

Hours in Levels or in Differences? - Empirical Investigation Based on Altig, Christiano, Eichenbaum and Lindé (2004)

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1 Introduction

Starting with the seminal paper of Kydland and Prescott, 1982, [4], and Prescott(1986), [3], macroeconomists assign a large role to shocks as a driving force of a business cycle. However, it is still questionable whether neutral technology shocks are indeed the most important type of shocks in determining aggregate fluctuations at business cycle frequencies.

There is a line in the literature that distinguishes another possible type of technological change that may compete with a standard neutral technology shock. This shock is analyzed in Fisher (2003), [7], and represents a shock to a technology of producing capital goods, or embodied technology shock. As mentioned in Gali and Rabanal (2004),[2], the reason to introduce such a shock may be an observation that the price of new equipment goods is highly volatile.

Altig, Christiano, Eichenbaum and Linde (2004),[1], (reads further ACEL) extend the model of Fisher (2003) by introducing a number of real and nominal frictions into the model. They use the theoretical model to identify capital-embodied and neutral shock, as well as a monetary policy shock within a structural VAR and estimate impulse responses using actual data. Also, they find the estimates of structural parameters of the theoretical model by simulating impulse response functions from the theoretical model and fitting them to empirical impulse response functions. ACEL use their VAR to analyze two different assumptions of the theoretical model - specific and non-specific capital. My application of their model is different. There is a well-know debate between researchers on the effect arising from modification of hours in the VAR. Gali (1999),[5] and Christiano, Eichenbaum and Vigfusson (2003),[8], (read further (CEV)) were the first who paid attention to the question - what transformation of hours should be used in the VAR. Gali

specified VAR model of hours and average labor productivity and used first differences of hours worked in the VAR. As a result, his estimates produced negative comovement of hours worked and labor productivity - the fact that was contrary to the standard implication of RBC models. CEV argued that it does not make sense to first difference the hours worked. He argued that based on a standard neoclassical model hours must be stationary, as they are constrained from above and below. Their evaluations of Gali's model with hours in levels produced positive comovement in hours and labor productivity. In ACEL, the authors follow CEV approach and use hours in levels in their VAR model. In this paper, in addition to replication the results of ACEL, I use hours in differences with long-run restriction and compare the results for both models.

I find that my estimates are very similar to the estimates in ACEL. Also, I find that the model with hours in differences fails to explain the variability of macroeconomic variables so well as the model with hours in levels does. The impulse responses of the model in differences are similar to the responses of the model in levels and produce insignificant response of hours to a shock to technology. Thus, this paper concludes that the theory developed and defended by Gali and his followers can not be supported, at least with the data used by Christiano et. al.

2 Identification of a VAR

ACEL use the following assumptions when they identify VAR and technology and monetary policy shocks:

- 1) Only technology shocks can affect labor productivity in the long run
- 2) Only embodied- capital technology shocks affect the price of investment in the long run
- 3) Monetary policy is defined as the following interest-rate rule:

$$R_t = f(\Omega_t) + \epsilon_{Rt}$$

where Ω_t is information available to monetary authorities at date t and ϵ_{Rt} is the shock to monetary policy. ACEL assume that the shock to monetary policy is orthogonal to the variables in Ω_t .

To understand how to implement these assumptions in ACEL, consider the following structural VAR:

$$A_0 Y_t = A(L) Y_{t-1} + u_t$$

where $Y_t = [\Delta p_{iv,t}, \Delta a_t, Y_{1,t}, R_t, Y_{2,t}]'$. The structure of Y_t is such that all the variables preceding R belong to the information set of monetary policy, Ω_t , which means the variables

in Y_1 do not react immediately to the shock to the monetary policy, and the variables contemporaneously affected by the monetary policy belong to Y_2 .

The fact that monetary policy shock is orthogonal to information in variables preceding R in vector Y implies that the last two elements of the matrix A_0 for the rows 1 to 8 are zeros. The fact that variables in Y_2 are affected by a monetary policy shock, although they do not have an effect on monetary policy, implies that the last element in the ninth row of A_0 is zero. The assumptions about the long-run influence of technology shocks on labor productivity and assumption of a long run effect on price of investment are transferred into VAR identification in a way similar to Shapiro and Watson (1988),[6]. Consider the assumption of the long run behavior of the investment price. The structural VAR implies that the following equation describes the evolution of p_{Iv} ¹

$$\Delta p_{Iv,t} = \alpha_1 + \sum_{j=1}^p A_{11}^j \Delta p_{Iv,t-j} + \sum_{j=0}^p A_{12}^j \Delta a_{t-j} + \sum_{j=0}^p A_{13}^j Y'_{1,t-j} + \sum_{j=1}^p A_{14}^j R_{t-j} + \sum_{j=1}^p A_{15}^j Y_{2,t-j} + \epsilon_{\Upsilon t} \quad (1)$$

where $\epsilon_{\Upsilon t}$ represents the shock to capital embodied technology. If all the shocks other than $\epsilon_{\Upsilon t}$ have no influence on p_{Iv} in the long-run, then the following conditions must hold:

$$\sum_{j=0}^p A_{1k}^j = 0$$

for l from 1 to 5. Estimation of the investment price equation with this restriction can be realized by regressing it on first differences up to lag $p - 1$ of the explanatory variables other than lags of p_{Iv} , i.e. estimate the following equation²:

$$\Delta p_{Iv,t} = \alpha_1 + \sum_{j=1}^p B_{11}^j \Delta p_{Iv,t-j} + \sum_{j=0}^{p-1} B_{12}^j \Delta^2 a_{t-j} + \sum_{j=0}^{p-1} B_{13}^j \Delta Y_{1,t-j} + \sum_{j=1}^{p-1} B_{14}^j \Delta R_{t-j} + \sum_{j=1}^{p-1} B_{15}^j \Delta Y_{2,t-j} + \epsilon_{\Upsilon t} \quad (2)$$

The coefficients in (2) are defined from the coefficients of the structural VAR as follows:

$$B_{1k}^j = \sum_{n=0}^j A_{1k}^n$$

and thus, after estimating the restricted form of equation (2), I derived the coefficients of the structural form VAR as follows:

$$A_{1k} = DB_{1k}$$

¹Notice that A in this equation is a (1×5) row vector, while $Y'_{1,t-j}$ is a (5×1) column vector

²One can see that the summation for the terms with R and Y_2 start from 1 as opposed to 0 for other variables. This reflects the fact that the last two coefficients in A_0 are equal to 0.

where D is a lower triangular matrix, and $A_{1k} = \{A_{1k}^j\}_{j=0}^{p-1}$ and $B_{1k} = \{B_{1k}^j\}_{j=0}^{p-1}$ and $A_{1k}^p = -B_{1k}^{p-1}$.

The similar approach is used to introduce assumptions about labor productivity. The second equation in the VAR relates labor productivity with other explanatory variables and the neutral technology shock, ϵ_{zt} :

$$\Delta a_t = \alpha_2 + \sum_{j=0}^p A_{21}^j \Delta p_{Iv,t-j} + \sum_{j=1}^p A_{22}^j \Delta a_{t-j} + \sum_{j=0}^p A_{23}^j Y'_{1,t-j} + \sum_{j=1}^p A_{24}^j R_{t-j} + \sum_{j=1}^p A_{25}^j Y_{2,t-j} + \epsilon_{zt}$$

The fact that both technology shocks can affect labor productivity in the long run, and no other shocks can influence productivity in the long-run, is implemented in the following transformation used for estimation:

$$\Delta a_t = \alpha_2 + \sum_{j=0}^{p-1} B_{21}^j \Delta p_{Iv,t-j} + \sum_{j=1}^p B_{22}^j \Delta a_{t-j} + \sum_{j=0}^{p-1} B_{23}^j \Delta Y_{1,t-j} + \sum_{j=1}^{p-1} B_{24}^j \Delta R_{t-j} + \sum_{j=1}^{p-1} B_{25}^j \Delta Y_{2,t-j} + \epsilon_{zt} \quad (3)$$

One can see that all explanatory variables except price of investment and lags of productivity are represented in differences and the similar discussion as one from p_{Iv} equation applies here.

In ACEL, hours worked are used in levels, and are included as the third element in Y_1 matrix, and thus the fifth element of the vector Y_t . In this paper, I want to reexamine the findings of ACEL. In addition to the standard specification, I use hours in first differences³. For the difference specification of hours, however, I need an additional assumption that hours worked can not be affected by technology and monetary policy shocks. In this case I proceed with the analysis similar to above. With the lower triangular identification of A_{33}^0 , as in ACEL, and hours the first variable in Y_1 , the equation to estimate for hours will look like

$$\begin{aligned} \Delta h_t = & \alpha_3^3 + \sum_{j=0}^p A_{33}^{3j} \Delta p_{Iv,t-j} + \sum_{j=0}^p A_{32}^{3j} \Delta a_{t-j} + \sum_{i=1}^{3-1} A_{33}^{i0} Y_{3,t}^i + \sum_{j=1}^p A_{33}^{3j} \Delta h_{t-j} + \\ & \sum_{i=1, i \neq 3}^6 \sum_{j=1}^p A_{33}^{ij} Y_{3,t-j}^i + \sum_{j=1}^p A_{34}^3 R_{t-j} + \sum_{j=1}^p A_{35}^3 Y_{2,t-j} + \epsilon_{1t}^3 \end{aligned}$$

I present this ordering here, but I also tried to estimate VAR with different ordering - I moved the variables of the logarithms of hours to the third place in VAR. There were no particular reason in the change of the variables, it was just the matter of convenience.

³I follow the debates of Gali and Christiano et. al. and call it difference specification

As I explain more carefully below, I found that the stability properties of the estimated VAR are not robust to the change of variables in the VAR.

Same as the first two equations for VAR, this equation will be transformed to account for the identification assumption that technology and monetary policy shocks do not affect hours in the long-run, as follows:

$$\begin{aligned} \Delta h_t = & \alpha_3^3 + \sum_{j=0}^{p-1} B_{33}^{3j} \Delta^2 p_{Iv,t-j} + \sum_{j=0}^{p-1} B_{32}^{3j} \Delta^2 a_{t-j} + \sum_{i=1}^{3-1} \sum_{j=0}^{p-1} B_{33}^{ij} \Delta Y_{3,t-j}^i + \sum_{j=1}^p B_{33}^{3j} \Delta h_{t-j} + \\ & \sum_{i=4}^6 \sum_{j=1}^p B_{33}^{ij} Y_{3,t-j}^i + \sum_{j=1}^p B_{34}^3 R_{t-j} + \sum_{j=1}^p B_{35}^3 Y_{2,t-j} + \epsilon_{1t}^3 \end{aligned}$$

Equations for other variables in Y_1 do not need specification assumptions a la Shapiro and Watson. Because we assume lower triangular matrix A_{33}^0 , these equations look like

$$\begin{aligned} Y_{1t}^i = & \alpha_3^i + \sum_{j=0}^p A_{31}^{ij} \Delta p_{Iv,t-j} + \sum_{j=0}^p A_{32}^{ij} \Delta a_{t-j} + \sum_{j=1}^p A_{33}^{ij} Y_{1t-j}^i + \sum_{j=0}^p A_{33}^{ij} Y_{1t-j}^{-i} + \\ & \sum_{j=1}^p A_{34}^i R_{t-j} + \sum_{j=1}^p A_{35}^i Y_{2,t-j} + \epsilon_{1t}^{-3} \end{aligned}$$

for $i = 1$ to 6 and $i \neq 3$, where X^{-i} implies a vector with all elements but $-i$.

Equation 9 in the VAR identifies the monetary policy shock:

$$R_t = \alpha_4 + \sum_{j=0}^p A_{41}^j \Delta p_{Iv,t-j} + \sum_{j=0}^p A_{42}^j \Delta a_{t-j} + \sum_{j=0}^p A_{43}^j Y_{1,t-j}' + \sum_{j=1}^p A_{44}^j R_{t-j} + \sum_{j=1}^p A_{45}^j Y_{2,t-j} + \epsilon_{Rt}$$

Equation in Y_2 is

$$Y_{2t} = \alpha_5 + \sum_{j=0}^p A_{51}^j \Delta p_{Iv,t-j} + \sum_{j=0}^p A_{52}^j \Delta a_{t-j} + \sum_{j=0}^p A_{53}^j Y_{1,t-j}' + \sum_{j=0}^p A_{54}^j R_{t-j} + \sum_{j=1}^p A_{55}^j Y_{2,t-j} + \epsilon_{2t}$$

Equations 9 and 10 need not be transformed, because we do not impose any assumptions on the long-run behavior of these variables.

Now let's turn to the estimation of the parameters of the VAR. It is easy to see that equations 1 – 8 in the VAR can not be consistently estimated equation by equation by OLS, because some explanatory variables are correlated with the error term. We follow ACEL and estimate these equations by TSLS. The instruments we take are: a constant, p lags of Y_t and estimated error terms from the previous equations.

3 Implications

For empirical analysis, I use the exact data used in the original paper, which are taken from DRI basics economics database. The data are quarterly for the period 1959 : 1 – 2001 : 4. For the price of investment, they use investment deflator. Output, consumption, investment and labor input are transformed into per capita terms by dividing by population 16 years old and over. Money variable is represented by the MZM series constructed by St. Louis Fed. I follow the strategy in ACEL and assume that vector Y_{1t} consists of the following variables: inflation, capacity utilization, hours, GDP/hours - real wage, Consumption/GDP and Investment/GP, while the only variable in the vector Y_{2t} is velocity⁴. The dynamics of the data series used to construct these variables is represented on the figure (1).

We see from the graphs on (1), the series for investment and consumption may indeed be cointegrated with GDP. One can not be so sure about the stationarity of the series that appear in the VAR in levels, however. Because this is beyond the scope of my research here, I do not perform tests for cointegration or stationarity of the series, similar to ACEL.

The dynamics of the modified data, representing the series used in the VAR, is represented on the graph (2). One can see that the series are basically trendless, although again it is hard to guarantee the stationarity of the series. Anyway, I proceed with the given specification of the model. The impulse response function from the two models I estimate⁵ are shown in figures (3-5).

When trying to perform ACEL analysis for difference specification of hours I encountered a few difficulties I had to overcome. As in ACEL, I estimated the VAR with 4 lags. The VAR was diverging when I put hours in differences, meaning the roots of the VAR were outside the unit circle. I checked several lag specifications of VAR and found that VAR with 3 lags does not produce such a problem. I have to admit that the property of explosiveness of the VAR was not robust to the choice of the number of lags. Also, I tried to move hours to the first place in Y_{1t} . The resulting IRFs were not identical, but very similar to the original specification with hours on the third place in Y_{1t} .

One can see from the figures of impulse responses that the effect of a monetary policy shock on the 11 variables listed in the figures for level specification is similar to the effect shown in ACEL. The most substantial difference is the effect on output and consumption, although qualitatively the effects are not too much different across the two specifications. Looking at the difference between level and difference specification one can see that the

⁴All the variables, except capacity utilization and federal funds rate, are represented in logarithms in the VAR

⁵level and difference specification of hours

difference is not huge. The most dependent to specification are output and consumption variables. The effect on hours does not seem to depend on specification - taking into consideration figures (6) - (11), which show confidence intervals of IRFs, one can see that the impact effect of neutral technology shock on hours is positive for level specification or insignificantly negative for difference specification. In my figures, I get significantly positive response of hours to both technology shocks for level specification and insignificantly negative effect of hours to technology shocks for difference specification of hours. I use a different method to calculate error bands for impulse response functions, than the one used in ACEL. Namely, ACEL use a symmetric confidence region, calculated as plus or minus one estimated standard deviation time 1.96. I think there is no reason to assume normality and symmetry of the confidence interval. I used two tailed asymmetric 95% confidence interval.

My results for the basic specifications are very similar to ACEL⁶. We see that interest rate drops by about 0.6% in response to the monetary policy shock - ACEL call this “liquidity effect”. The effect of monetary policy shock on almost all variables in about or larger than 1 year. The effect on price of investment is insignificantly different from 0. Inflation picks after output and consumption, in about 2 years after the policy shock occurs. Investment, output, consumption and hours rise in a hump shaped manner. IRFs from difference specification are very close to level specification.

The shock in the neutral technology leads to permanent rise in output and consumption, decrease in velocity and the price of investment, and increase in the wage rate. Hours rise after the neutral technology shock for the level specification. The IRFs resulting from the difference specification reveal that the main difference is in the effect on wage (significantly rises on impact) and velocity (no statistically significant response). Also, the effect on output, consumption, investment and many other variables, together with hours worked, is insignificantly different from zero.

The shock to the capital-embodied technology produces, similar to ACEL, negative permanent response of the wage, positive rise in consumption, decrease in investment price. It is interesting to note that the effect from the difference specification looks very similar to the level specification, with insignificant initial response of hours and positive response of hours afterwards. Inflation, investment price and velocity fall as a result of the shock, real wage effect is not significant.

⁶I assign all the differences in my results from that in ACEL to the change in the number of lags - ACEL use 4 lags, while I use 3 lags in the model. Although ACEL tests favor 4 lag specification, it produces diverging VAR when I use differences of hours instead of levels. Thus, I use 3 lags in this paper. My results for 4 lags and hours in levels are almost identical to the results in ACEL

4 Variance Decomposition Exercise

In this section, I repeat the effect of the volatility of the three shocks on the main 11 time series. To calculate this effect, the VAR estimates are used to generate detrended data. Detrending implies that no constant was used to generate the data and all the initial conditions were set to 0. My results are presented in Table (1) and figures (12)-(19). Numbers in Table (1) show the percentage variance of the HP 1600 filtered data. We see that the difference specification assigns a smaller role to the shocks to embodied capital and monetary policy in explaining fluctuations of the 11 macroeconomic variables, especially in the end of the period. The only shock that adds more or less to the variance of these variables is the neutral technology shock. This influence, however, seems smaller than implied by the level specification of hours in the VAR.

Similar to ACEL, for level specification we conclude that the effect of the shocks we study was more important for aggregate fluctuations of the variables at business cycle frequencies before 1980s. One can see that monetary policy shocks play a relatively big role in cyclical fluctuations of all the variables except real wage and price of investment. The largest influence is on the federal funds rate, velocity and capacity utilization. Investment specific shock plays comparatively large role in the cyclical volatility of the price of investment and small role for volatility of other variables. Neutral shock plays large role in the volatility of wages and inflation. Overall, the three shocks together explain more than a half of the variance of the investment price at a business cycle frequency and almost 50% of fluctuation in real wage and interest rate. The least effect of the three shocks is noticed on the volatility of consumption, hours and money growth.

5 Conclusion

This paper is based on the paper of ACEL, although it focuses on the empirical estimation of the VAR implied by the medium scale model with a number of nominal and real rigidities. The idea of the paper is different in that we do not study firm-specific capital idea, but we are trying to shed some light to a dilemma raised by Gali and Christiano et. al. on whether the log of hours in the VAR should be placed in levels or in differences. The result that emerges is that the model with hours in differences fails to explain the variability of macroeconomic variables so well as the model with hours in levels does. The impulse responses of the model in differences are not different too much from responses of the model in levels and do not suggest that hours fall in response to a shock to technology. Thus, we conclude that the data used in ACEL can not support the theory developed and defended by Gali and his followers.

References

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- [8] Christiano, Lawrence J. and Martin Eichenbaum, and Robert Vigfusson, “What Happens After a Technology Shock?”, 2003, Northwestern University, manuscript

6 Appendix

Figure 1: *The Dynamics of the Time Series in the VAR*

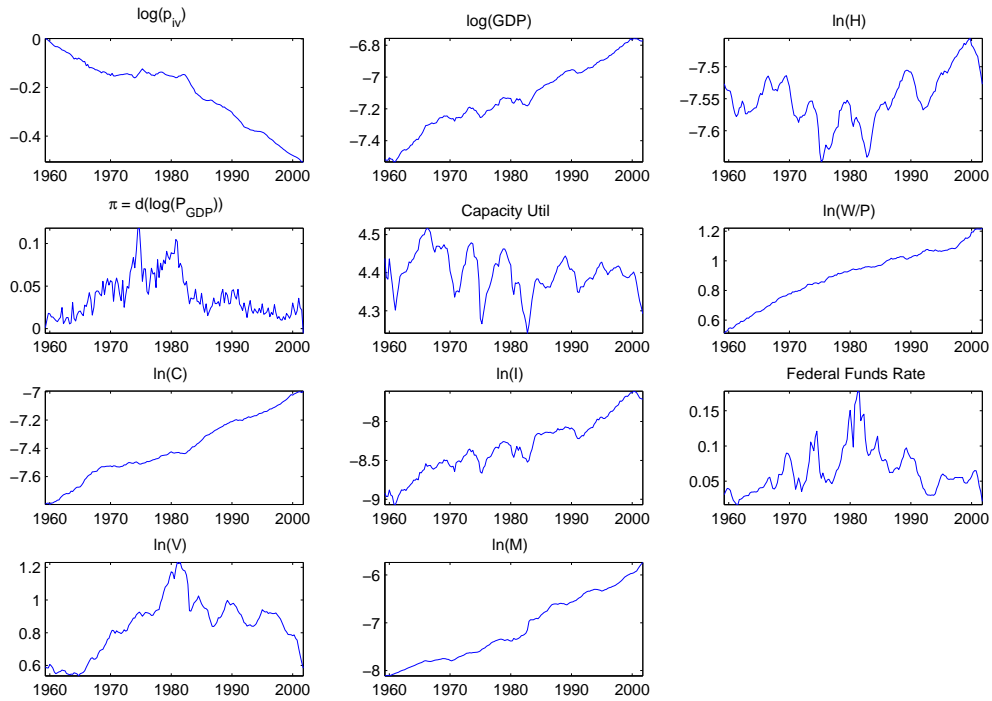


Figure 2: *The Dynamics of the Time Series in the VAR*

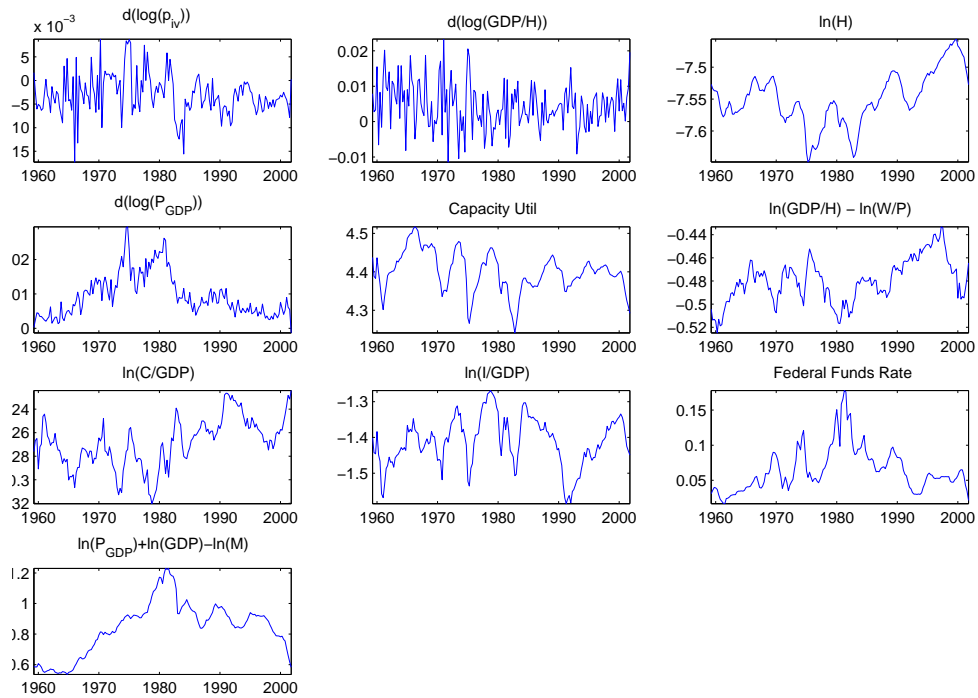


Figure 3: Comparison of IRFs from the two model specifications

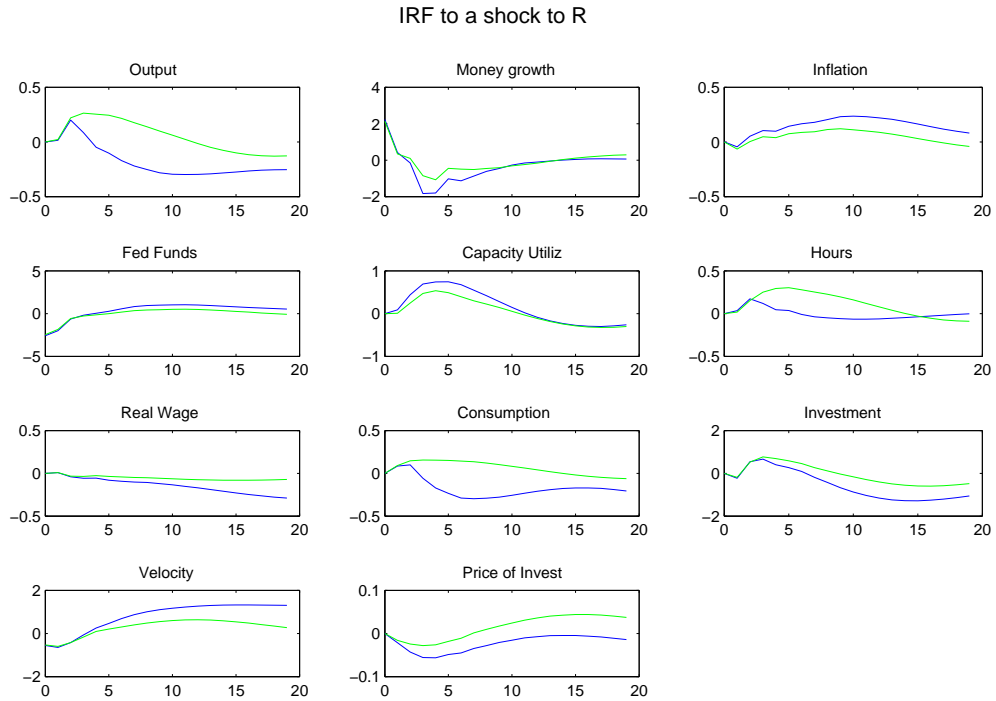


Figure 4: Comparison of IRFs from the two model specifications

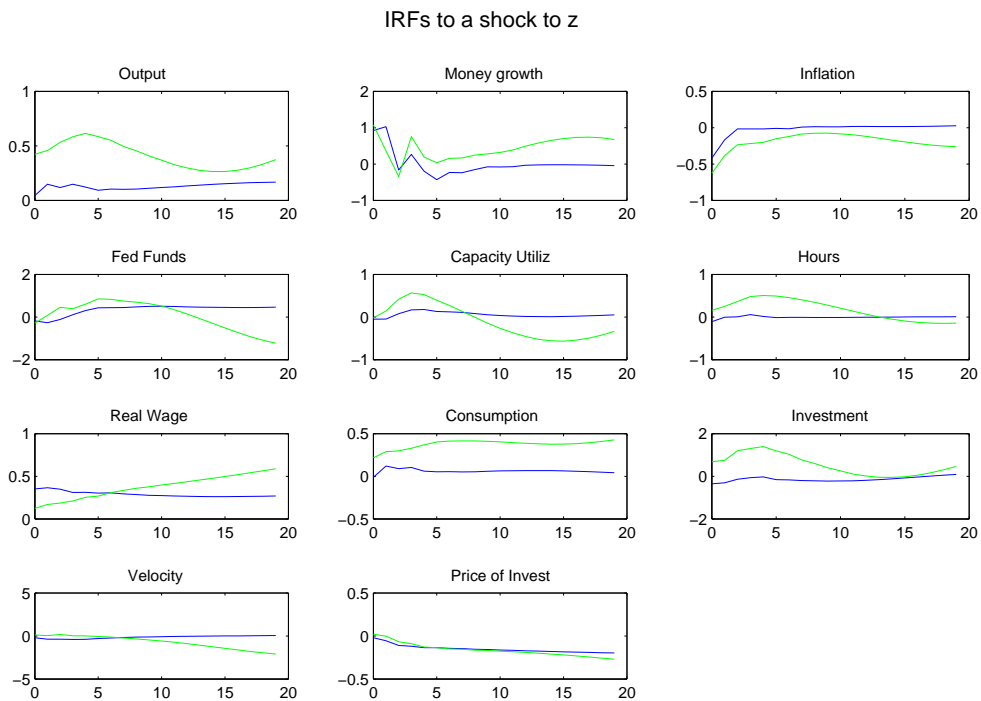


Figure 5: Comparison of IRFs from the two model specifications

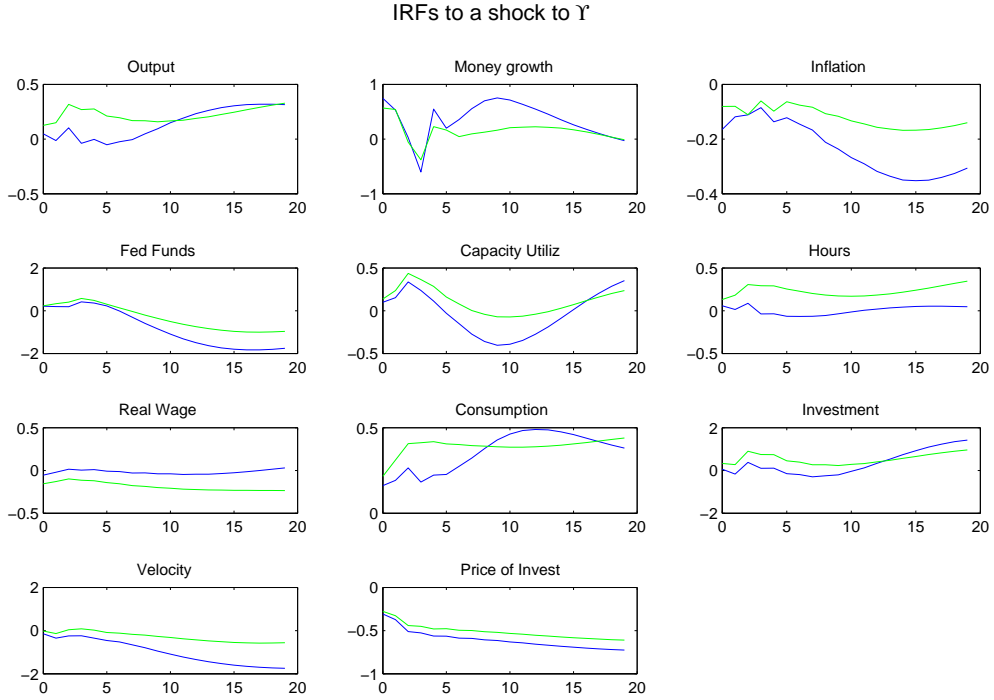


Table 1: Contribution of Shocks to Cyclical Variance,%

Shocks	Difference Specification				Level Specification			
	All 3	Embod.	Neut.	Monet.	All3	Embod.	Neut.	Monet.
Output	24.54	5.56	2.06	19.05	44.99	4.81	15.98	17.63
Money Growth	23.52	2.67	2.64	18.15	21.83	1.89	2.56	17.04
Inflation	29.32	5.83	14.33	16.40	41.09	1.73	31.71	8.14
Fed.Funds Rate	42.49	6.89	1.59	40.10	36.11	4.04	4.58	30.62
Capacity Util.	31.36	7.43	0.84	27.92	30.86	4.20	6.83	16.65
Hours	23.81	5.92	2.72	15.12	39.23	4.85	11.06	17.18
Real Wage	46.94	2.67	48.33	5.07	9.67	4.38	7.06	1.71
Consumption	19.84	6.69	1.12	16.48	67.97	23.97	13.95	19.32
Investment	27.24	7.14	0.83	25.77	29.22	3.76	7.24	14.05
Velocity	31.78	3.68	2.90	28.56	21.84	1.70	8.49	19.76
Investm. price	54.19	45.56	3.90	2.34	46.99	35.20	3.64	2.33

Figure 6: *IRFs with error bands, neutral technology shock, Z, level specification*

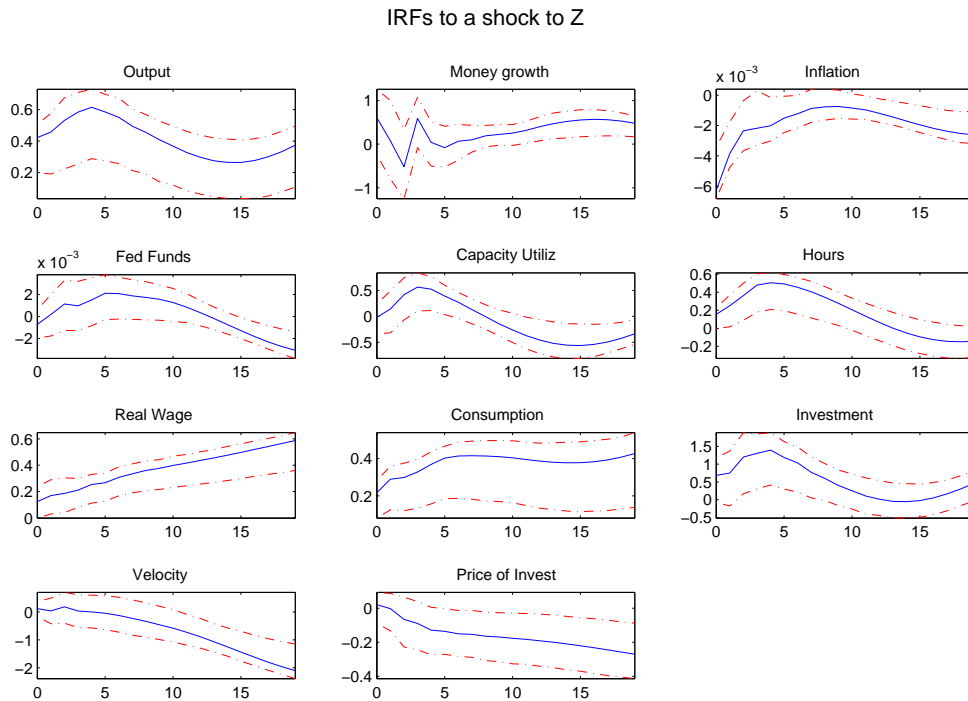


Figure 7: *IRFs with error bands, neutral technology shock, Z, difference specification*

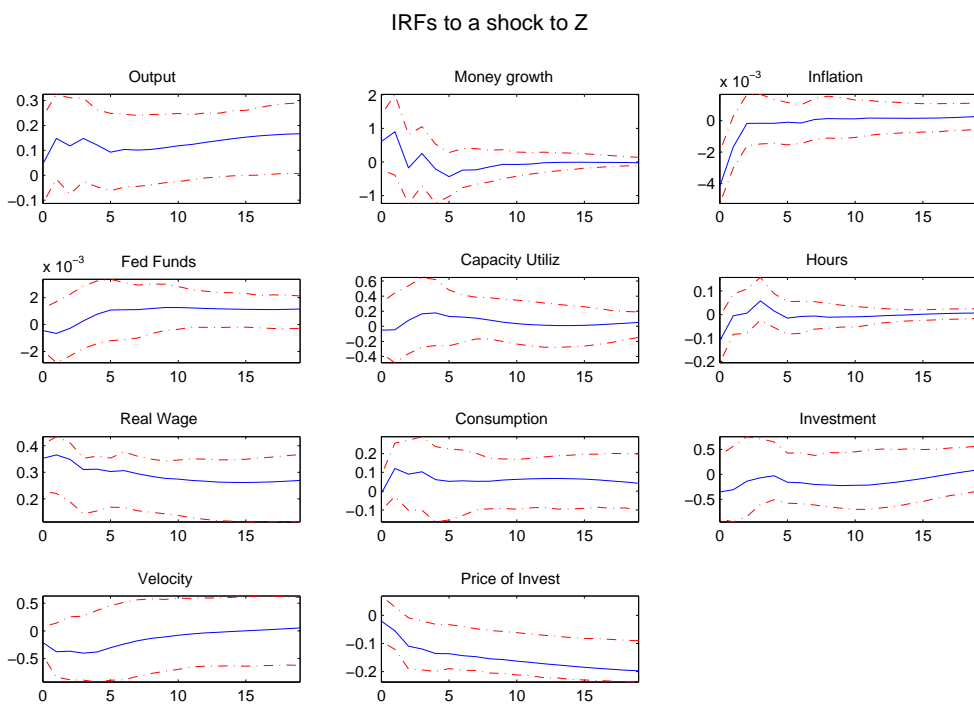


Figure 8: IRFs with error bands, capital embodied technology shock, Υ , level specification

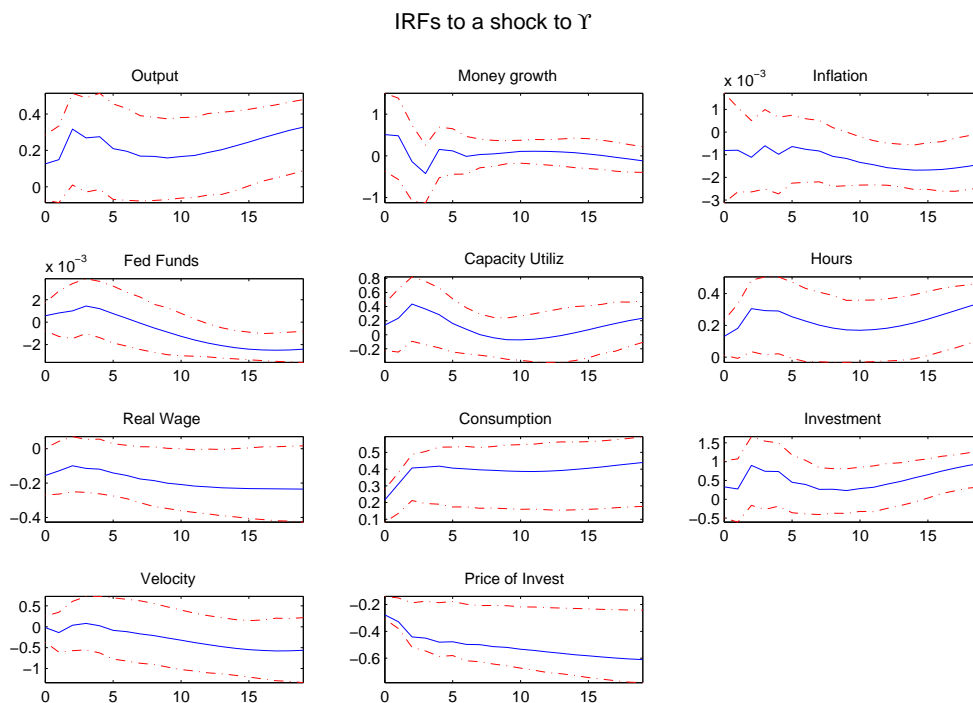


Figure 9: IRFs with error bands, capital embodied technology shock, Υ , difference specification

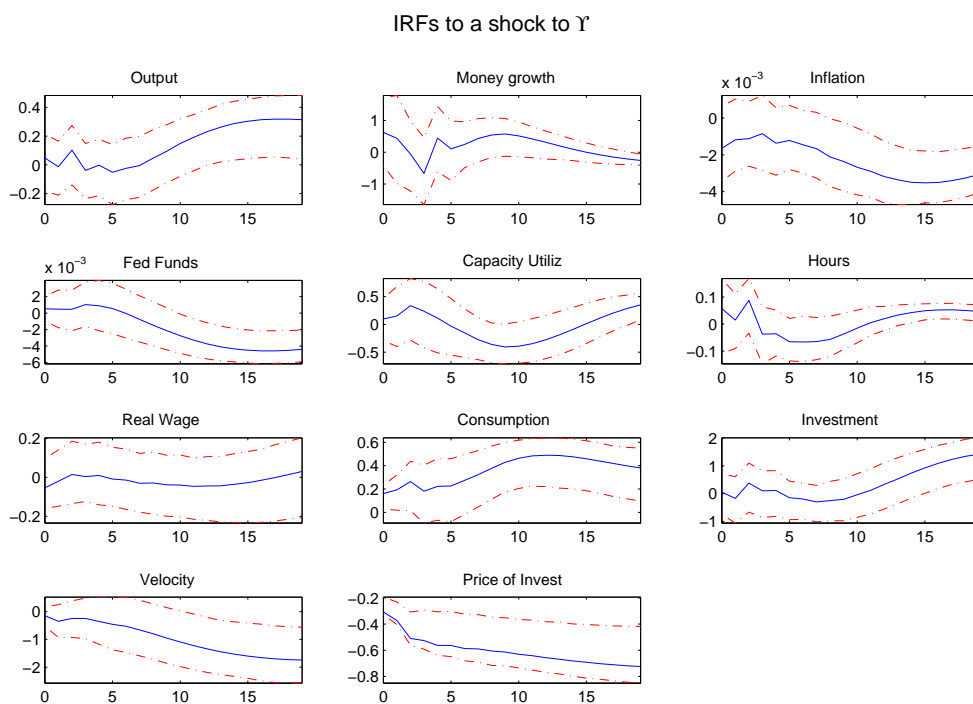


Figure 10: *IRFs with error bands, monetary policy technology shock, R, level specification*

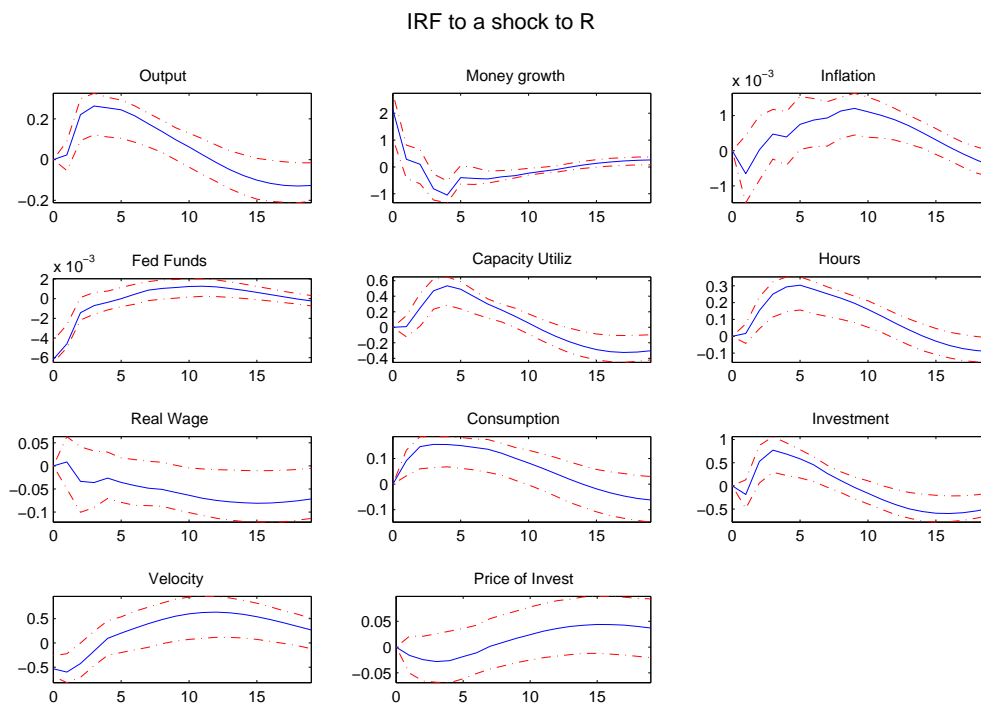


Figure 11: *IRFs with error bands, monetary policy technology shock, R, difference specification*

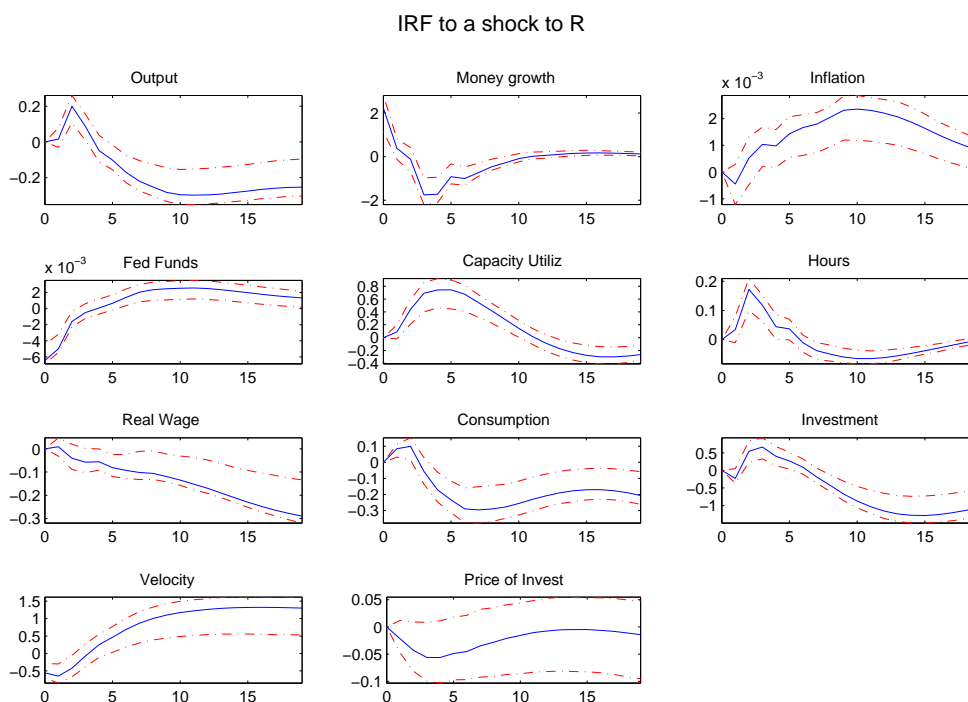


Figure 12: *Variance decomposition of historical data, monetary policy and technology shocks, level specification*

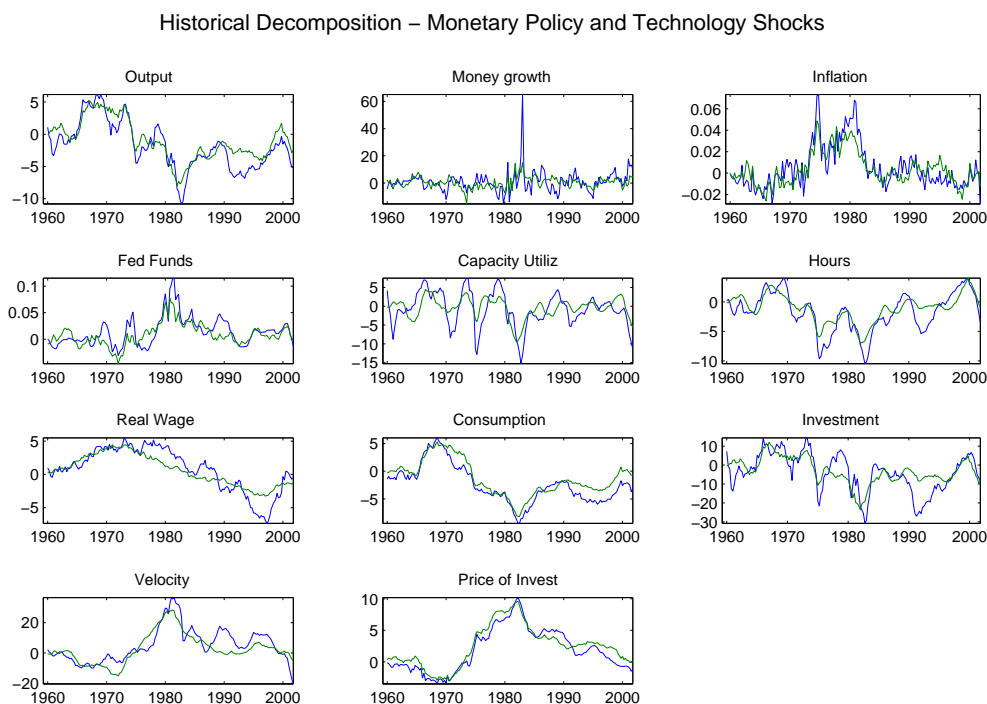


Figure 13: *Variance decomposition of historical data, monetary policy and technology shocks, difference specification*

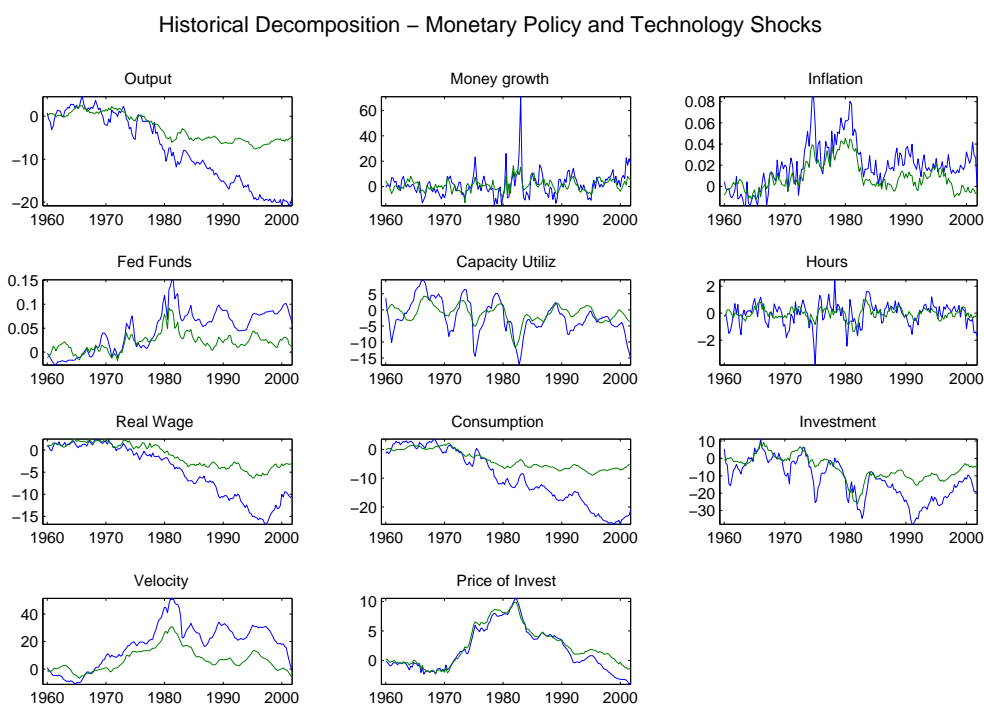


Figure 14: *Variance decomposition of historical data, neutral technology shock, Z, level specification*

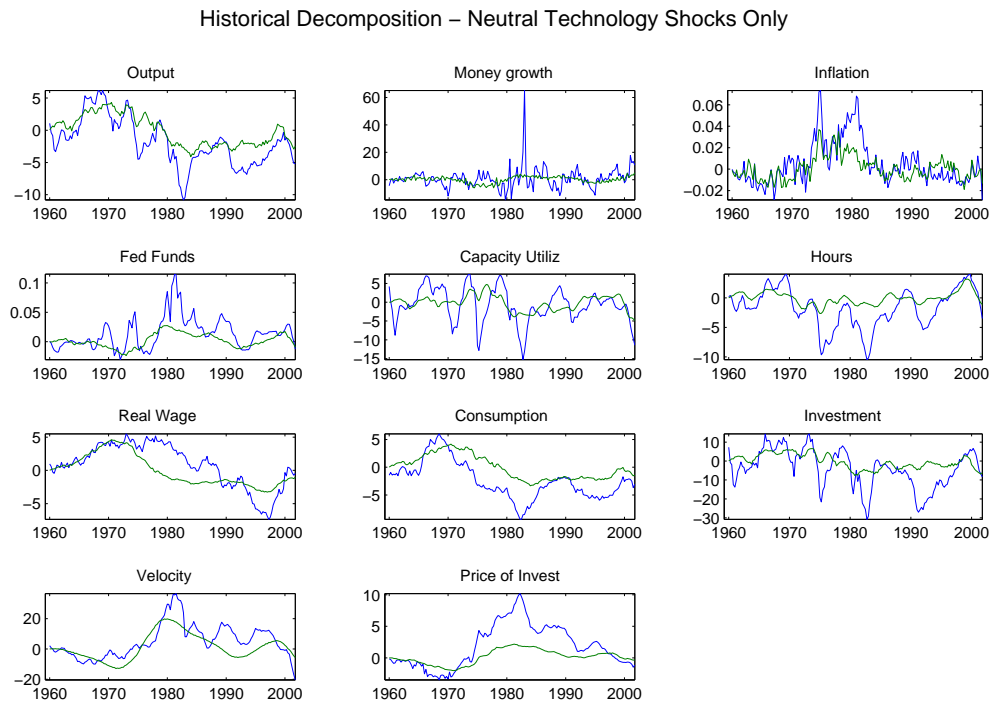


Figure 15: *Variance decomposition of historical data, neutral technology shock, Z, difference specification*

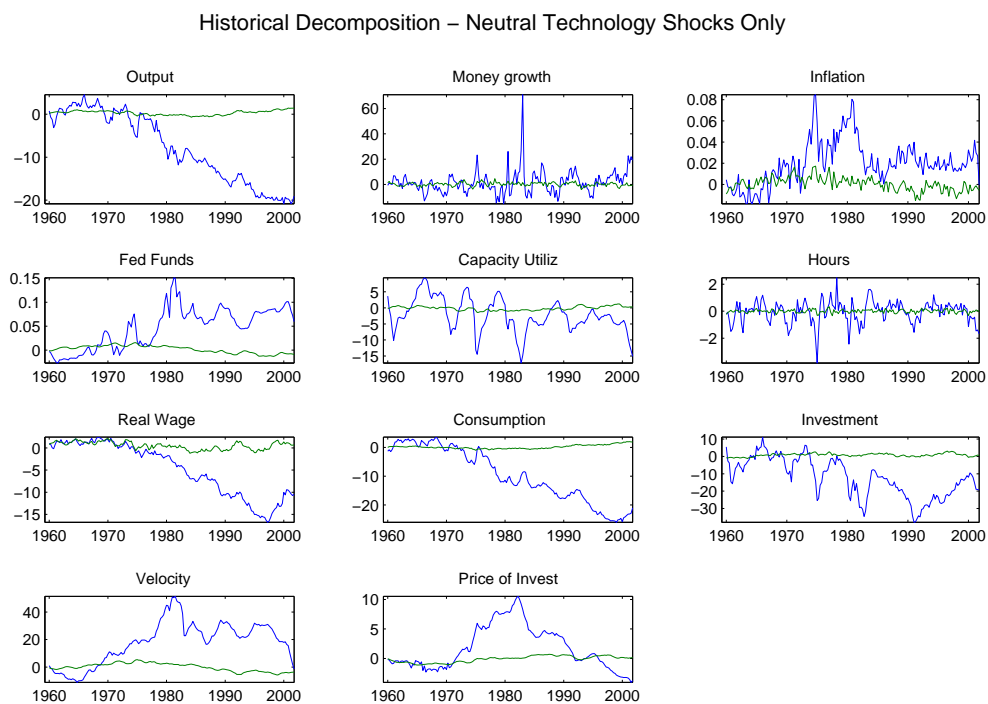


Figure 16: *Variance decomposition of historical data, neutral technology shock, Υ , level specification*

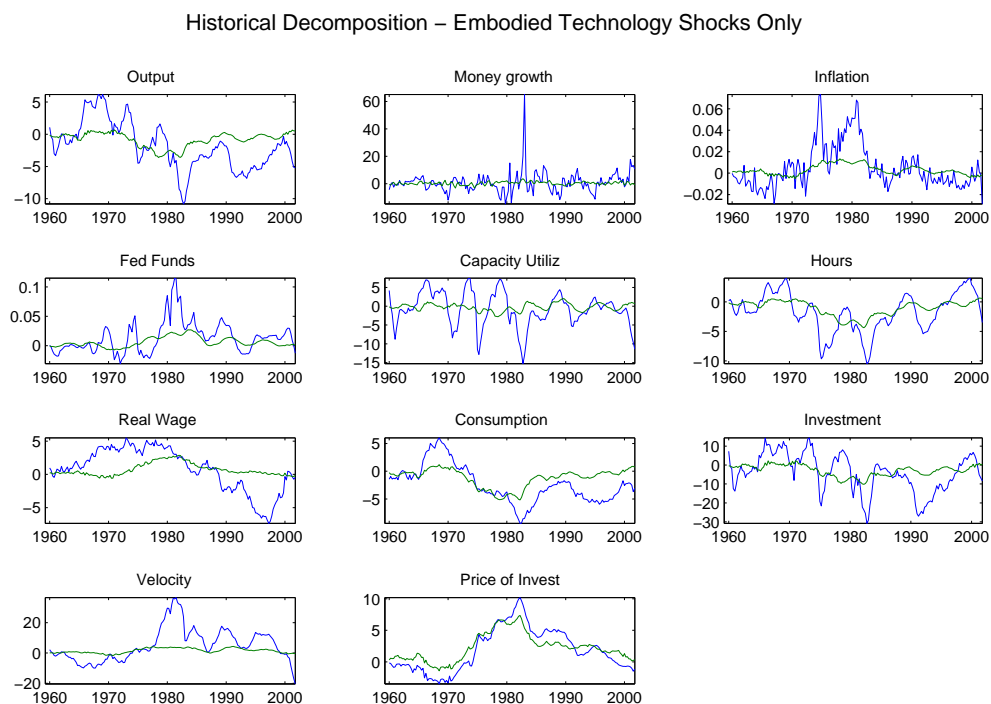


Figure 17: *Variance decomposition of historical data, neutral technology shock, Υ , difference specification*

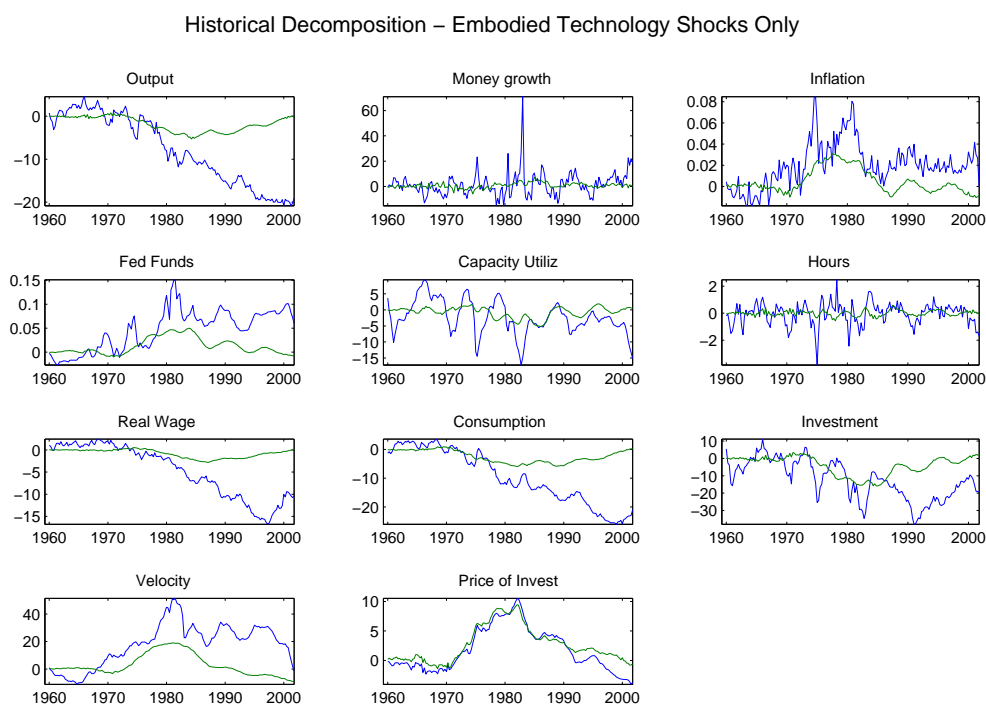


Figure 18: *Variance decomposition of historical data, monetary policy shock, level specification*

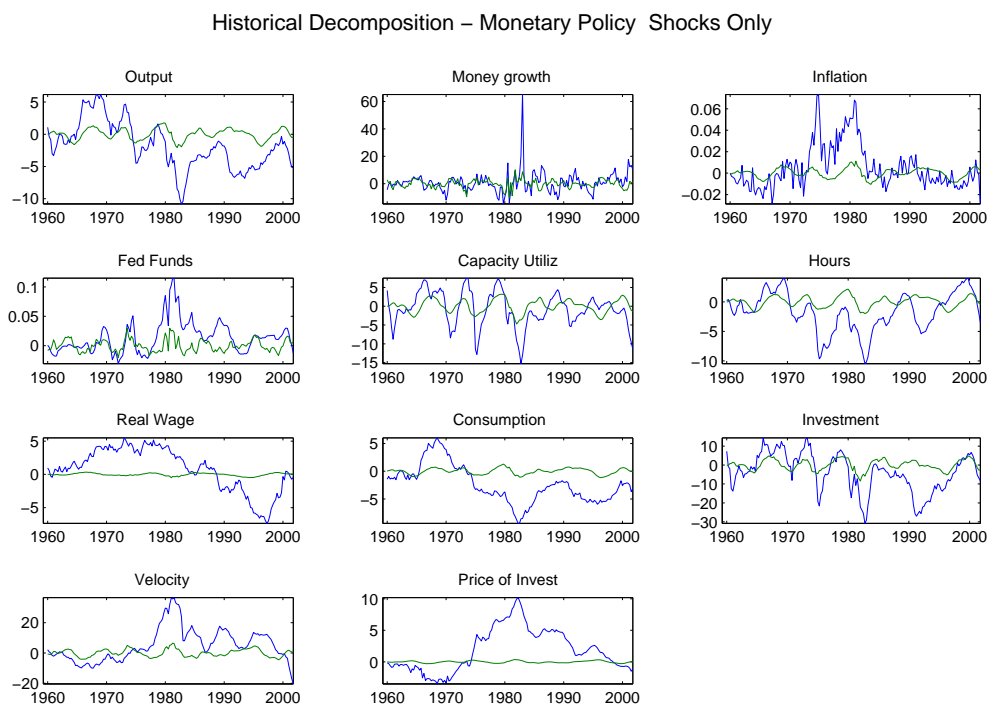


Figure 19: *Variance decomposition of historical data, monetary policy shock, difference specification*

