-1)} \left[\int_0^F Z_j^{\theta(\epsilon-1)} dj \right]^{-1} - \phi - L_{Zi}.$$

Differentiating under the assumption that firms take the term in brackets as given, we obtain

$$\frac{\partial \Pi_i}{\partial Z_i} = \frac{NE}{\epsilon}\theta\left(\epsilon - 1\right) Z_i^{\theta(\epsilon-1)-1} \left[\int_0^F Z_j^{\theta(\epsilon-1)} dj \right]^{-1}.$$

Substituting this expression into (31), taking logs and time derivatives of (30), and rearranging terms yields

$$r = \frac{NE}{\epsilon}\theta\left(\epsilon - 1\right) Z_i^{\theta(\epsilon-1)-1} \left[\int_0^F Z_j^{\theta(\epsilon-1)} dj \right]^{-1} \frac{K}{\alpha} - \frac{\dot{K}}{K}.$$

Imposing symmetry (i.e., $Z_i = K$ for all i) and using (9) yields (11) in the text. Suppose now that firms finance variety-expanding innovations by issuing equity. As argued in the text, entry is positive if the value of the firm is at least as large as its cost. That is,

$$V_i = \xi \Leftrightarrow L_N > 0. \tag{32}$$

The profit that accrues to an entrant is given by the expression that we derived for incumbents. Hence, the market value of firms' shares satisfies the arbitrage condition

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i}.$$

Using (32),we can write

$$r = \frac{1}{\xi} \left[\frac{NE}{\epsilon} Z_i^{\theta(\epsilon-1)} \left[\int_0^F Z_j^{\theta(\epsilon-1)} dj \right]^{-1} - \phi - \frac{\alpha}{K} \dot{Z}_i \right],$$

where we have made use of (9). Imposing symmetry yields (12) in the text.

Table 1. Benchmark Parameter Values

Parameter	Description	Notes
$\rho = 0.04$	subj. disc. rate	Unit of time = one year.
$\delta = 0.01315$	mortality rate/ prob. of death	This yields a life expectancy at birth of 76 years.
$\psi = 0.7$	elast. of utility w.r.t. children ($\psi>0$)	Values ranging from 0.2 to 1.2 were used without qualitatively affecting the results.
$\eta = 0.7$	elast. of instant. utility w.r.t. family size ($\eta>0$)	Values ranging from 0.2 to 8 were used without qualitatively affecting long run growth results. Still, the shape of the transition path depends on η and the response of x to changes in β differs with low versus high η. In all other experiments the long run effects are qualitatively similar for all variables.
$\beta = 40$	cost of reprod.	This yields $n = .006$. 1997 U.S. net pop. growth was .009.
$\epsilon = 2.5$	elast. of subst. betw. intermed. goods ($\epsilon>1$)	Estimates for ϵ range from 2.5 to 20 (Basu 1993 and Norrbin 1993).
$\theta = 0.3$	elast. of prod. w.r.t. quality ($1>\theta>0$)	Based on the symmetry condition, $1>\theta(\epsilon-1)$.
$\xi = 1200$	cost of vertical R&D (in units of labor)	Sutton (1992), p.104, estimates that the setup cost for the food industry is .03%- 2% of industry output. For the manuf. industry, this implies 14 - 943 (in units of L). These numbers serve as a guess as to a reasonable magnitude for ξ. Values ranging from 213.7 to 28000 were used without qualitatively affecting the results (allowing α to vary with ξ as explained below).
$\alpha = 325$	cost of entry (in units of labor)	Given ξ, α is chosen to focus on the case where vertical research depends positively on π, and to yield x and π consistent with U.S. data.
$\chi = 1.75$	degree of IR to variety of intermed. goods in prod.	Whether χ is greater than or less than one affects whether growth depends positively or negatively on population growth (eq. 14) and therefore affects the shape of the transition path for growth. Still, the parameter value does not greatly affect the long run results beyond pushing growth up as χ increases above 1.

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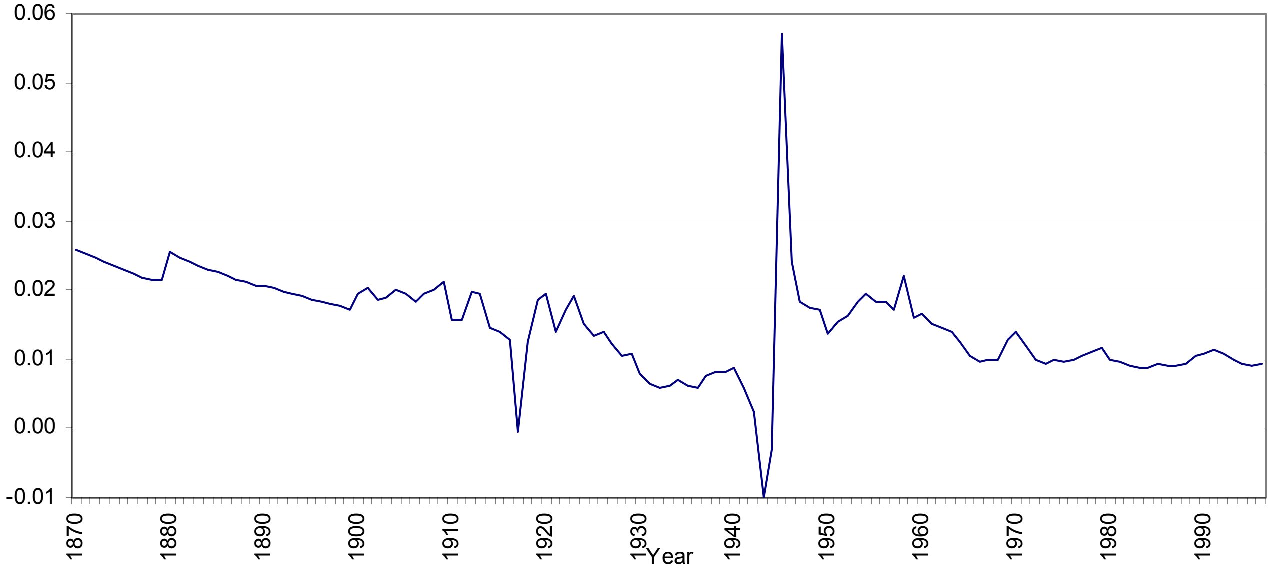
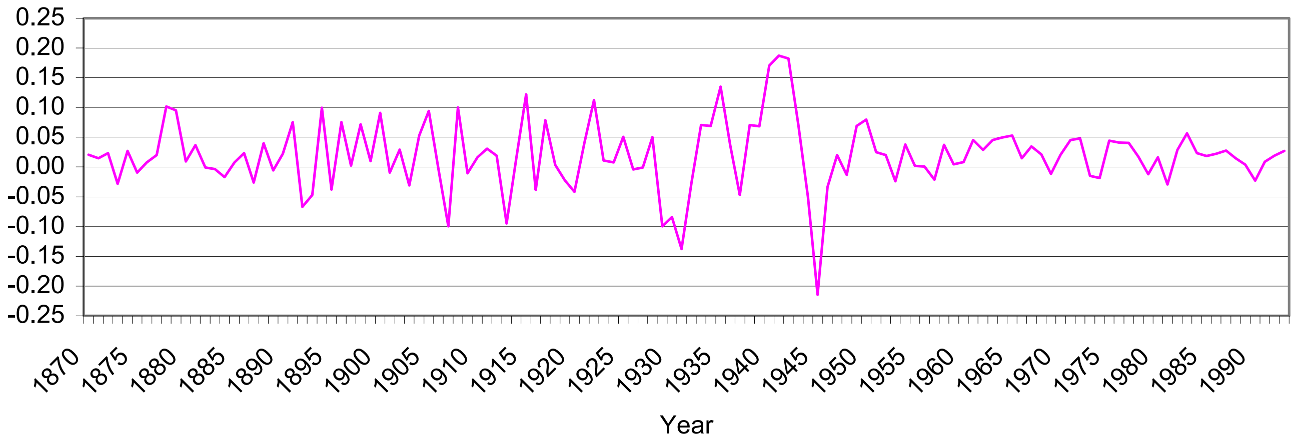
Figure 1: Net Population Growth

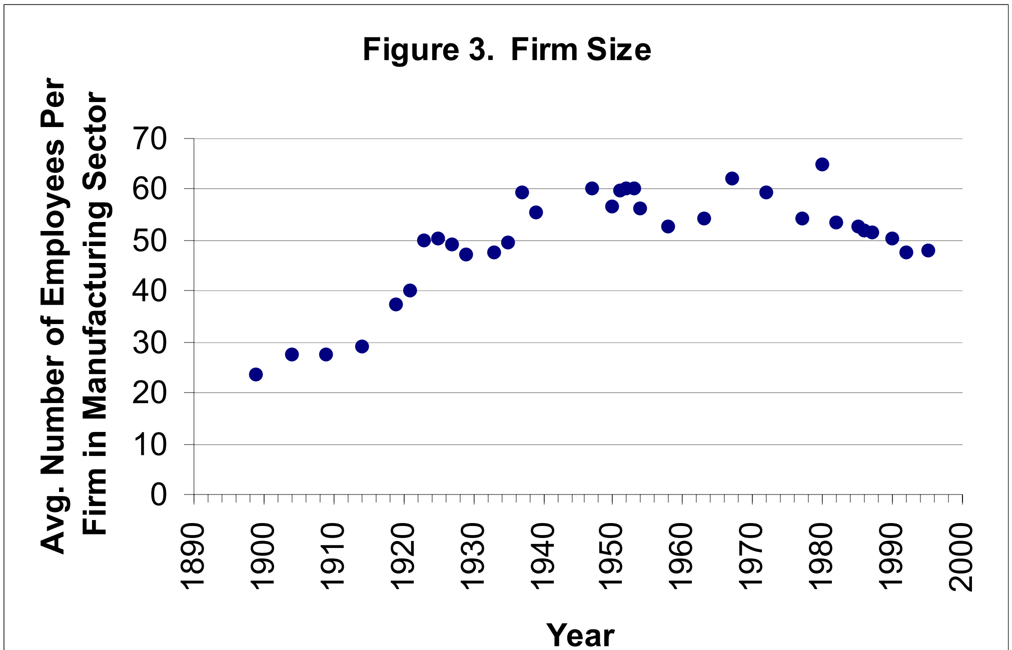


Sources: Statistical Abstract of the US: 1998, p. 8 and Historical Statistics of the United States: Colonial Times to 1970: Part 1, p. 8.

Figure 2: Per Capita GDP Growth Rate

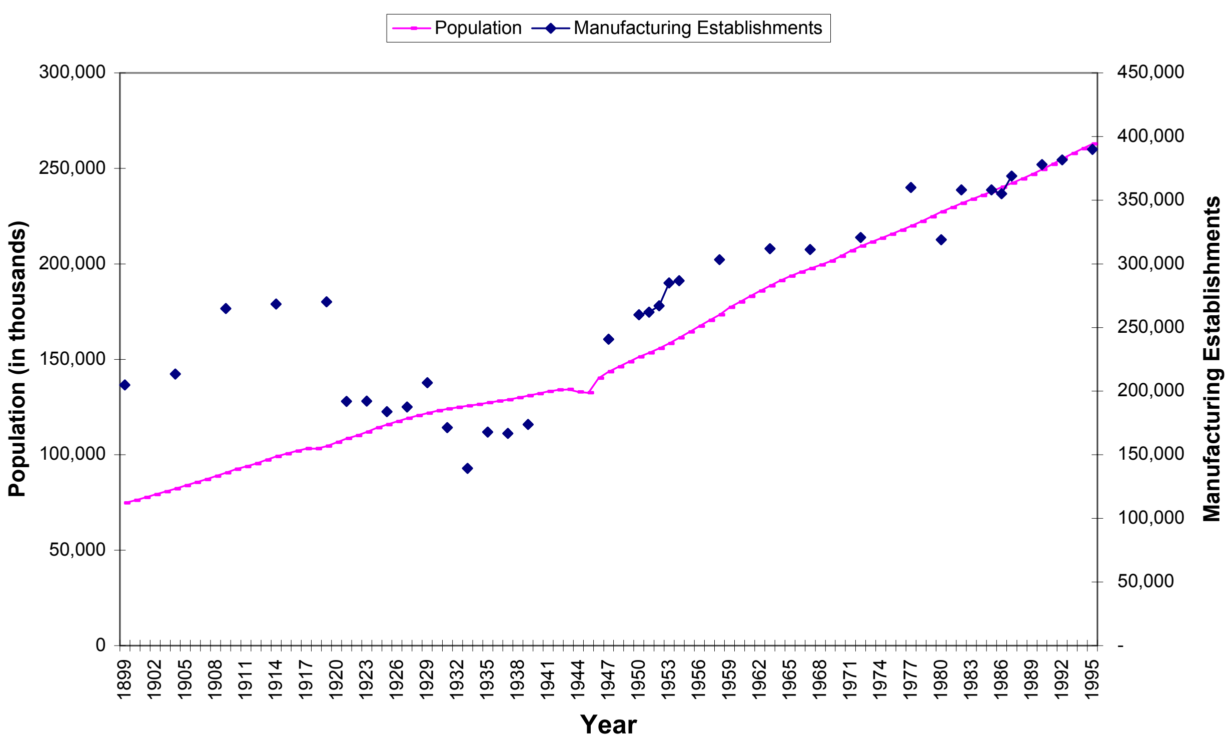


Source: Maddison (1995), AppendixD



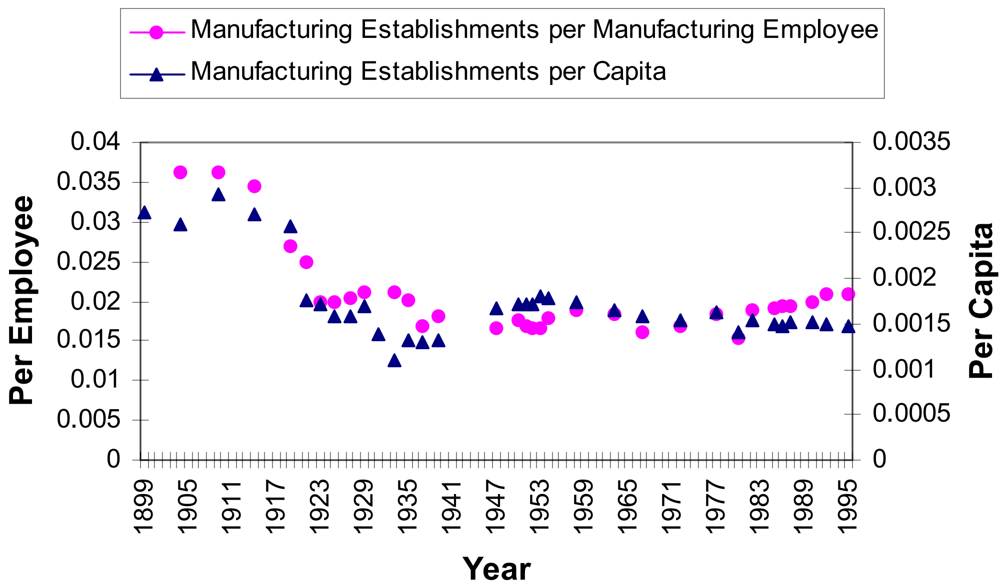
Source:1996 Annual Survey of Manufactures, Table 1a. p.1-7, Historical Statistics of the United States: Part 2, p. 666 and various issues of the Statistical Abstract of the United States

Figure 4. U.S. Population and Manufacturing Establishments



Source: Maddison (1995), plus sources from Figure 3.

Figure 5. Manufacturing Establishments



Sources: see Figure 4

Figure 6: Steady-state equilibrium

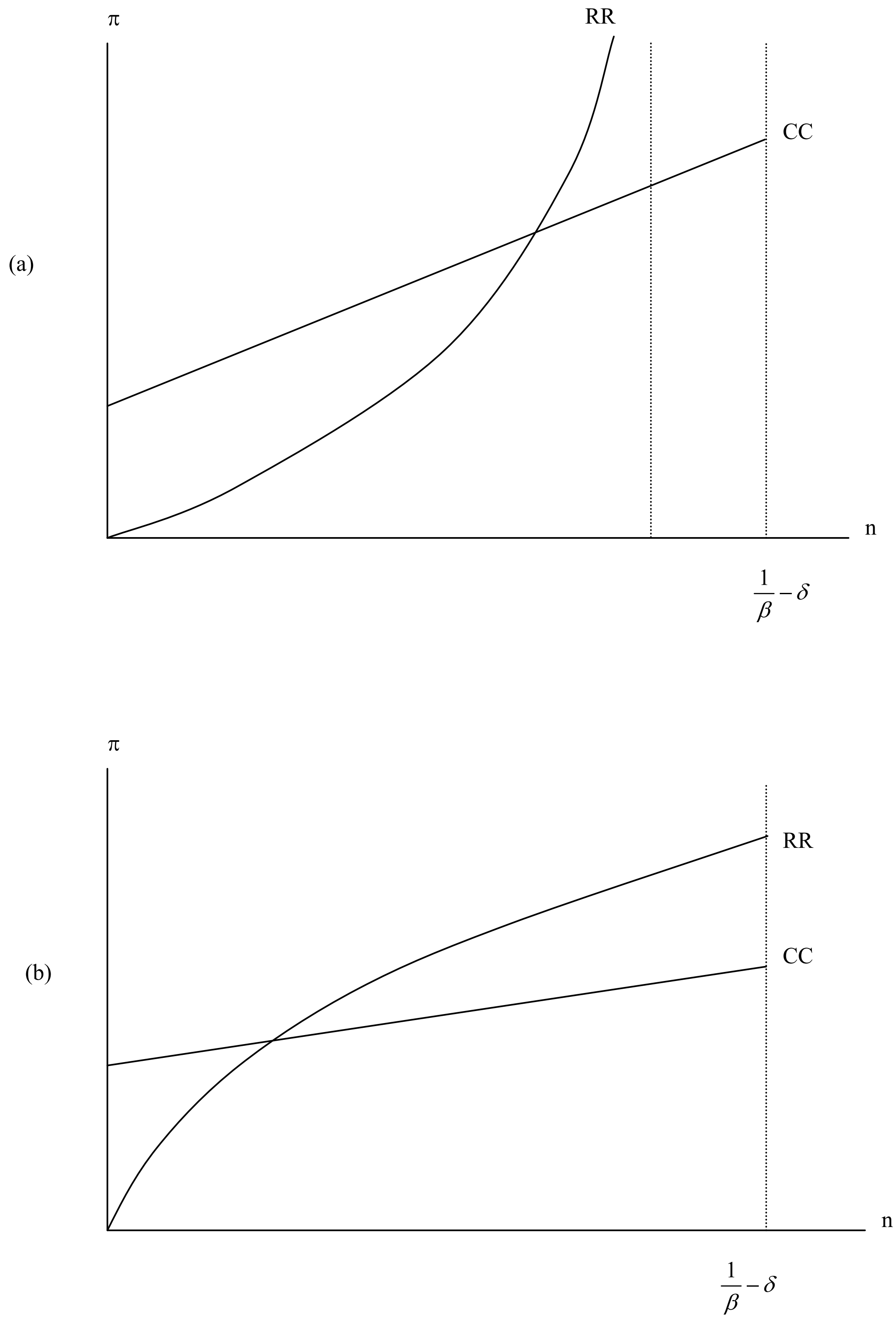


Figure 7: Phase diagram

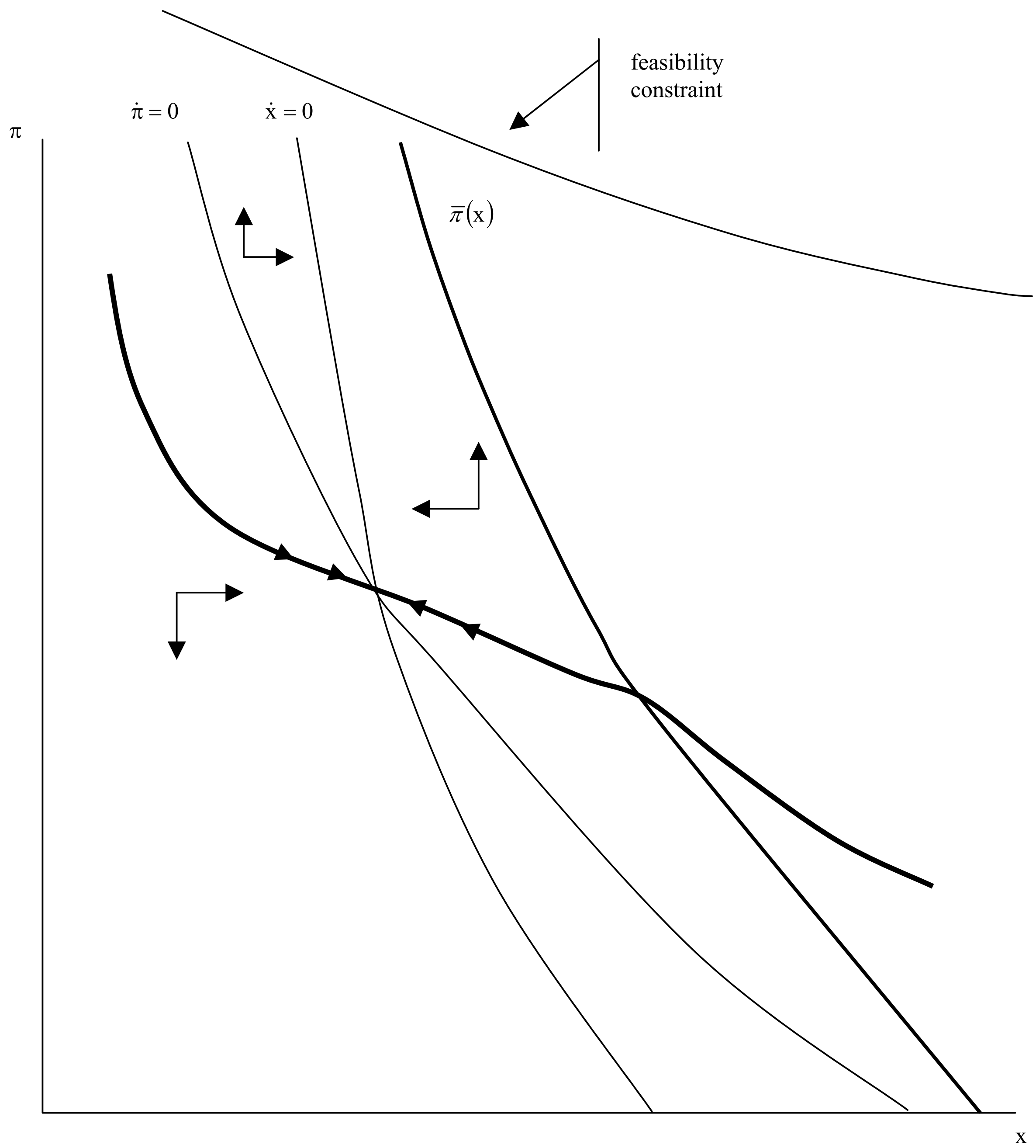


Figure 8. Transition Path to Steady-State

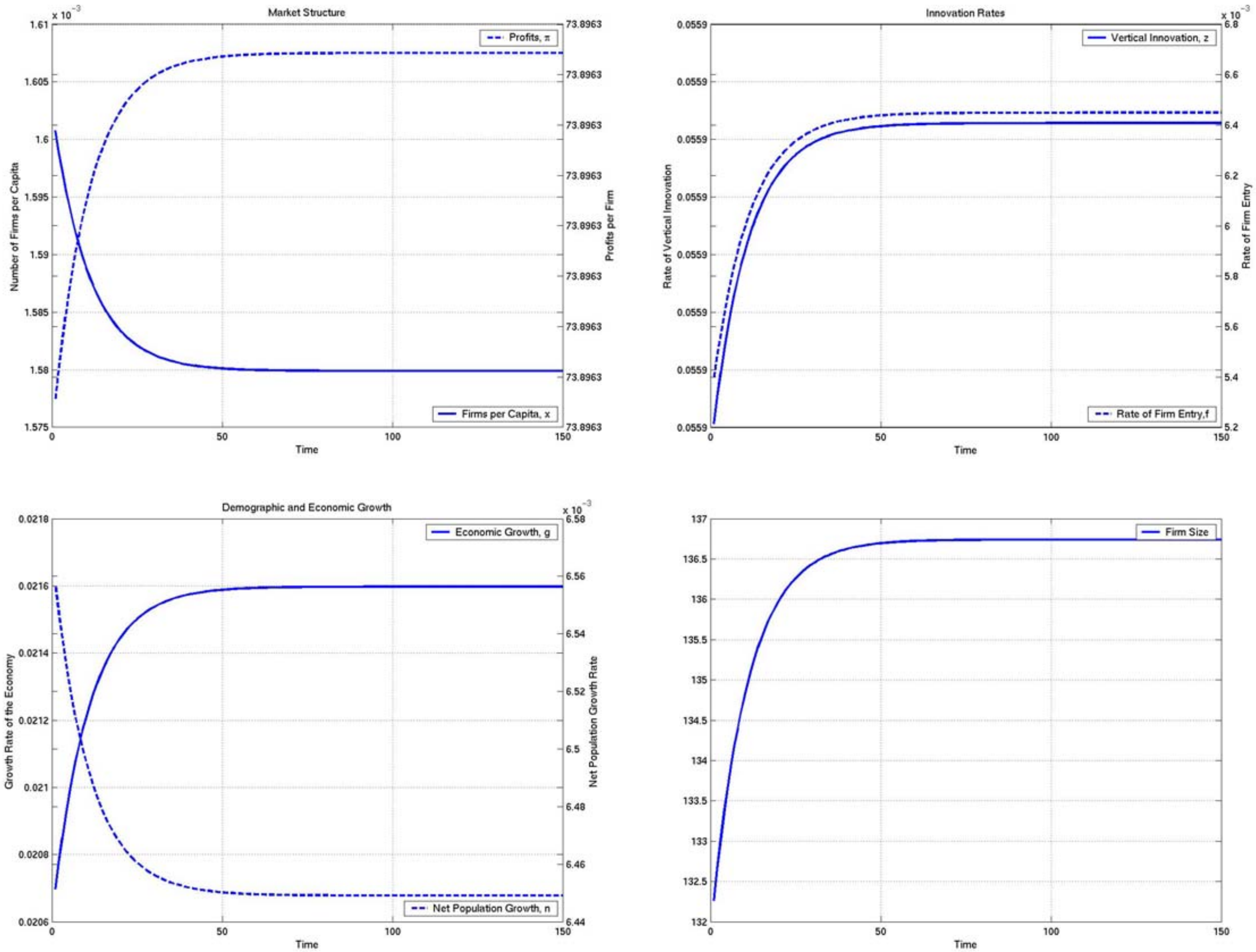


Figure 9 . Fall in Alpha

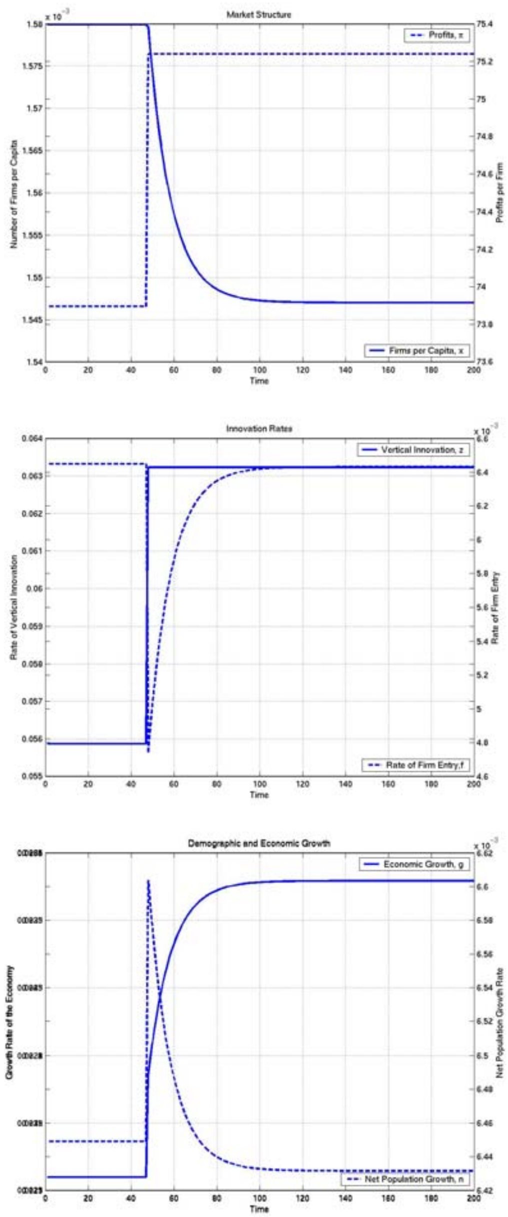


Figure 10. Fall in Zeta

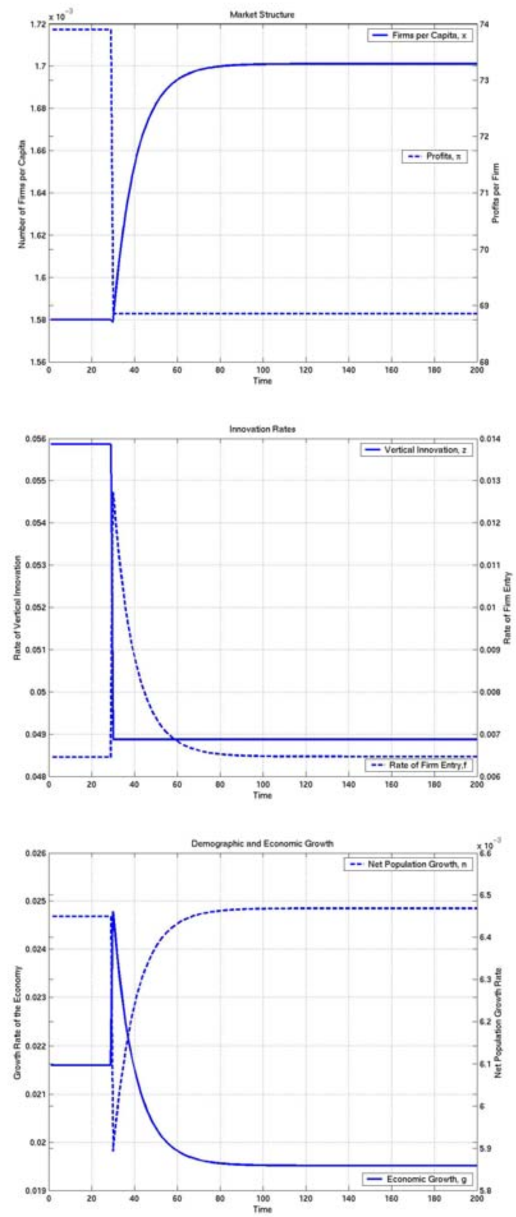


Figure 11. Fall in both alpha and zeta: Neutrality Result

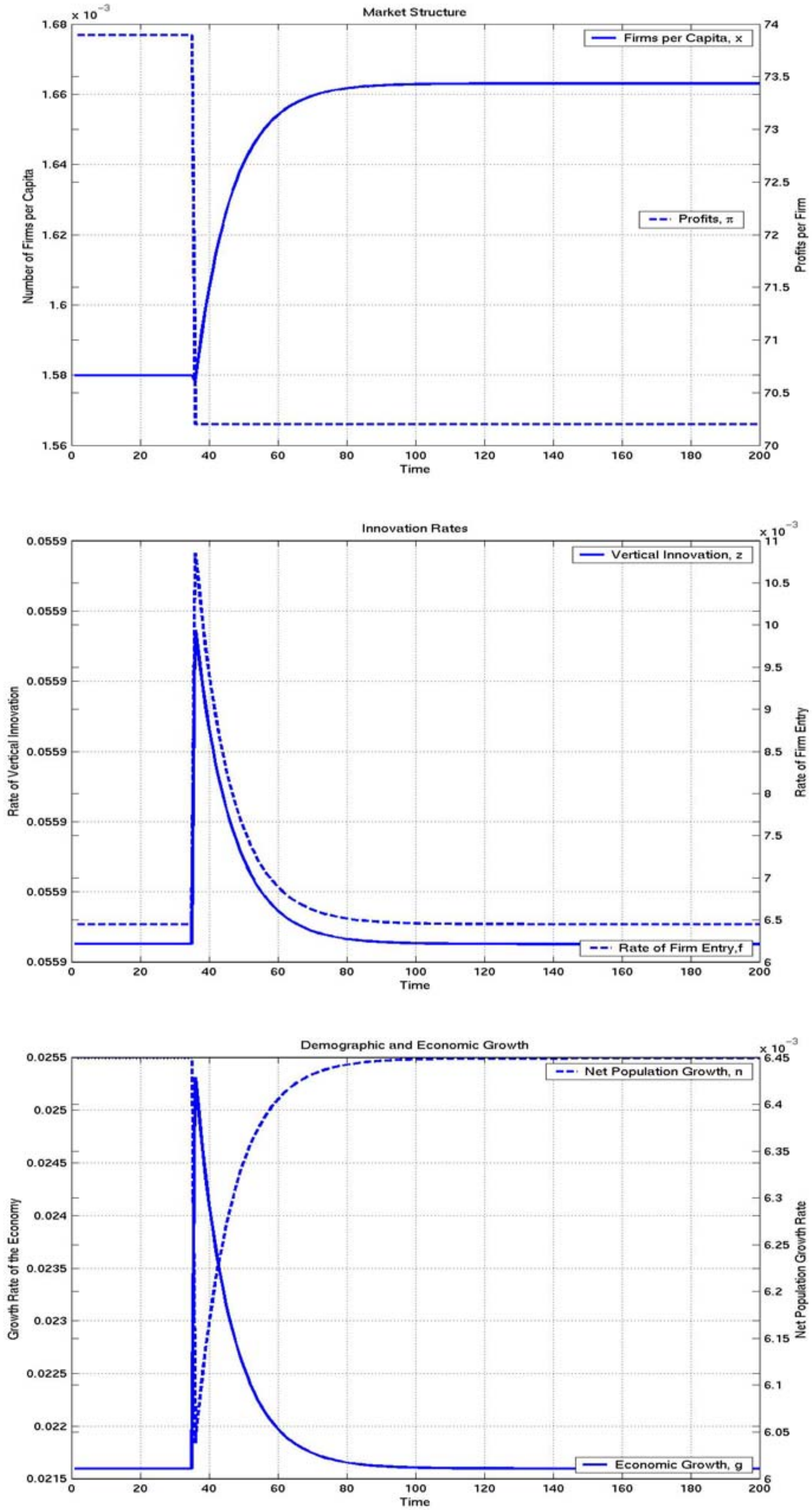


Figure 12. Decrease in Mortality, Delta

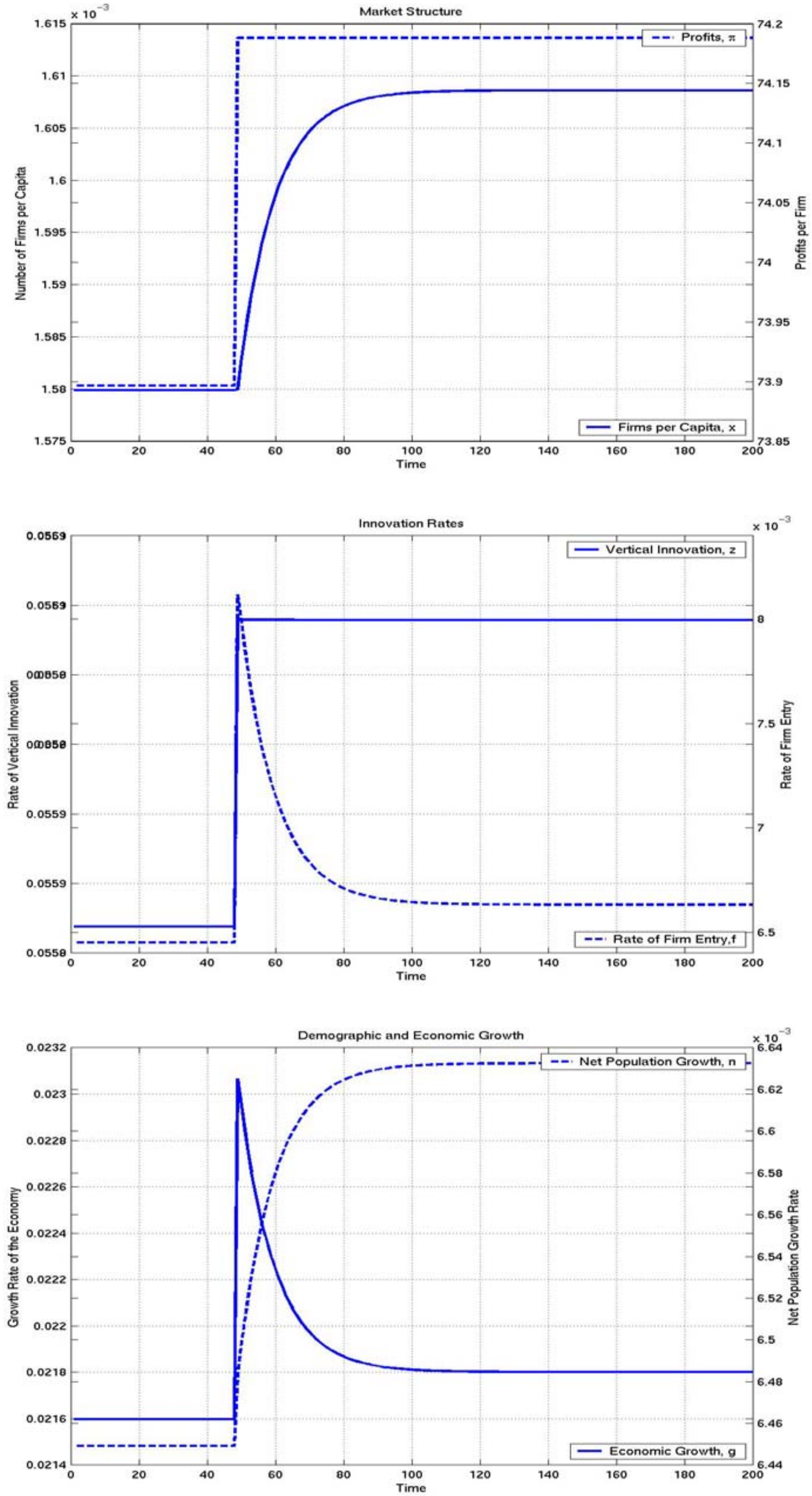


Figure 13. Increase in cost of reproduction, Beta