How Fast is Macroeconomic Uncertainty Resolved? Theory and Evidence from the Term Structure of Forecast Errors

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Motivation

- Uncertainty about macroeconomic variables such as GDP growth and inflation enters into the decision-making processes of governments, firms and individuals.

  - welfare implications of macroeconomic volatility, see Ramey and Ramey (1995, AER)
  - irreversibility and lags in investment decisions, see Kydland and Prescott (1982, Econometrica)
  - determination of asset prices, see Andersen et al. (2003, AER)
  - volatility and volume in asset markets, see Beber and Brandt (2006)
We consider two forms of macroeconomic uncertainty about the value of future macroeconomic variables.

1. Uncertainty about the future value of the macroeconomic variable of interest, as measured by mean-squared forecast error. This is the usual measure of uncertainty.

2. Disagreement about the optimal forecast based on currently available information, as measured by cross-sectional dispersion across forecasters. (See Bomberger, 1996, and discussion.)

These two measures each provide insights into the uncertainty faced by individuals in an economy.
Contributions of this paper

1. We develop a flexible framework for studying panels of survey forecasts containing multiple forecast horizons

   • Our model sheds light on the importance of measurement error to the forecasters in our panel

2. We propose a simple model for the cross-sectional dispersion of forecasts as a function of the horizon

   • We allow both for different signals and different models/priors across forecasters.
Sample root mean squared errors as a function of the forecast horizon: GDP growth in the G7 countries

Sample RMSE for GDP growth forecasts

- Canada
- France
- Germany
- Italy
- Japan
- USA
- UK
Sample root mean squared errors as a function of the forecast horizon: Inflation in the G7 countries

Sample RMSE for Inflation forecasts

- Canada
- France
- Germany
- Italy
- Japan
- USA
- UK
Detailed description of the data

- For each of the G7 countries, and for each target variable (real GDP growth and CPI inflation) we have $T = 14$ years of data (1991-2004) from Consensus Economics.

- In each year we have forecasts for $H = 24$ different forecast horizons.

- We have consensus forecasts and the cross-sectional dispersion of individual forecasts, but not the individual forecasts themselves.

- The realization of the target variable is the value reported by the IMF in the September of the following year.

  - Our measure of the target variable may be subject to measurement error - we do not explicitly model this.
Are the survey forecasts unbiased?

- Our use of the Consensus Economics survey forecasts in this study relies on the assumption that

\[ \hat{z}_{t,t-h} = E \left[ z_t | \tilde{F}_{t-h} \right] \]

i.e., that the forecasts are optimal under quadratic loss,

\[ L(z, \hat{z}) = (z - \hat{z})^2 \]

- We ran a battery of tests of forecast optimality: testing for bias, Mincer-Zarnowitz regressions, and testing for weakly increasing MSE as a function of the forecast horizon.

- Almost all of these tests failed to reject the null of optimality (unsurprising with \( T = 14 \)) and so we proceed as though the condition is satisfied.
Testing rationality of the consensus forecast

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<tr>
<th>Horizon</th>
<th>Bias</th>
<th>MZ p-values</th>
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<tr>
<td>2</td>
<td>0.07</td>
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<td>3</td>
<td>0.03</td>
<td>0.04**</td>
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<td>6</td>
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<td>0.08*</td>
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<td>0.01</td>
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<td>12</td>
<td>-0.34</td>
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<td>18</td>
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<tr>
<td>24</td>
<td>-0.09</td>
<td>0.37*</td>
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</tbody>
</table>

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Resolution of Macro Uncertainty
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Learning and uncertainty in our model

- We assume that forecasters know the DGP and its parameters, thus ruling out estimation error and learning as sources of forecast errors and dispersion.

- The primary source of uncertainty in the model concerns the future value of the variable of interest.

- When we extend to allow for measurement error, uncertainty exists also for current and past values of the variable of interest.

- Modelling the forecasters’ learning about the DGP and/or its parameters requires a long time series, whereas our sample is just $T = 14$ years.
Notation

\( y_t \) : monthly value of variable of interest

\( z_t = \sum_{j=0}^{11} y_{t-j} \) : annual value of variable of interest

\( \hat{z}_{t,t-h} \) : the consensus forecast of \( z_t \) made at time \( t - h \)

\( \tilde{F}_{t-h} \) : information available at time \( t - h \)

\( e_{t,t-h} = z_t - \hat{z}_{t,t-h} \) : consensus forecast error
The benchmark model

\[
\begin{align*}
z_t &= \sum_{j=0}^{11} y_{t-j} \\
y_t &= x_t + u_t \\
x_t &= \phi x_{t-1} + \epsilon_t, \quad |\phi| < 1 \\
\mathbf{u}_t &\sim iid \left(0, \begin{bmatrix} \sigma^2_u & 0 \\ 0 & \sigma^2_\epsilon \end{bmatrix}\right) \\
\hat{z}_{t,t-h} &= E[z_t | \mathcal{F}_{t-h}] \\
\mathcal{F}_{t-h} &= \sigma \left(\{x_s, y_s\}_{s=1}^{t-h}\right)
\end{align*}
\]
RMSE term structures under the benchmark model, for various levels of persistence.
Our benchmark model assumed that both the predictable and unpredictable components of the target variable are perfectly observed by the forecasters.

This implies that the squared forecast errors converge to zero as the horizon shrinks - this does not match the data.

We need to extend the model to allow for imperfect observation of the variable(s) of interest.

A more realistic framework would allow both components to be measured with noise. We use the Kalman filter to handle such an approach.
A state-space model

- The state equation for this model is unchanged:

\[
\begin{bmatrix}
0 & 0 \\
0 & \phi
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
x_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
u_t \\
\varepsilon_t
\end{bmatrix}
\sim iid \left(0, \begin{bmatrix}
\sigma_u^2 & 0 \\
0 & \sigma_\varepsilon^2
\end{bmatrix}\right)
\]

- The measurement equation is assumed to be:

\[
\begin{bmatrix}
\tilde{y}_t \\
\tilde{x}_t
\end{bmatrix}
= \begin{bmatrix}
y_t \\
x_t
\end{bmatrix}
+ \begin{bmatrix}
\eta_t \\
\psi_t
\end{bmatrix}
\sim iid \left(0, \begin{bmatrix}
\sigma_\eta^2 & 0 \\
0 & \sigma_\psi^2
\end{bmatrix}\right)
\]

- This is a standard state-space form and can be studied using textbook methods (see Harvey 1989 or Hamilton 1994).
Term structure of RMSE for various measurement error

\( \text{Sig-psi} = +\infty, \text{sig-eta} = k \times \text{sig-u} \) for various \( k \)
**Estimation method**

- Our vector of unknown parameters for the model of the “term structure” of mean squared consensus errors is \( \theta = [\sigma_u, \sigma_\varepsilon, \phi, \sigma_\eta, \sigma_\psi]' \).

- We estimate these parameters by GMM:

\[
\hat{\theta}_T \equiv \arg\min_{\theta \in \Theta} g_T (\theta)' W_T g_T (\theta)
\]

where

\[
g_T (\theta) \equiv \frac{1}{T} \sum_{t=1}^{T} \begin{bmatrix}
  e_{t, t-1}^2 - \text{MSE}_1 (\theta) \\
  \vdots \\
  e_{t, t-24}^2 - \text{MSE}_{24} (\theta)
\end{bmatrix}
\]

where \( \text{MSE}_h (\theta) \) is the model-implied mean-squared error for horizon \( h \).
The weight matrix used in the first stage is the identity matrix.

The second-stage estimates are use the efficient weight matrix, \( \hat{\mathbf{W}} \left( \hat{\theta}^{(1)}_T \right) \), which is based on the model-implied covariance matrix of the moments, obtained via simulation of 10,000 non-overlapping years of data.

After some experimentation we fixed \( \sigma_\eta = 2\sigma_u \) and set \( \sigma_\psi \rightarrow \infty \) to improve the identification of the model.

We initially used all 24 horizons for estimation, but in light of finite-sample studies of GMM estimators, we settled on estimating the model with just six forecast horizons: \( h = 1, 3, 6, 12, 18, 24 \).
### GMM parameter estimates: consensus model

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_u$</th>
<th>$\sigma_\varepsilon$</th>
<th>$\phi$</th>
<th>$J$</th>
<th>p-val</th>
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<tbody>
<tr>
<td>GDP growth</td>
<td>0.09</td>
<td>0.04</td>
<td>0.95</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.01</td>
<td>0.02</td>
<td>0.98</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.03)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Term structures of RMSE for GDP growth and inflation in the US.
Dispersion among forecasters

- Our second measure of macroeconomic “uncertainty” is measured as the differences in forecasts issued by individual forecasters:

\[
d_{t,t-h}^2 \equiv \frac{1}{N_{t,t-h}} \sum_{i=1}^{N_{t,t-h}} (\hat{z}_{i,t,t-h} - \bar{z}_{t,t-h})^2
\]

- As the following picture shows, the term structure of dispersion across countries has the same basic shape as the term structure of mean squared forecast errors.
Sample dispersion (std dev) as a function of the forecast horizon: GDP growth in the G7 countries.
Sample dispersion (std dev) as a function of the forecast horizon: Inflation in the G7 countries.

![Sample forecast dispersion for Inflation forecasts](image)

- Canada
- France
- Germany
- Italy
- Japan
- USA
- UK

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A model for dispersion

- The first source of dispersion is differences in signals:

\[
\tilde{y}_{i,t} = y_t + \eta_t + \nu_{i,t}
\]

\[
\begin{bmatrix}
\eta_t \\
\nu_t
\end{bmatrix} \sim iid \left( 0, \begin{bmatrix}
\sigma^2_{\eta} & 0 \\
0 & \sigma^2_{\nu}
\end{bmatrix} \right)
\]

But as the forecast survey results are published (with a lag) we also assume that the forecaster’s time \( t \) information set includes:

\[
\tilde{y}_{t-1} = y_{t-1} + \eta_{t-1}
\]

From these two measurement variables, the individual forecaster computes the optimal forecast using the Kalman filter:

\[
\hat{z}_{i,t,t-h} \equiv E \left[ z_t | \tilde{F}_{i,t-h} \right]
\]
Differences in signals & dispersion as h grows

- Allowing for differences in signals is a natural starting point for capturing dispersion, but it has an important counter-factual implication:

- As \( h \to \infty \), the value of any signal for predicting the target variable goes to zero, and so all forecasts converge to the unconditional mean:

\[
\hat{z}_{i,t,t-h}^* \equiv E \left[ z_t \mid \tilde{F}_{i,t-h} \right] \to E[z_t] \quad \text{as } h \to \infty
\]

\[
\text{so } d^2_{t,t-h} \equiv \frac{1}{N_{t,t-h}} \sum_{i=1}^{N_{t,t-h}} (\hat{z}_{i,t,t-h} - \bar{z}_{t,t-h})^2 \to 0 \quad \text{as } h \to \infty
\]

- Yet we saw that \( d^2_{t,t-h} \to \tilde{\delta} > 0 \) as \( h \to \infty \) for all countries and variables in our sample. So there must be another source of dispersion.
Differences in beliefs about long-run values

- One simple way of allowing for dispersion at long horizons is to allow the forecasters to have differing beliefs about the long-run average values of GDP growth and inflation.

- These differences can be motivated in a number of ways:
  - The use of different Bayesian priors, which affect the forecasts issued.
  - The use of different models for equilibrium rates of GDP growth and inflation.
  - The use of different samples of data, based on different beliefs about previous structural breaks, etc.

- This approach is a special case of allowing for different subjective probability densities across forecasters, see Pesaran and Weale (2005).
A model for differences in beliefs

- Forecaster $i$’s prior belief about the average value of $z_t$ is denoted $\mu_i$.

- We assume that forecaster $i$ “shrinks” the Kalman filter forecast towards his prior belief about the unconditional mean of $z_t$. The degree of shrinkage is governed by the parameter $\kappa^2 \geq 0$

\[
\hat{z}_{i,t-h,t} = \omega_h \mu_i + (1 - \omega_h) E \left[ z_t | \tilde{F}_{i,t-h} \right]
\]

where $\omega_h = \frac{E \left[ e_{i,t,t-h}^* \right]}{\kappa^2 + E \left[ e_{i,t,t-h}^* \right]}$

$e_{i,t-h,t}^* \equiv z_t - E \left[ z_t | \tilde{F}_{i,t-h} \right]$
The degree of “shrinkage”

- The weights, $\omega_h$, placed on the prior vary across $h$ in a manner consistent with standard forecast combinations: as the Kalman filter forecast becomes more accurate the weight attached to that forecast increases.

- For short horizons, $h \to 0$, the weight attached to the prior falls, while for long horizons the weight attached to the prior grows.

- For analytical tractability, and for better finite sample identification of $\kappa^2$, we impose that $\kappa^2$ is constant across all forecasters.
Model-implied dispersion

- We normalise $\bar{\mu} = 0$ since we cannot separately identify $\bar{\mu}$ and $\sigma_\mu^2$ from our data. This is reasonable if the number of “optimistic” forecasters is approximately equal to the number of “pessimistic” forecasters.

- Our model for dispersion is the unconditional expectation of $d_{t,t-h}^2$. We allow for a heteroskedastic residual term for our model of dispersion, with variance related to the level of the dispersion.

\[
d_{t,t-h}^2 = \delta_h^2(\theta) \cdot \lambda_{t,t-h}
\]

where

\[
\delta_h^2(\theta) \equiv E\left[d_{t,t-h}^2\right]
\]

\[
E\left[\lambda_{t,t-h}\right] = 1
\]

\[
V\left[\lambda_{t,t-h}\right] = \sigma_\lambda^2.
\]
Model-implied dispersion as a function of Sig-nu

With Sig-mu=0, kappa=+infinity, Sig-eta = 2*Sig-u.

Dispersion for various values of sig-nu (sig-mu=0, kappa=+inf)

- sig-nu = 3
- sig-nu = 1
- sig-nu = 0.5
- sig-nu = 0.1
- sig-nu = 0.05
- sig-nu = 0.01

horizon dispersion

24 21 18 15 12 9 6 3 1
0
0.05
0.1
0.15
0.2
0.25
0.3

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Model-implied dispersion as a function of Sig-mu

With Sig-nu=2*Sig-u, kappa=0.5*Sig-z, Sig-eta = 2*Sig-u.
Our vector of parameters for the model of the mean squared consensus errors and dispersion is $\theta = [\sigma_u, \sigma_\varepsilon, \phi, \sigma_\nu, \sigma_\mu, \kappa, \sigma_\lambda]'$. We estimate these parameters by SMM:

$$\hat{\theta}_T \equiv \arg \min_{\theta \in \Theta} g_T (\theta)' W_T g_T (\theta)$$

where $g_T (\theta) \equiv \frac{1}{T} \sum_{t=1}^{T} \begin{bmatrix} e^2_{t,t-1} - MSE_1 (\theta) \\ \vdots \\ d^2_{t,t-1} - \delta^2_{1} (\theta) \\ \vdots \\ (d^2_{t,t-1} - \sigma^2_{\delta,1} (\theta))^2 \\ \vdots \end{bmatrix}$

where $MSE_h (\theta)$, $\delta^2_h (\theta)$ and $\sigma^2_{\delta,h}$ are the model-implied MSE, dispersion, and dispersion variance.
The weight matrix used in the first stage is the identity matrix.

The second-stage estimates again use the efficient weight matrix, obtained via simulation of 1,000 non-overlapping years of data.

We again estimated the model with just six forecast horizons: $h = 1, 3, 6, 12, 18, 24$.

In our simulation-based estimation of $\delta_h^2(\theta)$ we set $N = 30$ and simulated 100 non-overlapping years of data.
GMM parameter estimates: consensus and dispersion model

<table>
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<th></th>
<th>$\sigma_u$</th>
<th>$\sigma_{\epsilon}$</th>
<th>$\phi$</th>
<th>$\sigma_v$</th>
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<th>$\kappa$</th>
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Term structures of dispersion for GDP growth and inflation in the US.
Interpreting the results from the dispersion model

- Our simple model of dispersion provides a reasonable fit to the dispersion term structure, though tests of over-identifying restrictions reject the specification for GDP growth dispersion.

- If we condition on our model being correctly specified, we can interpret these $J$-tests as testing the rationality of the unobserved individual forecasters in our sample:
  - Our results indicate that while the consensus forecast appears to be rational, the individual forecasts are not in many cases. Pesaran and Weale (2005) show that this is possible under certain conditions.
  - In fact, even when controlling for biases in individual forecasts (via the $\mu_i$ terms) we often reject our model.
Conclusions

- We have provided a simple model and estimation technique for obtaining the parameters of the data generating process, given forecast errors across a range of horizons: $E \left[ e_{t,t-h}^2 \right]$, $h = 1, 2, ..., H$.

- Consistent with evidence obtained from other sources, measurement errors are an important source of forecast error for GDP forecasts, but less so for inflation forecasts.

- We also proposed a simple model for dispersion in macroeconomic forecasts, allowing for two possible sources of dispersion

- Differences in beliefs about long-run values are substantially more important than differences in signals. A richer model for dispersion appears to be required for GDP forecasts.
Summary of results

- A simple model of GDP and inflation dynamics is sufficient to accurately describe the complete term structure of consensus mean squared forecast errors.
  - The estimated persistence parameters are consistent with estimates obtained using lower-frequency data. Our estimates are much more accurate than what can be obtained using only the realisations over the same time period.

- It appears that differences in prior beliefs about long-run averages explains almost all of the observed dispersion. A richer model of the determinants of dispersion may be required:
  - Allow for more behavioural considerations?
  - Allow greater heterogeneity across forecasters?
  - Allow dispersion to vary systematically with the level of the target variable?
Extensions

1. Extend dispersion (and MSE) model to consider “covariates”. For example, dispersion in our data set appears higher during periods of low GDP growth, so a “level effect” may be useful.

2. Formally model learning by forecasters: allow them to update estimates of parameters rather than assume them known.

3. Consider the impact of other non-stationarities in the data: structural breaks in GDP variance or the level of inflation, for example.

4. Combine our panel of forecasts with samples of data on the target variables over a longer sample but at a lower frequency.