ECON 327: Final Paper

Alice Ghezzi

Based on:

“Is the technology-driven real business cycle hypothesis dead?

Shocks and aggregate fluctuations revisited”

Francis and Ramey (2003)
Introduction

The purpose of this paper is twofold. First, we briefly review and replicate the findings of the recent literature about the role of (Hicksian-neutral) technology shocks as driving source of aggregate fluctuations in Real Business Cycle models. Second, we try to address some original questions, to our knowledge not yet discussed in the literature, regarding the correct specification of the estimated structural VAR, and their implication for estimation and interpretation of the results.

In particular, this work takes its moves from the replication of the working paper by Francis and Ramey (2003), “Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited”. The authors investigate both empirically and theoretically the validity of the findings by Gali (1999). Consistently with the literature, they find evidence of a persistent decline of hours in response to a positive technology shock. Finally, they try to address the objections raised by a recent strand of papers, starting with Christiano, Eichenbaum and Vigfusson (2003), both from a theoretical and an empirical perspective.

It is worth emphasizing that whether technology shocks cause hours to increase or decrease, could be crucial for solving the debate that sees RBC theories in opposition to the New Keynesian models. In particular, correctly evaluating the sign of the effect of technology shocks on hours in the data would determine if the substitution effect dominates the wealth effect as claimed by the RBC theory, or if the opposite applies as claimed by the New-Keynesians.

Instead of attempting a reconciliation of the two frameworks on a theoretical basis, we will focus on the methodological and empirical issues explored in Francis and Ramey (2003), considering the model as an established benchmark. This paper is organized as follows. The next section presents the replication of the results. We present further evidence in support of Gali (1999) findings, and control for the robustness of

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1 In the Appendix, we report the model's main equations and the steady state solutions. For the underlying motivations and theoretical foundations, we refer the reader to the seminal paper Gali (1999).
these arguments. In addition, we discuss the issues relative to the appropriate transformation for the series of hours worked, and introduce the new measure for the labor force constructed by Francis and Ramey (2004). Inspired by Fernald (2005), we provide additional analysis of the series involved in the estimation procedures in the following section, by testing for the presence of single and multiple breaks both in the mean and in the variance of the relevant variables. We subsequently explore the presence of GARCH effects in the VAR identified shocks and develop some seminal techniques for accounting for the presence of conditional heteroskedasticity in the estimated VARs. Finally, we conclude suggesting topics for further research.

**Empirical results**

We use quarterly data from 1947:1 to 2002:4 to estimate the model. For the labor productivity and the series of hours worked, we consider available data for both the business and the non-farm sectors, from the Bureau of Labor Statistics. The real wage measure is computed as in Francis and Ramey (2003), dividing the BLS measure of nominal hourly compensation in private business by the BLS deflator for private business. The capital tax series, was constructed by Jones (2002), and kindly provided by Craig Burnside. For the other macro aggregates we use chain-weighted 2000 dollar NIPA series. Following Francis and Ramey (2003), all relevant variables are put on a per capita basis. Given the recent work by Francis and Ramey (2005), instead of using only the “population age 16 and above” series, we provide results also for the per capita variables obtained by using a new measure for labor force, showing that this measure, adjusted for low-frequency demographic movements, possibly settles the debate about whether we should consider hours in levels or in first differences.

Based on the standard Real Business Cycle model, we estimate a structural VAR of labor productivity, $x_t$, and hours, $n_t$. We use the long-run identification scheme as in Gali (1999) and impose that only the

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2 We thank Neville Francis for providing the demographic-adjusted series for the U.S. Population.
technology shock, $e_t^z$, can have a long-run effect on labor productivity:

$$\begin{bmatrix} \Delta x_t \\ \hat{n}_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & 0 \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} e_t^z \\ e_t^m \end{bmatrix}$$

The identifying assumption, $C^{12}(L) = 0$, implies that technology shocks are the sole contributors of the unit root in labor productivity. For now, assume that $\hat{n}_t$ is an opportunely transformed, stationary measure for hours worked. In order to check for the robustness of the results, we estimate impulse response functions for the SVAR with variables in differences, both for business and non-farm measures. In Figure 1 we report the estimated impulse response functions, along with their confidence bands computed with bootstrap (1000 repetitions) for the business sector's series. The impulse response functions computed from an analogous SVAR for the non-farm sector give consistent results.

**Figure 1: Impulse Response Functions (business sector, first differences)**

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3 Notice that here we are also implicitly assuming that there is a unit root in labor productivity. This assumption is not necessary, though. In addition, ADF-GLS t-tests fail to reject the presence of a unit root against the presence of a linear trend.

4 Solid lines are the estimated IRFs, dashed bands are the 5-th and 95-th percentiles, dotted lines are 1-st and 99-th percentiles.

5 Please refer to the Matlab output.
We can further assess the validity of the technology shocks, identified using long-run identifications, by checking for an alternative way to identify the technology shock, involving the real wage. Indeed, since only a technology shock should have a permanent effect on real wages, we estimate a SVAR with real wages instead of labor productivity and find substantially similar results: hours goes down in response to a positive technology shock (see Figure 2).

**Figure 2: Alternative Identification scheme, real wages instead of labor productivity**

Overall, results appear to be robust, both to the use of different series for productivity and hours (business versus non-farm sector), and to the use of alternative identifying hypotheses, giving rise to a negative response of hours to a positive technology shock. However, in order to determine if what we called technology shock is actually not affected by other sources of noise, other than labor augmenting technical change, we need to refer to the equations for the balanced growth path of the economy.
Both technology shocks and permanent shifts in capital income tax rates can affect labor productivity in the long-run. Accordingly, the shocks identified using Galí’s assumption could include capital income tax rate shocks. Thus, Francis and Ramey (2003) control for capital taxes by including Jones (2002) series of average marginal capital tax rates as an exogenous variable in the SVAR and re-compute the impulse response functions. Figure 3 shows that when including this exogenous variable (dotted line) in the SVAR, results do not appear to be significantly different from the original estimations (solid line). Indeed, in this case, only the estimated coefficients of the VAR are affected, and we assume the bivariate system is still valid.

**Figure 3: Controlling for Capital Taxes**

While permanent shifts in technology should not affect long-run labor supply, Francis and Ramey (2003) emphasize that shifts in the share in government spending, preference shocks, labor income tax rates and capital income tax rates can all have permanent effects on labor. Therefore, in order to check for
the correct specification of the technology shock, we try to replicate their test for exogeneity.

Consider the following four sets of dummy variables, typically assumed to be exogenous shocks:

- Romer and Romer (1989) narrative approach: monetary dummies;
- Hoover and Perez (1994): oil shock dummies;
- Ramey and Shapiro (1998): war dates (defense expenses build-up prior to conflicts);
- Federal Fund Rate\(^6\) (see Bernanke and Blinder (1992));

We compute and report in Table 1 the p-values of the F-test for the joint significance of current and four lagged values of each set of dummies in explaining the identified technology shock. Whereas our results do not fully match those in Francis and Ramey (2003), some pattern can still be detected. Overall, the p-values for the technology shocks are higher than for the non-technology shock, suggesting that exogenous shocks are not correlated with the identified technology error term. However, the pattern we are not able to recognize in our results is that the exogeneity test should exhibits an inverted scheme in the case of the specification in levels, such that Francis and Ramey (2003) call for mis-specification of the model, when estimated à la Christiano, Eichenbaum and Vigfusson (2003).

<table>
<thead>
<tr>
<th>Difference specification, Business</th>
<th>Tech</th>
<th>Non-tec</th>
<th>Tech</th>
<th>Non-tec</th>
<th>Tech</th>
<th>Non-tec</th>
<th>Tech</th>
<th>Non-tec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference specification, Non-Farm</td>
<td>Tech</td>
<td>0.314</td>
<td>0.191</td>
<td>Romer</td>
<td>0.074</td>
<td>0.176</td>
<td>0.005</td>
<td>0.315</td>
</tr>
<tr>
<td>Difference specification, Non-Farm</td>
<td>Non-tec</td>
<td>0.176</td>
<td>0.005</td>
<td>Perez</td>
<td>0.004</td>
<td>0.002</td>
<td>0.315</td>
<td>0.066</td>
</tr>
<tr>
<td>Levels specification, Business</td>
<td>Tech</td>
<td>0.269</td>
<td>0.115</td>
<td>Ramey</td>
<td>0.022</td>
<td>0.206</td>
<td>0.007</td>
<td>0.311</td>
</tr>
<tr>
<td>Levels specification, Business</td>
<td>Non-tec</td>
<td>0.102</td>
<td>0.002</td>
<td>Shapiro</td>
<td>0.008</td>
<td>0.002</td>
<td>0.007</td>
<td>0.311</td>
</tr>
</tbody>
</table>

\(^6\) Including only lagged values.
The evidence calls forth further discussion about the convenient transformation to be applied to the available series of hours in order to have a stationary series. Indeed, several works addressing a similar question with different approaches found the same results: output’s and hours’ responses to a technology shock are negatively correlated. It seems that the estimated effects of technology shocks crucially depend on whether one estimates the structural VAR in levels or in differences. For instance, Gali and Rabanal (2004) found consistent results when estimating the SVAR with total hours. Nonetheless, when using hours per capita they find, consistently with the literature, that a positive technology shock lead to positive response of hours per capita when taken in levels, whereas the specification in first differences lead to opposite result.

Francis and Ramey (2004) claim that standard measure of hours per capita is significantly affected by low-frequency demographic and institutional trends in the measure used for computing per capita series. Accordingly, rather than taking the side of the levels- or the differences-argument on theoretical a-priori reasons, they find that adopting their new measure for population, adjusted for the working-age population, overturns some key results. In particular, positive technology shocks (identified through long-run restrictions) lead to a decrease in hours whether one assumes that hours per capita are stationary or non-stationary (see Figure 4).

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8 Corrected for government workers, younger population and school enrollment, older population, institutionalized population.
We report in Table 2 the results of ADF-GLS t-test for the presence of unit root on the variables involved in the estimation. Rather taking a stand regarding the more plausible model to assume, when constructing the test, report both the values for the hypothesis of no intercept and intercept in the augmented regression.

Table 2: ADF-GLS tests

<table>
<thead>
<tr>
<th>Optimal lag length</th>
<th>Hours (B)</th>
<th>Hours (NF)</th>
<th>Hours (B) New</th>
<th>Hours (NF) New</th>
</tr>
</thead>
<tbody>
<tr>
<td>No intercept, no trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAIC</td>
<td>-0.57</td>
<td>-2.82</td>
<td>-3.21</td>
<td>-0.63</td>
</tr>
<tr>
<td>4 lags</td>
<td>-0.37</td>
<td>-2.62</td>
<td>-3.00</td>
<td>-0.36</td>
</tr>
<tr>
<td>8 lags</td>
<td>-0.85</td>
<td>-1.64</td>
<td>-1.88</td>
<td>-1.06</td>
</tr>
<tr>
<td>5% CRIT. VALUE</td>
<td>-1.95</td>
<td>-1.95</td>
<td>-1.95</td>
<td>-1.95</td>
</tr>
<tr>
<td>Intercept, no trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAIC</td>
<td>-4.32</td>
<td>-2.98</td>
<td>-4.4</td>
<td>-5.64</td>
</tr>
<tr>
<td>4 lags</td>
<td>-3.73</td>
<td>-2.75</td>
<td>-4.65</td>
<td>-6.57</td>
</tr>
<tr>
<td>8 lags</td>
<td>-5.02</td>
<td>-1.76</td>
<td>-2.37</td>
<td>-5.42</td>
</tr>
<tr>
<td>5% CRIT. VALUE</td>
<td>-2.88</td>
<td>-2.88</td>
<td>-2.88</td>
<td>-2.88</td>
</tr>
</tbody>
</table>

This test statistic, developed by Elliott, Rothenberg and Stock (1996) is the most powerful test in the
presence of local-to-unity root, overcoming the affected ability to choose between specifications in levels or in first differences. In particular, the test has really good power properties for the case of no constant and no trend.

The modified measure for hours worked in the business sector exhibits a clearcut improvement. Whereas we could not reject the presence of a unit root in the old series, we can reject the null hypothesis for the measure obtained using the new series for labor force. Unfortunately, the results for the series of non-farm hours relies on the assumption we make about the true DGP. Overall, for the original series we reject the presence of a unit root. On the contrary, once we adjust for population, we fail to reject the null hypothesis. This could be due to the fact that the adjusted measure of population displays a slightly upward sloping trend. As a consequence, unit root tests might be affected.

It is important to recall that, as Pesavento and Rossi (2003) show, impulse response function estimates and confidence bands that rely on unit root pretests have bad small sample properties. As a consequence, impulse responses based on VARs estimated in levels or first differences have bad coverage properties as well.

**Structural breaks**

A completely different prospective for assessing the right specification of the series of hour worked takes into account the presence of structural breaks of the series used in the SVAR. Inspired by Fernald (2005), we test for the presence of structural breaks both in the mean and in the variance for labor productivity and hours worked. In the following analysis, we use the original data for the business sector as these are the one generally used in the literature, abstracting from the adjustment suggested by Francis and Ramey (2004).

Although hours per capita can not strictly speaking have a unit root, since they are a bounded series,
still the levels/differences dichotomy is not exhaustive and is not addressing the core problem. Even if a bounded process do not have a unit root, it could be still non covariance-stationary, which is the characteristic we are ultimately interested in in order to justify the legitimacy of the entire SVAR approach.

Fernald (2005) claim that “once we allow for (statistically and economically plausible) trend breaks in labor productivity, the treatment of hours is relatively unimportant”. Indeed, he argues that the key is the presence of a low-frequency comovement between the two series in the SVAR. For instance, the HIGH-LOW-HIGH pattern of hours per capita and productivity growth could give rise to positive estimated IRFs per se (Faust and Leeper (1997)). On the other hand, when taking into account for the presence of possible breaks in trend, hours worked fall in response to a technology innovation, both in small/large systems and in sub-periods.

In favor of the structural break framework, Blanchard and Quah (1989) claim that results are sensitive to how they model the post-1973 productivity slowdown. Fernald (2005) summarizes that the key is whether the actual small sample has a low-frequency correlation between the two series rather than whether the true DGP has a break. Indeed, any procedure that reduces the low-frequency comovement (such as using the demographics-adjusted series for population\(^9\) or detrending hours with a quadratic trend\(^10\)) has the ability to change the estimated responses.

When testing for the presence of structural breaks in the mean for the series of hours, we find significant results. We computed both QLR test for the presence of a single break and the Bai and Perron test for the presence of two and three breaks in the mean and in all cases we cannot reject the presence of a structural break. In Figure 5, we report both the plot with the series of hours, and the detected high-low-high pattern detected in Fernald (2005), and an alternative way of modeling structural break in the mean, by estimating the “better” break in a linear trend for the same data.

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From an analogous analysis for the growth of labor productivity, we get similar results, supporting the claim that we need to take into account of these breaks, in order to obtain reliable inference results.

In addition to testing for breaks in the mean, we tested for the presence of single and multiple breaks (up to 3 breaks) in the variance of the series taken in to account. Based on the results in Pitarakis (2002), under correctly specified conditional mean, testing for changes in the variance is equivalent to a mean-shift.
test implemented on the squared residual sequence. For instance consider the following true model: \( y_t = x_t \beta + u_t \), with \( u_t = \sigma \varepsilon_t \). The corresponding squared residuals sequence would be: \( z_t = (y_t - x_t \beta)^2 \). The corresponding LM statistic is given by:

\[
LM_T(k) = \frac{T}{k(T-k)} \frac{1}{\hat{s}_z^2} \left[ \sum_{t=1}^{k} z_t - \frac{k}{T} \sum_{t=1}^{T} z_t \right], \quad \text{with} \quad \hat{s}_z^2 = \frac{\sum_{t=1}^{T} (z_t - \bar{z})^2}{T}
\]

Note that under regularity assumptions and by the law of large numbers, the limiting distribution of the test statistic under the null hypothesis is given by:

\[
LM_T(\pi) \rightarrow \frac{[W(\pi) - \pi \cdot W(1)]^2}{\pi (1 - \pi)}
\]

We find significant evidence in favor of the presence of structural breaks in the variance, both for productivity and for hours. In Figure 6 we report the plot of the squared residuals, with the estimated breaks, in the case of 3 dates.

**Figure 6: Breaks in the volatility of productivity and hours**

**LABOR PRODUCTIVITY**
Recent research has emphasized that permanent changes in the variance lead to size distortions in conventional unit root test. In the case of slowly mean-reverting volatility, either because of deterministic permanent shifts or (near-)integrated volatility, non parametric corrected versions of the standard unit root test have been developed\textsuperscript{11}. On the other hand, when the heteroskedasticity follows a stationary GARCH specification, the invariance principle guarantees that the usual Dickey Fuller tests remain asymptotically valid (Boswijk (2005)).

**GARCH-type effect in macroeconomic time series**

Many papers have highlighted the fact that structural instability seems to be present in a wide variety of macroeconomic and financial time series (see Ang and Bekaert (2002) and Stock and Watson (1996)). The negative consequences of ignoring this instability for inference and forecasting has inspired a wide range of change-point models. Several approaches appear as an interesting substitutes to the structural

\textsuperscript{11} Cavaliere and Taylor (2004) define a sampling scheme. Beare (2004) applies the tests to the cumulative sum of weighted increments of the series to be tested.
VAR approach, especially in modeling macroeconomic time series: one can either assume a small number of (deterministic) breaks or stochastic regimes (Markov-Switching type of models), or alternatively allowing for time-varying parameters. However, in the literature we explored so far, no mention of the possible presence of ARCH effects in the structural errors were taken into account. We try to address this issue, supported by some preliminary univariate analysis of the identified shocks.

Consider the series of the identified technology shocks as plotted in Figure 7. We can detect a tendency for large (small) observations to be followed by other large (small) values, of unpredictable sign. In order to test for the normality of the series of the identified shocks, we compute skewness and kurtosis. The evident positive skewness (0.4102, versus 0 in N(0,1)) and the presence of thick tails (kurtosis = 4.3669, versus 3 in N(0,1)) is confirmed when we reject the normality of the series by Jarque-Bera test statistic (68.8805). The leptokurtic distribution and the evidence of volatility clustering suggests testing for the presence of ARCH effect.

When we test the null hypothesis of ARCH(1) versus no ARCH effects with the Lagrange Multiplier test proposed by Engle (1982) we fail to reject the null hypothesis (LM(1) = 0.0054, with a p value of 0.9415, even if in the case of estimated residuals we should not assume that LM(1) is asymptotically distributed as a $\chi^2_1$).

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12 Shaded areas correspond to NBER recessions.
This volatility clustering phenomenon has been often modeled in financial econometrics applications with conditional heteroskedasticity models. In particular, the parsimonious GARCH(p,q) specification introduced by Bollerslev (1986, 1987) can be expressed as follows:

\[ \text{Var}(e_t) = h_t = w_t + \sum_{j=1}^{q} a_j e_{t-j}^2 + \sum_{j=1}^{p} b_j h_{t-j} \]

As illustrated by Bollerslev (1990), the GARCH model can be seen as an ARMA model for conditional second moments, and the selection of the orders p and q may be addressed by traditional selection model techniques applied on the squared series. Because of the naturally restricted number of observations for macroeconomic time series constructed at low frequency, and guided by preliminary analysis on the series of the identified technology shock, we decide to model the conditional variance as a GARCH(1,1), allowing for non-zero constant conditional correlations (CCC), denoted by \( \rho_{1} \), across the two shocks identified by the SVAR.
For both the technology and the non-technology shock, the following should be estimated:

\[ \varepsilon_i^t = \mu_i + u_i^t, \quad \text{where} \quad i = z, m \]

\[ h_{ii}^t = \omega_i + \alpha_i (\varepsilon_{i, t-1}^2 + \beta_i h_{ii, t-1}) \]

\[ h_{ij}^t = \rho_{ij} (h_{ii}^t h_{jj}^t)^{1/2} \]

We provide some preliminary univariate results in Table 3. Since the identified technology shock is non serially correlated by construction, and its (partial-) autocorrelation functions do not display any significant peak, we model the conditional mean simply by including a (non significant) constant term. The coefficients relative to the GARCH component appear to be highly significative (also when we estimate a GARCH(1,2)).

**Table 3: GARCH(1,1) estimates for the identified technology shock**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.004342</td>
<td>0.052694</td>
<td>0.082399</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>RESID(-1)^2</td>
</tr>
<tr>
<td>GARCH(-1)</td>
</tr>
</tbody>
</table>

- R-squared: -0.000027
- Adjusted R-squared: -0.013664
- S.E. of regression: 0.838286
- Sum squared resid: 154.6990
- Log likelihood: -273.4260
Conclusions

In this paper, we briefly review and replicate the findings of the recent literature about the role of (Hicksian-neutral) technology shocks as driving source of aggregate fluctuations in Real Business Cycle models. We start with the replication of the main results in Francis and Ramey (2003) and consistently with the literature, we find evidence of a persistent decline of hours in response to a positive technology shock, controlling for the robustness of these arguments by using different measures and different long-run identification schemes. In addition, we also control for the new measure for the labor force constructed by Francis and Ramey (2004), providing a tentative solution to the puzzling evidence of the recent literature.

Based on Fernald (2005), we provide empirical evidence of the presence of single and multiple breaks both in the mean and in the variance of the relevant variables and discuss the drawbacks for estimation and inference.

The main original contribution of the paper is the attempt to model GARCH effects in the technology shocks identified through the SVAR. Despite the seminal treatment of the model, in a univariate framework, the results suggest promising empirical research on this topic. Namely, we would like to model the SVAR
and the GARCH effects jointly, avoiding the two step estimation procedure used above. Also, despite the reduced sample, we suggest the estimation of a bivariate system, assuming constant conditional correlation among the two structural shocks.

References


Basu, Fernald and Kimball (1999), Are technology improvements contractionary?, *Manuscript*.


Christiano, Eichenbaum and Vigfusson (2003), What happens after a technology shock?, NBER working paper 9819.


Francis and Ramey (2003), “Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited”


Jones (2002), Has fiscal policy helped stabilize the postwar U.S. economy?, *Journal of Monetary Economics* (49), 709-746.

Pesavento and Rossi (2003), "Do Technology Shocks Drive Hours Up or Down? A Little Evidence from an Agnostic Procedure", Working Papers 03-23, Duke University, Department of Economics.


Appendix: The model

\[ Y_t = (A_t, N_t)\alpha K^{1-\alpha} \quad \text{Production Function} \]

\[ A_t = \mu A_0, \quad \mu > 1 \quad \text{Technology Growth} \]

\[ K_{t+1} = (1-\delta) K_t + I_t \quad \text{Capital Accumulation} \]

\[ C_t + I_t + G_t < Y_t \quad \text{Resource Constraint} \]

\[ U(C_t, N_t) = \ln(C_t) + \varphi_t \ln(1-N_t) \quad \text{Utility Function} \]

\[ C_t + I_t = (1-\tau_{n,t}) W_t N_t + (1-\tau_{k,t}) r_t K_t + \delta \tau_{k,t} K_t - \psi_t \quad \text{Household Budget Constraint} \]

\[ G_t = \tau_{n,t} W_t N_t + \tau_{k,t} (r_t - \delta) K_t + \psi_t \quad \text{Government Budget Constraint} \]

Where \( Y_t \) is output, \( A_t \) is an exogenous process for labor augmenting technical change, \( K_t \) is capital, \( N_t \) is labor input, \( \delta \) is the depreciation rate, \( I_t \) is investment, \( C_t \) is consumption, \( G_t \) is government purchases, \( \varphi_t \) is a preference shifter, \( W_t \) is the real wage, \( r_t \) is the pre-tax return on capital, \( \tau_{n,t} \) is the tax on labor income, \( \tau_{k,t} \) is the tax on capital income, and \( \psi_t \) is a lump-sum tax. The representative consumer chooses capital, consumption and labor to maximize the expected present discounted value of utility, with discount factor \( \beta \). Consumers own the capital and rent it to firms. The government finances its spending through a combination of lump-sum taxes and distortionary labor and capital income taxes.
In order to solve for the steady-state balanced growth path, we consider all variables in terms of the technology process $A_t$. Lower case letters denote variables divided by $A_t$ and lower case letters with tildes denote variables divided by output $Y_t$. This yields to the following solution:

\[ 1 + (1 - \tau_k) \left[ (1 - \alpha) \left( \frac{k}{N} \right)^{-\alpha} - \delta \right] = \frac{\mu}{\beta} \quad \text{Intertemporal Euler Equation} \]

\[ \tilde{c} + \tilde{g} + \tilde{i} = 1 \quad \text{Resource Constraint (per unit of output)} \]

\[ \frac{1 - N}{N} = \frac{\phi}{\alpha(1 - \tau_n)} [1 - \tilde{i} - \tilde{g}] \quad \text{Intratemporal Euler Equation} \]

\[ \tilde{i} = (\mu + \delta - 1) \left( \frac{k}{N} \right)^{-\alpha} \quad \text{Investment Rate} \]

\[ Y = A N \left( \frac{k}{N} \right)^{1-\alpha} \quad \text{Production Function} \]

\[ \frac{Y}{N} = W = \alpha A \left( \frac{k}{N} \right)^{1-\alpha} \quad \text{Labor Productivity} \]