GREAT MODERATION AND SPECTRAL ANALYSIS: WHAT IF THE LONG-RUN VARIANCE HAS CHANGED?

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December 2005

Abstract

The starting point for this paper is the application of frequency domain approach to study the decline in the volatility of the U.S. real GDP growth since 1984. After observing the different behavior for the output growth in two sub-samples (1959-1979 and 1984-2004), the main focus of the paper is analyzing the difference in the long-run variance for the two series, testing for unit root and, more originally, constructing confidence intervals for the long-run variances. Our results support the hypothesis of a changing in the long-run variances in the two sub-samples, providing, thus, a statistical explanations for the "Great Moderation". Finally, using a Dynamic General Equilibrium Stochastic Model we test whether a better monetary policy, called "Good Policy" hypothesis, affects the spectrum of the output growth just in the business-cycles frequencies.

1 Introduction

The decline in the output volatility during the last twenty years is one of the most analyzed features in economics. A growing number of studies try to explain this evident phenomenon with different approaches: for example, Stock and

Watson, 2002, (SW) have investigated this issue through vector auto-regressive models, McConnel and Quiros, 2000, (MQ) have considered structural breaks in the economy to show the decline in output volatility, and Ahmed, Levin, and Wilson, 2002, (ALW) have used frequency domain methods to discuss about the causes of the so called "Great Moderation". Three different main facts have been proposed in the literature as explanations for the decline in the volatility of the business cycle. First, the "Good Practice" hypothesis focuses on the effects of improved business-practices, such as "just-in-time" inventory management, that have been facilitated by rapid advances in information technology. The "Good Policy" is the second explanation: according to this view, better monetary policy has tamed the business cycle, consistently with empirical studies that have documented systematic differences in monetary policy during the Volcker-Greenspan era compared with the previous period. Finally, the "Good Luck" hypothesis claims that the decline in the output volatility is not due to structural changes in the economy, but it may simply reflect a sharp drop in variance of exogenous disturbances hitting the U.S. economy.

In this paper the attention will be focused on the frequency domain analysis for the output growth following Ahmed, Levin, and Wilson's work. Similar results are derived and shown in Section 1. Moreover, three questions arise from the empirical findings and the main goal of this paper is to examine them in details. The first issue is to test the presence of a unit root in the output growth in the two sub-samples, in order to explain the different behavior of the spectra at low frequencies, as presented in Section 2. The results does not suggest a totally uniform evidence for the unit root process. Therefore, driven by the same question, in Section 3 confidence intervals for the long-run variance of the output growth are derived for each sub-sample. As it will be showed, it is possible to explain the different behavior of the spectra for the two sub-samples at low frequencies with a statistical significant change in the long-run variance for the process. Finally, in Section 4 an exercise using a business cycle theoretical model is computed in order to test whether changing in monetary policy affects the spectrum just in business cycle frequency, as claimed in ALW. All the results are summarized in the Conclusions.

2 Great moderation and spectral analysis

The decline in volatility of the output growth in U.S. economy since the mid-80s is immediately apparent, as it can be seen in Figure 1. Moreover, this evidence has been confirmed in previous works, using statistical methods; in fact, MQ find statistical significance for variance break around 1984, and, using an alternative Bayesian approach, Kim at all., 2001, also find a volatility break in real GDP growth at about the same time. Therefore, following ALW, the all sample 1958Q1-2004Q4 will be divided in two sub-samples 1959Q1-1979Q4 and 1984Q1-2004Q4. It is worth noticing some important features of the sub-samples. First, as in ALW, the period 1980-1983 is omitted, because the Bias-Perron test does non indicate a structural break exactly at 1984, but in the 1979-1984 range and because it is generally believed that monetary policy rule

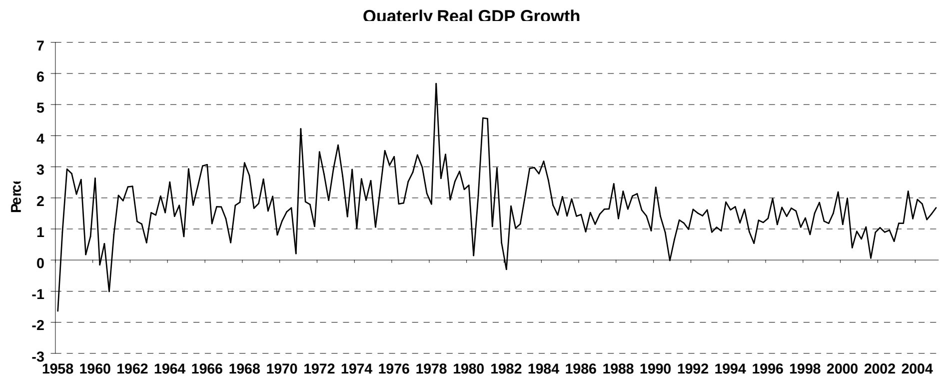

Figure 1:

being followed was quite different in the 1979-84 period from the other two periods. Second, the first sub-sample includes 1959, and the second sub-sample includes the period 2003-2005, both not included in the ALW dataset.

Figure 2 compares the series of the quarterly output growths in the two sub-samples, whereas Table 1 shows some statistics suggesting the decline in the volatility in the second sub-sample.

Output growth	Mean	Variance	Standard deviation
1959-1979	2.03	1.05	1.02
1984-2004	1.40	0.29	0.53

Table 1

To analyze the proprieties of the GDP growth, the spectra for the two sub-samples are estimated. Since the variance of output growth is given by integrating the spectrum $g(\omega)$ over all frequencies $-\pi \leq \omega \leq \pi$, the post-1984 decline in variance shows up as a downward shift in the spectrum, as it can be

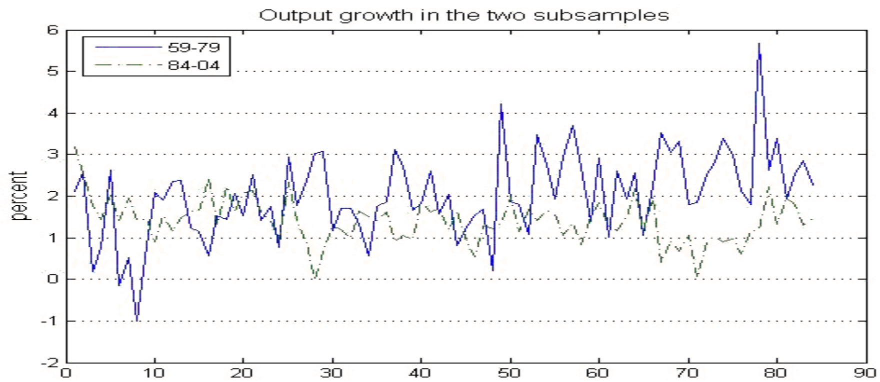

Figure 2:

seen in Figure 3. The three regions delimited by the verticals solid lines identify the three different frequency ranges: low, business-cycle, and high frequencies. As in Baxter and King (1995), the business cycle frequencies $(\frac{\pi}{16}, \frac{\pi}{3})$ correspond to cycles of 6 to 32 quarters.

Although the decreasing in the volatility is evident in the estimated spectra, it is worth introducing a different function of the spectra, the normalized spectrum, $h(\omega) = g(\omega)/\sigma_y^2$, which indicates the fraction of the total variance σ_y^2 occurring at each frequency ω. The normalized spectrum can be used to test the hypothesis for the great moderation. In fact, following ALW, the "Good Practice" hypothesis implies a change in the normalized spectrum at high frequencies, since management improvements have intra-quarter effects, the "Good Policy" hypothesis implies a change in the spectrum at business-cycle frequency, and the "Good Luck" hypothesis implies no changes in the normalized spectrum. Indeed, assuming that output growth is covariance stationary, Wold's theorem

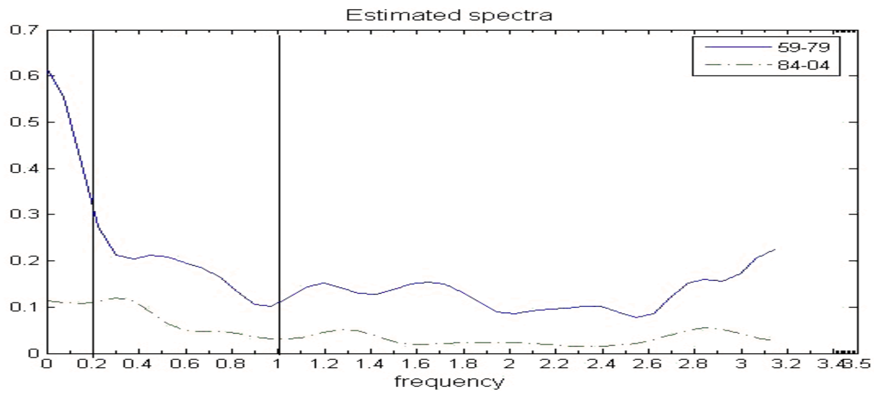

Figure 3:

indicates that it has a MA(∞) representation. Therefore, both the spectrum $g(\omega)$ and the variance of the output growth σ_y^2 are proportional to the variance of the shocks σ_ε^2. In formula,

$$g(\omega) = \frac{1}{2\pi}\theta(e^{i\omega})\sigma_\varepsilon^2\theta(e^{-i\omega}) \qquad \text{and} \qquad \sigma_y^2 = \sigma_\varepsilon^2\sum_{j=0}^{\infty}\theta_j^2$$

As a result, the normalized spectrum $h(\omega) = g(\omega)/\sigma_y^2$ is independent from σ_ε^2.

Analyzing the Figure 4, it is possible to compare the behavior of the two normalized spectra in the three frequency range. First, the output growth shows a similar shape for the two sub-sample at high frequencies. As a consequence, the "Good Practice" hypothesis seems to have weak evidence. Second, the normalized spectrum in the first sub-sample is slightly below than that one in the first sub-sample in business cycle frequency. According ALW, if this change were significant we could have same evidence for the "Good Policy" explanation. Third, the spectrum at low frequency in the first sub-sample is remarkably

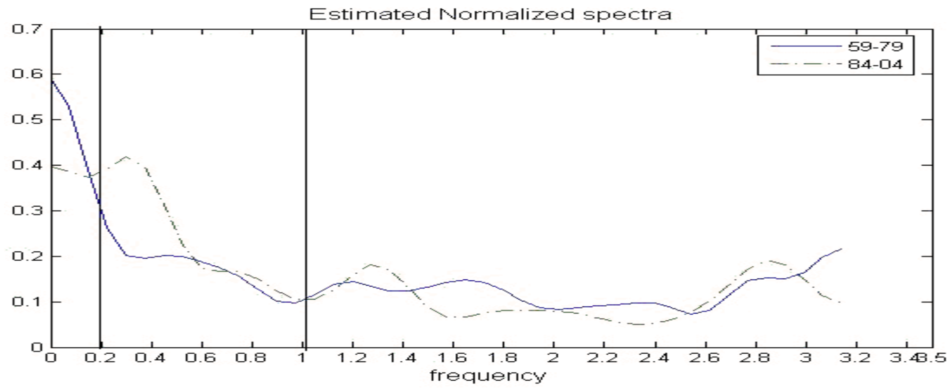

Figure 4:

greater than in the second sub-sample. In ALW, the difference is explained as a sample variation that affects the estimation of the spectrum at low frequencies. On the other hand, as it can be seen in Section 3 and 4, this discrepancy might be due to changes in the long-run behavior of the two process, and , thus, it might have an important role in explaining the decline on the volatility in the second period. Finally, if it is believed that the two normalized spectra are not significantly different in the two sub-sample, the "Good Luck" hypothesis should be considered as main cause of the "Great Moderation".

In order to following this approach more formally, two statistics will be introduced: the integrated spectrum and the normalized integrated spectrum. For a particular frequency range, the integrated spectrum $G(\omega_1, \omega_2) = 2\int_{\omega_1}^{\omega_2} g(\omega)d\omega$ indicates the variance attributable to the frequency range $\omega_1 \leq \mid \omega \mid \leq \omega_2$. The integrated spectrum can be estimated as:

$$\hat{G}(\omega_1,\omega_2) = \frac{\omega_2-\omega_1}{\pi}\hat{\Gamma}(0) + \frac{2}{\pi}\sum_{j=1}^{T-1}\hat{\Gamma}(j)\frac{[\sin(\omega_2 j)-\sin(\omega_1 j]}{j} \tag{1}$$

where $\hat{\Gamma}(j)$ represent the jth order sample autocovariance. This estimator is consistent and has an asymptotic normal distribution (Priestley,1982).

The integrated normalized spectrum $H(\omega_1,\omega_2) = G(\omega_1,\omega_2)/\sigma_y^2$ indicates the fraction of the variance attributable to the frequency range $\omega_1 \leq|\omega|\leq \omega_2$. Thus with $\omega_1 = 0$ and $\omega_2 = \pi$ the integrated spectrum has a value of unity. The estimated integrated spectrum is obtained by taking the integrated spectrum and dividing by the sample variance of Y.

$$\hat{H}(\omega_1,\omega_2) = \hat{G}(\omega_1,\omega_2)/\hat{\Gamma}(0) \equiv \hat{G}(\omega_1,\omega_2)/s_y^2. \tag{2}$$

It can be shown that the estimated integrated spectrum and the sample variance of Y have the following proprieties:

$$\begin{pmatrix}\hat{G}(\omega_1,\omega_2)\\ s_y^2\end{pmatrix} \sim \left[\begin{pmatrix}G(\omega_1,\omega_2)\\ \sigma_y^2\end{pmatrix}, \frac{1}{T}\begin{pmatrix}\Phi + eG^2(\omega_1,\omega_2) & \Phi + eG^2(\omega_1,\omega_2)\sigma_y^2\\ \Phi + eG^2(\omega_1,\omega_2)\sigma_y^2 & \Phi + e\sigma_y^2\end{pmatrix}\right] \tag{3}$$

with

$$\Phi = 8\pi\int_{\omega_1}^{\omega_2} g^2(\omega)d\omega \qquad and \qquad e = \left(\frac{E(\varepsilon_t^4)}{\sigma_\varepsilon^2} - 3\right) \tag{4}$$

and they can be estimated as

$$\hat{\Phi} = 8\pi \int_{\omega_1}^{\omega_2} \hat{I}^2(\omega) d\omega \qquad and \qquad \hat{e} = \left(\frac{\frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^4}{\left[\frac{1}{T}\sum_{t=1}^{T} \hat{\varepsilon}_t^2\right]^2} - 3 \right). \tag{5}$$

with $\hat{I}(\omega)$ is the estimate of the spectrum and $\hat{\varepsilon}_t$ are the residuals from an AR(p) model of y_t, with p chosen according information criteria.

Using (3), the deltha-method can be used to get the asymptotic distribution of the ratio of the estimated integrated spectrum to the sample variance, which is our estimate of the integrated normalized spectrum. Therefore, (2) is a consistent estimate of the integrated normalized spectrum, and its variance is given by:

$$Var\hat{H}(\omega_1, \omega_2) = \begin{pmatrix} \frac{1}{s_y^2} & -\frac{\hat{G}(\omega_1,\omega_2)}{s_y^2} \end{pmatrix} \begin{bmatrix} \hat{\Phi} + \hat{e}\hat{G}^2(\omega_1, \omega_2) & \hat{\Phi} + \hat{e}\hat{G}^2(\omega_1, \omega_2)\sigma_y^2 \\ \hat{\Phi} + \hat{e}\hat{G}^2(\omega_1, \omega_2)\sigma_y^2 & \hat{\Phi} + \hat{e}s_y^2 \end{bmatrix} \begin{pmatrix} \frac{1}{s_y^2} \\ -\frac{\hat{G}(\omega_1,\omega_2)}{s_y^2} \end{pmatrix}. \tag{6}$$

It is possible now testing the null hypothesis that the normalized integrated spectra are equal in the two sub-samples. Each rows in Table 2 provides the results of this test applied to each of the three frequency range. The first two columns report the estimates of the normalized integrated spectrum, with the standard error in parentheses. The third column gives the test statistic of the null hypothesis that the spectrum is equal in period 59-79 and in period 84-04, and the last column report the p-value.

Frequency	Integrated Normalized Spectrum period 59-79	Integrated Normalized Spectrum period 84-04	Ho: period 59-79=period 84-04 Ho: period 59-79≠period 84-04 Test	p-value
Real GDP growth				
Low	0.22 (0.14)	0.17 (0.13)	0.25	0.82
Business cycle	0.27 (0.10)	0.39 (0.17)	-0.61	0.54
High	0.51 (0.08)	0.44 (0.12)	0.47	0.64

Table 2

The results do not show a significant difference in the normalized integrated spectrum. Therefore, as in ALW, the "Good Luck" hypothesis seems reasonable, according frequency domain findings. However, in the next sections we deeply inspect some features of these results, namely, the decline in the low frequencies normalized spectrum in the second period, and the evidence that a better monetary policy affects just the business-cycle frequency, using a theoretical business cycle model.

3 Does the output growth show a unit root?

The changing in the volatility at low frequencies imply a different behavior of the series in the long-run. Therefore, it is worth testing the hypothesis of a presence of stochastic trend given by a unit root in the quarterly real GDP growth in each sub-samples, using different tests, such as Augmented Dickey

Fuller (ADF), and the efficient test for augmented unit root (ADF-GLS), as described in Elliott, Rothenberg, and Stock,1996.

The asymptotic distribution of the test statistics in presence of unit root depends from the specification of the true model, and, thus, it is crucial to choose the opportune data generator process, deciding whether including constant and deterministic trend in the specification. By the business cycle theory the economic activity fluctuates during the time through expansions and contractions: therefore, there is nothing in economic theory to suggest that output growth should exhibit a deterministic time trend. On the other hand, it is plausible to include the constant term in the models, since, if the process is stationary, it would have a positive mean, as the data suggest. This argues for including a constant term in the estimated regression, even though under the null hypothesis the true process does not contain a constant term. Therefore, for the ADF and ADF-GLS statistics, we test the true process :

TRUE PROCESS:

$$y_t = y_{t-1} + \sum_{j=1}^{p} \zeta_j \Delta y_{t-j} + u_t \quad u_t \sim i.i.d.(0, \delta^2)$$

and we estimated:

ESTIMATED REGRESSION:

$$y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^{p} \zeta_j \Delta y_{t-j} u_t$$

with p chosen according the Modified Information Criteria (MIC) proposed by Ng and Perron (2001) for both sub-samples.

The distributions of the t-statistics for testing $\rho = 1$ and $\alpha = 0$ is not standard.

Specification with constant	period 59-79	period 84-04	Critical Value at 5%
p from MIC	11	1	
ADF	-1.33	-4.87*	-2.93
ADF-GLS	-1.49	-1.21	-1.95

Table 3

Table 3 shows the results of these unit root tests: ADF reject the null hypothesis of unit root (*) for the output growth in the second sub-sample, whereas there is no evidence of a unit root in the first sub-sample. Moreover, the ADF-GLS test fails to rejects the presence of unit root in the both periods. In order to have further evidences to clarify the contrasting results, we run the ADF test with a model specifications without the constant term. Note, that since in this specification there is no a deterministic trend, ADF and ADF-GLS are numerically equivalent. Table 4 shows the result for this assumption. If we do not include the constant neither in the true process nor in the estimation, there is statistical evidence of presence of unit root in the both sub-samples.

Specification without constant	period 59-79	period 84-04	Critical Value at 5%
p from MIC	3	4	
ADF	-0.55	-0.97	-1.95

Table 4

As a conclusion, although the presence of unit root in the first sub-sample cannot be rejected for each specification, in the second sub-sample it significantly appears to depend on the specification of the model, in particular on whether the constant is assumed to be estimated in the regressions. Therefore, we cannot claim that the different behavior in the normalized spectrum at low frequencies of the output growth in the two sub-samples is due to a changing from a stationary process to a unit root unit process.

4 Confidence interval for the long-run variance

In Section 3 it was shown that there is not clear evidence of unit root in the processes of real GDP growth in the two different period 59-79 and 84-04. However, the main contribution to this paper is to find the confidence interval for the long-run variance in the two sub-sample, considering a local-to-unity root in the processes. Following Stock (1991) and Rossi (2004), we assume that the process for the output growth in each sub-sample has the form:

$$(1-\rho L)y_t = \mu + u_t \tag{7}$$

$$b(L)u_t = \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim i.i.d.(0,\delta^2) \tag{8}$$

with $b(L)$ a stationary lag polynomial.

Given this specification, we can write $u_t = C(L)\varepsilon_t$ with $C(L) = b(L)^{-1}$, being the polynomial for the MA(∞) representation of u_t. Since

y_t has a AR(1) representation with respect to u_t, iterating backward and assuming the initial condition $y_0 = 0$ we have,

$$y_T = \mu + \sum_{j=0}^{T} \rho^j u_{t-j} = \mu + \sum_{j=0}^{T} \rho^j C(L)\varepsilon_{t-j} = \mu + \sum_{j=0}^{T} \rho^j (\varepsilon_{t-j} + \psi_1 \varepsilon_{t-j-1} + \psi_2 \varepsilon_{t-j-2} + ...) \tag{9}$$

where ψ_j are the coefficients of the infinity MA polynomial $C(L)$.

Therefore, by the incorrelation of ε_t, the variance of y_t is:

$$Var(y_T) = \sum_{j=0}^{T} \rho^{2j} (1 + \psi_1^2 + \psi_2^2 + ...)\sigma_\varepsilon^2 = \sum_{j=0}^{T} \rho^{2j} \tilde{\psi} \sigma_\varepsilon^2 \tag{10}$$

with $\tilde{\psi} = (1 + \psi_1^2 + \psi_2^2 + ...)$.

Now, consider the local to unity parameter c defined such that $\rho = 1 + \frac{c}{T}$.

Then,

$\rho^{2j} = \left(1 + \frac{c}{T}\right)^{2j} = \left[\left(1 + \frac{c}{T}\right)^{\frac{j}{T}T}\right]^2 = \left[\left(1 + \frac{c}{T}\right)^{sT}\right]^2 = e^{2cs}$,

where we defined $s = \frac{j}{T}$ and we used the definition $e^x = \left(1 + \frac{x}{T}\right)^T$.

Then, the long-run variance of the process can be approximated as:

$$Var(y_T) = \sum_{j=0}^{T} e^{2cs} \tilde{\psi} \sigma_\varepsilon^2 \simeq \int_0^1 e^{2cs} \tilde{\psi} \sigma_\varepsilon^2 ds = \tilde{\psi} \sigma_\varepsilon^2 \frac{1}{2c} \left(e^{2c} - 1\right) \tag{11}$$

In order to construct the confidence interval for the long-run variance, estimate of $\sigma_\varepsilon^2, \tilde{\psi}$ are needed.

The model specified in (9) and (10) is equivalent to:

$$y_t = \tilde{\mu} + \alpha(1)y_t + \sum_{j=0}^{k} \alpha_{j-1}^{*} \Delta y_{t-j} + \varepsilon_t \tag{12}$$

with $\alpha(1) = \left(1 + \frac{c}{T} b(1)\right)$, $\tilde{\mu} = -\frac{c}{T}\mu\ b(1)$ and $\alpha_j^{*} = -\sum_{i=j+1}^{k} \alpha_i$.

From the Augmented Dickey Fuller regression is it possible to estimate the parameters $\hat{\alpha}(1)$ and $\hat{\alpha}_{j-1}^{*}$ with OLS. Moreover, a consistent estimate for $b(1)$ is given by, see Rossi(2004),

$$\hat{b}(1) = \left(1 - \sum_{j=0}^{k} \alpha_{j-1}^{*}\right) \tag{13}$$

Then, we have an estimate for , $\hat{c} = \left(\frac{\hat{\alpha}(1)-1}{\hat{b}(1)}\right) T$.

Following Stock (1991), having an estimate for c and therefore for $\rho = 1 + \frac{c}{T}$, by (9) it is possible compute an estimate of the series u_t. Then, we can estimate the coefficients of the polynomial $b(L)$ regressing

$$u_t = -\sum_{j=1}^{k} b_j u_{t-j} + \varepsilon_t \tag{14}$$

using OLS estimator and noticing that $b_0 = 1$. Using this regression, we can also estimate σ_ε^2 as the average of the sum of the squared residuals.

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_t \tag{15}$$

Once the estimate of the polynomial $b(L)$ is computed, it is possible obtaining an estimate for the coefficients of the MA(∞) polynomial $C(L)$, through the convolution formula. Then, having an estimate for the coefficients $\psi_1, \psi_2, ...$, an estimate of $\tilde{\psi}$ is trivial to derive.

Thus, since estimates for σ_ε^2 and $\tilde{\psi}$ are available, the confidence interval for the Long-run variance can be computed, using the confidence interval for the local to unity parameter derived by Stock (1991).

The results for the confidence interval for the long-run variance for the output growth in the two periods 59-79 and 84-04 are shown in Table 5.

	period 59-79	period 84-04
sample variance	1.046	0.286
$\hat{\rho}$	0.930	0.960
Confidence Interval for LR var at 95%	[0.194; ∞]	[0.027; ∞]
Confidence Interval for LR var at 90%	[0.227; ∞]	[0.033; ∞]
Confidence Interval for LR var at 80%	[0.283; ∞]	[0.046; ∞]
Confidence Interval for LR var at 70%	[0.340; ∞]	[0.064; ∞]

Table 5

First, notice that the upper bound of the confidence intervals is infinity, consistently with the results in Section 3, were a unit root was found in both period when we include the constant in the specification of the ADF regression. Most importantly, the sample variance in the second sub-sample (0.286) falls outside the confidence interval in the boundary of the confident intervals for the long-run variance at 70% of significance, [0.340; ∞]: therefore, there is statistical evidence that the long-run variance of the process has changed. Moreover, even though we fails to rejects the hypothesis that the long run variance is changed at

80% or more of significance, the sample variance in period 2 lies in the boundary of the confidence interval for the long-run variance in period 1. These results might explain the discrepancy on the normalized spectrum at low frequencies and, above all, they offer a new view on the cause of the "Great Moderation": in fact, the reduction in the volatility of the output growth after the mid-80s might be due to the change in the long-run variance of the process.

5 Spectral analysis in a real business-cycle model

In ALW the three different hypothesis "Good Practice", "Good Policy", and "Good Luck" for the great moderation are tested using spectral analysis, assuming that an improvement in the inventory management affects the spectrum just in the high frequencies range, the decline in the volatility of the shocks hitting the economy has no impact in the normalized spectrum, and a better monetary policy affects the spectrum just in the business cycle frequencies. In this Section, the last assumption is tested considering a theoretic real business-cycle model, where only the parameters of the monetary policy rules are changed, and estimating the spectrum of the simulated output growth series.

5.1 The model

In order to illustrate how the spectrum of the simulated output growth series under two different monetary rules looks like, we consider a modified standard New Keynesian framework with staggered price setting a la Calvo, introduced by Gali and Rabanal (2004). The equilibrium dynamics in standard New Keynesian model can be summarized as follow. On the demand side, output is determined by a forward-looking IS-type equation:

$$y_t = E_t \{y_{t+1}\} - \sigma (r_t - E_t \{\pi_{t+1}\}) \tag{16}$$

where y_t denotes the log-output, r_t is the nominal interest rate, and $\pi_t \equiv p_t - p_{t-1}$ denotes the rate of inflation between $t-1$ and t. Parameter σ can be interpreted a measure of the sensitivity of aggregate demand to changes in interest rate.

Inflation evolves according to a forward-looking new Keynesian Phillips curve:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa (y_t - \bar{y}_t) \tag{17}$$

where $\bar{y}_t$ is the natural level of output (or potential output), defined as the one that would prevail in the absence of nominal frictions.

The simulated model slightly changes from the basis New Keynesian model: as in Gali and Rabanal (2004), we can summarize the set of equilibrium condition of the model as following. The demand side of the model is represented by the Euler-equation

$$b\Delta y_t = E_t \{\Delta y_{t+1}\} - (1-b) (r_t - E_t \{\pi_{t+1}\}) + (1-\rho_g)(1-b)g_t \tag{18}$$

which modifies equation (17) above by allowing for some external habit formation, indexed by parameter b, and introducing a preference shock $\{g_t\}$ which follows an AR(1) process with parameter ρ_g. Preferences are separable between consumption and hours, and logarithmic in quasidifference of consumption in order to preserve the balanced growth path propriety. Aggregate output and hours are related by the simple log-linear production function

$$y_t = a_t + n_t \tag{19}$$

where a_t represents an exogenous technology parameter, that is assumed to evolve as an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t$.

Introducing a tilde to denote variables normalized by current productivity, in order to introduce stationarity, we have:

$$\tilde{y}_t = n_t \tag{20}$$

Log-linearitation of the output price-setting condition around the zero inflation steady-state yields an equation describing the dynamics of inflation as a function of the deviations of the average log-markup from its steady-state level, denoted by μ_t^p

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \{\pi_{t+1}\} + \kappa_p \left(\mu_t^p - u_t\right). \tag{21}$$

Notice that $\mu_t^p = -\log\left(\frac{W_t}{P_t A_t}\right) \equiv -\tilde{\omega}_t$ is the price markup, where $\tilde{\omega}_t = \omega_t - a_t$

is the real wage per efficiency unit. u_t denotes exogenous variations in the desiderated price markup.

Log-linearitation of the optimal wage-setting condition yields the following equation for the dynamics of the normalized real wage:

$$\tilde{\omega}_t = \frac{1}{1+\beta}\tilde{\omega}_{t-1} + \frac{\beta}{1+\beta}E_t\left\{\tilde{\omega}_{t+1}\right\} - \frac{1}{1+\beta}\Delta a_t + \frac{\beta}{1+\beta}E_t\left\{\Delta a_{t+1}\right\} + \quad (22)$$

$$+\frac{\eta_w}{1+\beta}\pi_{t-1} - \frac{1+\beta\eta_w}{1+\beta}\pi_t + \frac{\beta}{1+\beta}E_t\left\{\pi_{t+1}\right\} - \frac{\kappa_w}{1+\beta}\left(\mu_t^w - v_t\right). \quad (1)$$

Notice that $\mu_t^w = \tilde{\omega}_t - \left(\frac{1}{1+b}\tilde{y}_t - \frac{b}{1+b}\tilde{y}_{t-1} - g_t + \frac{b}{1+b}\Delta a_t + \varphi n_t\right)$ is the wage markup. v_t denotes exogenous variations in the desiderated wage markup.

Finally, to close the model we assume that monetary authority adjust interest rates in response to changes in inflation and output growth according to the rule

$$r_t = \phi_r r_{t-1} + (1-\phi_r)\phi_\pi \pi_t + (1-\phi_y)\phi_y \Delta y_t + z_t \quad (23)$$

where z_t is an exogenous monetary shock.

5.2 Simulation

Gali and Rabanal provides an estimation with bayesian methods for the mean of the posterior distribution of all the parameters of the model. In order to isolate the effect of changing in the monetary policy in the economy, the exercise proceeds as the following. First, we consider two sets for the parameter characterizing different monetary policy. For this purpose, Clarida, Gali and Gerlter estimated the value of ϕ_r, ϕ_π, and ϕ_y, in the pre-Volcker era (Period 1) and in the post-Volker era (Period 2). These sets of estimation are shown in Table 6.

	Period 1	Period 2
ϕ_r	0.68	0.79
ϕ_π	0.83	2.15
ϕ_y	0.27	0.93

Table 6

Second, fixing all the others parameter of the model as the mean of the posterior distribution estimated by Gali and Rabanal, we simulated two sample of 84 observations for the output growth corresponding to the two different monetary policy sets of parameter. The plot for the simulated series are shown in Figure 5. As expected, the visual inspection of the series shows a bigger volatility for the output growth corresponding to the pre-Volcker monetary policy with respect to the volatility for the simulated growth with post-Volcker monetary policy. This analysis is confirmed by the statistics for the two series reported in Table 7. In fact, the volatility in the Period 1 simulation is more than double than the volatility in the Period 2 simulation. Moreover, it is worth underling that the variance in the data is twice bigger than variance in the simulation for both the pre-Volcker era (59-79 and Period 1) and post-Volcker era (84-04 and Period 2). However, the task of the exercise is to compare the spectrum of the simulated output growth due only to changing in the monetary policy and,

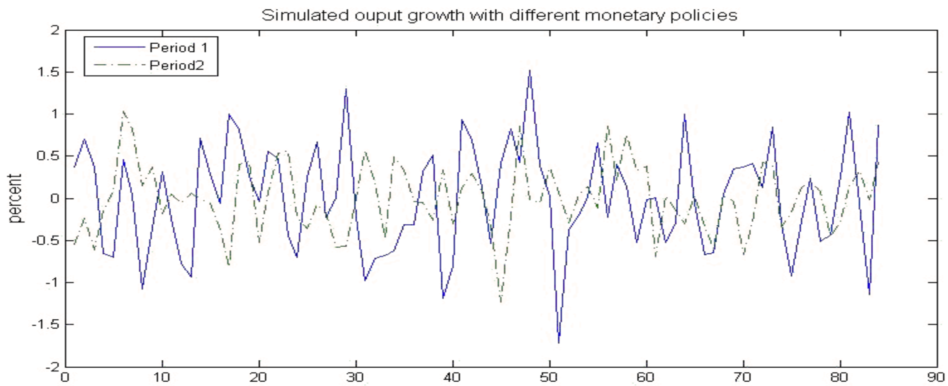

Figure 5:

since the decrease in the volatility in the two simulations (42%) is even bigger than in the two sub-samples (28%), the discrepancy in the spectra should be even more remarked.

Output growth	Mean	Variance	Standard deviation
Period 1 simulation	0.00	0.397	0.63
1959-1979 sample	2.03	1.05	1.02
Period 2 simulation	0.00	0.169	0.41
1984-2004 sample	1.40	0.29	0.53

Table 7

5.3 Spectral analysis

Finally, we estimate the spectra and the normalized spectra for the two simulations and we test the hypothesis of changing in the normalized spectrum, using the normalized integrated spectrum statistics, as in Section 2.

In Figure 6, the decline on the volatility is evident since the estimated spectrum in the second simulations is lower than the first one. In order to analyze which frequencies explain the biggest part of the volatility, we estimate the normalized integrated spectrum. Analyzing Figure 7, two important features are worth being underlined: first, most of the variance of both series is due to frequencies in the range [1.4-1.8] that corresponds to 3.5 and 4.5 quarters. Therefore, the variance of the simulated output growths depends for the most part to the component of yearly periodicity. Second, the two normalized spectra appear to be very close in each of the three frequencies range (low, business-cycle, high). Hence, there is no evidence for claiming that monetary policy affects the spectra in business-cycle frequencies. It is possible to formally test this hypothesis with the normalized integrated spectrum. The results of the test are presented in Table 8: as in Table 2,the first two columns report the estimates of the normalized integrated spectrum, with the standard error in parenthesis, the third column gives the test statistic of the null hypothesis that the spectrum is equal in Period 1 and in Period 2, and the last column report the p-value. The test imply that we fails to rejects that the normalized spectrum are equal for the two simulation in each of the frequencies range: the conclusion of this exercise contrasts the theory such that a better monetary policy affects the spectrum at business-cycle frequencies.

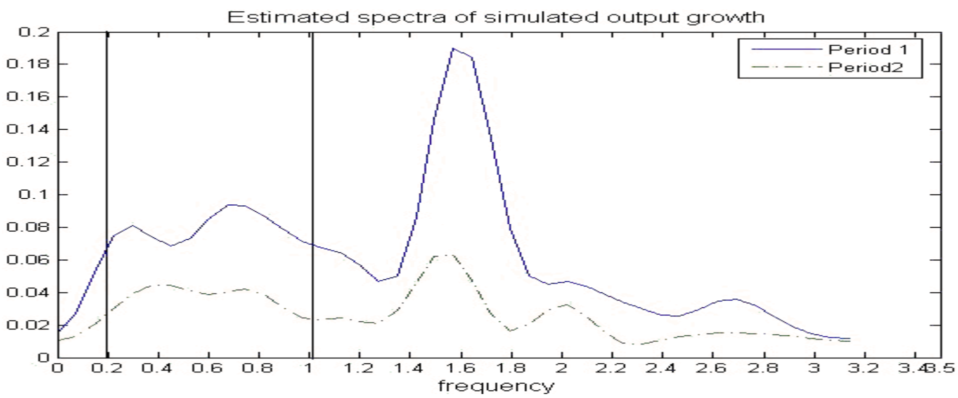

Figure 6:

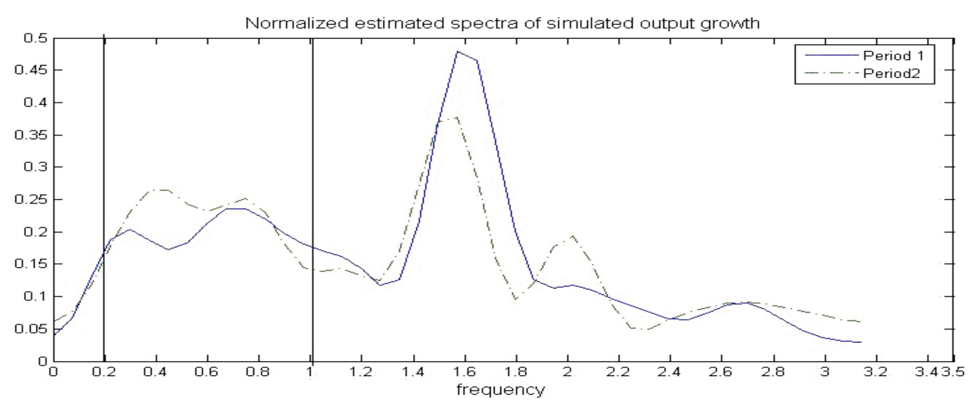

Figure 7:

Frequency	Integrated Normalized Spectrum Period 1simul	 Period 2 simul	Ho: Period 1 simul=Period 2 simul Ho: Period 1 simul≠Period 2 simul Test	 p-value
output growth				
Low	0.008 (0.03)	0.02 (0.03)	-0.31	0.76
Business cycle	0.38 (0.11)	0.39 (0.14)	-0.07	0.94
High	0.61 (0.14)	0.59 (0.13)	0.13	0.90

Table 8

5.4 A new scenario

In order reinforce the results in Section 5.3, we run a similar exercise, in which the parameter of the monetary policies in first period are chosen to obtain a bigger variance in the output growth, fixing the parameters of the monetary policies in the second period as in Clarida, Gali and Gartler and all the parameters of the models as estimated by Gali and Rabanal.

Using the parameter as in Table 9, the two output growth series are simulated from the model. Figure 8 and Table 10 show the bigger volatility of the series using the new set of parameter for Period 1. The increasing in variance can be explained by the decreasing of ϕ_π, that measures the aggressiveness of the monetary authority in response to increasing in the expected inflation, and by the increasing of ϕ_r, that is a measure of the correlations of the interest rate with its lagged value.

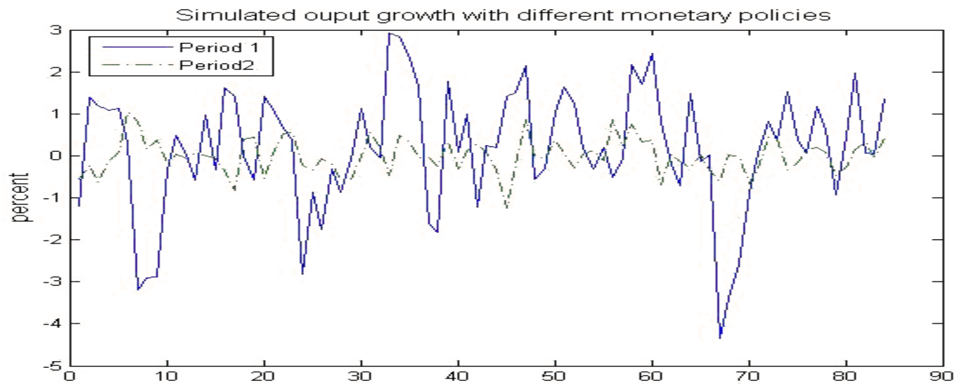

Figure 8:

	Period 1 scenario	Period 2
ϕ_r	0.85	0.79
ϕ_π	0.7	2.15
ϕ_y	0.3	0.93

Table 9

Output growth	Mean	Variance	Standard deviation
Period 1 -new scenario	0.20	2.097	1.44
1959-1979 sample	2.03	1.05	1.02
Period 2 - original simulation	0.00	0.169	0.41
1984-2004 sample	1.40	0.29	0.53

Table 10.

As before, the estimated spectra and the estimated normalized spectra are plotted, in Figure 9 and 10. Again, since the variance in the first period is much grater than in the second period, the estimated spectrum for Period 1 is above

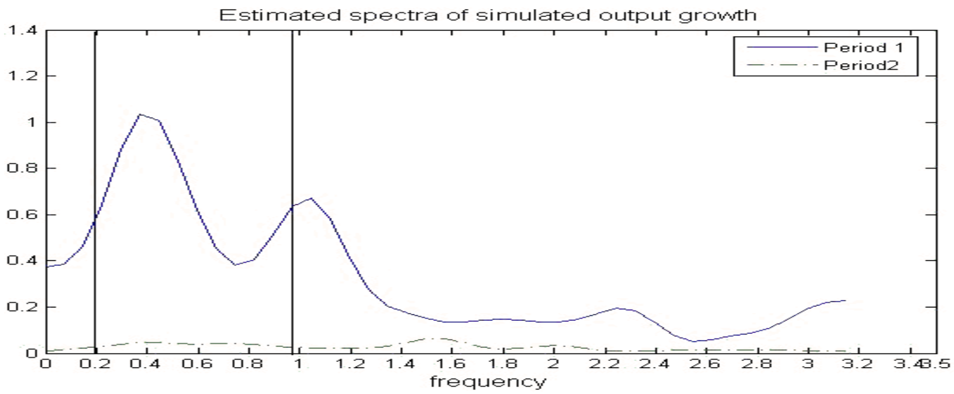

Figure 9:

that one in the second 2. However, in this new scenario, most of the variance for the output growth in the first period is due to business cycles frequencies and only a small part in the high frequencies range, in contrast with the simulation called Period 2, as it can be seen in Figure 10. The results for the normalized integrated spectrum tests are consistent to those one in the previous spectrum. A dramatic change in the monetary policy, as assumed in this scenario, does not modified the normalized spectrum in business cycle frequencies. However, at 10% of significance the test rejects that the normalized spectra are the same in the high frequencies range. If in the previous subsection we have seen that changing on monetary policy can imply no changing in the normalized spectrum in all range of frequencies, with this exercise we have found an example of different monetary policies that have an impact on high frequencies and not in the business cycle frequencies.

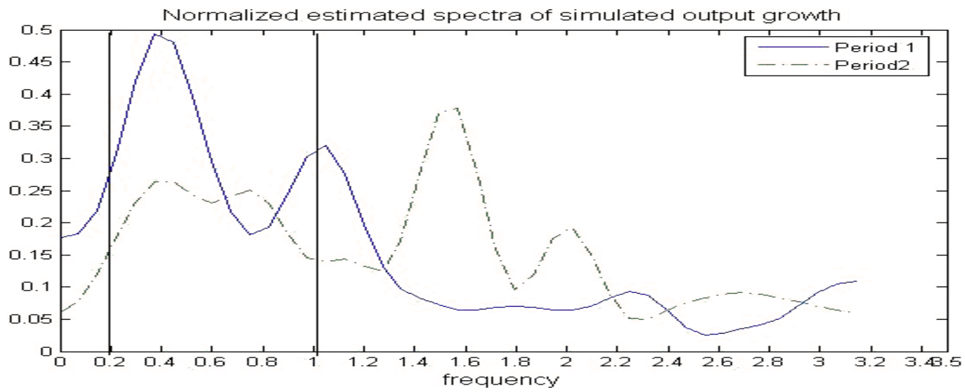

Figure 10:

Frequency	Integrated Normalized Spectrum		Ho: Period 1 simul=Period 2 simul Ho: Period 1 simul≠Period 2 simul	
	Period 1 simul	Period 2 simul	Test	p-value
output growth				
Low	0.07 (0.06)	0.02 (0.03)	0.61	0.54
Business cycle	0.60 (0.06)	0.39 (0.14)	1.40	0.16
High	0.34 (0.03)	0.59 (0.13)	-1.68	0.09

Table 8

6 Conclusions

The behavior of the spectrum for the output growth estimated in two sub-sample provides some important suggestions about the causes of the decline in the volatility in the economy after mid-80s. In particular, the decline in the normalized spectrum for the second sub-sample (1984-2004) at low frequencies could imply a changing in the long-run features of the process. We have shown that this view has a fundament, since we have found that the long-run variance of the output growth process has statistically changed. According this result, the "Great Moderation" can be justified by a changing in the moment of the stochastic process.

Moreover, the normalized spectrum has been used to test the three different hypothesis, "Good Luck", Good Practice", and "Good Policy", with the assumption that each of this hypothesis has effect just on a particular range of the frequencies. However, computing a simple exercise using a theoretical real-business cycle model, we can find example according which the thesis that changing in monetary policies change the normalized spectrum just in business cycle is rejected.

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