Price Volatility, Spread Variability, and the Role of Alternative Market Mechanisms

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Abstract

We study the role of the trade-execution process in the endogenous generation of serial dependence in the conditional variance of transaction returns. The algorithm of the Chicago Mercantile Exchange's Globex automated futures trading system is directly compared with the traditional open outcry auction method in a computerized experimental setting, which allows the examination of the communication of volatility from the process of generating bids and offers through to transactions prices. We find that returns based on transactions done through the open outcry auction mechanism exhibit little or no serial dependence in conditional volatility, while the conditional variance of returns from Globex is time-dependent. Unconditional volatility is larger on average in the open outcry auction setting than in Globex. Temporal dependence in price volatility is traced to serial correlation in the market spread induced by the electronic order book in Globex, which is the distinguishing feature of the system.

1. Introduction

The purpose of this paper is to characterize a link between the endogenous generation of serial dependence in the conditional variance of transactions returns and market-trading structure. Due to the intractable complexity involved in the functioning of a complete financial market, our focus is necessarily going to be somewhat more narrow. Stoll [1990] describes the market as a communications system consisting of three components: an information system, an order-routing system, and a trade-execution system. In this paper, we shall concentrate on the volatility effects in the price distribution directly traceable to the trade-execution mechanism.

Our interest in the trade-execution system stems in large part from the increasing pace of automation of that function in futures markets. The use of various automated systems to trade futures contracts and options on futures contracts, replacing the traditional method of open outcry auction, is becoming common worldwide. Six such foreign futures markets from Tokyo to London are reviewed by the U.S. General Accounting Office [1990], and an additional 16 automated systems are documented in Domowitz [1990], including stock trading systems. Interestingly, Stoll [1990] notes that the functioning of such automated markets is remarkably similar across different financial instruments, stocks, futures, and options, thus broadening the interest of the analysis of any one particular system.

The trade-execution mechanisms compared in this study are the traditional open outcry auction system of futures trading and the algorithm governing trade execution in the new Globex futures trading system. The Chicago Mercantile Exchange (CME) and Reuters first proposed the Globex trading system in May 1988. Globex is to be an automated matching system for trading CME futures and options during a separate trading session, which precedes regular floor-trading hours. A small number of CME currency contracts are to be traded on the system initially. Plans for expansion include the trading of all CME contracts, as well as other domestic and even foreign exchanges' contracts via Globex. The partnership with Reuters ensures the potential exposure of Globex trading opportunities to markets in over 100 countries.

The Globex algorithm and the rules of open outcry auction provide alternative mechanisms through which we can study the volatility transmission process in a simple market microstructure setting. In effect, a controlled computerized experiment is set up, holding all aspects of the information and order-generation environment constant while varying only
the trading mechanism. Order generation is characterized by an aggregate process, much as in Garman [1976], with Poisson arrival of bids and asks to the market and a pricing rule due to Cohen, Maier, Schwartz, and Whitcomb [1978a], henceforth CMSW. For each offer generated, the resulting bid or ask is then passed through the different market mechanisms. The experimental design permits examination of the role of the market spread, in particular, in the communication of volatility. We document the influence of market structure in determining the serial correlation properties of polynomial functions of the spread process, which in turn determine the degree of serial dependence in the conditional second moments of transactions returns. We find that returns based on transactions done through the open outcry auction mechanism exhibit little or no serial dependence in conditional volatility, while the conditional variance of returns from Globex is not only time-dependent, but seems to be particularly persistent in a time-series sense. These results also have very interesting implications relating to the recent empirical literature documenting serial dependence in the volatility of asset returns.

Details of the Globex algorithm are not widely known, and a description is given in Section 2. Rules governing the open outcry trading process also are summarized there. The design of the computer experiment is described in Section 3. The time-series properties of the conditional means and variances of transactions returns are examined in Section 4. Section 5 is devoted to further study of the inputs to the Globex and open outcry auction systems, in an attempt to document the mechanism of the endogenous generation of serial dependence in return variances. Section 6 contains some concluding remarks.

2. The Globex System and Rules for Floor Trading

Under the Globex system, bids and offers are matched based on precise criteria of price, quantity, and time. Only good-till-cancelled limit orders and explicit instructions to attempt to hit a bid or lift an offer are accepted. The matching system maintains an electronic "book" of bids and offers. The existence of this order book, and the rules by which trades are processed as the users interact with the book through their bid and offer submissions, form the substantive difference between an automated execution system and the open outcry auction market system. The purpose of this section is to provide a clear statement of the Globex order-matching algorithm that in turn is used to produce the price series examined below. The following description of the Globex rules is taken from Domowitz [1990]; additional information on Globex operations and market-efficiency properties may be found therein.

Globex is a strict price and time priority system. Trades take place at the price of orders standing on the electronic order book. This guarantees that all new orders are filled at the best available price at the time of order entry. Only limit orders are accepted. A buy and sell order may be matched if they are for the same instrument and the buy order price is higher than the price of the sell order. The maximum possible quantity will be traded by completely filling the order on one side (buy or sell); at the conclusion of the matching process, there will be no additional trade potential before the submission. The algorithm is formalized as follows:
1. Order eligibility
   A new order is eligible to be matched with a standing order, and a trade will result whenever the following conditions occur:
   1.1 One order is a buy order and the other is a sell order.
   1.2 The two orders are for the same contract.
   1.3 The price of the buy order is greater than or equal to the price of the sell order.

2. Transaction price determination
   If an order match is possible according to the criteria of Rule 1, the trade will take place at the price of the standing order.

3. Trade quantity
   If an order match is possible according to Rule 1, then the trade will take place for a quantity equal to the smaller of:
   3.1 The remaining quantity of the new order.
   3.2 The remaining quantity of the standing order.

4. Market exposure
   If there are multiple standing orders eligible for matching against a new order, then matching will be considered in priority sequence until one of the following conditions is attained:
   4.1 The new order is completely filled.
   4.2 All eligible standing orders have been considered.

5. Standing order priority
   5.1 Price: for buy orders, higher price is higher priority; for sell orders, lower price is higher priority.
   5.2 Time: within the same price and quantity type, older orders have higher priority.

There also is a standing order priority rule for so-called primary versus secondary (or “more” in Globex terminology) orders, but the protocol is a bit complicated and is not relevant to the work reported on here; see Domowitz [1990] for details and examples. Similarly, there are special rules governing the setting of an opening price in the Globex system and pertaining to the changing or placing of an order “on hold.” The latter concerns time priorities and also is not relevant, while analysis of the opening is quite a separate problem from that considered in this paper; see, for example, Cohen and Schwartz [1989] and Stoll [1990] for discussion of many of the issues involved.

An example may help to clarify the details of the matching algorithm. Consider the following sets of bids and offers for one specific contract:
<table>
<thead>
<tr>
<th>Time</th>
<th>Bid</th>
<th>Bid quantity</th>
<th>Offer</th>
<th>Offer quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00</td>
<td>58.86</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:01</td>
<td>59.70</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:02</td>
<td>58.87</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:03</td>
<td>59.18</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:04</td>
<td>59.18</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:05</td>
<td>59.19</td>
<td>10</td>
<td>59.18</td>
<td>1</td>
</tr>
<tr>
<td>11:06</td>
<td>59.18</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At 11:02, there are 5 contracts bid at 58.86 and 1 at 58.87, with 2 offered at 59.70. No trades take place, as the bids and offer do not cross. The bid of 58.86 remains on the book, although there exists a better one. The spread is now 0.83. Two offers then are sequentially entered at 59.18 for 6 and 5 contracts; the offer for 6 contracts arrives first. We have noted this at times 11:03 and 11:04, respectively, but these offers could have been made virtually simultaneously. Computers in general, and Globex in particular, do not allow ties in terms of time, and fractions of a second have meaning within an automated trade execution system. A bid is then made at 59.19 for 10 contracts. The 11:03 offer is cleared first at a price of 59.18, the price of the standing order. Four contracts are left at the bid of 59.19, and they are matched with 4 of the 5 contracts offered at 11:04. The record will show 10 contracts traded at a price of 59.18. The remaining contract offered at 59.18 stays on the book and retains its time priority; i.e., should an appropriate bid arrive, this contract will be traded before the new contract offered at 59.18 at 11:06. The new spread at 11:05 is 0.31, which is unchanged at 11:06.

Orders are exposed to the entire Globex market by means of the electronic order book, and this book comprises the main difference between the automated system and our formalization of the floor trading process. The model of floor trading corresponds to the description of financial auction markets outlined by Teweles and Bradley [1987, pp. 176–77]. The highest bid and the lowest offer have precedence in all cases. When two bids or offers enter the market at the same time, the bid or offer with time priority has precedence. Although “matching” rules can prevail for so-called simultaneous bids and offers, i.e., when time priority cannot be clearly established, the computer does not allow for ties in terms of time and such rules therefore are not relevant for the purpose of our simulations. A contract is traded when the outstanding bid or offer is accepted. Acceptance in the computer simulation can occur when an order is crossed, just as in Globex, with the price of the transaction being at the price of the standing order. A new auction starts with each new completed transaction and the priority of bids and offers does not carry over from auction to auction. Unlike Globex, orders are not consigned to a book if they are away from the market or superseded by a better bid or offer before a contra order valid for a transaction enters the market.

This model of floor trading ignores possible informational externalities available to traders in the trading pit. It might be said that there is an

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1The priority rules in Teweles and Bradley [1987] are distilled from the New York Stock Exchange Guide rules concerning open outcry auction practices.
“implicit” book in the pit, known through physical trading activity and face-to-face contact. We believe this to be an arguable point at best. On the other hand, we have not built in potential negative aspects of pit trading. As noted, for example, by Stoll [1990], transactions in different parts of an active futures pit may not follow the time sequence in which they were entered by customers and may even violate price priority. Further, large orders may be executed at a price that is inferior to a standing small order. Our model of trading is based on exchange rules and preserves both price and time priority.

3. Bid and Offer Generation

Generation of bids and offers as inputs to the floor trading and Globex matching processes is based on the simulation model of Cohen, Maier, Schwartz, and Whitcomb [1978a, 1983]. The goal of CMSW was to model the entire agency/auction process of the specialist market. We are concerned with the automated market structure of Globex and the open outcry auction structure of the floor, as described in the last section. We rely, therefore, only on their general setup with respect to new order generation. In order to avoid useless repetition, the discussion below relates to offers only. The generation of bids is completely analogous.

Offers arrive to the floor or Globex according to a Poisson process with mean arrival rate of 100 orders per period. The simulation lasts for as many periods as required to generate 5,000 transactions. The use of a Poisson arrival process is consistent with the theoretical models of Garman [1976] and CMSW [1978b]. Following CMSW [1978a], offer quantities are generated from a standardized Gamma distribution with the mean set equal to 10 contracts; see, e.g., Johnson and Kotz [1970, p. 166]. The role of quantity in keeping Globex orders on the electronic order book is evident from the example of the last section. Explicit modeling of the transactions quantity process is beyond the scope of this paper; however, some discussion of the issues involved is contained in Section 6.

Offers are priced according to the relation

\[ \log P^o_t = \log P^{mb}_t + \epsilon^o_t, \]  \hspace{1cm} (1)

where \( P^{mb}_t \) denotes the current market’s best bid, and \( \epsilon^o_t \) is an innovation drawn from the so-called Yawl distribution of CMSW [1983]. The Yawl distribution depends on the logarithmic spread of the current market’s best bid and ask quotations at the time of the offer generation, i.e.,

\(^3\) Alternatively, we could have considered periods of an “hour,” with six hours in a “trading day.” Since we are not concerned with end-of-day effects in this paper, it is more convenient and without loss of generality to standardize the number of observations available for analysis across the different experimental designs.

\(^4\) Sufficient conditions for convergence of individual demand processes to a Poisson arrival process as modeled in Garman [1976] include: (i) a large number of market agents; (ii) agents act independently in selecting the timing of their orders; (iii) no small subset of agents dominates overall order generation; and (iv) no agent can generate an infinite number of orders in a finite period of time. Of these assumptions, (ii) may seem the most questionable; see Loeve [1963, p. 317] for necessary conditions in the abstract.

83
\( S_t = \log P_t^{ma} - \log P_t^{mb} \). Formally, the density for \( e_t^a \) is given by

\[
F^a(e_t^a) = \begin{cases} 
0 & e_t^a < -C \\
Y_t + (Y_t/C)e_t^a & -C \leq e_t^a < 0 \\
BY_t/(S_t)e_t^a & 0 \leq e_t^a < S_t \\
BY_t - (BY_t/A)(e_t^a - S_t) & S_t \leq e_t^a < A + S_t \\
0 & A + S_t \leq e_t^a
\end{cases}
\] (2)

where \( Y_t = 2/(C + B(A + S_t)) \) to guarantee that the density integrates to unity. The parameters \( A, B, \) and \( C \) are varied over the simulation runs.

The offer innovation density tends to zero mass as \( e_t^a \) approaches the market bid and then jumps to a local maximum at the market bid. This is consistent with the order-placement strategy results of CMSW [1981]. The time-varying structure of the distribution as a function of the market spread provides another rationale for this choice for the price generating process, however. As in Grossman and Stiglitz [1976], changes in market quotes are taken to reflect a noisy signal of changes in available, but possibly unobservable, information. These changes feed back into the order-generation process through a density of orders linked to current market prices through the spread. Thus, the random variables associated with supply that are presented to the market mechanism for execution are conditioned by current prices, much as are the price probability functions of Garman [1976]. The models of Glosten [1989] and Madhavan [1990] also specify a form of feedback from the best bid and offer to the generation of new bids and asks. In those models, market-makers provide bid/ask quotations to traders, who trade at the market bid and offer. Market-maker bids and offers are revised only after a transaction has been made, consistent with our experimental design. In setting the quotes, the market-makers use the information contained in the trading price history, which includes the best bids and asks since strict price priority prevails in the trading game that is modeled. In equilibrium, the quote is related directly to the previous quote, much as in equation (1).

Madhavan summarizes this conditioning by stating that the size of the spread forms a sufficient statistic for the entire history of trading. In the simulation model used here, bids and offers are conditioned on the spread by the simple artefact of making the bid and offer distributions directly dependent on the spread.\(^5\) As a more practical matter, use of a distribution that varies with the spread avoids excess negative serial correlation in returns and instability in the returns process, both of which arise through the use of a fixed distribution

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\(^4\)By symmetry, the price of a bid is given by \( \log P_t^b = \log P_t^{mb} + e_t^b \), where \( e_t^b \) is drawn from a distribution that is the mirror image of that given for \( e_t^a \) by equation (2). Given this method of bid and offer generation, there is no systematic drift in prices and trends are short-lived, as documented in Section 4.

\(^5\)We believe the results reported below to be robust to alternative order-generation processes, which depend on the spread. Generalizations to offer price distributions that do not take into account the current market spread are less obvious.
of order prices; see CMSW [1983].

Most important for our purpose here is the fact that such an order-generation process offers a simple experimental setting in which to study the differential transmission of order volatility to transactions price volatility through different market mechanisms. In particular, the variance for the offer distribution conditioned on the spread is found to be

\[
\text{Var}\left( e^a \right) = \left( \frac{1}{12} \right) Y_t \left[ C^3 + B (A + S_s) (A + S_s) (A + 2S_s) + S_s^3 \right] - \left( \frac{1}{36} \right) Y_t \left[ C^4 + B (A + S_s) (A + 2S_s) (2C^2 + B (A + S_s) (A + 2S_s)) \right].
\]

The instantaneous change in the variance obviously depends upon moments of the spread process. The offer process conditioned upon current information is, therefore, heteroscedastic.

The heteroscedasticity of offers as modeled in the generation process does not imply time-dependence in the second moments of offers nor of the returns of transactions prices, however. For a fixed spread, the offer-innovations process is independent and identically distributed. If the spread varies, but is serially independent, the offer process is heteroscedastic, but the second moments are not serially correlated. In that case, any serial correlation in the second moments of transaction price returns must arise from the trade-execution mechanism of the market. Finally, the market mechanism translating bids and offers into transaction prices may transmit some degree of correlation to the spread process, but the amount of dependence induced may vary between market mechanisms. In that case, the degree of serial dependence in second moments of transaction returns will vary between markets in a heuristically predictable fashion. We elaborate upon this theme and fully describe such predictions in Section 5.

The simulation proceeds in the following way. Bid and offer sequences are generated according to the model above using five different parameterizations of the bid and offer innovation distributions, with \((A, B, C)\) set equal to \((1, 1, 1)\), \((2, 1, 1)\), \((1, 2, 1)\), \((1, 1, 2)\), and \((2, 1, 3)\), respectively. Separate series are generated for entry into Globex and the floor because there is feedback to the order-generation process through the spread. The best orders extant in the two markets may differ at any instant of time, given the difference in order-execution rules.\(^6\) This instantaneous difference affects new-order generation through the order-innovations distributions, which depend on the spread.

As bids and offers are generated, they are transmitted in sequence to the Globex or floor-matching algorithm. Series of the best bids, best offers, the

\(^6\)For example, consider a scenario in which an order to sell at 10 arrives to both the floor and Globex, followed by an order to sell at 9. If the bid outstanding in both markets is 8, the spread has now been reduced by 1 in both markets. Suppose now that a bid of 9½ arrives in both markets. Transactions occur. In Globex, the order to sell at 10 has been recorded in the limit order book, while the higher ask is superseded on the floor. The Globex spread is now 10 - 8 = 2. The floor spread remains at 9 - 8 = 1 until new orders arrive.
spread, and the transaction prices are recorded and matched in terms of transactions-price time. In other words, for each of the 5,000 transactions across the 10 experiments we record $P_t$, $P_t^{ma}$, $P_t^{mh}$, and $P_t^{ma} - P_t^{mh}$.

4. Transactions Prices Dependence

Turning to the simulation results, Figures 1 and 2 illustrate the time series of the logarithmic transactions prices for the floor and Globex trading systems, respectively; i.e., $\log P_t$, $t = 1, \ldots, 5,000$. Both price series are for the parameters in the Yawil distribution for the bid and offer processes set at $(A, B, C) = (2, 1, 3)$. Without attempting a formal calibration to the actual prices for any particular time period or speculative asset, the long-run dynamic patterns appear reasonable. Both market mechanisms lead to price series characterized by long persistent cycles. Indeed, the $t$-statistic for a unit root in the autoregressive polynomial in the univariate time series representation for each of the two series calculated using 10 lags equals .627 and $-1.182$ under floor and Globex trading, respectively, none of which are close to the conventionally 1 or 5 percent critical values of $-2.58$ and $-1.95$; see Fuller [1976].

Figure 1. Log transactions prices (Floor: $A = 2$, $B = 1$, $C = 3$)

![Graph](image)

$^7$Including a constant in the autoregression, the $t$-statistics for a unit root become $- .960$ and $.431$, respectively, both of which are far below the 1 and 5 percent critical values of $-3.43$ and $-2.86$; see Fuller [1976].
These results for \((A,B,C) = (2,1,3)\) are very much in line with the findings for the other parameter settings investigated, and, in the subsequent empirical analysis, we shall therefore concentrate on characterizing the dynamic properties of the first differences of the logarithmic transactions prices only. The two resulting continuously compounded percentage return series are illustrated in Figures 3 and 4; i.e., \(100\Delta \log P_t = 100\log(P_t/P_{t-1}), t = 1, \ldots, 5,000\).

Summary statistics for both return series in Figures 3 and 4, together with the results obtained with the four other parameter values for \((A,B,C)\), are given in Table 1. Since the arrival rates for the bid and offer processes are equal and for the same average quantity, none of the simulated transactions price processes show any significant drift.\(^\star\) Not surprisingly, the unconditional variances are generally seen to be lower for Globex than for floor trading. The presence of “old” limit orders tends to reduce the overall variability in the transactions price process. At the same time, very large price changes between transactions, or “outliers,” are also seen to be more likely under Globex trading. The sample kurtosis, \(b_4\), for the unconditional Globex distributions

\(^\star\) Using conventional statistical inference procedures, the estimates for the mean, \(\mu\), are approximately normal with variance equal to \(\sigma^2/5,000\).
range from 6.82 to 19.6, whereas the unconditional returns distributions under floor trading is approximately normal according to both the skewness and kurtosis coefficients, $b_3$ and $b_4$.

It follows also from Table 1 that the autocorrelations persist somewhat longer under Globex than under floor trading.\(^\text{10,11}\)

Probably the most striking difference between the two trade execution systems pertains to the heteroscedasticity induced in the price processes. For instance, for $((A,B,C) = (2,1,3))$ the Ljung and Box [1978] portmanteau test, $Q_{20}^{(1)}$, for up to 20th order serial correlation in the squared returns process, $(\Delta \log P)^2$, $t = 1, \ldots, 5,000$, equals 30.1 for the floor, compared to a highly significant 644.3 under Globex trading. This pronounced serially dependent

\(^{1}\)From Jarque and Bera [1980], under the null hypothesis of normality, the $b_3$ and $b_4$ statistics are both approximately normal with means and variances given by 0.0 and 6/5,000, and 3.0 and 24/5,000, respectively.

\(^{10}\)For $\Delta \log P$, i.i.d. the standardized 4th order sample autocorrelation, $\sqrt{5,000}p_4$, is approximately standard normally distributed, and the joint Ljung and Box [1978] portmanteau test, $Q_{20}$, for the first 20 autocorrelations being equal to zero, has an asymptotic chi-squared distribution with 20 degrees of freedom.

\(^{11}\)The low-order negative sample autocorrelations are consistent with the theory and empirical evidence for NYSE transactions prices in Niederhauser and Osborne [1966].
conditional heteroscedasticity in the Globex return series is also apparent from Figure 4, and carries over to all the other parameter values for \((A,B,C)\) in Table 1.\(^{12}\)

Thus, to gain some further understanding regarding the temporal dependence built into the transactions price series through the trading mechanisms, a series of univariate time-series models was estimated. This analysis is, therefore, distinct from that in the next section by our explicit reliance on a restricted information set. In the usual spirit of univariate time-series analysis, the information set, \(\mathcal{F}_t\), is taken to be the \(\sigma\)-algebra generated by the history of own returns; i.e., \(\Delta \log P_{t-i}, i = 0, 1, \ldots, t - 1\). Our motivation for this is to link the results directly to the empirical finance literature relying on such techniques, most notably in the modeling of time-varying volatility; see, for instance, Bollerslev, Chou, and Kroner [1991] for a recent survey of the extensive literature on empirical applications of the Autoregressive Conditional Heteroscedastic, or ARCH, class of models to financial data.

\(^{12}\)This is also consistent with the unconditional leptokurtosis for the Globex returns process documented above.
Table 1. Transaction prices, summary statistics

<table>
<thead>
<tr>
<th></th>
<th>FL.</th>
<th>GL.</th>
<th>FL.</th>
<th>GL.</th>
<th>FL.</th>
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<tr>
<td>A</td>
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<td>2</td>
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<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
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<tr>
<td>C</td>
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<td>2</td>
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<tr>
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<td>-0.000</td>
<td>-0.002</td>
<td>-0.001</td>
<td>0.008</td>
<td>-0.000</td>
<td>0.006</td>
<td>-0.013</td>
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<td>$\sigma^2$</td>
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<td>0.399</td>
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<td>0.024</td>
<td>0.007</td>
<td>0.571</td>
<td>0.896</td>
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<td>$b_1$</td>
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<td>0.084</td>
<td>0.006</td>
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<td>-0.206</td>
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<tr>
<td>$b_2$</td>
<td>3.36</td>
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<td>-0.227</td>
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<td>$\rho_3$</td>
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<td>268.4</td>
<td>71.6</td>
<td>267.4</td>
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<td>366.6</td>
<td>637.5</td>
<td>129.0</td>
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<td>71.6</td>
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<td>113.3</td>
<td>366.6</td>
<td>637.5</td>
<td>129.0</td>
</tr>
</tbody>
</table>

Key: FL and GL refer to floor and Globex trading, respectively. $\mu$ and $\sigma^2$ denote the sample mean and variance of the distribution for $100 \Delta \log P_t$, while $b_1$ and $b_2$ refer to the sample skewness and kurtosis, respectively. $\rho_i$ and $\rho_i^{(3)}$ give the $i$th order-sample autocorrelation for $\Delta \log P_t$ and $(\Delta \log P_t)^2$ with the corresponding Ljung-Box statistics for up to 20th-order serial correlation denoted by $Q_{20}$ and $Q_{20}^{(3)}$.

To ease comparisons across parameter settings and trading systems, a simple third-order Moving Average, or MA(3), model was estimated for the conditional mean for all the series, i.e.,

$$E(100 \Delta \log P_t | \Psi_{t-1}) = \mu_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3},$$  \(4\)

where

$$\varepsilon_t = 100 \Delta \log P_t - \mu_t,$$  \(5\)

denotes the time $t$ innovation, or shock to the conditional mean.

The results discussed in Table 1 indicate a lack of temporal dependence in the conditional variance of the transactions price returns under floor trading. In fact, as the discussion in Section 5 below suggests, the only conditional heteroscedasticity present in floor returns arise from conditioning on current period information. Thus, conditional on $\Psi_{t-1}$, the floor returns variance is constant,\(^{13}\)

$$\text{Var}(100 \Delta \log P_t | \Psi_{t-1}) = \sigma^2 = \omega.$$  \(6\)

For the Globex trading prices, the assumption of conditional

\(^{13}\)We did not attempt to estimate models with temporally dependent variances for the floor, since it is well-known that the nonlinear estimation procedures required often experience convergence difficulties in the absence of such temporal dependence.
homooscedasticity is obviously unrealistic. To characterize the temporal variation in the conditional variance, a first-order Generalized ARCH, or GARCH (1,1), model is therefore employed,

$$\text{Var} (100\Delta \log P_t | \Psi_{t-1}) = \sigma_t^2 = \omega + \alpha_t \varepsilon_{t-1}^2 + \beta_t \sigma_{t-1}^2.$$  \hspace{1cm} (7)

Following the seminal paper by Engle [1982] introducing the ARCH class of models, the GARCH \((p,q)\) model proposed by Bollerslev [1986] is readily interpretable as a flexible ARMA-type model for conditional second-order moments, and the particularly simple GARCH (1,1) formulation in (7) has already found wide use in the modeling of volatility clustering in financial data; see Bollerslev, Chou, and Kroner [1991].

In order to calculate Maximum Likelihood Estimates (MLE) for the homoscedastic MA(3) model in (4), (5), and (6), or the MA(3) \(-\text{GARCH (1,1)}\) formulation in (3), (4), and (7), the auxiliary assumption of conditional normality was imposed across all the estimated models, i.e.,

$$100\Delta \log P_t | \Psi_{t-1} \sim N(\mu_t, \sigma_t^2).$$  \hspace{1cm} (8)

Even though the conditional normality assumption in (8) might be violated empirically, provided the conditional mean in (4) and the conditional variance in (6) or (7) are correctly specified, the resulting Quasi MLE

<table>
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<th>Table 2. Transaction prices, model estimates</th>
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</tr>
</tbody>
</table>

\(\mu\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
-2.54 & 0.43 \\ 0.352 & 0.059
\end{array}
\end{align*}

\(\beta\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
-2.10 & -2.40 \\ 0.020 & 0.012
\end{array}
\end{align*}

\(\theta\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
0.09 & -2.81 \\ 0.010 & 0.012
\end{array}
\end{align*}

\(\phi\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
0.10 & -0.003 \\ 0.008 & -0.109
\end{array}
\end{align*}

\(\omega\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
18.9 & 1.08 \\ 0.211 & -0.009
\end{array}
\end{align*}

\(\alpha_t\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
-0.136 & -0.174 \\ 0.001 & -0.005
\end{array}
\end{align*}

\(\beta_t\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
0.830 & -0.827 \\ 0.018 & -0.010
\end{array}
\end{align*}

\(b_2\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
0.010 & -0.009 \\ 0.017 & 0.050
\end{array}
\end{align*}

\(b_3\) \hspace{1cm} \begin{align*}
\begin{array}{cc}
3.51 & 4.35 \\ 4.45 & 4.06
\end{array}
\end{align*}

\(Q_{10}\) \hspace{1cm} \begin{align*}
14.0 & 44.2 \\ 17.0 & 38.9
\end{align*}

\(Q_{10}^{(2)}\) \hspace{1cm} \begin{align*}
15.8 & 21.7 \\ 43.9 & 25.1
\end{align*}

Key: F1. and GL. refer to floor and Globex trading. Normal quasi-maximum likelihood estimates with robust standard errors in parentheses for the transaction price returns. \(100\Delta \log P_t\). To make the numbers comparable, the estimates for \(\mu\) and \(\omega\) and the corresponding standard errors have all been multiplied by 100. \(b_2\) and \(b_3\) give the sample skewness and kurtosis for the standardized residuals, and \(Q_{10}\) and \(Q_{10}^{(2)}\) the corresponding Ljung-Box portmanteau tests for up to 20th-order serial correlation.
(QMLE) given in Table 2 are generally consistent and asymptotically normally distributed; see, e.g., Domowitz and White [1982], and Bollerslev and Wooldridge [1991]. The corresponding robust standard errors are reported in parentheses.

From the portmanteau tests for the standardized residuals, $\varepsilon_i \sigma_i - 1$, the MA (3) model is seen to capture most of the serial dependence in the data under both trading systems. For all the Globex price series, the estimates for the GARCH (1,1) parameters are also highly significant, and the model does a remarkably good job of tracking the time-varying conditional variances. The drop in the Ljung-Box portmanteau tests for additional serial correlation in the standardized squared residuals, $\varepsilon_i^2 / \sigma_i - 2$, from the corresponding statistics for the raw squared Globex returns in Table 1 is dramatic, and for none of the models in Table 2 is there evidence of any remaining serial dependence in the second-order moments.

Interestingly, the estimates for $\alpha_1 + \beta_1$, corresponding to the autoregressive parameter in the ARMA (1,1) representation for $\varepsilon_i$ as in Bollerslev [1986], equal .966, 1.001, .960, 1.038, and .975 across the five different models. These values are all suggestive of persistence in the conditional variance process as in the so-called Integrated GARCH (1,1), or IGARCH (1,1), model in which $\alpha_1 + \beta_1 \equiv 1$. In fact, a similar empirical regularity holds true with most high frequency financial data. The diffusion approximation results in Nelson [1991] provide a possible explanation for this systematic finding, but we shall not pursue these ideas further here.

5. Endogenous Generation of Serial Dependence in Return Variance

The estimates and inference procedures of the last section highlight the important difference between the two trading mechanisms in terms of the properties of transaction prices. Returns based on transactions done through the floor mechanism exhibit little or no serial dependence in conditional volatility, while the conditional variance of returns from Globex is not only time-dependent, but also seems to be particularly persistent in a time-series sense. Although the basic difference between the two market systems lies in the presence or absence of an electronic order book, it is difficult to pin down a single source or explanation for the results of the last section. The reason for the confusion lies in the recursiveness of the overall market, defined to include the order-generation process. A rough sketch of the volatility transmission process may help to clarify the problem and suggest means of empirical investigation.

We know from equation (3) that the variance of the order-innovation process is conditionally heteroscedastic if the information set includes the contemporaneous spread. This is a theoretical result arising from the construction of the market that does not require empirical verification. The information set containing the spread at time $t$ is not the same as the information set employed in the last section. Volatility of transactions returns is serially dependent on time $t - 1$ information only if the spread itself is serially correlated. The stochastic properties of the spread depend on the
trade-execution mechanism, however. The degree of this serial correlation in both the spread and its own second moment, which enters the variance of the order-innovation process as well, can be determined empirically for both systems. Examination of equations (1) and (3) suggests that a high degree of serial correlation in the spread and its higher moments should also result in correspondingly higher degrees of serial correlation in the conditional variances of the bids and offers. This feature also is amenable to empirical verification within the experimental design here. Finally, the volatility of the order process is transmitted to the transaction prices and returns, as well as to the variance of the returns process through complicated nonlinear filters called Floor or Globex. The transactions prices, determined through the design of the execution system, in turn determine the current best bid and offer, which conditions the next round of bids and offers through the spread. Time-varying volatility conditional on past information is thus endogenously determined as a function of trade-execution-system characteristics.

Since the last step in the volatility chain formed the basis of investigation in the last section, it is convenient to track backward through the process from an empirical point of view. Summary statistics for the time-series behavior of the logarithmic first differences of the market bids and asks corresponding to the 5,000 transactions prices analyzed in Tables 1 and 2 are reported in Tables 3 and 4; i.e., 100Δlog P^{mb}_t and 100Δlog P^{md}_t, t = 1, . . . , 5,000. The properties of the bid and offer distributions are very similar within each experiment defined by the order-distribution parameter settings. This is a function of the symmetry built into the simulation design. Whereas the shape and the dependence in the transactions returns distributions showed some sensitivity to the parameter values (A, B, C), the serial-correlation properties for the levels of the bid-and-ask returns distributions are almost identical across all sets of parameters and the two market mechanisms. The first-order autocorrelations, ρ₁, are typically around −0.15, while the higher-order autocorrelations are effectively zero. On the other hand, the serial-correlation properties of the squared first differences of the market bids and asks differ greatly across the two market mechanisms. For instance, for (A, B, C) = (2,1,3), the first-order correlation coefficient for the squared floor bid returns, ρ^{(2)}₁, is less than one third of that generated by Globex, while the differences among the higher-order correlations are even larger. Similar results are obtained across experiments and for the correlations in the squared offer returns.

This set of results is entirely consistent with the findings in the last section. Following the logic of the endogenous volatility transmission mechanism outlined above, the bid-ask spreads generated via Globex should show substantially more serial dependence in both the levels and the squares than those produced by the floor market. This is confirmed by Table 5, where summary statistics for the bid-ask spread distribution are reported; i.e., P^{md}_t − P^{mb}_t, t = 1, . . . , 5,000. Details of the following discussion are based on the first experiment with (A,B,C) = (1,1,1), but again very similar results hold true for the other parameter values. The first-order autocorrelation of the spread level, ρ₁, equals 0.200 for the floor, versus 0.768 under Globex.

14See footnote 5.
### Table 3. Bid prices, summary statistics

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Key: See Table 1.

### Table 4. Ask prices, summary statistics

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Key: See Table 1.
Further, the spread autocorrelations from the floor rapidly fall to zero, while those from Globex indicate a high degree of persistence. The story repeats itself with respect to the autocorrelation sequence of the squares of the spread. The first-order floor correlation, $\rho_1^{(0)}$, is barely 20 percent of that computed from the Globex data. Higher-order floor spread correlations are effectively zero, while the Globex correlations remain high through the fifth lag, the longest lag reported in the table.

These results, combined with those of the last section, confirm the general content of the discussion of endogenous volatility generation above. There is one remaining empirical issue to examine, however. The cross-correlation between squared current transaction-price returns and the contemporaneous square of the bid-ask spread may be high even for the floor mechanism, given equation (3), but the cross-correlations between squared transaction returns and the lagged spread should be low, given the results above. The former is not obvious, because the floor trade-execution mechanism is a nonlinear filter through which the bid-offer sequence must pass through to transactions prices. Contemporaneous correlations are not necessarily preserved by nonlinear filters. Similarly, although the results thus far predict a strong relationship between squared transaction returns and the lagged spreads under Globex, we have no information concerning contemporaneous correlations. These sample correlations are all summarized in Table 6.

As expected, the correlations between the spread and the levels of the returns, $\xi$, are zero both contemporaneously, $i = 0$, and at all lags, $i = 1, 2, 3$.

Interestingly, the correlations between the spread and the contemporaneous

Table 5. Bid-ask spreads, summary statistics

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<td>$b_4$</td>
<td>3.37</td>
<td>4.95</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>.200</td>
<td>.768</td>
</tr>
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<td>$\rho_2$</td>
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</tr>
<tr>
<td>$\rho_3$</td>
<td>.028</td>
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<tr>
<td>$\rho_4$</td>
<td>.027</td>
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<tr>
<td>$\rho_5$</td>
<td>.021</td>
<td>.242</td>
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<tr>
<td>$Q_{2a}$</td>
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<td>7153</td>
</tr>
<tr>
<td>$Q_{2a}^{(0)}$</td>
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<td>.812</td>
</tr>
<tr>
<td>$Q_{2a}^{(0)}$</td>
<td>.046</td>
<td>.650</td>
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<tr>
<td>$Q_{2a}^{(0)}$</td>
<td>.033</td>
<td>.410</td>
</tr>
<tr>
<td>$Q_{2a}^{(0)}$</td>
<td>.027</td>
<td>.399</td>
</tr>
</tbody>
</table>

Key: Same notation as in Table 1 for the bid-ask spread distribution, $P_{b} - P_{a}$. 
Table 6. Transaction prices, bid-ask spreads, correlations

<table>
<thead>
<tr>
<th></th>
<th>FL</th>
<th>GL</th>
<th>FL</th>
<th>GL</th>
<th>FL</th>
<th>GL</th>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccccccc}
\xi_0 & .010 & -.000 & .003 & -.015 & .007 & -.008 & .026 & -.002 & -.007 & .026 \\
\xi_1 & -.015 & -.020 & -.006 & .012 & -.030 & -.013 & .005 & -.006 & -.020 & -.030 \\
\xi_2 & .001 & .005 & .004 & -.013 & -.010 & .006 & .019 & -.019 & -.015 & -.023 \\
\xi_3 & -.007 & .003 & .016 & .004 & -.033 & -.008 & .006 & -.035 & .020 & .010 \\
\xi_{1(2)} & .420 & .240 & .387 & .269 & .365 & .205 & .556 & .366 & .500 & .440 \\
\xi_{1(3)} & .099 & .262 & .125 & .292 & .294 & .270 & .093 & .213 & .111 & .330 \\
\xi_{2(3)} & .016 & .251 & .020 & .317 & .023 & .246 & .022 & .158 & .067 & .267 \\
\xi_{2(3)} & .019 & .218 & .007 & .258 & .007 & .173 & .008 & .131 & .078 & .251 \\
\end{array}
\]

Key: FL and GL refer to floor and Globex trading. \( \xi \) denotes the sample correlation between \( \Delta \log P_t \) and the time \( t-i \) bid-ask spread, \( P_{t-i}^{\text{Ask}} - P_{t-i}^{\text{Bid}} \). \( \xi^{(2)} \) gives the sample correlation between \( (\Delta \log P_t)^2 \) and the time \( t-i \) spread.

Squared returns, \( \xi^{(2)} \), are highly significant for all the experiments under both systems. The predictions regarding the correlations between the squared returns and past values of the spread are also confirmed by the simulations. Under Globex \( \xi^{(2)}, i = 1, 2, 3 \), are all highly significant across the five experiments, while the same correlations die off fairly quickly under floor trading.

Figure 5. Bid-ask spreads, returns (Floor: A = 1, B = 1, C = 1)
Figure 6. Bid-ask spreads, returns (Globex: A = 1, B = 1, C = 1)

The relationship between the heteroscedasticity in the transaction prices and the bid-ask spread distribution is also illustrated in Figures 5-8, where the spread and returns are graphed together for the two market mechanisms and \((A, B, C)\) equal to \((1, 1, 1)\) and \((1, 1, 2)\), respectively. In the figures for the other parameter settings are similar, and are omitted for lack of space. In spite of the correlation with the contemporaneous spread, it is interesting to note the lack of any apparent time-varying volatility in the returns processes for the two floor-trading schemes depicted in Figures 5 and 7. At the same time, Figures 6 and 8 for Globex both show very significant clustering of volatility through time. It is also immediately obvious from the two figures for Globex that periods of high transaction return volatility are characterized by high bid-ask spreads, while low volatility and narrow spreads go together.

We shall not attempt any formal multivariate modeling of this linkage here, but future work exploring the relationship even further holds the promise of important new insight into the workings of the different market mechanisms.

6. Conclusion

Theoretical work by Glosten [1989] and Madhavan [1990], and informational arguments along the lines of Grossman and Stiglitz [1976] link the current market spread to new bid and offer generation. Using the

\textsuperscript{13}In order to make the returns and spreads fit on the same plots, 2, 1, 3, and 10, respectively, were added to the four-spread time series.
aggregate orders model of CMSW [1978a], we have introduced the possibility that temporal dependence in the conditional volatility of bids and offers can be induced through changes in the spread. Conditional heteroscedasticity may then be transmitted to transactions prices through the market mechanism governing trade execution. Neither of these possibilities was certain before the empirical analysis undertaken here, however. Serially dependent forms of heteroscedasticity in the bid and offer distributions require serial dependence in the spread process and its higher moments. It is worth noting that such dependence was not built into the experimental design by the offer-generating process, however. The market mechanisms considered here are complicated nonlinear filters, and it is well-known that nonlinear filters do not preserve moment properties, conditional or otherwise.

We find that returns based on transactions done through the floor mechanism exhibit little or no serial dependence in conditional volatility, while the conditional variance of returns from Globex is time-dependent. The market mechanism itself appears to induce serial correlation in the spread through the workings of the electronic order book, which distinguishes the Globex system from open outcry auction. Further, the temporal dependence of the conditional heteroscedasticity induced in the bid and offer distributions is
passed through to transactions prices, despite the nonlinearity of the Globex filter. Although we can document conditional heteroscedasticity based on current-period information for the floor, as the model suggests, there is no evidence of temporal dependence in the second moment of transactions returns conditioned on previous history.

There are several directions in which this research can be extended. Serial correlation in order arrival is a natural extension with respect to the experimental design. By designating periods to be “hours” and calling six hours a trading “day,” end-of-day effects can be examined. This is of interest in the context of Globex, because the Globex book is cleared at the end of a trading day. Such an extension also relates to existing work by Admati and Pfleiderer [1988], Foster and Viswanathan [1990], and Stoll and Whaley [1990] on volatility effects traceable to openings and closings of the market. The estimation of multivariate GARCH models is difficult, but such an effort would allow the direct testing of the influence of the spread on the second moment of transactions returns by including the spread process directly into the expression for the conditional variance.

Most promising, however, may be the inclusion of quantity as both an important independent variable to study and as a means to augment the
experimental design. The work of Stoll [1989], Glosten [1989], and Madhaven [1990] supports the inclusion of quantity in the spread process. The combination of experimental design changes and difficulties in the multivariate modeling of conditional second moments of both price and quantity place this effort outside the scope of the present paper. Nevertheless, the examination of volume effects on volatility already suggested by the theoretical literature (see, e.g., Tauchen and Pitts [1983]) is an obvious extension of the analysis conducted here.

Finally, the combination of the finding of conditional volatility movements and lower unconditional variances of transactions returns in Globex relative to the floor has implications with respect to market efficiency. In the particular case of Globex, efficiency comparisons with open outcry auction are crucial with respect to regulatory approval under Regulation 1.38 of the Commodity Futures Trading Commission. The issues are complex, however, as noted in Domowitz [1989, 1990]. Volatility comparisons must be accompanied by other evidence pertaining to the possibility that Globex offers an equally open and competitive method of trade execution relative to open outcry auction.
References


