

# ROUGHING IT UP: INCLUDING JUMP COMPONENTS IN THE MEASUREMENT, MODELING, AND FORECASTING OF RETURN VOLATILITY

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*Abstract*—A growing literature documents important gains in asset return volatility forecasting via use of realized variation measures constructed from high-frequency returns. We progress by using newly developed bipower variation measures and corresponding nonparametric tests for jumps. Our empirical analyses of exchange rates, equity index returns, and bond yields suggest that the volatility jump component is both highly important and distinctly less persistent than the continuous component, and that separating the rough jump moves from the smooth continuous moves results in significant out-of-sample volatility forecast improvements. Moreover, many of the significant jumps are associated with specific macroeconomic news announcements.

## I. Introduction

VOLATILITY is central to asset pricing, asset allocation, and risk management. In contrast to the estimation of expected returns, which generally requires long timespans of data, the results of Merton (1980) and Nelson (1992) suggest that volatility may be estimated arbitrarily well through the use of sufficiently finely sampled high-frequency returns over any fixed time interval. However, the assumption of a continuous sample path diffusion underlying the theoretical results is invariably violated in practice. Thus, despite the increased availability of high-frequency data for a host of different financial instruments, practical complications have hampered the implementation of direct high-frequency volatility modeling and filtering procedures.<sup>1</sup>

In response, Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2001) (henceforth AB DL), Barndorff-Nielsen and Shephard (2002a,b), and Meddahi

(2002), among others, have recently advocated the use of nonparametric realized volatility, or variation, measures to conveniently circumvent the data complications while retaining most of the relevant information in the intraday data for measuring, modeling, and forecasting volatilities over daily and longer horizons. Indeed, the empirical results in AB DL (2003) strongly suggest that simple models of realized volatility outperform the popular GARCH and related stochastic volatility models in out-of-sample forecasting.<sup>2</sup>

At the same time, recent parametric studies have suggested the importance of explicitly allowing for jumps, or discontinuities, in the estimation of specific stochastic volatility models and in the pricing of options and other derivatives.<sup>3</sup> In particular, it appears that many (log) price processes are best described by a combination of a smooth and very slowly mean-reverting continuous sample path process and a much less persistent jump component.<sup>4</sup> Thus far, however, the nonparametric realized volatility literature has paid comparatively little attention to jumps, and related, to distinguishing jump from nonjump movements.

Set against this backdrop, in the present paper we seek to further advance the nonparametric realized volatility approach through the development of a practical nonparametric procedure for separately measuring the continuous sample path variation and the discontinuous jump part of the quadratic variation process. Our approach builds directly on the new theoretical results in Barndorff-Nielsen and Shephard (2004a, 2006) involving so-called bipower variation measures constructed from the summation of appropriately scaled cross-products of adjacent high-frequency absolute returns.<sup>5</sup> Implementing these ideas empirically with more than a decade of five-minute high-frequency returns for the DM/\$ foreign exchange market, the S&P 500 market index, and the thirty-year U.S. Treasury yield, we shed new light on the dynamics and comparative magnitudes of jumps

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<sup>1</sup> See, among others, Andersen and Bollerslev (1997), Dacorogna et al. (2001), Engle (2000), Russell and Engle (2005), and Rydberg and Shephard (2003).

<sup>2</sup> These empirical findings are further corroborated by the analytical results for specific stochastic volatility models reported in Andersen, Bollerslev, and Meddahi (2004).

<sup>3</sup> See, among others, Andersen, Benzoni, and Lund (2002), Bates (2000), Chan and Maheu (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Drost, Nijman, and Werker (1998), Eraker (2004), Eraker, Johannes, and Polson (2003), Johannes (2004), Johannes, Kumar, and Polson (1999), Maheu and McCurdy (2004), Khalaf, Saphores, and Bilodeau (2003), and Pan (2002).

<sup>4</sup> Earlier influential work on homoskedastic jump-diffusions includes Merton (1976), Ball and Torous (1983), Beckers (1981), and Jarrow and Rosenfeld (1984). More recently, Jorion (1988) and Vlaar and Palm (1993) incorporated jumps in the estimation of discrete-time ARCH and GARCH models. See also the discussion in Das (2002).

<sup>5</sup> This approach is distinctly different from the recent work of Ait-Sahalia (2002), who relies on direct estimates of the transition density function.

across the different markets. We also demonstrate important gains in terms of volatility forecast accuracy by explicitly differentiating the jump and continuous sample path components. These gains obtain at daily, weekly, and even monthly forecast horizons. Our new HAR-RV-CJ forecasting model incorporating the jumps builds directly on the heterogeneous AR model for the realized volatility, or HAR-RV model, due to Müller et al. (1997) and Corsi (2003), in which the realized volatility is parameterized as a linear function of the lagged realized volatilities over different horizons.

The paper proceeds as follows. In the next section we briefly review the relevant bipower variation theory. In section III we describe our high-frequency data, extract a preliminary measure of jumps, and describe their features. In section IV we describe the HAR-RV volatility forecasting model, modify it to allow and control for jumps (producing our HAR-RV-J model), and assess its empirical performance. In section V we significantly refine the jump estimator by shrinking it toward zero in a fashion motivated by recently developed powerful asymptotic theory, and by robustifying it to market microstructure noise (as motivated by the extensive simulation evidence in Huang and Tauchen, 2005). We then illustrate that many of the jumps so identified are associated with macroeconomic news. In section VI we build a more refined model that makes full use of our refined jump estimates by incorporating jump and nonjump components separately. The new model (HAR-RV-CJ) includes the earlier HAR-RV-J as a special and potentially restrictive case and produces additional forecast enhancements. We conclude in section VII with several suggestions for future research.

## II. Theoretical Framework

Let  $p(t)$  denote a logarithmic asset price at time  $t$ . The continuous-time jump diffusion process traditionally used in asset pricing is conveniently expressed in stochastic differential equation (sde) form as

$$\begin{aligned} dp(t) &= \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), \\ 0 &\leq t \leq T, \end{aligned} \quad (1)$$

where  $\mu(t)$  is a continuous and locally bounded variation process,  $\sigma(t)$  is a strictly positive stochastic volatility process with a sample path that is right continuous and has well-defined left limits (allowing for occasional jumps in volatility),  $W(t)$  is a standard Brownian motion, and  $q(t)$  is a counting process with (possibly) time-varying intensity  $\lambda(t)$ . That is,  $P[dq(t) = 1] = \lambda(t)dt$ , where  $\kappa(t) \equiv p(t) - p(t-)$  refers to the size of the corresponding discrete jumps in the logarithmic price process. The quadratic variation for the cumulative return process,  $r(t) \equiv p(t) - p(0)$ , is then

$$[r, r]_t = \int_0^t \sigma^2(s)ds + \sum_{0 < s \leq t} \kappa^2(s), \quad (2)$$

where by definition the summation consists of the  $q(t)$  squared jumps that occurred between time 0 and time  $t$ . Of course, in the absence of jumps, or  $q(t) \equiv 0$ , the summation vanishes and the quadratic variation simply equals the integrated volatility of the continuous sample path component.

Several recent studies concerned with the direct estimation of continuous time stochastic volatility models have highlighted the importance of explicitly incorporating jumps in the price process along the lines of equation (1).<sup>6</sup> Moreover, the specific parametric model estimates reported in this literature have generally suggested that any dynamic dependence in the size or occurrence of the jumps is much less persistent than the dependence in the continuous sample path volatility process. Here we take a complementary nonparametric approach, squarely in the tradition of the realized volatility literature but specifically distinguishing jump from nonjump movements, relying on both the recent emergence of high-frequency data and powerful asymptotic theory.

### A. High-Frequency Data, Bipower Variation, and Jumps

Let the discretely sampled  $\Delta$ -period returns be denoted by  $r_{t,\Delta} \equiv p(t) - p(t - \Delta)$ . For ease of notation we normalize the daily time interval to unity and label the corresponding discretely sampled daily returns by a single time subscript,  $r_{t+1} \equiv r_{t+1,1}$ . Also, we define the daily *realized volatility*, or *variation*, by the summation of the corresponding  $1/\Delta$  high-frequency intradaily squared returns,<sup>7</sup>

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r_{t+j\cdot\Delta,\Delta}^2, \quad (3)$$

where for notational simplicity and without loss of generality  $1/\Delta$  is assumed to be an integer. Then, as emphasized in Andersen and Bollerslev (1998), ABDL (2001), Barndorff-Nielsen and Shephard (2002a, b), and Comte and Renault (1998), among others, it follows directly by the theory of quadratic variation that the realized variation converges uniformly in probability to the increment of the quadratic variation process as the sampling frequency of the underlying returns increases. That is,

$$RV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s)ds + \sum_{t < s \leq t+1} \kappa^2(s), \quad (4)$$

for  $\Delta \rightarrow 0$ . Thus, in the absence of jumps the realized variation is consistent for the integrated volatility that figures prominently in the stochastic volatility option pricing

<sup>6</sup> See, among others, Andersen, Benzoni, and Lund (2002), Eraker, Johannes, and Polson (2003), Eraker (2004), Johannes (2004), and Johannes, Kumar, and Polson (1999).

<sup>7</sup> We will use the terms realized volatility and realized variation interchangeably.

literature. This result, in part, motivates the modeling and forecasting procedures for realized volatilities advocated in ABDL (2003). It is clear, however, that in general the realized volatility will inherit the dynamics of both the continuous sample path process and the jump process. Although this does not impinge upon the theoretical justification for directly modeling and forecasting  $RV_{t+1}(\Delta)$  through simple procedures that do not distinguish jump and nonjump contributions to volatility, it does suggest that superior forecasting models may be constructed by separately measuring and modeling the two components in equation (4).

Building on this intuition, the present paper seeks to improve on the predictive models developed in ABDL (2003) through the use of new and powerful asymptotic results (for  $\Delta \rightarrow 0$ ) of Barndorff-Nielsen and Shephard (2004a, 2006) that allow for separate (nonparametric) identification of the two components of the quadratic variation process. Specifically, define the standardized *realized bi-power variation* as

$$BV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} |r_{t+j\Delta, \Delta}| |r_{t+(j-1)\Delta, \Delta}|, \quad (5)$$

where  $\mu_1 \equiv \sqrt{2/\pi} = E(|Z|)$  denotes the mean of the absolute value of standard normally distributed random variable,  $Z$ . It is then possible to show that for  $\Delta \rightarrow 0$ ,<sup>8</sup>

$$BV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s) ds. \quad (6)$$

Hence, as first noted by Barndorff-Nielsen and Shephard (2004a), combining the results in equations (4) and (6), the contribution to the quadratic variation process due to the discontinuities (jumps) in the underlying price process may be consistently estimated (for  $\Delta \rightarrow 0$ ) by

$$RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{t < s \leq t+1} \kappa^2(s). \quad (7)$$

<sup>8</sup> Corresponding general asymptotic results for so-called realized power variation measures have recently been established by Barndorff-Nielsen and Shephard (2003, 2004a); see also Barndorff-Nielsen, Graversen, and Shephard (2004) for a survey of related results. In particular, it follows that in general for  $0 < p < 2$  and  $\Delta \rightarrow 0$ ,

$$RPV_{t+1}(\Delta, p) \equiv \mu_p^{-1} \Delta^{1-p/2} \sum_{j=1}^{1/\Delta} |r_{t+j\Delta, \Delta}|^p \rightarrow \int_t^{t+1} \sigma^p(s) ds,$$

where  $\mu_p \equiv 2^{p/2} \Gamma(1/2(p+1))/\Gamma(1/2) = E(|Z|^p)$ . Hence, the impact of the discontinuous jump process disappears in the limit for the power variation measures with  $0 < p < 2$ . In contrast,  $RPV_{t+1}(\Delta, p)$  diverges to infinity for  $p > 2$ , while  $RPV_{t+1}(\Delta, 2) \equiv RV_{t+1}(\Delta)$  converges to the integrated volatility plus the sum of the squared jumps, as in equation (4). Related expressions for the conditional moments of different powers of absolute returns have also been utilized by Ait-Sahalia (2004) in the formulation of a GMM-type estimator for specific parametric homoskedastic jump-diffusion models.

This is the central insight on which the theoretical and empirical results of this paper build.

Of course, nothing prevents the estimates of the squared jumps defined by the right side of equation (7) from becoming negative in a given finite ( $\Delta > 0$ ) sample. Thus, following the suggestion of Barndorff-Nielsen and Shephard (2004a), we truncate the actual empirical measurements at zero,

$$J_{t+1}(\Delta) \equiv \max[RV_{t+1}(\Delta) - BV_{t+1}(\Delta), 0], \quad (8)$$

to ensure that all of the daily estimates are nonnegative.

### III. Data and Summary Statistics

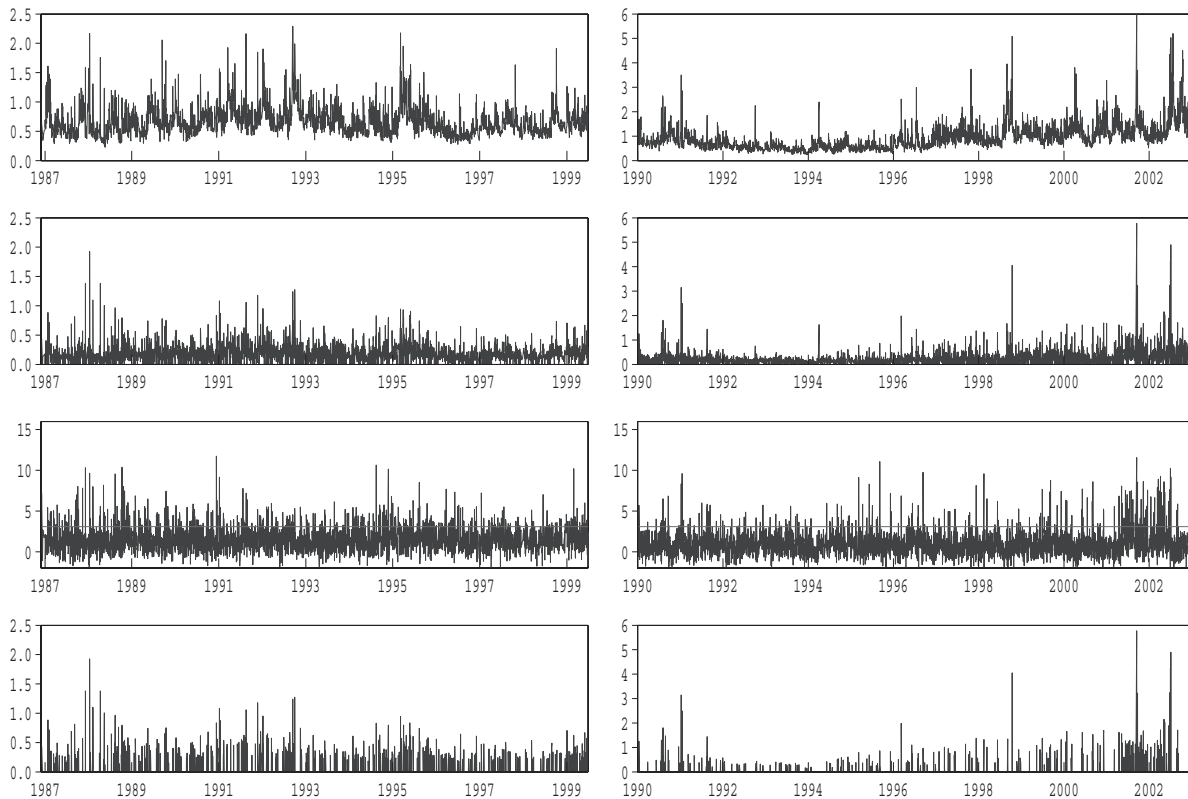
To highlight the generality of our empirical results related to the improved forecasting performance obtained by separately measuring the contribution to the overall variation coming from the discontinuous price movements, we present the results for three different markets. We begin this section with a brief discussion of the data sources, followed by a summary of the most salient features of the resulting realized volatility and jump series for each of three markets.

#### A. Data Description

We present the results for three markets: the foreign exchange spot market (DM/\$), the equity futures market (U.S. S&P 500 index), and the interest rate futures market (thirty-year U.S. Treasury yield). The DM/\$ volatilities range from December 1986 through June 1999, for a total of 3,045 daily observations. The underlying high-frequency spot quotations were kindly provided by Olsen & Associates in Zurich, Switzerland. This same series has been previously analyzed in ABDL (2001, 2003). The S&P 500 volatility measurements are based on tick-by-tick transactions prices from the Chicago Mercantile Exchange (CME) augmented with overnight prices from the GLOBEX automated trade execution system, from January 1990 through December 2002. The T-bond volatilities are similarly constructed from tick-by-tick transactions prices for the thirty-year U.S. Treasury bond futures contract traded on the Chicago Board of Trade (CBOT), again from January 1990 through December 2002. After removing holidays and other inactive trading days, we have a total of 3,213 observations for each of the two futures markets.<sup>9</sup> A more detailed description of the S&P and T-bond data is available in Andersen, Bollerslev, Diebold, and Vega (2005), where the same high-frequency data are analyzed from a very different perspective. All of the volatility measures are based on linearly interpolated logarithmic five-minute returns, as in Müller et

<sup>9</sup> We explicitly exclude all days with sequences of more than twenty consecutive five-minute intervals of no new prices for the S&P 500, and forty consecutive five-minute intervals of no new prices for the T-bond market.

FIGURE 1.—(A) DAILY DM/\$ REALIZED VOLATILITIES AND JUMPS; (B) DAILY S&amp;P 500 REALIZED VOLATILITIES AND JUMPS



Key: The top panel shows daily realized volatility in standard deviation form, or  $RV_t^{1/2}$ . The second panel graphs the jump component defined in equation (8),  $J_t^{1/2}$ . The third panel shows the  $Z_{1,\alpha}(\Delta)$  statistic, with the 0.999 significance level indicated by the horizontal line. The bottom panel graphs the significant jumps corresponding to  $\alpha = 0.999$ , or  $J_{t,0.999}^{1/2}$ . See the text for details.

al. (1990) and Dacorogna et al. (1993).<sup>10</sup> For the foreign exchange market this results in a total of  $1/\Delta = 288$  high-frequency return observations per day, while the two futures contracts are actively traded for  $1/\Delta = 97$  five-minute intervals per day. For notational simplicity, we omit the explicit reference to  $\Delta$  in the following, referring to the five-minute realized volatility and jump measures defined by equations (3) and (8) as  $RV_t$  and  $J_t$ , respectively.

### B. Realized Volatilities and Jumps

The first panels in figures 1A–C show the resulting three daily realized volatility series in standard deviation form, or  $RV_t^{1/2}$ . Each of the three series clearly exhibits a high degree

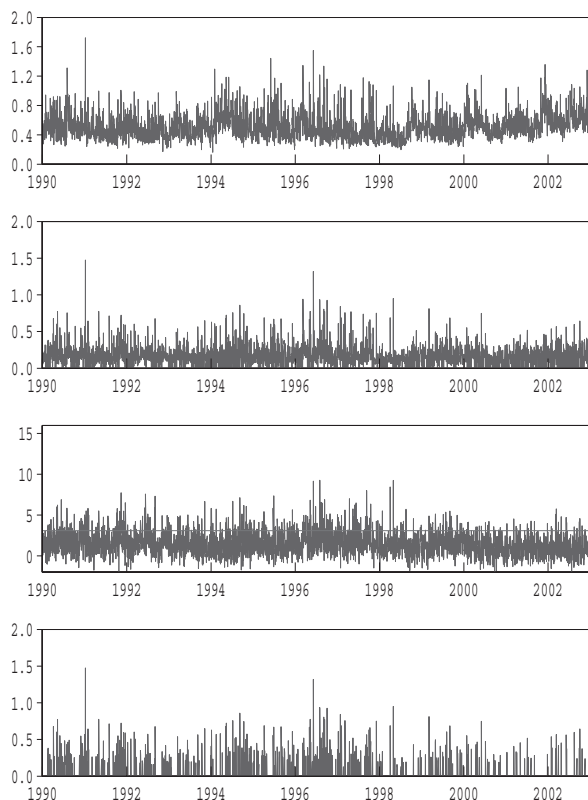
<sup>10</sup> To mitigate the impact of market microstructure frictions in the construction of unbiased and efficient realized volatility measurements, a number of recent studies have proposed ways of “optimally” choosing  $\Delta$  (e.g., Ait-Sahalia, Mykland, & Zhang, 2005; Bandi & Russell, 2004, 2006), subsampling schemes (e.g., Zhang, Ait-Sahalia, & Mykland, 2005; Zhang, 2004), prefiltering (e.g., Andreou & Ghysels, 2002; Areal & Taylor, 2002; Bollen & Inder, 2002; Corsi, Zumbach, Müller, & Dacorogna, 2001; Oomen, 2006), Fourier methods (Barucci & Reno, 2002; Malliavin & Mancino, 2002), or other kernel type estimators (e.g., Barndorff-Nielsen, Hansen, Lunde, & Shephard, 2006; Hansen & Lunde, 2006; Zhou, 1996). For now we simply follow ABDL (2000, 2001), along with most of the existing empirical literature, in the use of unweighted five-minute returns for each of the three actively traded markets analyzed here. However, we will return to a more detailed discussion of the market microstructure issue and pertinent jump measurements in section V below.

of own serial correlation. This is confirmed by the Ljung-Box statistics for up to tenth-order serial correlation reported in tables 1A–C equal to 5,714, 12,184, and 1,718, respectively. Similar results obtain for the realized variances and logarithmic transformations reported in the first and third columns in the tables. Comparing the volatility across the three markets, the S&P 500 returns are the most volatile, followed by the exchange rate returns. Also, consistent with earlier evidence for the foreign exchange market in ABDL (2001), and related findings for individual stocks in Andersen, Bollerslev, Diebold, and Ebens (2001) and the S&P 500 in Deo, Hurvich, and Lu (2006) and Martens, van Dijk, and de Pooter (2004), the logarithmic standard deviations are generally much closer to being normally distributed than are the raw realized volatility series. Hence, from a modeling perspective, the logarithmic realized volatilities are more amenable to the use of standard time series procedures.<sup>11</sup>

The second panels in figures 1A–C display the separate measurements of the jump components (again in standard deviation form) based on the truncated estimator in equation

<sup>11</sup> Modeling and forecasting log volatility also has the virtue of automatically imposing nonnegativity of fitted and forecasted volatilities.

FIGURE 1C.—DAILY U.S. T-BOND REALIZED VOLATILITIES AND JUMPS



Key: The top panel shows daily realized volatility in standard deviation form, or  $RV_t^{1/2}$ . The second panel graphs the jump component defined in equation (8),  $J_t^{1/2}$ . The third panel shows the  $Z_{1,t}(\Delta)$  statistic, with the 0.999 significance level indicated by the horizontal line. The bottom panel graphs the significant jumps corresponding to  $\alpha = 0.999$ , or  $J_{t,0.999}^{1/2}$ . See the text for details.

(8).<sup>12</sup> As is evident from the figures, many of the largest realized volatilities are directly associated with jumps in the underlying price process. Some of the largest jumps in the DM/\$ market occurred during the earlier 1986–88 part of the sample, while the size of the jumps for the S&P 500 has increased significantly over the most recent 2001–02 period. In contrast, the size of the jumps in the T-bond market seem to be much more evenly distributed throughout the sample. Overall, both the size and occurrence of jumps appear to be much more predictable for the S&P 500 than for the other two markets.

These visual observations are readily confirmed by the standard Ljung-Box portmanteau statistics for up to tenth-order serial correlation in the  $J_t$ ,  $J_t^{1/2}$ , and  $\log(J_t + 1)$  series reported in the last three columns in tables 1A–C. It is noteworthy that although the Ljung-Box statistics for the jumps are generally significant at conventional significance levels (especially for the jumps expressed in standard deviation or logarithmic form), the actual values

<sup>12</sup> The difference between the daily realized variation and bipower variation measures result in negative estimates for the squared daily jumps on 30.6%, 27.9%, and 18.3% of the days for each of the three markets, respectively. As discussed below, in the absence of jumps, the difference should be negative asymptotically ( $\Delta \rightarrow 0$ ) for half of the days in the sample.

are markedly lower than the corresponding test statistics for the realized volatility series reported in the first three columns. This indicates decidedly less own dynamic dependence in the portion of the overall quadratic variation originating from the discontinuous sample path price process compared to the dynamic dependence in the continuous sample path price movements. The numbers in the table also indicate that the jumps are relatively least important for the DM/\$ market, with the mean of the  $J_t$  series accounting for 0.072 of the mean of  $RV_t$ , while the same ratios for the S&P 500 and T-bond markets equal 0.144 and 0.126, respectively.

Motivated by these observations, we now put the idea of separately measuring the jump component to work in the construction of new and simple-to-implement realized volatility forecasting models. More specifically, we follow ABDL (2003) in directly estimating a set of time series models for each of the different realized volatility measures in tables 1A–C; i.e.,  $RV_t$ ,  $RV_t^{1/2}$ , and  $\log(RV_t)$ . Then, in order to assess the added value of separately measuring the jump component in forecasting the realized volatilities, we simply include the raw  $J_t$ ,  $J_t^{1/2}$ , and  $\log(J_t + 1)$  jump series as additional explanatory variables in the various forecasting regressions.

#### IV. Accounting for Jumps in Realized Volatility Modeling and Forecasting

A number of empirical studies have argued for the importance of long-memory dependence in financial market volatility. Several different parametric ARCH and stochastic volatility formulations have also been proposed in the literature for capturing this phenomenon (e.g., Andersen & Bollerslev, 1997; Baillie, Bollerslev, & Mikkelsen, 1996; Breidt, Crato, & de Lima, 1998; Dacorogna et al., 2001; Ding, Granger, & Engle, 1993; Robinson, 1991). These same empirical observations have similarly motivated the estimation of long-memory type ARFIMA models for realized volatilities in ABDL (2003), Areal and Taylor (2002), Deo, Hurvich, and Lu (2006), Koopman, Jungbacker, and Hol (2005), Martens, van Dijk, and de Pooter (2004), Pong, Shackleton, Taylor, and Xu (2004), and Thomakos and Wang (2003), among others.

Here we eschew such complicated fractionally integrated long-memory formulations and rely instead on the simple-to-estimate HAR-RV class of volatility models proposed by Corsi (2003). The HAR-RV formulation is based on a straightforward extension of the so-called Heterogeneous ARCH, or HARARCH, class of models analyzed by Müller et al. (1997), in which the conditional variance of the discretely sampled returns is parameterized as a linear function of the lagged squared returns over the identical return horizon together with the squared returns over longer and/or

TABLE 1A.—SUMMARY STATISTICS FOR DAILY DM/\$ REALIZED VOLATILITIES AND JUMPS

	$RV_t$	$RV_t^{1/2}$	$\log(RV_t)$	$J_t$	$J_t^{1/2}$	$\log(J_t + 1)$
Mean	0.508	0.670	-0.915	0.037	0.129	0.033
St. dev.	0.453	0.245	0.657	0.110	0.142	0.072
Skewness	3.925	1.784	0.408	16.52	2.496	7.787
Kurtosis	26.88	8.516	3.475	434.2	18.20	108.5
Min.	0.052	0.227	-2.961	0.000	0.000	0.000
Max.	5.245	2.290	1.657	3.566	1.889	1.519
LB <sub>10</sub>	3,786	5,714	7,060	16.58	119.4	63.19

TABLE 1B.—SUMMARY STATISTICS FOR DAILY S&P 500 REALIZED VOLATILITIES AND JUMPS

	$RV_t$	$RV_t^{1/2}$	$\log(RV_t)$	$J_t$	$J_t^{1/2}$	$\log(J_t + 1)$
Mean	1.137	0.927	-0.400	0.164	0.232	0.097
St. dev.	1.848	0.527	0.965	0.964	0.332	0.237
Skewness	7.672	2.545	0.375	20.68	5.585	6.386
Kurtosis	95.79	14.93	3.125	551.9	59.69	59.27
Min.	0.058	0.240	-2.850	0.000	0.000	0.000
Max.	36.42	6.035	3.595	31.88	5.646	3.493
LB <sub>10</sub>	5,750	12,184	15,992	558.0	1,868	2,295

TABLE 1C.—SUMMARY STATISTICS FOR DAILY U.S. T-BOND REALIZED VOLATILITIES AND JUMPS

	$RV_t$	$RV_t^{1/2}$	$\log(RV_t)$	$J_t$	$J_t^{1/2}$	$\log(J_t + 1)$
Mean	0.286	0.506	-1.468	0.036	0.146	0.033
St. dev.	0.222	0.173	0.638	0.069	0.120	0.055
Skewness	3.051	1.352	0.262	8.732	1.667	5.662
Kurtosis	20.05	6.129	3.081	144.6	10.02	57.42
Min.	0.026	0.163	-3.633	0.000	0.000	0.000
Max.	2.968	1.723	1.088	1.714	1.309	0.998
LB <sub>10</sub>	1,022	1,718	2,238	20.53	34.10	26.95

Key: The first six rows in each of the panels report the sample mean, standard deviation, skewness, and kurtosis, along with the sample minimum and maximum. The rows labeled LB<sub>10</sub> give the Ljung-Box test statistic for up to tenth-order serial correlation. The daily realized volatilities and jumps for the DM/\$ in panel A are constructed from five-minute returns spanning the period from December 1986 through June 1999, for a total of 3,045 daily observations. The daily realized volatilities and jumps for the S&P 500 and U.S. T-bonds in panels B and C are based on five-minute returns from January 1990 through December 2002, for a total of 3,213 observations.

shorter return horizons.<sup>13</sup> Although the HAR structure does not formally possess long memory, the mixing of relatively few volatility components is capable of reproducing a remarkably slow volatility autocorrelation decay that is almost indistinguishable from that of a hyperbolic (long-memory) pattern over most empirically relevant forecast horizons.<sup>14</sup>

A. The HAR-RV-J Model

To define the HAR-RV model, let the multiperiod *normalized* realized variation, defined by the sum of the corresponding one-period measures, be denoted by

$$RV_{t,t+h} = h^{-1}[RV_{t+1} + RV_{t+2} + \dots + RV_{t+h}], \quad (9)$$

<sup>13</sup> Müller et al. (1997) heuristically motivate the HAR model through the existence of distinct groups of traders with different investment horizons.

<sup>14</sup> Mixtures of low-order ARMA models have similarly been used in approximating and forecasting long-memory dependence in the conditional mean by Basak, Chan, and Palma (2001), Cox (1991), Hsu and Breidt (2003), Man (2003), O’Connell (1971), and Tiao and Tsay (1994), among others. The component GARCH model in Engle and Lee (1999) and the multifactor continuous time stochastic volatility model in Gallant, Hsu, and Tauchen (1999) are both motivated by similar considerations; see also the discussion of the related multifractal regime-switching models in Calvet and Fisher (2001, 2002).

where  $h = 1, 2, \dots$ . Note that, by definition  $RV_{t,t+1} \equiv RV_{t+1}$ . Also, provided that the expectations exist,  $E(RV_{t,t+h}) \equiv E(RV_{t+1})$  for all  $h$ . For ease of reference, we will refer to these normalized measures for  $h = 5$  and  $h = 22$  as the weekly and monthly volatilities, respectively. The daily HAR-RV model of Corsi (2003) may then be expressed as<sup>15</sup>

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \epsilon_{t+1}, \quad (10)$$

where  $t = 1, 2, \dots, T$ . Of course, realized volatilities over other horizons could easily be included as additional explanatory variables on the right side of the regression

<sup>15</sup> The time series of realized volatilities in this and all of the subsequent HAR-RV regressions are implicitly assumed to be stationary. Formal tests for a unit root in  $RV_{t+1}$  easily reject the null hypothesis of nonstationarity for each of the three markets. Also, the standard log-periodogram estimates of the degree of fractional integration in  $RV_{t+1}$  equal 0.347, 0.383, and 0.437, respectively, with a theoretical asymptotic standard error of 0.087.

equation, but the daily, weekly, and monthly measures employed here afford a natural economic interpretation.<sup>16</sup>

This HAR-RV model for one-day volatilities extends straightforwardly to longer horizons,  $RV_{t,t+h}$ . Moreover, given the separate nonparametric measurements of the jump component discussed above, the corresponding time series is readily included as an additional explanatory variable, resulting in the new HAR-RV-J model,

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_t + \varepsilon_{t,t+h}. \quad (11)$$

With observations every period and longer forecast horizons, or  $h > 1$ , the error term will generally be serially correlated up to (at least) order  $h - 1$ . This will not affect the consistency of the regression coefficient estimates, but the corresponding standard errors of the estimates obviously need to be adjusted. In the results discussed below, we rely on the Bartlett/Newey-West heteroskedasticity consistent covariance matrix estimator with 5, 10, and 44 lags for the daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 22$ ) regression estimates, respectively.

Turning to the results reported in the first three columns in tables 2A–C, the estimates for  $\beta_D$ ,  $\beta_W$ , and  $\beta_M$  confirm the existence of highly persistent volatility dependence. Interestingly, the relative importance of the daily volatility component decreases from the daily to the weekly to the monthly regressions, whereas the monthly volatility component tends to be relatively more important for the longer-run monthly regressions. Importantly, the estimates of the jump component,  $\beta_J$ , are systematically negative across all models and markets, and with few exceptions, overwhelmingly significant.<sup>17</sup> Thus, whereas the realized volatilities are generally highly persistent, the impact of the lagged realized volatility is significantly reduced by the jump component. For instance, for the daily DM/\$ realized volatility a unit increase in the daily realized volatility implies an average increase in the volatility on the following day of  $0.430 + 0.196/5 + 0.244/22 = 0.480$  for days where  $J_t = 0$ , whereas for days in which part of the realized volatility comes from the jump component the increase in the volatility on the following day is reduced by  $-0.486$  times the jump component. In other words, if the realized volatility is entirely attributable to jumps, it carries no predictive power for the following day's realized volatility. Similarly for the other two markets: the combined impact of a jump for forecasting the next day's realized volatility equals  $0.341 + 0.485/5 + 0.165/22 - 0.472 = -0.027$  and  $0.074 + 0.317/5 + 0.358/22 - 0.152 = -0.002$ , respectively.

<sup>16</sup> Related mixed data sampling, or MIDAS in the terminology of Ghysels, Santa-Clara, and Valkanov (2004), regressions have recently been estimated by Ghysels, Santa-Clara, and Valkanov (2006).

<sup>17</sup> Note that nothing prevents the forecasts of the realized volatilities from the HAR-RV-J model with  $\beta_J < 0$  from becoming negative. We did not find this to be a problem for any of our in-sample model estimates, however. A more complicated multiplicative error structure, along the lines of Engle (2002) and Engle and Gallo (2006), could be employed to ensure positivity of the conditional expectations.

Comparing the  $R^2$ 's of the HAR-RV-J models to the  $R^2$ 's of the "standard" HAR model reported in the last row, in which the jump component is absent and the realized volatilities on the right side but not the left side of equation (11) are replaced by the corresponding lagged squared daily, weekly, and monthly returns clearly highlights the added value of the high-frequency data. Although the coefficient estimates of the  $\beta_D$ ,  $\beta_W$ , and  $\beta_M$  coefficients in the "standard" HAR models (available upon request) generally align fairly closely with those of the HAR-RV-J models reported in the tables, the explained variation is systematically lower.<sup>18</sup> Importantly, the gains afforded by the use of the high-frequency-based realized volatilities are not restricted to the daily and weekly horizons. In fact, the longer-run monthly forecasts result in the largest relative increases in the  $R^2$ 's, with those for the S&P 500 and T-bonds tripling for the HAR-RV-J models relative to those from the HAR models based on the coarser daily, weekly, and monthly squared returns. These large gains in forecast accuracy through the use of realized volatilities are, of course, entirely consistent with the earlier empirical evidence in ABDL (2003), Bollerslev and Wright (2001), and Martens (2002), among others, and further corroborated by the analytical results of Andersen, Bollerslev, and Meddahi (2004).

### B. Nonlinear HAR-RV-J Models

Practical use of volatility models and forecasts often involves standard deviations as opposed to variances. The second set of columns in tables 2A–C thus reports the parameter estimates and  $R^2$ 's of the corresponding HAR-RV-J model cast in standard deviation form,

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_D RV_t^{1/2} + \beta_W (RV_{t-5,t})^{1/2} + \beta_M (RV_{t-22,t})^{1/2} + \beta_J J_t^{1/2} + \varepsilon_{t,t+h}. \quad (12)$$

The qualitative features and ordering of the different parameter estimates are generally the same as for the variance formulation in equation (11). In particular, the estimates for  $\beta_J$  are systematically negative. Similarly, the  $R^2$ 's indicate quite dramatic gains for the high-frequency-based HAR-RV-J model relative to the standard HAR model. The more robust volatility measurements provided by the standard deviations also result in higher  $R^2$ 's than for the variance-based models reported in the first three columns.<sup>19</sup>

As noted in table 1 above, the logarithmic daily realized volatilities are approximately unconditionally normally

<sup>18</sup> Note that although the relative magnitudes of the  $R^2$ 's for a given volatility series are directly comparable across the two models, as discussed in Andersen, Bollerslev, and Meddahi (2005), the measurement errors in the left-hand-side realized volatility invariably result in a systematic downward bias in the reported  $R^2$ 's vis-à-vis the inherent predictability in the true latent quadratic variation process.

<sup>19</sup> The  $R^2 = 0.431$  for the daily HAR-RV-J model for the DM/\$ realized volatility series in the fourth column in table 2A also exceeds the comparable in-sample one-day-ahead  $R^2 = 0.355$  for the long-memory VAR model reported in ABDL (2003).

TABLE 2A.—DAILY, WEEKLY, AND MONTHLY DM/\$ HAR-RV-J REGRESSIONS

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_t + \varepsilon_{t,t+h}$$

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_D RV_t^{1/2} + \beta_W (RV_{t-5,t})^{1/2} + \beta_M (RV_{t-22,t})^{1/2} + \beta_J J_t^{1/2} + \varepsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \beta_0 + \beta_D \log(RV_t) + \beta_W \log(RV_{t-5,t}) + \beta_M \log(RV_{t-22,t}) + \beta_J \log(J_t + 1) + \varepsilon_{t,t+h}$$

<i>h</i>	<i>RV<sub>t,t+h</sub></i>			<i>(RV<sub>t,t+h</sub>)<sup>1/2</sup></i>			<i>log(RV<sub>t,t+h</sub>)</i>		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.083 (0.015)	0.132 (0.018)	0.231 (0.025)	0.096 (0.015)	0.159 (0.021)	0.293 (0.034)	-0.122 (0.021)	-0.142 (0.030)	-0.269 (0.048)
$\beta_D$	0.430 (0.043)	0.222 (0.040)	0.110 (0.022)	0.392 (0.033)	0.216 (0.028)	0.124 (0.020)	0.408 (0.030)	0.252 (0.029)	0.162 (0.027)
$\beta_W$	0.196 (0.063)	0.216 (0.055)	0.218 (0.043)	0.248 (0.046)	0.264 (0.050)	0.243 (0.040)	0.271 (0.042)	0.265 (0.051)	0.221 (0.039)
$\beta_M$	0.244 (0.061)	0.323 (0.068)	0.225 (0.062)	0.223 (0.041)	0.296 (0.056)	0.219 (0.063)	0.212 (0.036)	0.284 (0.054)	0.226 (0.065)
$\beta_J$	-0.486 (0.096)	-0.297 (0.070)	-0.166 (0.056)	-0.103 (0.039)	-0.047 (0.031)	-0.026 (0.029)	-0.762 (0.161)	-0.590 (0.176)	-0.395 (0.178)
$R^2_{HAR-RV-J}$	0.364	0.417	0.353	0.431	0.472	0.386	0.476	0.502	0.407
$R^2_{HAR}$	0.252	0.261	0.215	0.261	0.272	0.225	0.160	0.159	0.138

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-J volatility forecast regressions. The realized volatilities and jumps are constructed from five-minute returns spanning the period from December 1986 through June 1999, for a total of 3,045 daily observations. The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively. The last two rows labeled  $R^2_{HAR-RV-J}$  and  $R^2_{HAR}$  are for the HAR-RV-J model and a "standard" HAR model with no jumps and with the realized volatilities on the right side of the regression replaced with the corresponding lagged daily, weekly, and monthly squared returns.

TABLE 2B.—DAILY, WEEKLY, AND MONTHLY S&P 500 HAR-RV-J REGRESSIONS

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_t + \varepsilon_{t,t+h}$$

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_D RV_t^{1/2} + \beta_W (RV_{t-5,t})^{1/2} + \beta_M (RV_{t-22,t})^{1/2} + \beta_J J_t^{1/2} + \varepsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \beta_0 + \beta_D \log(RV_t) + \beta_W \log(RV_{t-5,t}) + \beta_M \log(RV_{t-22,t}) + \beta_J \log(J_t + 1) + \varepsilon_{t,t+h}$$

<i>h</i>	<i>RV<sub>t,t+h</sub></i>			<i>(RV<sub>t,t+h</sub>)<sup>1/2</sup></i>			<i>log(RV<sub>t,t+h</sub>)</i>		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.088 (0.054)	0.186 (0.066)	0.387 (0.074)	0.060 (0.020)	0.102 (0.030)	0.205 (0.038)	-0.065 (0.015)	0.001 (0.019)	0.022 (0.036)
$\beta_D$	0.341 (0.094)	0.220 (0.064)	0.109 (0.040)	0.375 (0.041)	0.262 (0.038)	0.169 (0.029)	0.348 (0.028)	0.235 (0.025)	0.164 (0.024)
$\beta_W$	0.485 (0.111)	0.430 (0.097)	0.287 (0.093)	0.354 (0.064)	0.395 (0.067)	0.291 (0.071)	0.329 (0.041)	0.363 (0.050)	0.260 (0.049)
$\beta_M$	0.165 (0.067)	0.220 (0.079)	0.279 (0.088)	0.239 (0.040)	0.270 (0.058)	0.357 (0.069)	0.285 (0.031)	0.334 (0.046)	0.444 (0.054)
$\beta_J$	-0.472 (0.102)	-0.228 (0.078)	-0.075 (0.067)	-0.213 (0.051)	-0.134 (0.041)	-0.054 (0.046)	-0.260 (0.062)	-0.187 (0.064)	-0.105 (0.080)
$R^2_{HAR-RV-J}$	0.415	0.569	0.474	0.604	0.697	0.634	0.693	0.761	0.727
$R^2_{HAR}$	0.248	0.239	0.159	0.322	0.320	0.275	0.197	0.219	0.218

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-J volatility forecast regressions. The realized volatilities and jumps are constructed from five-minute returns spanning the period from January 1990 through December 2002, for a total of 3,213 daily observations. The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively. The last two rows labeled  $R^2_{HAR-RV-J}$  and  $R^2_{HAR}$  are for the HAR-RV-J model and a "standard" HAR model with no jumps and with the realized volatilities on the right side of the regression replaced with the corresponding lagged daily, weekly, and monthly squared returns.

TABLE 2C.—DAILY, WEEKLY, AND MONTHLY U.S. T-BOND HAR-RV-J REGRESSIONS

$$RV_{t,t+h} = \beta_0 + \beta_D RV_t + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_t + \varepsilon_{t,t+h}$$

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_D RV_t^{1/2} + \beta_W (RV_{t-5,t})^{1/2} + \beta_M (RV_{t-22,t})^{1/2} + \beta_J J_t^{1/2} + \varepsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \beta_0 + \beta_D \log(RV_t) + \beta_W \log(RV_{t-5,t}) + \beta_M \log(RV_{t-22,t}) + \beta_J \log(J_t + 1) + \varepsilon_{t,t+h}$$

<i>h</i>	<i>RV<sub>t,t+h</sub></i>			<i>(RV<sub>t,t+h</sub>)<sup>1/2</sup></i>			<i>log(RV<sub>t,t+h</sub>)</i>		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.077 (0.012)	0.088 (0.013)	0.126 (0.017)	0.118 (0.017)	0.152 (0.021)	0.222 (0.031)	-0.353 (0.044)	-0.323 (0.051)	-0.473 (0.080)
$\beta_D$	0.074 (0.031)	0.084 (0.016)	0.044 (0.011)	0.066 (0.026)	0.082 (0.014)	0.044 (0.009)	0.104 (0.024)	0.100 (0.015)	0.063 (0.012)
$\beta_W$	0.317 (0.050)	0.217 (0.043)	0.160 (0.040)	0.319 (0.045)	0.215 (0.039)	0.162 (0.036)	0.324 (0.043)	0.206 (0.039)	0.154 (0.032)
$\beta_M$	0.358 (0.056)	0.416 (0.055)	0.373 (0.070)	0.369 (0.046)	0.429 (0.051)	0.388 (0.068)	0.379 (0.046)	0.440 (0.053)	0.395 (0.070)
$\beta_J$	-0.152 (0.099)	-0.202 (0.042)	-0.140 (0.034)	-0.047 (0.032)	-0.062 (0.018)	-0.042 (0.015)	-0.869 (0.240)	-0.693 (0.129)	-0.545 (0.123)
$R^2_{HAR-RV-J}$	0.130	0.308	0.340	0.171	0.332	0.360	0.200	0.351	0.379
$R^2_{HAR}$	0.067	0.128	0.105	0.072	0.117	0.093	0.025	0.042	0.035

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-J volatility forecast regressions. The realized volatilities and jumps are constructed from five-minute returns spanning the period from January 1990 through December 2002, for a total of 3,213 daily observations. The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively. The last two rows labeled  $R^2_{HAR-RV-J}$  and  $R^2_{HAR}$  are for the HAR-RV-J model and a "standard" HAR model with no jumps and with the realized volatilities on the right side of the regression replaced with the corresponding lagged daily, weekly, and monthly squared returns.



distributed for each of the three markets. This empirical regularity motivated ABDL (2003) to model the logarithmic realized volatilities, in turn allowing for the use of standard normal distribution theory and related mixture models.<sup>20</sup> Guided by this same idea, we report in the last three columns of tables 2A–C the estimates of the logarithmic HAR-RV-J model,

$$\begin{aligned} \log(RV_{t,t+h}) &= \beta_0 + \beta_D \log(RV_t) + \beta_W \log(RV_{t-5}) \\ &+ \beta_M \log(RV_{t-22}) \\ &+ \beta_J \log(J_t + 1) + \varepsilon_{t,t+h}. \end{aligned} \tag{13}$$

The estimates are again directly in line with those of the HAR-RV-J models for  $RV_{t,t+h}$  and  $(RV_{t,t+h})^{1/2}$  discussed earlier. In particular, the  $\beta_D$  coefficients are generally the largest in the daily models, the  $\beta_W$ 's are the most important in the weekly models, and the  $\beta_M$ 's in the monthly models. At the same time, the negative estimates of the  $\beta_J$  coefficients temper the persistency in the forecasts, suggesting that jumps in the price processes tend to be associated with short-lived bursts in volatility.

**V. Shrinkage Estimation and Microstructure Noise Correction**

The empirical results discussed thus far rely on the simple nonparametric jump estimates defined by the difference between the realized volatility and the bipower variation. As discussed in section II, the theoretical justification for those measurements is based on the notion of increasingly finer sampled returns, or  $\Delta \rightarrow 0$ . Of course, any practical implementation with a fixed sampling frequency, or  $\Delta > 0$ , is invariably subject to measurement error. The nonnegativity truncation in equation (8) alleviates part of this finite-sample problem by eliminating theoretically infeasible negative estimates for the squared jumps. However, the resulting  $J_t^{1/2}$  series depicted in figures 1A–C arguably also exhibit an unreasonably large number of nonzero small positive values. It may be desirable to treat these small jumps as measurement errors, or part of the continuous sample path variation process, associating only large values of  $RV_t(\Delta) - BV_t(\Delta)$  with the jump component. The next subsection provides a theoretical framework for doing so.

*A. Asymptotic Distribution Theory*

The distributional results developed in Barndorff-Nielsen and Shephard (2004a, 2006) and extended in Barndorff-

Nielsen, Graversen, Jacod, et al. (2006) imply that, under sufficient regularity, frictionless market conditions and in the absence of jumps in the price path,

$$\Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) \int_t^{t+1} \sigma^4(s)ds]^{1/2}} \Rightarrow N(0, 1), \tag{14}$$

for  $\Delta \rightarrow 0$ . Hence, an abnormally large value of this standardized difference between  $RV_{t+1}(\Delta)$  and  $BV_{t+1}(\Delta)$  is naturally interpreted as evidence in favor of a “significant” jump over the  $[t, t + 1]$  time interval. Of course, the *integrated quarticity* that appears in the denominator needs to be estimated in order to actually implement this statistic. In parallel to the arguments underlying the robust estimation of the integrated volatility by the realized bipower variation, it is possible to show that even in the presence of jumps, the integrated quarticity may be consistently estimated by the normalized sum of the product of  $n \geq 3$  adjacent absolute returns raised to the power of  $4/n$ . In particular, on defining the standardized *realized tripower quarticity* measure,

$$\begin{aligned} TQ_{t+1}(\Delta) & \\ \equiv \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-1)\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-2)\cdot\Delta,\Delta}|^{4/3}, \end{aligned} \tag{15}$$

where  $\mu_{4/3} \equiv 2^{2/3} \cdot \Gamma(7/6) \cdot \Gamma(1/2)^{-1} = E(|Z|^{4/3})$ . It follows that for  $\Delta \rightarrow 0$ ,

$$TQ_{t+1}(\Delta) \Rightarrow \int_t^{t+1} \sigma^4(s)ds. \tag{16}$$

Combining the results in equations (14)–(16), the “significant” jumps may therefore be identified by comparing realizations of the feasible test statistics,<sup>21</sup>

<sup>21</sup> Similar results were obtained by using the robust *realized quad-power quarticity* measure advocated in Barndorff-Nielsen and Shephard (2004a, 2006),

$$QQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_4^{-4} \sum_{j=4}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}| |r_{t+(j-1)\cdot\Delta,\Delta}| |r_{t+(j-2)\cdot\Delta,\Delta}| |r_{t+(j-3)\cdot\Delta,\Delta}|.$$

Note, however, that the realized quarticity,

$$RQ_{t+1}(\Delta) \equiv RPV_{t+1}(\Delta, 4) \equiv \Delta^{-1} \mu_4^{-1} \sum_{j=1}^{1/\Delta} r_{t+j\cdot\Delta,\Delta}^4,$$

used in estimating the integrated quarticity by Barndorff-Nielsen and Shephard (2002a) and Andersen, Bollerslev, and Meddahi (2005) is not consistent in the presence of jumps, which in turn would result in a complete loss of power for the corresponding test statistic obtained by replacing  $TQ_{t+1}(\Delta)$  in equation (17) with  $RQ_{t+1}(\Delta)$ .

<sup>20</sup> This same transformation has subsequently been used for other markets by Deo, Hurvich, and Lu (2006), Koopman, Jungbacker, and Hol (2005), and Martens, van Dijk, and de Pooter (2004) among others. Of course, the log-normal distribution isn't closed under temporal aggregation. Thus, if the daily logarithmic realized volatilities are normally distributed, the weekly and monthly volatilities can not also be log-normally distributed. However, as argued by Barndorff-Nielsen and Shephard (2002a) and Forsberg and Bollerslev (2002), the log-normal volatility distributions may be closely approximated by Inverse Gaussian distributions, which are formally closed under temporal aggregation.

$$W_{t+1}(\Delta) \equiv \Delta^{-1/2} \frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5)TQ_{t+1}(\Delta)]^{1/2}}, \quad (17)$$

to a standard normal distribution.

The extensive simulation-based evidence for specific parametric continuous time diffusions reported in Huang and Tauchen (2005) suggests that the  $W_{t+1}(\Delta)$  statistic defined in equation (17) tends to over-reject the null hypothesis of no jumps for large critical values. At the same time, following the approach advocated by Barndorff-Nielsen and Shephard (2004b), different variance stabilizing transforms for the joint asymptotic distribution of the realized volatility and bipower variation measures generally produce test statistics with improved finite-sample performance. In particular, on applying the delta rule to the joint bivariate distribution, Huang and Tauchen (2005) find that the ratio-statistic,

$$Z_{t+1}(\Delta) \equiv \Delta^{-1/2} \times \frac{[RV_{t+1}(\Delta) - BV_{t+1}(\Delta)]RV_{t+1}(\Delta)^{-1}}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5) \max\{1, TQ_{t+1}(\Delta)BV_{t+1}(\Delta)^{-2}\}]^{1/2}}, \quad (18)$$

where the max adjustment follows by a Jensen's inequality argument as in Barndorff-Nielsen and Shephard (2004b), is very closely approximated by a standard normal distribution throughout its entire support in samples of the size relevant here.<sup>22</sup> Moreover, they find that the ratio-statistic in equation (18) also has reasonable power against several empirically realistic calibrated stochastic volatility jump diffusion models.

Hence we naturally identify the "significant" jumps by the realizations of  $Z_{t+1}(\Delta)$  in excess of some critical value, say  $\Phi_\alpha$ ,

$$J_{t+1,\alpha}(\Delta) \equiv I[Z_{t+1}(\Delta) > \Phi_\alpha] \cdot [RV_{t+1}(\Delta) - BV_{t+1}(\Delta)], \quad (19)$$

where  $I[\cdot]$  denotes the indicator function.<sup>23</sup> To ensure that the estimated continuous sample path component variation and jump variation sum to the total realized variation, we estimate the former component as the residual,

$$C_{t+1,\alpha}(\Delta) \equiv I[Z_{t+1}(\Delta) \leq \Phi_\alpha] \cdot RV_{t+1}(\Delta) + I[Z_{t+1}(\Delta) > \Phi_\alpha] \cdot BV_{t+1}(\Delta). \quad (20)$$

<sup>22</sup> In an earlier version of this paper, we relied on the log-based statistic,

$$U_{t+1}(\Delta) \equiv \Delta^{-1/2} \frac{\log(RV_{t+1}(\Delta)) - \log(BV_{t+1}(\Delta))}{[(\mu_1^{-4} + 2\mu_1^{-2} - 5)TQ_{t+1}(\Delta)BV_{t+1}(\Delta)^{-2}]^{1/2}},$$

which produced qualitatively very similar results.

<sup>23</sup> As noted in personal communication with Neil Shephard, this may alternatively be interpreted as a shrinkage estimator for the jump component.

Note that for  $\Phi_\alpha > 0$ , the definitions in equations (19) and (20) automatically guarantee that both  $J_{t+1,\alpha}(\Delta)$  and  $C_{t+1,\alpha}(\Delta)$  are positive.<sup>24</sup> Of course, the nonnegativity truncation imposed in equation (8) underlying the empirical jump measurements employed in the preceding two sections corresponds directly to  $\alpha = 0.5$ , or  $J_{t,0.5}$ .

### B. Market Microstructure Noise

As already discussed in section IIIA, a host of practical market microstructure frictions, including the use of discrete price grid points and bid-ask spreads, invariably renders fictitious the assumption of a continuously observed logarithmic price process following a semimartingale. Hence, following Ait-Sahalia, Mykland, and Zhang (2005), Bandi and Russell (2006), and Zhang, Mykland, and Ait-Sahalia (2005), among others, assume that the observed price process is "contaminated" by a market microstructure noise component, say  $p(t) = p^*(t) + v(t)$ , where  $p^*(t)$  refers to the true (latent) semimartingale logarithmic price process that would obtain in the absence of any frictions, while  $v(t)$  denotes an i.i.d. white-noise component.<sup>25</sup> The discretely sampled  $\Delta$ -period observed returns,

$$r_{t,\Delta} \equiv p^*(t) - p^*(t - \Delta) + v(t) - v(t - \Delta) \equiv r_{t,\Delta}^* + \eta_{t,\Delta}, \quad (21)$$

then equal the true (latent) returns plus the first-order moving-average process,  $\eta_{t,\Delta}$ . Assuming that the variance of  $v(t)$  does not depend upon  $\Delta$ , the noise term will eventually (for  $\Delta \rightarrow 0$ ) dominate the contribution to the overall realized variation in equation (3) coming from the squared true (latent) high-frequency returns, formally rendering  $RV_{t+1}(\Delta)$  inconsistent as a measure of the quadratic variation of  $p^*(t)$ . In practice, the impact of the market microstructure noise is most easily controlled through the choice of  $\Delta$ . Our choice of a five-minute sampling frequency for the very active markets analyzed here is motivated by this bias-variance tradeoff, as the bias in the realized variation measure in equation (3) appears to largely vanish at this frequency.

By analogous arguments, the noise term will generally result in an upward bias in the new bipower variation measure in equation (5) for  $\Delta$  "too small," as  $E(|r_{t,\Delta}^*|) < E(|r_{t,\Delta}^* + \eta_{t,\Delta}|)$ . The first-order serial correlation in  $\eta_{t,\Delta}$  further implies that any two adjacent observed returns, say  $r_{t+j\cdot\Delta,\Delta}$  and  $r_{t+(j-1)\cdot\Delta,\Delta}$ , will be serially correlated. In comparison to the realized variation measure based on the sum

<sup>24</sup> It is possible that, by specifying  $\alpha(\Delta) \rightarrow 1$  as an explicit function of  $\Delta \rightarrow 0$ , this approach may formally be shown to result in period-by-period consistent (as  $\Delta \rightarrow 0$ ) estimates of the jump component. Of course, data limitations restrict the sampling frequency ( $\Delta > 0$ ), rendering such a result of limited practical use.

<sup>25</sup> More complicated non-i.i.d. market microstructure noise components have been analyzed in the realized volatility setting by Bandi and Russell (2004) and Hansen and Lunde (2006), among others.

of the squared high-frequency returns, this spuriously induced first-order serial correlation will therefore result in an additional source of bias in the  $BV_{t+1}(\Delta)$  measure. Of course, similar arguments apply to the tripower quarticity measure in equation (15). As discussed at length in Huang and Tauchen (2005), this in turn implies that in the presence of market microstructure noise, the jump test statistics discussed in the previous section will generally be biased *against* finding jumps. In particular, it is possible to show that in the absence of jumps,  $\lim_{\Delta \rightarrow \infty} [RV_{t+1}(\Delta) - BV_{t+1}(\Delta)] = \kappa < 0$ , so that the  $W_{t+1}(\Delta)$  test statistic defined in equation (17) will be negatively biased. Although comparable analytical results are not available for the ratio-statistic in equation (18), the numerical calculations and extensive simulation evidence reported in Huang and Tauchen (2005) confirm that for small values of  $\Delta$ , the test tends to be undersized, and this tendency to under-reject further deteriorates with the magnitude of the variance of the  $v(t)$  noise component.

The spurious serial correlation in the observed returns defined in equation (21) is, however, readily broken through the use of staggered, or skip-one, returns. Specifically, replacing the sum of the absolute adjacent returns in equation (5) with the corresponding staggered absolute returns, a modified realized bipower variation measure may be defined by

$$BV_{1,t+1}(\Delta) \equiv \mu_1^{-2} (1 - 2\Delta)^{-1} \sum_{j=3}^{1/\Delta} |r_{t+j \cdot \Delta, \Delta}| |r_{t+(j-2) \cdot \Delta, \Delta}|, \quad (22)$$

where the normalization factor in front of the sum reflects the loss of two observations due to the staggering. Of course, higher-order serial dependence could be broken in an analogous fashion by further increasing the lag length. Similarly, the integrated quarticity may alternatively be estimated by the staggered realized tripower quarticity,

$$TQ_{1,t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} (1 - 4\Delta)^{-1} \sum_{j=5}^{1/\Delta} |r_{t+j \cdot \Delta, \Delta}|^{4/3} |r_{t+(j-2) \cdot \Delta, \Delta}|^{4/3} |r_{t+(j-4) \cdot \Delta, \Delta}|^{4/3}. \quad (23)$$

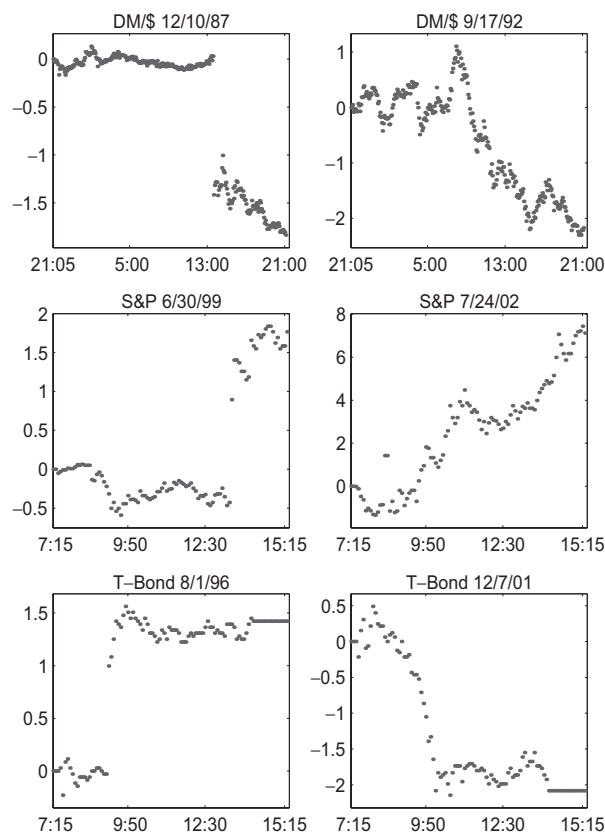
Importantly, as shown by Barndorff-Nielsen and Shephard (2004a), in the absence of the noise component, these staggered realized variation measures remain consistent for the corresponding integrated variation measures. Consequently, the asymptotic distribution of the test statistic obtained by replacing  $BV_{t+1}(\Delta)$  and  $TQ_{t+1}(\Delta)$  in equation (18) with their staggered counterparts,  $BV_{1,t+1}(\Delta)$  and  $TQ_{1,t+1}(\Delta)$ , respectively, say  $Z_{1,t+1}(\Delta)$ , will also be asymptotically (for  $\Delta \rightarrow 0$ ) standard normally distributed. However, following the discussion above, the staggering should help alleviate the confounding influences of the market

microstructure noise, resulting in empirically more accurate finite-sample approximations. This conjecture is indeed confirmed by the comprehensive simulation results reported in Huang and Tauchen (2005), which show that the ratio-statistic calculated with the staggered realized bipower and tripower variation measures performs admirably for a wide range of market microstructure contaminants.<sup>26</sup>

Hence, in the empirical results reported below we rely on the  $J_{t,\alpha}(\Delta)$  and  $C_{t,\alpha}(\Delta)$  measures previously defined in equations (19) and (20) calculated on the basis of the staggered  $Z_{1,t}(\Delta)$  statistic. To facilitate the notation, we will again omit the explicit reference to the sampling frequency,  $\Delta$ , simply referring to the jump and continuous sample path variability measures calculated from the five-minute returns as  $J_{t,\alpha}$  and  $C_{t,\alpha}$ , respectively. Subsequently, we shall summarize various features of these jump measurements for values of  $\alpha$  ranging from 0.5 to 0.9999, or  $\Phi_\alpha$  ranging from 0.0 to 3.719.

<sup>26</sup> Quoting from the conclusion of Huang and Tauchen (2005): “The Monte Carlo evidence suggests that, under the arguable realistic scenarios considered here, the recently developed tests for jumps perform impressively and are not easily fooled.”

FIGURE 2.—INTRADAY PRICE MOVEMENTS



Key: The figure graphs the five-minute intraday price increments for days with large jump statistics  $Z_{1,t}(\Delta)$  (left-side panels), and days with large daily price moves but numerically small jump statistics (right-side panels).

### C. Jump Measurements and Macroeconomic News

Before summarizing the full-sample time series evidence, it is instructive to look at a few specific days to illustrate the working of the jump statistic. To this end, figure 2 displays the five-minute increments in the logarithmic prices for a highly significant jump day and a day with a large continuous price move for each of the three markets. For ease of comparison, the logarithmic price has been normalized to zero at the beginning of each day, so that a unit increment corresponds to a 1% return in all of the graphs.

The first panel shows the movements in the DM/\$ exchange rate on December 10, 1987. The  $Z_{1,t}$  statistics for this day equals 10.315, thus indicating a highly significant jump. The timing of the jump, as evidenced by the apparent discontinuity at 13:30 GMT, corresponds exactly to the 8:30 EST release of the U.S. trade deficit for the month of October.<sup>27</sup> Quoting from the *Wall Street Journal*: “The trade gap swelled to a record \$17.63 billion in October, sending the dollar and bonds plunging.” The second panel in the first row depicts a similar large daily decline in the value of the dollar on September 17, 1992. In fact, this is the day in the entire sample with the highest value of  $BV_{1,t} = 4.037$ . At the same time, the  $Z_{1,t}$  statistic for this day equals  $-0.326$ , and thus in spite of the overall large daily move, does not signify any jump(s). This particular day succeeds the day following the temporary withdrawal of the British pound from the European Monetary System, and it has previously been highlighted in the study by Barndorff-Nielsen and Shephard (2006). Again, quoting from the *Wall Street Journal*: “The dollar and the pound each sank more than 2% against the mark as nervousness persisted in the currency market.” Of course, without the benefit of the intraday high-frequency data, the day-to-day price moves in the first two panels would look almost identical.

Turning to the second row in the figure, the first panel shows the five-minute movements in the S&P 500 on June 30, 1999. As suggested by visual inspection of the plot, the  $Z_{1,t} = 7.659$  statistic is again highly significant. Moreover, the apparent timing of the jump at 13:15 CST, or 14:15 EST, corresponds exactly to the time of the 1/4% increase in the Fed funds rate on that day. That rate hike was accompanied by a statement by the Fed that it “might not raise rates again in the near term due to conflicting forces in the economy,” which apparently was viewed as a positive sign by the market. In contrast, on July 24, 2002, as depicted in the panel on the right,  $Z_{1,t} = -0.704$ , while  $BV_{1,t}$  achieves its maximum value of 29.247. The abnormally large daily return of 7.157 is also the largest over the whole sample. Yet, this “rough” daily move is made up of the sum of many “smooth” intraday price moves, with no apparent jump(s) in

the process. Interestingly, the NYSE also saw a record trading volume of 2.77 billion shares on that day.

The last row in the figure refers to the T-bond market. The apparent timing of the highly significant jump,  $Z_{1,t} = 6.877$ , on August 1, 1996, corresponds directly to the release of the National Association of Purchasing Managers (NAPM) index at 9:00 CST, or 10:00 EST.<sup>28</sup> In contrast, the second T-bond panel for December 7, 2001, again depicts a large daily, but generally “smooth” intraday, price move. Interestingly, most of the movements occurred in the morning following news of higher than expected joblessness. While this did not result in an immediate jump in the T-bond price, it reassured most economists that the Fed would cut its rate at the next board meeting the following business day, which in fact it did. According to the *Wall Street Journal*: “Economists said the jobs report removed any lingering doubts that the Federal Reserve will reduce interest rates for the 11th time in the past 12 months when it meets tomorrow.”

The direct association of the highly significant jump days in figure 2 with readily identifiable macroeconomic news affirms earlier case studies for the DM/\$ foreign exchange market in Barndorff-Nielsen and Shephard (2006) and is directly in line with the aforementioned evidence in Andersen, Bollerslev, Diebold, and Vega (2003, 2005), among others, documenting significant intradaily price moves in response to a host of macroeconomic news announcements. Similarly, Johannes (2004) readily associates the majority of the estimated jumps in a parametric jump-diffusion model for daily interest rates with specific macroeconomic news events. At the same time, informal inspection suggests that not all of the jump days identified by large values of the nonparametric high-frequency  $Z_{1,t}$  statistic are as easily linked to specific “news” arrivals. Indeed, it would be interesting, but beyond the scope of the present paper, to attempt a more systematic characterization of the types of events that cause the different markets to jump. Instead, we turn to a discussion of various summary statistics related to the time series of significant jumps employed in our subsequent volatility forecasting models.<sup>29</sup>

### D. Jump Dynamics

To begin, the first row in tables 3A–C reports the proportion of days with significant jumps for each of the three

<sup>28</sup> The results in Andersen, Bollerslev, Diebold, and Vega (2005) again indicate that, for the T-bond market, news about the NAPM index results in the overall highest five-minute return regression  $R^2$  among all of the regularly scheduled macroeconomic announcements.

<sup>29</sup> Obviously, it is literally impossible to obtain perfect jump detection on the basis of discretely observed data. On very volatile days smaller jumps may be “hidden,” resulting in a loss of power, or an increase in the type II error rate. On the other hand, on very tranquil days the inference will be more precise, and certain “microstructure effects” may be identified as jumps, resulting in an increase in the type I error rate. The Huang and Tauchen (2005) study provides further guidelines for understanding these tradeoffs. Meanwhile, as discussed in section VI, from a practical perspective the most relevant question arguably remains: What are the gains in forecast accuracy (if any) to our particular approach of disentangling jump and nonjump components?

<sup>27</sup> The systematic news announcement analysis in Andersen, Bollerslev, Diebold, and Vega (2003, 2005) also points to the U.S. trade balance as one of the most important regularly scheduled macroeconomic news releases for the foreign exchange market.

TABLE 3A.—SUMMARY STATISTICS FOR SIGNIFICANT DAILY DM/\$ JUMPS

$\alpha$	0.500	0.950	0.990	0.999	0.9999
Prop.	0.859	0.409	0.254	0.137	0.083
Mean.	0.059	0.047	0.037	0.028	0.021
St. dev.	0.136	0.137	0.135	0.131	0.127
LB <sub>10</sub> , $J_{t,\alpha}$	65.49	26.30	6.197	3.129	2.414
LR, $I(J_{t,\alpha} > 0)$	0.746	2.525	0.224	0.994	0.776
LB <sub>10</sub> , $D_{t,\alpha}$	10.78	9.900	7.821	6.230	19.95
LB <sub>10</sub> , $J_{t,\alpha}^+$	73.62	116.4	94.19	87.69	34.57

TABLE 3B.—SUMMARY STATISTICS FOR SIGNIFICANT DAILY S&P 500 JUMPS

$\alpha$	0.500	0.950	0.990	0.999	0.9999
Prop.	0.737	0.255	0.141	0.076	0.051
Mean.	0.163	0.132	0.111	0.095	0.086
St. dev.	0.961	0.961	0.958	0.953	0.950
LB <sub>10</sub> , $J_{t,\alpha}$	300.6	271.9	266.4	260.9	221.6
LR, $I(J_{t,\alpha} > 0)$	2.415	1.483	12.83	8.418	7.824
LB <sub>10</sub> , $D_{t,\alpha}$	50.83	31.47	22.67	36.18	49.25
LB <sub>10</sub> , $J_{t,\alpha}^+$	320.8	146.0	77.06	35.11	25.49

TABLE 3C.—SUMMARY STATISTICS FOR SIGNIFICANT DAILY U.S. T-BOND JUMPS

$\alpha$	0.500	0.950	0.990	0.999	0.9999
Prop.	0.860	0.418	0.254	0.132	0.076
Mean.	0.048	0.038	0.030	0.021	0.016
St. dev.	0.094	0.096	0.096	0.090	0.085
LB <sub>10</sub> , $J_{t,\alpha}$	30.34	30.37	27.85	19.80	18.85
LR, $I(J_{t,\alpha} > 0)$	4.746	21.62	13.69	3.743	1.913
LB <sub>10</sub> , $D_{t,\alpha}$	45.55	100.1	59.86	103.3	81.42
LB <sub>10</sub> , $J_{t,\alpha}^+$	21.23	17.18	15.18	9.090	11.98

Key: The significant jumps for each of the three markets,  $J_{t,\alpha}$ , are determined by equation (19) along with the staggered bipower and tripower variation measures in equations (22) and (23), respectively. The first row in each of the panels gives the proportion significant jump days for each of the different  $\alpha$ 's. The next two rows report the corresponding mean and standard deviation of the jump series, while the row labeled LB<sub>10</sub>,  $J_{t,\alpha}$  gives the Ljung-Box tests for up to tenth-order serial correlation. LR,  $I(J_{t,\alpha} > 0)$  denotes the likelihood ratio test for *i.i.d* jump occurrences against a first Markov chain, while LB<sub>10</sub>,  $D_{t,\alpha}$ , and LB<sub>10</sub>,  $J_{t,\alpha}^+$  refer to the Ljung-Box tests for serial correlation in the corresponding durations, or times between jumps, and the sizes of the significant jumps, respectively.

markets based on the  $Z_{1,t}$  statistic as a function of the significance level,  $\alpha$ . Although the use of  $\alpha$ 's in excess of 0.5 has the intended effect of reducing the number of days with jumps, the procedure still identifies many more significant jumps than would be expected if the underlying price process was continuous. Comparing the jump intensities across the three markets, the foreign exchange and the T-bond markets generally exhibit the highest proportion of jumps, whereas the stock market has the lowest.<sup>30</sup> For instance, employing a cutoff of  $\alpha = 0.999$ , or  $\Phi_\alpha = 3.090$ , results in 417, 244, and 424 significant jumps for each of the three markets respectively, all of which far exceed the expected three jumps for a continuous price process (0.001 times 3,045 and 3,213, respectively). Indeed, all of the daily jump proportions are much higher than the jump intensities estimated with specific parametric jump diffusion models applied to daily or coarser-frequency returns, which typically suggest only a few jumps a year; see, for example, the estimates for the S&P 500 in Andersen, Benzoni, and Lund (2002). Intuitively, just as the stock market crash of 1987 and the corresponding large negative daily return on Octo-

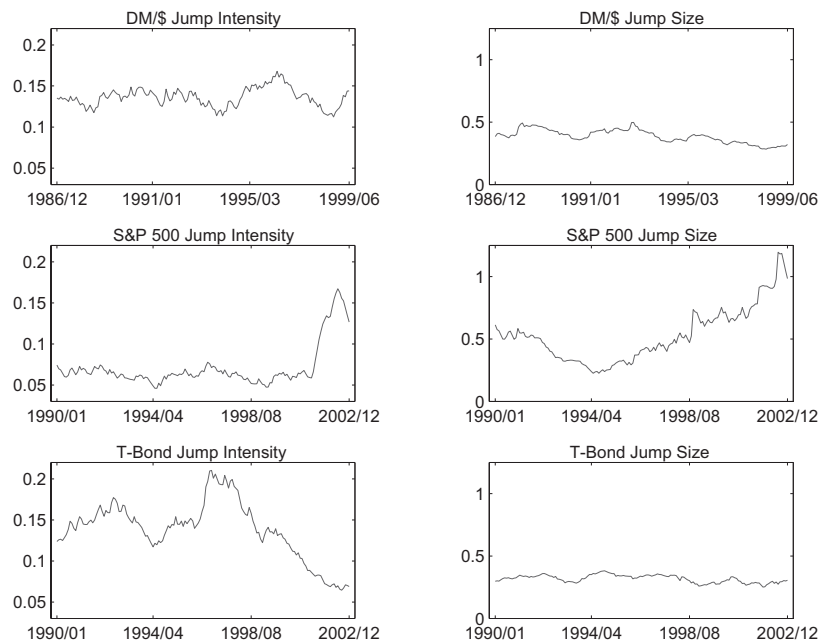
ber 17 is not visible in the time series of annual equity returns, many of the jumps identified by the high-frequency-based realized variation measures employed here will invariably be hidden in the coarser daily or lower-frequency returns.

Turning to the second and third rows in the table, it is noteworthy that although the proportion of jumps depends importantly on the particular choice of  $\alpha$ , the sample means and standard deviations of the resulting jump time series are not nearly as sensitive to the significance level. This observation is further corroborated by the time series plots for each of the three markets in the third and fourth panels in figures 1A–1C, which show the  $Z_{1,t}$  statistics and a horizontal line at 3.090, along with the resulting significant jumps, or  $J_{t,0.999}^{1/2}$ . It is evident that the test statistic generally picks out the largest values of  $J_t^{1/2}$  as being significant, so that the sample means and standard deviations of the time series depicted in the second and the fourth panels are all fairly close.

The Ljung-Box statistics for up to tenth-order serial correlation in the  $J_{t,\alpha}$  series for the S&P 500 reported in the fourth row in table 3B are all highly significant, regardless of the choice of  $\alpha$ . This contrasts with most of the parametric jump-diffusion model estimates reported in the recent literature, which as previously noted suggest very little, or

<sup>30</sup> This ordering among the three markets is again consistent with the evidence in Andersen, Bollerslev, Diebold, and Vega (2005), showing that equity markets generally respond the least to macroeconomic news announcements.

FIGURE 3.—SMOOTHED JUMP INTENSITIES AND JUMP SIZES



Key: The figure graphs the exponentially smoothed (with a smoothing parameter of 0.94) average monthly jump intensities and sizes for the significant jumps based on  $\alpha = 0.999$ . The jump sizes are expressed in standard deviation form, or  $J_{t,0.999}^{1/2}$ .

no, predictable variation in the jump process. Still, it is noteworthy that the values of the Ljung-Box statistics for the significant S&P 500 jumps are all much less than the corresponding statistics for the realized variation series reported in table 1B. The corresponding Ljung-Box tests for the DM/\$ and T-bond jump series are not nearly as large, and generally insignificant for the jumps defined by  $\alpha$ 's in excess of 0.990.

These findings are further corroborated by Christoffersen's (1998) likelihood ratio test for the null of i.i.d. jumps against a first-order Markov alternative reported in the fifth row. Under the null of no dependence, this test statistic should be asymptotically *chi*-square distributed with one degree of freedom. None of the test statistics for  $\alpha$  equal to 0.999 or 0.9999 for the DM/\$ and T-bond markets exceed the corresponding 95% critical value of 3.84, while the tests for the S&P 500 are highly significant.

Interestingly, when looking beyond the own linear dependence and the Ljung-Box test for the  $J_{t,\alpha}$  series, a somewhat different picture emerges. In particular, decomposing the  $J_{t,\alpha}$  series into the times between jumps and the sizes of the corresponding jumps, there appears to be strong evidence for clustering in the occurrences of the significant jumps for both the S&P 500 and T-bond markets, as evidenced by the Ljung-Box test for up to tenth-order serial correlation in the durations between the jumps, denoted by  $LB_{10}, D_{t,\alpha}$  in table 3. Similarly, the Ljung-Box tests for serial correlation in the time series of only the significant jumps, denoted by  $LB_{10}, J_{t,\alpha}^+$  in table 3, strongly suggest that large (small) jumps tend to cluster together in time with other large (small) jumps for both the DM/\$ and S&P 500 markets. In contrast, for the

T-bond market and  $\alpha = 0.999$ , only the durations but not the sizes of the jumps appear to cluster in time.

The more complex dynamic dependence in the significant jump time series is further illustrated in figure 3, which plots the smoothed jump intensities and jump sizes for each of the three markets. The graphs are constructed by exponentially smoothing (with a smoothing parameter of 0.94) the average monthly jump intensities and sizes of the significant jumps based on  $\alpha = 0.999$ . The jump sizes are again expressed in standard deviation form, or  $J_{t,0.999}^{1/2}$ . From the very first panel the DM/\$ jump intensities are approximately constant throughout the sample. Similarly, the smoothed jump sizes for the T-bond market depicted in the last panel vary very little over the sample period. In contrast, all of the other four panels suggest the existence of potentially important temporal dependence in the jump arrival processes and jump sizes.<sup>31</sup> Of course, exponential smoothing automatically induces some serial correlation, so that the appearance of dependence gleaned from the figure should be carefully interpreted. Note also that the common scale enforced on the three jump-size panels tends to hide the subtle, but

<sup>31</sup> It would be interesting, but beyond the scope of the present paper, to further explore the formulation of parametric jump-diffusion models best designed to capture such nonlinear dependence. For instance, following Bates (2000), Pan (2002), and Eraker (2004), the jump intensity could be specified as a function of the instantaneous diffusion volatility. In the notation of equation (1),  $\lambda(t) = \lambda_0 + \lambda_1\sigma(t)$ . Similarly, the size of the jumps could be allowed to depend on the volatility and/or lagged past jump sizes, as in, for example,  $\kappa^2(t) = \kappa_0 + \kappa_1\sigma(t) + \kappa_2\kappa^2(t-1)$ . The recent discrete-time parametric model estimates reported in Chan and Maheu (2002) and McCurdy and Maheu (2004) also point to the existence of time-varying jump intensities in U.S. equity index returns.

systematic, decline in the sizes of the jumps for the DM/\$ market over the sample period, as evidenced by the highly significant  $LB_{10}$ ,  $J_{t,\alpha}^+$  in table 3; see also the plot for the raw jump series in figure 1A.

Instead, we next turn to a simple extension of the HAR-RV-J volatility forecasting model introduced in section IV, in which we incorporate the time series of significant jumps as additional explanatory variables in a straightforward linear fashion.

### VI. Accounting for Jumps in Realized Volatility Modeling and Forecasting, Revisited

The regression estimates of the HAR-RV-J model reported in section IV show that inclusion of the simple consistent daily jump measure corresponding to  $\alpha = 0.5$  as an additional explanatory variable over and above the daily realized volatilities results in highly significant and negative estimates of the jump coefficient. These results are, of course, entirely consistent with the summary statistics for the jump measurements discussed above, which indicate markedly less own (linear) serial correlation in the significant jump series in comparison to the realized volatility series. Building on these results, the present section extends the HAR-RV-J model by explicitly decomposing the realized volatilities that appear as explanatory variables on the right side into the continuous sample path variability and the jump variation utilizing the separate nonparametric measurements based on the  $Z_{1,t}$  statistic along with equations (19) and (20), respectively. In so doing, we rely exclusively on  $\alpha = 0.999$ , and the jump series depicted in the bottom panels of figures 1A–C.<sup>32</sup> To facilitate the exposition, we omit the 0.999 subscript on the  $J_{t,0.999}$  and  $C_{t,0.999}$  series in what follows.

#### A. The HAR-RV-CJ Model

Defining the normalized multiperiod jump and continuous sample path variability measures,

$$J_{t,t+h} = h^{-1}[J_{t+1} + J_{t+2} + \dots + J_{t+h}], \quad (24)$$

and

$$C_{t,t+h} = h^{-1}[C_{t+1} + C_{t+2} + \dots + C_{t+h}], \quad (25)$$

respectively, the new HAR-RV-CJ model may be expressed as

$$RV_{t,t+h} = \beta_0 + \beta_{CD}C_t + \beta_{CW}C_{t-5,t} + \beta_{CM}C_{t-22,t} + \beta_{JD}J_t + \beta_{JW}J_{t-5,t} + \beta_{JM}J_{t-22,t} + \varepsilon_{t,t+h}. \quad (26)$$

The model obviously nests the HAR-RV-J model in equation (11) for  $\beta_D = \beta_{CD} + \beta_{JD}$ ,  $\beta_W = \beta_{CW} + \beta_{JW}$ ,  $\beta_M =$

$\beta_{CM} + \beta_{JM}$ , and  $\beta_J = \beta_{JD}$ , but in general it allows for a more flexible dynamic lag structure.<sup>33</sup>

Turning to the empirical estimates in the first three columns in tables 4A–C, most of the jump coefficient estimates are insignificant. In other words, the predictability in the HAR-RV realized volatility regressions is almost exclusively due to the continuous sample path components. For the DM/\$ and the S&P 500, the HAR-RV-CJ models typically result in relatively modest increases in the  $R^2$  of less than 0.01 in absolute value compared to the HAR-RV-J models in tables 2A–B, whereas for the T-bond market the improvements are closer to 0.02, or about 4%–5% in a relative sense. The tests for first-, sixth-, and twenty-third-order serial correlation in the residuals from the estimated ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 22$ ) regressions also indicate that the HAR-RV-CJ models have eliminated most of the strong serial correlation in the  $RV_{t,t+h}$  series.<sup>34</sup> Still, some statistically significant autocorrelations remain at higher lags for some of the models, suggesting that further refinements might be possible.

These same qualitative results carry over to the nonlinear HAR-RV-CJ models cast in standard deviation and logarithmic form; that is,

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_{CD}C_t^{1/2} + \beta_{CW}(C_{t-5,t})^{1/2} + \beta_{CM}(C_{t-22,t})^{1/2} + \beta_{JD}J_t^{1/2} + \beta_{JW}(J_{t-5,t})^{1/2} + \beta_{JM}(J_{t-22,t})^{1/2} + \varepsilon_{t,t+h}, \quad (27)$$

and

$$\log(RV_{t,t+h}) = \beta_0 + \beta_{CD} \log(C_t) + \beta_{CW} \log(C_{t-5,t}) + \beta_{CM} \log(C_{t-22,t}) + \beta_{JD} \log(J_t + 1) + \beta_{JW} \log(J_{t-5,t} + 1) + \beta_{JM} \log(C_{t-22,t} + 1) + \varepsilon_{t,t+h}, \quad (28)$$

respectively. The jump coefficient estimates, reported in the last six columns in tables 4A–C, are again insignificant for most of the markets and forecast horizons. In contrast, the estimates of  $\beta_{CD}$ ,  $\beta_{CW}$ , and  $\beta_{CM}$ , which quantify the impact of the continuous sample path variability on the total future variation, are all generally highly significant.

To further illustrate the predictability afforded by the HAR-RV-CJ model, figures 4A–C plot the daily, weekly, and monthly realized volatilities (again in standard-deviation form) together with the corresponding forecasts from the model in equation (27). The close coherence

<sup>33</sup> For the two models to be nested, the (implicit) choice of  $\alpha$  employed in the measurements of  $J_{t,t+h}$  and  $C_{t,t+h}$  should, of course, be the same across models.

<sup>34</sup> The lag one, six, and twenty-three autocorrelations of the residuals from the three DM/\$ HAR-RV-CJ models are 0.014,  $-0.026$ , and 0.003, respectively. For the S&P 500, the same residual autocorrelations are  $-0.011$ , 0.007, and  $-0.081$ , while for T-bonds the autocorrelations are 0.003,  $-0.006$ , and  $-0.060$ , respectively.

<sup>32</sup> By “optimally” choosing  $\alpha$ , it may be possible to further improve upon the empirical results reported below. However, for simplicity and to guard against obvious data snooping biases, we simply restrict  $\alpha = 0.999$ .

TABLE 4A.—DAILY, WEEKLY, AND MONTHLY DM/\$ HAR-RV-CJ REGRESSIONS

$$RV_{t,t+h} = \beta_0 + \beta_{CD}C_t + \beta_{CW}C_{t-5,t} + \beta_{CM}C_{t-22,t} + \beta_{JD}J_t + \beta_{JW}J_{t-5,t} + \beta_{JM}J_{t-22,t} + \epsilon_{t,t+h}$$

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_{CD}C_t^{1/2} + \beta_{CW}(C_{t-5,t})^{1/2} + \beta_{CM}(C_{t-22,t})^{1/2} + \beta_{JD}J_t^{1/2} + \beta_{JW}(J_{t-5,t})^{1/2} + \beta_{JM}(J_{t-22,t})^{1/2} + \epsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \beta_0 + \beta_{CD}\log(C_t) + \beta_{CW}\log(C_{t-5,t}) + \beta_{CM}\log(C_{t-22,t}) + \beta_{JD}\log(J_t + 1) + \beta_{JW}\log(J_{t-5,t} + 1) + \beta_{JM}\log(J_{t-22,t} + 1) + \epsilon_{t,t+h}$$

<i>h</i>	<i>RV<sub>t,t+h</sub></i>			<i>(RV<sub>t,t+h</sub>)<sup>1/2</sup></i>			<i>log(RV<sub>t,t+h</sub>)</i>		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.083 (0.015)	0.131 (0.018)	0.231 (0.025)	0.096 (0.015)	0.158 (0.021)	0.292 (0.034)	-0.095 (0.024)	-0.114 (0.036)	-0.249 (0.057)
$\beta_{CD}$	0.407 (0.044)	0.210 (0.040)	0.101 (0.021)	0.397 (0.032)	0.222 (0.029)	0.127 (0.019)	0.369 (0.026)	0.205 (0.021)	0.130 (0.016)
$\beta_{CW}$	0.256 (0.077)	0.271 (0.054)	0.259 (0.046)	0.264 (0.048)	0.289 (0.051)	0.264 (0.042)	0.295 (0.039)	0.318 (0.048)	0.258 (0.040)
$\beta_{CM}$	0.226 (0.072)	0.308 (0.078)	0.217 (0.074)	0.212 (0.044)	0.281 (0.060)	0.205 (0.068)	0.217 (0.036)	0.270 (0.055)	0.213 (0.071)
$\beta_{JD}$	0.096 (0.089)	0.006 (0.040)	-0.002 (0.017)	0.022 (0.027)	0.001 (0.017)	0.003 (0.010)	0.043 (0.111)	0.024 (0.076)	-0.004 (0.044)
$\beta_{JW}$	-0.191 (0.168)	-0.179 (0.199)	-0.073 (0.125)	-0.006 (0.033)	0.001 (0.044)	0.002 (0.028)	-0.076 (0.239)	-0.317 (0.327)	-0.127 (0.242)
$\beta_{JM}$	-0.001 (0.329)	0.055 (0.460)	-0.014 (0.604)	-0.034 (0.057)	-0.011 (0.087)	0.014 (0.127)	-0.690 (0.408)	-0.301 (0.668)	-0.261 (0.990)
$R^2_{HAR-RV-CJ}$	0.368	0.427	0.361	0.443	0.486	0.397	0.485	0.514	0.415

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-CJ volatility forecast regressions. All of the realized volatility measures are constructed from five-minute returns spanning the period from December 1986 through June 1999, for a total of 3,045 daily observations. The weekly and monthly measures are the scaled sums of the corresponding daily measures. The significant daily jump and continuous sample path variability measures are based on equations (19) and (20), respectively, along with the staggered power variation measures in equations (22) and (23), using a critical value of  $\alpha = 0.999$ . The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively.

TABLE 4B.—DAILY, WEEKLY, AND MONTHLY S&P 500 HAR-RV-CJ REGRESSIONS

$$RV_{t,t+h} = \beta_0 + \beta_{CD}C_t + \beta_{CW}C_{t-5,t} + \beta_{CM}C_{t-22,t} + \beta_{JD}J_t + \beta_{JW}J_{t-5,t} + \beta_{JM}J_{t-22,t} + \epsilon_{t,t+h}$$

$$(RV_{t,t+h})^{1/2} = \beta_0 + \beta_{CD}C_t^{1/2} + \beta_{CW}(C_{t-5,t})^{1/2} + \beta_{CM}(C_{t-22,t})^{1/2} + \beta_{JD}J_t^{1/2} + \beta_{JW}(J_{t-5,t})^{1/2} + \beta_{JM}(J_{t-22,t})^{1/2} + \epsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \beta_0 + \beta_{CD}\log(C_t) + \beta_{CW}\log(C_{t-5,t}) + \beta_{CM}\log(C_{t-22,t}) + \beta_{JD}\log(J_t + 1) + \beta_{JW}\log(J_{t-5,t} + 1) + \beta_{JM}\log(J_{t-22,t} + 1) + \epsilon_{t,t+h}$$

<i>h</i>	<i>RV<sub>t,t+h</sub></i>			<i>(RV<sub>t,t+h</sub>)<sup>1/2</sup></i>			<i>log(RV<sub>t,t+h</sub>)</i>		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.143 (0.040)	0.222 (0.057)	0.393 (0.075)	0.062 (0.018)	0.103 (0.028)	0.202 (0.037)	-0.063 (0.013)	0.003 (0.019)	0.026 (0.036)
$\beta_{CD}$	0.356 (0.067)	0.224 (0.043)	0.135 (0.023)	0.381 (0.041)	0.262 (0.031)	0.183 (0.024)	0.320 (0.028)	0.224 (0.022)	0.162 (0.020)
$\beta_{CW}$	0.426 (0.120)	0.413 (0.114)	0.204 (0.070)	0.367 (0.063)	0.413 (0.072)	0.272 (0.061)	0.368 (0.043)	0.383 (0.053)	0.274 (0.049)
$\beta_{CM}$	0.111 (0.063)	0.168 (0.076)	0.319 (0.070)	0.163 (0.042)	0.206 (0.062)	0.322 (0.065)	0.246 (0.032)	0.297 (0.049)	0.403 (0.056)
$\beta_{JD}$	-0.153 (0.063)	-0.016 (0.049)	0.005 (0.022)	-0.043 (0.043)	-0.013 (0.027)	0.005 (0.017)	-0.006 (0.066)	-0.027 (0.049)	0.018 (0.031)
$\beta_{JW}$	0.465 (0.233)	0.362 (0.205)	0.456 (0.287)	0.082 (0.071)	0.096 (0.075)	0.132 (0.113)	0.062 (0.105)	0.163 (0.126)	0.198 (0.176)
$\beta_{JM}$	0.355 (0.304)	0.458 (0.448)	0.215 (0.202)	0.133 (0.054)	0.170 (0.084)	0.190 (0.105)	0.207 (0.085)	0.233 (0.136)	0.246 (0.201)
$R^2_{HAR-RV-CJ}$	0.421	0.574	0.478	0.613	0.700	0.639	0.696	0.763	0.722

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-CJ volatility forecast regressions. All of the realized volatility measures are constructed from five-minute returns spanning the period from January 1990 through December 2002, for a total of 3,213 daily observations. The weekly and monthly measures are the scaled sums of the corresponding daily measures. The significant daily jump and continuous sample path variability measures are based on equations (19) and (20), respectively, along with the staggered power variation measures in equations (22) and (23), using a critical value of  $\alpha = 0.999$ . The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively.

between the different pairs of realizations and forecasts is immediately evident across all of the markets and forecast horizons. Visual inspection of the graphs also shows that the volatility in the U.S. T-bond market is the least predictable, followed by the DM/\$, and then the S&P 500. Nonetheless, the forecasts of the T-bond volatilities still track the overall patterns fairly well, especially for the longer weekly and monthly horizons.

All told, these results underscore the potential benefit from a volatility forecasting perspective of separately mea-

suring the individual components of the realized volatility. It is possible that even further improvements may be obtained by a more structured approach in which the jump component,  $J_t$ , and the continuous sample path component,  $C_t$ , are each modeled separately. These individual models for  $J_t$  and  $C_t$  could then be used in the construction of separate out-of-sample forecasts for each of the components, as well as combined forecasts of the total realized variation process,  $RV_{t,t+h} = C_{t,t+h} + J_{t,t+h}$ . We leave further work along these lines for future research.



TABLE 4C.—DAILY, WEEKLY, AND MONTHLY U.S. T-BOND HAR-RV-CJ REGRESSIONS

$$RV_{i,t+h} = \beta_0 + \beta_{CD}C_t + \beta_{CW}C_{t-5,t} + \beta_{CM}C_{t-22,t} + \beta_{JD}J_t + \beta_{JW}J_{t-5,t} + \beta_{JM}J_{t-22,t} + \varepsilon_{i,t+h}$$

$$(RV_{i,t+h})^{1/2} = \beta_0 + \beta_{CD}C_t^{1/2} + \beta_{CW}(C_{t-5,t})^{1/2} + \beta_{CM}(C_{t-22,t})^{1/2} + \beta_{JD}J_t^{1/2} + \beta_{JW}(J_{t-5,t})^{1/2} + \beta_{JM}(J_{t-22,t})^{1/2} + \varepsilon_{i,t+h}$$

$$\log(RV_{i,t+h}) = \beta_0 + \beta_{CD} \log(C_t) + \beta_{CW} \log(C_{t-5,t}) + \beta_{CM} \log(C_{t-22,t}) + \beta_{JD} \log(J_t + 1) + \beta_{JW} \log(J_{t-5,t} + 1) + \beta_{JM} \log(J_{t-22,t} + 1) + \varepsilon_{i,t+h}$$

<i>h</i>	$RV_{i,t+h}$			$(RV_{i,t+h})^{1/2}$			$\log(RV_{i,t+h})$		
	1	5	22	1	5	22	1	5	22
$\beta_0$	0.085 (0.011)	0.095 (0.012)	0.133 (0.017)	0.133 (0.016)	0.166 (0.019)	0.236 (0.031)	-0.337 (0.040)	-0.335 (0.052)	-0.473 (0.079)
$\beta_{CD}$	0.107 (0.031)	0.064 (0.015)	0.031 (0.006)	0.087 (0.025)	0.069 (0.013)	0.034 (0.006)	0.091 (0.022)	0.068 (0.012)	0.036 (0.007)
$\beta_{CW}$	0.299 (0.051)	0.238 (0.047)	0.196 (0.037)	0.306 (0.045)	0.223 (0.042)	0.180 (0.033)	0.297 (0.043)	0.203 (0.042)	0.168 (0.030)
$\beta_{CM}$	0.366 (0.062)	0.426 (0.062)	0.369 (0.068)	0.367 (0.048)	0.428 (0.055)	0.380 (0.065)	0.389 (0.046)	0.439 (0.055)	0.382 (0.064)
$\beta_{JD}$	-0.136 (0.055)	-0.010 (0.021)	-0.019 (0.008)	-0.080 (0.026)	-0.006 (0.012)	-0.007 (0.006)	-0.769 (0.185)	-0.090 (0.082)	-0.091 (0.041)
$\beta_{JW}$	0.230 (0.122)	0.050 (0.081)	-0.075 (0.067)	0.090 (0.033)	0.043 (0.029)	-0.004 (0.025)	0.775 (0.390)	0.227 (0.298)	-0.289 (0.271)
$\beta_{JM}$	-0.271 (0.177)	-0.145 (0.216)	-0.116 (0.245)	-0.113 (0.045)	-0.076 (0.058)	-0.057 (0.075)	-1.319 (0.589)	-0.477 (0.773)	-0.034 (0.918)
$R_{HAR-RV-CJ}^2$	0.144	0.325	0.377	0.192	0.353	0.393	0.222	0.365	0.400

Key: The table reports the OLS estimates for daily (*h* = 1) and overlapping weekly (*h* = 5) and monthly (*h* = 22) HAR-RV-CJ volatility forecast regressions. All of the realized volatility measures are constructed from five-minute returns spanning the period from January 1990 through December 2002, for a total of 3,213 daily observations. The weekly and monthly measures are the scaled sums of the corresponding daily measures. The significant daily jump and continuous sample path variability measures are based on equations (19) and (20), respectively, along with the staggered power variation measures in equations (22) and (23), using a critical value of  $\alpha = 0.999$ . The standard errors reported in parentheses are based on a Newey-West/Bartlett correction allowing for serial correlation of up to order 5 (*h* = 1), 10 (*h* = 5), and 44 (*h* = 22), respectively.

### VII. Concluding Remarks

Building on recent theoretical results in Barndorff-Nielsen and Shephard (2004a, 2006) for bipower variation measures, we provide a simple and easy-to-implement practical framework for measuring “significant” jumps in financial asset prices. Applying the theory to more than a decade of high-frequency prices from the foreign exchange, equity, and fixed-income markets, we find that the procedure works well empirically. Our nonparametric measurements suggest that jumps dynamics are much less persistent (and predictable) than continuous sample path dynamics. In addition, the high-frequency data underlying our estimates allow us to identify many more jumps than do the parametric models based on daily or courser-frequency data hitherto reported in the literature. It also appears that many of the most significant jumps are readily associated with specific macroeconomic news announcements. Finally, when separately including the continuous sample path and jump variability measures in a simple linear volatility forecasting model, we find that only the continuous part has predictive power, in turn resulting in significant gains relative to the simple realized volatility forecasting models advocated in some of the recent literature.

The ideas and empirical results presented here are suggestive of several interesting extensions. First, it seems natural that jump risk may be priced differently from easier-to-hedge continuous price variability; see, for example, Santa-Clara and Yan (2004). Hence, separately modeling and forecasting the continuous sample path, or integrated volatility, and jump components of the quadratic variation process, as discussed above, is likely to

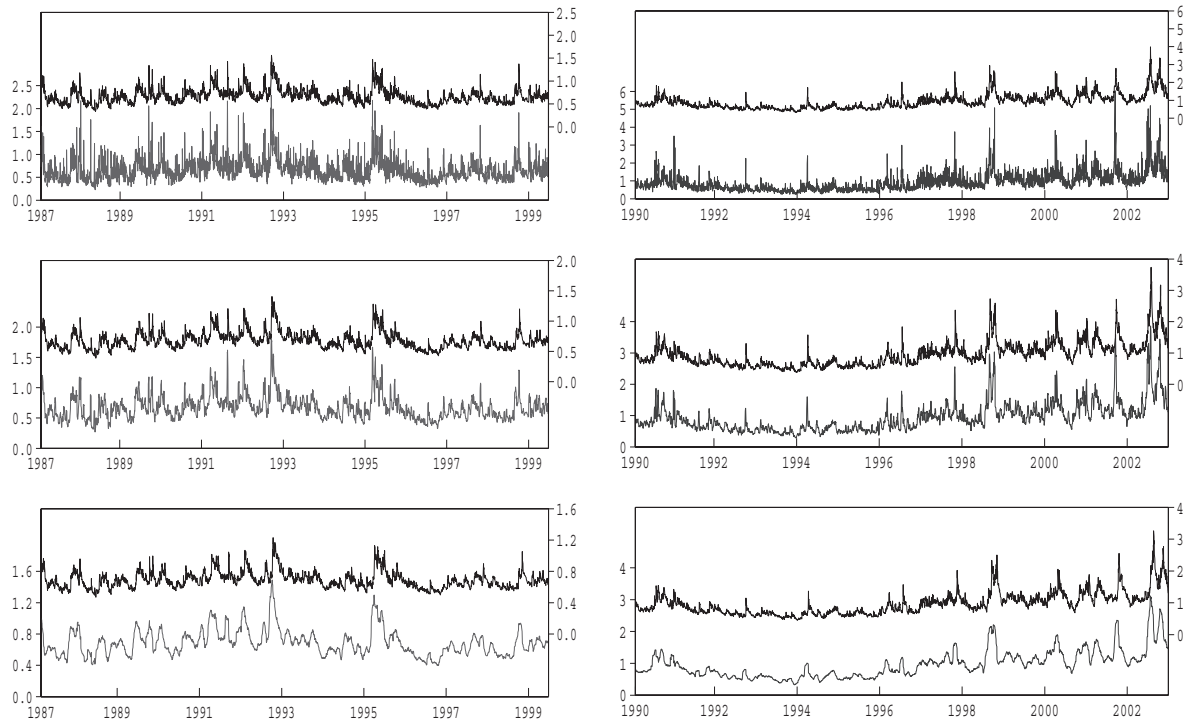
result in important improvements in derivatives and other pricing decisions.

Second, our choice of a five-minute sampling frequency and the new skip-one realized bipower and tripower variation measures to mitigate the market microstructure frictions in the high-frequency data were based on intuitive but informal considerations. It would be interesting to further investigate the “optimal” choice of sampling frequency, or the use of optimal subsampling schemes in the construction of the bi- and tripower variation measures. The related results for the realized variation measures in, for example, Bandi and Russell (2004, 2006), Hansen and Lunde (2006), and Zhang, Mykland, and Ait-Sahalia (2005), should be helpful.

Third, if interest centers exclusively on volatility forecasting, the use of more traditional robust power variation measures defined by the sum of absolute high-frequency returns raised to powers less than two might afford additional gains over and above the improvements provided by the bipower variation and significant jump measures used here; the recent empirical results in Forsberg and Ghysels (2004) are suggestive.

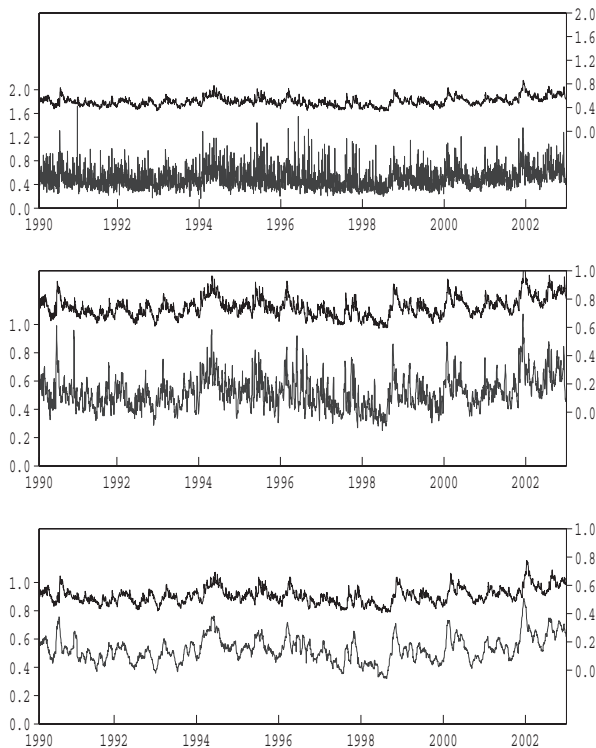
Fourth, casual empirical observation suggests that very large price moves, or jumps, often occur simultaneously across different markets. It would be interesting to extend the present analysis to a multivariate framework explicitly incorporating such commonalities through the use of quadratic covariation and appropriately defined copower variation measures; the abstract theoretical results in Barndorff-Nielsen, Graversen, Jacod, et al. (2006) are intriguing. In addition to allowing for more accurate statistical identification of the most important, or significant,

FIGURE 4.—(A) DAILY, WEEKLY, AND MONTHLY DM/\$ REALIZED VOLATILITIES AND HAR-RV-CJ FORECASTS (B) DAILY, WEEKLY, AND MONTHLY S&P 500 REALIZED VOLATILITIES AND HAR-RV-CJ FORECASTS



Key: The top, middle, and bottom panels show daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 22$ ) realized volatilities,  $RV_{i,t+h}^{1/2}$  (left scale), and the corresponding forecasts from the HAR-RV-CJ model in standard deviation form in equation (27) (right scale). See the text for details.

FIGURE 4C.—DAILY, WEEKLY, AND MONTHLY U.S. T-BOND REALIZED VOLATILITIES AND HAR-RV-CJ FORECASTS



Key: The top, middle, and bottom panels show daily ( $h = 1$ ), weekly ( $h = 5$ ), and monthly ( $h = 22$ ) realized volatilities,  $RV_{i,t+h}^{1/2}$  (left scale), and the corresponding forecasts from the HAR-RV-CJ model in standard deviation form in equation (27) (right scale). See the text for details.

jumps, this should also enhance our understanding of the underlying economic influences that “drive” financial markets and prices.

Last, and not at all least, additional exploration of variations on the shrinkage estimation undertaken here would be of great interest. We simply shrink raw jump estimates toward zero by setting all those less than some arbitrary threshold (motivated by classical asymptotic distribution theory) to zero. This “hard threshold” contrasts with the “soft thresholds” typical of Bayesian shrinkage, which would involve coaxing, but not forcing, of raw jump estimates toward zero. A thorough Bayesian analysis must, however, await future work.

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