

# Identifying the Sources of Instabilities in Macroeconomic Fluctuations

Atsushi Inoue and Barbara Rossi

NCSU      Duke University

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## Abstract

This paper investigates the sources of the substantial decrease in output growth volatility in the mid-1980s by identifying which of the structural parameters in a representative New Keynesian and Structural VAR models changed. Overturning conventional wisdom, we show that the Great Moderation was not only due to changes in shocks volatilities but also to changes in monetary policy parameters as well as in private sector's parameters. The Great Moderation was previously attributed to "good luck" because the alternative sources of instabilities appear to have offsetting effects on output volatility, and therefore were impossible to detect using existing techniques.

**Keywords:** Instability, Model Evaluation, Great Moderation.

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# 1 Introduction

This paper analyzes the sources of instabilities in US macroeconomic fluctuations in the post-war period. Perhaps the most important feature of the data is the sharp decline in the volatility of output (as well as consumption and investment) documented by McConnell and Perez-Quiros (2000) and Stock and Watson (2002) among others, and labeled “the Great Moderation”. The causes of the decline are highly debated, and so far no consensus has been reached. In particular, it is still unclear whether the decline in volatility was due to changes in policy or changes in the unanticipated shocks hitting the economy. Stock and Watson (2002, 2003) perform counterfactual VAR and New Keynesian model analyses and conclude that the Great Moderation was mainly caused by a decrease in the volatility of the shocks. Therefore, they attribute most of the Great Moderation to "good luck". Similarly, Justiniano and Primiceri (2008) and Sims and Zha (2006) do not find a big role for monetary policy in explaining the decline in output volatility in an estimated DSGE model.<sup>1</sup> On the other hand, Boivin and Giannoni (2006) find instead an important role for monetary policy during the Great Moderation (see also Clarida, Gali and Gertler, 2000, and Cogley and Sargent, 2001, 2005).

This paper challenges the existing empirical findings on three grounds. First, we show that the existing literature is plagued by the "one at a time" approach to parameter instability criticized by Cogley (2007). In fact, all the above mentioned papers compare models in which a given subset of parameters change, while the other parameters are assumed to be constant. If the parameters that are assumed to be constant are in reality time-varying, this procedure may incorrectly attribute the time variation to the wrong source.<sup>2</sup> In particular, in the effort to shed light on the causes of the Great Moderation, recent research performs counterfactuals by allowing either the parameters in the monetary policy reaction function *or*

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<sup>1</sup>The literature on estimating structural macroeconomic models with time-varying parameters also includes Owyang and Ramey (2004), and Primiceri (2005). Gurkaynak et al. (2005), Fernald (2007) and Ireland (2001) instead apply tests for structural breaks to macroeconomic data. Although Justiniano and Primiceri's (2008) results have been interpreted as favorable to the "good luck" hypothesis, and we follow that interpretation here, there are two nuances. First, they do find a very important role for policy in the decline in the variability of both interest rates and inflation. Second, Justiniano and Primiceri (2006) are more keen on advocating policy of a different type: regulatory changes which had a huge effect on the volatility of residential investment, and hence output.

<sup>2</sup>Note that allowing all the parameters to be time-varying and repeatedly using structural break tests to identify which parameters are time-varying does not solve the problem either. In fact, one would incur size distortions, as we discuss in this paper.

the variance of the shocks to change over time, while keeping the slope of both the Phillips curve and the IS equation constant, and similar criticisms may apply. In addition, some crucial parameters are calibrated rather than estimated, and therefore are implicitly treated as constant as well. Second, most of these analyses involve descriptive statistics and not formal tests. For example, counterfactual analyses are not formal tests to search for the set of parameters that changed at the time of the Great Moderation, and therefore are not completely informative on its sources. We indeed demonstrate that counterfactual analyses are incapable of detecting all sources of instabilities in the data. Third, most of the literature has focused on VAR estimation, whereas structural models' estimation might be more suitable for identifying the sources of the changes that are of interest for policy purposes.<sup>3</sup>

This paper addresses these shortcomings in the following ways. First, we use a new approach to directly estimate the set of stable parameters. The advantage of our procedure relative to the existing literature is to provide a tool that can be used by researchers to search for the set of stable parameters, and does not rely on a “one-at-a-time”, ad-hoc approach. Our procedure offers a convenient alternative to testing all possible subsets of parameters to determine the set of unstable parameters, and can exactly identify which parameters changed at the time of the Great Moderation. Second, our results formally demonstrate that there were significant changes in the transmission mechanism at the time of the Great Moderation. However, such changes were undetectable by existing counterfactual analyses, as many parameter changes can have offsetting effects on output volatility. Thus, counterfactuals where only the shocks volatilities change may fit the data well even though they are not the only parameters that changed in reality. In fact, there is no reason why we should think that the changes in policy parameters offset each other or offset changes in private sector's parameters, as one could as well think that changes in some policy parameters offset the effects of changes in shock variances and that the bulk of the Great Moderation is explained by the changes in the remaining policy parameters. In addition, our procedure can exactly identify which parameters have changed, and therefore can provide an economic interpretation of the changes in the economy. Third, we focus on a representative New Keynesian model, although we do verify that our main results are robust to standard VAR estimation as well as larger-scale DSGE model estimation.

Overall, our results are noteworthy for three reasons. First, they imply that not only the variance of the unpredictable shocks has decreased during the Great Moderation (as

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<sup>3</sup>Notable exceptions are Justiniano and Primiceri (2008), and Fernandez-Villaverde and Rubio-Ramirez (2007), who estimate a medium scale DSGE model.

in Stock and Watson, 2002, Justiniano and Primiceri, 2008, and Sims and Zha, 2006), but that the transmission mechanism has changed as well. While the existence of instabilities in the parameters of the monetary policy reaction function is a well-known fact (see Clarida, Galí and Gertler, 2000), the existence of a link between the Great Moderation and changes in monetary policy was never formally established. In a closely related paper, Boivin and Giannoni (2006) consider counterfactual analyses in Structural VARs and a small scale New-Keynesian model, and are the first to show that the shape of the impulse responses changed before and after the Great Moderation. They conclude that this empirical evidence suggests a change in the transmission mechanism, and that changes in monetary policy and the private sector behavior might have played an important role. However, looking at whether impulse responses have changed is not a formal test, and also masks whether individual parameters have changed in offsetting ways, thus becoming subject to the same criticisms that we point out in this paper. In addition, changes in the persistence of the autoregressive shocks, which should also be considered as "good luck", do change the shape of the impulse responses and would incorrectly be attributed by this methodology to either policy or private sector's changes. Second, our analysis is informative regarding which specific parameters significantly changed at the time of the Great Moderation. The empirical results of our new test show that at the time of the Great Moderation only some of the variances of the structural disturbances changed, and that many monetary policy and other structural parameters present in the transmission mechanism changed as well. These findings provide important and useful guidelines for understanding the mechanism behind the Great Moderation and for modeling theoretical structural macroeconomic models that are consistent with the data. Third, this paper makes a methodological contribution by introducing a new procedure to search for the set of stable parameters. Such a procedure can potentially be applied to other areas of research, where it is important to establish which sets of parameters have changed over time. Note that, as a consequence, our approach could be used to identify which additional shocks the model needs to describe the data.

Our paper is closely related to Justiniano and Primiceri (2008) and Sims and Zha (2006), but has important differences as well. First, they both rely on counterfactual analyses, whose potential drawbacks we discuss. In addition, they mainly consider a specific parametric form of time variation, namely autoregressive and Markov switching specifications, respectively, for specific subsets of the parameters. In contrast, our framework allows for a one-time break at an unknown date, thus being consistent with the original framework of McConnell and Perez-Quiros (2000), and allows for the possibility that all parameters might change.

We also show that our results are robust to multiple breaks and smooth time variation specifications similar to Justiniano and Primiceri (2008). Sims and Zha (2006), unlike us, do not consider a fully structural DSGE model. In addition, our results are more distantly related to Lubik and Surico (2006), Fernandez-Villaverde and Rubio-Ramirez (2007), Cogley and Sbordone (2005) and Ireland (2001). Lubik and Surico (2006) show that a shift in monetary policy can generate the decline in the volatility of inflation and real activity in reduced-form models. However, their analysis is based on calibrations and Monte Carlo simulations, not on estimated DSGE models. Cogley and Sbordone (2005) investigate the stability of the Phillips curve in the face of changes elsewhere in the economy, and do so by estimating parameters in ad-hoc sub-samples, whereas we let the data identify the sub-samples as well as the parameters that change in the structural model. Fernandez-Villaverde and Rubio-Ramirez (2007) separately investigate either the stability of the monetary policy or the stability of the parameters controlling nominal rigidities and indexation of prices and wages, while keeping all the other parameters constant. Ireland (2001) attempts to identify which parameters are time-varying by applying structural break tests to each of the parameters separately. However, we show that when used repeatedly to test structural change in more than one subset of parameters, such tests lead to size distortions in the overall procedure. Finally, the recent works by Liu et al. (2009) and Bianchi (2009) have analyzed the sources of the Great Moderation in a regime-switching model, which assumes that the data are stationary but switching between two regimes; our work assumes instead that there is a one-time permanent change in the parameters at the time of the Great Moderation. In addition, they assume that only a subset of the parameters can change (either monetary policy or volatilities) whereas we allow all parameters to possibly change.<sup>4</sup>

The paper is organized as follows. The next section provides an overview of the model, Section 3 informally describes the new techniques that we utilize, and Section 4 presents the main empirical results. Section 5 discusses robustness analyses, in particular to larger scale DSGE models and Structural VARs. Section 6 concludes. The technical details, including the formal proofs of the desirable properties of our technique, are relegated to the Appendix.

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<sup>4</sup>Our paper is more distantly related to Sims (2001) and Ghysels and Hall (1990), which evaluate whether parameters in standard economic models can be interpreted as "structural".

## 2 The Model

The model we focus on is a New Keynesian model with features such as in Clarida, Gali and Gertler (1999), Woodford (2003), Gurkaynak et al. (2005), Ireland (2004, 2007) and widely used in the recent literature on monetary policy and business cycles. In particular, the model is based on Ireland (2007). The model concisely describes the monetary policy of the Federal Reserve and the optimizing behavior of households and firms in the U.S. economy by including a Phillips curve, an IS equation and a monetary policy reaction function. The model also allows for a variety of features that have been deemed important to match theoretical models with the empirical data, namely habit formation, and forward-looking price setting.<sup>5</sup>

The model is log-linearized around a steady state where consumption, output, and the marginal utility of consumption grow at the rate of technological process (a random walk with drift). The model is as follows. Let  $\hat{y}_t$ ,  $\hat{\pi}_t$ ,  $\hat{e}_t$ ,  $\hat{z}_t$ ,  $\hat{a}_t$ ,  $\hat{v}_t$ ,  $\hat{\lambda}_t$  denote the deviation of output, inflation, the cost-push shock, technology, the preference shock, the transitory monetary policy shock, and the marginal utility of consumption from their steady state levels.

The core of the model is formed by the following three equilibrium conditions:

(1) the IS/Euler equation:

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{r}_t - E_t \hat{\pi}_{t+1}, \quad (1)$$

where  $\hat{\lambda}_t = \frac{\gamma_t z}{(z-\gamma_t)(z-\beta\gamma_t)} \hat{y}_{t-1} - \frac{(z^2 + \beta\gamma_t^2)}{(z-\gamma_t)(z-\beta\gamma_t)} \hat{y}_t + \frac{\beta\gamma_t z}{(z-\gamma_t)(z-\beta\gamma_t)} E_t \hat{y}_{t+1} + \frac{(z-\gamma_t)(z-\beta\gamma_t \rho_{a,t})}{(z-\gamma_t)(z-\beta\gamma_t)} \hat{a}_t - \frac{\gamma_t z}{(z-\gamma_t)(z-\beta\gamma_t)} \hat{z}_t$ ;

(2) the Phillips curve:

$$(1 + \beta\alpha_t) \hat{\pi}_t = \alpha_t \hat{\pi}_{t-1} + \beta E_t \hat{\pi}_{t+1} + \psi_t (\hat{a}_t - \hat{\lambda}_t) - \hat{e}_t - \alpha_t \hat{\pi}_t^*; \quad (2)$$

(3) and the Monetary Policy Reaction Function:

$$\hat{r}_t - \hat{r}_{t-1} = \rho_{\pi,t} \hat{\pi}_t + \rho_{gy,t} \hat{g}_t^y + \varepsilon_{r,t}, \quad (3)$$

where the monetary policy shock has both a persistent and a transitory component:  $\varepsilon_{r,t} =$

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<sup>5</sup>Fuhrer (2000) demonstrated that habit formation is important to fit the data. See also Fuhrer and Rudebusch (2004).

$\widehat{v}_t - \widehat{\pi}_t^*$ ,  $\widehat{v}_t = \rho_{v,t}\widehat{v}_{t-1} + \sigma_{v,t}\varepsilon_{vt}$ ,  $\widehat{\pi}_t^* = \sigma_{\pi,t}\varepsilon_{\pi,t}$ , and the following hold:

$$\begin{aligned}\widehat{g}_t^y &= \widehat{y}_t - \widehat{y}_{t-1} + \widehat{z}_t, \\ \widehat{a}_t &= \rho_{a,t}\widehat{a}_{t-1} + \sigma_{a,t}\varepsilon_{at}, \\ \widehat{e}_t &= \rho_{e,t}\widehat{e}_{t-1} + \sigma_{e,t}\varepsilon_{et}, \\ \widehat{z}_t &= \sigma_{z,t}\varepsilon_{zt}, \\ \widehat{v}_t &= \rho_{v,t}\widehat{v}_{t-1} + \sigma_{v,t}\varepsilon_{vt}.\end{aligned}$$

The model builds on a series of parameters:  $z$  (the steady state level of technology),  $\beta$  (the discount factor),  $\gamma_t$  (the habit formation),  $\alpha_t$  (the parameter measuring the extent to which price setting is backward or forward looking:  $\alpha_t = 0$  means purely forward looking),  $\psi_t$  (a function of the magnitude of the adjustment cost and of the long-run level of the cost-push shock),  $\rho_{\pi,t}$  (the Fed's inflation aversion),  $\rho_{gy,t}$  (the Fed's aversion to the output gap),  $\rho_{v,t}$  (the persistence of the monetary policy shock),  $\sigma_{v,t}$  (the standard deviation of the monetary policy shock),  $\rho_{a,t}$  (the persistence of the preference shock),  $\sigma_{a,t}$  (the standard deviation of the preference shock),  $\rho_{e,t}$  (the persistence of the cost-push shock),  $\sigma_{e,t}$  (the standard deviation of the cost-push shock), and  $\sigma_{z,t}$  (the standard deviation of the aggregate technology shock).

### 3 Methodology

This section informally describes the new methodology that allows us to estimate the set of stable parameters. Further details, including a formal description of the algorithm and proofs, are relegated to the Appendix.

To estimate the set of stable parameters, we propose a recursive procedure. First, we test the joint null hypothesis that all parameters are stable using any test for structural breaks.<sup>6</sup> In our main empirical application, we will utilize the well-known QLR stability test (Andrews, 1993). If the test does not reject, then we conclude that all the parameters belong to the set of stable parameters. If it does, we calculate the p-values of the individual test statistics for testing whether each of the parameters is stable. We note that it is important that such individual tests do not rely on the maintained assumption that the other parameters be constant over time. Therefore, the individual tests should be implemented by allowing all the other parameters to be time-varying.

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<sup>6</sup>Note that the main requirement for the procedure to work is that the test for structural break be consistent. See the Appendix. Most of the tests developed in the literature satisfy this assumption.

Next, we start by eliminating from the set of stable parameters the parameter with the lowest p-value, that is the parameter that is most likely to be unstable, then test whether the remaining parameters are jointly stable.<sup>7</sup> If they are, then the set of stable parameters includes such parameters; otherwise, we eliminate the parameter with the second lowest p-value from the set, and continue this procedure until the joint test on the remaining parameters does not reject stability. This recursive methodology will identify the set of constant parameters, which we refer to as the “Estimate of the Set of Stable Parameters”, or ESS. See Algorithm 1 in the Appendix for a detailed step by step description of the ESS procedure. A very useful advantage of our methodology is that it allows researchers to conduct a more exhaustive analysis of the possible sets of unstable parameters while remaining conveniently implementable in practice. This is made possible by using the step-by-step procedure, which avoids considering all possible  $2^r$  configurations for the set of stable parameters, where  $r$  is the number of parameters.

It is important to realize that the ESS procedure guarantees that the set of stable parameters is estimated with a pre-specified probability level. Other procedures do not fulfill this requirement. In particular, judging whether a set of parameters is constant or not by the "naïve approach" of simply looking at their individual stability t-tests is misleading. We believe this is an important point, as the "naïve approach" appears to be a common approach in practice (e.g. Ireland, 2001). To see why the “naïve approach” is inappropriate, consider a model with only two parameters. In the “naïve approach” one would apply a test for structural change to each of the two parameters. Because the testing procedure is repeated twice, one rejects the null hypothesis that the parameters are both stable with an overall probability level that is higher than the significance level of the testing procedure. As a result, one may inadvertently find more unstable parameters than there actually are. In contrast, in our proposed ESS procedure, one jointly tests for constancy of the two parameters first. Thus, one does not reject the null hypothesis of no structural change more often than the significance level specified for the joint test when there is no structural change. When the joint test rejects the joint hypothesis that the two parameters are constant, our procedure then identifies which parameters are unstable (if there are any) by applying a test of structural change to each of the two parameters in the second step. The size distortion of

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<sup>7</sup>Alternative procedures could involve calculating joint tests for every subset of parameters and use that information to choose which parameters to eliminate in the sequential procedure. However, this procedure is more computationally burdensome, especially when applied to the estimation of DSGE models, and our methodology provides a convenient alternative in these situations.

the “naïve approach” worsens as the number of parameters grows, which is important because macroeconomic models may have more than fifteen parameters. A Not-for-Publication Appendix discusses this technical issue more in detail, and provides Monte Carlo simulations to evaluate the performance of the naïve procedure relative to the ESS procedure.

Finally, some technical remarks on the practical implementation of the method. The data are quarterly time series of per-capita GDP growth in real terms, the first difference of the GDP price deflator, and the three month U.S. Treasury bill interest rate minus inflation from 1959:1 through 2004:2. We focus on this sample so that our results are directly comparable with those in Stock and Watson (2003), Ireland (2007), and Smets and Wouters (2007), although we discuss the robustness to a sample up to 2007 in Section 5. We implement our procedure using the QLR stability test. At each step of the procedure, we need to estimate the model’s parameters. As it is customary in the literature, we calibrate a small number of the model parameters to well-known values. We calibrate the discount factor  $\beta$  to be 0.9995, and  $z$  to be 1.0047, a combination that ensures that the steady-state rate of output growth  $z$  and the steady state ratio of the nominal interest rate to the inflation rate  $z/\beta$  implied by the theoretical model match the average values of the same two variables as measured in the data. See Ireland (2007) for more details.<sup>8</sup> Overall, the model has 13 remaining parameters to be estimated, and we allow all of them to be possibly time varying:

$$\Theta_t = \{\gamma_t, \alpha_t, \rho_{\pi,t}, \rho_{gy,t}, \rho_{a,t}, \rho_{e,t}, \rho_{v,t}, \sigma_{a,t}, \sigma_{e,t}, \sigma_{z,t}, \sigma_{v,t}, \sigma_{\pi,t}, \psi_t\}$$

Our analysis focuses on the situation in which there is a single, unanticipated, and once-and-for-all shift in some of the parameters of the structural model at an unknown time, and in which there is an immediate convergence to a rational-expectations equilibrium after the regime change. Thus, the parameters are assumed to change at one unknown date, that is estimated via our procedure to be around 1980:4 (around the time of the Great Moderation).

$$\Theta_t = \begin{cases} \Theta_1 & \text{if } t < 1980 : 3 \\ \Theta_2 & \text{if } t \geq 1980 : 4 \end{cases}$$

This justifies implementing our procedure with the QLR stability test.<sup>9</sup> A plot of the estimated QLR test statistic over possible break-dates uncovers that there is substantial uncer-

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<sup>8</sup>Although the discount factor  $\beta$  is calibrated and constant, note however that we are not imposing a constant discount factor because the discount factor itself is multiplied by a time-varying preference shock. We let both the serial correlation and the variance of the preference shock to possibly change following a structural break, and we estimate them from the data.

<sup>9</sup>For computational simplicity we fixed the break date in Algorithm 1 to be the estimated one, and applied

tainty regarding the time of the break, as the plot is relatively flat between 1980 and 1984. We thus decided to use the estimated break date 1980:4 although we checked that our main results are robust to the choice of a break in 1984:1.<sup>10</sup> The estimated time of the break that we use is similar to that in Boivin and Giannoni (2006), who estimated the time of the break to be in 1979:3. In the VAR analysis in Section 5.3, the estimated time of the break is more similar to that in the VAR analysis in Stock and Watson (2001, 2004) and it is 1985:2.

## 4 Empirical Results

In this section, we shed light on the major source of fluctuations in US macroeconomic post-war data, namely the Great Moderation. McConnell and Perez-Quiros (2000) noticed that the volatility of output and inflation fell considerably in the US starting from 1984. Our objective is to analyze which parameters changed at the time of the Great Moderation by using our novel methodology. We show that our procedure overturns the main economic conclusions reached by Stock and Watson (2003), Sims and Zha (2006), Justiniano and Primiceri (2008), and Smets and Wouters (2007). We demonstrate that the main reason why our results are substantially different from the existing literature is that the latter utilizes counterfactual comparison procedures that only consider selected subsets of time-varying parameters.

Section 5 shows that our main results are robust to other specifications of the structural models, most notably different specification of monetary policy, larger scale DSGE models, Structural VARs, as well as multiple breaks and smoothly time-varying parameters in Structural VARs, and more recent data. The Not-for-Publication Appendix reports parameter estimates, standard errors and other details.

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a standard Chow test for structural break; however, since we evaluate the Chow test statistic at the estimated break date, the p-values are calculated using Andrews' (1993) critical values. Note that the LR-like statistics in Andrews (1993), eq. (4.5), simply becomes the likelihood ratio test calculated as the difference between the constrained model (that is, the loglikelihood estimated over the full sample) and the unconstrained one (that is, the weighted average of the loglikelihood estimated separately over all possible two sub-samples, weighted according to the percentage of observations in each of the two sub-samples). Andrews' (1993) is very computationally convenient in this case because once the break date is estimated, we only need to estimate the model twice (before and after the break) and not take averages over all possible unknown break dates, as we would do for the Andrews and Ploberger's (1994) Exp-W and Ave-W tests.

<sup>10</sup>When we impose a break in 1984:1, we find that the estimated set of stable parameters again suggests a combination of policy, shocks and private sector's parameter changes similar to what we find with a break date of 1980:4.

## 4.1 The ESS methodology

Table 1 reports the estimated set of stable parameters obtained by using our ESS procedure,  $\mathcal{S}$ . In order to give an economic interpretation of the sources of time variation, we divide the parameters in three groups:

(i) the private sector parameters, including:  $\gamma$  (the habit formation parameter);  $\alpha$  (the parameter indexing the degree of backward looking price behavior);<sup>11</sup>  $\psi$  (the slope of the Phillips curve);

(ii) the monetary policy parameters, including:  $\rho_{gy}$  (the Fed's output gap reaction); and  $\rho_\pi$  (the degree of inflation aversion of the Fed);

(iii) the shocks parameters, including:  $\sigma_z$  (the standard deviation of the aggregate technology shock),  $\rho_a$  and  $\sigma_a$  (the persistence and standard deviation of the preference shock);  $\rho_e, \sigma_e$  (the persistence and standard deviation of the cost-push shock);  $\rho_v, \sigma_v$  (the serial correlation and standard deviation of the persistent monetary policy shock); and  $\sigma_\pi$  (the standard deviation of the i.i.d. monetary policy shock).

Note that, while we include the serial correlation and standard deviation of the monetary policy shock among the shocks parameters, it could be argued that they are actually function of the monetary policy, and thus should be included among the monetary policy parameters. In fact, one could think of the policy shock as something under the control of the policy maker, and therefore part of policy, or as outside the control of the policy maker (due to lags in data arrival, differences between the target and effective funds rates, timing of policy meetings within the year, etc.) and therefore be part of the exogenous shocks. While it is plausible to think that policy shocks are shocks for the econometrician but are intended outcomes of the optimal decision of the policy maker, the reason for the existence of these shocks is still an open question in the literature. Similarly,  $\rho_v$  could be interpreted as the degree of monetary policy smoothing. In this section, we present results emphasizing the interpretation that monetary policy shocks are exogenous, although our main empirical conclusions would only be strengthened if these parameters were considered part of monetary policy. Readers can thus interpret our results as a lower bound for the importance of monetary policy.

Table 1 shows that the unstable parameters include not only the volatilities and degree of persistence of most shocks, but also the parameters of the monetary policy reaction

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<sup>11</sup> $\alpha = 0$  denotes the case in which the Phillips curve is purely forward looking. In our estimates,  $\alpha$  is close to zero – see also Galí and Gertler (2000).

function, as well as many other structural parameters governing the behavior of the private sector. In particular, we find instabilities in the volatilities of the monetary policy shock (both transitory and persistent), as well as the volatility of the cost-push and preference parameters ( $\sigma_\pi, \sigma_v, \sigma_e, \sigma_a$ ); the volatility of the technology shocks is instead constant over time ( $\sigma_z$ ). The persistence of these shocks ( $\rho_v, \rho_a$ ) as well as the persistence of the cost-push shock ( $\rho_e$ ) have also significantly changed over time. However, this is not the whole picture, as we also find that some monetary policy parameters as well as other structural parameters have changed. In particular, the parameter governing the extent to which prices are backward or forward looking ( $\alpha$ ), the adjustment cost ( $\psi$ ) and the habit parameter ( $\gamma$ ) also have changed. More importantly, both the Central Bank's inflation aversion ( $\rho_\pi$ ) and the output gap reaction parameter ( $\rho_{gy}$ ) have significantly changed.

Overall, we find empirical evidence in favor of changes in the transmission mechanism, in particular changes in monetary policy as well as other structural parameters, in addition to shocks volatilities. These results overturn the commonly held view that only the variance of the unpredictable shocks has decreased during the Great Moderation, and that monetary policy played no role. In addition, our analysis is informative regarding which specific parameters significantly changed at the time of the Great Moderation, thus providing valuable information for understanding the Great Moderation itself and for modeling theoretical structural macroeconomic models that are consistent with the data.

## 4.2 Comparison with existing methodologies

Our findings are substantially different from the conventional picture that emerges from the literature, namely the "good luck hypothesis" endorsed by Stock and Watson (2001, 2003), Sims and Zha (2006), Smets and Wouters (2007), and Justiniano and Primiceri (2008). This crucial finding deserves a careful investigation to understand why our results are substantially different from theirs. We will do so by considering each of the traditional methods used in the literature, and highlight their limitations. We will argue that some of the time variation that they find is due to estimation error. In addition, we demonstrate that many parameter changes have offsetting effects on output volatility, so that existing methods that only focus on a limited number of subsets for their counterfactual comparisons are incapable of detecting them.

### A. Counterfactual Analyses

Counterfactual analyses are the most common tool utilized by researchers to assess the

sources of the Great Moderation. Counterfactual analyses involve evaluating different scenarios for the volatilities of output and inflation in the second sub-sample by changing either the volatilities of the shocks, or the policy parameters, or the private sector’s parameters, one at a time. The most likely scenario (or “best model”) is the one that best replicates the substantial drop in volatility (Stock and Watson, 2001, and Smets and Wouters, 2007) or the one with the highest posterior probability (Sims and Zha, 2006, and Justiniano and Primiceri, 2008). In particular, Smets and Wouters (2007) show that the scenario closest to the actual realization in the data is one in which the economy faced shocks with lower variances, while keeping the policy and the structural parameters the same. They show that scenarios in which only either the policy parameters or the structural parameters have changed would be incapable of replicating the actual volatilities observed in the data. Similar conclusions have been reached by Stock and Watson (2003), Justiniano and Primiceri (2008) and Sims and Zha (2006). However, while the counterfactual analysis approach is a clever descriptive device, it is not a formal statistical testing procedure. In fact, we will show that both counterfactual scenarios in which either the shock processes or the unstable parameters identified by ESS change are consistent with the data. However, the latter is statistically the best fitting model. Furthermore, researchers have typically used counterfactual analyses to look at scenarios in which either all the volatilities of the shocks change, or all the policy parameters change, or all the other structural parameters change. In other words, all the parameters in a specific category are required to change. Realistic situations in which some of the volatilities and some of the policy parameters change are completely ignored. This substantially limits the ability of these methods to identify the actual set of unstable parameters unless it exclusively contains all of the parameters in a specific category. Our procedure instead provides a convenient way to consider potential instabilities in all the parameters while, at the same time, avoiding to consider all possible subsets of parameters (which would be computationally infeasible, involving  $2^r$  possible combinations, where  $r$  is the number of parameters).

Let us start by demonstrating that the set of stable parameters estimated by ESS is a better description of the data than the “good luck” scenario. Panel A in Table 2 shows that *counterfactual analyses* in which the parameters describing the shock processes<sup>12</sup> are kept constant (labeled "Shocks<sub>C</sub>")<sup>13</sup> are substantially at odds with the data, as they would

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<sup>12</sup>Shock parameters include  $\rho_v, \sigma_v, \rho_a, \sigma_a, \rho_e, \sigma_e, \sigma_z$ .

<sup>13</sup>To clarify, the column labeled "Shocks<sub>C</sub>" reports the standard deviation of selected macroeconomic variables when shocks volatilities are kept constant but all other parameters are allowed to change.

predict a volatility of output growth of 1.58 when instead the volatility dropped to 0.88 (left panel, labeled "Estimates");<sup>14</sup> instead, the counterfactuals with constant policy parameters are not at odds with the data (labeled "Policy<sub>C</sub>"). This is the stylized fact in the existing literature, which therefore concludes that the evidence favors the "good luck" hypothesis. However, one should not conclude that only the shocks have been constant, as indeed also the counterfactual where the set of unstable parameters identified by ESS are kept constant (labeled "ESS<sub>C</sub>") is at odds with the data. Indeed, the table shows that the latter is at odds with the data, based on marginal likelihood measures, to a degree similar to that of the counterfactual with constant shocks (see the row labeled "Likelihood"). Panel B considers the *fit* of the various scenarios rather than their counterfactuals. It reports data moments and likelihood estimates in the second sub-sample for the scenarios in which the only parameters that are allowed to change across sub-samples are either the shocks (labeled "Shocks"), policy (labeled "Policy") or the set of ESS unstable parameters (labeled "ESS"). For example, the column labeled "Shocks" reports standard deviations for the selected macroeconomic variables when shocks volatilities are allowed to change but all other parameters are kept constant. Note that while allowing only the shocks to change seems to replicate almost perfectly the drop in output volatility, the likelihood worsens substantially from 1,261 to 1,241. When we consider instead only allowing the ESS unstable parameters to change, the likelihood does not change substantially, while still replicating a large decrease in output volatility.

We next proceed to shed light on why the counterfactual analyses and our ESS procedure reach different conclusions. As we will show, the main reason is that many parameter changes have offsetting effects on output volatility. The parameters that ESS identifies as unstable but that are not shocks parameters *do significantly* affect output volatility, but *some increase* it and *some decrease* it, in such a way that the total net effect is negligible. This, however, does not mean that the latter parameters have not changed, as the counterfactual analyses would conclude. Counterfactual analyses are simply incapable of distinguishing such parameter changes, as their effects on output volatility cancel out. However, there is no reason why we should think that the changes in policy parameters offset each other, as one could as well think that changes in some policy parameters offset the effects of changes in shock variances and that the bulk of the Great Moderation is explained by the

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<sup>14</sup>Note, that in this exercise, "Estimates" means estimates of the unconstrained model in the two sub-samples.

changes in the remaining policy parameters.<sup>15</sup> Table 3 substantiates these points, while at the same time providing substantial evidence that the statistical significance of the structural breaks in the monetary policy parameters does indeed result in economic significance, and that monetary policy played an important role. The table reports the marginal percentage increase (decrease, if negative) in the standard deviation of output, inflation and the interest rate calculated by progressively letting all the parameters change from their second sub-sample values to their first sub-sample values, according to the ordering of the ESS procedure. For example, the volatility of the monetary policy shock ( $\sigma_v$ ) is among the parameters that ESS identifies to be unstable (see Table 1). Note that allowing  $\sigma_v$  to change from its value in the second sub-sample to its value in the first sub-sample, one would observe a 62% increase in the standard deviation of output. Note that, while the change in the volatility of the monetary policy shock parameter is capable of explaining a large fraction of the increase in volatility, considering changes in both the inflation aversion and the output gap parameters ( $\rho_\pi, \rho_{gy}$ ) almost exactly offsets the increase in the volatility (their joint contribution is about  $-56\%$ ). Changes in the persistence of the preference shock and the volatility of the technology shock ( $\rho_a, \sigma_z$ ) produce an additional increase in the volatility of output growth of about 79%, and changes in private sector's parameters ( $\gamma, \alpha, \psi$ ) overall increase output volatility substantially too – about 30%.

This explains why policy counterfactuals appear unable to explain the change in output volatility: the overall additional effect of policy parameter changes on output volatility might be obscured by changes in other parameters, even though each of the policy parameters individually accounts for substantial changes. It is therefore incorrect to conclude that only the shock processes have changed, and to attribute the Great Moderation solely to "good luck". Counterfactuals where only the shocks parameters change may fit the data well even though they are not the only parameters that changed in reality. In fact, there is no reason why we should think that the changes in policy parameters offset changes in private sector's parameters, as one could as well think that changes in some policy parameters offset the effects of changes in shock variances and that the bulk of the Great Moderation is explained by the changes in the remaining private sector parameters.

Note that including the monetary policy shocks parameters among the policy parameters

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<sup>15</sup>In other words, the fact that these changes have offsetting effects on output volatility does *not* mean that they are unimportant: there are alternative sets of parameters beyond those associated with the "good luck" hypothesis that are plausible, thus challenging the thesis that good luck provides the "best" explanation for the Great Moderation.

only reinforces our conclusions, as  $\sigma_v$  alone contributed 62% to the increase in the volatility of output growth.

Table 4 investigates the sources of the Great Moderation. We consider changes in the standard deviation of five structural shocks: the transitory monetary policy shock, the technology shock, the preference shock, the cost-push shock and the persistent monetary policy shock. The last column of Table 4 shows the relative contribution of each shock to the total reduction in the variance of GDP growth. The biggest relative contribution to the decrease in volatility is by far that associated with persistent monetary policy shock (about 78%). While the monetary policy shock seems to explain most of such decrease, the cost-push, preference and technology shocks played a role as well.

## B. Standard Stability Tests

It is important to note that the ESS methodology used in this paper is very different from existing tests for parameter stability. In fact, standard tests for parameter stability only assess whether a *given* subset of the parameters is constant: they thus require researchers to take a stand on which parameters are constant and which might possibly be time varying prior to implementing the test. Therefore, they cannot be used to *search* for the set of stable parameters, which is instead the goal of this paper.

There is only one case in which the standard tests' results would be the same as our ESS procedure's, and that is when the set of stable parameters contains only one element. This is indeed the case in the empirical results shown in Table 1 (see the column labeled "Individual p-value", which displays the p-values of the individual parameter tests). We must stress, however, that this is just a coincidence. For example, in the Smets and Wouters' (2007) model, the discrepancy between standard stability tests and the ESS procedure will be much more striking, as will be shown in Section 5, where the standard stability test would identify nine more parameters to be unstable relative to the ESS procedure. Similarly, individual tests in the VAR analysis in Section 5.3 find four more unstable parameters than there really are.

The technical reason why standard stability tests on individual parameters fail to provide reliable inference on the set of stable parameters is related to the traditional problem of multiple hypotheses tests. Performing tests multiple times without correcting the size of the test results in incorrect inference, as eventually the test will reject the null hypothesis with very high probability. The Not-for-Publication Appendix discusses these issues more in detail, and provides Monte Carlo simulations to quantitatively show how unreliable inference

becomes in those circumstances.

### **C. Impulse-response Analyses**

Another qualitative analysis to assess whether the transmission mechanism has changed is to compare impulse responses across sub-samples, as suggested by Boivin and Giannoni (2006). They claim that if only the volatilities of the shocks had changed during the Great Moderation, then the normalized impulse responses would not have changed at all. However, a change in their shape would indicate a change in the transmission mechanism. Figure 1, Panels A and B, shows impulse responses from the model estimated separately in the two sub-samples. Some of the impulse responses seem to have changed shape, especially the output and inflation responses to monetary policy, suggesting that monetary policy might have become more effective in stabilizing the economy. However, this approach, while intuitively appealing, has similar drawbacks to counterfactual analyses: it is a clever descriptive device, but it is not a formal test, and looking at whether impulse responses have changed can mask whether individual parameters have changed in offsetting ways. This approach is therefore subject to the same criticisms that we point out in this paper. An additional drawback of impulse-response analyses is that changes in the persistence of the autoregressive parameters in the shocks' processes, which should be classified as "good luck", would change the shape of the impulse responses, and would incorrectly be attributed to either policy or private sector's changes. This approach is also not informative regarding exactly which parameters have changed, as impulse responses are functions of all the parameters in the model. Thus, it does not really address exactly how the transmission mechanism has changed, nor whether the changes are statistically significant, which are instead the advantages of the ESS methodology.

### **D. Alternative Interpretations**

The main conclusion of this section is that, since the set of unstable parameters includes not only parameters describing shock volatilities but also monetary policy parameters as well as parameters describing the private sector's tastes and technologies, then if we have faith that the models are well-specified, the interpretation is that the Great Moderation is due in fact to all three of these factors: "good luck", "good policy", and shifts in tastes and technologies that also apparently insulate the economy from shocks. However, alternative interpretations are possible.

One alternative interpretation, more favorable to Stock and Watson (2002, 2003), Justiniano and Primiceri (2006) and Sims and Zha (2006) is that it could theoretically be possible

that the changes in private sector and policy parameters have largely offsetting effects on volatility. That is, suppose that there have indeed been shifts in the structure of tastes and technologies that have changed the way in which exogenous shocks affect the economy. But suppose, at the same time, that the Federal Reserve has adjusted its policies so as to exactly offset those changes — now the net effect vanishes. But, finally, volatility still falls during the Great Moderation because the shocks themselves have become smaller or less frequent. While it is true that this scenario is characterized by shifts in a wide range of parameters, we would not say that shifts in monetary policy play a big role in explaining the Great Moderation; instead, the Federal Reserve adjusted its policies appropriately in response to changes it observed in firm's price setting behavior or in household's consumption-savings behavior, and ultimately, it is "good luck" that explains the Great Moderation. However, one could as well think that changes in some policy parameters offset the effects of changes in shock variances and that the bulk of the Great Moderation is explained by the changes in the remaining private sector parameters, as discussed in Section 4.2. What is clear from the empirical results is that the scenario where both policy and private sector parameters are constant leads to a sizeable decrease in the likelihood relative to the case in which all the parameters identified by ESS change (the likelihood drops from 1,261 to 1,241, as Table 2 shows). A likelihood ratio test would reject the null hypothesis that only the shocks changed at conventional critical values.<sup>16</sup> Note that, instead, a likelihood ratio test would not reject the null hypothesis that only the ESS parameters changed at conventional critical values.<sup>17</sup> The fact that the likelihood does fall sharply when private and policy parameters are held fixed implies that, even if these changes might have had offsetting effects on macroeconomic volatility, they are important for understanding changes in other dynamic features of the data.

Alternatively, one might view the fact that so many of the DSGE models' parameters exhibit instability — including parameters describing tastes and technologies that are supposed to be "structural" — as evidence that those models just don't fit the data very well and don't avoid falling victim to the Lucas critique. However, the definition of 'structural parameters' (in the sense of the Lucas critique) is that these parameters are policy invariant, not necessarily time invariant. As technologies and/or preferences change, structural

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<sup>16</sup>The estimated likelihood ratio statistic is 40; the 5% critical value of the QLR test with 5 degrees of freedom is 18.35, and the 1% critical value is 22.49.

<sup>17</sup>The estimated likelihood ratio statistic is 6; the 5% critical value of the QLR test with 12 degrees of freedom is 30.16, and the 1% critical value is 35.07.

parameters may vary as well. Along those lines, the next section will consider VAR models. It is interesting that the VAR — what we might call a "purely backward-looking model" — exhibit less instability. Viewed in this light, the results could be related to Fuhrer and Rudebusch (2004) among others, who show that backward-looking models fit the data much better than forward-looking ones, even if they might not be appropriate for analyzing the effects of large but perfectly credible changes in policy regimes.<sup>18</sup> In addition, we tested whether Ireland's (2007) New Keynesian model is mis-specified by applying the information matrix test of White (1982) to the post Great Moderation sample. Because the asymptotic approximation is known to be poor, we used bootstrap critical values. The bootstrap  $p$ -value of the information matrix test with 200 bootstrap iterations is 0.975. This result supports our interpretation that the presence of parameter change in this model does not necessarily mean that the model is mis-specified.

## 5 Robustness Issues

This section has two objectives. First, we analyze whether our results are robust to considering a different monetary policy reaction function in which the Central Bank's long-run inflation target reacts to shocks, as well as considering a larger scale DSGE model. Second, we evaluate the robustness of our results in VAR models: the structural New Keynesian model in Section 1 imposes strong parametric restrictions on the data, which might fail to hold; by imposing weaker identifying assumptions, Structural VAR inference might be less efficient but more robust to model mis-specification.

Our analysis shows that the main conclusions of Section 4 are strikingly robust in all these alternative cases. In particular, considering larger scale models, alternative monetary policy rules, or VARs does not change our conclusion that changes in monetary policy and in private sector's behavior did play a substantial role during the Great Moderation. Unreported results also show that our VAR results are robust to multiple breaks<sup>19</sup> and that

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<sup>18</sup>Similarly, lack of time variation does not imply that a parameter is structural: there are plenty of reduced form parameters for which we cannot reject stability, and yet it may well be that these parameters may change under a policy different from those enacted in the past.

<sup>19</sup>More in detail, we examined the possibility that all the parameters change smoothly over time, rather than abruptly, and allowed the parameters of the Structural VAR model to evolve according to a random walk, thus allowing them to have multiple breaks rather than a one-time break. In this framework, the ESS procedure is implemented as follows. Since we assume that the parameters evolve over time according to a random walk, we utilize the Nyblom (1989) stability test, which is designed for this purpose. Consistency

our main results in Section 4 are robust when using data up to 2007.<sup>20</sup>

## 5.1 Alternative Monetary Policy Specification

A first concern is that long run inflation might be time-varying, and that the monetary policy authority's inflation target might be changing over time as well (see Blinder, 1982, Ireland, 2007, and Cogley and Sbordone, 2008). We therefore allow the Central Bank's inflation target to adjust in response to other shocks hitting the economy, along the lines of Ireland (2007). This feature is particularly appealing for our purposes, as it allows for an additional source of time-variation in monetary policy and makes our conclusions robust to possible mis-specification of the monetary policy rule due to a time-varying long-run inflation target of the Central Bank. We will refer to this model as the "endogenous inflation target" model. In this model, the Central Bank can systematically adjust its long run inflation target ( $\pi_t^*$ ) to the cost push shock  $\varepsilon_{e,t}$  and the technology shock  $\varepsilon_{z,t}$ . The parameters influencing monetary policy include, in addition to the same parameters discussed in Section 2, also the long-term inflation target parameters, that is the reaction to technology and cost-push shocks,  $\delta_z, \delta_e$ , according to the following monetary policy reaction function:

$$\begin{aligned}\widehat{r}_t - \widehat{r}_{t-1} &= \rho_{\pi,t} \widehat{\pi}_t + \rho_{gy,t} \widehat{g}_t^y - \widehat{\pi}_t^* + \widehat{v}_t \\ \widehat{v}_t &= \rho_{v,t} \widehat{v}_{t-1} + \varepsilon_{v,t} \\ \widehat{\pi}_t^* &= \sigma_{\pi,t} \varepsilon_{\pi t} - \delta_{e,t} \varepsilon_{et} - \delta_{z,t} \varepsilon_{zt}\end{aligned}\tag{4}$$

of the Nyblom (1989) test for subsets of parameters has been established by Li and Muller (2008). The set of stable parameters in this case is:  $S = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{12}, \sigma_{22}, \sigma_{33}\}$ . The results are therefore very similar to those we obtained in the case of a one-time break. In fact, all the parameters in the conditional mean are stable. The instability is mainly in the volatilities of the monetary policy shock and in the degree of inflation aversion of the central bank. In addition, also the reaction of output to inflation seems unstable over time. Overall, therefore, our main results are robust to this alternative way of modeling time variation.

<sup>20</sup>We re-estimate the model in Section 2 with more recent data. One possibility would be to analyze data including the financial crisis of 2008-2009. However, due to revisions in the data and a new base year for GDP in 2007, the ability to construct fully updated series that match exactly those used in Section 4 is limited. Most of the data are similar to Ireland (2007) up to 2007:2, after which there was the change in the GDP base year. We thus decided to end the sample in 2007:2. We find that, even if we consider a larger sample, the largest break estimated in the sample continues to be the same as that identified in Section 4. Results show that, again, instabilities are widespread. Now the set of stable parameters includes only the Central Bank's inflation aversion.

where  $\varepsilon_{v,t}$  and  $\varepsilon_{\pi,t}$  are i.i.d. shocks with variance  $\sigma_{v,t}^2$  and  $\sigma_{\pi,t}^2$ , respectively. We thus re-estimate the model in Section 2 where eq. (3) is replaced by (4). The additional parameters to be estimated are:  $\sigma_{\pi,t}$  (the standard deviation of the shock to the inflation target),  $\delta_{e,t}$  (the reaction of the time-varying inflation target to the shock to the time varying elasticity of demand for each intermediate good), and  $\delta_{z,t}$  (the reaction of the time-varying inflation target to the temporary shock to aggregate technology).

Panel A in Table 5 reports results for stability tests on individual parameters as well as the ESS procedure. The table shows that, in the endogenous inflation target model, the set of stable parameters is  $\mathcal{S} = \{\gamma, \rho_v, \delta_e, \alpha\}$ . Quite interestingly, the table shows that instabilities do not affect the way the Central Bank reacts to the cost-push shock (as  $\delta_e$  belongs to the set of stable parameters), although they affect how the Central Bank reacts to productivity shocks (as  $\delta_z$  is unstable). That is, the way in which the Central Bank has responded to productivity shocks over the last few decades is time-varying. Panel B in Table 5 shows that while the technology shock now seems to explain most of the decrease in the volatility of output growth, the transitory monetary policy shock still plays a role, although much smaller than in the presence of an exogenous inflation target.

Tables 6 and 7 also confirm that our main counterfactual comparisons conclusions do carry over to the model with endogenous inflation targeting as well. In particular, Table 7 shows that the shock to the long run inflation target contributed to a small increase in the volatility of GDP growth. This may be related to the fact that a large part of the Great Moderation period is marked by a secular disinflationary trend in the US economy. A lengthy, gradual disinflation may appear in estimation as a series of negative inflation target shocks and thus it will seem like the inflation target shock variance has increased.

## 5.2 A Larger Scale DSGE Model

A second concern is whether the main conclusions of the paper carry over when considering a larger scale DSGE model. The goal of this section is to verify that these concern do not significantly affect our main conclusions. We consider a larger scale DSGE model following Smets and Wouters (2007) and estimate the model using exactly the same Bayesian methods that they use.<sup>21</sup> This model has attracted a lot of attention in the literature, and has become

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<sup>21</sup>Note that we utilize the Bayesian methods mainly as estimation devices, using the well-known correspondence between Bayesian and classical Maximum Likelihood estimates in large samples. Standard results ensure that the effect of the priors should disappear asymptotically, and Bayesian methods could be used within our procedure mainly as computation devices. See Bickel and Doksum (2007, Section 5.5) for exam-

widely used in Central Banks as well as academia (see, for instance, del Negro, Schorfheide, Smets and Wouters, 2007). The model includes a number of real and nominal frictions, and has a variety of shocks that can be given a structural interpretation. Table 8 briefly describes the parameters of the model; for brevity, we refer to Smets and Wouters (2007) and a Not-for-Publication Appendix to this paper for a detailed description of the model, the priors and the data. The main difference between the model that we consider and that in Smets and Wouters (2007) is that we allow all the parameters to be possibly time-varying.

Table 9 reports p-values of t-tests for structural breaks on individual parameters and p-values of the ESS procedure, as well as the estimated set of stable parameters,  $\mathcal{S}$ . It is clear that, again, the unstable parameters include not only the volatilities and degree of persistence of most shocks, but also some parameters in the monetary policy reaction function, as well as many other structural parameters. In particular, we find instabilities in the volatilities of the monetary policy, productivity, price markup, exogenous spending, investment specific technology, and interest rate wedge shocks ( $\sigma_r, \sigma_a, \sigma_p, \sigma_g, \sigma_I, \sigma_b$ ); the volatility of the mark-up shock ( $\sigma_w$ ) instead is constant over time. The persistence of these shocks ( $\rho_g, \rho_a, \rho_{ga}, \rho_b, \rho_w, \mu_w$ ) has also significantly changed over time. However, this is not the whole picture, as we also find that some monetary policy parameters as well as other structural parameters have changed. In particular, the elasticity of the capital utilization adjustment cost function ( $\psi$ ), the degree of indexation to past inflation ( $\iota_p$ ), the degree of price stickiness ( $\xi_p$ ), the steady state growth rate ( $\bar{\gamma}$ ) and elasticity of the capital adjustment cost function ( $\varphi$ ) also have changed. More importantly, especially both the Central Bank's degree of smoothing ( $\rho$ ) and the output gap reaction parameter ( $r_y$ ) have significantly changed.

We next turn to the counterfactual analyses in Table 10(a,b). Panel A in Table 10(a) shows that counterfactual analyses in which the shock processes are kept constant (labeled "Shocks<sub>C</sub>") are substantially at odds with the actual data (left panel, labeled "Estimates"); instead, the counterfactuals with constant policy parameters are not (labeled "Policy<sub>C</sub>"). As in Section 4, counterfactuals where the unstable parameters identified by the ESS procedure are kept constant is even more at odds with the data than the counterfactual with constant shocks (see the row labeled "Marg. Likel."). Panel B considers the fit of the various scenarios rather than their counterfactuals. It shows that, while allowing only the shocks to change seems to replicate perfectly the volatility of output, nevertheless the likelihood worsens substantially from  $-375$  to  $-542$ . By allowing only the ESS unstable parameters to change,

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ple. In finite samples, however, the choice of prior distributions is important and can create discrepancies between the Bayesian estimation results and maximum likelihood estimation results.

the likelihood decreases only slightly, from  $-375$  to  $-419$ , while still replicating most of the decrease in output volatility.

Table 10(b) sheds light on why the counterfactual analyses and our ESS procedure reach different conclusions. The table reports the marginal percentage increase (decrease, if negative) in the standard deviation of output, inflation and the interest rate calculated by progressively letting all the parameters change from their second sub-sample to their first sub-sample values, according to the ordering of the ESS procedure.<sup>22</sup> Note that the unstable parameters that ESS identifies beyond the shock processes  $(\rho, \xi_p, \bar{\gamma}, \psi, \rho, \iota_p, r_y)$  jointly only account for a negligible increase in output volatility (about 8%). This explains why both the ESS and shock counterfactuals appear to explain the change in output volatility equally well. However, while the overall effect of such additional parameter changes on output volatility is negligible, each of them individually accounts for substantial changes (for example,  $\varphi$  explains 19%,  $\psi$  explains 11%, whereas  $\rho$  and  $r_y$  explain  $-16\%$  and  $-11\%$  respectively). It follows that it is incorrect to conclude that only the shock processes have changed, and to attribute the Great Moderation solely to "good luck" as the literature has done. For example, by comparing our results with Smets and Wouters (2007, section V.D), we note that the consequence of an increase in the volatilities of productivity and price markup shocks (which they highlight as the major source of time variation) is only 8% and 4% respectively. On the other hand, the consequences of a change in  $\varphi$  and  $\psi$  (which were deemed unimportant by Smets and Wouters) are 19% and 11% respectively. The contribution of the latter parameters to the overall increase in output volatility are obscured by the opposite contribution of changes in the monetary policy parameters  $\rho$  and  $r_y$ , which decrease output volatility in the order of  $-16\%$  and  $-11\%$ .<sup>23</sup>

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<sup>22</sup>For example, if one estimates the model imposing all the parameter values to be equal to their first sub-sample values, except for allowing  $\sigma_r$  to change, one would observe a 27% increase in the standard deviation of output. Similarly, if one allowed both  $\sigma_r$  and  $\sigma_a$  to change, one would explain an additional 8% of the increase in the standard deviation of output, and together they would explain about 35% of the increase in the volatility.

<sup>23</sup>To some extent, our findings reconcile the Smets and Wouters (2007) results with those of Boivin and Giannoni (2006). Boivin and Giannoni (2006) find that, by qualitatively comparing counterfactual impulse response functions in a Structural VAR, changes in monetary policy and private sector's behavior can account for some of the Great Moderation. Our results point in the same direction. A difference between our results and Boivin and Giannoni (2006) is that, similarly to Smets and Wouters (2007), we do not find instabilities in the Central Bank's inflation aversion. Section 5.3 will show that a Structural VAR does find instabilities in the Central Bank's inflation aversion. This suggests that the differences arise from the different identification of monetary policy in the structural DSGE model versus a Structural VAR.

### 5.3 VAR Analysis

We consider a parsimonious VAR with structural breaks in both coefficients and volatilities. Similar VARs have been estimated by Stock and Watson (2001), Primiceri (2005), Cogley and Sargent (2005), and Boivin and Giannoni (2006), among others, to investigate the sources of the Great Moderation. We use our procedure to investigate which parameters are time varying – the conditional mean parameters or the volatilities – and which equations of the model contain most of the instability. We consider two VARs: a reduced form VAR with GDP growth, inflation, and the nominal interest rate; and a Structural VAR (SVAR), where the shocks are identified according to a recursive ordering scheme. In both cases, we estimate the subset of stable parameters and discuss the economic implications of our results.

We consider a VAR in three observables for two reasons. First, it is a parsimonious and widely used benchmark. Second, the observables are consistent with the smaller scale DSGE model considered previously. As a starting point, we consider the reduced-form VAR, where  $y_t$  is per-capita GDP growth in real terms,  $\pi_t$  is the GDP price deflator, and  $r_t$  is the three month U.S. Treasury bill interest rate:<sup>24</sup>

$$\begin{pmatrix} r_t \\ y_t \\ \pi_t \end{pmatrix} = \underbrace{\begin{pmatrix} k_{11} \\ k_{22} \\ k_{33} \end{pmatrix}}_K + \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_A \begin{pmatrix} r_{t-1} \\ y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} \quad (5)$$

$$\text{where } V(u_t) = \Omega = \begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{12} & \omega_{22} & \omega_{23} \\ \omega_{13} & \omega_{23} & \omega_{33} \end{pmatrix}.$$

The lag length is chosen according to the BIC (implemented with a maximum lag length of four lags) and is equal to one. We let both sets of parameters, that is those in the conditional mean ( $K, A$ ) and those in the covariance matrix ( $\Omega$ ), to possibly change over time according to a one-time structural break at an unknown date. We use the QLR test for structural breaks in our ESS procedure. The estimated break date is 1985:2 in the reduced form VAR, and we use that date in our analysis.

Table 11, Panel A, shows the results. The joint hypothesis of stability in all the parameters is strongly rejected. Interestingly, the stability in the conditional mean parameters ( $k$ 's and  $a$ 's) is not rejected at the 10% significance level, whereas the stability of the variance

<sup>24</sup>For comparability, the time series of the variables are the same as in the previous section. That is, they are calculated in deviations from their steady state levels.

parameters ( $\omega \in \Omega$ ) is rejected. However, we cannot really rely on such tests, as they repeatedly test hypotheses without taking into account the "recursive" nature of the procedure. Furthermore, such tests do not identify which parameters are time-varying. We therefore apply our ESS procedure. We find that the biggest evidence of parameter instability comes from the variance of all three reduced form shocks. In fact, the only parameters that do not belong to the set of stable parameters are  $\omega_{11}, \omega_{22}, \omega_{33}$ . All other parameters in the conditional mean of the VAR ( $k$ 's and  $a$ 's) appear to be stable, as well as the contemporaneous covariances between the endogenous variables ( $\omega_{12}, \omega_{13}, \omega_{23}$ ). Note, however, that the correlations among the macroeconomic variables are unstable.

To overcome the problem that the covariances of the reduced form shocks do not separately identify the transmission mechanism from the variance of the structural shocks, we next consider a Structural VAR. We identify the structural shocks according to the recursive VAR identification used, among others, by Stock and Watson (2002), Primiceri (2005) and Boivin and Giannoni (2006):

$$\underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} = \underbrace{\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & 0 & \sigma_{33} \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} \eta_{r,t} \\ \eta_{y,t} \\ \eta_{\pi,t} \end{pmatrix}}_{\eta_t} \quad (6)$$

where  $\eta_t \sim iid(0, I)$  represents the structural shocks and  $I$  is the identity matrix. We will refer to the equations in (6) as follows: the first equation represents the monetary policy rule, the second is the output equation and the third is the inflation equation.

Panel B in Table 11 shows the empirical results, which are striking. The set of stable parameters does *not* contain  $\{\sigma_{11}, \sigma_{12}, \sigma_{13}\}$ , that is the parameters of the monetary policy reaction function, as well as  $\sigma_{22}$ , the variance of the shock to the output equation. Therefore, unlike the reduced form VAR case, the Structural VAR analysis too uncovers the very interesting result that time variation appears to affect both the transmission mechanism and the impulse. This evidence seems to suggest, again, that changes in monetary policy might be linked to the Great Moderation, as well as an exogenous reduction in the variance of the output equation. The relationship that appears to be the most stable over time is that of inflation, which is the only equation for which all the parameters ( $k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{33}$ ) belong to the set of stable parameters.<sup>25</sup>

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<sup>25</sup>Note that, since for example  $\omega_{12} = \sigma_{12}\sigma_{22} + \sigma_{13}\sigma_{23}$ , one might expect instabilities in  $\sigma_{12}, \sigma_{22}, \sigma_{13}, \sigma_{23}$  to generate instabilities in  $\omega_{12}$ . However, such instabilities can cancel out among themselves (positive changes

We proceed to quantitatively evaluate the role of instabilities in each of the parameters in contributing to the Great Moderation. We estimate the overall decline in the standard deviation of GDP growth that one would have observed by progressively allowing each of the parameters excluded from the set of stable parameters to be time-varying in the Structural VAR. The order in which we allow the parameters to become time-varying is determined by ESS (see Algorithm 1). Table 12 shows the results. The column labeled “Unstable parameters” reports the parameters that are progressively allowed to be time-varying; in each row, all the parameters listed in the previous rows as well as the parameter indicated in that row are allowed to be time-varying.<sup>26</sup> Allowing for time-variation in the parameters that do not belong to the estimated set of stable parameters explains about 40% of the reduction in GDP volatility. Most of the reduction is attributed to a decrease in the variance of output shocks (14%), a decline in the variability of the monetary policy shocks (19%), as well as a change in the monetary policy reaction to inflation (5%). Therefore, again, the VAR analysis corroborates our previous result that changes in monetary policy played an important role during the Great Moderation, due to monetary policy becoming both less averse to inflation and less volatile. The former is a change in the way the Fed systematically affects other macroeconomic variables, the latter is a change in the way the Fed directly induces variability in the economy. These two effects are arguably very different, as, according to Stock and Watson (2002), the former reflects “propagation”, whereas the latter reflects “impulse”. However, both reflect a change in monetary policy, and neither may reflect “good luck”. Next, we allow other parameters to be time-varying even if, according to our procedure, the instability is not statistically significant. The column labeled “Other parameters” denotes other parameters that are progressively allowed to be time-varying beyond those that belong to the estimated set of unstable parameters. We find that the parameters that contribute the most to the reduction are: the persistence of GDP growth ( $a_{22}$ ), the volatility of the inflation shock ( $\sigma_{33}$ ), the reaction of output to interest rates ( $a_{21}$ ), and the average inflation rate ( $k_{33}$ ). Note that changes in some of the parameters contribute to a substantial *increase* in the variance of GDP growth, such as  $\sigma_{23}$  and  $a_{13}$ .<sup>27</sup>

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offsetting negative changes) and "big" instabilities might be multiplied with "small" instabilities to generate an overall "small" instability. Nothing guarantees that statistically significant breaks in  $\sigma$ 's should result in statistically significant breaks in  $\omega$ 's.

<sup>26</sup>For example, in the row corresponding to  $\sigma_{13}$ , all the parameters belonging to the unstable set (that is,  $\sigma_{22}, \sigma_{11}, \sigma_{12}, \sigma_{13}$ ) are time-varying.

<sup>27</sup>One might wonder why the VAR and the New Keynesian model sometimes find different results. It is possible that the New Keynesian model is mis-specified and the VAR is not, which would explain the

## 6 Conclusions

This paper investigates which, among the many parameters that characterize a representative DSGE model, are stable at the time of the Great Moderation. Our findings challenge conventional wisdom (Stock and Watson, 2001, 2003, Justiniano and Primiceri, 2008, Sims and Zha, 2006) by showing that the Great Moderation was not only due to changes in the volatilities of the shocks, but also to changes in monetary policy as well as private sector's parameters.

We demonstrate that the difference between our results and the previous literature is due to the fact that the neglected sources of instabilities have offsetting effects on output volatility, and therefore were impossible to detect by using standard counterfactual analyses. Thus, counterfactuals where only the shocks volatilities change appear to fit the data well even though they are not the only parameters that changed in reality. In fact, there is no reason why we should think that the changes in policy parameters offset each other, as one could as well think that changes in some policy parameters offset changes in shock variances and that the bulk of the Great Moderation is explained by the changes in the remaining policy or private sector's parameters. In addition, we show that standard structural break tests or impulse response comparisons are subject to similar shortcomings.

It appears that the volatility of macroeconomic aggregate increased again after the 2008-2009 crisis, thus marking the end of the Great Moderation. However, understanding the historical data (and the Great Moderation in particular) is important to understand the transmission mechanisms driving the economy, and may carry valuable lessons for the future. We conjecture that the methods proposed in this paper might turn out to be useful for investigating the sources of instability in the latest crisis as well, once sufficient data become available. We believe that our techniques and results shed light on the economic causes of such instabilities, and offer important insights to guide the future theoretical development of macroeconomic models.

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difference. However, even if the model is correctly specified and the SVAR's identifying assumptions are consistent with the model, it still might be possible that the two may point to different sources of instability (see Benati and Surico, 2007).

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## 7 Appendix - Detailed Description of the Methodology

This section presents details on our econometric procedure to estimate the set of stable parameters. We will show that our testing procedure controls size, is consistent and provides an estimate of the set of stable parameters that contains the true set with a pre-specified probability level.

Consider a general parametric model with parameters  $\theta_t \in \Theta \subseteq \mathbb{R}^p$  for  $t = 1, 2, \dots$ , where some (or all) of the elements are time-varying. Let  $s \in \{0, 1\}^p$  denote a parameter selection vector and  $\theta(s)$  denotes the subset of  $\theta$  selected by the selection vector  $s$ , where  $s(i)$  denotes the  $i$ -th element of such vector.<sup>28</sup> We also let  $s^*$  denote the population selection vector that selects only the constant parameters, so that  $\theta(s^*)$  denote the vector of constant parameters. Note that it is possible that  $s^*$  is the vector of ones, in which case all parameters are stable, or  $s^*$  is the vector of zeros, in which case all the parameters are time-varying. The problem considered in this paper is that it is not known which parameters are time-varying and which are stable. In other words,  $s^*$  is unknown. We will propose a sequential procedure that uses sample information to estimate  $s^*$  by an estimator  $\hat{s} \in \{0, 1\}^p$ . With our procedure, the estimator  $\hat{s}$  will be equal to  $s^*$  with a pre-specified probability level.

Let the observed sample be  $W = \{W_t : 1 \leq t \leq T\}$ , and let  $e_i$  be the  $p \times 1$  vector whose  $i$ -th element is one and the other elements are zero,  $\mathbf{1}_{p \times 1}$  be the  $p \times 1$  vector of ones,  $\mathbf{0}_{p \times 1}$  be the  $p \times 1$  vector of zeros, and  $I_n$  denote the  $(n \times n)$  identity matrix. Also, let  $\mathcal{T}_T(W; s)$  be a size- $\alpha$ , consistent test statistic for testing the null hypothesis that the subset of the parameters,  $\theta(s)$ , are constant over time ( $H_0(s) : \theta_t(s) = \theta(s)$ ) against the alternative that they are time-varying ( $H_A(s) : \theta_t(s) \neq \theta(s)$ ), allowing for the rest of the parameters,  $\theta(\mathbf{1}_{p \times 1} - s)$ , to be time-varying.<sup>29</sup> For notational simplicity, we will omit the dependence of the test statistic on the observed sample, and use  $\mathcal{T}_T(s)$ . For example,  $\mathcal{T}_T(e_i)$  will denote the individual stability test on the  $i$ -th parameter  $\theta(e_i)$ , and  $\mathcal{T}_T(\mathbf{1}_{p \times 1})$  will denote the joint stability test on all the parameters in  $\theta$ . Finally, let  $k_\alpha(s)$  denote the size- $\alpha$  critical value of  $\mathcal{T}_T(s)$ , and  $pv(s)$  denotes its p-value. For example, in the case of a one-time structural break at an unknown time, when using Andrews' (1993) QLR test statistic,  $k_\alpha(s)$  is the critical value of such test at level  $\alpha$  for testing a number of restrictions equal to the number of non-zero elements in  $s$ .

<sup>28</sup>For example, when  $p = 3$ ,  $s_0 = (0, 1, 0)$  indicates that the second element of  $\theta$  is not time-varying and the first and third elements are time-varying in population.

<sup>29</sup>For example, in the case of a one-time structural break at an unknown fraction of the sample size  $[\pi T]$ :  $H_A(s) : \theta_t(s) = \theta_1(s) \cdot 1(t \leq \pi T) + \theta_2(s) \cdot 1(t > \pi T)$ , where  $\pi \in \Pi \subset (0, 1)$ .

In what follows, we will discuss a procedure to construct an estimate of the set of stable parameters that equals the true set of stable parameters with a pre-specified probability level. We call this procedure the “Estimate of the Set of Stable parameters” procedure (“ESS”). We formalize the ESS procedure in the following Algorithm:

**Algorithm 1 (The ESS procedure)** *Step 0.* Initially, let  $s_0 = \mathbf{1}_{p \times 1}$ . Test  $H_0^{(0)}(s_0)$  against  $H_A^{(0)}(s_0)$  at significance level  $\alpha$  by using the test  $\mathcal{T}_T(s_0)$ . If the test does not reject, let  $\widehat{s}_{ESS} = s_0$ . If the test rejects, calculate the vector of test statistics  $\mathcal{T}_T(e_i)$  for  $i = 1, \dots, p$ , and order them such that their estimated values are increasing:  $pv(e_1) \leq pv(e_2) \leq \dots \leq pv(e_p)$ . Without loss of generality, let  $e_1$  identify the parameter with the smallest p-value.<sup>30</sup> Continue to step 1.

*Step 1.* Without loss of generality, let  $s_1 = [0, \mathbf{1}_{1 \times (p-1)}]'$ . Test  $H_0^{(1)}(s_1)$  against  $H_A^{(1)}(s_1)$  at significance level  $\alpha$  by using  $\mathcal{T}_T(s_1)$ . If the test does not reject, let  $\widehat{s}_{ESS} = s_1$ . If the test rejects, let  $e_2$  identify the parameter with the second smallest p-value, and continue to step 2. (..)

*Step j.* Without loss of generality, let  $s_j = [\mathbf{0}_{1 \times j}, \mathbf{1}_{1 \times (p-j)}]'$ . Test  $H_0^{(j)}(s_j)$  against  $H_A^{(j)}(s_j)$  at significance level  $\alpha$  by using  $\mathcal{T}_T(s_j)$ . If the test does not reject, let  $\widehat{s}_{ESS} = s_j$ . If the test rejects, let  $e_j$  identify the parameter with the j-th smallest p-value, and continue to step (j+1). (..)

*Step (p-1).* Without loss of generality, let  $s_{p-1} = [\mathbf{0}_{1 \times (p-1)}, 1]'$ . Test  $H_0^{(p-1)}(s_{p-1})$  against  $H_A^{(p-1)}(s_{p-1})$  at significance level  $\alpha$  by using  $\mathcal{T}_T(s_{p-1})$ . If the test does not reject, let  $\widehat{s}_{ESS} = s_{p-1}$ . If the test rejects, let  $\widehat{s}_{ESS} = \mathbf{0}_{p \times 1}$ .

We now prove that Algorithm 1 provides an estimate of the set of stable parameters that equals the true set with probability  $(1 - \alpha)$ . More precisely,  $\widehat{s}_{ESS}$  defined in Algorithm 1 equals the true set of stable parameters with probability  $(1 - \alpha)$ . The Not-for-Publication Appendix shows that  $\widehat{s}_{ESS}$  can be viewed as a confidence set for the set of stable parameters with coverage probability  $(1 - \alpha)$ . It is different from conventional confidence sets because the parameter of interest is discrete and the confidence set is a singleton. In addition, the Not-for-publication Appendix also shows that there are size distortions in existing tests for structural breaks when used repeatedly to test structural change in more than one subsets of parameters.

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<sup>30</sup>Actually, one does not even need to compute p-values: one may simply pick the estimated test statistic which is the largest. Because the degrees of freedom for testing each parameter individually are the same, and therefore the critical values are, the largest test statistic has the smallest p-value.

*Assumption 1.* For any two vectors  $a$  and  $b$ , let  $\max(a, b)$  denote the vector whose  $i$ -th element is the maximum of  $a(i)$  and  $b(i)$ , where  $a(i)$  denotes the  $i$ -th element of  $a$ . For all  $s^* \in \{0, 1\}^p$  such that  $s^* \neq 0_{p \times 1}$ ,  $\mathcal{T}_T(s) \xrightarrow{d} D(s)$  if  $s = s^*$  and  $\mathcal{T}_T(s) \rightarrow \infty$  a.s. if  $|\max(s, s^*)| > |s^*|$ , where  $|s|$  denotes the number of components in  $s$  that are different from zero.<sup>31</sup>

**Proposition 2 (Properties of the ESS procedure)** *Let Assumption 1 hold, and let  $\hat{s}$  be estimated as described by Algorithm 1. Then:*

$$\lim_{T \rightarrow \infty} \Pr \{ \hat{s}_{ESS} = s^* \} = 1 - \alpha \quad (7)$$

for any  $s^* \in \{0, 1\}^p$  such that  $s^* \neq 0_{p \times 1}$ ,

$$\lim_{T \rightarrow \infty} \Pr \{ \hat{s}_{ESS} = s^* \} = 1 \quad (8)$$

for  $s^* = 0_{p \times 1}$ , and

$$\lim_{T \rightarrow \infty} \Pr \{ \hat{s}_{ESS} \neq s^* \text{ and } \hat{s}_{ESS} \geq s^* \} = 0. \quad (9)$$

for any  $s^* \in \{0, 1\}^p$ .

**Proof of Proposition (2).** Let  $k_\alpha(s)$  denote the critical value of  $\mathcal{T}_T(s)$  of the null distribution  $D(s)$  at the level of significance,  $\alpha$ . Let  $p^* = p - |s^*|$  be the number of unstable parameters. Because the test is consistent by assumption,

$$\lim_{T \rightarrow \infty} P(T(s_j) > k_\alpha(s_j)) = 1, \quad (10)$$

for any  $s_j$  such that  $|\max(s_j, s^*)| > |s^*|$  for  $j = 1, 2, \dots, p - p^* - 1$ . For the same reason,  $\lim_{T \rightarrow \infty} P(T(e_i) > k_\alpha(e_j)) = 1$  if the  $i$ th parameter is not stable. Thus, if  $p - p^* > 0$ ,

$$\lim_{T \rightarrow \infty} P(I_j \in \{i : s^*(i) = 0\}) = 1, \quad (11)$$

for  $j = 1, \dots, p - p^*$ , where  $I_j = \{i : \mathcal{T}_T(e_i) \text{ is among the } j \text{ largest among } \{\mathcal{T}_T(e_\ell)\}_{\ell=1}^p\}$ . In particular (11) implies that

$$\lim_{T \rightarrow \infty} P(I_{p-p^*} = \{i : s^*(i) = 0\}) = 1. \quad (12)$$

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<sup>31</sup>Assumption 1 requires that  $\mathcal{T}_T(s)$  has a well-defined asymptotic distribution under the null hypothesis, and diverges to positive infinity when testing a subset of parameters which includes at least one unstable parameter (under the alternative hypothesis of parameter instability). This assumption is satisfied by many tests for structural breaks including Andrews' (1993) QLR test, and Andrews and Ploberger's (1994) Exp-W and Mean-W tests.

We will consider three cases: (i)  $p^* = 0$ , (ii)  $p^* > 0$  and  $p > p^*$  and (iii)  $p^* > 0$  and  $p = p^*$ .

First, suppose that  $p^* = 0$ . Then  $\lim_{T \rightarrow \infty} Pr(\mathcal{T}_T(s_0) < k_\alpha(s_0)) = \lim_{T \rightarrow \infty} Pr(\mathcal{T}_T(s^*) < k_\alpha(s^*)) = 1 - \alpha$ , thus proving (7) under  $H_0^{(0)}(s_0)$ . When  $p^* = 0$ , (8) does not apply and (9) trivially holds.

Next, suppose that  $p^* > 0$  and  $p - p^* > 0$ . Because we eliminate only one stable parameter in each step, we have  $|\max(s_j, s^*)| > |s^*| = p - p^*$  for any  $s_j$  and  $j = 0, 1, 2, \dots, p^* - 1$ . Thus it follows from (10) that

$$\lim_{T \rightarrow \infty} P(\mathcal{T}_T(s_j) > k_\alpha \quad \forall \quad j = 0, 1, \dots, p - p^* - 1) = 1. \quad (13)$$

By (12) and (13), we have

$$\lim_{T \rightarrow \infty} P(s_{p-p^*} = s^*) = 1 \quad (14)$$

Now

$$\begin{aligned} & \lim_{T \rightarrow \infty} P(\hat{s}_{ESS} = s^*) \\ &= \lim_{T \rightarrow \infty} P(\mathcal{T}_T(\hat{s}_{ESS}) < k_\alpha(\hat{s}_{ESS}) | \text{The test rejects the null at steps } 0, 1, \dots, p - p^* - 1) \\ &= \lim_{T \rightarrow \infty} P(\mathcal{T}_T(s^*) < k_\alpha(s^*) | \text{The test rejects the null at steps } 0, 1, \dots, p - p^* - 1 \text{ and } s_{p-p^*} = s^*) \\ &= \lim_{T \rightarrow \infty} P(\mathcal{T}_T(s^*) < k_\alpha(s^*)) \\ &= 1 - \alpha \end{aligned} \quad (15)$$

where the third equality follows from (13) and (14) and the last from Assumption 1. (7) holds. (9) follows from (10).

Lastly, suppose that  $p = p^*$ . Then the null hypotheses in steps  $0, 1, 2, \dots, p^*$  are all rejected by (10) and each of the  $p^*$  unstable parameters is selected in these steps with probability approaching one by (12). Therefore  $\hat{s}_{ESS}$  converges almost surely to  $s^*$ , and (8) and (9) hold.

■

## 8 Tables and Figures

**Table 1. Empirical Results for the Representative New Keynesian Model**

Model	Individual	ESS
Parameters:	p-value	p-value
$\rho_e$	0	0
$\sigma_v$	0	0
$\alpha$	0	0
$\sigma_a$	0	0
$\sigma_\pi$	0	0
$\rho_a$	0	0
$\gamma$	0	0
$\psi$	0	0
$\rho_{gy}$	0	0
$\sigma_e$	0	0
$\rho_v$	0	0
$\rho_\pi$	0	0
$\sigma_z$	0.76	0.76

Set of Stable Parameters

(90% probability level):

$$\mathcal{S} = \{\sigma_z\}$$

Note. The table reports p-values of the QLR stability test on individual parameters (labeled "Individual p-value"). It also reports both the set of stable parameters obtained by the ESS procedure as well as the p-values at each step of our procedure (labeled "ESS p-value"). The tests are implemented in a Wald form, using standard errors obtained by bootstrap with 1,000 replications.

**Table 2. Counterfactual Comparisons for  
the Representative New Keynesian Model**

PANEL A. Standard Counterfactual Comparisons					
	Estimates		Counterfactual for 1980:4-2004:4		
	1959:1-1980:3	1980:4-2004:4	Shocks <sub>C</sub>	Policy <sub>C</sub>	ESS <sub>C</sub>
S.D. Output	1.44	0.88	1.58	0.73	1.24
S.D. Inflation	0.91	0.47	0.85	0.44	0.78
S.D. Interest Rate	0.38	0.29	0.37	0.31	0.37
Likelihood		1,261	1,218	1,247	1,229
PANEL B. Moments and Likelihood Comparisons					
	Estimates		Fit in 1980:4-2004:4: selected constant param.		
	- -	1980:4-2004:4	Shocks	Policy	ESS
S.D. Output		0.88	0.79	1.76	1.25
S.D. Inflation		0.47	0.49	0.95	0.67
S.D. Interest Rate		0.29	0.33	0.36	0.30
Likelihood		1,261	1,241	1,208	1,258

Note. Panel A reports model implied standard deviations (S.D.) and likelihood in observable variables (Output, Inflation and the Interest Rate) in sub-samples estimated from the model ("Estimates") and model implied counterfactual standard deviations and likelihood on the second sub-sample ("Counterfactual for 1984:1-2004:4"). For example, the column labeled "Shocks<sub>C</sub>" reports the standard deviation of selected macroeconomic variables when shocks volatilities are kept constant but all other parameters are allowed to change. Panel B shows the standard deviations (S.D.) of observables as well as the same moments estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure. For example, the column labeled "Shocks" reports standard deviations for the selected macroeconomic variables when shocks processes are allowed to change but all other parameters are kept constant. Changes in monetary policy include changes in  $\rho_\pi$  and  $\rho_{gy}$ . The table also reports the likelihood estimated using only the parameter values in the relevant sub-sample ("Estimates") or the likelihood estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure.

**Table 3. Counterfactuals for the  
Representative New Keynesian model**

Parameter:	Output	Inflation	Interest Rate
No change: (actual s.d.)	0.88	0.47	0.29
Unstable Parameters:	% Contribution to Change		
$\rho_e$	7%	10%	0%
$\sigma_v$	62%	33%	83%
$\alpha$	-3%	16%	7%
$\sigma_a$	-15%	-3%	-84%
$\sigma_\pi$	3%	12%	26%
$\rho_a$	17%	2%	76%
$\gamma$	32%	0%	-9%
$\psi$	0%	0%	0%
$\rho_{gy}$	-43%	2%	17%
$\sigma_e$	-2%	-5%	0%
$\rho_v$	5%	4%	-8%
$\rho_\pi$	-13%	-23%	5%
Stable Parameters:			
$\sigma_z$	49%	52%	3%
All change: (actual s.d.)	1.44	0.91	0.38

Set of Stable Parameters (90% probability level):

$$\mathcal{S} = \{\sigma_z\}$$

Note. The table shows the percentage contribution to the increase or decrease in the volatilities of output, inflation and the interest rate when we progressively allow each of the parameters to be time varying (ordered according to the p-values of the individual t-tests for parameter instabilities).

**Table 4. Standard Deviations of Macroeconomic Shocks  
for the New Keynesian model**

	1959:1-1980:3	1980:4-2004:1	$s_2/s_1$	Relative Contribution to GDP Variance Reduction
Transitory Monetary Target*	0.060	0.060	1.000	0
Technology Shock*	1.714	1.126	0.657	0.060
Preference Shock	146.9	4.435	0.030	0.065
Cost-Push Shock	0.168	0.141	0.837	0.197
Persistent Monetary Shock	0.393	0.243	0.617	0.783

Note. The standard deviations are multiplied by 100. For the processes with asterisk, the variance of the disturbance term, not the variance of the process, is reported because they are unit-root processes. To be consistent with Stock and Watson (2002), the break date is 1980:Q4.

**Table 5. Empirical Results for the Representative New Keynesian Model with Endogenous Inflation Target**

Model Parameters:	Panel A. Stability Analysis		Panel B. Counterfactual Analysis			
	Individual p-value	ESS p-value	Parameter: No change:	Output	Inflation	Interest Rate
				1.08	0.40	0.28
			Unstable Parameters:			
$\psi$	0	0	$\psi$	0%	0%	0%
$\delta_z$	0	0	$\delta_z$	-18%	12%	-30%
$\sigma_a$	0	0	$\sigma_a$	-14%	-1%	-399%
$\sigma_\pi$	0	0	$\sigma_\pi$	-2%	-3%	-109%
$\rho_a$	0	0	$\rho_a$	23%	1%	533%
$\rho_\pi$	0	0	$\rho_\pi$	0%	14%	-44%
$\rho_e$	0	0	$\rho_e$	2%	1%	0%
$\sigma_v$	0	0	$\sigma_v$	40%	8%	121%
$\sigma_e$	0	0	$\sigma_e$	3%	2%	4%
$\rho_{gy}$	0	0	$\rho_{gy}$	-24%	4%	84%
$\sigma_z$	0	0	$\sigma_z$	94%	62%	-39%
			Stable Parameters:			
$\gamma$	0.22	0.43	$\gamma$	-4%	0%	0%
$\rho_v$	0.25	0.67	$\rho_v$	1%	0%	-19%
$\delta_e$	1	1	$\delta_e$	0%	0%	0%
$\alpha$	1	1	$\alpha$	0%	0%	0%
	Set of Stable Parameters (90% probability level):		All change:	1.73	1.60	0.30
	$\mathcal{S} = \{\gamma, \rho_v, \delta_e, \alpha\}$					

Note. Panel A reports p-values of the QLR stability test on individual parameters. It also reports both the set of stable parameters obtained by the ESS procedure as well as the p-values at each step of our procedure. The tests are implemented in a Wald form, using standard errors obtained by bootstrap with 1,000 replications. Panel B reports the percentage contribution to the increase or decrease in the volatilities of output, inflation and the interest rate when we progressively allow each of the parameters to be time varying (ordered according to the p-values of the individual t-tests for parameter instabilities).

**Table 6. Counterfactual Comparisons for the  
Representative New Keynesian Model with Endogenous Inflation Target**

PANEL A. Standard Counterfactual Comparisons					
	Estimates		Counterfactual for 1980:4-2004:4		
	1959:1-1980:3	1980:4-2004:4	Shocks <sub>C</sub>	Policy <sub>C</sub>	ESS <sub>C</sub>
S.D. Output	1.73	1.08	2.72	0.83	1.75
S.D. Inflation	1.60	0.40	0.78	0.78	1.60
S.D. Interest Rate	0.30	0.28	0.38	0.30	0.31
Likelihood		1,257	1,119	1,215	1,227
PANEL B. Moments and Likelihood Comparisons					
	Estimates		Fit in 1980:4-2004:4: selected constant param.		
	- -	1980:4-2004:4	Shocks	Policy	ESS
S.D. Output	1.73	1.08	0.81	2.65	1.05
S.D. Inflation	1.60	0.40	0.78	0.78	0.40
S.D. Interest Rate	0.30	0.28	0.30	0.38	0.28
Likelihood		1,257	1,246	1,109	1,257

Note. Panel A reports model implied standard deviations (S.D.) and likelihood in observable variables (Output, Inflation and the Interest Rate) in sub-samples estimated from the model ("Estimates") and model implied counterfactual standard deviations and likelihood on the second sub-sample ("Counterfactual for 1980:4-2004:4"). For example, the column labeled "Shocks<sub>C</sub>" reports the standard deviation of selected macroeconomic variables when shocks volatilities are kept constant but all other parameters are allowed to change. Panel B shows the standard deviations (S.D.) of observables as well as the same moments estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure. For example, the column labeled "Shocks" reports standard deviations for the selected macroeconomic variables when shocks processes are allowed to change but all other parameters are kept constant. Changes in monetary policy include changes in  $\rho_\pi$ ,  $\rho_{gy}$ ,  $\delta_e$ ,  $\delta_z$ . The table also reports the likelihood estimated using only the parameter values in the relevant sub-sample ("Estimates") or the likelihood estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure.

**Table 7. Standard Deviations of Macroeconomic Shocks  
for the New Keynesian Model with Endogenous Inflation Target**

	1959:1-1980:3	1980:4-2004:1	$s_2/s_1$	Relative Contribution to GDP Variance Reduction
Inflation Target*	0.130	0.086	0.659	-0.179
Technology Shock*	1.562	1.103	0.706	0.720
Preference Shock	5.118	14.123	2.759	0.117
Cost-Push Shock	0.084	0.084	0.999	-0.000
Monetary Shock	0.269	0.244	0.906	0.041

Note. The standard deviations are multiplied by 100. For the processes with asterisk (\*), the variance of the disturbance term, not the variance of the process, is reported because they are unit-root processes. To be consistent with Stock and Watson (2002), the break date is 1980:Q4.

**Table 8. Description of Parameters  
in the Smets and Wouters' (2007) Model**

Structural Parameters	Description
$\varphi$	Elasticity of capital adjustment cost function
$\sigma_c$	Intertemporal elasticity of substitution
$\lambda$	Habit parameter
$\xi_w$	Degree of wage stickiness (Calvo param.)
$\sigma_\ell$	Elasticity of labor supply
$\xi_p$	Degree of price stickiness (Calvo param.)
$\iota_w$	Degree of indexation to past wages
$\iota_p$	Degree of indexation to past inflation
$\psi$	Elasticity of capital utilization
$\Phi$	One plus share of fixed cost in production
$r_\pi$	Fed's inflation aversion
$\rho$	Fed's degree of smoothing
$r_y$	Fed's output gap reaction
$r_{\Delta y}$	Fed's reaction to output gap difference
$\bar{\pi}$	Quarterly steady state inflation rate (annualized)
$(\beta^{-1} - 1) 100$	Discount rate (annual basis)
$\bar{\ell}$	Steady state hours worked
$\bar{\gamma}$	Quarterly trend growth rate
$\alpha$	Capital share in production
$\sigma_r, \sigma_a, \sigma_p, \sigma_w$	S.D. of mon. policy, productivity, price and wage mark-up shocks
$\sigma_g, \sigma_I, \sigma_b$	S.D. of exog. spending, invest. specific, int. rate wedge shocks
$\rho_r, \rho_a, \rho_p, \rho_w$	Persistence of mon. policy, productivity, price and wage mark-up shocks
$\rho_g, \rho_I, \rho_b$	Persistence of exog. spending, invest. specific, int. rate wedge shocks
$\rho_{ga}$	Degree to which exog. spending responds to productivity shock
$\mu_p, \mu_w$	Moving average parameters in the price and wage mark-up processes

Note. The table provides a description of the parameters in the Smets and Wouters' (2007) model.

**Table 9. Empirical Results for the Smets and Wouters' (2007) Model**

Parameter:	Individual	ESS	Parameter:	Individual	ESS
	p-value	p-value		p-value	p-value
$\sigma_r$	0	0	$\rho_w$	0.31	0.07
$\sigma_a$	0	0	$\rho_p$	0.33	1
$\sigma_p$	0	0	$\bar{\ell}$	0.35	1
$\varphi$	0.01	0	$\rho_r$	0.40	1
$\sigma_g$	0.02	0	$r_{\Delta y}$	0.46	1
$\xi_p$	0.02	0	$\sigma_\ell$	0.47	1
$\bar{\gamma}$	0.03	0	$\Phi$	0.48	0.84
$\psi$	0.04	0	$\alpha$	0.53	0.70
$\rho$	0.05	0	$\xi_w$	0.57	0.64
$\sigma_I$	0.06	0	$\mu_p$	0.59	1
$\rho_g$	0.09	0.05	$\beta^{-1} - 1$	0.61	1
$\rho_a$	0.13	0.05	$\iota_w$	0.65	1
$\rho_{ga}$	0.14	0.06	$\sigma_c$	0.69	1
$\iota_p$	0.18	0.08	$\rho_I$	0.70	1
$r_y$	0.20	0.03	$\sigma_w$	0.76	1
$\mu_w$	0.20	0.08	$\bar{\pi}$	0.79	1
$\sigma_b$	0.20	0	$\lambda$	0.80	1
$\rho_b$	0.22	0.03	$r_\pi$	0.85	1

Set of Stable Parameters (90% probability level):

$$\mathcal{S} = \{\rho_p, \bar{\ell}, \rho_r, r_{\Delta y}, \sigma_\ell, \Phi, \alpha, \xi_w, \mu_p, \beta, \iota_w, \sigma_c, \rho_I, \sigma_w, \bar{\pi}, \lambda, r_\pi\}$$

Note. The table reports p-values of the QLR stability test on individual parameters. It also reports both the set of stable parameters obtained by the ESS procedure as well as the p-values at each step of our procedure. The tests are implemented in the Wald form.

**Table 10(a). Counterfactual Comparisons for the Smets and Wouters' (2007) Model**

PANEL A. Standard Counterfactual Comparisons					
	Estimates		Counterfactual for 1984:1-2004:4		
	1966:1-1983:4	1984:1-2004:4	Shocks <sub>C</sub>	Policy <sub>C</sub>	ESS <sub>C</sub>
S.D. Output	1.20	0.73	1.23	0.70	1.34
S.D. Inflation	1.01	0.34	0.78	0.39	1.54
S.D. Interest Rate	1.06	0.37	0.83	0.56	1.56
Marg. Likel.		-375	-518	-431	-536

PANEL B. Moments and Likelihood Comparisons					
	Estimates		Fit in 1984:1-2004:4: selected constant param.		
	--	1984:1-2004:4	Shocks	Policy	ESS
S.D. Output		0.73	0.73	0.67	0.66
S.D. Inflation		0.34	0.46	0.39	0.26
S.D. Interest Rate		0.37	0.45	0.46	0.30
Marg. Likel.		-375	-543	-418	-419

Note. Panel A reports model implied standard deviations (S.D.) and Marginal Likelihood (Marg. Likel.) in observables in sub-samples ("Estimates") and model implied counterfactual standard deviations and likelihood on the second sub-sample ("Counterfactual for 1984:1-2004:4"). For example, the column labeled "Shocks<sub>C</sub>" reports the standard deviation of selected macroeconomic variables when shocks volatilities are kept constant but all other parameters are allowed to change. Panel B shows the standard deviations (S.D.) of observable variables (Output, Inflation and the Interest Rate) in the data as well as the same moments estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure. For example, the column labeled "Shocks" reports standard deviations for the selected macroeconomic variables when shocks volatilities are allowed to change but all other parameters are kept constant. Changes in monetary policy include changes in  $\rho$ ,  $r_\pi$ ,  $r_y$ ,  $r_{\Delta y}$ ,  $\rho_r$ . The table also reports the Marginal Likelihood estimated using only the parameter values in the relevant sub-sample (under the column labeled "Estimates") or the likelihood estimated in models allowing only changes in either the shock processes, monetary policy or those identified by the ESS procedure.

**Table 10(b). Counterfactuals for Smets and Wouters' (2007) Model**

Parameter:	Counterfactual			Parameter:	Counterfactual		
	Output	Inflation	Interest Rate		Output	Inflation	Interest Rate
No change:	0.73	0.34	0.37				
Unstable Parameters:				Stable Parameters:			
$\sigma_r$	27 %	10 %	17 %	$\rho_p$	-5 %	-5 %	-2 %
$\sigma_a$	8 %	2 %	4 %	$\bar{\ell}$	0 %	0 %	0 %
$\sigma_p$	4 %	16 %	2 %	$\rho_r$	-3 %	0 %	0 %
$\varphi$	19 %	0 %	-8 %	$r_{\Delta y}$	-3 %	0 %	0 %
$\sigma_g$	13 %	0 %	1 %	$\sigma_\ell$	6 %	36 %	32 %
$\xi_p$	-2 %	12 %	1 %	$\Phi$	0 %	-6 %	-6 %
$\bar{\gamma}$	0 %	0 %	0 %	$\alpha$	-6 %	1 %	1 %
$\psi$	11 %	0 %	0 %	$\xi_w$	-12 %	-107 %	-94 %
$\rho$	-16 %	-10 %	14 %	$\mu_p$	1 %	2 %	1 %
$\sigma_I$	12 %	1 %	5 %	$\beta^{-1}-1$	0 %	0 %	0 %
$\rho_g$	4 %	0 %	0 %	$\iota_w$	0 %	1 %	1 %
$\rho_a$	4 %	0 %	0 %	$\sigma_c$	-3 %	1 %	-1 %
$\rho_{ga}$	10 %	-1 %	-2 %	$\rho_I$	-1 %	0 %	-1 %
$\iota_p$	7 %	10 %	4 %	$\sigma_w$	-1 %	-8 %	-6 %
$r_y$	-11 %	10 %	9 %	$\bar{\pi}$	0 %	0 %	0 %
$\mu_w$	-7 %	-26 %	-16 %	$\lambda$	-1 %	0 %	-1 %
$\sigma_b$	11 %	0 %	2 %	$r_\pi$	0 %	9 %	7 %
$\rho_b$	23 %	1 %	9 %				
$\rho_w$	12 %	154 %	129 %				
				All change:	1.20	1.01	1.06
Set of Stable Parameters (90% probability level):							
$S = \{\rho_p, \bar{\ell}, \rho_r, r_{\Delta y}, \sigma_\ell, \Phi, \alpha, \xi_w, \mu_p, \beta, \iota_w, \sigma_c, \rho_I, \sigma_w, \bar{\pi}, \lambda, r_\pi\}$							

Note. The table shows the percentage contribution to the increase or decrease in the volatilities of output, inflation and the interest rate when we progressively allow each of the parameters to be time varying (ordered according to the p-values of the individual t-tests for parameter instabilities).

**Table 11. Empirical Results for the VAR**

Panel A. Reduced form VAR		Panel B. Structural VAR	
Parameter	Individual p-value	Parameter	Individual p-value
$k_{11}$	1	$k_{11}$	1
$a_{11}$	1	$a_{11}$	1
$a_{12}$	1	$a_{12}$	1
$a_{13}$	1	$a_{13}$	1
$k_{22}$	1	$k_{22}$	1
$a_{21}$	1	$a_{21}$	1
$a_{22}$	1	$a_{22}$	1
$a_{23}$	1	$a_{23}$	1
$k_{33}$	1	$k_{33}$	1
$a_{31}$	1	$a_{31}$	1
$a_{32}$	1	$a_{32}$	1
$a_{33}$	0.70	$a_{33}$	0.70
$\omega_{11}$	0	$\sigma_{11}$	0.71
$\omega_{12}$	1	$\sigma_{12}$	1
$\omega_{13}$	1	$\sigma_{13}$	1
$\omega_{22}$	0.04	$\sigma_{22}$	0
$\omega_{23}$	1	$\sigma_{23}$	1
$\omega_{33}$	0.04	$\sigma_{33}$	1
Joint test – all param:	0	Joint test – all param:	0
Joint test – all $a, k$ :	0.11	Joint test – all $a, k$ :	0.11
Joint test – all $\omega$ :	0	Joint test – all $\sigma$ :	0
Set of stable parameters (90% probability level):		Set of stable parameters (90% probability level):	
$\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23},$		$\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23},$	
$k_{33}, a_{31}, a_{32}, a_{33}, \omega_{12}, \omega_{13}, \omega_{23}\}$		$k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{23}, \sigma_{33}\}$	

Note. The table reports p-values of the QLR test on individual parameters for both the reduced form VAR (eq. (5), in panel A), and the Structural VAR (eq. (6), in panel B). The table also reports the set of stable parameters obtained by the ESS procedure. Subscripts denote:  $i = 1$  for the real interest rate,  $i = 2$  for GDP growth,  $i = 3$  for inflation.

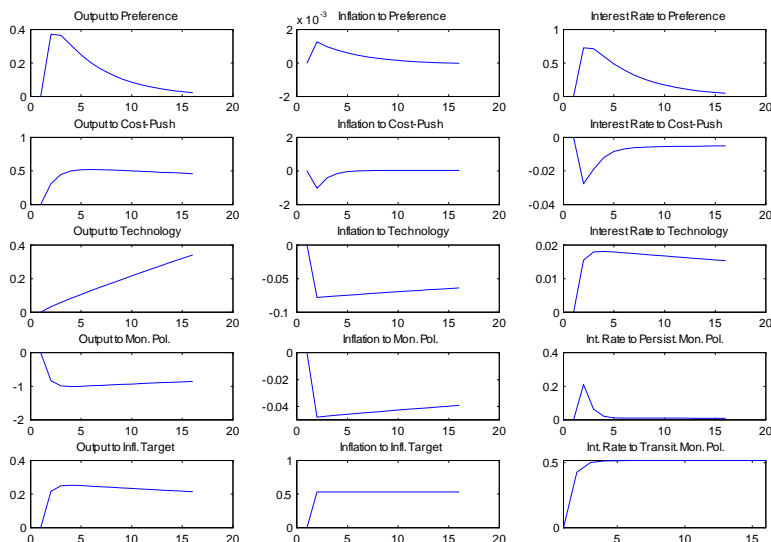
**Table 12. Contribution to the Reduction in  
Standard Deviation of GDP Growth in SVAR**

<i>Unstable parameters:</i>	<i>% contribution to change in std(GDP):</i>
None	0
$\sigma_{22}$	-14%
$\sigma_{11}$	-19%
$\sigma_{12}$	0%
$\sigma_{13}$	-5%
<i>Additional parameters:</i>	
$\sigma_{23}$	23%
$\sigma_{33}$	-32%
$k_{11}$	0%
$a_{11}$	6%
$a_{12}$	-4%
$a_{13}$	85%
$k_{22}$	10%
$a_{21}$	-94%
$a_{22}$	-22%
$a_{23}$	1%
$k_{33}$	-25%
$a_{31}$	2%
$a_{32}$	2%
$a_{33}$	-14%

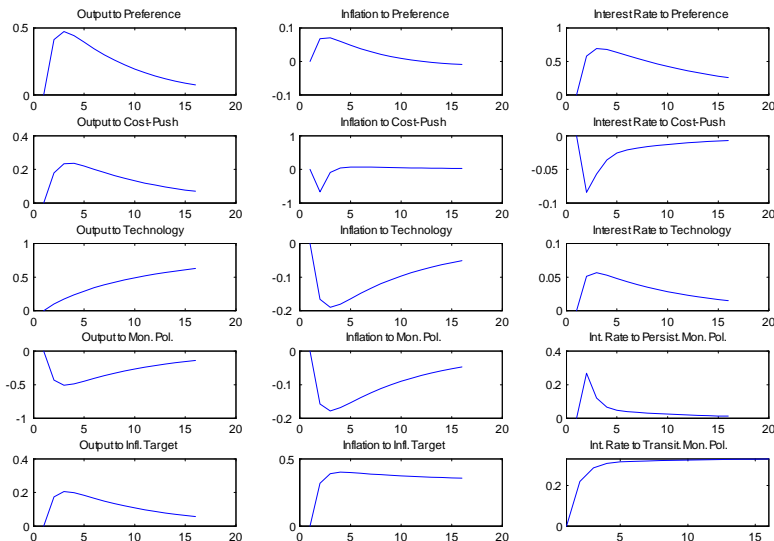
Note. The table shows the percentage contribution to the change in the standard deviation of GDP growth after 1985 obtained by estimating the SVAR (6) imposing that a certain subset of the parameters is stable. The column labeled “Unstable parameters” reports the parameters that are progressively allowed to be time-varying. Therefore, in each row, all the parameters listed in the previous rows as well as the parameter indicated in that row are allowed to be time-varying. For example, the row corresponding to  $\sigma_{13}$  allows all the parameters belonging to the unstable set ( $\sigma_{22}, \sigma_{11}, \sigma_{12}, \sigma_{13}$ ) to be time-varying. The order in which the unstable parameters are excluded by the set of stable parameters is determined by Algorithm 1. The column labeled “Other parameters” shows results where other parameters (beyond those that belong to the set of unstable parameters) are progressively allowed to be time-varying.

**Figure 1. Impulse Responses in the Representative New Keynesian Model**

**Panel A. Responses before 1980:4**



**Panel B. Responses after 1980:4**



Note to Figure 1. The figure reports impulse responses from the representative New Keynesian model. Each panel shows the percentage-point response of one of the model's variables to a one-standard deviation shock. The inflation and interest rates are expressed in annualized terms. Panel A shows impulse responses before the estimated break date, and Panel B shows impulse responses after the estimated break date.