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**NO 955 / OCTOBER 2008**

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**MONETARY POLICY  
AND STOCK MARKET  
BOOM-BUST CYCLES**

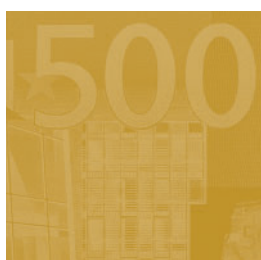
by Lawrence Christiano, Cosmin Ilut,  
Roberto Motto and Massimo Rostagno





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# MONETARY POLICY AND STOCK MARKET BOOM-BUST CYCLES<sup>1</sup>

by Lawrence Christiano<sup>2</sup>, Cosmin Ilut<sup>2</sup>,  
Roberto Motto<sup>3</sup> and Massimo Rostagno<sup>3</sup>



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## Abstract

We explore the dynamic effects of news about a future technology improvement which turns out ex post to be overoptimistic. We find that it is difficult to generate a boom-bust cycle (a period in which stock prices, consumption, investment and employment all rise and then crash) in response to such a news shock, in a standard real business cycle model. However, a monetized version of the model which stresses sticky wages and a Taylor-rule based monetary policy naturally generates a welfare-reducing boom-bust cycle in response to a news shock. We explore the possibility that integrating credit growth into monetary policy may result in improved performance. We discuss the robustness of our analysis to alternative specifications of the labor market, in which wage-setting frictions do not distort on going firm/worker relations.

**Keywords:** DSGE Models, Monetary Policy, Asset price boom-busts.

**JEL Classification:** C11, C51, E5, E13, E32

## Non-Technical Summary

Sustained and accelerating trends in asset prices have always been a matter of central concern for economic policies. What is most worrying in many episodes of asset price run-ups, as they unfold, is the possibility that they might come to a sudden stop followed by a crashing reversal. Economic history tells many such stories of asset price booms followed by resounding busts.

What are the basic forces driving these boom-bust episodes? Are they driven by economic fundamentals, or are they “bubbles”? The boom phase is generally associated with strong output, employment, consumption and investment, while there is substantial economic weakness in the bust phase. Does this association reflect causality going from volatility in the stock market to the real economy, or does causality go the other way? Or, is it that both are the outcome of some other factor, perhaps the nature of monetary policy?

To answer some of these questions, we study models that have been useful in the analysis of US and Euro Area business cycles and we apply a simple idea. The idea is that the boom phase is triggered by a signal which leads agents to rationally expect an improvement in technology in the future. Although the signal agents receive is informative, it is not perfectly accurate. Occasionally, the signal turns out to be false and the bust phase of the cycle begins when agents finally realise the actual state of technology. While the trigger of the boom-bust cycle in our analysis is in some ways simplistic, it has the advantage of allowing us to highlight a result that we think is likely to survive in more realistic settings.

We find that – within the confines of a set of models that fit the data well – it is hard to account for a boom-bust episode (an episode in which consumption, investment, output, employment and the stock market all rise sharply and then crash) without introducing nominal frictions. When we introduce sticky nominal wages and a central bank blindly wedded to a conventional Taylor rule, a theory of boom-busts emerges naturally. In our environment, this combination suboptimally converts what would otherwise be a mild economic fluctuation into a major boom-bust episode.

The logic is simple. In our model, when agents receive a signal of improved future technology it is efficient for investment, consumption and employment to all rise on impact, and then fall when expectations are disappointed. In the efficient allocations, the size of the boom is sharply limited by the rise in the real wage that occurs as the shadow cost of labour increases with higher work effort. When there are frictions in setting nominal wages, however, a rise in the real wage requires that inflation be allowed to drift down. However, in the model, under the standard Taylor rule, the central bank resists the tendency of inflation to decelerate by cutting the interest rate. In our exercise, this cut in the interest rate fuels the asset price boom and makes the economic expansion – and the subsequent fall-off – much larger than is socially optimal.

In our model, asset price booms are correlated with strong credit growth. We show that a modification of the Taylor rule in the direction of having policy ‘lean against the wind’ – by tightening when credit growth is strong – would raise welfare by reducing the magnitude of the boom-bust cycle.

## 1. Introduction

Inflation has receded from center stage as a major problem, and attention has shifted to other concerns. One concern that has received increased attention is volatility in asset markets. A look at the data reveals the reason. Figure 1 displays monthly observations on the S&P500 (converted into real terms using the CPI) for the period 1870 to early 2006. Note the recent dramatic boom and bust. Two other pronounced “boom-bust” episodes are evident: the one that begins in the early 1920s and busts near the start of the Great Depression, and another one that begins in the mid 1950s and busts in the 1970s. These observations raise several questions. What are the basic forces driving the boom-bust episodes? Are they driven by economic fundamentals, or are they bubbles? The boom phase is associated with strong output, employment, consumption and investment, while there is substantial economic weakness (in one case, the biggest recorded recession in US history) in the bust phase. Does this association reflect causality going from volatility in the stock market to the real economy, or does causality go the other way? Or, is it that both are the outcome of some other factor, perhaps the nature of monetary policy? The analysis of this paper lends support to the latter hypothesis.

We study models that have been useful in the analysis of US and Euro Area business cycles. We adopt the fundamentals perspective on boom-busts suggested by the work of Beaudry and Portier (2000,2003,2004) and recently extended in the analysis of Jaimovich and Rebelo (2006). The idea is that the boom phase is triggered by a signal which leads agents to rationally expect an improvement in technology in the future. Although the signal agents see is informative, it is not perfect. Occasionally, the signal turns out to be false and the bust phase of the cycle begins when people find this out. As an example, we have in mind the signals that led firms to invest

heavily in fiber-optic cable, only to be disappointed later by low profits. Another example is the signals that led Motorola to launch satellites into orbit in the expectation (later disappointed) that the satellites would be profitable as cell phone usage expanded. Although our analysis is based on rational expectations, we suspect that the same basic results would go through under other theories of how agents can become optimistic in ways that turn out ex post to be exaggerated.

Our notion of what triggers a boom-bust cycle is very stylized: the signal occurs on a particular date and people learn that it is exactly false on another particular date. In more realistic scenarios, people form expectations based on an accumulation of various signals. If people's expectations are in fact overoptimistic, they come to this realization only slowly and over time. Although the trigger of the boom-bust cycle in our analysis is in some ways simplistic, it has the advantage of allowing us to highlight a result that we think is likely to survive in more realistic settings.

Our results are as follows. We find that - within the confines of a set of models that fit the data well - it is hard to account for a boom-bust episode (an episode in which consumption, investment, output, employment and the stock market all rise sharply and then crash) without introducing nominal frictions. However, when we introduce an inflation targeting central bank and sticky nominal wages, a theory of boom-busts emerges naturally. In our environment, inflation targeting suboptimally converts what would otherwise be a small economic fluctuation into a major boom-bust episode. In this sense, our analysis is consistent with the view that boom-bust episodes are in large part caused by monetary policy.

In our model, we represent monetary policy by an empirically estimated Taylor rule. Because that rule satisfies the Taylor principle, we refer to it loosely as an 'inflation targeting' rule. Inflation targeting has the powerful attraction of anchoring expectations in New Keynesian models. However, our analysis suggests that there

are also costs. The analysis suggests that it is desirable to modify the standard inflation targeting approach to monetary policy in favor of a policy that does not promote boom-busts. In our model, boom-bust episodes are correlated with strong credit growth. So, a policy which not only targets inflation, but also ‘leans against the wind’ by tightening monetary policy when credit growth is strong would reduce some of the costs associated with pure inflation targeting.

Our results are based on model simulations, and so the credibility of the findings depends on the plausibility of our model. For the purpose of our results here, the two cornerstones of our model are sticky wages and an estimated Taylor rule. That the latter is a reasonable way to capture monetary policy has almost become axiomatic. Sticky wages have also emerged in recent years as a key feature of models that fit the data well (see, for example, the discussion in Christiano, Eichenbaum and Evans (2005).) The view that wage-setting frictions are key to understanding aggregate fluctuations is also reached by a very different type of empirical analysis in Galí, Gertler and Lopes-Salido (forthcoming).

That sticky wages and inflation targeting are uneasy bedfellows is easy to see. When wages are sticky, an inflation targeting central bank in effect targets the real wage. This produces inefficient outcomes when shocks occur which require an adjustment to the real wage (Erceg, Henderson and Levin (2000).) For example, suppose a shock - a positive oil price shock, say - occurs which reduces the value marginal product of labor. Preventing a large fall in employment under these circumstances would require a drop in the real wage. With sticky wages and an inflation-targeting central bank, the required fall in the real wage would not occur and employment would be inefficiently low.

That sticky wages and inflation targeting can cause the economy’s response to an oil shock to be inefficient is well known. What is less well known is that the

interaction of sticky wages and inflation targeting in the form of a standard Taylor rule can trigger a boom-bust episode. The logic is simple. In our model, when agents receive a signal of improved future technology it is efficient for investment, consumption and employment to all rise a little, and then fall when expectations are disappointed. In the efficient allocations, the size of the boom is sharply limited by the rise in the real wage that occurs as the shadow cost of labor increases with higher work effort.<sup>1</sup> When there are frictions in setting nominal wages, however, a rise in the real wage requires that inflation be allowed to drift down. The inflation targeting central bank, seeing this drift down in inflation, cuts the interest rate to keep inflation on target. In our model, this cut in the interest rate triggers a credit boom and makes the economic expansion much bigger than is socially optimal. In a situation like this, a central bank that ‘leans against the wind’ when credit expands sharply would raise welfare by reducing the magnitude of the boom-bust cycle.

The notion that inflation targeting increases the likelihood of stock market boom-bust episodes contradicts conventional wisdom. We take it that the conventional wisdom is defined by the work of Bernanke and Gertler (2000), who argue that an inflation-targeting monetary authority automatically stabilizes the stock market. The reason for this is that in the Bernanke-Gertler environment, inflation tends to rise in a stock market boom, so that an inflation targeter would raise interest rates, moderating the rise in stock prices.

So, the behavior of inflation in the boom phase of a boom-bust cycle is the crucial factor that distinguishes our story from the conventional wisdom. To assess the two perspectives, consider Figure 2, which displays inflation and the stock market in the three major US boom-bust episodes in the 20th century. In each case, in-

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<sup>1</sup>Our definition of the ‘efficient allocations’ is conventional. They are the allocations that solve the Ramsey problem.

flation is either falling or at least not rising during the initial phase of the boom. Additional evidence is presented in Adalid and Detken (2006). They identify asset price boom/bust episodes in 18 OECD countries since the 1970s. In their Table A1, they show that inflation is on average weak during the boom-phase of these episodes. In sum, the proposition that inflation is weak during the boom phase of boom-bust cycles receives support from US and OECD data.

So far, we have stressed that integrating nominal variables and inflation targeting into the analysis is merely helpful for understanding boom-bust episodes. In fact, if one is not to wander too far from current standard models, it is essential.

To clarify this point, it is useful to think of the standard real business cycle model that emerges when we strip away all monetary factors from our model. If we take a completely standard version of such a model, a signal shock is completely incapable of generating a boom-bust that resembles anything like what we see. Households in effect react to the signal by going on vacation: consumption jumps, work goes down and investment falls. When households realize the expected shock will not occur they in effect find themselves in a situation with a low initial capital stock. The dynamic response to this is familiar: they increase employment and investment and reduce consumption. We identify the price of equity in the data with the price of capital in the model. In the simplest version of the real business cycle model this is fixed by technology at unity, so we have no movements in the stock prices. It is hard to imagine a less successful model of a boom-bust cycle!

We then add habit persistence and costs of adjusting the flow of investment (these are two features that have been found useful for understanding postwar business cycles) and find that we make substantial progress towards a successful model. However, this model also has a major failing that initially came to us as a surprise: in the boom phase of the cycle, stock prices in the model fall and in the bust phase they



rise. The reason for this counterfactual implication is simple. Investment expands in response to the signal about future productivity because agents expect investment to be high in the future, when technology is high. Under these circumstances, the strategy of cutting investment now and raising it in the future when technology is high is inefficient because it entails heavy adjustment costs in the future. In effect, high expected future investment adds an extra payoff to current investment in the form of reduced future adjustment costs. This corresponds to a reduction in the current marginal cost of producing capital goods. In a competitive market, this reduction in marginal cost is passed on to consumers in the form of a lower price.

So, our real business cycle model cannot simultaneously generate a rise in the price of capital and a rise in investment, in response to a signal about future productivity. The real business cycle model has two additional shortcomings. It generates an extremely large jump in the real interest rate and it generates very little persistence. It really only generates a boom-bust pattern in consumption, investment, employment and output when the signal is about a shock that will occur four quarters in the future. If the signal is about a shock, say, 12 quarters in the future, agents go on an 8 quarter vacation and then begin working roughly in the 9th quarter. But, as we see in Figure 1, stock market booms last considerably longer than one year. So, while a real model takes us part way in understanding a boom-bust cycle, there are significant shortcomings.

When we introduce monetary factors and an inflation targeting central bank, these shortcomings disappear. The monetary expansion produced in the wake of a signal about higher future productivity generates a boom in the stock price. The monetary response is associated with very little volatility in the real interest rate, and the boom bust cycle is highly persistent. In addition, the monetary response greatly amplifies the magnitude of fluctuations in real quantities. Actually the boom-bust

produced in the monetary model is so much larger than it is in the real business cycle model that it is not an exaggeration to say that our boom-bust episode is primarily a monetary phenomenon.

A feature of the simple monetary model is that wage frictions affect employment on the intensive margin. As a result, the model is vulnerable to the Barro (1977) critique. The problem is that in practice, variations on the intensive margin occur in long-lasting relationships between workers and firms, and such relationships offer many opportunities to undo the effects of idiosyncrasies in wage setting. To investigate whether our analysis is robust to alternative specifications of the labor market which avoid the Barro critique, we explore a recent proposal of Gertler and Trigari (2006) and Gertler, Sala and Trigari (2006). In this formulation, wage setting frictions have allocational consequences via their impact on employer efforts to recruit new workers and they do not distort on-going worker/firm relations. We find that with this alternative formulation of the labor market, our analysis goes through qualitatively, though the findings are less dramatic quantitatively. We conjecture that this does not reflect a weakness in the mechanism outlined in this paper, but instead reflects that the alternative model requires additional modifications to magnify its monetary non-neutralities.

After analyzing the simple monetary model and its robustness to the Barro critique, we move on to the full monetary model of Christiano, Motto and Rostagno (2006). That model incorporates a banking sector and the financial frictions in Bernanke, Gertler and Gilchrist (1999) into the simple monetary model. The resulting model is interesting for two reasons. First, we use the model to investigate the robustness of our findings for boom-busts. We feed the model the same signal about future technology that we fed to our real business cycle and simple monetary models. We find that the full and simple monetary models behave quite similarly. The

second reason it is interesting to study boom-bust episodes in the Christiano, Motto and Rostagno (2006) model is that the model has implications for different monetary aggregates as well as credit. Discussions of boom-bust cycles often focus on the behavior of money and credit during a boom. These discussions often emphasize the importance of distinguishing between money and credit (see, for example, Eichengreen and Mitchener (2003)). They show, for example, that credit grew very rapidly during the 1920s, but M2 showed weak and declining growth. Interestingly, when we feed the signal shock to the model of Christiano, Motto and Rostagno (2006) we find that credit rises strongly during the boom, though the predictions are ambiguous for the monetary aggregates, with some showing strength and others weakness. This is broadly consistent with some existing empirical studies.

Our analysis has more general implications. It is already well known that monetary policy plays an important role in the transmission of fundamental shocks. We can add that monetary policy is also very important in the transmission of expectational shocks.

Following is a brief outline of the paper. The next section describes the real business cycle version of our model in which all monetary factors have been stripped away. Numerical simulations are used to develop the model's implications for boom-bust cycles. Section 3 introduces the smallest number of monetary factors that will allow the model to successfully generate a boom-bust cycle. In this simple monetary model, monetary policy is assumed to follow a Taylor rule, which has been estimated using post war US data. This Taylor rule places positive weight on the output gap and incorporates 'interest smoothing' in that the interest rate is also a function of the lagged interest rate. The monetary policy rule is an inflation targeting rule in the sense that the coefficient on expected inflation satisfies the 'Taylor Principle' in being larger than unity. In our estimate, it is 1.95. We show that the amplitude of

fluctuation of variables in the boom-bust cycle of the simple monetary model is much greater than it is in the version of the model with optimal monetary policy. This is the basis for our conclusion that the boom-bust in the monetary model is inefficient in a welfare sense. Section 4 investigates robustness to the Barro critique. Section 5 presents the implications for a boom-bust model of the full model monetary model whose pieces have been studied up to now. The paper closes with a brief conclusion. Three appendices discuss the technical details of our analysis.

## 2. Real Business Cycle Model

This section explores the limits of a simple Real Business Cycle explanation of a boom-bust episode. We show that preferences and investment adjustment costs that have become standard in successful empirical models of business cycles move us part way to a full qualitative explanation of a boom-bust episode. However, we are not successful producing a rise in the price of capital in the boom phase of the cycle. In addition, we will see that it is hard to generate a boom that is much longer than one year. Finally, we will see that the model generates extreme fluctuations in the real rate of interest.

### 2.1. The Model

The preferences of the representative household are given by:

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left\{ \log(C_{t+l} - bC_{t+l-1}) - \psi_L \frac{h_t^{1+\sigma_L}}{1+\sigma_L} \right\}$$

Here,  $h_t$  is hours worked,  $C_t$  is consumption and the amount of time that is available is unity. When  $b > 0$  then there is habit persistence in preferences. The resource

constraint is

$$I_t + C_t \leq Y_t, \quad (2.1)$$

where  $I_t$  is investment,  $C_t$  is consumption and  $Y_t$  is output of goods.

Output  $Y_t$  is produced using the technology

$$Y_t = \epsilon_t K_t^\alpha (z_t h_t)^{1-\alpha}, \quad (2.2)$$

where  $\epsilon_t$  represents a stochastic shock to technology and  $z_t$  follows a deterministic growth path,

$$z_t = z_{t-1} \exp(\mu_z). \quad (2.3)$$

The law of motion of  $\epsilon_t$  will be described shortly.

We consider two specifications of adjustment costs in investment. According to one, adjustment costs are in terms of the change in the flow of investment:

$$K_{t+1} = (1 - \delta)K_t + (1 - S\left(\frac{I_t}{I_{t-1}}\right))I_t, \quad (2.4)$$

where

$$S(x) = \frac{a}{2}(x - \exp(\mu_z))^2,$$

with  $a > 0$ . We refer to this specification of adjustment costs as the ‘flow specification’. In our second model of investment the adjustment costs are in terms of the level of investment:

$$K_{t+1} = (1 - \delta)K_t + I_t - \Phi\left(\frac{I_t}{K_t}\right)K_t, \quad (2.5)$$

where

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{1}{2\delta\sigma_\Phi}\left(\frac{I_t}{K_t} - \eta\right)^2, \quad (2.6)$$

where  $\eta$  is the steady state investment to capital ratio. Here, the parameter,  $\sigma_{\Phi} > 0$ , is the elasticity of the investment-capital ratio with respect to Tobin's  $q$ . We refer to this specification of adjustment costs as the 'level specification'.

Throughout the analysis, we consider the following impulse. Up until period 1, the economy is in a steady state. In period  $t = 1$ , a signal occurs which suggests  $\epsilon_t$  will be high in period  $t = 1 + p$ . But, when period  $1 + p$  occurs, the expected rise in technology in fact does not happen. A time series representation for  $\epsilon_t$  which captures this possibility is:

$$\log \epsilon_t = \rho \log \epsilon_{t-1} + \varepsilon_{t-p} + \xi_t, \quad (2.7)$$

where  $\varepsilon_t$  and  $\xi_t$  are uncorrelated over time and with each other. For example, suppose  $p = 1$ . Then, if the realized value of  $\varepsilon_1$  is high value, this shifts up the expected value of  $\log \epsilon_2$ . But, if  $\xi_2 = -\varepsilon_1$ , then the high expected value of  $\log \epsilon_2$  does not materialize.

We consider the following parameterization,

$$\begin{aligned} \beta &= 1.01358^{-0.25}, \mu_z = 1.0136^{0.25}, b = 0.63, a = 15.1, \\ \alpha &= 0.40, \delta = 0.025, \psi_L = 109.82, \sigma_L = 1, \rho = 0.83, p = 4. \end{aligned} \quad (2.8)$$

The steady state of the model associated with these parameters is:

$$\frac{C}{Y} = 0.64, \frac{K}{Y} = 12.59, l = 0.092$$

We interpret the time unit of the model as being one quarter. This model is a special case of the model estimated in Christiano, Motto and Rostagno (2006) using US data. In the above list, the parameters  $a$  and  $\rho$  were estimated;  $\mu_z$  was estimated based on the average growth rate of output;  $\beta$  was selected so that given  $\mu_z$  the model matches the average real return on three-month Treasury bills;  $\sigma_L$  was simply set to produce a Frish labor supply of unity;  $b$  was taken from Christiano, Eichenbaum and

Evans (2004);  $\alpha$  and  $\delta$  were chosen to allow the model to match several ratios (see Christiano, Motto and Rostagno (2006)).

## 2.2. Results

Consider the line with circles in Figure 3. This line displays the response of the ‘Baseline RBC’ model to a signal in period 1 that technology will jump in period 5 by 1 percent. Then,  $\xi_5 = -0.01$ , so that the impact of the signal on  $\epsilon_t$  is cancelled and no change ever happens to actual technology. Note how in the figure output, investment and hours worked all rise until period 4 and then slowly decline. The price of capital falls despite the anticipated rise in the payoff associated with capital. This fall is discussed in further detail below, although perhaps it is not surprising in view of the spike in the interest rate on one period consumption loans taken in period 4. This jump in the interest rate is extraordinarily large. In the period before the anticipated jump in technology, the real rate jumps by more than 10 percentage points, at an annual rate.

The solid line in Figure 3 and the results in Figures 5-8 allow us to diagnose the economics underlying the line with circles in Figure 3. In Figures 5-8, the circled line in Figure 3 is reproduced for comparison. The solid line in Figure 3 displays the response of the variables in the case when the technology shock is realized. This shows the scenario which agents expect when they see the signal in period 1. Their response has several interesting features. First, the rise in investment in period 4, the first period in which investment can benefit from the higher expected rate of return, is not especially larger than the rise in other periods, such as period 5. We suspect that the failure of investment to rise more in period 4 reflects the consumption smoothing motive. Period 4 is a period of relatively low productivity, and while

high investment then would benefit from the high period 5 rate of return, raising investment in period 4 is costly in terms of consumption. The very high period 4 real interest rate is an indicator of just how costly consumption then is. Second, hours worked drops sharply in the period when the technology shock is realized. The drop in employment in our simulation reflects the importance of the wealth effect on labor. This wealth effect is not felt in periods before 5 because of high interest rate before then. Commenting on an earlier draft of our work, Jaimovich and Rebelo (2006) conjecture that this drop is counterfactual, and they propose an alternative specification of utility in which it does not occur. An alternative possibility is that the sharp movement in employment reflects the absence of labor market frictions in our model. For example, we suspect that if we incorporate a simple model of labor search frictions the drop in employment will be greatly attenuated while the basic message of the paper will be unaffected (see Blanchard and Gali (2006)).

Figure 4 allows us to assess the role of habit persistence in the responses in Figure 3. Three things are worth emphasizing based on this figure. First, Figure 4 shows that  $b > 0$  is a key reason why consumption rises in periods before period 5 in Figure 3. Households, understanding that in period 5 they will want to consume at a high level, experience a jump in the marginal utility of consumption in earlier periods because of habit persistence. This can be seen in the expression for the marginal utility of period  $t$  consumption,  $\lambda_t$ , which is increasing in future consumption:

$$\lambda_t = \frac{1}{C_t - bC_{t-1}} - b\beta E_t \frac{1}{C_{t+1} - bC_t}. \quad (2.9)$$

Second, the early jump in the marginal utility of consumption induced by the presence of habit persistence also explains why the employment response to the technology signal is relatively strong in the presence of habit persistence (the bottom left

graph in Figure 4). To see this, consider the intratemporal Euler equation:

$$\lambda_t \times MPL_t = MUL_t, \quad (2.10)$$

where  $MPL_t$  denotes the marginal product of labor and  $MUL_t$  denotes the marginal utility of leisure. From this expression it is clear that with a normal specification of preferences, it is not possible for both consumption and labor to rise in response to a future technology shock. The rise in labor would reduce  $MPL_t$  and increase  $MUL_t$ , while the rise in consumption would ordinarily reduce  $\lambda_t$ .<sup>2</sup> With habit persistence, this logic is broken because the anticipated rise in  $t + 1$  consumption raises  $\lambda_t$ . Third, note that the employment response without habit persistence, though weak, is positive. The reason for this is that in the absence of habit persistence, households find it optimal to reduce consumption in order to make room for an increase in investment. The fall in consumption raises  $\lambda_t$  and is the reason why employment rises in Figure 4, even when  $b = 0$ .

Figure 5 shows what happens when there are no adjustment costs in investment. In this case, there is no cost to the strategy of simply waiting until later to raise investment. This strategy has the advantage of permitting consumption smoothing. One way to see this is to note how the real rate of interest hardly moves when there are no investment adjustment costs.

Figure 6 shows what happens when we adopt the level specification of adjustment costs. With this specification, investment falls in response to the signal. This makes room for additional consumption which reduces  $\lambda_t$  and accounts for the weak response of employment after the signal (bottom left graph in Figure 6). So, Figure 6 indicates

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<sup>2</sup>This logic was stressed by Barro and King (1984), who argued that it would be difficult to square the procyclical movement of consumption and labor with the acyclical behavior of wages under standard preferences.

that the flow specification of adjustment costs in our baseline real business cycle model plays an important role in producing the responses in Figure 3.

A final experiment was motivated by the fact that in practice the boom phase of a boom-bust cycle often lasts considerably more than 4 quarters. To investigate whether the real business cycle model can generate a longer boom phase, we considered an example in which there is a period of 3 years from the date of the signal to the bust. Figure 7 displays simulation results for the case when  $p = 12$ . Notice that we have in fact not lengthened the boom phase very much because output, employment and investment actually only begin to rise about 4 quarters before the bust. In addition, the model no longer generates a rise in consumption in response to the signal. According to the figure, consumption falls (after a very brief rise) in the first 9 quarters. Evidently, households follow a strategy similar to the one in Figure 4, when there is no habit. There, consumption falls in order to increase the resources available for investment. Households with habit persistence do not mind following a similar strategy as long as they can do so over a long enough period of time. With time, habit stocks fall, thus mitigating the pain of reducing consumption.

### 2.3. The Price of Capital

To understand the response of the price of capital to a signal about future productivity, we study two model equations that characterize the dynamics of  $P_{k',t}$ . One equation is the present discounted value of future payoffs from capital. This is derived by focusing on the demand for capital. The other implication flows from the fact that capital is produced in the model, and corresponds to what is sometimes referred to as the model's Tobin's  $q$  relation.

Let  $\mu_t$  denote the multiplier on (2.4) and  $\lambda_t$  the multiplier on the resource con-

straint in the Lagrangian representation of the planning problem. The first order conditions for consumption and labor are (2.9) and (2.10), respectively. The first order condition with respect to  $K_{t+1}$  is:

$$\mu_t = \beta \left[ \lambda_{t+1} \alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + \mu_{t+1} (1 - \delta) \right].$$

Note that the object on the right side of the equality is the marginal utility of an extra unit of  $K_{t+1}$ . It is tomorrow's marginal physical product of capital, converted to marginal utility terms by multiplying by  $\lambda_{t+1}$  plus the value of the undepreciated part of  $K_{t+1}$  that is left over for use in subsequent periods, which is converted into marginal utility terms by  $\mu_{t+1}$ . Divide both sides of the first order condition for  $K_{t+1}$  with respect to  $\lambda_t$  and rearrange:

$$\frac{\mu_t}{\lambda_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \left[ \alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha} + \frac{\mu_{t+1}}{\lambda_{t+1}} (1 - \delta) \right].$$

Now, recall that  $\mu_t$  is the marginal utility of  $K_{t+1}$ , loosely,  $dU/dK_{t+1}$ . Similarly,  $\lambda_t$  is the marginal utility of  $C_t$ , loosely  $dU/dC_t$ . Thus, the ratio is the consumption cost of a unit of  $K_{t+1}$ , or the price of capital,  $P_{k',t}$ :

$$P_{k',t} = \frac{\mu_t}{\lambda_t} = \frac{\frac{dU_t}{dK_{t+1}}}{\frac{dU_t}{dC_t}} = \frac{dC_t}{dK_{t+1}}.$$

Substituting this into the first order condition for  $K_{t+1}$ , we obtain:

$$P_{k',t} = E_t \frac{\beta \lambda_{t+1}}{\lambda_t} \left[ r_{t+1}^k \alpha + P_{k',t+1} (1 - \delta) \right],$$

where

$$r_{t+1}^k = \alpha (K_{t+1})^{\alpha-1} (z_{t+1} h_{t+1})^{1-\alpha}.$$

Iterating this expression forward, we obtain:

$$P_{k',t} = E_t \sum_{i=1}^{\infty} \left( \prod_{j=1}^i \frac{\beta \lambda_{t+j}}{\lambda_{t+j-1}} \right) (1 - \delta)^{i-1} r_{t+i}^k \quad (2.11)$$

Focusing on the effect of the signal on future  $r_t^k$ 's creates the expectation that  $P_{k',t}$  should jump in response to a signal about future productivity. However, we saw in Figure 3 that the real interest rate jumps in response to such a signal, and this drives  $P_{k',t}$  in the other direction. Since this expression highlights two conflicting forces on  $P_{k',t}$ , it is not particularly useful for understanding why it is that the force driving  $P_{k',t}$  down dominates in our simulations.

We obtain the model's Tobin's  $q$  relation by working the first order condition for investment:

$$-\lambda_t + \mu_t(1 - S\left(\frac{I_t}{I_{t-1}}\right)) - \mu_t S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} + \beta \mu_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 0.$$

Rewriting this, taking into account the definition of the price of capital,

$$P_{K',t} = \frac{1 - \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} P_{K',t+1} \right] \left[ S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right]}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}} \quad (2.12)$$

The right side of the above expression is the marginal cost of an extra unit of capital. This marginal cost is the sum of two pieces. We see the first by ignoring the expression after the minus sign in the numerator. The resulting expression for  $P_{K',t}$  is the usual marginal cost term that occurs with level adjustment costs. It is the ratio of the consumption cost of a unit of investment goods,  $dC_t/dI_t$  (which is unity), divided by the marginal productivity (in producing new capital) of an extra investment good,  $dK_{t+1}/dI_t$ . To see that this is indeed the marginal cost of producing new capital, note that this corresponds to

$$\frac{\frac{dC_t}{dI_t}}{\frac{dK_{t+1}}{dI_t}} = \frac{dC_t}{dK_{t+1}},$$

i.e., the consumption cost of capital. If we just focus on this part of (2.12), the puzzle about why  $P_{K',t}$  drops during a boom only deepens. This is because, with the growth rate of investment high (see Figure 3, which shows that  $I_t/I_{t-1}$  is high for several periods), the first term after the equality should unambiguously be high during the boom. Both  $S$  and  $S'$  rise, and this by itself makes  $P_{K',t}$  rise.

Now consider the other term in the numerator of (2.12). The term in square brackets unambiguously rises after a positive signal about future technology because future growth in investment increases both  $S$  and  $S'$ . The square bracketed term contributes to a fall in  $P_{K',t}$ . The intuition for this is straightforward. In the wake of a positive signal about productivity, producers of new capital understand that investment will be high in the future. When there are adjustment costs, this means that there is an extra benefit to investing today: the reduction of adjustment costs when investment is made high in the future. In a competitive market, suppliers of capital will respond to this by bidding down the price of capital. That is what happens in the equilibrium of our real business cycle model.

It is similar for the central planner, who understands that a signal about positive future technology implies that building more capital today generates a future 'kickback', in the form of reduced adjustment costs in the future. This kickback is properly thought of as a reduction to the marginal cost of producing current capital, and is fundamentally the reason the planner is motivated to increase current investment. This reasoning suggests to us that there will not be a simple perturbation of the real business cycle model which will generate a rise in investment and a rise in the price of capital after a signal about future technology. This motivates us to consider the monetary version of the model in the next subsection.

### 3. Introducing Nominal Features and an Inflation-Targeting Central Bank

We now modify our model to introduce monetary policy and wage/price frictions. In the first subsection below, we present the model. In the second subsection we present our numerical results.

#### 3.1. Simple Monetary Model

To accommodate price-setting, we adopt the usual assumption that a representative final good producer manufactures final output using the following linear homogenous technology:

$$Y_t = \left[ \int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \quad 1 \leq \lambda_f < \infty, \quad (3.1)$$

Intermediate good  $j$  is produced by a price-setting monopolist according to the following technology:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^\alpha (z_t h_{jt})^{1-\alpha} - \Phi z_t & \text{if } \epsilon_t K_{jt}^\alpha (z_t h_{jt})^{1-\alpha} > \Phi z_t \\ 0, & \text{otherwise} \end{cases}, \quad 0 < \alpha < 1, \quad (3.2)$$

where  $\Phi z_t$  is a fixed cost and  $K_{jt}$  and  $h_{jt}$  denote the services of capital and homogeneous labor. Capital and labor services are hired in competitive markets at nominal prices,  $P_t r_t^k$ , and  $W_t$ , respectively.

In (3.2), the shock to technology,  $\epsilon_t$ , has the time series representation in (2.7). We adopt a variant of Calvo sticky prices. In each period,  $t$ , a fraction of intermediate-goods firms,  $1 - \xi_p$ , can reoptimize their price. If the  $i^{\text{th}}$  firm in period  $t$  cannot reoptimize, then it sets price according to:

$$P_{it} = \tilde{\pi}_t P_{i,t-1},$$

where

$$\tilde{\pi}_t = \pi_{t-1}^\ell \bar{\pi}^{1-\ell}. \quad (3.3)$$

Here,  $\pi_t$  denotes the gross rate of inflation,  $\pi_t = P_t/P_{t-1}$ , and  $\bar{\pi}$  denotes steady state inflation. If the  $i^{th}$  firm is able to optimize its price at time  $t$ , it chooses  $P_{i,t} = \tilde{P}_t$  to optimize discounted profits:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \lambda_{t+j} [P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} (Y_{i,t+j} + \Phi z_{t+j})]. \quad (3.4)$$

Here,  $\lambda_{t+j}$  is the multiplier on firm profits in the household's budget constraint. Also,  $P_{i,t+j}$ ,  $j > 0$  denotes the price of a firm that sets  $P_{i,t} = \tilde{P}_t$  and does not reoptimize between  $t+1, \dots, t+j$ . The equilibrium conditions associated with firms are displayed in the appendix.

In this environment, firms set prices as an increasing function of marginal cost. A firm that has an opportunity to set price will take into account future marginal costs because of the possibility that they may not be able to reoptimize again soon.

We model the labor market in the way suggested by Erceg, Henderson and Levin (2000). The homogeneous labor employed by firms in (3.2) is produced from specialized labor inputs according to the following linear homogeneous technology:

$$h_t = \left[ \int_0^1 (l_{t,i})^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \quad 1 \leq \lambda_w. \quad (3.5)$$

We suppose that this technology is operated by perfectly competitive labor contractors, who hire specialized labor from households at wage,  $W_{jt}$ , and sell homogenous labor services to the intermediate good firms at wage,  $W_t$ . Optimization by labor contractors leads to the following demand for  $l_{t,i}$ :

$$l_{t,i} = \left( \frac{W_{t,i}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} h_t, \quad 1 \leq \lambda_w. \quad (3.6)$$

The  $j^{\text{th}}$  household maximizes utility

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \psi_L \frac{l_{t,j}^{1+\sigma_L}}{1+\sigma_L} - v \frac{\left( \frac{P_{t+l}C_{t+l}}{M_{t+l}^d} \right)^{1-\sigma_q}}{1-\sigma_q} \right\} \quad (3.7)$$

subject to the constraint

$$P_t(C_t + I_t) + M_{t+1}^d - M_t^d + T_{t+1} \leq W_{t,j}l_{t,j} + P_t r_t^k K_t + (1 + R_t^e) T_t + A_{j,t}, \quad (3.8)$$

where  $M_t^d$  denotes the household's beginning-of-period stock of money and  $T_t$  denotes nominal bonds issued in period  $t-1$ , which earn interest,  $R_t^e$ , in period  $t$ . This nominal interest rate is known at  $t-1$ . In the interest of simplifying, we suppose that  $v$  in (3.7) is positive, but so small that the distortions to consumption, labor and capital first order conditions introduced by money can be ignored. Later, we will consider a model in which  $v$  is chosen to match money velocity data. The  $j^{\text{th}}$  household is the monopoly supplier of differentiated labor,  $l_{j,t}$ . Given the labor demand curve, (3.6), and absent any price frictions, the household would set its wage rate,  $W_{j,t}$ , as a fixed markup,  $\lambda_w$ , above its marginal cost:

$$W_{j,t} = \lambda_w P_t \frac{\psi_L l_{t,j}^{\sigma_L}}{\lambda_t} = \lambda_w P_t \frac{\psi_L l_{t,j}^{\sigma_L}}{\lambda_t},$$

where  $\lambda_t$  denotes the marginal utility of consumption, defined in (2.9). In fact, the household is subject to the same Calvo frictions faced by intermediate good producers in setting their prices. In particular, in any given period the  $j^{\text{th}}$  household can reoptimize its wage rate with probability,  $1 - \xi_w$ . With probability  $\xi_w$  it cannot reoptimize, in which case it sets its wage rate as follows:

$$W_{j,t} = \tilde{\pi}_{w,t} \mu_z W_{j,t-1},$$

where

$$\tilde{\pi}_{w,t} \equiv (\pi_{t-1})^{\iota_w} \bar{\pi}^{1-\iota_w}. \quad (3.9)$$

In (3.8), the variable,  $A_{j,t}$  denotes the net payoff from insurance contracts on the risk that a household cannot reoptimize its wage rate,  $W_t^j$ . The existence of these insurance contracts have the consequence that in equilibrium all households have the same level of consumption, capital and money holdings. We have imposed this equilibrium outcome on the notation by dropping the  $j$  subscript.

The household's problem is to maximize (3.7) subject to the demand for labor, (3.6), the Calvo wage-setting frictions, and (2.4). Households set their wage as an increasing function of the marginal cost of working. The presence of wage frictions leads households who have the opportunity to reoptimize their wage, to take into account future expected marginal costs. The sluggishness of wages and the fact that households are required to satisfy (3.6) implies that in the short run, employment is largely demand-determined.

The monetary authority controls the supply of money,  $M_t^s$ . It does so to implement a following Taylor rule. The target interest rate is:

$$R_t^* = \frac{\mu_z \pi}{\beta} + \alpha_\pi [E_t(\pi_{t+1}) - \bar{\pi}] + \alpha_y \log\left(\frac{Y_t}{Y_t^+}\right) + u_t, \quad (3.10)$$

where  $Y_t^+$  is aggregate output on a nonstochastic steady state growth path and  $u_t$  is an iid monetary policy shock. The monetary authority manipulates the money supply to ensure that the equilibrium nominal rate of interest,  $R_t$ , satisfies:

$$R_t = \rho_i R_{t-1} + (1 - \rho_i) R_t^*. \quad (3.11)$$

The parameter values for the model are the ones in the real business cycle model, plus the following:

$$\begin{aligned} \lambda_f &= 1.20, \quad \lambda_w = 1.05, \quad \xi_p = 0.63, \quad \xi_w = 0.81, \quad \iota = 0.84, \\ \nu_w &= 0.13, \quad \rho_i = 0.81, \quad \alpha_\pi = 1.95, \quad \alpha_y = 0.18, \quad v = 0. \end{aligned} \quad (3.12)$$

Our monetary model is a special case of a more general model which was estimated in US data in Christiano, Motto and Rostagno (2006). All but the first two parameters above were estimated there.

### 3.2. Results

Figure 8 displays the results of the response to a signal in period 1 about a shock in period 13 (indicated by the ‘\*’), which ultimately does not occur. The results for the real business cycle model in Figure 7 are reproduced here to facilitate comparison. Both models have costs in adjusting the flow of investment, as well as habit persistence in consumption. Still, the two models display strikingly different responses to the signal shock in period 1. First, the magnitude of the responses in output, hours, consumption and investment in the monetary model are more than three times what they are in the real business cycle model. Second, consumption booms in the immediate period after the signal. Third, the risk free rate moves by only a small amount, and it falls rather than rising as in the real business cycle model. Fourth, although inflation initially rises, eventually it falls by 0.8 of one percent. Fifth, and perhaps most significantly, the stock price rises.

The simple monetary model behaves so differently from the real business cycle model because the monetary policy rule, (3.11), is suboptimal for our model economy. With optimal monetary policy, the response of the allocations in the simple monetary model would have been virtually the same as the responses in the real business cycle model. In particular, there would have been a boom, but only a very small one. To understand how we reached this conclusion, consider the starred lines in Figure 9 (the solid lines reproduce the solid lines in Figure 8 for convenience). The starred lines were obtained by first deleting the monetary policy rule, (3.11), from the simple

monetary model. Of course, this renders the model unable to determine values for the endogenous variables. In effect, there are now many configurations of the endogenous variables which satisfy the remaining equilibrium conditions (i.e., the resource constraint, the necessary conditions associated with optimality, etc.) From this set of possible equilibria, we selected the Ramsey equilibrium, the one associated with the highest level of expected household utility. The allocations in this Ramsey equilibrium are what is achieved with the best possible monetary policy, when we do not place any constraints on what form monetary policy takes on.<sup>3</sup> Note how the Ramsey equilibrium responses of output, investment, consumption, hours worked, the real interest rate, and the price of capital virtually coincide with what they are in the real business cycle. That is, had monetary policy in the simple monetary model been optimal, the allocations would have been essentially the same as those in the RBC model.<sup>4</sup>

What is it about (3.11) that causes it transform what would have been a minor fluctuation into a substantial, welfare-reducing, boom-bust cycle? A clue lies in the fact that the real wage rises during the boom under the optimal policy, while it falls under (3.11). During the boom of the simple monetary model, employment is inefficiently high. This equilibrium outcome is made possible in part by the low real wage (which signals employers that the marginal cost of labor is low) and the fact that

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<sup>3</sup>Technical details on the computation of optimal monetary policy are provided in Appendix B.

<sup>4</sup>The reasons the allocations in the RBC model and in the Ramsey equilibrium do not coincide exactly is that there are too many frictions for monetary policy to undo. Monetary policy is only one instrument, but there are several frictions, including sticky prices and sticky wages. Other frictions include the distortions induced by the presence of market power in firms and households. If these frictions were small (or, undone by a suitable choice of taxes) then if there were either only sticky prices or only sticky wages, the Ramsey equilibrium allocations would coincide with the allocations in the RBC model.

employment is demand-determined. The high real wage in the Ramsey equilibrium, by contrast, sends the right signal to employers and discourages employment. Since wages are relatively sticky compared to prices, an efficient way to achieve a higher real wage is to let inflation drop. But, the monetary authority who follows the inflation-targeting strategy, (3.11), is reluctant to allow this to happen. Such a monetary authority responds to inflation weakness by shifting to a looser monetary policy stance. This, in our model, is the reason the boom-bust cycle is amplified. Thus, the suboptimally large boom-bust cycle in the model is the outcome of the interaction between sticky wages and the inflation targeting monetary policy rule, (3.11).

To gain further intuition into the boom-bust cycle in the simple monetary model, consider Figure 11 (there is no Figure 10 in the paper). Figure 11 shows what would have happened if the rise in productivity that is signalled is actually realized. In effect, the figure shows what expectations agents have as they respond to the signal about future productivity. Note that they expect a much bigger and more persistent decline in the nominal interest rate than actually occurs in the equilibrium in Figure 8. Note too, that there is a sharp drop in employment when the technology shock is realized, as in the real business cycle model. This is similar to what we saw in the real business cycle analysis in Figure 3.

Figures 12 and 13 explore the role of sticky prices and wages in our analysis, respectively, by repeating our experiment twice. In the first case,  $\xi_p = 0.01$  and all other parameters are held fixed at their baseline levels. In Figure 13,  $\xi_w = 0.01$  and all other parameters are held constant at their baseline values. The experiments convey a strong message: sticky wages are crucial to our result, not sticky prices. When wages are flexible, the boom-bust in the simple monetary model closely resembles what it is in the real business cycle model. We verified that this result is not an

artifact of the fact that  $\xi_p$  is smaller than  $\xi_w$  in our baseline parameterization. For example, when we set  $\xi_p = 0.95$  and  $\xi_w = 0.01$ , the results for quantities are similar to what we see in Figure 13.

In Figures 14 and 15 we attempt to investigate the specific role played by policy in generating the boom-bust cycle. Figure 14 reports what happens when the monetary policy rule focuses less on inflation, by setting  $\alpha_\pi = 1.05$ . The real quantities now fluctuate much as they do in the real business cycle model. This is consistent with our general theme, that inflation targeting is at the heart of why we get a boom-bust in our model.

Figure 15 displays a different way of assessing the role of monetary policy in our equilibrium. This experiment focuses specifically on the impact of the sharp drop in the interest rate that agents expect according to the solid line in Figure 11. For this experiment, we add an iid monetary policy shock to the policy rule, (3.11). We suppose that the shock has the same time series representation as (2.7), except  $\rho = 0$ . We set  $p = 10$ , and considered a signal which creates the expectation that there will be a 100 basis point negative shock to the quarterly rate of interest in period 11. We imposed that that shock is in fact not realized. The idea is to create a policy-induced move in the actual interest rate, together with a prior expectation that the rate would fall even more. Figure 15 indicates that this anticipated monetary loosening creates a substantial boom right away in the model. The boom accounts for almost all of the monetary part of the boom-bust in the simple monetary model. The vertical difference between the two lines in Figure 15 is roughly the size of the boom in the real business cycle model. The results in Figure 15 suggests the following loose characterization of our boom-bust result in the simple monetary model: one-third of the boom-bust cycle is the efficient response to a signal about technology, and two-thirds of the boom-bust cycle is the inefficient consequence of suboptimality in

the monetary policy rule. This suboptimality operates by creating an expectation of a substantial future anticipated loosening in monetary policy.

In sum, analysis with the simple monetary model suggests that successfully generating a substantial boom-bust episode requires a monetary policy rule which assigns substantial weight to inflation and which incorporates sticky wages.

#### **4. Robustness to an Alternative Specification of the Labor Market**

The boom phase of our analysis is characterized by a low real wage, which encourages firms to expand employment, while workers are required to supply whatever labor is demanded at the given wage. It is unclear exactly how one should map this labor market specification into actual labor market arrangements, where workers spend a substantial fraction of the time in repeated relationships with firms. Still, the setup is vulnerable to the Barro (1977) critique. Because labor effort in the model is varied on the intensive margin, it is tempting to interpret the model as a metaphor for actual firm-worker relationships. But, as Barro emphasized, the notion that idiosyncrasies in the setting of wages should have allocational consequences for firms and workers in long-lasting relationships seems implausible. There are simply too many opportunities for people in long-lasting relationships to undo the effects of wage frictions.<sup>5</sup>

In recent years an alternative approach to labor markets has emerged, starting with the work of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005a,b). Gertler and Trigari (2006) build on this work (particularly, the insight in Hall (2005a)) to show how the wage setting frictions that have proved so

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<sup>5</sup>This point was stressed more recently by Goodfriend and King (1997).

useful in fitting models to macroeconomic data may be recast so as to avoid running afoul of the Barro critique. We follow Gertler, Sala and Trigari (2006) (GST) in incorporating a version of the Gertler and Trigari (2006) (GT) labor market model into the monetary model of the previous section. In GST, wage setting frictions have no impact on on-going worker employer relations because (i) they have no direct impact on the intensive labor margin, and (ii) they do not induce worker-employer separations since, by assumption, these occur only in response to exogenous events.<sup>6</sup> Wage setting frictions do have allocational effects in the GT and GST models, but these effects operate by their impact on firms' incentives to recruit new workers.

The basic logic stressed throughout this paper, that wage-setting frictions and inflation targeting can trigger boom-busts, also goes through in this alternative formulation of the labor market. The low real wage in the boom phase generates an excess of economic activity because the low real wage causes too many resources to be allocated to recruiting workers.

#### 4.1. The Model

The labor market in our alternative labor market model is a slightly modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions.<sup>7</sup> In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with the

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<sup>6</sup>The specification that the job separation rate is constant and the finding rate endogenous is consistent with findings reported in Hall (2005b,c) and Shimer (2005a,b), who report that the job finding rate is very cyclical and the job separation rate is substantially less so.

<sup>7</sup>We work with Taylor frictions because we want to be in a position to check the accuracy of our linear approximations by comparing them with higher-order approximations. It is our impression that the linearization strategy adopted by GST for their Calvo-specification of wage frictions does not easily generalize to higher orders.

model used elsewhere in the paper. In what follows, we first provide an overview and after that we present the detailed decision problems of agents in the labor market.

#### 4.1.1. Overview

As before, we adopt the Dixit-Stiglitz specification of goods production. A representative, competitive final good firm aggregates differentiated intermediate goods into a homogeneous final good. Intermediate goods are supplied by monopolists, who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions assumed in the previous section. The novelty in the present environment is that labor services are supplied to the homogeneous labor market by ‘employment agencies’. An alternative (perhaps more natural) formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem. As it is, the equilibrium conditions associated with the behavior of intermediate good firms are unchanged from what they are elsewhere in this paper.

Each employment agency retains a large number of workers. At the beginning of the period, some workers are randomly selected to separate from the firm and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the firm in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every  $N$  periods. Each agency is permanently allocated to one of

$N$  different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort,  $1/N$  of the agencies bargain in each period. Agencies in cohorts that do not bargain in the current period simply apply the wage implied for that period by the outcome of the most recent bargaining round.

Once a wage rate is determined - whether by Nash bargaining or not - we assume that each matched worker-firm pair finds it optimal to proceed with the relationship in that period. In our calculations, we verify that this assumption is correct, by confirming that the wage rate in each worker-agency relationship lies inside the bargaining set associated with that relationship.

Next, the intensity of labor effort is determined according to a particular efficiency criterion. To explain this, we discuss the implications of increased intensity for the worker and for the employment agency. The utility function of the household in the alternative labor market model is a modified version of (3.7):

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left\{ \log(C_{t+l} - bC_{t+l-1}) - \gamma \frac{\psi_t^{1+\sigma_L}}{1+\sigma_L} L_t \right\}, \quad \gamma, \omega > 0, \quad (4.1)$$

where  $L_t$  is the fraction of members of the household that are working and  $\psi_t$  is the intensity with which each worker works. As in GST, we follow the family household construct of Merz (1995) in supposing that each household has a large number of workers. Although the individual worker's labor market experience - whether employed or unemployed - is determined in part by idiosyncratic shocks, the household has sufficiently many workers that the total fraction of workers employed,  $L_t$ , as well as the fractions allocated among the different cohorts,  $l_t^i$ ,  $i = 0, \dots, N - 1$ , is the same for each household. We suppose that all the household's workers are supplied inelastically to the labor market. Each worker passes randomly from employment with a particular agency to unemployment and back to employment according to

exogenous probabilities described below. The household's budget constraint is:

$$P_t(C_t + I_t) + M_{t+1}^d - M_t^d + T_{t+1} \leq (1 - L_t) P_t b^u z_t + \sum_{i=0}^{N-1} W_t^i l_t^i \psi_t + P_t r_t^k K_t + (1 + R_t^e) T_t, \quad (4.2)$$

where  $W_t^i$  is the nominal wage rate earned by workers in cohort  $i = 0, \dots, N - 1$ . The index,  $i$ , indicates the number of periods in the past when bargaining occurred most recently. Note that we implicitly assume that labor intensity is the same in each employment agency, regardless of cohort. This is explained below. The presence of the term involving  $b^u$  indicates the assumption that unemployed workers receive a payment of  $b^u z_t$  final goods. The marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is  $\gamma \psi_t^{\sigma_L}$ . This is converted to currency units by dividing by the multiplier,  $\lambda_t$ , on (4.2) in the Lagrangian representation of the household's problem.

Let the price of labor services,  $W_t$ , denote the marginal gain to the employment agency that occurs when an individual worker raises labor intensity by one unit. Because the employment agency is competitive in the supply of labor services,  $W_t$  is taken as given and is the same for all agencies, regardless of which cohort it is in. Labor intensity equates the worker's marginal cost to the agency's marginal benefit:

$$W_t = \gamma \frac{\psi_t^{\sigma_L}}{\lambda_t}. \quad (4.3)$$

Labor intensity is the same for all cohorts because none of the variables in (4.3) is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (4.3).

### 4.1.2. Details of the Labor Market Model

Employment agencies in the  $i^{th}$  cohort which does not renegotiate its wage in period  $t$  sets the period  $t$  wage,  $W_{i,t}$ , as in (3.9):

$$W_{i,t} = \tilde{\pi}_{w,t} \mu_z W_{i,t-1}, \quad \tilde{\pi}_{w,t} \equiv (\pi_{t-1})^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,2}}, \quad (4.4)$$

for  $i = 1, \dots, N - 1$ . It is convenient to define

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1} \mu_z^j, & j > 0 \\ 1 & j = 0 \end{cases}. \quad (4.5)$$

After wages are set, employment agencies in cohort  $i$  supply labor services,  $l_t^i \psi_t$ , into competitive labor markets. In addition, they post vacancies to attract new workers in the next period.

To understand how agencies bargain and how they make their employment decisions, it is useful to consider  $F(l_t^0, \omega_t)$ , the value function of the representative employment agency in the cohort that negotiates its wage in the current period.<sup>8</sup> The arguments of  $F$  are the agency's workforce after beginning-of-period separations and new arrivals,  $l_t^0$ , and an arbitrary value for the nominal wage rate,  $\omega_t$ . To simplify notation, we leave out arguments of  $F$  that correspond to economy-wide variables. Then,

$$\begin{aligned} F(l_t^0, \omega_t) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\lambda_{t+j}}{\lambda_t} \max_{x_{t+j}^j} \left[ (W_{t+j} - \Gamma_{t,j} \omega_t) \psi_{t+j} - P_{t+j} z_{t+j} \frac{\kappa}{2} (x_{t+j}^j)^2 \right] l_{t+j}^j \\ &\quad + \beta^N E_t \frac{\lambda_{t+N}}{\lambda_t} F(l_{t+N}^0, \tilde{W}_{t+N}), \end{aligned} \quad (4.6)$$

where  $\psi_t$  is assumed to satisfy (4.3). Also,  $\tilde{W}_{t+N}$  denotes the Nash bargaining wage rate that will be negotiated when the agency next has an opportunity to do so. At

<sup>8</sup>The value function for firms in other cohorts is defined analogously, and appears in the appendix.

time  $t$ , the agency takes this as given. The law of motion of an agency's work force is:

$$l_{t+1}^{i+1} = (x_t^i + \rho) l_t^i, \quad (4.7)$$

for  $i = 0, 1, \dots, N - 1$ , with the understanding here and throughout that  $i = N$  is to be interpreted as  $i = 0$ . Here,  $x_t^i$  is the period  $t$  hiring rate of an agency in cohort  $i$ . Expression (4.7) is deterministic, reflecting the assumption that the agency employs a large number of workers. Finally, the expression,

$$z_{t+i} \frac{\kappa}{2} (x_{t+i}^i)^2 l_{t+i}^i, \quad i = 0, 1, \dots, N - 1,$$

denotes the cost, in units of the final good, associated with a hiring rate,  $x_t^i$ . We include the state of technology,  $z_t$ , which grows at deterministic gross rate,  $\mu_z > 1$ , to assure that growth is balanced in steady state.

The firm's hiring rate is chosen to solve the problem in (4.6). It is easy to verify:

$$F(l_t^0, \omega_t) = J(\omega_t) l_t^0, \quad (4.8)$$

where  $J(\omega_t)$  is not a function of  $l_t^0$ . The function,  $J(\omega_t)$ , is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is  $\omega_t$ .

We now turn to the worker. The period  $t$  value of being a worker in an agency in cohort  $i$  is  $V_t^i$ :

$$V_t^i = \Gamma_{t-i,i} \tilde{W}_{t-i} \psi_t - \gamma \frac{\psi_t^{1+\sigma_L}}{(1+\sigma_L) \lambda_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [\rho V_{t+1}^{i+1} + (1-\rho) U_{t+1}], \quad (4.9)$$

for  $i = 1, \dots, N - 1$ . Here,  $\rho$  is the probability of remaining with the firm in the next period and  $U_t$  is the value of being unemployed in period  $t$ . The values,  $V_t^i$  and  $U_t$ , pertain to the beginning of period  $t$ , after job separation and job finding has

occurred. For workers employed by agencies in cohort  $i = 0$ , the value function is  $V^0(\omega_t)$ , where  $\omega_t$  is an arbitrary value for the current period wage rate,

$$V^0(\omega_t) = \omega_t \psi_t - \gamma \frac{\psi_t^{1+\sigma_L}}{(1+\sigma_L)\lambda_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [\rho V_{t+1}^1 + (1-\rho)U_{t+1}]. \quad (4.10)$$

The notation makes the dependence of  $V^0$  on  $\omega_t$  explicit to simplify the discussion of the Nash bargaining problem below.

The value of being an unemployed worker is  $U_t$  :

$$U_t = P_t z_t b^u + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [f_t V_{t+1}^x + (1-f_t)U_{t+1}], \quad (4.11)$$

where  $f_t$  is the probability that an unemployed worker will land a job in period  $t+1$ . Also,  $V_t^x$  is the period  $t+1$  value function of a worker who finds a job, before it is known which agency it is found with:

$$V_t^x = \sum_{i=0}^{N-1} \frac{x_{t-1}^i l_{t-1}^i}{m_{t-1}} V_t^{i+1}, \quad (4.12)$$

where the total number of new matches is given by:

$$m_t = \sum_{j=0}^{N-1} x_t^j l_t^j. \quad (4.13)$$

In (4.12),

$$\frac{x_{t-1}^i l_{t-1}^i}{m_{t-1}}$$

is the probability of finding a job in an agency which was of type  $i$  in the previous period, conditional on being a worker who finds a job in  $t$ .

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m (1 - L_t)^\sigma v_t^{1-\sigma}, \quad (4.14)$$

where

$$L_t = \sum_{j=0}^{N-1} l_t^j,$$

and where  $v_t$  denotes the total number of vacancies in the economy. Total hours worked is:

$$h_t = \psi_t \sum_{j=0}^{N-1} l_t^j. \quad (4.15)$$

The job finding rate is:

$$f_t = \frac{m_t}{1 - L_t}. \quad (4.16)$$

The  $i = 0$  cohort of agencies in period  $t$  solve the following Nash bargaining problem:

$$\max_{\omega_t} (V^0(\omega_t) - U_t)^\eta J(\omega_t)^{(1-\eta)}, \quad (4.17)$$

where  $V^0(\omega_t) - U_t$  is the match surplus enjoyed by a worker. We denote the solution to this problem by  $\tilde{W}_t$ . Note that  $\tilde{W}_t$  takes into account that intensity will be chosen according to (4.3) as well as (4.4).

The resource constraint in the model must be modified to accommodate resources paid to unemployed workers and resources used up in posting vacancies:

$$C_t + I_t + b^u z_t (1 - L_t) + z_t \frac{\kappa}{2} \sum_{j=0}^{N-1} (x_t^j)^2 l_t^j = Y_t, \quad (4.18)$$

where  $Y_t$  is the quantity of final goods, (3.1).

After scaling the variables that grow, the equilibrium conditions associated with the labor market can be combined with the non-labor market conditions of the model in the previous section to solve the overall model. For complete details about how this economy is scaled to account for growth, and for the other equations, see the Appendix.

## 4.2. Quantitative Results

In the first subsection below, we report the parameterization of the model. In the second subsection, we discuss some properties of the equilibrium of the alternative

labor market model. The results for boom-bust episodes are presented in the third subsection.

#### 4.2.1. Parameterization of Models

To accommodate the fact that the average duration of unemployment is a little over two months, we adopt a bi-monthly time period for the model. For comparability with the standard sticky wage model in the previous section, we study a bimonthly version of that model too. The parameters that the two models have in common were set as follows:

$$\begin{aligned}\lambda_f &= 1.20, \lambda_w = 1.05, \xi_p = \frac{5}{6}, \iota = 0.84, \iota_w = 0.13, \alpha_\pi = 1.95, \alpha_y = 0.18, \\ \beta &= 1.01358^{-(1/6)}, \mu_z = 1.0136^{(1/6)}, b = 0.63, a = 15.1, \alpha = 0.40, \\ \delta &= \frac{2}{3}0.025.\end{aligned}$$

Here, only  $\xi_p$ ,  $\beta$ ,  $\delta$ ,  $\mu_z$  are adjusted relative to their values in the analysis of the previous section (see (3.12)). The parameter,  $\xi_p$ , was set to imply a mean time between price changes of one year.<sup>9</sup> The inertia parameter in the monetary policy rule,  $\rho_i$ , was set to 0.60 in our benchmark parameterization of the model with the alternative labor market. We found that the degree of over reaction to a signal about future productivity is sensitive to this parameter, with a smaller value making the over-reaction large.

Regarding the parameters that apply specifically to our standard sticky wage model, we set  $\xi_w = 0.87$ , which implies that wages remain unchanged on average for

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<sup>9</sup>We preferred to set  $\xi_p$  to a lower value. However, when we tried we encountered numerical problems in computing the Ramsey equilibrium for the alternative labor market model. We hope to solve this numerical problem for the next draft.

7.89 bi-monthly periods, or 1.3 years. The values of  $\psi_L$  and  $\sigma_L$  were set as before, as specified in (2.8).

Now consider the parameters that apply specifically to the alternative labor market model. The disutility of work intensity parameter,  $\gamma$ , was set so as to imply a value for labor intensity equal to unity in steady state,  $\psi = 1$ . In addition, we follow GT in setting the hiring rate adjustment cost parameter,  $\kappa$ , so that the steady state value of the job finding rate,  $f$ , implies that jobs last on average 2.22 months, or 1.11 model periods (i.e.,  $f = 0.9$ ). We also follow GT in our setting of the bargaining power parameter,  $\eta$ , the elasticity of matches to unemployment,  $\sigma$ , the degree of convexity in the disutility from work intensity,  $\sigma_L$ , and the constant term,  $\sigma_m$ , in the matching function:

$$b^u = 0.3, \rho = 0.9333, \eta = \frac{1}{2}, \sigma_m = 0.1, \sigma = \frac{1}{2}, \sigma_L = 13.5, \kappa = 54.42.$$

Following GT, the value for  $\rho$  is chosen to imply that jobs on average last about 2.5 years. Finally, we must select a value for  $N$ , the length of time between wage negotiations. Following Dixon and Kara (2006) we set  $N = 15$  so that the average age of existing wage contracts in the alternative labor market model is equal to the average age (i.e., 7.89 periods) of existing contracts in the standard sticky price model.<sup>10</sup>

#### 4.2.2. Properties of the Alternative Labor Market Model

Before evaluating the model's implications for boom-bust episodes, we display some of its basic steady state and dynamic properties. For the dynamic properties, we find that money has bigger real effects in the standard sticky wage model than it does in

<sup>10</sup>Dixon and Kara (2006) report that the average age of a wage contract when there is staggering as in Taylor is  $(N + 1)/2$ .

the alternative labor market model. The dynamic response to a technology shock is similar across the two models.

Consider the steady state first. At the parameter values discussed in the previous subsection, we find:

$$\frac{C}{C+I} = 0.63, \quad \frac{K}{C+I} = 19.51, \quad \bar{b} \equiv \frac{b^u}{w\tilde{w}\psi - \gamma \frac{\psi^{1+\sigma_L}}{(1+\sigma_L)\lambda_z}} = 0.11, \quad (4.19)$$

$$\frac{\frac{\kappa}{2}(1-\rho)^2 L}{C+I} = 0.026, \quad w\tilde{w} = 3.0, \quad \underline{w} = 0.43, \quad \bar{w}^0 = 3.27, \quad \bar{w}^{14} = 6.97, \quad L = 0.93.$$

where absence of a time subscript indicates the steady state value of the variable. The product,  $w\tilde{w}$ , is the steady state value of the real wage paid by employment agencies to workers, scaled by  $z_t$ , and  $\lambda_z$  is the steady state value of  $\lambda_t P_t z_t$ , where  $\lambda_t$  is the multiplier on the household's budget constraint. The object,  $\bar{b}$ , is the ratio of the flow value of unemployment to the flow value of employment for a worker. This ratio has attracted much attention, particularly with Hagedorn and Minovskii (2006)'s demonstration that a higher value of  $\bar{b}$  increases the volatility of labor market variables. Our setting for this ratio is quite small (it corresponds to the value estimated by GST). For example, Shimer (2005a) argues that an empirically plausible value for  $\bar{b}$  lies in the neighborhood of 0.4. The consumption to GDP ( $\equiv C+I$ ) ratio for the model is in line with standard estimates for the US, and the capital-output ratio corresponds roughly to the familiar US (annualized) value of 3. Our ratio of recruitment costs to GDP of 2.6 percent is somewhat larger than the value of 0.01. The steady state unemployment rate in the model,  $1 - 0.93 = 0.07$ , corresponds to the value used in GT. Our steady state unemployment rate is somewhat high if we compare it with measured unemployment. However, the concept of unemployment in the model is broader and includes all (working age) people not working.

The object,  $\underline{w}$ , is the real wage (scaled by  $z_t$ ) that makes a worker indifferent

between employment (i.e., matching with an agency and staying with it until exogenously separated) and unemployment. The object,  $\bar{w}^i$ , is the scaled real wage that makes an employment agency that last renegotiated its wage  $i$  periods ago just indifferent between matching with a worker and not matching (see Appendix B for details). The latter object is a function of  $i$  because it is calculated under the assumption that when the next renegotiation takes place, the firm reverts to the scaled steady state real wage,  $w\check{w}$ . Our dynamic calculations assume that the real wage paid to workers lies inside the interval,  $[\underline{w}_t, \bar{w}_t^i]$ ,  $i = 0, \dots, N - 1$ , for all  $t$ . Note from (4.19) that this is the case in steady state. This is also true for the dynamic simulations reported in the paper. The low value of  $\underline{w}$  reflects the relatively low value of  $\bar{b}$  used in the model.<sup>11</sup>

Turning to the model's dynamic properties, we consider the dynamic effects of a monetary policy shock,  $u_t$  (see (3.10) and (3.11)) and of a (conventional) technology shock,  $\xi_t$  (see (2.7)). Consider Figure 16 first. It displays dynamic responses to the same monetary policy shock in each of the bimonthly version of our standard sticky wage model and the alternative labor market model. The response of the equilibrium interest rate appears in the 2,5 panel of the figure. This response is slightly different between the two models because, according to (3.11), the equilibrium interest rate responds to output and expected inflation, as well as to a monetary policy shock.

Note that the real effects of monetary policy are substantially larger than in the standard sticky wage model. For example, the percent response of GDP is three times larger in the latter model than it is in the alternative labor market model. Similarly, the response of inflation in the alternative labor market model is larger. The figure allows us to trace the economics of the monetary transmission mechanism

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<sup>11</sup>We attempted to raise  $\bar{b}$  by increasing  $b^u$ . However, when we did so we ran into numerical difficulties computing the Ramsey equilibria for our model. We are working to fix this.

in the alternative labor market model. Thus, note how the real price of labor (that is,  $W_t/P_t$ , the real wage paid for a unit of homogeneous labor by intermediate good firms to employment agencies) jumps in response to an expansionary monetary policy shock. This jump reflects that the demand for labor rises as the demand for goods rises in response to the expansionary monetary policy shock. Employment agencies respond by posting higher vacancies and this leads, in subsequent periods, to an expansion in the number of people employed (see ‘labor’). The posting of vacancies is further encouraged by the fact that the average, across firm cohorts, of the real wage paid to workers falls. This fall reflects the rise in inflation and the wage-setting frictions. The intensity of work effort rises in response to the rise in the price of labor because intensity is set efficiently (see (4.3)). The product of intensity and labor corresponds to hours worked. Note that the response of hours worked in the alternative labor supply model is smaller than the response in the sticky wage model. Finally, a result that plays an important role in boom-bust dynamics is that the price of capital rises in response to an expansionary monetary policy shock. The effect is roughly the same in the two models.

We investigated the possibility that the weaker effect of monetary policy shocks in the alternative labor market model reflects our low value of  $\bar{b}$ . To investigate this, we set  $b^u = 2$ ,<sup>12</sup> which implies  $\bar{b} = 0.70$  (a value suggested by Hall). The peak output effect of a monetary policy shock rises from a little over 0.01 percent in the low  $\bar{b}$  model (see Figure 16) to a little under 0.015 in the high  $\bar{b}$  economy. Although the change in the value of  $b^u$  has the expected qualitative effect, it does essentially nothing to close the quantitative gap in the output response of the alternative labor market and standard sticky wage economies.

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<sup>12</sup>Given that we hold  $f$  constant, this implies  $\kappa = 20.92$ .

Now consider Figure 17, which displays the response of the two models to a standard technology shock,  $\xi_t$ . The figure displays the response of the two models, when monetary policy is specified as in (3.10) and (3.11), as well as when it is specified optimally (the computation of the Ramsey equilibrium is explained in Appendix C). Consider the Ramsey equilibrium responses, which are quite similar, first. The responses of output, hours worked, and inflation are essentially the same. The initial response of hours worked in the Ramsey equilibrium is negative, and turns positive afterward. The composition of output differs between the two models, with consumption rising relatively more strongly in the alternative labor market model. In both models, the nominal rate of interest is reduced sharply in the wake of a technology shock. The interest rate is reduced by more in the alternative labor market model, presumably to compensate for the weaker monetary non-neutrality in that model, which was documented in Figure 16. Note how the interest rate cuts are so large that the zero lower bound on the nominal interest rate is violated. Strictly speaking, these are not equilibria. We report them nevertheless, as evidence on the properties of the models.

The responses of output and employment to a technology shock when monetary policy is described by (3.10) and (3.11) are similar in the two models, though weaker than in the Ramsey equilibria. The weaker responses presumably reflect that monetary policy is less expansionary in the wake of a technology shock than it is in the Ramsey equilibria. Note how the interest rate falls less in the equilibrium than in the Ramsey responses. As in the Ramsey equilibrium, consumption responds relatively strongly in the alternative labor market model. A big difference in the two models lies in the inflation rate, which falls more in the alternative labor market model.

The figures allow us to trace the economics of the transmission of a technology shock in the alternative labor market model. Note how the price of labor falls sharply

in the equilibrium of this model. This fall reflects the inadequacy of aggregate demand in the wake of a technology shock. As a result, there is a persistent decline in the demand for labor by intermediate good firms. Employment agencies react by reducing vacancies. At the same time, the real wage soars because of the wage frictions and the drop in inflation. This provides an additional incentive for employment agencies to reduce vacancies. The result is a persistent decline in employment. The intensity of work effort falls because of the drop in the price of labor, and the combined effects result in a persistent drop in total hours worked. It is interesting that the drop is similar across the two models, even though the rise in the real wage in the standard sticky wage model is substantially smaller than what it is in the alternative labor market model.

#### 4.2.3. Results for Boom-Bust

The results in Figure 18 show the dynamic response of our models to a  $\xi_t$  shock that occurs in period 1, followed by  $\varepsilon_{t+p} = -\xi_t$  for  $p = 12$ . Thus, there is a signal that technology will improve two years in the future, a signal that in the end turns out to be false. The figure displays the responses of the variables in our alternative labor market and standard sticky wage models. In each case, we display the responses for the version of the model in which policy is governed by (3.10) and (3.11), as well as for the version in which policy is optimal. Note that the Ramsey equilibria for the two models are for the most part very similar (compare the circles and the stars). Interestingly, in both models the Ramsey equilibria are characterized by a rise in the real wage and a fall in the price of capital. Although the behavior of these variables is qualitatively similar, there are clearly important quantitative differences. At the same time, the ‘real wage’ in the two models clearly refers to at least slightly different

concepts and so comparisons of the real wage must be made with caution.

Now consider the equilibria in which policy is governed by (3.10) and (3.11). Our earlier result that output overreacts to a signal shock in the standard sticky wage model is reproduced in the bimonthly version of that model. The percent increase in output, at its peak, is over three times the corresponding Ramsey increase. The alternative labor market model also exhibits over reaction, though one that is quantitatively smaller. In the equilibrium, the percent increase in output at its peak is 0.85 percent, versus 0.47 percent in the Ramsey equilibrium. There is a similar overreaction in hours worked. In both models, the real wage is low in the boom phase, consistent with our diagnosis that a key problem is the failure of the equilibrium to reproduce the rise in the real wage that occurs in the Ramsey equilibrium. In the alternative labor market model, the fall in the real wage combines with a rise in the price of capital to create an incentive to increase the hiring rate. This leads to a rise in employment. In addition, the increased demand for goods leads to a rise in the demand for labor, which produces a rise in the price of labor. This provides additional incentive to increase the hiring rate and its leads to an increase in the intensity of work.

Recall that in the sticky wage model, there are two crucial reasons that output over reacts to a signal about future technology: sticky wages and inflation targeting. To investigate the role of inflation targeting, we redid the computations setting  $\alpha_\pi = 1.01$ . As before, the overreaction does not occur with this change. In fact, output, consumption and investment actually under react - these variables rise less than they do in the Ramsey equilibrium. We also redid the calculations with flexible wages by supposing that Nash bargaining occurs in each period in each agency. We found that responses are reduced, and more nearly resemble the responses in the Ramsey equilibrium. For example, at its peak output rises by 0.50 percent, versus 0.47 in the

Ramsey equilibrium. Similarly, at their peaks investment, consumption and hours worked rise 0.88, 0.34 and 0.47 percent, respectively. The corresponding Ramsey figures are 0.98, 0.23 and 0.45 percent.<sup>13</sup>

In sum, the overreaction to an anticipated technology shock that is so pronounced in our sticky wage model is also a feature of the alternative labor market model. However, the quantitative effects are smaller. As discussed above, our model of boom-bust is driven by two things: a technology shock that is never realized and a monetary policy response. The alternative labor market model exhibits a similar dynamic response to technology shocks as our standard sticky wage model. However, the real effects of monetary policy shocks are weaker in the alternative labor market model, and this is the reason the boom-bust in the alternative labor market model is quantitatively weaker. This suggests that factors which enhance the nonneutrality of money in the alternative labor market model would result in a quantitatively larger boom-bust cycle.

## 5. Full Monetary Model

We consider the full monetary model of Christiano, Motto and Rostagno(2006). That model introduces into the simple monetary model of section 3 a banking sector following Chari, Christiano, and Eichenbaum (1995) and the financial frictions in Bernanke, Gertler and Gilchrist (1999). In the model, there are two financing requirements: (i) intermediate good firms require funding in order to pay wages and capital rental costs and (ii) the capital is owned and rented out by entrepreneurs, who do not have enough wealth ('net worth') on their own to acquire the capital

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<sup>13</sup>The peaks in output, consumption and hours worked occur in period 12, while it occurs in period 11 for investment.

stock and so they must borrow. The working capital lending in (i) is financed by demand deposits issued by banks to households. The lending in (ii) is financed by savings deposits and time deposits issued to households. Demand deposits and savings deposits pay interest, but relatively little, because they generate utility services to households. Following Chari, Christiano, and Eichenbaum (1995), the provision of demand and savings deposits by banks requires that they use capital and labor resources, as well as reserves. Time deposits do not generate utility services. In addition to holding demand, savings and time deposits, households also hold currency because they generate utility services. Our measure of M1 in the model is the sum of currency plus demand deposits. Our broader measure of money (we could call it M2 or M3) is the sum of M1 and savings deposits. Total credit is the sum of the lending done in (i) and (ii). Credit differs from money in that it includes time deposits and does not include currency. Finally, the monetary base in the model is the sum of currency plus bank reserves. A key feature of the model that allows it to make contact with the literature on boom-busts is that it includes various monetary aggregates as well as credit, and that there are nontrivial differences between these aggregates.

Figure 19 displays the response of the full model to the signal shock, and includes the response of the simple model for ease of comparison. Note that the full model displays a weaker response, though qualitatively the results are similar. Figure 20 displays the response of various financial variables to the signal shock. The top left figure shows that the monetary base grows throughout the boom, and then falls in the period it is realized that the shock will not happen. M1 growth is strong throughout the expansion, while M3 growth falls consistently throughout the boom. Thus, the behavior of the monetary aggregates is inconsistent. However, credit growth is strong throughout the boom and remains high during the bust. That credit growth is con-

sistently strong throughout the boom-bust episode while monetary aggregates send conflicting signals reflects portfolio shifts among the monetary aggregates. Moreover, as noted above, not all the expansion in credit shows up in the monetary aggregates.

Figure 21 shows what happens when we include credit growth in the policy rule, (3.11), with a coefficient of 3. The strong growth of credit leads to a tightening of monetary policy during the boom, and causes the boom to more nearly resemble the efficient response of the economy to the signal about future productivity.

In sum, we have simulated the response to a signal shock of a model with considerably more frictions than those in our simple monetary model. The basic qualitative findings of our simple model are robust to this added complexity.

## 6. Conclusions

Wage setting frictions deserve to be taken seriously because they are a feature of models that fit the data well. It is known that with sticky wages, a policy of targeting inflation can lead to suboptimal results by making the real wage overly rigid. For example, if there is a negative oil shock which requires that the real wage fall, a policy of preventing the price level from rising will cause the real wage to be too high and employment too low. In this paper we have shown that the type of boom-bust episodes we have observed may be another example of the type of suboptimal outcome that can occur in an economy with sticky wages and monetary targeting. Indeed, we argued that within the current class of DSGE models that have been fit to data, it is hard to find another way to account for boom-bust episodes.

We have not addressed the factors that make inflation targeting attractive. These are principally the advantages that inflation targeting has for anchoring inflation expectations. We are sympathetic to the view that these advantages argue in favor

of some sort of inflation-targeting in monetary policy. However, our results suggest that it may be optimal to mitigate some of the negative consequences of inflation targeting by also taking into account other variables. In particular, we have examined the possibility that a policy which reacts to credit growth in addition to inflation may reduce the likelihood that monetary policy inadvertently contributes to boom-bust episodes.

Our argument remains tentative, even within the confines of the monetary economies that we studied in this paper. We only showed that a monetary policy which feeds back onto credit growth moves the response of the economy to a certain signal shock closer to the optimal response. Such a policy would substantially reduce the violence of boom-bust episodes. However, a final assessment of the value of integrating credit growth into monetary policy awaits an examination of how the response of the economy to other shocks is affected. This work is currently under way. The model of Christiano, Motto and Rostagno (2006) is ideally suited for this. Not only does it contain a variety of credit measures, so that a serious consideration of the consequences of integrating credit into monetary policy is possible. In addition, that model is estimated with many shocks, making it possible to see how integrating credit growth into monetary policy affects the transmission of shocks.

## A. Appendix A: The Equations Characterizing Equilibrium in the Simple Monetary Model

Our simple monetary model is quite standard and the derivation of the equilibrium conditions can be found in several places (see, especially, Yun (1996) and Erceg, Henderson, and Levin (2000)). For the sake of completeness, we list the equations of the model here. To define the seven equations that characterize price and wage optimization, we must define several auxiliary variables,  $p_t^*$ ,  $w_t^*$ ,  $F_{p,t}$ ,  $F_{w,t}$ ,  $w_t^*$ . The three equations associated with optimal price setting are:

$$p_t^* - \left[ (1 - \xi_p) \left( \frac{1 - \xi_p \left( \frac{\pi_t^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right)^{\lambda_f} + \xi_p \left( \frac{\pi_{t-1}^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}} = 0, \quad (\text{A.1})$$

$$E_t \left\{ \lambda_{z,t} (p_t^*)^{\frac{\lambda_f}{\lambda_f-1}} \left[ \epsilon_t \left( \frac{k_{t-1}}{\mu_z} \right)^\alpha \left( (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right)^{1-\alpha} - \phi \right] + \left( \frac{\pi_t^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0, \quad (\text{A.2})$$

$$\lambda_{z,t} \lambda_f (p_t^*)^{\frac{\lambda_f}{\lambda_f-1}} \left[ \epsilon_t \left( \frac{k_{t-1}}{\mu_z} \right)^\alpha \left( (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right)^{1-\alpha} - \phi \right] s_t + \quad (\text{A.3})$$

$$\beta \xi_p \left( \frac{\pi_t^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \left[ \frac{1 - \xi_p \left( \frac{\pi_t^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} F_{p,t+1} - F_{p,t} \left[ \frac{1 - \xi_p \left( \frac{\pi_t^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} = 0.$$

Let  $\pi_{w,t}$  denote the rate of wage inflation:

$$\pi_{w,t} = \frac{\tilde{w}_t \mu_z \pi_t}{\tilde{w}_{t-1}},$$

where  $\tilde{w}_t$  is the real wage, scaled by  $z_t$ . The equations that relate to wage optimization are:

$$E_t \left\{ \lambda_{z,t} \frac{(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t}{\lambda_w} + \beta \xi_w \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}} \frac{\left( \frac{\tilde{w}_t}{\tilde{w}_{t+1} \pi_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} \right\} = 0 \quad (\text{A.4})$$

$$E_t \left\{ \left[ (w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t \right]^{1+\sigma_L} \right. \\ \left. + \beta \xi_w \left( \frac{\tilde{\pi}_{w,t+1}}{\tilde{w}_{t+1} \pi_{t+1}} \tilde{w}_t \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} \frac{1}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t+1}}{\tilde{w}_{t+1} \pi_{t+1}} \tilde{w}_t \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w (1+\sigma_L)} \tilde{w}_{t+1} F_{w,t+1} \right. \\ \left. - \frac{1}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\tilde{w}_t \pi_t} \tilde{w}_{t-1} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w (1+\sigma_L)} \tilde{w}_t F_{w,t} \right\} = 0, \quad (\text{A.5})$$

and

$$w_t^* = \left[ (1 - \xi_w) \left( \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\tilde{w}_t \pi_t} \tilde{w}_{t-1} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w} + \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\tilde{w}_t \pi_t} \tilde{w}_{t-1} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \quad (\text{A.6})$$

To characterize the cross-sectional average utility of households, we require an additional variable related to wage dispersion,  $w_t^+$  :

$$w_t^+ = \left[ (1 - \xi_w) \left( \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\tilde{w}_t \pi_t} \tilde{w}_{t-1} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w (1+\sigma_L)} + \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\tilde{w}_t \pi_t} \tilde{w}_{t-1} w_{t-1}^+ \right)^{\frac{\lambda_w (1+\sigma_L)}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w (1+\sigma_L)}}. \quad (\text{A.7})$$

The fact that real marginal cost,  $s_t$ , is the real wage divided by the marginal productivity of labor is represented as follows:

$$s_t = \frac{\tilde{w}_t}{(1 - \alpha) \epsilon_t} \left( \frac{\mu_z (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_t}{k_{t-1}} \right)^\alpha, \quad (\text{A.8})$$

where  $h_t$  denotes the unweighted average of household employment, and the adjustment,  $(w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_t$ , is required to convert household average work,  $h_t$ , into effective average labor effort. Also,  $k_t$  denotes the capital stock, scaled by  $z_{t-1}$ . A similar adjustment is required to define an aggregate resource constraint:

$$c_t + I_t = (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left\{ \epsilon_t \left( \frac{k_{t-1}}{\mu_z} \right)^\alpha \left[ (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_t \right]^{1-\alpha} - \phi \right\}. \quad (\text{A.9})$$

The capital accumulation equation is:

$$k_t - (1 - \delta) \mu_z^{-1} k_{t-1} = \left[ 1 - \frac{S'' \mu_z^2}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \quad (\text{A.10})$$

where we interpret  $I_t$  as investment scaled by  $z_t$ .

The equation that defines the nominal rate of interest is:

$$E_t \left\{ \beta \frac{1}{\pi_{t+1} \mu_z} \lambda_{z,t+1} (1 + R_t) - \lambda_{z,t} \right\} = 0, \quad (\text{A.11})$$

and the marginal utility of consumption,  $\lambda_{z,t}$ , is:

$$E_t \left[ \lambda_{z,t} - \frac{\mu_z}{c_t \mu_z - b c_{t-1}} + b \beta \frac{1}{\mu_z c_{t+1} - b c_t} \right] = 0. \quad (\text{A.12})$$

Here,  $\lambda_{z,t}$  has been scaled by multiplying by  $z_t$ . The period  $t$  first order condition associated with the choice of  $k_{t+1}$  is:

$$-\lambda_{z,t} + E_t \lambda_{z,t+1} \beta \frac{1}{\mu_z q_t} \left[ \alpha \epsilon_{t+1} \left( \frac{\mu_z (w_{t+1}^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_{t+1}}{k_t} \right)^{1-\alpha} s_{t+1} + q_{t+1} (1 - \delta) \right] = 0. \quad (\text{A.13})$$

The first order necessary condition associated with the optimal choice of  $I_t$  is:

$$E_t \left\{ \lambda_{zt} q_t \left[ 1 - \frac{S'' \mu_z^2}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - S'' \mu_z^2 \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] - \lambda_{zt} + \beta \lambda_{z,t+1} q_{t+1} S'' \mu_z^2 \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} = 0. \quad (\text{A.14})$$

The cross-sectional average of household period utility is:

$$u(c_t, c_{t-1}, w_t^*, w_t^+, h_t) = \log(c_t \mu_z - b c_{t-1}) - \frac{\psi_L}{1 + \sigma_L} \left( \frac{w_t^*}{w_t^+} \right)^{\frac{\lambda_w(1+\sigma_L)}{\lambda_w - 1}} h_t^{1+\sigma_L}. \quad (\text{A.15})$$

The appearance of  $w_t^*/w_t^+$  reflects that household labor effort varies across the population with the variation in individual wages. When  $\sigma_L > 0$  this matters for average utility, and  $w_t^*/w_t^+$  provides the necessary adjustment. When  $\sigma_L = 0$  then utility is linear in hours and no adjustment is required. Accordingly, in this case (A.6) and (A.7) coincide, so that  $w_t^*/w_t^+ = 1$ .

The 15 endogenous variables to be determined are:

$$\pi_t, c_t, h_t, k_{t+1}, p_t^*, F_{p,t}, s_t, R_t, w_t^+, w_t^*, F_{w,t}, \tilde{w}_t, \lambda_{z,t}, I_t, q_t.$$

We have listed 14 equations above, (A.1)-(A.14). When the equations are augmented by a monetary policy rule, then we can solve for our 15 endogenous variables. This corresponds to what we call our simple monetary model. Alternatively, the 14 equations can be augmented by the first order conditions associated with the Lagrangian representation of the Ramsey problem. The paper discusses the dynamic properties of the resulting Ramsey equilibrium. Technical notes related to this appear in Appendix C.

## B. Appendix B: Equilibrium with Alternative Labor Market

This section displays the equilibrium conditions used to solve the version of the model with Gertler-Trigari labor markets. It then provides formulas for the steady state.

## B.1. Equilibrium Conditions Needed to Solve the Model

To define  $J$  in (4.8), it is convenient to first denote the growth rate of employment from  $t$  to  $t + j$  for an agency that renegotiates in period  $t$  :

$$\Omega_{t,j} = \begin{cases} \prod_{l=0}^{j-1} (x_{t+l}^l + \rho) & j > 0 \\ 1 & j = 0 \end{cases}.$$

Then,

$$J(\omega_t) = E_t \sum_{j=0}^{N-1} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \max_{x_{t+j}^j} \left[ (W_{t+j} - \Gamma_{t,j}\omega_t) \psi_{t+j} - P_{t+j} z_{t+j} \frac{\kappa}{2} (x_{t+j}^j)^2 \right] \Omega_{t,j} \right\} + \beta^N E_t \frac{\lambda_{t+N}}{\lambda_t} J(\tilde{W}_{t+N}) \Omega_{t,N}. \quad (B.1)$$

After some algebra, it can be verified that optimality associated with  $x_t^i$  implies:

$$P_t z_t \kappa x_t^i = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (W_{t+1} - \Gamma_{t-i,i+1} \tilde{W}_{t-i}) \psi_{t+1} + P_{t+1} z_{t+1} \frac{\kappa}{2} (x_{t+1}^{i+1})^2 + \rho P_{t+1} z_{t+1} \kappa x_{t+1}^{i+1} \right], \quad (B.2)$$

for  $i = 0, 1, \dots, N - 2$ . Similarly, optimality for the hiring decision of a firm that is in the last period before the current contract expires implies:

$$P_t z_t \kappa x_t^{N-1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (W_{t+1} - \tilde{W}_{t+1}) \psi_{t+1} + P_{t+1} z_{t+1} \frac{\kappa}{2} (x_{t+1}^0)^2 + \rho P_{t+1} z_{t+1} \kappa x_{t+1}^0 \right]. \quad (B.3)$$

Note the familiar result here, that the value of the increase in period  $t+1$  employment (i.e., the expression on the right side of the equality in (B.2) and (B.3)) resulting from an increase in period  $t$  hiring is captured by a function of the hiring decision in  $t + 1$ .

The derivative of  $J$  with respect to  $\omega_t$ , evaluated at  $\tilde{W}_t$ , is, after taking into account the envelope conditions associated with  $x_{t+i}^i$ ,

$$J_{w,t} = -\left\{\psi_t + \sum_{j=1}^{N-1} \beta^j E_t \frac{\lambda_{t+j}}{\lambda_t} \Omega_{t,j} \Gamma_{t,j} \psi_{t+j}\right\}, \quad (\text{B.4})$$

where  $J_{w,t}$  denotes the derivative of  $J(\omega_t)$  evaluated at  $\omega_t = \tilde{W}_t$ .

It is easily verified that the marginal value to the worker of the wage is:

$$V_{w,t} = \psi_t + \sum_{j=1}^{N-1} (\beta\rho)^j E_t \Gamma_{t,j} \frac{\lambda_{t+j}}{\lambda_t} \psi_{t+j}, \quad (\text{B.5})$$

where  $V_{w,t}$  denotes the derivative of  $V^0(\omega_t)$  in (4.10) with respect to  $\omega_t$ , evaluated at  $\omega_t = \tilde{W}_t$ . The difference in the way the agency and the worker discount future wages reflects their asymmetric positions. Both take into account the way the wage contract evolves over time, with  $\Gamma_{t,j}$ . However, the worker includes  $\rho$  in the discounting, to take into account the fact that it could separate. The firm is bound to the contract for all periods of the contract, and so  $\rho$  does not enter its discounting. However, the number of workers covered by the contract will change over time as new workers arrive and old workers separate. This factor is accounted for by the presence of  $\Omega_{t,j}$ .

The first order necessary condition associated with the Nash bargaining problem, (4.17) is:

$$\eta V_{w,t} J_t = (1 - \eta) [V_t^0 - U_t] J_{w,t}, \quad (\text{B.6})$$

where  $J_t$  and  $V_t^0$  denote  $J(\omega_t)$  and  $V^0(\omega_t)$  evaluated at  $\omega_t = \tilde{W}_t$ .

Before the preceding equations can be used to solve the model, they must be scaled. We adopt the following scaling of variables:

$$\begin{aligned} J_{z,t} &= \frac{J_t}{z_t P_t}, \quad \lambda_{z,t+1} = \lambda_{t+1} z_{t+1} P_{t+1}, \quad w_t = \frac{\tilde{W}_t}{W_t}, \quad \tilde{w}_t = \frac{W_t}{z_t P_t}, \\ V_{z,t}^i &= \frac{V_t^i}{P_t z_t}, \quad U_{z,t} = \frac{U_t}{P_t z_t}, \quad V_{z,t}^x = \frac{V_t^x}{P_t z_t}. \end{aligned}$$

The following relations, which make use of (4.5), are useful:

$$\frac{\Gamma_{t,j}\tilde{W}_t}{P_{t+j}z_{t+j}} = G_{t,j}w_t\tilde{w}_t, \quad G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}} & j > 0 \\ 1 & j = 0 \end{cases}$$

$$\frac{\Gamma_{t-i,i+1}\tilde{W}_{t-i}}{P_{t+1}z_{t+1}} = G_{t-i,i+1}w_{t-i}\tilde{w}_{t-i}$$

$$\frac{\Gamma_{t-i,i}\tilde{W}_{t-i}}{P_t z_t} = G_{t-i,i}w_{t-i}\tilde{w}_{t-i}.$$

Divide (B.1) by  $P_t z_t$  :

$$J_{z,t} = E_t \sum_{j=0}^{N-1} \beta^j \frac{\lambda_{z,t+j}}{\lambda_{z,t}} \left[ (\tilde{w}_{t+j} - G_{t,j}w_t\tilde{w}_t) \psi_{t+j} - \frac{\kappa}{2} (x_{t+j}^j)^2 \right] \Omega_{t,j} \quad (\text{B.7})$$

$$+ \beta^N E_t \frac{\lambda_{z,t+N}}{\lambda_{z,t}} J_{z,t+N} \Omega_{t,N}.$$

Equations (B.2) and (B.3) can be written

$$\kappa x_t^i = \beta E_t \frac{\lambda_{z,t+1}}{\lambda_{z,t}} [(\tilde{w}_{t+1} - G_{t-i,i+1}w_{t-i}\tilde{w}_{t-i}) \psi_{t+1} + \frac{\kappa}{2} (x_{t+1}^{i+1})^2 + \rho \kappa x_{t+1}^{i+1}], \quad (\text{B.8})$$

for  $i = 0, 1, \dots, N-2$ , and

$$\kappa x_t^{N-1} = \beta E_t \frac{\lambda_{z,t+1}}{\lambda_{z,t}} [(\tilde{w}_{t+1} - w_{t+1}\tilde{w}_{t+1}) \psi_{t+1} + \frac{\kappa}{2} (x_{t+1}^0)^2 + \rho \kappa x_{t+1}^0]. \quad (\text{B.9})$$

The conditions for  $V_t^i$  are:

$$V_{z,t}^i = \psi_t G_{t-i,i}w_{t-i}\tilde{w}_{t-i} - \gamma \frac{\psi_t^{1+\sigma_L}}{(1+\sigma_L)\lambda_{z,t}} + \beta E_t \frac{\lambda_{z,t+1}}{\lambda_{z,t}} [\rho V_{z,t+1}^{i+1} + (1-\rho)U_{z,t+1}], \quad (\text{B.10})$$

for  $i = 0, \dots, N-1$ . Here, the weight in front of  $w_{t-i}\tilde{w}_{t-i}$  is understood to be unity, when  $i = 0$ . The condition for  $J_{w,t}$ , in terms of scaled variables, is:

$$J_{w,t} = -\left\{ \psi_t + \sum_{j=1}^{N-1} \beta^j E_t \frac{\lambda_{z,t+j}}{\lambda_{z,t}} G_{t,j} \Omega_{t,j} \psi_{t+j} \right\}. \quad (\text{B.11})$$

The condition for  $U_t$  becomes:

$$U_{z,t} = b^u + \beta E_t \frac{\lambda_{z,t+1}}{\lambda_{z,t}} [f_t V_{z,t+1}^x + (1 - f_t) U_{z,t+1}]. \quad (\text{B.12})$$

The condition for  $V_t^x$  becomes:

$$V_{z,t}^x = \sum_{j=0}^{N-1} \frac{x_{t-1}^j l_{t-1}^j}{m_{t-1}} V_{z,t}^{j+1}. \quad (\text{B.13})$$

The condition for efficient intensity becomes:

$$\tilde{w}_t = \gamma \frac{\psi_t^{\sigma L}}{\lambda_{z,t}}. \quad (\text{B.14})$$

The following six conditions are unaffected, but repeated here for convenience:

$$m_t = \sum_{j=0}^{N-1} x_t^j l_t^j, \quad (\text{B.15})$$

$$m_t = \sigma_m (1 - L_t)^\sigma v_t^{1-\sigma}, \quad (\text{B.16})$$

$$h_t = \psi_t \sum_{j=0}^{N-1} l_t^j, \quad (\text{B.17})$$

$$L_t = \sum_{j=0}^{N-1} l_t^j, \quad (\text{B.18})$$

$$f_t = \frac{m_t}{1 - L_t}, \quad (\text{B.19})$$

and

$$l_t^{i+1} = (x_{t-1}^i + \rho) l_{t-1}^i, \quad (\text{B.20})$$

for  $i = 0, \dots, N - 1$ . The expression for  $V_{w,t}$  becomes:

$$V_{w,t} = \psi_t + \sum_{j=1}^{N-1} (\beta \rho)^j E_t \frac{\lambda_{z,t+j}}{\lambda_{z,t}} G_{t,j} \psi_{t+j}. \quad (\text{B.21})$$

Finally, the Nash Bargaining condition becomes:

$$\eta V_{w,t} J_{z,t} + (1 - \eta) [V_{z,t}^0 - U_{z,t}] J_{w,t} = 0. \quad (\text{B.22})$$

The previous  $12 + 3N$  equations contain the following  $15 + 3N$  endogenous variables:

$$J_{z,t}, J_{w,t}, V_{w,t}, V_{z,t}^i, x_t^i, V_{z,t}^x, m_t, v_t, w_t, \tilde{w}_t, U_{z,t}, l_t^i, f_t, \lambda_{z,t}, h_t, \pi_t, \psi_t, L_t. \quad (\text{B.23})$$

Regarding the equilibrium conditions associated with the rest of the model, we drop the equations in Appendix A which pertain to wage setting, i.e., (A.4), (A.5), (A.6), (A.7). The 8 variables in that section, that are not listed in (B.23) are:

$$p_t^*, F_{p,t}, k_t, s_t, c_t, I_t, R_t, q_t.$$

The 11 equations that remain are (A.1), (A.2), (A.3), (A.8), (A.10), (A.11), (A.12), (A.13), (A.14), (3.11) with  $w_t^* \equiv 1$  and

$$c_t + I_t + (1 - L_t) b^u + \frac{\kappa}{2} \sum_{j=0}^{N-1} (x_t^j)^2 l_t^j = (p_t^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left\{ \epsilon_t \left( \frac{k_{t-1}}{\mu_z} \right)^\alpha h_t^{1-\alpha} - \phi \right\}. \quad (\text{B.24})$$

Thus, we have  $23 + 3N$  equations in  $23 + 3N$  unknowns.

## B.2. Equilibrium Conditions on the Bargaining Set

In the previous section we assumed that agencies and individual workers only separate when it happens for exogenous reasons. Implicitly, we assumed that the scaled wage,

$$w_t^i \equiv \frac{W_t^i}{z_t P_t},$$

paid by an employment agency which has renegotiated most recently  $i$  periods in the past is always inside the bargaining set,  $[\underline{w}_t^i, \bar{w}_t^i]$ ,  $i = 0, 1, \dots, N - 1$ . Here,  $\bar{w}_t^i$  has the property that if  $w_t^i > \bar{w}_t^i$  then the agency prefers not to employ the worker and  $\underline{w}_t^i$

has the property that if  $w_t^i < \underline{w}_t^i$  then the worker prefers to be unemployed. We now describe our strategy for computing  $\underline{w}_t^i$  and  $\bar{w}_t^i$ .

The lower bound,  $\underline{w}_t^i$ , sets the surplus of a worker in an agency in cohort  $i$  to zero. By (B.10):

$$U_{z,t} = \psi_t \underline{w}_t^i - \gamma \frac{\psi_t^{1+\sigma_L}}{(1+\sigma_L) \lambda_{z,t}} + \beta E_t \frac{\lambda_{z,t+1}}{\lambda_{z,t}} [\rho V_{z,t+1}^{i+1} + (1-\rho) U_{z,t+1}],$$

for  $i = 0, \dots, N-1$ . In steady state, this is

$$\underline{w} = U_z + \gamma \frac{\psi^{1+\sigma_L}}{(1+\sigma_L) \lambda_z} - \beta [\rho V_z + (1-\rho) U_z],$$

where a variable without time subscript denotes its steady state value. We now consider the upper bound,  $\bar{w}_t^i$ , which sets the surplus of an agency in cohort  $i$  to zero,  $i = 0, \dots, N-1$ . From (B.7)

$$\begin{aligned} 0 = & E_t \sum_{j=0}^{N-1-i} \beta^j \frac{\lambda_{z,t+j}}{\lambda_{z,t}} \left[ (\tilde{w}_{t+j} - G_{t,j} \bar{w}_t^i) \psi_{t+j} - \frac{\kappa}{2} (x_{t+j}^j)^2 \right] \Omega_{t,j} \\ & + \beta^{N-i} E_t \frac{\lambda_{z,t+N-i}}{\lambda_{z,t}} J_{z,t+N-i} \Omega_{t,N-i}, \end{aligned}$$

for  $i = 0, \dots, N-1$ . In steady state:

$$0 = \frac{1 - \beta^{N-i}}{1 - \beta} \left[ \tilde{w} - \bar{w}^i - \frac{\kappa}{2} (1 - \rho)^2 \right] + \beta^{N-i} J_z,$$

or,

$$\bar{w}^i = \left[ \tilde{w} - \frac{\kappa}{2} (1 - \rho)^2 \right] - \frac{1 - \beta}{1 - \beta^{i-N}} J_z,$$

which is a monotonically increasing function of  $i$ .

For the dynamic economy, the additional unknowns are the  $2N$  variables composed of  $\underline{w}_t^i$  and  $\bar{w}_t^i$  for  $i = 0, 1, \dots, N-1$ . We have an equal number of equations to solve for them.

### B.3. Steady State

Our strategy for computing the steady state replaces the job finding rate,  $f$ , as an unknown, by  $\kappa$ . In addition, we impose that the steady state value of intensity is  $\psi = 1$ , and we solve for the value of  $\gamma$  that is consistent with this restriction.

#### B.3.1. Labor market equations

In steady state,  $l_t^i = l_t^j$  for all  $i, j, t, l$ , so that

$$x = 1 - \rho, \quad \Omega_{t,j} = 1.$$

Expression (B.7) reduces to:

$$J_z = \frac{1}{1 - \beta} \left[ \tilde{w} - w\tilde{w} - \frac{\kappa}{2} (1 - \rho)^2 \right]. \quad (\text{B.25})$$

The first order conditions, (B.8) and (B.9) are all identical in steady state, and given the value of  $x$  obtained above (as well as  $\psi = 1$ ), they reduce to:

$$\kappa (1 - \rho) = \beta [\tilde{w} - w\tilde{w} + \frac{\kappa}{2} (1 - \rho)^2 + \rho\kappa (1 - \rho)],$$

or,

$$w = 1 + \kappa (1 - \rho) \frac{\frac{1}{2} (1 - \rho) + (\beta\rho - 1) \frac{1}{\beta}}{\tilde{w}} \quad (\text{B.26})$$

which gives  $w$  as a function of  $\tilde{w}$ .

The derivative of the marginal value of a worker is, in steady state:

$$J_w = -\frac{1 - \beta^N}{1 - \beta}.$$

Equation (B.10) becomes:

$$V_z = w\tilde{w} - \gamma \frac{1}{(1 + \sigma_L) \lambda_z} + \beta [\rho V_z + (1 - \rho) U_z], \quad (\text{B.27})$$

where the  $i$  superscript on  $V_z$  has been dropped because all the equations in (B.10) are the same, and so they have the same solution. Equation (B.13) implies:

$$V_z^x = V_z. \quad (\text{B.28})$$

Equation (B.12) becomes

$$U_z = b^u + \beta[fV_z^x + (1 - f)U_z]. \quad (\text{B.29})$$

Equation (B.17) implies

$$L = h = Nl, \quad (\text{B.30})$$

so that equation (B.15) reduces to:

$$m = (1 - \rho)h. \quad (\text{B.31})$$

Also,

$$m = \sigma_m (1 - L)^\sigma v^{1-\sigma}.$$

Also,

$$f = \frac{m}{1 - L}. \quad (\text{B.32})$$

Also,

$$V_w = \frac{1 - (\beta\rho)^N}{1 - \beta\rho}.$$

The first order condition associated with the Nash problem is:

$$\eta V_w J_z + (1 - \eta) [V_z - U_z] J_w = 0, \quad (\text{B.33})$$

#### B.4. Non-labor market equations

In steady state,

$$p_t^* = 1.$$

Also,

$$F_p = \frac{\lambda_z \left[ \left( \frac{k}{\mu_z} \right)^\alpha h^{1-\alpha} - \phi \right]}{1 - \beta \xi_p},$$

and

$$F_p = \frac{\lambda_z \lambda_f \left[ \left( \frac{k}{\mu_z} \right)^\alpha h^{1-\alpha} - \phi \right] s}{1 - \beta \xi_p} = \frac{\lambda_z \left[ \left( \frac{k}{\mu_z} \right)^\alpha h^{1-\alpha} - \phi \right]}{1 - \beta \xi_p}$$

so that

$$s = \frac{1}{\lambda_f}.$$

and

$$\tilde{\pi}_w = \bar{\pi}.$$

The marginal cost equation reduces to:

$$\frac{1}{\lambda_f} = \frac{\tilde{w}}{1 - \alpha} \left( \frac{\mu_z h}{k} \right)^\alpha.$$

The resource constraint in steady state is:

$$\begin{aligned} c + I + (1 - L) b^u + \frac{\kappa}{2} (1 - \rho)^2 L &= \left( \frac{k}{\mu_z} \right)^\alpha h^{1-\alpha} - \phi & \text{(B.34)} \\ &= \frac{1}{\lambda_f} \left( \frac{k}{\mu_z} \right)^\alpha h^{1-\alpha}, \end{aligned}$$

by the condition that profits are zero in steady state. That is profits in steady state are, before scaling:

$$P_t Y_t - P_t s_t (Y_t + \phi z_t).$$

Dividing by  $P_t z_t$ , we obtain in steady state:

$$(\lambda_f - 1) Y_z = \phi,$$

where  $Y_z$  is  $c + I$  (i.e., it is gross output scaled by  $z_t$ ). From the capital accumulation equation:

$$k [1 - (1 - \delta) \mu_z^{-1}] = I.$$

The nominal interest rate is, in steady state:

$$1 + R = \frac{\pi\mu_z}{\beta}.$$

Also,

$$\lambda_z = \frac{1}{c} \frac{\mu_z - b\beta}{\mu_z - b}.$$

The investment intertemporal equation gives:

$$\frac{\mu_z}{\beta} = \alpha \left( \frac{\mu_z h}{k} \right)^{1-\alpha} \frac{1}{\lambda_f} + 1 - \delta. \quad (\text{B.35})$$

The adjustment for the investment equation is:

$$q = 1.$$

## B.5. Steady State Solution

As noted above, we treat  $f$  as known and we set  $\psi = 1$  (so that  $h = L$ ). These two restrictions allows us to solve for  $\gamma$  and  $\kappa$ . The value of  $\kappa$  is found by a non-linear, one-dimensional search. Fix an arbitrary value of  $\kappa$ . First compute  $J_w$ ,  $V_w$ ,  $R$  and  $x$  using the trivial expressions above. Use (B.35) to compute the labor-capital ratio,  $h/k$ :

$$\frac{h}{k} = \frac{1}{\mu_z} \left\{ \frac{\lambda_f}{\alpha} \left[ \frac{\mu_z}{\beta} - (1 - \delta) \right] \right\}^{\frac{1}{1-\alpha}}.$$

Then, from the law of motion for investment:

$$\frac{I}{k} = 1 - \frac{1 - \delta}{\mu_z}.$$

Solve for  $\tilde{w}$  using

$$\tilde{w} = \frac{1 - \alpha}{\lambda_f \left( \frac{\mu_z h}{k} \right)^\alpha}.$$

Now, fix an arbitrary value of  $\gamma$ . From the efficiency condition on intensity and  $\tilde{w}$ , compute  $\lambda_z$  :

$$\tilde{w} = \frac{\gamma}{\lambda_z}.$$

Given  $\lambda_z$  and

$$\lambda_z = \frac{1}{c} \frac{\mu_z - b\beta}{\mu_z - b},$$

compute  $c$ . Adjust  $\gamma$  until the resource constraint:

$$\frac{c}{k} = \left[ \frac{1}{\lambda_f} \left( \frac{k}{\mu_z h} \right)^\alpha - \frac{\kappa}{2} (1 - \rho)^2 \right] \frac{h}{k} - \frac{1 - h}{k} b^u - \left( 1 - \frac{1 - \delta}{\mu_z} \right),$$

is satisfied.

Use (B.26) compute  $w$  and use (B.25) to compute  $J_z$ . Then use (B.27), (B.28) and (B.29) to solve for  $V_z^x$ ,  $V_z$ , and  $U_z$ . After using (B.28) substitute out for  $V_z^x$ , these equations are:

$$V_z = \frac{w\tilde{w} - \gamma \frac{1}{(1+\sigma_L)\lambda_z} + \beta(1-\rho)U_z}{1-\beta\rho},$$

$$U_z = \frac{b^u}{1-\beta(1-f)} + \frac{\beta f}{1-\beta(1-f)} \frac{w\tilde{w} - \gamma \frac{1}{(1+\sigma_L)\lambda_z} + \beta(1-\rho)U_z}{1-\beta\rho},$$

so that

$$U_z = \frac{b^u + \frac{\beta f \left[ w\tilde{w} - \frac{\gamma}{(1+\sigma_L)\lambda_z} \right]}{1-\beta\rho}}{1-\beta(1-f) - \beta f \frac{\beta(1-\rho)}{1-\beta\rho}}. \quad (\text{B.36})$$

Equation (B.36) can be used to solve for  $U_z$  and then (??) is solved or  $V_z$ . Combining (B.31) and (B.32) we obtain the steady state level of employment:

$$L = h = \frac{f}{f + 1 - \rho}.$$

Then, (B.31) and (B.30) are solved for  $m$  and  $l$ . Given  $h$ ,  $F_p$ ,  $k$ ,  $c$ ,  $\lambda_z$  and  $I$  can be computed. For  $F_p$ , the steady state zero-profit condition must be applied. Adjust  $\kappa$  until (B.33) is satisfied.

## C. Appendix C: Ramsey-Optimal Policy

We find the Ramsey-optimal allocations for our economy using the computer code and strategy used in Levin, Lopez-Salido, (2004) and Levin, Onatski, Williams and Williams (2005). For completeness, we briefly describe this strategy below. Let  $x_t$  denote a set of  $N$  endogenous variables in a dynamic economic model. (According to Appendix A, in our simple monetary model,  $N = 15$ .) Let the private sector equilibrium conditions be represented by the following  $N - 1$  conditions:

$$\sum_{s^{t+1}|s^t} \frac{\mu(s^{t+1})}{\mu(s^t)} f(x(s^t), x(s^{t+1}), s_t, s_{t+1}) = 0, \quad (\text{C.1})$$

for all  $t$  and all  $s^t$ . Here,  $s^t$  denotes a history:

$$s^t = (s_0, s_1, \dots, s_t),$$

and  $s_t$  denotes the time  $t$  realization of uncertainty, which can take on  $n$  possible values:

$$\begin{aligned} s_t &\in \{s(1), \dots, s(n)\} \\ \mu(s^t) &= \text{prob}[s^t], \end{aligned}$$

so that  $\mu(s^{t+1})/\mu(s^t)$  is the probability of history  $s^{t+1}$ , conditional on  $s^t$ .

Suppose preferences over  $x(s^t)$  are follows:

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mu(s^t) U(x(s^t), s_t). \quad (\text{C.2})$$

In our simple monetary model,  $U$  is given by (A.15). The Ramsey problem is to maximize preferences by choice of  $x(s^t)$  for each  $s^t$ , subject to (C.1). We express the Ramsey problem in Lagrangian form as follows:

$$\max \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \mu(s^t) \left\{ U(x(s^t), s_t) + \underbrace{\lambda(s^t)}_{1 \times N-1} \sum_{s^{t+1}|s^t} \frac{\mu(s^{t+1})}{\mu(s^t)} \underbrace{f(x(s^t), x(s^{t+1}), s_t, s_{t+1})}_{N-1 \times 1} \right\},$$

where  $\lambda(s^t)$  is the row vector of multipliers on the equilibrium conditions. Consider a particular history,  $s^t = (s^{t-1}, s_t)$ , with  $t > 0$ . The first order necessary condition for optimality of  $x(s^t)$  is

$$\underbrace{U_1(x(s^t), s_t)}_{1 \times N} + \underbrace{\lambda(s^t)}_{1 \times N-1} \sum_{s^{t+1}|s^t} \frac{\mu(s^{t+1})}{\mu(s^t)} \underbrace{f_1(x(s^t), x(s^{t+1}), s_t, s_{t+1})}_{N-1 \times N} + \beta^{-1} \underbrace{\lambda(s^{t-1})}_{1 \times N-1} \underbrace{f_2(x(s^{t-1}), x(s^t), s_{t-1}, s_t)}_{N-1 \times N} = \underbrace{0}_{1 \times N} \quad (\text{C.3})$$

after dividing by  $\mu(s^t) \beta^t$ . In less notationally-intensive notation,

$$U_1(x_t, s_t) + \lambda_t E_t f_1(x_t, x_{t+1}, s_t, s_{t+1}) + \beta^{-1} \lambda_{t-1} f_2(x_{t-1}, x_t, s_{t-1}, s_t) = 0.$$

The first order necessary condition for optimality at  $t = 0$  is (C.3) with  $\lambda_{-1} \equiv 0$ . The time-consistency problem occurs when the multipliers associated with the Ramsey problem are non-zero after date 0. Following the Ramsey equilibrium in such a future  $t$  requires respecting  $\lambda_{t-1}$ . However, if  $\lambda_{t-1} \neq 0$ , then utility can be increased at  $t$  by restarting the Ramsey problem with  $\lambda_{t-1} = 0$ . When we study Ramsey allocations, we assume there is a commitment technology that prevents the authorities from acting on this incentive to deviate from the Ramsey plan.

The equations that characterize the Ramsey equilibrium are the  $N - 1$  equations, (C.1), and the  $N$  equations (C.3). The unknowns are the  $N$  elements of  $x$  and the  $N - 1$  multipliers,  $\lambda$ . The equations, (C.3) are computed symbolically using the software prepared for Levin, Lopez-Salido, (2004) and Levin, Onatski, Williams and Williams (2005). The resulting system of equations is then solved by perturbation around steady state using the software package, Dynare.

To apply the perturbation method, we require the nonstochastic steady state value of  $x$ . We compute this in two steps. First, fix one of the elements of  $x$ , say the inflation rate,  $\pi$ . We then solve for the remaining  $N - 1$  elements of  $x$  by imposing the

$N - 1$  equations, (C.1). In the next step we compute the  $N - 1$  vector of multipliers using the steady state version of (C.3):

$$U_1 + \lambda [f_1 + \beta^{-1}f_2] = 0,$$

where a function without an explicit argument is understood to mean it is evaluated in steady state.<sup>14</sup> Write

$$\begin{aligned} Y &= U_1' \\ X &= [f_1 + \beta^{-1}f_2]' \\ \beta &= \lambda', \end{aligned}$$

so that  $Y$  is an  $N \times 1$  column vector,  $X$  is an  $N \times (N - 1)$  matrix and  $\beta$  is an  $(N - 1) \times 1$  column vector. Compute  $\beta$  and  $u$  as

$$\begin{aligned} \beta &= (X'X)^{-1} X'Y \\ u &= Y - X\beta. \end{aligned}$$

Note that this regression will not in general fit perfectly, because there are  $N - 1$  ‘explanatory variables’ and  $N$  elements of  $Y$  to ‘explain’. We vary the value of  $\pi$  until  $\max |u_i| = 0$ . This completes the discussion of the calculation of the steady state and of the algorithm for computing Ramsey allocations.

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<sup>14</sup>This step is potentially very cumbersome, but has been made relatively easy by the software produced for Levin, Lopez-Salido, (2004) and Levin, Onatski, Williams and Williams (2005). This software endogenously writes the code necessary to solve for the multipliers.

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Figure 1: S&P500 Divided by CPI (Shiller)

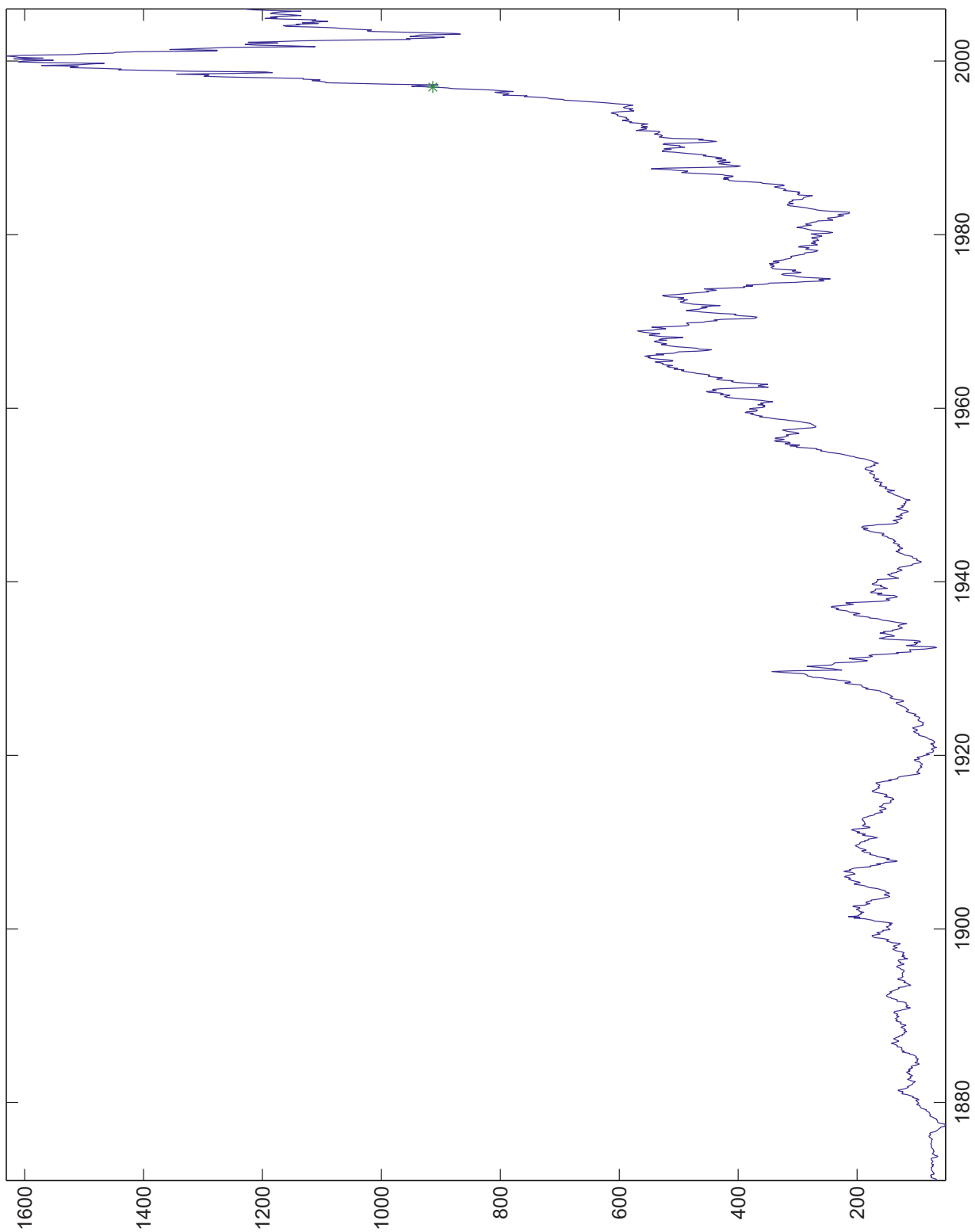


Fig 2A: Standardized inflation and stock price in interwar period

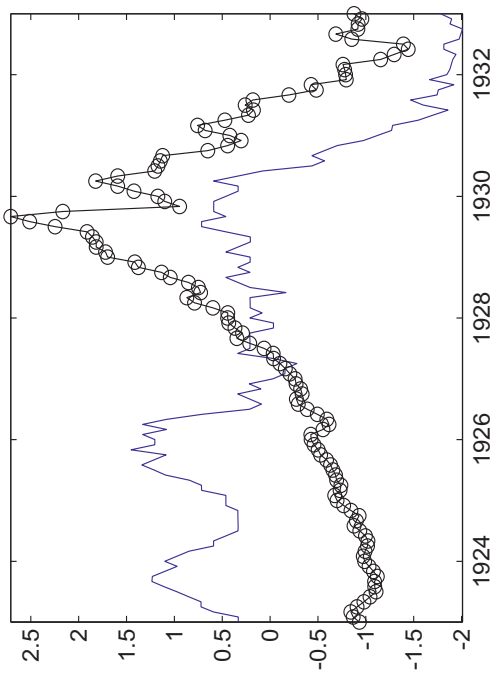


Fig 2B: Standardized inflation and stock price in 1950's to 1970's period

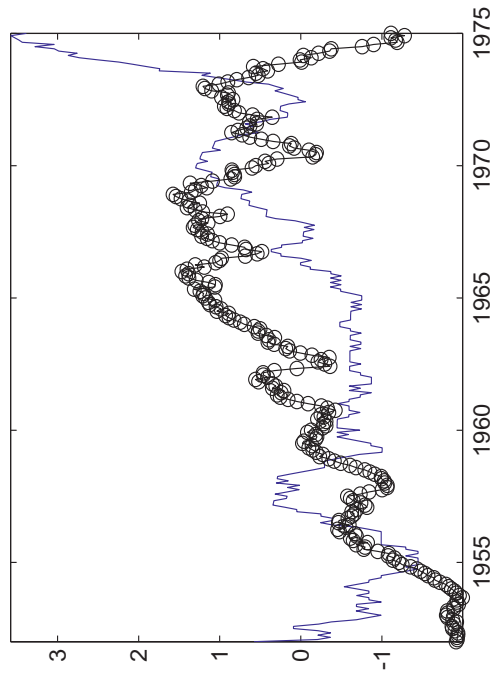
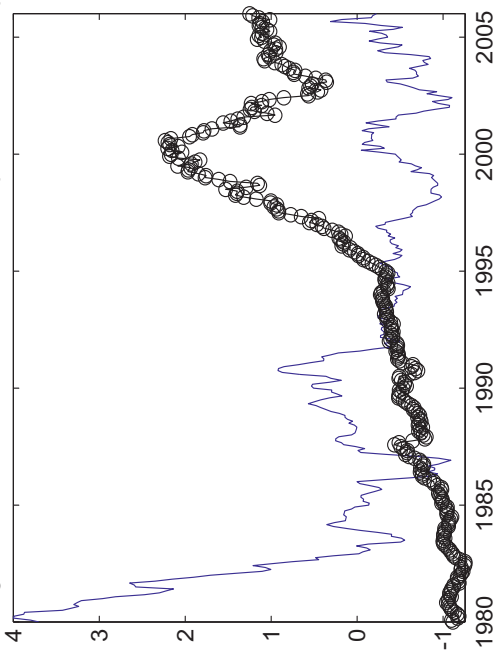


Fig 2C: Standardized inflation and stock price in interwar period



— Inflation  
—○— S&P500

Figure 3: Real Business Cycle Model with Habit and CEE Investment Adjustment Costs  
 Baseline - Tech Shock Not Realized, Perturbation - Tech Shock Realized in Period 5

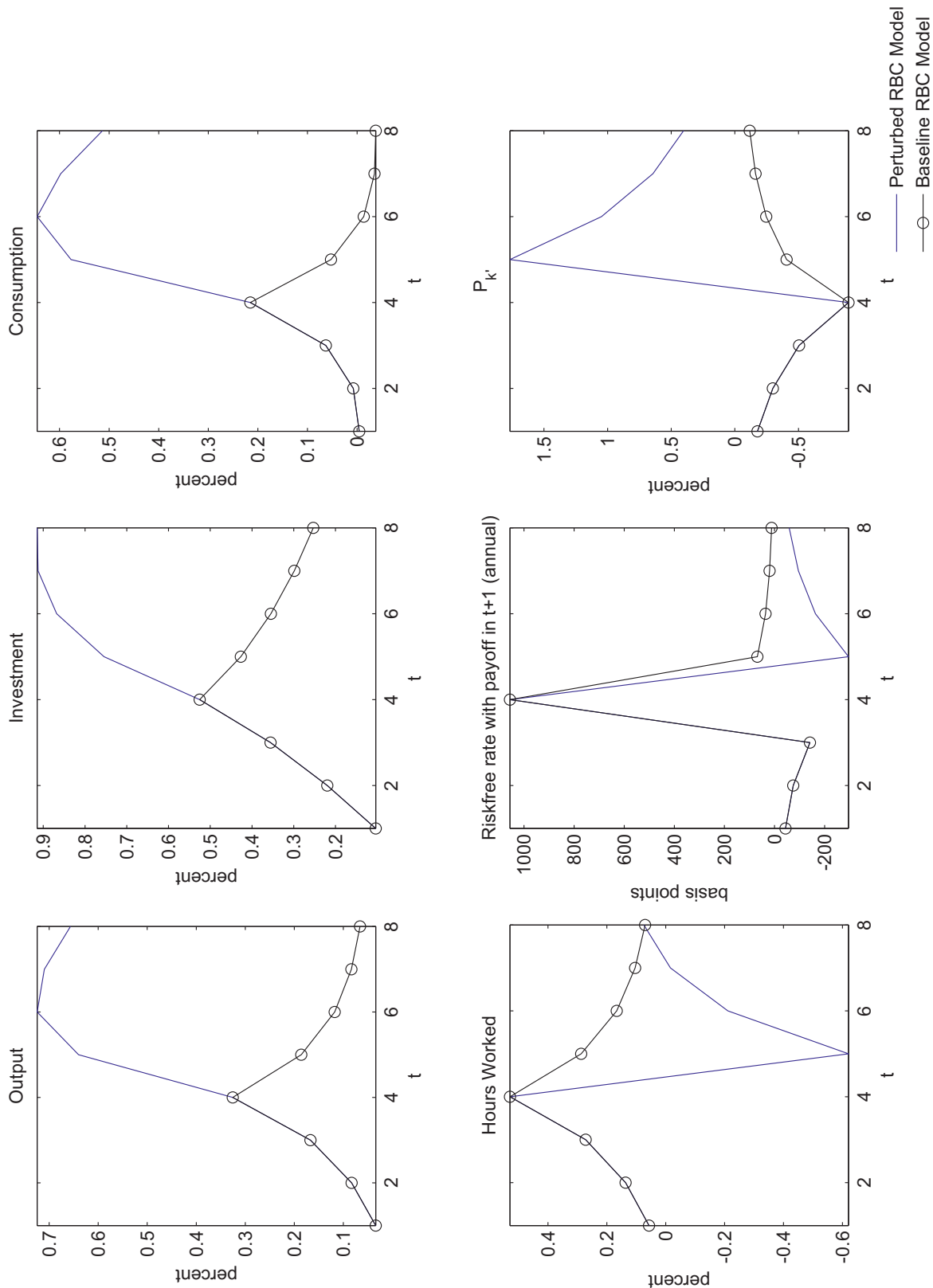


Figure 4: Real Business Cycle Model without Habit and with CEE Investment Adjustment Costs  
Technology Shock Not Realized in Period 5

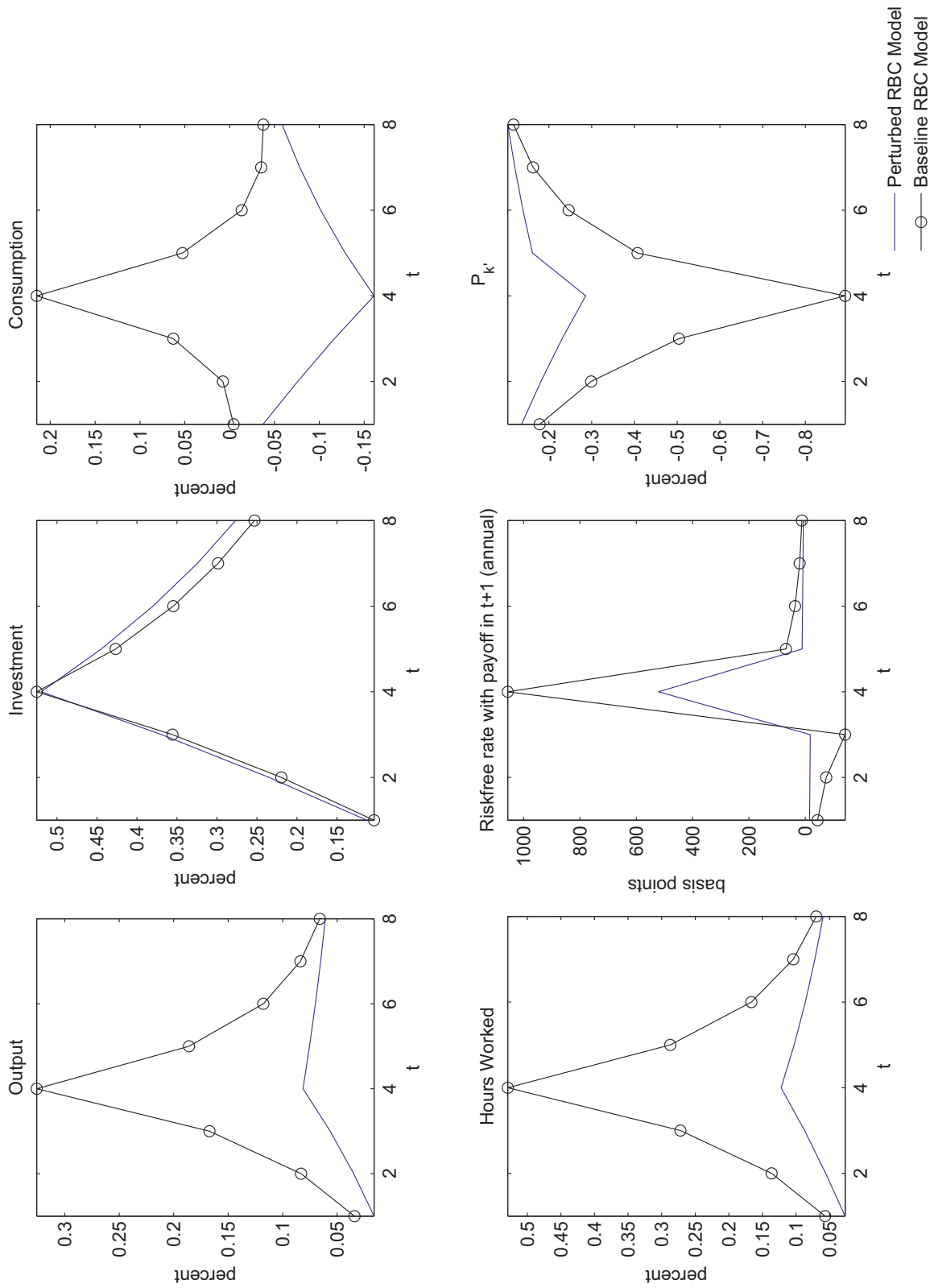


Figure 5: Real Business Cycle Model with Habit and Without Investment Adjustment Costs  
Technology Shock not Realized in Period 5

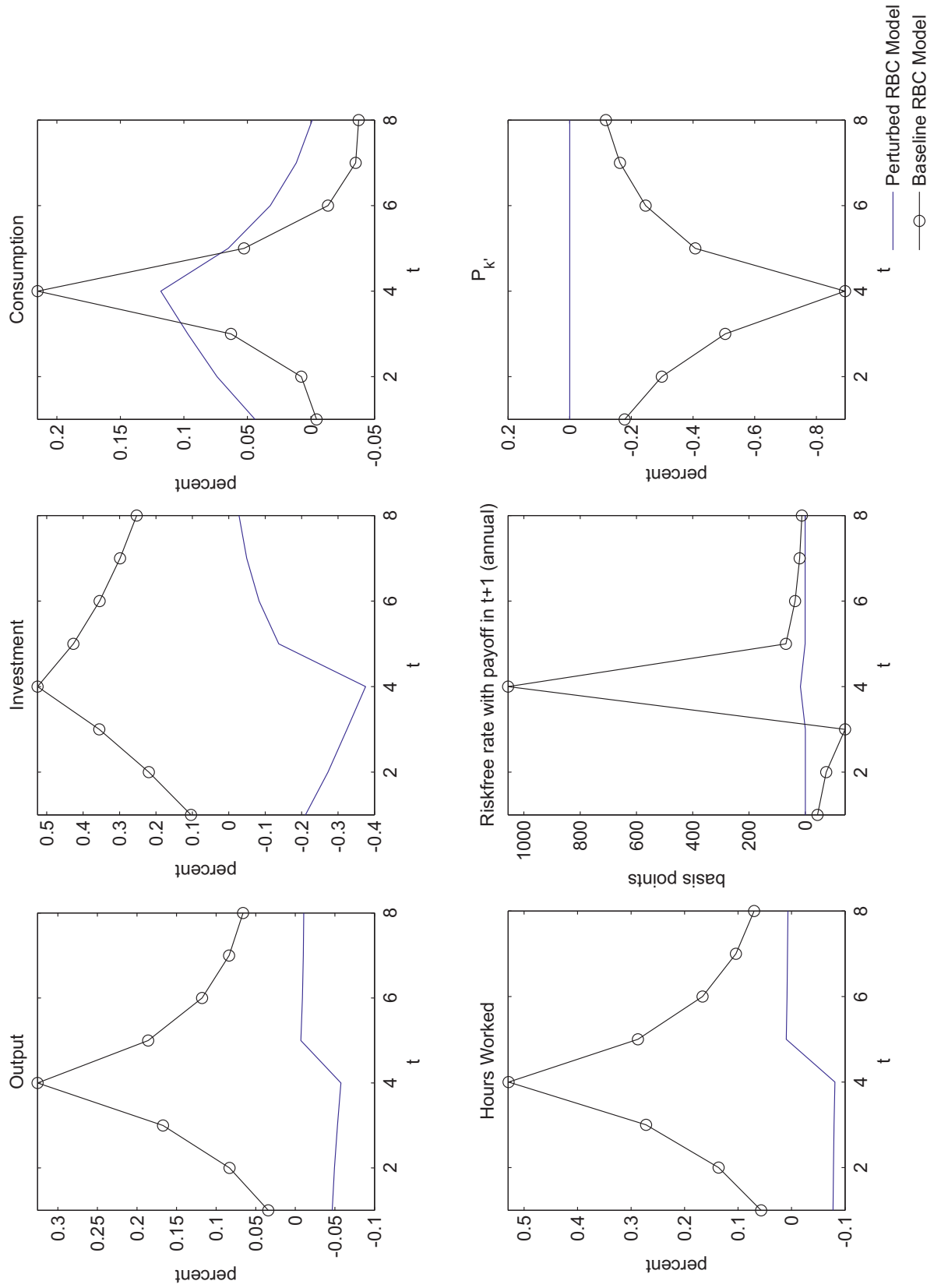


Figure 6: Real Business Cycle Model with Habit and with Level Investment Adjustment Costs  
 Technology Shock Not Realized in Period 5

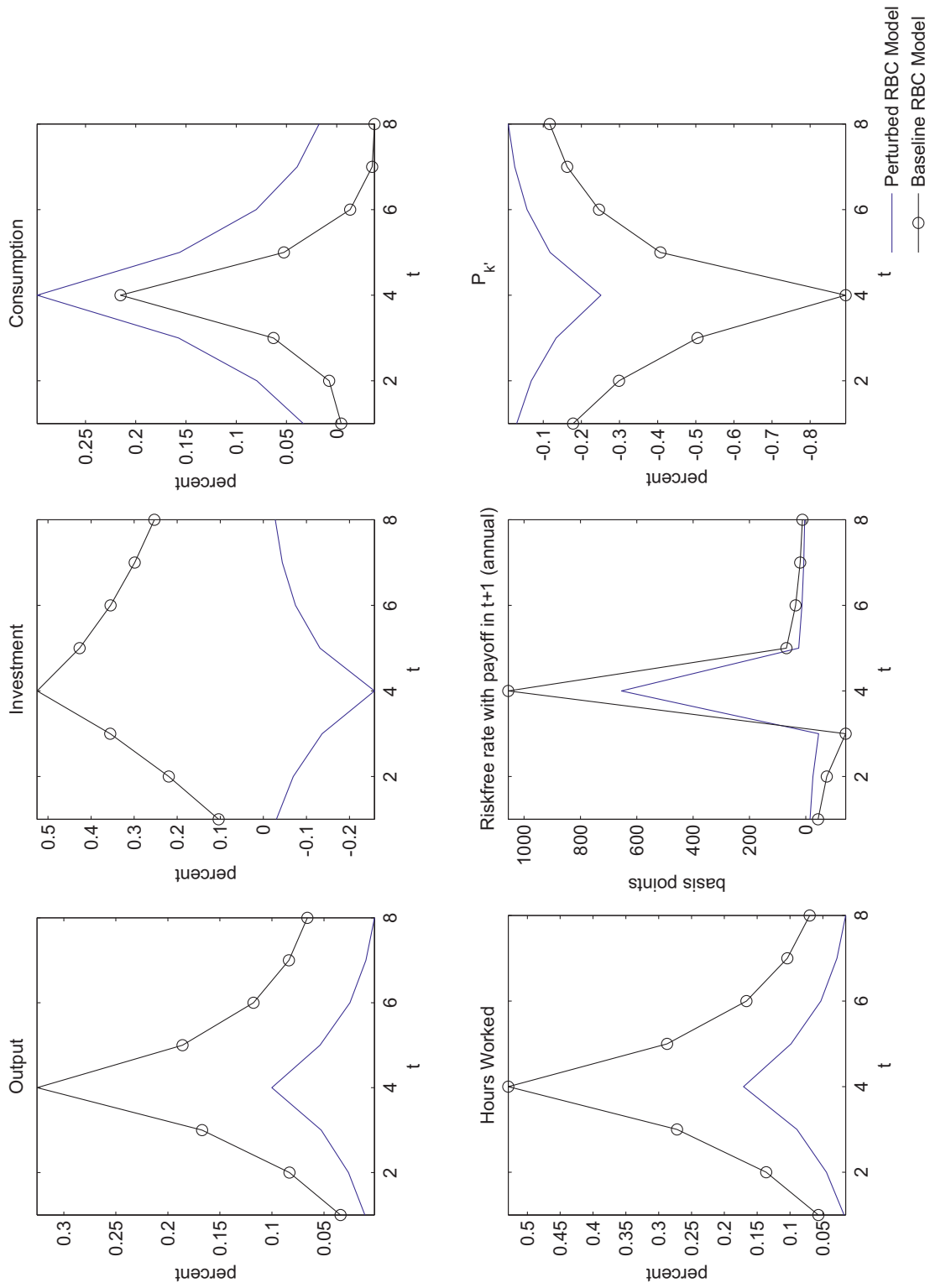


Figure 7: Real Business Cycle Model with Habit Persistence and Flow Adjustment Costs  
Perturbed - Technology Shock Expected in Period 13, But Not Realized

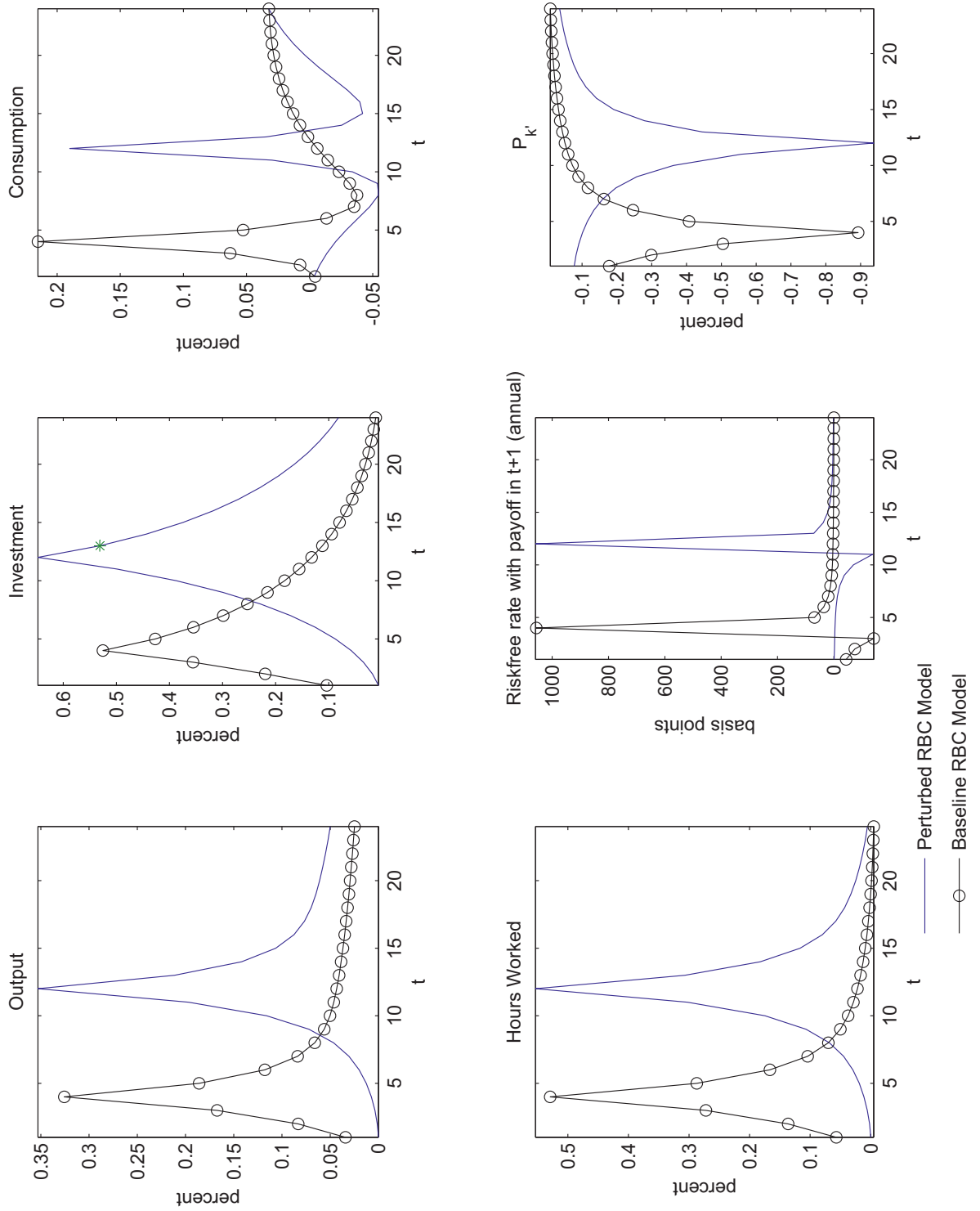
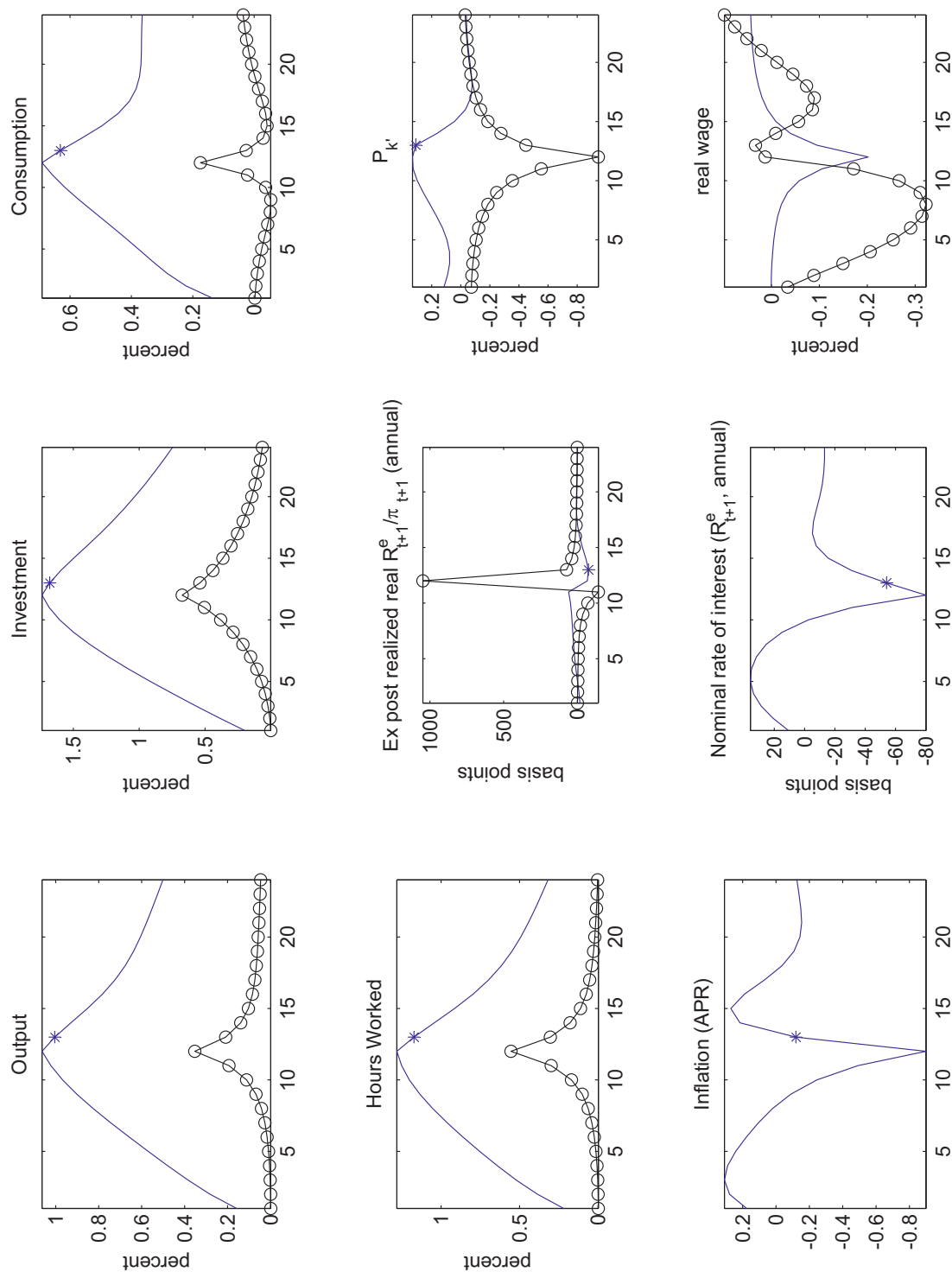


Figure 8: RBC and Simple Monetary Model  
Expectation of Technology Shock in Period 13 Not Realized



Note: subscript on nominal rate of interest indicates date of payoff.  $R_{t+1}^e$  is graphed at date  $t$ .  $\pi_t$  indicates gross change in price level from  $t-1$  to  $t$ .

Figure 9: Simple Monetary Model and Associated Ramsey Equilibrium

—\*— Ramsey Equilibrium of Simple Monetary Model  
— Simple Monetary Model

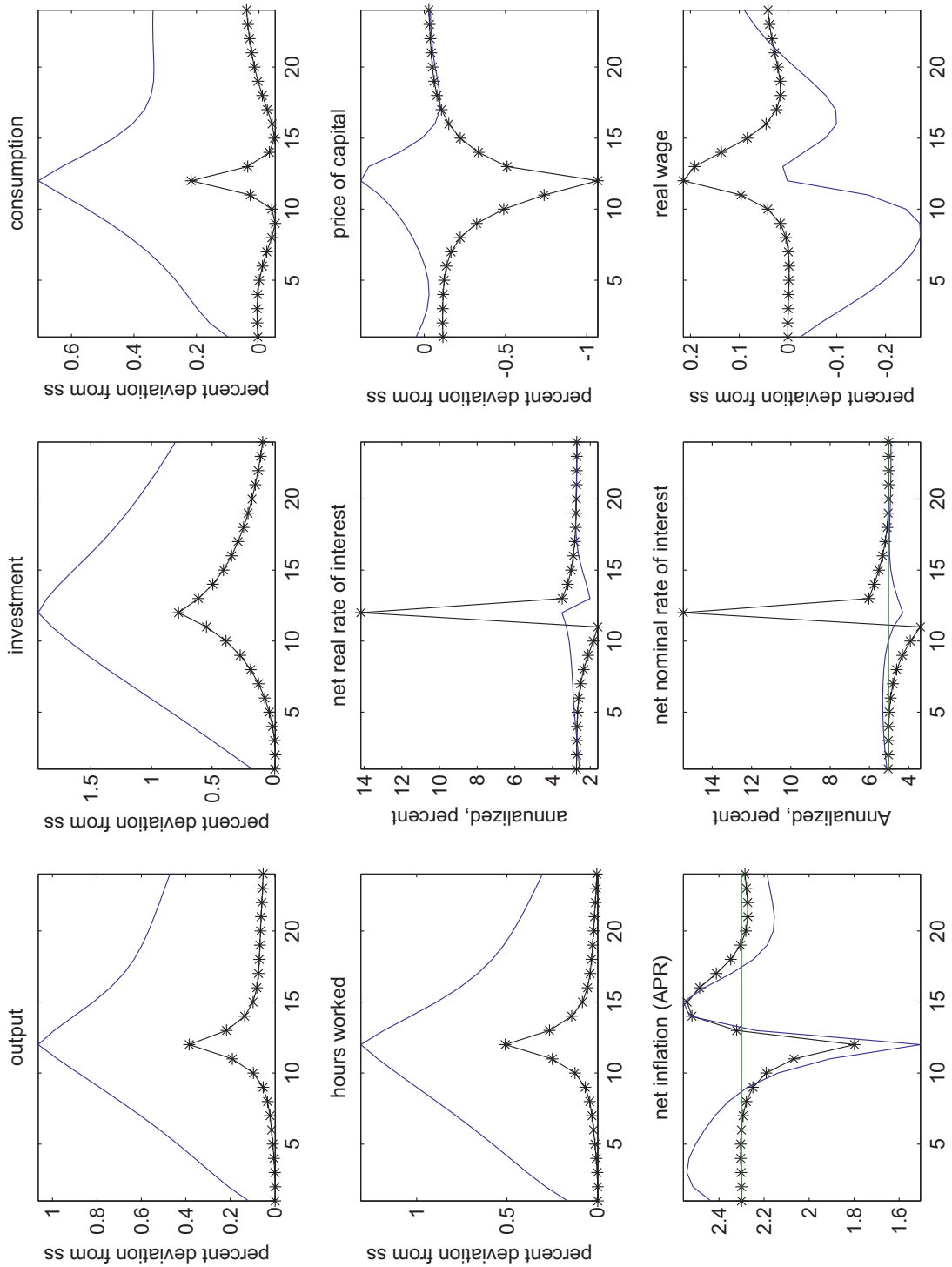
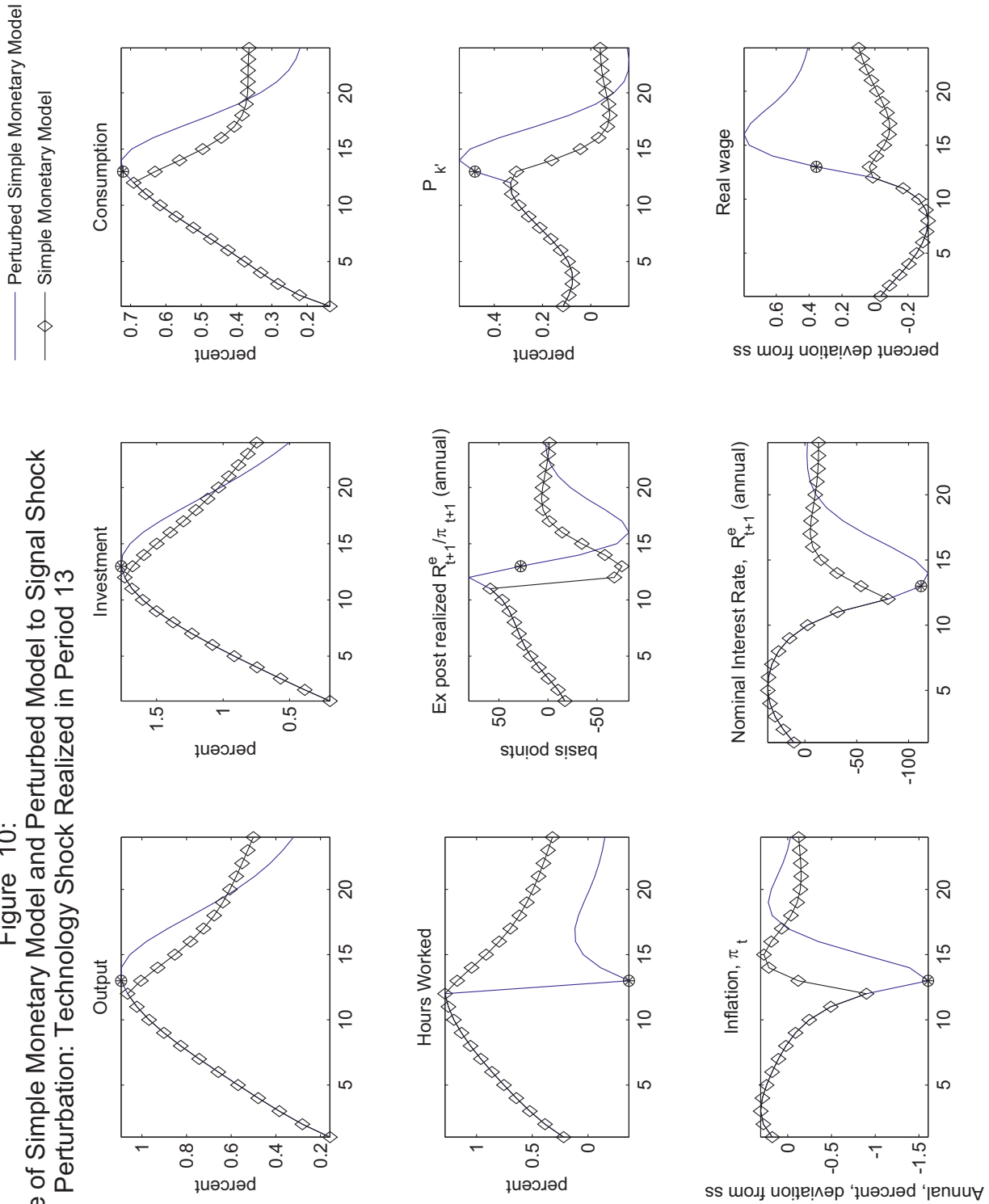
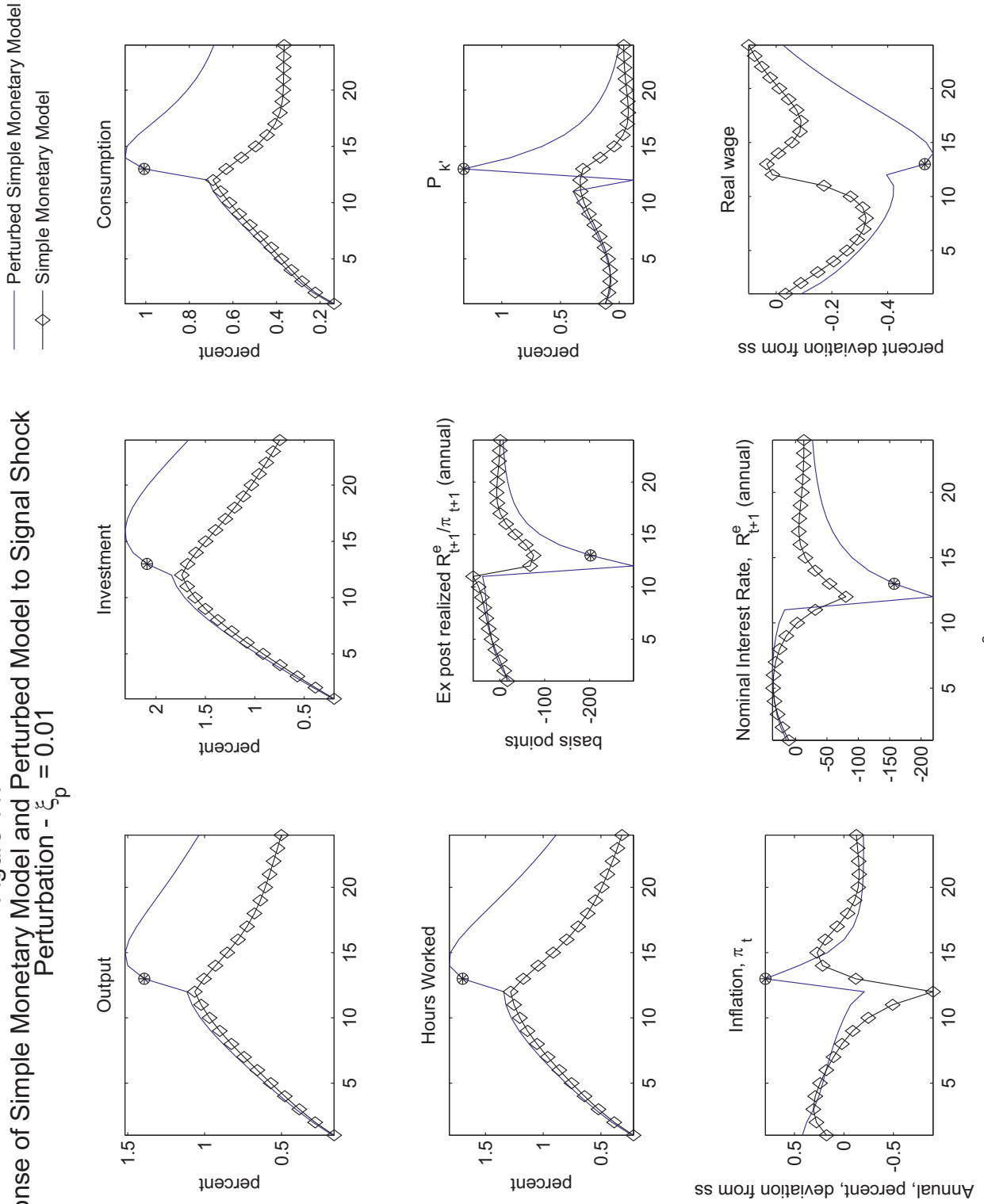


Figure 10:  
 Response of Simple Monetary Model and Perturbed Model to Signal Shock  
 Perturbation: Technology Shock Realized in Period 13



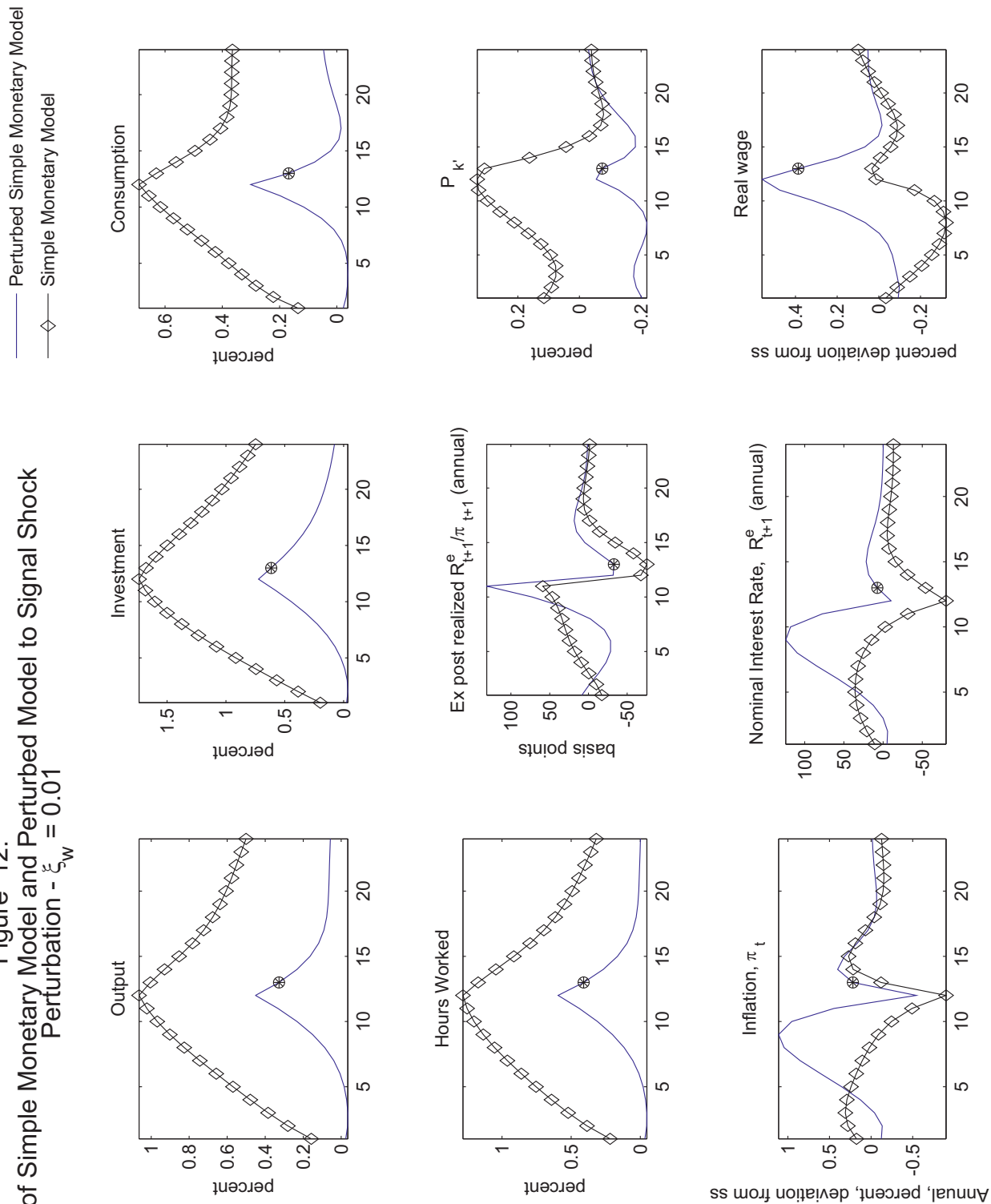
Note: subscript on nominal rate of interest indicates date of payoff.  $R_{t+1}^e$  is graphed at date  $t$ .  $\pi_t$  indicates gross change in price level from  $t-1$  to  $t$ .

Figure 11:  
Response of Simple Monetary Model and Perturbed Model to Signal Shock  
Perturbation -  $\xi_p = 0.01$



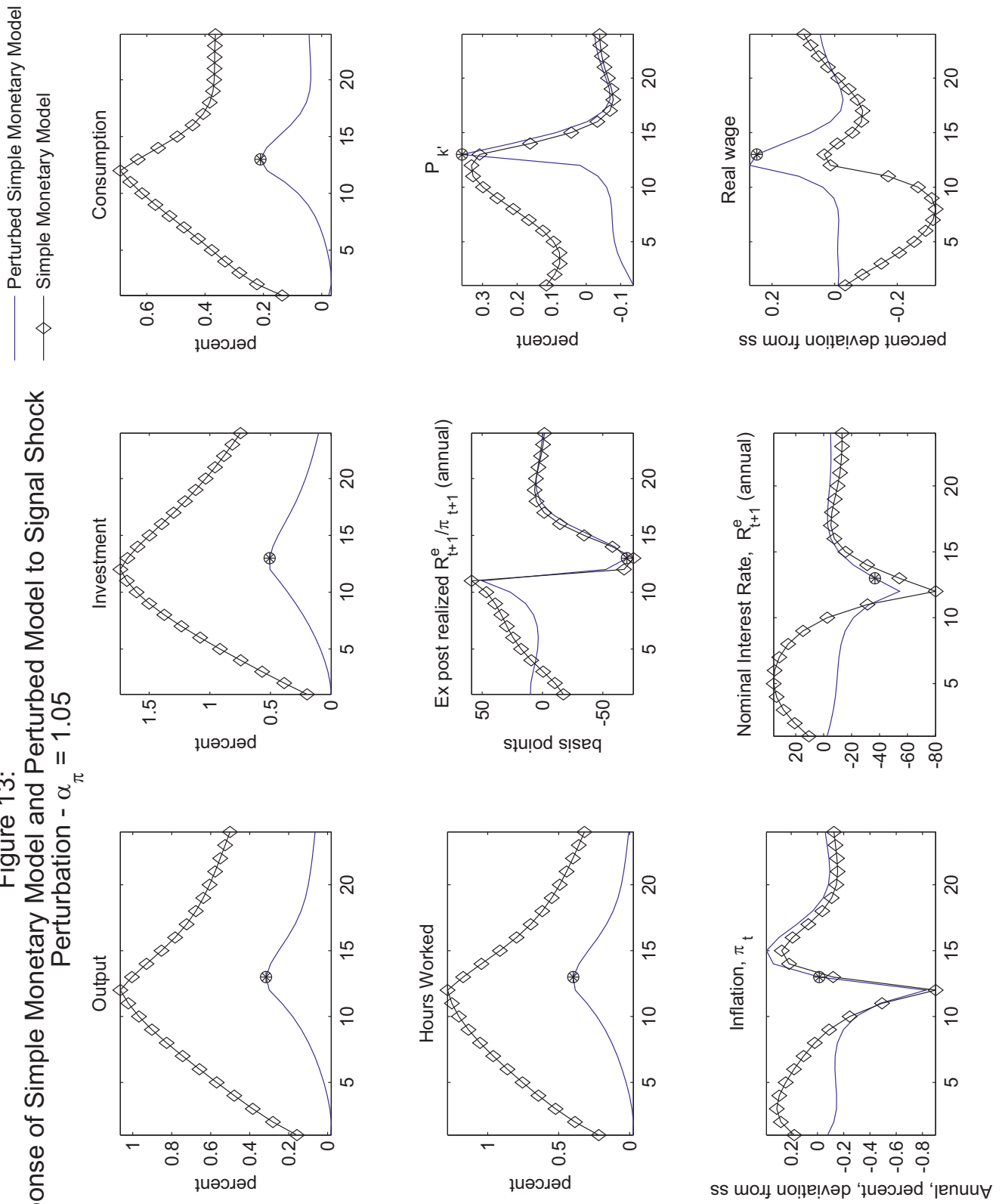
Note: subscript on nominal rate of interest indicates date of payoff.  $R_{t+1}^e$  is graphed at date  $t$ .  $\pi_t$  indicates gross change in price level from  $t-1$  to  $t$ .

Figure 12:  
 Response of Simple Monetary Model and Perturbed Model to Signal Shock  
 Perturbation -  $\xi_w = 0.01$



Note: subscript on nominal rate of interest indicates date of payoff.  $R_{t+1}^e$  is graphed at date  $t$ .  $\pi_t$  indicates gross change in price level from  $t-1$  to  $t$ .

Figure 13:  
 Response of Simple Monetary Model and Perturbed Model to Signal Shock  
 Perturbation -  $\alpha_\pi = 1.05$



— Perturbed Simple Monetary Model  
 —◇— Simple Monetary Model

Figure 14:  
 Response of Simple Monetary Model and Perturbed Model to Signal Shock  
 Perturbation - 100 Basis Point Negative Policy Shock to Interest Rate in Period 11 that is Not Realized

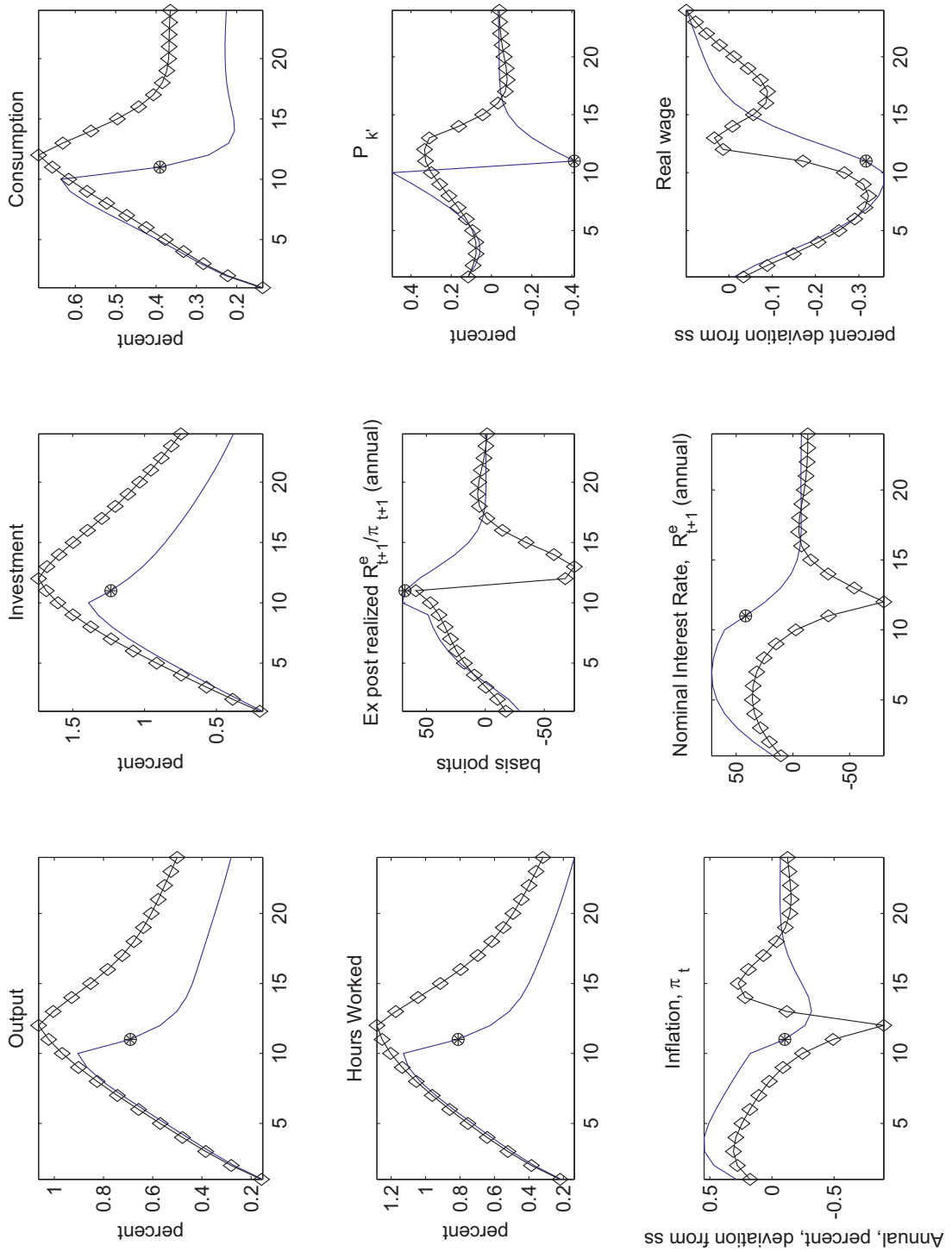


Figure 15: Response to a monetary policy shock

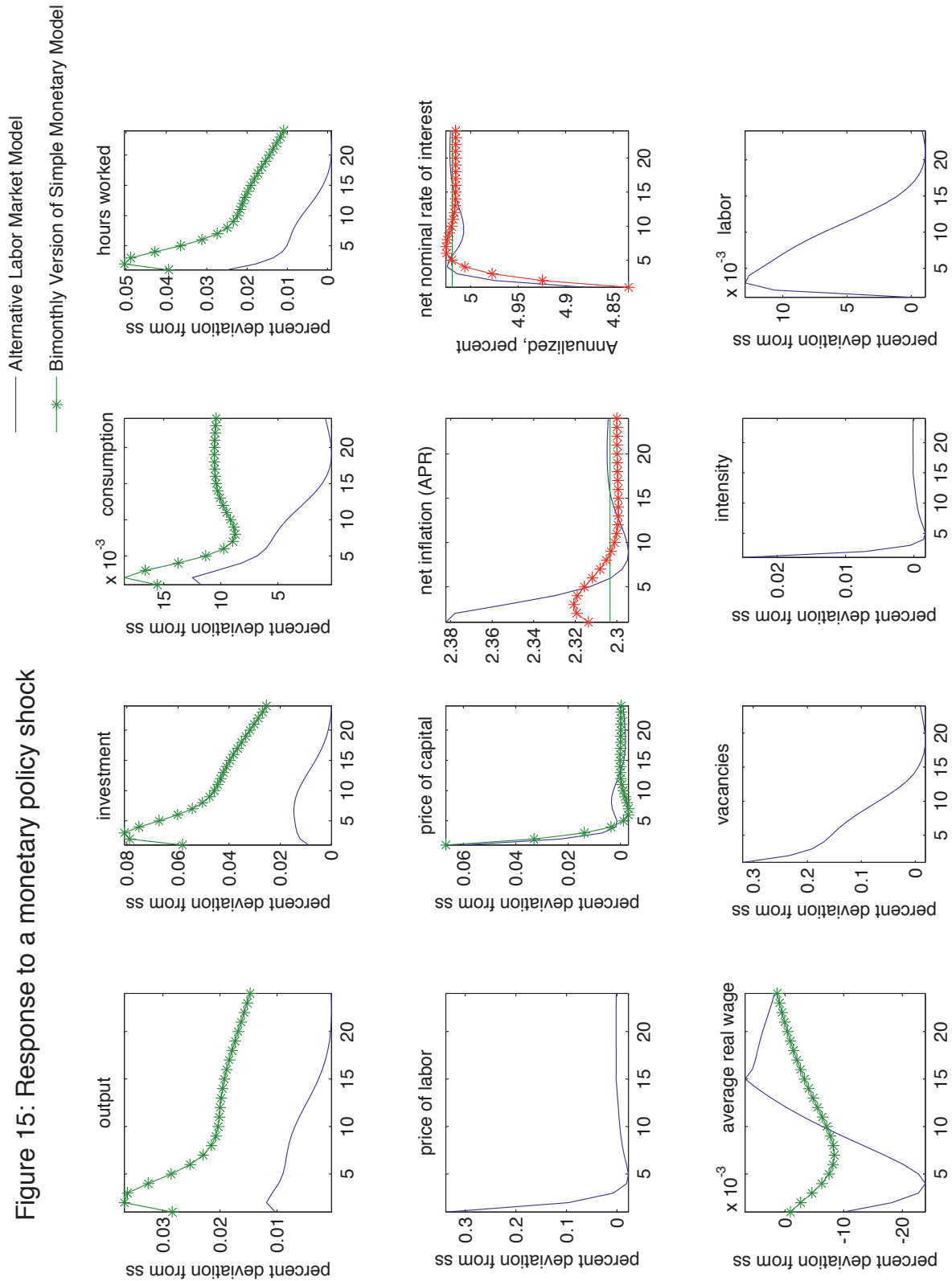


Figure 16: Response to a contemporaneous positive technology shock

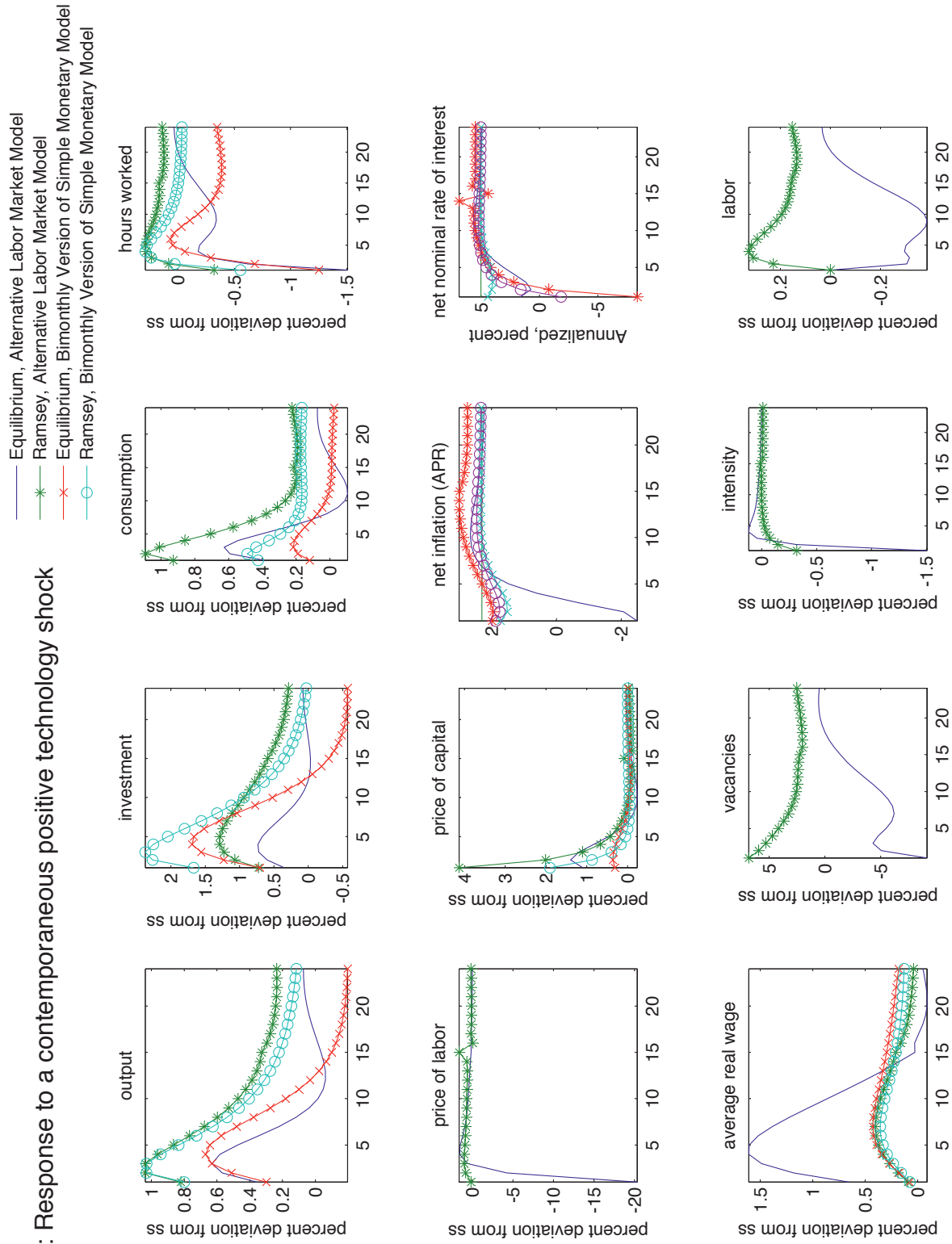


Figure 17: Response to a signal (not realized) of future technology shock

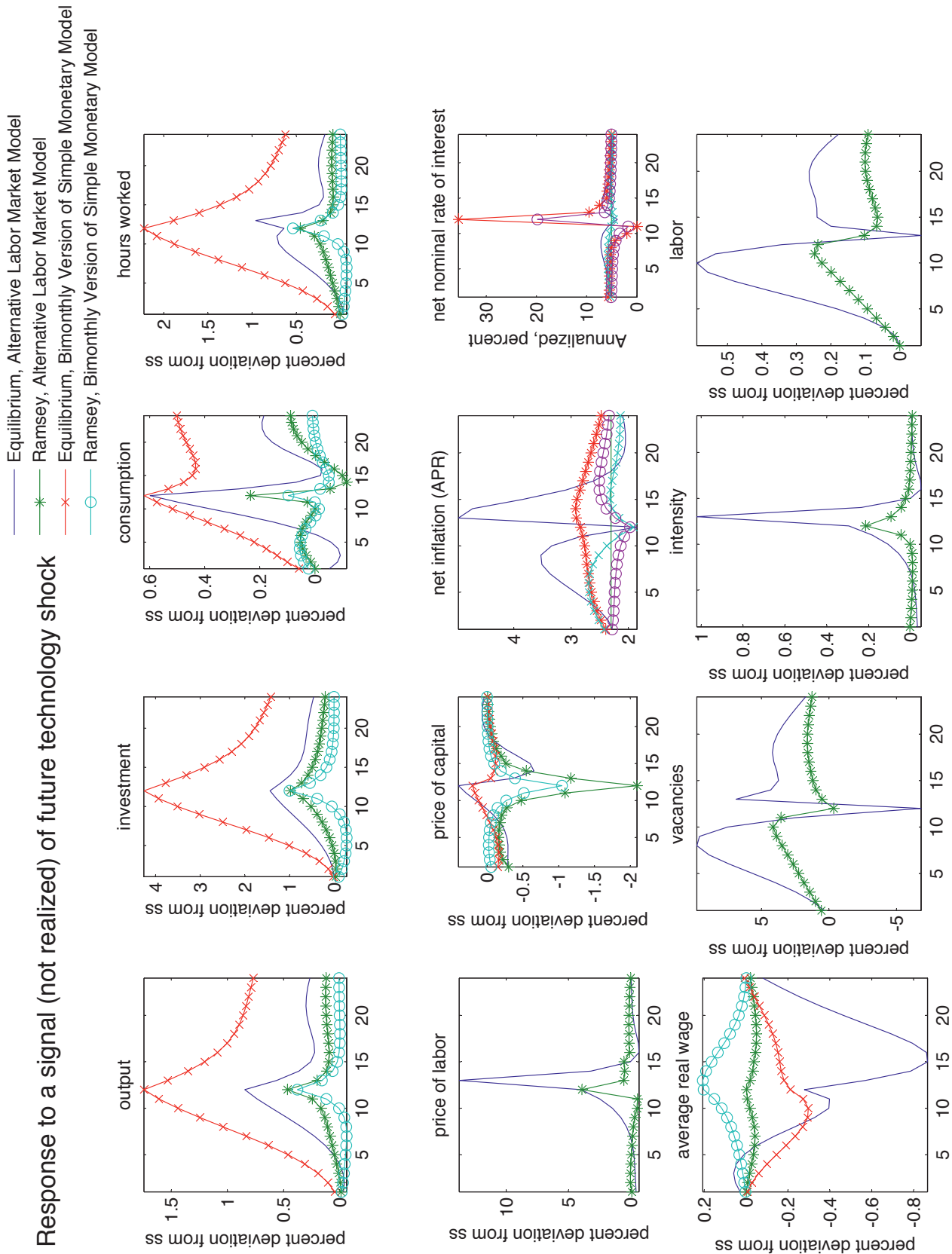


Figure 18: Response of Full and Simple Monetary Model to Signal Shock

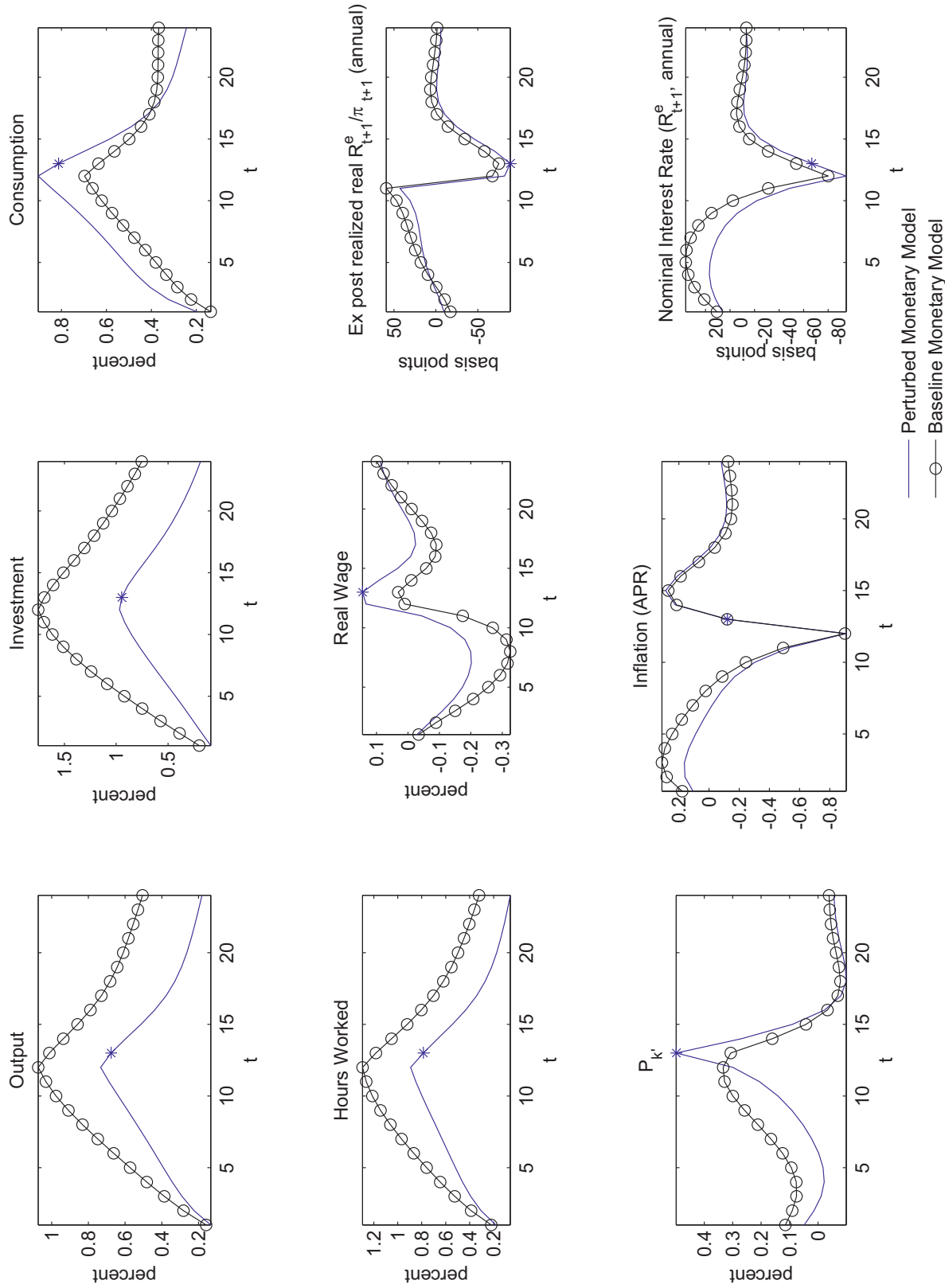


Figure 19: Behavior of Money, Credit and Net Worth in Full Monetary Model

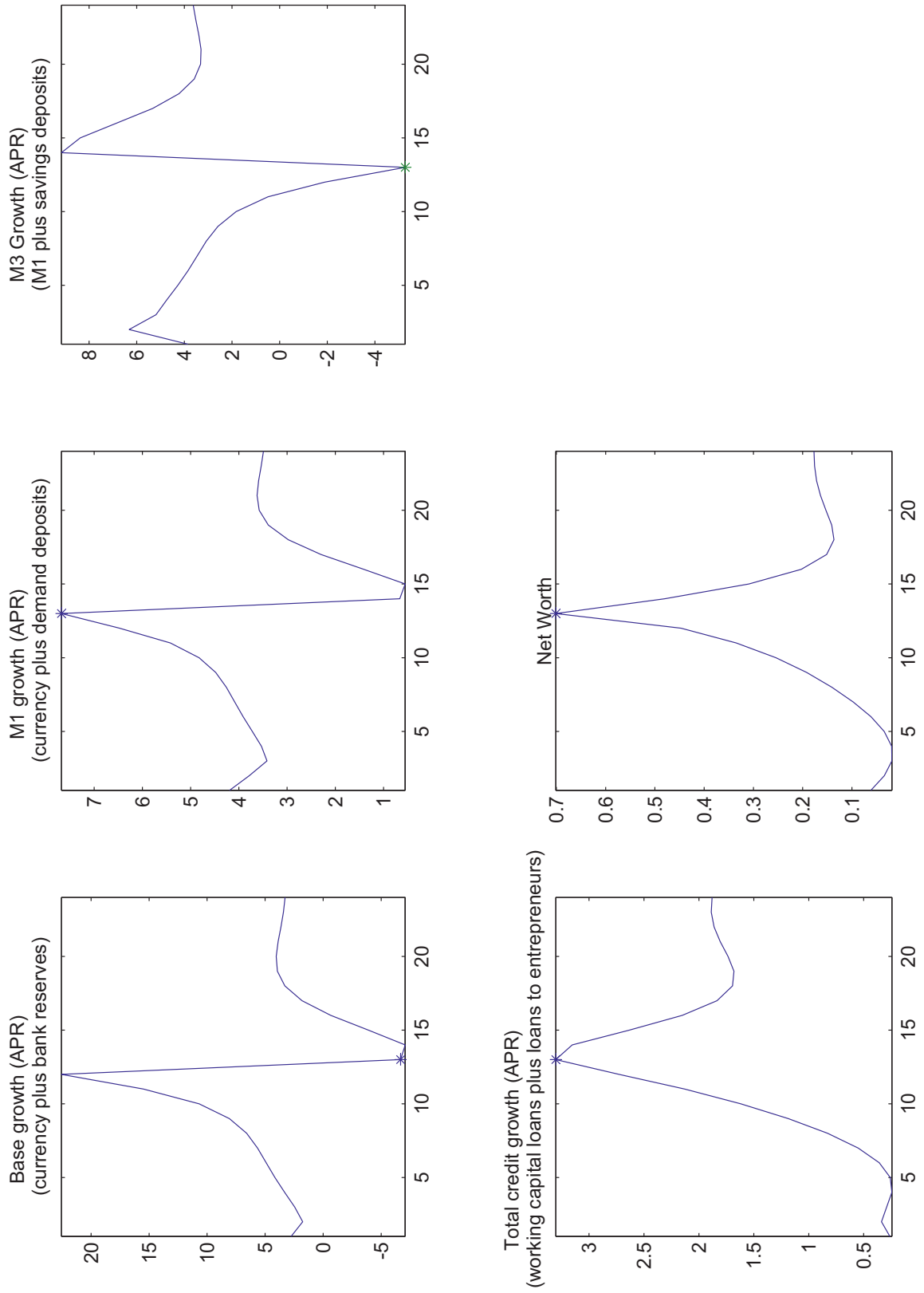
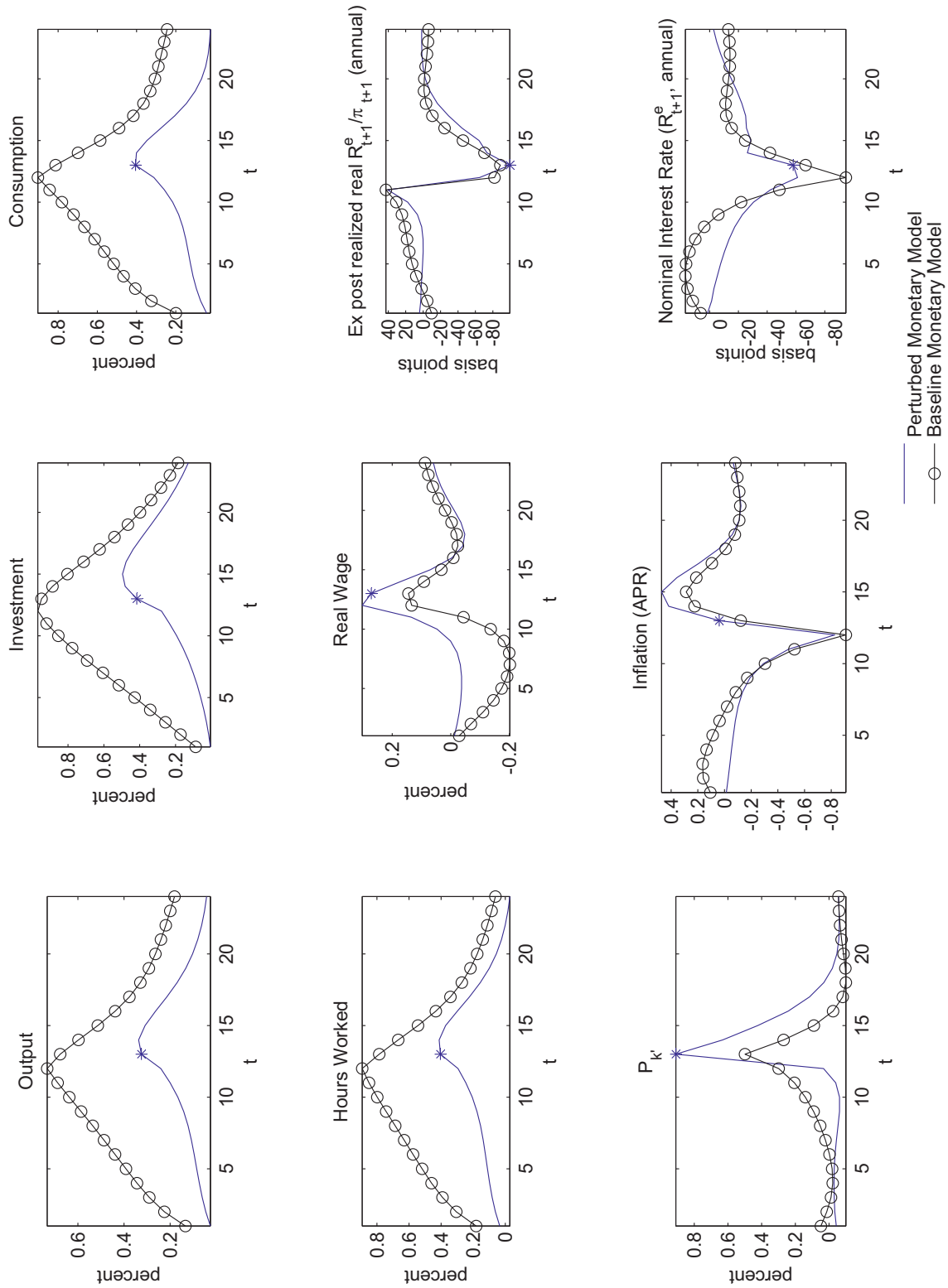


Figure 20: Response of Full Monetary Model and Perturbed Model to Signal Shock  
 Perturbation - Monetary Policy Response to Credit Growth



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