



# A reduced form framework for modeling volatility of speculative prices based on realized variation measures<sup>☆</sup>

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## ABSTRACT

Building on realized variance and bipower variation measures constructed from high-frequency financial prices, we propose a simple reduced form framework for effectively incorporating intraday data into the modeling of daily return volatility. We decompose the total daily return variability into the continuous sample path variance, the variation arising from discontinuous jumps that occur during the trading day, as well as the overnight return variance. Our empirical results, based on long samples of high-frequency equity and bond futures returns, suggest that the dynamic dependencies in the daily continuous sample path variability are well described by an approximate long-memory HAR–GARCH model, while the overnight returns may be modeled by an augmented GARCH type structure. The dynamic dependencies in the non-parametrically identified significant jumps appear to be well described by the combination of an ACH model for the time-varying jump intensities coupled with a relatively simple log-linear structure for the jump sizes. Finally, we discuss how the resulting reduced form model structure for each of the three components may be used in the construction of out-of-sample forecasts for the total return volatility.

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## 1. Introduction

A burgeoning literature concerned with modeling and forecasting the dynamic dependencies in financial market volatility has emerged over the past two decades. Until fairly recently, most of the empirical results in the literature were based on the use of

daily, or coarser frequency data, coupled with formulations within the GARCH or stochastic volatility class of models; for a recent survey see Andersen et al. (2006). Meanwhile, somewhat of a paradigm shift has started to occur in which high-frequency data is now incorporated into longer-run volatility modeling and forecasting problems through the use of simple reduced-form time series models for non-parametric daily realized volatility measures based on the summation of intraday squared returns; see, e.g., Andersen et al. (2003) and the supportive theoretical results in Andersen et al. (2004).<sup>3</sup> Further, decomposing the total daily return variability into its continuous and discontinuous components based on the bipower variation measures developed by Barndorff-Nielsen and Shephard (2004a, 2006), the empirical results in Andersen et al. (2007) suggest that most of the predictable variation in the volatility stems from the strong own dynamic dependencies in the continuous price path variability, while the predictability of the (squared) jumps is typically minor. The present paper takes this analysis one step further by developing, estimating and implementing separate reduced-form time series forecasting models

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<sup>3</sup> Closely related empirical findings have been reported in Anderson and Vahid (2007), Areal and Taylor (2002), Corsi (2004), Deo et al. (2006), Koopman et al. (2005), Martens et al. (2004), Pong et al. (2004) and Thomakos and Wang (2003), among others.

for each of the different components that make up the total daily price variation.

Following the analysis in Andersen et al. (2007), we begin by decomposing the total return variability over the trading day into its continuous sample path variation and the variation due to jumps based on the bipower variation measure developed by Barndorff-Nielsen and Shephard (2004a, 2006). Our empirical results with a fifteen year sample of high-frequency intraday S&P 500 and T-Bond futures returns confirm earlier findings that the dynamic dependencies in the daily continuous sample path variability is well described by an approximate long-memory Heterogeneous AR (HAR) model, as originally proposed by Corsi (2004). Meanwhile, careful analysis of the non-parametrically identified jumps reveals some new and interesting dynamic dependencies vis-a-vis the results reported in the existing literature. In particular, while the time series of statistically significant squared jumps appear to be approximately serially uncorrelated, the times between jumps and the sizes of the jumps are both autocorrelated.<sup>4</sup> We successfully model these dependencies by the combination of an Autoregressive Conditional Hazard (ACH) model, as developed by Hamilton and Jordà (2002), for the time-varying jump intensities, coupled with a log-linear model with GARCH errors for the size of the jumps.<sup>5</sup>

The two separate model structures described above effectively account for the variability over the active part of the trading day when the market is formally open. However, the opening price typically differs from the closing price from the previous day, and the corresponding overnight return often accounts for a non-trivial fraction of the total daily return. The most common approach for dealing with this issue when modeling and forecasting realized volatilities is to scale the intraday measures and/or model forecasts by a constant to make them unconditionally unbiased for the total daily variation; see, e.g., Martens (2002), Fleming et al. (2003) and Koopman et al. (2005). An alternative approach based on minimizing the mean square error for the realized variance over the whole day has also been advocated by Hansen and Lunde (2005). Instead, we treat the overnight returns as a time series of regularly occurring jumps. We model these by a discrete-time GARCH model in which the conditional variance explicitly depends on the continuous sample path variation over the immediately preceding active part of the trading day.

We also show how the three separate models discussed above may be combined in the construction of recursive forecasts for the total daily and longer horizon return volatility.<sup>6</sup> Comparing both in- and out-of-sample daily, weekly and monthly forecasts to those from other discrete-time volatility models, including a standard GARCH(1, 1) model and the HAR-RV model, our results suggest that the more detailed modeling approach developed here can in fact result in important forecast improvements.

Our paper is most directly related to Bollerslev et al. (in this issue), who estimate a discrete-time model for the joint dynamics of daily S&P 500 returns, realized variance and bipower variation. However, in contrast to the present paper, the former paper makes no attempt at separately identifying or modeling the dynamics of the jump and the overnight return components. Closely related results have also been reported in independent

work by Lanne (2006). Our paper is also related to the concurrent work of Tauchen and Zhou (2006), who document time-varying jump intensities based on the same realized variation measures and test statistics used here. Discrete-time GARCH models incorporating Poisson jump processes with time-varying jump intensities based solely on daily data have also previously been estimated by Chan and Maheu (2002) and Maheu and McCurdy (2004), while earlier work by Neely (1999) highlights the potential benefits from removing jumps when forecasting volatility using GARCH type models.

At a somewhat broader level our results also speak to the vast finance literature based on continuous-time methods and corresponding parametric models. In particular, the compound Poisson model of Merton (1976) and the many subsequent studies that rely on time-invariant jump-diffusions, are all at odds with the empirical findings reported here. On the other hand, the more recent studies by Andersen et al. (2002), Chernov et al. (2003) and Eraker et al. (2003) that explicitly allow for time-varying jump intensities all report difficulties in precisely estimating the process from daily data. Meanwhile, consistent with the empirical results for the high-frequency realized variation measures reported here, Bates (2000), Pan (2002), Carr and Wun (2003) and Eraker (2004) all point to the existence of time-varying jump intensities when on incorporating additional information from options data.

The rest of the paper is organized as follows. Section 2 sets up the notation and reviews the jump detection statistic used in revealing the latent jump processes. Section 3 reports the initial empirical evidence for the distinct dynamic characteristics of the different components that make up the total daily return variation. Section 4 models the continuous sample path variance, while Sections 5 and 6 develop our models for the discrete jump contribution and the overnight return dynamics, respectively. Section 7 discusses the construction of forecasts and compares the results to those from other procedures. Section 8 concludes.

## 2. Jump detection test statistics

### 2.1. General setup and notation

We assume that the scalar logarithmic asset price within the active part of the trading day follows a standard jump-diffusion process

$$dp(\tau) = \mu(\tau)d\tau + \sigma(\tau-)dw(\tau) + \kappa(\tau)dq(\tau), \quad (1)$$

where  $\tau \in \mathbb{R}^+$ , and the time scale is normalized so that the unit interval corresponds to a trading day;  $\mu(\tau)$  denotes the drift term with a continuous and locally finite variation sample path;  $\sigma(\tau) > 0$  is the spot volatility process, assumed to be càdlàg;  $w(\tau)$  is a standard Brownian motion;  $\kappa(\tau)dq(\tau)$  refers to the pure jump part, where  $dq(\tau) = 1$  if there is a jump at time  $\tau$  and 0 otherwise, where the jumps occur with potentially time-varying jump intensity  $\lambda(\tau)$ , and size  $\kappa(\tau)$ . We denote the corresponding discrete-time within-day geometric returns by

$$r_{t,j} = p(t - 1 + j/M) - p(t - 1 + (j - 1)/M), \quad j = 1, 2, \dots, M, \quad (2)$$

where  $t \in \mathbb{N}^+$ , and  $M$  refers the number of (equally spaced) return observations over the trading day.

The continuous-time diffusion process above only applies for the active part of the trading day. However, the opening price on one day typically differs from the closing price recorded on the previous day. In fact, as discussed further below, it is natural to think of the overnight returns as random jumps occurring at the deterministic times  $t = 1, 2, \dots$ . As such, the total return for day  $t$  equals

$$r_t = r_{t,n} + \sum_{j=1}^M r_{t,j} = r_{t,n} + r_{t,d}, \quad (3)$$

<sup>4</sup> The occurrences of jumps in the T-Bond market also appear to be related to the releases of macroeconomic news announcements, as documented in, e.g., Andersen et al. (2007) and Johannes (2004).

<sup>5</sup> The idea of modeling the jump process in terms of the occurrence and the size of the jumps has a natural precedent in the bin model for tick-by-tick transaction prices proposed by Rydberg and Shephard (2003).

<sup>6</sup> The separate model estimates and accompanying forecasts reported here ignore any contemporaneous dependencies among the innovations to the continuous sample path variability and jump equations. Incorporating this into a fully efficient multivariate system estimation is severely complicated by the fact that the time series of significant jumps are effectively censored and the corresponding equations only estimated based on a subset of the sample.

where  $r_{t,n}$  denotes the overnight logarithmic price change from day  $t - 1$  to day  $t$ , and we follow the convention of measuring the daily returns as close-to-close.

## 2.2. Realized variation measures

The volatility over the active part of the trading day  $t$  is measured by the quadratic variation

$$QV_t = \int_{t-1}^t \sigma^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2. \quad (4)$$

The first integrated variance term represents the contribution from the continuous price path, while  $N_t$  gives the number of jumps over day  $t$ , and  $\sum_{j=1}^{N_t} \kappa_{t,j}^2$  accounts for the corresponding contribution to the variance from the within-day jumps.

The quadratic variation process and its separate components are, of course, not directly observable. Instead, we resort to recently popularized model-free non-parametric consistent measures, including the now familiar realized variance

$$RV_t(M) = \sum_{j=1}^M r_{t,j}^2. \quad (5)$$

As noted in Andersen and Bollerslev (1998), Comte and Renault (1998), Andersen et al. (2001b, 2003) and Barndorff-Nielsen and Shephard (2001, 2002), among others,  $RV_t(M)$  converges uniformly in probability to  $QV_t$  as the sampling frequency goes to infinity

$$RV_t(M) \xrightarrow[M \rightarrow \infty]{PM} \int_{t-1}^t \sigma^2(s)ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2, \quad (6)$$

or equivalently, the length of the return interval goes to zero.

Meanwhile, a host of practical market microstructure complications prevents us from sampling too frequently while maintaining the fundamental semimartingale assumption underlying Eq. (1). Ways in which to best deal with these complications and the practical choice of  $M$  have been the subject of intensive recent research efforts; see, e.g., Ait-Sahalia et al. (2005), Bandi and Russell (2008), Barndorff-Nielsen et al. (2008), Hansen and Lunde (2006) and Zhang (2006). In the analysis reported on below, we simply follow most of the existing empirical literature in the use of a fixed five-minute sampling frequency, corresponding to  $M$  equal to 80 and 79 for each of the two markets that we study.

In order to separately measure the jump part, we rely on the realized bipower variation measure developed by Barndorff-Nielsen and Shephard (2004a, 2006),

$$RBV_{1,t} = \mu_1^{-2} \left( \frac{M}{M-2} \right) \sum_{j=3}^M |r_{t,j-2}| |r_{t,j}|, \quad (7)$$

where  $\mu_a = E(|Z|^a)$  for  $Z \sim N(0, 1)$ . The bipower variation measure defined above involves an additional stagger relative to the measure originally considered in Barndorff-Nielsen and Shephard (2004a), which helps render it robust to certain types of market microstructure noise; see Huang and Tauchen (2005) for some initial analytical investigations and simulation-based evidence along these lines. Barndorff-Nielsen et al. (2006), Barndorff-Nielsen et al. (2006) and Jacod (2008) show that  $RBV_{1,t}(M)$  converges in probability to the integrated variance, under the general assumption that the logarithmic price process is a Brownian semimartingale with a finite-activity jump process, or an infinite-activity  $\alpha$ -stable jump process with the Blumenthal–Gettoor index  $\alpha < 2$ . Consequently, the difference between the realized variance and the realized bipower variation consistently estimates the part of the quadratic variation due to jumps

$$RV_t(M) - RBV_{1,t}(M) \xrightarrow[M \rightarrow \infty]{PM} \sum_{j=1}^{N_t} \kappa_{t,j}^2. \quad (8)$$

Moreover, under similar regularity conditions, except  $\alpha < 1$ , the test statistic

$$Z_t = \frac{RV_t - RBV_{1,t}}{RV_t} \sqrt{\left( \left( \frac{\pi}{2} \right)^2 + \pi - 5 \right) \frac{1}{M} \max \left( 1, \frac{RTQ_t}{RBV_{1,t}^2} \right)}, \quad (9)$$

where

$$RTQ_{1,t} = M \mu_{4/3}^{-3} \left( \frac{M}{M-4} \right) \sum_{j=5}^M |r_{t,j-4}|^{4/3} |r_{t,j-2}|^{4/3} |r_{t,j}|^{4/3}, \quad (10)$$

is asymptotically standard normally distributed under the null hypothesis of no within-day jumps.<sup>7</sup>

Based on the above jump detection test statistic, the realized measure of the jump contribution to the quadratic variation of the logarithmic price process is then measured by

$$J_t(M) = I(Z_t > \Phi_\alpha) \cdot (RV_t(M) - RBV_{1,t}(M)), \quad (11)$$

where  $I(\cdot)$  denotes the indicator function and  $\Phi_\alpha$  refers to the appropriate critical value from the standard normal distribution. Accordingly, our realized measure for the integrated variance is defined by

$$C_t(M) = I(Z_t \leq \Phi_\alpha) \cdot RV_t(M) + I(Z_t > \Phi_\alpha) \cdot RBV_{1,t}(M). \quad (12)$$

This definition automatically ensures that the non-parametric measures for the jump and continuous components add up to  $RV_t(M)$ . This same decomposition of the within day variance has also previously been explored by Andersen et al. (2007), among others. Of course, the actual implementation requires a choice of  $\alpha$ . In the results reported on below, we use a critical value of  $\alpha = 0.99$ , but very similar results (available upon request) were obtained for other values of  $\alpha$ .<sup>8</sup> For notational simplicity, we will refer to these empirical measures as  $RV_t$ ,  $C_t$  and  $J_t$  in the sequel.

## 3. Data and summary statistics

### 3.1. Data

Our data consist of five-minute prices for the S&P 500 futures (SP) and 30-year US treasury bond futures (US) contracts. The raw transaction prices for both contracts were obtained from Price-Data. The sample period for both assets begins on January 2, 1990, and ends on February 4, 2005. The intraday five-minute prices for the SP contracts span the time interval from 9:35 to 16:15 (EST), resulting in  $M = 80$  non-overlapping return observations per day. The five-minute prices for the US contracts cover the period from 8:25 to 15:00 (EST), for a total of  $M = 79$  intraday returns. Our use of a five-minute sampling frequency parallels many previous studies in the literature, and as discussed further in Andersen et al. (2007), for the two contracts analyzed here strikes a reasonable balance between the desire for as finely sampled observations as possible on the one hand, and robustness to contaminating market microstructure influences on the other.<sup>9</sup>

<sup>7</sup> Huang and Tauchen (2005) report extensive simulation evidence showing that this particular jump detection test statistic exhibits excellent size and power properties for a one-factor logarithmic stochastic volatility model augmented with compound Poisson jumps. Also, while the original proofs for asymptotic normality in the above cited papers relied on  $\alpha < 1$ , Zhang (2007) has recently extended the results to allow for  $1 < \alpha < 3/2$ .

<sup>8</sup> In the actual implementation we also imposed a hard lower bound of 0.001 on the daily  $C_t(M)$ .

<sup>9</sup> For further details concerning the previous-tick method used in the construction of the five-minute returns and the specific contract rollover scheme, see Wasserfallen and Zimmermann (1985), Dacorogna et al. (2001), Andersen et al. (2007) and Fleming et al. (2003), respectively. For SP around 98% of the prices occur within one minute of each five-minute mark, while for US the proportion is around 86%.

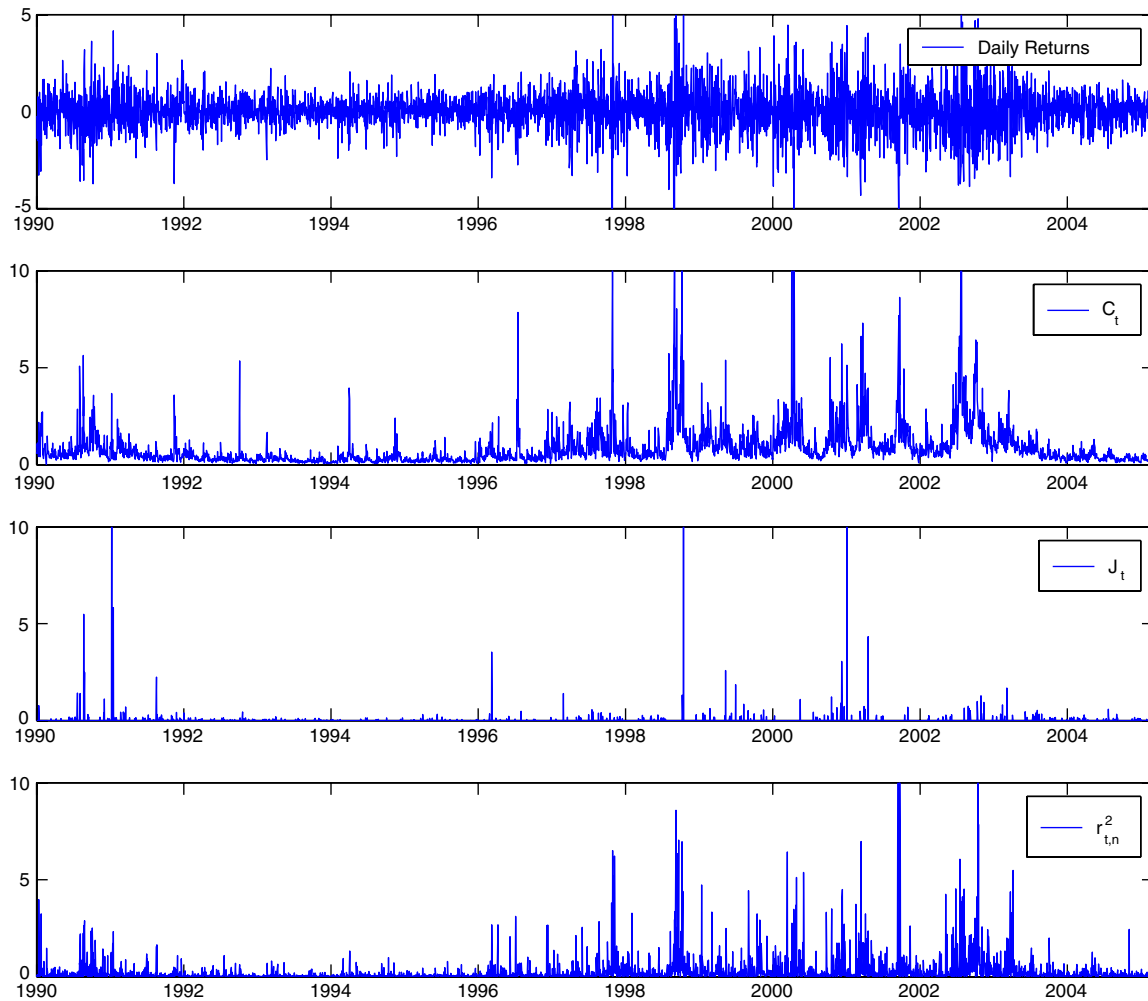


Fig. 1. Daily returns and variation components for SP.

**Table 1**  
Descriptive statistics for SP.

			$C_t$	$J_t$	$I_t$	$S_t$	$r^2_{t,n}$			
	Mean		0.856	0.042	0.086	0.491	0.261			
	Std. dev.		1.098	0.610	0.281	2.025	0.865			
	Skewness		5.274	34.77	2.947	10.37	16.44			
	Kurtosis		47.20	1390	9.683	123.8	483.3			
	Min		0.004	0.000	0.000	0.006	0.000			
	Max		14.33	27.59	1.000	27.59	31.38			
	Obs.		3801	3801	3801	328	3800			
Ljung–Box Q-statistics										
Lags	$C_t$		$J_t$		$I_t$		$S_t$		$r^2_{t,n}$	
5	6109	(0.000)	2.773	(0.735)	7.361	(0.195)	3.606	(0.607)	577.3	(0.000)
10	10039	(0.000)	24.69	(0.006)	15.50	(0.115)	6.190	(0.799)	727.3	(0.000)
15	12629	(0.000)	24.79	(0.053)	32.95	(0.005)	6.884	(0.961)	861.2	(0.000)
20	15050	(0.000)	26.75	(0.143)	37.47	(0.010)	7.077	(0.996)	962.9	(0.000)

### 3.2. Summary statistics

To get an idea about the properties of the different components that make up the total daily return variance for each of the two markets, we plot in Figs. 1 and 2 the daily return  $r_t$ , our measure for the continuous sample path variation  $C_t$ , the sum of the within day squared jumps  $J_t$ , and the overnight squared returns  $r^2_{t,n}$ . The figures clearly indicate rather distinct dynamic dependencies in each of the different components, with the jump time series appearing noticeably more erratic and less predictable than the other series.

To better understand these dependencies, we further decompose the  $J_t$  series into two separate processes: one describing the occurrence of jumps, and the other the size of the squared jump(s) within the day when at least one jump occurs. We denote these two processes by  $I_t$  and  $S_t$ , respectively. More precisely,  $Pr(J_t = 0 | \mathcal{F}_{t-1}) = Pr(I_t = 0 | \mathcal{F}_{t-1})$ , while  $Pr(0 < J_t \leq j | \mathcal{F}_{t-1}) = Pr(I_t = 1 | \mathcal{F}_{t-1}) \cdot Pr(S_t \leq j | \mathcal{F}_{t-1}, I_t = 1)$ . The resulting summary statistics reported in Tables 1 and 2 do indeed reveal some significant dynamic dependencies in the  $I_t$  and  $S_t$  series that are largely masked in the corresponding  $J_t$  series. Of course, the Ljung–Box Q-statistics reported in the tables only capture own linear dependencies.

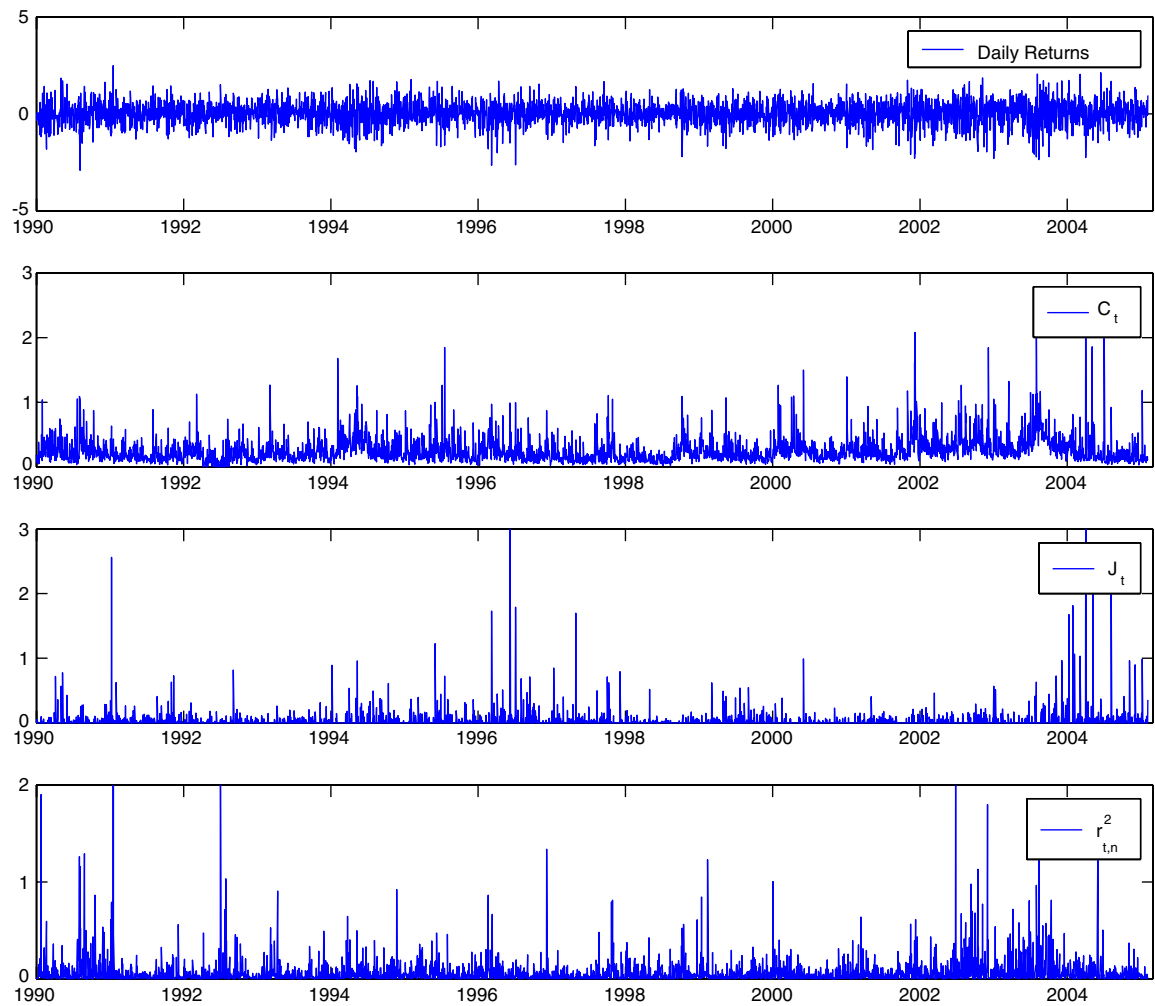


Fig. 2. Daily returns and variation components for US.

Table 2  
Descriptive statistics for US.

		$C_t$	$J_t$	$I_t$	$S_t$	$r_{t,n}^2$				
	Mean	0.253	0.037	0.255	0.143	0.066				
	Std. dev.	0.205	0.158	0.436	0.286	0.160				
	Skewness	3.312	13.72	1.123	7.798	13.46				
	Kurtosis	23.99	281.5	2.261	88.71	340.9				
	Min	0.001	0.000	0.000	0.009	0.000				
	Max	2.742	4.519	1.000	4.519	5.271				
	Obs.	3781	3781	3781	965	3780				
Ljung–Box Q-statistics										
Lags	$C_t$		$J_t$		$S_t$		$r_{t,n}^2$			
5	1492	(0.000)	5.511	(0.357)	38.11	(0.000)	4.648	(0.460)	72.6	(0.000)
10	2444	(0.000)	7.946	(0.634)	58.27	(0.000)	38.15	(0.000)	128.8	(0.000)
15	3192	(0.000)	10.42	(0.793)	93.10	(0.000)	44.12	(0.000)	152.2	(0.000)
20	3939	(0.000)	158.2	(0.000)	142.5	(0.000)	63.82	(0.000)	182.8	(0.000)

As discussed further in Section 5, there are also strong non-linear dynamic dependencies embedded in the series for both markets. The reduced form models for each of the different components discussed next are explicitly designed to account for these features.

4. Continuous sample path variation

We start by detailing our model for the strongly serially correlated continuous sample path variation process,  $C_t$ . The HAR–RV

model first proposed by Corsi (2004), and further developed by Andersen et al. (2007), provides a particular convenient framework for modeling these dependencies.<sup>10</sup> The specific HAR–C model adopted here takes the form,

<sup>10</sup> Following Müller et al. (1997), the HAR type specification is sometimes given a structural interpretation as arising from the interaction of agents with different investment horizons. We merely view the HAR–C model as providing a convenient, or “poor-man’s”, approximation to long-memory.



**Table 3**  
HAR–C model estimates.

	Homoskedastic		GARCH(2, 1)	
	SP	US	SP	US
$\beta_0$	−0.085 (0.011) [0.000]	−0.166 (0.052) [0.001]	−0.097 (0.010) [0.000]	−0.249 (0.038) [0.000]
$\beta_{CD}$	0.349 (0.024) [0.000]	0.144 (0.043) [0.001]	0.331 (0.020) [0.000]	0.109 (0.020) [0.000]
$\beta_{CW}$	0.338 (0.034) [0.000]	0.315 (0.062) [0.000]	0.369 (0.030) [0.000]	0.327 (0.036) [0.000]
$\beta_{CM}$	0.262 (0.029) [0.000]	0.501 (0.061) [0.000]	0.252 (0.025) [0.000]	0.444 (0.039) [0.000]
$\beta_{JD}$	−0.109 (0.071) [0.126]	−0.342 (0.135) [0.012]	−0.087 (0.078) [0.267]	−0.278 (0.116) [0.017]
$\beta_{JW}$	−0.118 (0.077) [0.124]	0.099 (0.224) [0.657]	−0.081 (0.100) [0.418]	0.281 (0.221) [0.205]
$\beta_{JM}$	−0.122 (0.088) [0.165]	−0.820 (0.402) [0.041]	−0.175 (0.099) [0.079]	−1.460 (0.329) [0.000]
$\omega$	–	–	0.049 (0.015) [0.001]	0.015 (0.003) [0.000]
$\alpha_1$	–	–	0.111 (0.027) [0.000]	0.129 (0.026) [0.000]
$\alpha_2$	–	–	−0.024 (0.032) [0.457]	−0.090 (0.026) [0.000]
$\beta_1$	–	–	0.720 (0.076) [0.000]	0.921 (0.015) [0.000]
$\nu$	–	–	7.696 (0.740) [0.000]	7.370 (0.822) [0.000]
Log L	−2805.434	−3890.313	−2671.580	−3341.914
Obs.	3779	3759	3779	3759

$$\begin{aligned} \log(C_{t+1}) = & \beta_0 + \beta_{CD} \log(C_t) + \beta_{CW} \log(C_{t-5,t}) \\ & + \beta_{CM} \log(C_{t-22,t}) + \beta_{JD} \log(J_t + 1) + \beta_{JW} \log(J_{t-5,t} + 1) \\ & + \beta_{JM} \log(J_{t-22,t} + 1) + \epsilon_{t+1,C}, \end{aligned} \quad (13)$$

where  $C_{t-h,t} \equiv h^{-1}[C_{t-h+1} + C_{t-h+2} + \dots + C_t]$  and  $J_{t-h,t} \equiv h^{-1}[J_{t-h+1} + J_{t-h+2} + \dots + J_t]$ . The logarithmic transform obviously prevents the implied continuous variation defined by exponentiating  $C_{t+1}$  from becoming negative.<sup>11</sup>

The left columns in Table 3 report the resulting OLS parameter estimates, together with robust standard errors in parentheses and *p*-values in square brackets. The estimation results confirm the strong own dynamic dependencies in  $C_t$ . Consistent with the results reported in Andersen et al. (2007) the coefficient estimates associated with the lagged squared jumps for SP are generally insignificant, albeit all negative, while there is some evidence of significant anti-persistent effects of the squared jumps for US. Meanwhile, the Ljung–Box *Q*-statistics for the squared and absolute residuals (available upon request) reveal clear evidence for significant conditional heteroskedasticity.

Hence, we augment the basic HAR–C model above with a GARCH error structure for the time-varying volatility-of-volatility.<sup>12</sup> Further, to allow for the possibility of fat-tails, we estimate the model under the assumption of conditionally *t*-distribution errors as in Bollerslev (1987). After some experimentation, we found that a GARCH(2, 1) model provided a good fit for both markets,

$$\begin{aligned} \epsilon_{t+1,C} = & \sigma_{t+1,C} \cdot \sqrt{\frac{\nu-2}{\nu}} \cdot z_{t+1,C}, \quad z_{t+1,C} \sim t(\nu) \\ \sigma_{t+1,C}^2 = & \omega_C + \alpha_{1,C} \epsilon_{t,C}^2 + \alpha_{2,C} \epsilon_{t-1,C}^2 + \beta_{1,C} \sigma_{t,C}^2. \end{aligned} \quad (14)$$

The estimates from this augmented model are reported in the right columns of Table 3. The conditional mean parameters are generally close to the previously reported OLS estimates.<sup>13</sup> The coefficient estimates associated with the past squared jumps again suggest

that, everything else equal, large jumps tend to lower the future continuous sample path volatility, and particularly so for US.<sup>14</sup> The residual diagnostics (available upon request) also confirm that the estimated GARCH(2, 1) models adequately account for the conditional heteroskedasticity.

## 5. Jump variation

Our model for the trading-time jump variation consists of two parts: a model for the occurrence of jumps coupled with a model for the squared jump sizes.<sup>15</sup> We begin by a discussion of our model for jump occurrences.

### 5.1. The ACH model

Let  $\{t_0, t_1, \dots, t_n, \dots\}$  denote the random arrival times, or days, associated with significant jumps. The Autoregressive Conditional Duration (ACD) model proposed by Engle and Russell (1998), is ideally suited for modeling dynamic dependencies in the jump-durations  $d_i = t_i - t_{i-1}$ , or the number of days between two adjacent significant jumps. However, the ACD model only updates the conditional expected durations on event, or jump, days. From a forecasting perspective, it is desirable to continuously incorporate new information as it becomes available. The autoregressive conditional hazard (ACH) model of Hamilton and Jordà (2002) was explicitly designed with this objective in mind.<sup>16</sup>

In order to more formally define the ACH model, let  $N(t)$  denote the counting process representing the number of jump days that have occurred up until time  $t$ . Also, define the hazard rate

$$h_t = \Pr[N(t) \neq N(t-1) | \mathcal{F}_{t-1}]. \quad (15)$$

The relationship between the hazard rate and the expected duration, say  $\psi_N(t-1)$ , if no new information occurs between jump days, is then given by,

$$\psi_{N(t-1)} = \sum_{j=1}^{\infty} j(1-h_t)^{j-1} h_t = \frac{1}{h_t}. \quad (16)$$

<sup>11</sup> Moreover, the unconditional distributions of realized logarithmic volatilities often appear approximately normal; see, e.g., Andersen et al. (2001b,a) and Barndorff-Nielsen and Shephard (2004b), among others.

<sup>12</sup> The presence of time-varying volatility-of-volatility is consistent with most of the continuous-time stochastic volatility models used in the asset pricing finance literature. For instance, in the square-root affine, or Heston, diffusion model, the conditional variance of the future instantaneous variance is an affine function of the current instantaneous variance and the current instantaneous variance squared; see, e.g., Bollerslev and Zhou (2002).

<sup>13</sup> Since the model is formulated in terms of  $\log(C_{t+1})$ , the form of the conditional heteroskedasticity plays a direct role in determining the expected value of  $C_{t+1}$ . Specifically, under the simplifying assumption of conditional normality, or  $\nu = \infty$ ,  $E[C_{t+1} | \mathcal{F}_t] = \exp\{E[\log(C_{t+1}) | \mathcal{F}_t] + 1/2 \text{Var}[\log(C_{t+1}) | \mathcal{F}_t]\}$ . We will return to a discussion of the numerical procedure that we actually use in the calculation of the expectations from the more general model in the forecasting section below.

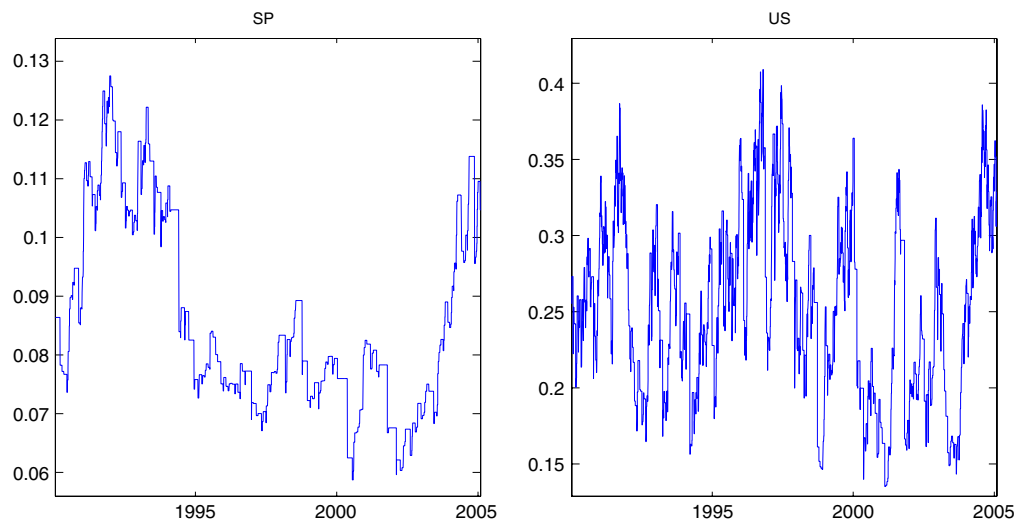
<sup>14</sup> Although some of the estimated coefficients are not significantly different from zero at the usual five percent level, we purposely maintain the same HAR–C GARCH(2, 1) specification for both markets. Also note, that even though the ARCH(2) coefficients are estimated to be negative for both markets, the implied coefficients in the infinite ARCH representations are all positive, so that the models are indeed well-defined.

<sup>15</sup> The model-free approach used here only identifies days with at least one significant jump and in turn the sum of the within day squared jump(s). Further refinements along the lines of Andersen et al. (2006) for actually estimating each of the individual significant jumps could be used in the formulation of even richer reduced form models.

<sup>16</sup> Bowsher (2007) has recently developed a more general econometric modeling framework for continuous-time conditional intensity-based multivariate processes.

**Table 4**  
ACH model estimates.

	ACH(1, 1)		Augmented ACH	
	SP	US	SP	US
$\omega$	0.227 (0.241) [0.346]	0.212 (0.084) [0.012]	–	–
$\alpha_1$	0.038 (0.021) [0.061]	0.088 (0.020) [0.000]	0.056 (0.029) [0.053]	0.088 (0.019) [0.000]
$\beta_1$	0.942 (0.036) [0.000]	0.858 (0.035) [0.000]	0.900 (0.059) [0.000]	0.859 (0.033) [0.000]
$\delta_0$	–	–	0.654 (2.771) [0.813]	–0.282 (0.423) [0.505]
$\delta_M$	–	–	0.320 (1.695) [0.850]	0.412 (0.298) [0.166]
$\delta_T$	–	–	1.117 (1.731) [0.519]	0.387 (0.271) [0.154]
$\delta_W$	–	–	2.979 (1.962) [0.129]	0.681 (0.299) [0.023]
$\delta_{Th}$	–	–	1.144 (1.666) [0.492]	0.325 (0.266) [0.222]
$\delta_{ES}$	–	–	0.629 (0.575) [0.274]	0.199 (0.094) [0.034]
$\delta_{CPI}$	–	–	0.757 (0.629) [0.229]	0.396 (0.089) [0.000]
Log $L$	–1107.797	–2111.457	–1106.136	–2098.906
Obs.	3792	3778	3792	3778



**Fig. 3.** Conditional hazard rates from ACH(1, 1) model.

The ACH model directly parameterizes the hazard rate,  $h_t$ , allowing it to depend on any relevant time  $t - 1$  information.

To illustrate, consider the simple ACH(1, 1) model without any information updating between jump days,

$$h_t = \frac{1}{\psi_{N(t)-1}}, \quad (17)$$

$$\psi_{N(t)} = \omega + \alpha_1 d_{N(t)-1} + \beta_1 \psi_{N(t)-1}.$$

Under appropriate distributional assumptions this ACH(1, 1) model is asymptotically equivalent to the ACD(1, 1) model, which parameterizes the conditional durations as  $\psi_i = \omega + \alpha_1 d_{i-1} + \beta_1 \psi_{i-1}$ ; see Hamilton and Jordà (2002) for further details. The parameter estimates from this ACH(1, 1) model are reported in the first part of Table 4. The estimates confirm the existence of strong persistence in the hazard rates, or equivalently the durations, in both markets. The Ljung–Box  $Q$ -statistics for any remaining own dynamic dependencies in the standardized durations implied by the model (available upon request),  $d_i/\hat{\psi}_i$ , are generally also insignificant.

The second set of estimates reported in Table 4 augments the basic ACH(1, 1) model by four weekday dummies for Monday, Tuesday, Wednesday and Thursday. We explicitly exclude the Friday dummy to avoid singularity, so that the estimated coefficients represent the effects relative to Friday. In addition, we include the logarithm of the number of days to the next nearest news announcements of the Employment Report (representing the real side of the economy) and the Consumer Price Index

(representing the nominal side of the economy).<sup>17</sup> Specifically,

$$h_t = \frac{1}{\alpha_1 d_{N(t)-1} + \beta_1 \psi_{N(t)-1} + \delta' z_{t-1}}, \quad (18)$$

$$\delta' z_{t-1} = \delta_0 + \delta_M D_M + \delta_T D_T + \delta_W D_W + \delta_{Th} D_{Th} + \delta_{ES} \log(n_{ES} + 1) + \delta_{CPI} \log(n_{CPI} + 1).$$

Consistent with the extant news announcement literature, the results suggest a statistically significant decreasing hazard for the occurrence of jumps in the US market as a function of the number of days until the release of one of the two announcements. Also, the corresponding coefficients for SP are both positive, albeit insignificant. The Monday through Thursday weekday dummies are all positive, but they do not indicate any statistically significant day-of-the-week effects in the jump occurrences. Nonetheless, in order to highlight the added flexibility afforded by the augmented ACH model, we maintain this as our preferred specification for both markets.<sup>18</sup>

To better illustrate the workings of the two different ACH specifications, Figs. 3 and 4 plot the resulting implied conditional hazard rates,  $\hat{h}_t$ . Comparing the two figures, the impact of the day-to-day updating for the latter set of plots is immediately evident.

<sup>17</sup> The results in Andersen et al. (2007) suggest that these are the two most important macroeconomic news announcements.

<sup>18</sup> We also experimented with augmenting the ACH model by  $C_t$  and  $J_t$ , but the estimated hazard rates did not appear plausible, so we decided not to include any of these variables in our final model specification.

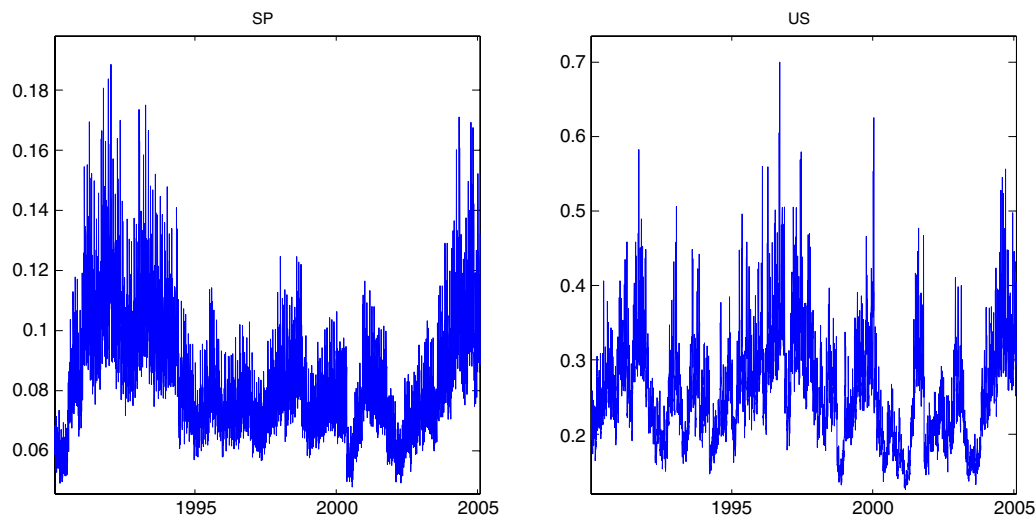


Fig. 4. Conditional hazard rates from augmented ACH model.

**Table 5**  
HAR-J model estimates.

	Homoskedastic		GARCH(1, 1)	
	SP	US	SP	US
$\beta_0$	−1.093 (0.072) [0.000]	−1.479 (0.113) [0.000]	−1.299 (0.122) [0.000]	−1.535 (0.114) [0.000]
$\beta_{CD}$	0.281 (0.114) [0.014]	−0.057 (0.053) [0.280]	0.172 (0.122) [0.158]	−0.085 (0.051) [0.095]
$\beta_{CW}$	0.460 (0.185) [0.013]	0.269 (0.112) [0.017]	0.440 (0.172) [0.011]	0.338 (0.099) [0.001]
$\beta_{CM}$	0.345 (0.150) [0.022]	0.489 (0.111) [0.000]	0.343 (0.147) [0.020]	0.474 (0.103) [0.000]
$\omega$	–	–	0.068 (0.110) [0.540]	0.013 (0.007) [0.046]
$\alpha_1$	–	–	0.039 (0.042) [0.355]	0.035 (0.010) [0.000]
$\beta_1$	–	–	0.866 (0.009) [0.000]	0.950 (0.010) [0.000]
$\nu$	–	–	3.265 (0.659) [0.000]	6.430 (1.608) [0.000]
Log L	−387.854	−1281.353	−353.162	−1250.054
Obs.	327	960	327	960

Still, the two sets of figures reveal the same general patterns in the estimated hazard rates, with jumps appearing more than twice as likely for US compared to SP over most of the sample. Again, we believe that this is partly due to the much bigger impact of macroeconomic news announcements for the fixed-income market. There is also a pronounced tendency for even fewer jumps in the equity market during the middle part of the sample, almost akin to a level shift in the estimated hazard rates. It is not clear what drives this change.

## 5.2. The HAR-J model

Most continuous-time parametric jump–diffusion models assume that the size of the jumps are *i.i.d.* distributed through time. By directly observing the squared jumps, or more precisely the realized measure of the sum of within-the-day squared jumps, the present framework affords us much greater flexibility in terms of modeling the jump sizes.

Following the same basic idea underlying the HAR-C model, we parameterize the conditional jump sizes as a function of the past continuous sample path variations.<sup>19</sup> In particular,

$$\log(S_{t(i)}) = \beta_0 + \beta_{CD} \log(C_{t(i)-1}) + \beta_{CW} \log(C_{t(i)-5,t(i)}) + \beta_{CM} \log(C_{t(i)-22,t(i)}) + \epsilon_{t(i)}, \quad (19)$$

where  $t(i)$  maps the jump counter  $i$  into the corresponding trading day  $t$ , so that the lagged variation measures on the right-hand-side are always measured in calendar time relative to the

time of the jump. The estimation results from this model are reported in the left columns in Table 5. As seen from the table, the one month lagged continuous volatility generally have the most explanatory power. Also, the size of the jumps for US are much more persistent than for SP. Meanwhile, the Ljung–Box Q-statistics for the squared and absolute residuals (available upon request) again clearly indicate the existence of conditional heteroskedasticity in the residuals from the model for both markets.

We therefore augment the basic HAR-J model above with a GARCH(1, 1)- $t$  error structure,

$$\epsilon_{t(i)} = \sigma_{t(i)} \cdot \sqrt{\frac{\nu-2}{\nu}} \cdot z_{t(i)}, \quad z_{t(i)} \sim t(\nu), \quad (20)$$

$$\sigma_{t(i)}^2 = \omega + \alpha_1 \epsilon_{t(i-1)}^2 + \beta_1 \sigma_{t(i-1)}^2.$$

The estimates from this preferred model are reported in the right columns in Table 5. The results confirm the existence of significant GARCH effects. Otherwise, the estimated dependencies in the conditional mean are directly in line with those for the homoskedastic model.

## 6. Overnight return variance

The realized variation measures and corresponding reduced form models developed above pertains to the return variation observed during the regular trading hours when the exchanges are open. However, as previously noted, the opening price on one day typically differs from the closing price on the previous day. Since most investors hold their portfolios over longer inter-day horizons, the corresponding overnight return variability

<sup>19</sup> We also tried including various lags of  $\log(S_t)$ , as well as the raw and expected durations. All of these other variables turned out to be insignificant.



**Table 6**  
Overnight GARCH model estimates.

	Unrestricted		Restricted	
	SP	US	SP	US
$\mu$	0.015 (0.005) [0.004]	0.006 (0.004) [0.121]	0.015 (0.005) [0.003]	0.006 (0.004) [0.105]
$\omega_n$	−0.001 (0.001) [0.233]	0.002 (0.001) [0.014]	−0.001 (0.001) [0.199]	0.002 (0.001) [0.020]
$\alpha_{1,n}$	0.041 (0.012) [0.001]	0.045 (0.011) [0.000]	0.044 (0.012) [0.000]	0.046 (0.010) [0.000]
$\beta_{1,n}$	0.817 (0.024) [0.000]	0.852 (0.027) [0.000]	0.806 (0.025) [0.000]	0.854 (0.027) [0.000]
$\beta_{CP}$	0.040 (0.009) [0.000]	0.014 (0.006) [0.013]	–	–
$\beta_{CN}$	0.045 (0.007) [0.000]	0.023 (0.006) [0.000]	–	–
$\beta_{JP}$	−0.031 (0.012) [0.010]	−0.000 (0.011) [0.964]	–	–
$\beta_{JN}$	0.023 (0.022) [0.305]	0.005 (0.009) [0.574]	–	–
$\beta_C$	–	–	0.045 (0.007) [0.000]	0.019 (0.005) [0.000]
$\nu$	5.004 (0.434) [0.000]	7.906 (0.747) [0.000]	4.944 (0.422) [0.000]	7.847 (0.7303) [0.000]
Log L	−1863.415	−8.696	−1865.451	−10.300
Obs.	3799	3779	3799	3779

will directly affect the risks of their positions. In particular, the proportion of the total daily variation due to the over-night returns, as measured by the sample means of  $r_{t,n}^2/(RV_t + r_{t,n}^2)$ , equals 0.160 and 0.165 for the SP and US markets, respectively.

Two common ways of dealing with this non-trivial overnight variation have emerged in the realized volatility literature. The first approach simply scales up the daytime realized variation measures to provide an unbiased estimate for the variation over the whole-day. This is the method used in, e.g., Martens (2002), Fleming et al. (2003) and Koopman et al. (2005). Alternatively, the overnight squared returns may be added to the within-day realized variation so that it covers the whole day. This approach, along with its pros and cons, is discussed further in Hansen and Lunde (2005), who also propose an improved estimator by optimally weighting, in a minimum mean-square-error sense, the daytime realized volatility and the squared overnight return. Both of these approaches implicitly assumes that the overnight squared returns may somehow be viewed as part of the same process that generates the within day realized volatility. Here we take a different approach and directly model the overnight returns, or jumps, by a separate discrete time model.<sup>20</sup>

### 6.1. The GARCH- $t$ model

The summary statistics previously reported and discussed in Section 3.2, not surprisingly, indicate the presence of serial correlation in the squared overnight returns. This naturally suggests a GARCH type approach for capturing these dependencies. Since the overnight returns are separated by the returns during the regular trading hours, we include the immediately preceding daytime realized volatility as an additional explanatory variable in the conditional variance equation. Moreover, since the continuous and discrete sample-path variation over the day may affect the subsequent overnight return differently, we split up the realized volatility into  $C_t$  and  $J_t$ . Furthermore, to allow for the possibility that positive and negative daytime shocks may have different effects, we condition the estimated coefficients for  $C_t$  and  $J_t$  on the sign of the daytime return,  $r_{t,d}$ . The resulting specification for the overnight returns takes the form,

$$\begin{aligned}
 r_{t+1,n} &= \mu + \epsilon_{t+1,n} \\
 \epsilon_{t+1,n} &= \sigma_{t+1,n} \cdot \sqrt{\frac{v-2}{v}} \cdot z_{t+1,n}, \quad z_{t+1,n} \sim t(v) \\
 \sigma_{t+1,n}^2 &= \omega_n + \alpha_{1,n} \epsilon_{t,n}^2 + \beta_{1,n} \sigma_{t,n}^2 + \beta_{CP} C_t^P \\
 &\quad + \beta_{CN} C_t^N + \beta_{JP} J_t^P + \beta_{JN} J_t^N,
 \end{aligned} \tag{21}$$

<sup>20</sup> The studies by Chan et al. (1991) and Martens (2002), which estimate individual discrete-time models for the trading-time and overnight returns, provide an earlier precedent.

where  $C_t^P = C_t \cdot I(r_{t,d} > 0)$ ,  $C_t^N = C_t \cdot I(r_{t,d} < 0)$ , and similarly for  $J_t^P$  and  $J_t^N$ .

The left columns in Table 6 report the estimation results. As expected, the estimates for  $\alpha_1$  and  $\beta_1$  are both highly statistically significant and broadly in line with the typical daily GARCH(1, 1) model estimates, although their sums are slightly less than what is generally found with daily returns. Of course, some of this “lack” in persistence is made up by significant positive dependence on the within day realized continuous sample path variation,  $C_t^P$  and  $C_t^N$ . Interestingly, the jump components,  $J_t^P$  and  $J_t^N$ , are generally not significant. Furthermore, the Wald test for the hypothesis of no volatility asymmetry, or  $\beta_{CP} = \beta_{CN}$ , equal 0.267 and 1.647 for each of the two markets respectively, with corresponding asymptotic  $p$ -values of 0.606 and 0.199.<sup>21</sup>

The right columns in Table 6 report the estimation results from the more parsimonious GARCH(1, 1)- $t$  model obtained by eliminating the jump components and combining the positive and negative continuous variation components,

$$\sigma_{t+1,n}^2 = \omega_n + \alpha_{1,n} \epsilon_{t,n}^2 + \beta_{1,n} \sigma_{t,n}^2 + \beta_C C_t. \tag{22}$$

The estimated parameters are directly in line with those from the earlier more general specification, and the corresponding values for the maximized log likelihood functions are also close to those for the unrestricted models. We consequently maintain this simpler model as our preferred specification for the overnight return variation.

## 7. Forecasting

One of the many potential useful applications of the reduced form modeling framework developed above relates to volatility forecasting. In particular, consider the question of calculating one-day-ahead return volatility forecasts, or  $\text{Var}(r_{t+1} | \mathcal{F}_t)$ . The standard GARCH based approach directly parameterizes this conditional expectation as a function of its own past value(s) and the lagged squared return(s). This, of course, does not include any high-frequency information. On the other hand, the now popular HAR-RV model parameterizes the conditional variance as a distributed lag of the past realized variation measures. While this does incorporate high-frequency information into the resulting forecasts, the traditional HAR-RV model does not distinguish between the continuous sample path variation and the discontinuous jump part. However, as discussed at length above, the dynamic dependencies in these two components are very different. Moreover, the standard approaches of scaling the

<sup>21</sup> On estimating the same GARCH models under the assumption of conditionally normal errors, the asymmetry appears significant for SP, indirectly suggesting that the effect is associated with the tails of the distribution.

**Table 7**  
In-sample forecast statistics.

	Horizon	RMSE			MAE		
		1	5	22	1	5	22
SP	GARCH	1.519	0.995	0.869	0.626	0.506	0.504
	HAR–RV	1.477	0.943	0.827	0.556	0.453	0.466
	HAR–CJN	1.412	0.855	0.748	0.542	0.417	0.420
US	GARCH	0.325	0.170	0.119	0.176	0.112	0.087
	HAR–RV	0.325	0.169	0.118	0.175	0.112	0.086
	HAR–CJN	0.322	0.167	0.120	0.176	0.114	0.086

realized volatilities or treating the overnight return as another intraday return in order to get an unbiased measure for the full day variance both ignore the distinct dynamic dependencies in the overnight returns.

In contrast, the framework proposed here explicitly decomposes the conditional variance into three separate components,<sup>22</sup>

$$\text{Var}(r_{t+1}|\mathcal{F}_t) = E(C_{t+1}|\mathcal{F}_t) + E(J_{t+1}|\mathcal{F}_t) + \text{Var}(r_{t+1,n}|\mathcal{F}_t). \quad (23)$$

The last term on the right-hand-side comes directly from the GARCH– $t$  model discussed in the previous section. As for the second term, our results suggest that the occurrences of jumps and the sizes of the jumps are independent. Thus,

$$\begin{aligned} E[J_{t+1}|\mathcal{F}_t] &= \int_0^\infty j dP(0 < J_{t+1} \leq j|\mathcal{F}_t) \\ &= \int_0^\infty j dP(S_{t+1} \leq j|\mathcal{F}_t, I_{t+1} = 1) \cdot P(I_{t+1} = 1|\mathcal{F}_t) \\ &= \left[ \int_0^\infty j f(S_{t+1} \leq j|\mathcal{F}_t, I_{t+1} = 1) dj \right] \cdot P(I_{t+1} = 1|\mathcal{F}_t) \\ &= E(S_{t+1}|\mathcal{F}_t, I_{t+1} = 1) \cdot h_{t+1}. \end{aligned}$$

Forecast for the hazard rate,  $h_t$ , follows directly from the estimated ACH models. Since the models for  $S_{t+1}$  and  $C_{t+1}$  are formulated in logarithmic terms, the two conditional expectations  $E(S_{t+1}|\mathcal{F}_t, I_{t+1} = 1)$  and  $E(C_{t+1}|\mathcal{F}_t)$  will both involve a Jensen's inequality type correction. However, numerical evaluations of these expectations are easily accomplished by means of simulations. Similarly, even though the highly non-linear dynamic dependencies among the different model components render closed-form expressions for the multi-step ahead conditional expectations,  $\text{Var}(r_{t+h}|\mathcal{F}_t)$  for  $h > 1$ , infeasible, these are relatively easy to compute by means of recursive simulations.<sup>23</sup>

To assess the accuracy of the HAR–CJN model forecasts, we compare the predictions to the actual realized variation measures; i.e., for the one-day horizon forecasts  $RV_{t+1} + r_{t+1,n}^2$ . In addition to the one-day forecasts, we also calculate one-week and one-month forecasts defined by the average of the forecasts from 1 to 5, and 1 to 22 days ahead, respectively. As a benchmark comparison, we consider the forecasts from a simple GARCH(1, 1) model estimated on the daily returns, and an HAR–RV model properly scaled by the contribution from the overnight return so that the forecasts are unconditionally unbiased.<sup>24</sup> The first subsection below discuss the results for the full sample period, labeled in-sample, while the subsequent section reports on the results from a true out-of-sample forecast comparison.

<sup>22</sup> The validity of this decomposition for the conditional expectations implicitly assumes that the aforementioned convergence in probability of the realized variation measures implies convergence in mean. The assumption of a bounded return process, or a weaker uniform integrability condition, is sufficient to ensure that this holds; see, e.g., the discussion in Andersen et al. (2003).

<sup>23</sup> In the results reported on below we rely on a total of 10,000 replications in calculating the expectations. To minimize the impact of large influential outliers and stabilize the algorithm, we also trim any simulated values more than twice the largest in-sample observation. Further details are available upon request.

<sup>24</sup> An alternative to these popular forecasting models and the HAR–CJN model developed here suggested by one of the referees would be to project  $RV_{t+1} + r_{t+1,n}^2$

## 7.1. In-sample forecasts

To begin, Table 7 reports the standard Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) for the forecasts from each of the three different models based on the data over the full sample period.<sup>25</sup> As is clear from the table, these in-sample summary statistics clearly favor the more complicated HAR–CJN model for SP. Meanwhile, the in-sample RMSE and MAE for US are not as clear-cut.

In order to further analyze relative performance of the HAR–CJN model, we also estimate a series of Mincer–Zarnowitz style regressions. In particular, for the one-day-ahead forecasts,

$$(RV_{t+1} + r_{t+1,n}^2) = b_0 + b_1 V_{t,\text{GARCH}} + b_2 V_{t,\text{HAR–RV}} + b_3 V_{t,\text{HAR–CJN}} + \epsilon_{t+1} \quad (24)$$

where  $V_{t,\mathcal{M}}$  refers to the time  $t$  one-day-ahead forecast from model  $\mathcal{M}$ . In addition to the one-day-ahead forecasts, we run the same regressions for the 5- and 22-steps ahead forecasts, appropriately correcting the standard errors of the parameter estimates for the serial correlation in the residuals induced by the overlap in the data. Following the discussion in, e.g., Anderson and Vahid (2007), these regressions are naturally interpreted as volatility forecast encompassing regressions, in the sense that a coefficient significantly different from zero implies that the information in that particular model forecast is not encompassed in the forecasts by the two other models. As a further robustness check, we also report the results from the simple Mincer–Zarnowitz regressions, in which the ex-post variation measures are regressed on a constant and one of the three individual model forecasts in isolation.

The results from these joint and individual Mincer–Zarnowitz regressions are all reported in Table 8. In the joint regressions for SP the forecasts from the HAR–CJN model invariably receives a weight indistinguishably different from unity in a statistical sense, while the estimated coefficients for the other two model forecasts are close to zero and insignificant, indicating that the HAR–CJN forecasts encompasses the forecasts from other two models. The individual SP regressions reported in the bottom part of the table further corroborate these findings. In particular, the  $R^2$ 's from the HAR–CJN models are always the highest,<sup>26</sup> with the estimated intercept and slope coefficients very close to zero and unity, respectively. The corresponding in-sample results for US generally also favor the HAR–CJN model, although the differences among the three model forecasts are not as large.

## 7.2. Out-of-sample forecasts

Even though the loss functions used in evaluating the forecasts discussed in the previous section formally differ from the likelihood functions used in estimating the models, the in-sample comparisons may seem to tilted toward making the more complicated HAR–CJN model perform well. Hence, in order to more closely mimic a real-world forecast situation, we also report on the results obtained by re-estimating all of the models with data up until the end of 1999, retaining the last five years of the sample from January 2, 2000 to February 4, 2005 for out-of-sample forecast

on all of the variables in the time  $t$  information set, including information about the jumps and the overnight returns. While this procedure might perform well in a pure forecasting sense, it would obviously be completely void of any detailed information about the individual components that make up the total daily variation.

<sup>25</sup> Patton (2006) has recently cautioned against the use of the MAE criteria with a noisy volatility proxy. The realized volatility measures that we use here effectively mitigate these concerns.

<sup>26</sup> As discussed in Andersen et al. (2004, 2005), the reported  $R^2$ 's understate the true degree of predictability due to the measurement errors in the realized volatility proxies. This does not, however, impede any cross model comparisons.

**Table 8**  
In-sample Mincer–Zarnowitz regressions.

Horizon	SP			US		
	1	5	22	1	5	22
Joint regressions						
Const.	−0.030 (0.045)	0.025 (0.055)	0.140 (0.083)	−0.004 (0.020)	−0.015 (0.024)	−0.012 (0.033)
GARCH	−0.096 (0.106)	−0.136 (0.106)	−0.022 (0.150)	0.524 (0.110)	0.459 (0.109)	0.358 (0.128)
HAR–RV	−0.103 (0.071)	−0.189 (0.123)	−0.410 (0.226)	−0.302 (0.148)	0.003 (0.187)	0.097 (0.156)
HAR–CJN	1.264 (0.142)	1.353 (0.161)	1.363 (0.216)	0.774 (0.097)	0.565 (0.120)	0.539 (0.132)
R <sup>2</sup>	0.399	0.601	0.564	0.123	0.310	0.408
Individual regressions						
Const.	0.044 (0.046)	0.127 (0.061)	0.257 (0.073)	−0.023 (0.019)	−0.009 (0.025)	0.022 (0.036)
GARCH	0.939 (0.050)	0.868 (0.068)	0.750 (0.062)	1.093 (0.059)	1.053 (0.077)	0.963 (0.109)
R <sup>2</sup>	0.301	0.460	0.450	0.098	0.259	0.324
Intercept	−0.066 (0.081)	−0.098 (0.083)	−0.083 (0.104)	−0.007 (0.021)	−0.018 (0.027)	−0.031 (0.037)
HAR–RV	1.150 (0.089)	1.180 (0.095)	1.169 (0.111)	1.043 (0.062)	1.076 (0.080)	1.113 (0.111)
R <sup>2</sup>	0.346	0.523	0.473	0.095	0.265	0.345
Const.	−0.061 (0.050)	−0.022 (0.056)	0.046 (0.066)	0.038 (0.016)	0.044 (0.019)	0.042 (0.030)
HAR–CJN	1.091 (0.058)	1.068 (0.065)	1.024 (0.072)	0.870 (0.045)	0.838 (0.052)	0.805 (0.080)
R <sup>2</sup>	0.397	0.579	0.558	0.117	0.291	0.385

**Table 9**  
Out-of-sample forecast statistics.

Horizon		RMSE			MAE		
		1	5	22	1	5	22
SP	GARCH	1.929 (0.001)	1.269 (0.004)	1.130 (0.069)	0.836 (0.000)	0.669 (0.000)	0.694 (0.019)
	HAR–RV	1.868 (0.002)	1.234 (0.004)	1.147 (0.009)	0.717 (0.224)	0.600 (0.002)	0.642 (0.005)
	HAR–CJN	1.793	1.127	1.055	0.705	0.557	0.586
US	GARCH	0.375 (0.044)	0.199 (0.107)	0.150 (0.133)	0.193 (0.305)	0.130 (0.191)	0.105 (0.436)
	HAR–RV	0.375 (0.000)	0.198 (0.010)	0.151 (0.041)	0.192 (0.251)	0.130 (0.041)	0.108 (0.135)
	HAR–CJN	0.368	0.188	0.132	0.190	0.124	0.098

comparisons.<sup>27</sup> Due to the relatively time consuming calculations involved in the estimation of the non-linear models, we did not re-estimate the models on a rolling basis over the out-of-sample period, instead simply freezing all of the parameters at their estimates based on the full 1990–1999 in-sample period.

The out-of-sample results essentially affirm the earlier in-sample findings. The RMSEs and MAEs for SP reported in Table 9 again achieve their lowest values across all horizons for the HAR–CJN models. The out-of-sample values for US are also the lowest for the HAR–CJN model, although the numerical differences are not particularly large. Interestingly, however, limiting the out-of-sample forecast comparisons for US to the last two years of the sample, which tend to exhibit both larger and more frequent jumps, results in sharper differences among the RMSE and MAE criteria. As such, this indirectly suggests that the benefits from a forecasting perspective from separately modeling the two volatility components is to some extent period specific.

Several procedures to formally test for the statistical significance of the observed differences in the RMSE and MAE criteria and the superior predictive ability of the underlying forecasting models have recently been proposed in the literature. As a simple guide we here rely on the easy-to-calculate Diebold and Mariano (1995) test involving a pairwise comparison of the forecasts from each of the two traditional models to the forecasts from the HAR–CJN model.<sup>28</sup> The test is based on the heteroskedasticity and autocorrelation consistent *t*-statistic for the sample mean of  $L_{t,\text{HAR-CJN}} - L_{t,\mathcal{M}}$ , where  $L_{t,\mathcal{M}}$  denotes the time *t* squared or absolute loss from the

particular model  $\mathcal{M}$ . Many of the corresponding *p*-values reported in parentheses in Table 9 do indeed indicate statistically significant superior out-of-sample performance of the HAR–CJN model.

The out-of-sample Mincer–Zarnowitz regressions adjusted for the in-sample parameter estimation error uncertainty following West and McCracken (1998) reported in Table 10 generally also favor the HAR–CJN model. Although the high degree of co-linearity among the three forecasts render most of the estimated coefficients for the joint encompassing regressions rather imprecise, the individual regressions all achieve their highest *R*<sup>2</sup>'s for the HAR–CJN model. Moreover, the estimated intercept and slope coefficients for the individual HAR–CJN regressions are all close to zero and unity, respectively.

To further appreciate these results and the basic features of the different models, Figs. 5 and 6 plot the one-day ahead out-of-sample forecasts. The overall level of the forecasts obviously matches fairly closely across the three models for both of the markets. Consistent with the results from the Mincer–Zarnowitz regressions, it also appears more difficult to discern any sharp differences in the three US forecasts. Nonetheless, the HAR–CJN based forecasts do seem to adapt more quickly to changes in the volatility than do the GARCH and, to a lesser degree, the HAR–RV, based forecasts. Not surprisingly, on comparing the forecasts to the actual realization in Figs. 1 and 2, all of the models miss the very largest observations which inherently must represent genuine large volatility innovations.

## 8. Conclusion

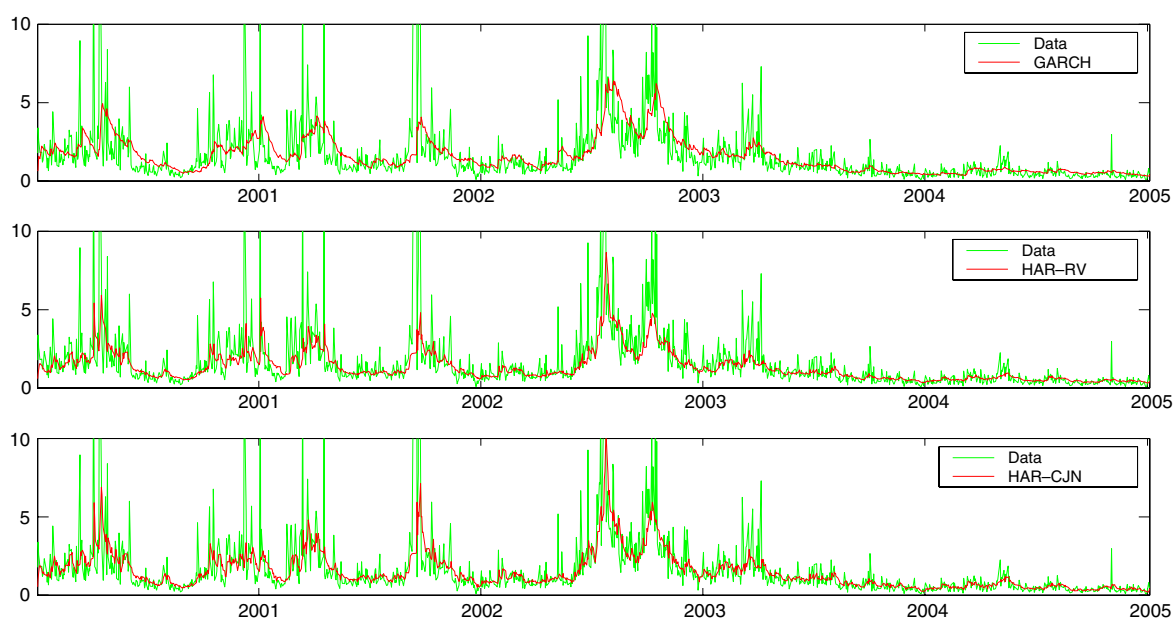
We use two fifteen-year samples of high-frequency intraday data for the S&P 500 and T-Bond futures markets along with the model-free bipower variation measures and corresponding jump statistics of Barndorff-Nielsen and Shephard (2004a, 2006) to non-parametrically identify and measure the daily continuous sample path variation and squared jumps. Directly in line with earlier

<sup>27</sup> We also experimented with other out-of-sample periods, resulting in the same basic conclusions. Further details concerning these additional robustness checks are available upon request.

<sup>28</sup> Although the Diebold and Mariano (1995) test does not explicitly account for the effect of estimation uncertainty, the out-of-sample version of the test coincides with the generally valid test for equal unconditional predictive ability recently developed by Giacomini and White (2006).

**Table 10**  
Out-of-sample Mincer–Zarnowitz regressions.

Horizon	SP			US		
	1	5	22	1	5	22
Joint regressions						
Const.	0.008 (0.117)	0.114 (0.170)	0.394 (0.250)	−0.029 (0.062)	−0.039 (0.072)	−0.050 (0.100)
GARCH	−0.194 (0.270)	−0.187 (0.262)	−0.097 (0.294)	0.852 (0.301)	0.762 (0.279)	0.516 (0.329)
HAR–RV	−0.238 (0.594)	−0.813 (0.779)	−2.428 (1.545)	−0.709 (0.379)	−0.361 (0.404)	−0.061 (0.446)
HAR–CJN	1.515 (0.542)	2.019 (0.716)	3.333 (1.321)	1.018 (0.247)	0.797 (0.265)	0.795 (0.402)
R <sup>2</sup>	0.375	0.568	0.518	0.124	0.321	0.413
Individual regressions						
Const.	0.077 (0.124)	0.201 (0.187)	0.487 (0.259)	−0.049 (0.060)	−0.034 (0.079)	0.016 (0.102)
GARCH	0.989 (0.099)	0.915 (0.154)	0.746 (0.119)	1.282 (0.166)	1.250 (0.224)	1.137 (0.288)
R <sup>2</sup>	0.260	0.404	0.348	0.094	0.262	0.310
Const.	−0.128 (0.150)	−0.043 (0.202)	0.182 (0.296)	0.025 (0.070)	0.002 (0.085)	−0.033 (0.107)
HAR–RV	1.325 (0.144)	1.300 (0.200)	1.217 (0.194)	1.062 (0.183)	1.139 (0.234)	1.286 (0.304)
R <sup>2</sup>	0.346	0.506	0.402	0.084	0.250	0.333
Const.	−0.078 (0.144)	−0.008 (0.168)	0.170 (0.232)	0.048 (0.084)	0.048 (0.064)	0.012 (0.098)
HAR–CJN	1.154 (0.124)	1.157 (0.156)	1.137 (0.150)	0.944 (0.138)	0.948 (0.162)	1.058 (0.257)
R <sup>2</sup>	0.371	0.554	0.466	0.109	0.287	0.383



**Fig. 5.** One-day-ahead out-of-sample forecasts for SP.

studies we find that the volatility associated with the continuous price movements within the day is a highly persistent process for both markets. Counter to a number of previous studies, however, we detect important dynamic dependencies in both the times between significant jumps and the sizes of the jumps. Further, the time series of overnight returns, or price jumps, associated with the change in the closing price from one day to the opening price of the next exhibits strong volatility clustering. To satisfactorily account for these dependencies, we formulate and estimate a combination of several reduced form time series models. In addition, we compare and contrast the forecasting performance of the estimated models for each of the three non-parametrically identified volatility components to other commonly used volatility forecasting models.

Looking ahead, our estimation results for the ACH model indicate that the occurrence of jumps in the T-Bond market is directly related to certain macroeconomic news releases. In this regard, it would be interesting to more systematically investigate the economic determinants behind the apparent discontinuities.

What is it that causes financial markets to jump? The reduced form modeling setup developed here provides a particular convenient framework for further exploring this important question.

In the model diagnostics and forecast comparisons presented in the paper, we have primarily focused on mean square error type criteria. However, separately modeling the intraday jumps and the overnight returns are likely to prove especially beneficial for better understanding the tails of the return distributions. It would be interesting to more directly analyze this issue, and the model's ability to capture the more extreme tail behavior and corresponding expected shortfalls, as would be of interest in many practical risk management situations.

As previously noted, the specification and estimation of empirically realistic continuous-time jump–diffusion models have been the subject of extensive recent research efforts. In this regard, the relatively simple reduced form model structures for each of the different variation measures developed here could also be used as auxiliary models in an indirect inference setting to more effectively estimate and discriminate among some of these



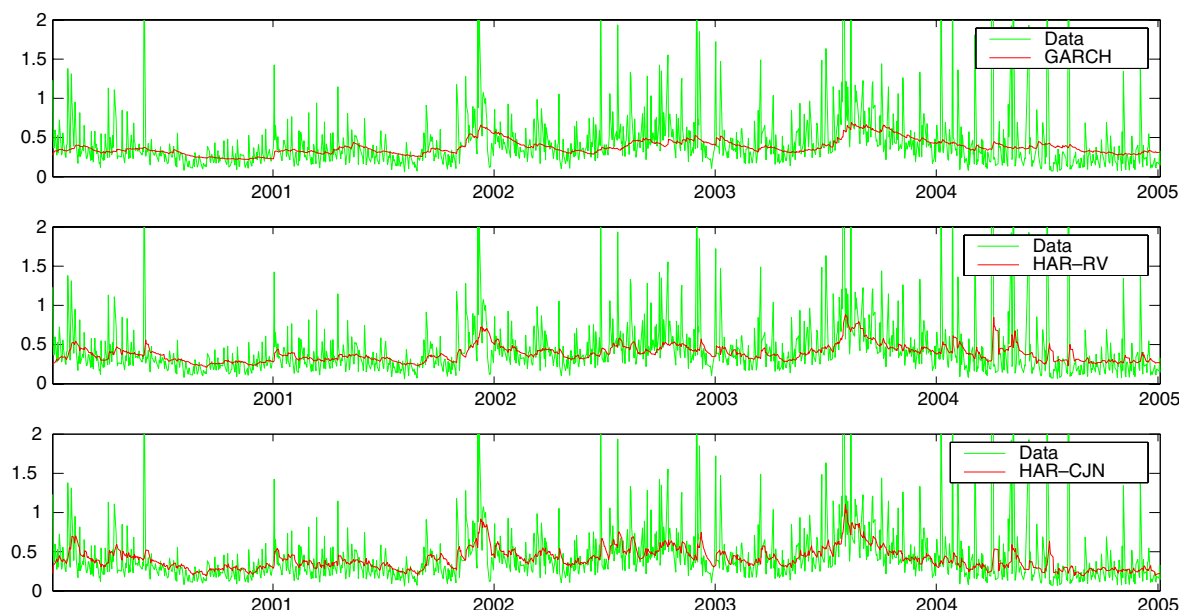


Fig. 6. One-day-ahead out-of-sample forecasts for US.

competing continuous-time specifications, naturally extending the earlier realized variation based inferential procedures of Barndorff-Nielsen and Shephard (2002) and Bollerslev and Zhou (2002).

In a related context, the recent studies by Santa-Clara and Yan (2004) and Todorov (2006) suggest that the premia required by investors in options markets to compensate for jump and continuous volatility risks differ. By easily allowing for different risk premia associated with the future risks originating from the continuous sample path price process and the harder-to-hedge intraday jump and overnight components, it is possible that our relatively simple-to-implement reduced form forecasting model may be used in the calculation of more accurate derivatives prices.

We leave further work along these lines for future research.

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