

A multivariate generalized ARCH approach to modeling risk premia in forward foreign exchange rate markets

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Assuming that daily spot exchange rates follow a martingale process, we derive the implied time series process for the vector of 30-day forward rate forecast errors from using weekly data. The conditional second moment matrix of this vector is modeled as a multivariate generalized ARCH process. The estimated model is used to test the hypothesis that the risk premium is a linear function of the conditional variances and covariances as suggested by the standard asset pricing theory literature. Little support is found for this theory; instead lagged changes in the forward rate appear to be correlated with the 'risk premium.'

This study examines spot and forward exchange rates at a weekly level for four different currencies. It is shown that the vector of forward market forecast errors can be parameterized as a vector moving average (MA) process where the MA coefficients can be theoretically determined from knowledge of the martingale behavior of exchange rates. The conditional covariance matrix is then estimated by assuming a multivariate GARCH structure which depends on a relatively small number of parameters. A range of LM tests confirms that the model provides an adequate description of the first- and second-order moments of the conditional density of the data. The vector MA process is then used to provide some bounds on the magnitude of the risk premium. Series of tests are also applied to the estimated model to test for the inclusion of terms that would be implied by a time varying risk premium. The results are not consistent with any standard model of asset pricing, but do provide evidence for the existence of this type of effect.

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Our study is related to those of Bollerslev (1988) and Diebold and Nerlove (1989), who used different multivariate ARCH modeling techniques to describe the behavior of a vector of spot exchange rates. The study is also related to the various attempts in the literature at estimating time varying risk premium models in the forward foreign exchange market, *e.g.*, Domowitz and Hakkio (1985) and Hodrick (1989). However these studies used monthly data which typically do not possess much ARCH type behavior. By using weekly data, which are rich in ARCH effects, we are able to provide a relatively simple model for the forward rate forecast errors and to provide a specification that allows several models of the time varying risk premium to be directly tested.

Before describing our analysis in more detail it is worth recalling that many recent studies of the current floating exchange rate regime have concluded that spot rates and forward rates appear to contain a unit root in their autoregressive univariate time series representations: *e.g.*, Meese and Singleton (1982) and Baillie and Bollerslev (1989a). While changes in the levels or logarithms of nominal exchange rates are approximately uncorrelated over time their distributions tend to be unimodal and symmetric but with fatter tails than the normal. This phenomenon was originally detected in a variety of asset price series by Mandelbrot (1963) and Fama (1965) and in exchange rate data by Westerfield (1977) and McFarland *et al.* (1982), who tried various parametric distributions to describe the *unconditional* distribution of changes in the exchange rate. However, the important development of modeling time dependent *conditional* heteroskedasticity, *i.e.*, the ARCH model of Engle (1982), has led to this methodology being used to describe the volatility in exchange rates; see among many others, Engle and Bollerslev (1986), Bollerslev (1987), Hsieh (1989), Diebold and Nerlove (1989), McCurdy and Morgan (1987, 1988, 1989), and Milhøj (1987). In particular Baillie and Bollerslev (1989b) show that apart from specific day of the week and holiday effects daily spot exchange rate series for the 1980s are well described by a simple martingale difference sequence with a GARCH structure for the conditional variance. However, on sampling the data on a weekly, fortnightly, and monthly basis Baillie and Bollerslev (1989b) note a sharp decline in the ARCH effects and a tendency towards normality as the observation period increases. Other studies such as Boothe and Glassman (1987), Domowitz and Hakkio (1985), and Hodrick (1989) have also observed only minimal ARCH effects and approximate normality with monthly spot exchange rate data.

While most of the above studies have concentrated on the univariate time series characteristics and stylized facts of spot exchange rates, an even larger literature has developed on the relationship with forward foreign exchange markets and whether or not the forward rate is an unbiased and efficient predictor of the corresponding future spot rate. Of course, the unbiasedness hypothesis critically depends upon the twin assumptions of rational expectations and risk neutrality of the representative agent in the foreign exchange market. With weekly data and forward rates of one- or three-month contract periods, the sampling time interval of the data is finer than the maturity time of the forward contract, and researchers have incorporated estimation techniques that treat this overlapping data problem. Based on these techniques a consensus has emerged that the unbiasedness hypothesis can generally be rejected. See Geweke and Feige (1979), Hansen and Hodrick (1980), Hakkio (1981), Baillie *et al.* (1983), Hodrick and Srivastava (1984), Hsieh (1984), and Baillie (1989) for a variety of econometric studies that

have reached this conclusion. Rejection of the hypothesis is generally attributed to the presence of a time varying risk premium, which has led to an intensive search for the proper specification of such a risk premium; see Hodrick (1987) for an excellent recent survey. In particular the intertemporal capital asset pricing model of Lucas (1978) implies that the risk premium is determined by the conditional covariance between the return on a long position in the forward market and the marginal rate of substitution between current and future consumption. However, most studies such as Dornbusch (1982) and Hansen and Hodrick (1983) assume an underlying model where the conditional variances and covariances of the future spot rates are time invariant. Indeed, this may well explain the apparent lack of success in finding empirically significant risk premia terms. To quote Hodrick (1987, pp. 67), '... modeling the conditional variance may be a fruitful direction to pursue in order to understand the nature of the rejection of the unbiasedness hypothesis and to determine whether the rejection is due to a time varying risk premium.' On the other hand, when using monthly data the conditional heteroskedasticity or time dependence between the conditional covariances of the exchange rate and the forcing or fundamental variables, such as consumption and prices, are frequently of a very small magnitude; see Domowitz and Hakkio (1985), Engel and Rodrigues (1989), Hodrick (1989), Kaminsky and Peruga (1990), and Mark (1988).

The purpose of this paper is to consider the interdependence between the conditional second moments for a set of weekly exchange rate series through a multivariate ARCH type framework. Given the empirically reasonable assumption that the first differences in nominal exchange rates follow an uncorrelated process, together with risk neutrality and rational expectations, it is possible to derive the implied model for the forward rate forecast error. The details of this approach are discussed in Section I. Of course, if the martingale difference hypothesis for each of the underlying spot rates is true, imposing the implied coefficients directly in the estimation of the model for the forward rate forecast errors is likely to enhance the power properties of any test for a time varying risk premium. Section II of the paper discusses the joint estimation of such a model for a set of four different currencies versus the US dollar. Interestingly, this implied model for the forward rate forecast errors cannot be rejected empirically. The most appropriate general model specification allows the conditional covariance matrix to change over time according to a simple multivariate GARCH model. In general the parameterization of multivariate ARCH models is somewhat problematic, but following Bollerslev (1988), the assumption of constant conditional correlations together with a GARCH(1, 1) model for each of the four conditional variances is seen to provide a satisfactory model that survives a variety of diagnostic tests. For all the currencies examined, the magnitude of the risk premium appears to be relatively small when compared to the implied model for the associated error from forecasting the future spot rate. Section III describes the application of a series of tests for the presence of a time varying risk premium in the estimated multivariate GARCH model motivated by the discrete time asset pricing model. One such set of tests incorporates explicit functions of the time varying conditional variances and covariances in explaining the risk premium. However, empirically these tests gain only little support for any of the four currencies we examine. Instead, the contemporaneous forward rates appear to be correlated with the risk premium. Finally, Section IV provides a few concluding remarks.

I. A simple model for the implied forward rate forecast error

Let s_{it} denote the logarithm of the spot exchange rate for the i th currency and f_{it} the logarithm of the corresponding forward exchange rate of k period maturity time. The model considered in this paper takes the form

$$\langle 1 \rangle \quad s_{it+k} - f_{it} = \delta_{it} + \mu_{it+k}.$$

The left-hand side of $\langle 1 \rangle$ gives the forward rate forecast error, which is equal to the return on a long position in the forward market where no resources must be sacrificed before time $t+k$. There is substantial empirical evidence that both spot and forward exchange rates are integrated of order one, that is, they need to be first differenced to become covariance stationary; see Meese and Singleton (1982), Baillie and Bollerslev (1989a), and Hakkio and Rush (1989). The latter two papers have also noted that the forward rate forecast errors appear to be stationary. Thus, spot and forward exchange rates are cointegrated in the sense of Engle and Granger (1987) with a cointegrating factor of approximately unity. Hence standard asymptotically based procedures are generally valid when conducting inference on equation $\langle 1 \rangle$. The term μ_{it+k} on the right-hand side of $\langle 1 \rangle$ represents the random innovations in the period between the market setting the forward exchange rate and the actual realization of the spot rate k periods later. Under the assumption of rational expectations $E_t(\mu_{it+k})=0$, where $E_t(\cdot)$ denotes the conditional expectation given all the available information up through time t , say ψ_t . Finally, δ_{it} represents a possibly time varying risk premium. Under the assumption that the representative agent operates under rational expectations and risk neutrality $\delta_{it}=0$, then the forward rate will be an unbiased predictor of the future spot rate.

With weekly data there is now well documented evidence that the unbiasedness hypothesis can be rejected; see for instance Hansen and Hodrick (1980, 1983), Hakkio (1981), Baillie *et al.* (1983), and Hsieh (1984). This paper investigates the possibility that the rejection is due to the existence of a time varying risk premium.

However, the purpose of the present section is to consider the time series modeling of μ_{it+k} under appropriate assumptions about the process generating s_{it} . The actual tests for the time varying risk premium will be left until Section III. In order to detect relatively short-lived risk premia, or market inefficiencies, it appears important to have high frequency data. However, it is well known that when the sampling interval is finer than the forecast interval, *i.e.*, $k > 1$, the forecast errors will be serially correlated; see Hansen and Hodrick (1980), Hakkio (1981), and Baillie (1989). In particular, the autocorrelation function for μ_{it} , say γ_j , will equal zero only for $j \geq [k] + 1$, where $[k]$ denotes the closest integer smaller than k . Only in situations where the forecast horizon is less than or equal to the sampling interval should the forecast errors μ_{it+k} be serially uncorrelated. The two approaches for dealing with this problem have been to either estimate subject to cross equation restrictions on a vector time series model, or to use a generalized method of moments estimator in a single equation framework; see Hansen and Hodrick (1980, 1983), Hakkio (1981), and Baillie *et al.* (1983). This study takes a different tack since the main concern is the appropriate modeling of the first and second moments jointly.

The simplest model in accordance with the autocorrelation structure for μ_{it+k} is

of course the $[k]$ th-order moving average process

$$\langle 2 \rangle \quad \mu_{it+k} = \sum_{j=1}^{[k]} \theta_j \varepsilon_{it+k-j} + \varepsilon_{it+k},$$

where the innovations, ε_{it+k} , are serially uncorrelated with mean zero and finite unconditional variance; see Ansley *et al.* (1977) and Baillie (1989). To simplify the subsequent estimation, we now assume that the discrete time spot exchange rate, s_{it+k} , is obtained by taking point observations of the continuous time random walk process

$$\langle 3 \rangle \quad ds_i(t) = \sigma_i dz_i(t),$$

where $z_i(t)$ is a unit Wiener process. Several recent studies have documented the approximate random walk behavior of spot exchange rates on a daily or weekly basis; see, for instance, Meese and Singleton (1982), Hsieh (1988), Diebold and Nerlove (1989), Baillie and Bollerslev (1989b), Bollerslev (1987, 1988), and Milhøj (1987). In fact, for the validity of the subsequent analysis it is only necessary to impose the much weaker condition that the daily spot rate is a martingale difference sequence. The stronger assumption in $\langle 3 \rangle$ merely serves to clarify the exposition.

In particular, given $\langle 3 \rangle$ it follows immediately from the definition of μ_{it+k} in $\langle 1 \rangle$ that,

$$\langle 4 \rangle \quad \mu_{it+k} = s_{it+k} - s_{it} = \sigma_i \int_{\tau=0}^{\tau=k} dz_i(t+\tau).$$

Therefore, from $\langle 4 \rangle$ the j th-order autocorrelation coefficient for μ_{it+k} equals $\gamma_j = (k-j)k^{-1}$, and the $[k]$ moving average parameters in $\langle 2 \rangle$ can be found by solving the $[k]$ nonlinear equations,

$$\langle 5 \rangle \quad (k-j)k^{-1} = (\theta_j + \theta_1 \theta_{j-1} + \dots + \theta_{[k]-j} \theta_{[k]}) (1 + \theta_1^2 + \dots + \theta_{[k]}^2)^{-1},$$

$$j = 1, 2, \dots, [k].$$

In general, $2^{[k]}$ solutions to $\langle 5 \rangle$ will exist, but the invertible solution will be unique; see Harvey (1981) for a discussion of invertibility. The above derivation of $\langle 5 \rangle$ based on the distributional assumptions in $\langle 3 \rangle$ would also imply conditional homoskedasticity and normality of the error term in $\langle 2 \rangle$; *i.e.*, ε_{it} . As already mentioned, this is a much stronger condition than needed for μ to follow the moving average process in $\langle 5 \rangle$. It is easy to show that $\langle 5 \rangle$ is valid as long as the discrete time process $\partial s_{it} = \int_{\tau=0}^1 ds_i(t+\tau)$ is serially uncorrelated which allows for departures from conditional homoskedasticity and normality in ε_{it} .

For example, with one-month forward contracts and weekly data, as analyzed in this paper, the typical maturity of the contract equals four weeks and two days, *i.e.*, $k = [4\frac{2}{5}] = 4$, and μ_{it+k} in $\langle 1 \rangle$ will be MA(4) rather than MA(3); a point previously noted by Hakkio (1981). In this situation the unique invertible solution to $\langle 5 \rangle$ gives rise to the four MA parameters,

$$\langle 6 \rangle \quad \bar{\theta}_1 = 0.8366, \quad \bar{\theta}_2 = 0.7728, \quad \bar{\theta}_3 = 0.6863, \quad \bar{\theta}_4 = 0.2577,$$

with corresponding roots of $0.1909 \pm 1.1724i$ and $-1.5226 \pm 0.6575i$.

We now turn to the estimation of a multivariate system with these roots imposed. As noted previously, if the martingale difference hypothesis is true,

imposing these implied coefficients directly in the estimation of the model is likely to enhance the power properties of a test for a time varying risk premium when compared to the standard type adjustment for overlapping data suggested by Hansen and Hodrick (1980).

II. Estimation of a multivariate ARCH model

In order to deal with a multivariate ARCH model of manageable size it was decided to restrict attention to four major European currencies, the UK pound, the West German deutschemark, the Swiss franc, and the French franc, all against the US dollar. The deutschemark and the French franc are the two major currencies in the European Monetary System (EMS), while both the pound and the Swiss franc are freely floating. In order to provide a tractable model for exchange rate interdependencies and to detect possibly short-lived time dependent risk premia, it seems important to use finely sampled data. Consequently, the data used in this study are weekly spot and one-month or 30-day forward rates. The data are opening bid prices from the New York Foreign Exchange Market between March 1, 1980 and February 2, 1989, and constitute a total of 462 weekly observations. The data were obtained from Data Resources Incorporated in Boston, MA and are available from the authors on request. Analogously to Baillie *et al.* (1983) and Hansen and Hodrick (1983), the forward rates are taken on Tuesdays and the spot rates four weeks and two days later on Thursdays.¹ Thus, μ_{it+k} in <1> will be MA(4), and under the assumption that the spot rate follows a martingale difference sequence together with rational expectations and the absence of a time varying risk premium, the corresponding four moving average parameters are given by <6>.

On imposing these implied MA coefficients the following SUR system was estimated for the set of all four currencies,²

$$\langle 7 \rangle \quad s_{t+4} - f_t = b_0 + \sum_{j=1}^4 \bar{\Theta}_j \varepsilon_{t+4-j} + \varepsilon_{t+4}$$

$$\varepsilon_t | \psi_{t-1} \sim N(0, \Omega),$$

where $s_t = (s_{UKt}, s_{WGr}, s_{SWt}, s_{FRt})'$, $f_t = (f_{UKt}, f_{WGr}, f_{SWt}, f_{FRt})'$. b_0 is a four-dimensional column vector of constants, $\bar{\Theta}_j$ are 4×4 diagonal matrices with diagonal elements $\bar{\theta}_j$ and ε_t is a normally distributed four-dimensional vector random error with covariance matrix Ω .

Results of applying the Ljung and Box (1978) portmanteau test to each of the four series of residuals reported in the first row of Table 1 do not reveal any significant serial correlation in addition to the imposed MA(4) process. However, even though the residuals might be serially uncorrelated, they are clearly not independent through time or across countries. The application of the Ljung-Box statistic in Table 1 to each series of squared residuals reveals comprehensive rejection of the assumption of conditional homoskedasticity. Furthermore, apart from the UK/France relationship the cross products of the squared residuals also indicate substantial ARCH effects in the conditional covariances. As noted by Cumby and Huizinga (1988), Domowitz and Hakkio (1986), and Diebold (1986) among others, the presence of time varying higher moments generally induces a bias in the traditional test statistics for the absence of autocorrelation, as well as a loss in asymptotic efficiency. Despite this caveat the reported values of the Ljung-Box

TABLE 1. Ljung-Box tests for homoskedastic SUR model.

		UK	WG	SW	FR
$Q(20)$		19.926 (0.463)	18.999 (0.522)	25.634 (0.178)	22.014 (0.340)
$Q^2(20)$	UK	101.441 (0.000)	—	—	—
	WG	63.384 (0.000)	34.408 (0.023)	—	—
	SW	81.444 (0.000)	40.761 (0.004)	42.226 (0.003)	—
	FR	70.106 (0.000)	40.966 (0.004)	50.385 (0.000)	44.544 (0.001)

$Q(20)$ and $Q^2(20)$ denotes the Ljung-Box tests for up to 20th-order serial correlation in $\hat{\varepsilon}_{it}$ and $\hat{\varepsilon}_{it}\hat{\varepsilon}_{jt}$, respectively. Asymptotic p -values in parentheses.

statistics for the squared residuals are sufficiently large to question the specification of the model.

Several authors have discussed the difficulties that arise in deciding on the most appropriate parameterization of the conditional covariance matrix. In this paper we follow the suggestion in Bollerslev (1988) and assume the conditional correlations to be constant, so that all the variations over time in the conditional covariances are due to changes in each of the corresponding two conditional variances. This assumption considerably simplifies the estimation and inference procedures and seems more amenable than the factor ARCH model when the multivariate GARCH formulation is used and possibly more than one factor is present. Preliminary estimation revealed that each of the four conditional variances are well approximated by a simple GARCH(1, 1) model; see Bollerslev (1986). Thus, the four-dimensional GARCH model estimated here takes the form,

$$\langle 8 \rangle \quad s_{t+4} - f_t = b_0 + \sum_{j=1}^4 \bar{\Theta}_j \varepsilon_{t+4-j} + \varepsilon_{t+4},$$

$$\varepsilon_{it} | \psi_{t-1} \sim N(0, H_t), \quad \{H_t\}_{ij} = h_{ijt}, \quad i, j = 1, \dots, 4,$$

$$h_{iit} = \omega_i + \alpha_{i1} \varepsilon_{it}^2 + \beta_{i1} h_{iit-1},$$

$$h_{ijt} = \rho_{ij} h_{iit}^{1/2} h_{jjt}^{1/2}, \quad i \neq j,$$

where $\rho_{ij} = \text{Corr}(\varepsilon_{it}, \varepsilon_{jt} | \psi_{t-1})$ denotes the conditional correlation which is assumed to be constant. Note that a set of necessary and sufficient conditions for the model to be well defined and H_t positive definite for all t is that each of the conditional variances, h_{iit} , are positive and that the constant matrix of conditional correlations is positive definite. These conditions are easy to impose and verify.

The maximum likelihood estimates of the model in $\langle 8 \rangle$ obtained by the Berndt *et al.* (1974) algorithm are given in Table 2, where the returns for numerical and comparison purposes have been converted to monthly percentage rates by multiplication with 100.

The estimates of the intercepts in each equation are negative but insignificant,

TABLE 2. Maximum likelihood estimates.

$$s_{t+4} - f_t = b_0 + \sum_{j=1}^4 \bar{\theta}_j \varepsilon_{t+4-j} + \varepsilon_{t+4}$$

$$\varepsilon_t | \psi_{t-1} \sim N(0, H_t), \quad \{H_t\}_{ij} = h_{ijt}$$

$$h_{iit} = \omega_i + \alpha_{i1} \varepsilon_{it}^2 + \beta_{i1} h_{iit-1}$$

$$h_{ijt} = \rho_{ij} h_{iit}^{1/2} h_{jtt}^{1/2}, \quad i \neq j$$

Mean					
<i>i</i>	<i>b</i> ₀	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\bar{\theta}_4$
UK	-0.109 (0.304)	0.837	0.773	0.686	0.258
WG	-0.496 (0.274)	0.837	0.773	0.686	0.258
SW	-0.582 (0.311)	0.837	0.773	0.686	0.258
FR	-0.262 (0.292)	0.837	0.773	0.686	0.258

Variance				
<i>i, j</i>	ω_i	α_{i1}	β_{i1}	ρ_{ij}
UK	0.237 (0.114)	0.042 (0.018)	0.893 (0.041)	—
UK, WG	—	—	—	0.688 (0.023)
UK, SW	—	—	—	0.661 (0.025)
UK, FR	—	—	—	0.660 (0.023)
WG	1.086 (0.406)	0.062 (0.022)	0.659 (0.116)	—
WG, SW	—	—	—	0.921 (0.007)
WG, FR	—	—	—	0.932 (0.005)
SW	0.287 (0.131)	0.043 (0.014)	0.892 (0.037)	—
SW, FR	—	—	—	0.872 (0.010)
FR	1.033 (0.278)	0.099 (0.021)	0.626 (0.080)	—

Asymptotic standard errors in parentheses.

which is presumably due to the systematic depreciation of the US dollar over the early and latest part of the sample.

Strong own country GARCH effects are apparent for all four currencies. Also, the Likelihood Ratio (LR) test statistic for the model versus the homoskedastic SUR system in <7> equals 58.230, which is significant at any reasonable level for the asymptotic χ^2_8 distribution under the null hypothesis of no ARCH effects.

The estimated conditional correlations, ρ_{ij} , are also highly individually significant. In fact, the asymptotic χ^2_6 LR test statistic for absence of any cross-country dependence, *i.e.*, $\rho_{ij}=0$ for all *i* and *j*, equals 2128.561. It is also interesting to see that the conditional correlations are particularly large between the deutschemark, the dominant EMS currency, and the other currencies, whereas the British pound exhibits the lowest overall conditional correlations with the other countries. These findings are very much in line with the estimates reported in Bollerslev (1988) for spot rates.

A particularly interesting feature of model <8> is the interpretation of the imposed MA(4) coefficients and their contribution to the variance of the forward rate forecast errors. Although not a formal test, a simple and revealing check for the presence of time varying risk premia is available from comparing sample estimates of the residual variances with those implied by the MA(4) process. If the assumption that spot rates follow a martingale is true and the risk premium is constant, then the total variation in $s_{it+4} - f_{it}$ should be $(1 + \theta^2_1 + \dots + \theta^2_4) = 2.833$ times greater than the total variation in ϵ_{it+4} . The sample values from the estimated model for the UK, West Germany, Switzerland, and France are 3.236, 3.221, 3.378, and 3.270, respectively. Thus, it appears that the presence of a time varying risk premium adds between 14 per cent to 19 per cent of the total variability to that already explained by the MA(4) process.

In order to get a general idea about the descriptive validity of the model a series of misspecification tests were also performed. As previously mentioned, although the Ljung-Box test and presumably its multivariate extension due to Hosking (1980) are inappropriate in the presence of autocorrelation and heteroskedasticity they are nevertheless indicative of substantial misspecification. Table 3 presents different values of the Hosking (1980) multivariate portmanteau test for cross-equation autocorrelation in the standardized residuals. It should be noted that the Hosking statistics are asymptotically equivalent to an LM test for a vector AR(*p*)

TABLE 3. Hosking portmanteau tests for multivariate GARCH model.

$$H(p) = T^2 \sum_{j=1}^p (T-j)^{-1} \text{tr}(\hat{C}_j \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1})$$

$$\hat{C}_j = T^{-1} \sum_{t=j+1}^T (\hat{\epsilon}_t \hat{H}_t^{-1/2})(\hat{\epsilon}_{t-j} \hat{H}_{t-j}^{-1/2})$$

<i>p</i>	1	2	3	5	10
<i>H(p)</i>	25.457	40.372	55.132	109.682	183.910
	(0.062)	(0.147)	(0.223)	(0.078)	(0.095)

Asymptotic *p*-values from the χ^2_{16p} distribution in parentheses.

TABLE 4. Ljung-Box tests for multivariate GARCH model.

		UK	WG	SW	FR
Q(20)		20.072 (0.453)	18.653 (0.544)	25.411 (0.186)	19.381 (0.497)
Q ² (20)	UK	28.851 (0.091)	—	—	—
	WG	27.925 (0.111)	15.930 (0.721)	—	—
	SW	31.843 (0.045)	18.152 (0.577)	17.264 (0.636)	—
	FR	28.951 (0.089)	19.131 (0.513)	22.581 (0.310)	20.380 (0.434)

Q(20) and Q²(20) denotes the Ljung-Box tests for up to 20th-order serial correlation in $\hat{\varepsilon}_{it}\hat{h}_{it}^{-1/2}$ and $\hat{\varepsilon}_{it}\hat{h}_{it}^{-1/2}\hat{\varepsilon}_{jt}\hat{h}_{jt}^{-1/2}$, respectively. Asymptotic *p*-values in parentheses.

or vector MA(*p*) process against the null hypothesis of a vector white noise process. Under the null hypothesis, the test statistics have an asymptotic χ^2_{16p} distribution, where *p* denotes the number of lags. For lags 1, 2, 3, 5, and 10 the statistic failed to detect any serial dependence in the vector of standardized residuals. As Table 4 indicates, the Ljung-Box test applied to the normalized residuals and their cross products also fails to detect any serious misspecifications of the serial correlation or ARCH components of the model. Both the diagonal and the off diagonal elements in the lower part of the table are much smaller than the corresponding statistics for the homoskedastic model in Table 1.

Specific Lagrange Multiplier (LM) tests, calculated as TR^2 from the first BHHH regression towards the augmented model, confirm these findings. In particular, the asymptotic χ^2_4 LM test statistics for a freely estimated MA(4) process, without imposing the parameter values given in <6> take the values 2.800, 7.263, 4.588, and 6.454 for the UK, West Germany, Switzerland, and France, respectively, and none are significant at the usual 5 per cent level. This corresponds to the asymptotic χ^2_{16} LR test equal to 17.476 for a freely estimated multivariate model with MA(4) errors. Also, the asymptotic χ^2_1 LM test statistics for the inclusion of ε_{it}^2 in each of the four conditional variances, *i.e.*, a GARCH(1, 2) process, are all insignificant taking on the values 0.300, 0.268, 0.019, and 0.009, respectively.

A further test for specification in models involving time dependent conditional heteroskedasticity due to Pagan and Sabau (1987) was also calculated. The test is one of model consistency, and requires the postulated conditional variance process to conform to the pattern in the second moments of the residuals. The Pagan-Sabau tests reported here are computed from the OLS regressions

$$\hat{\varepsilon}_{it}\hat{\varepsilon}_{jt} = b_{ij0} + b_{ij1}\hat{h}_{ijt}, \quad i, j = 1, \dots, 4$$

Under the null hypothesis of model consistency, the estimates of b_{ij1} should not be significantly different from unity for all *i, j*. The *t*-statistics for the hypothesis $b_{ij1} = 1$, reported in Table 5, were calculated by using White type standard errors that are robust to the presence of heteroskedasticity, in the above regressions. Two out of the ten statistics were significant at the 0.05 per cent level, both cases involving West Germany.

TABLE 5. Pagan-Sabau consistency tests.

	UK	WG	SW	FR
UK	1.259 (0.204)	—	—	—
WG	1.885 (0.060)	2.120 (0.034)	—	—
SW	1.713 (0.087)	2.102 (0.036)	0.912 (0.362)	—
FR	1.551 (0.121)	1.841 (0.066)	1.873 (0.061)	1.124 (0.261)

Asymptotic p -values in parentheses.

In conclusion, there is reason to believe that the relatively simple formulation in <8> provides a reasonably good description of the properties of the vector of the weekly observations on the one-month forward rate forecast errors.

III. Tests for a time varying risk premium

Following Hansen and Hodrick (1983) and Hodrick and Srivastava (1984) a popular discrete time asset pricing model in the context of modeling time varying risk premia in the forward foreign exchange market gives rise to the equation

$$\begin{aligned}
 \langle 9 \rangle \quad E_t(s_{it+k}) - f_{it} &= \delta_{it} \\
 &= -1/2 \text{Var}_t(s_{it+k}) - \text{Cov}_t(s_{it+k}, \log(p_t/p_{t+k})) \\
 &\quad + \gamma \text{Cov}_t(s_{it+k}, \log(c_{t+k}/c_t)),
 \end{aligned}$$

where c_t denotes consumption and p_t the price level at time t . This formulation assumes the representative agent to have a utility function with constant relative risk aversion and for the variables to be jointly log normally distributed. The parameter $\gamma \geq 0$ denotes the constant coefficient of relative risk aversion. The relative lack of success in modeling a time varying risk premium in previous studies may therefore well have been due to the assumption of constant conditional variances and covariances which can be relaxed with the aid of the ARCH specification.

One such direct attempt at estimating <9> can be found in Kaminsky and Peruga (1990) who use monthly data on consumption and prices and replace the conditional covariance terms with specific numerical values. However, subsequent attempts by these authors at modeling the conditional variance of s_{it+k} as time varying with an ARCH model were unsatisfactory due to the minimal amount of conditional heteroskedasticity apparent with monthly foreign exchange rate data. Unfortunately, the weekly model estimated here does not allow a direct test of <1>, since it would require a sequence of weekly measurements on both consumption and prices. However, since the time paths of consumption and prices are relatively smooth it is reasonable to suppose that the influence of the second and third terms on the right-hand side of <9> will be small. Consequently, it is reasonable to proxy the risk premium by the first term, the conditional variance of the future spot

exchange rate. Hence this study reports a series of tests for the significance of various proxies for a time varying risk premium based on the conditional variance in the multivariate GARCH model estimated in <8>. A similar indirect approach is taken in the recent papers by McCurdy and Morgan (1988, 1989) in modeling the basis in foreign currency futures markets.

The TR^2 LM test statistics for the inclusion of the own conditional variance for ε_{it} , i.e., h_{it} , in explaining any deviation of the forward rate forecast error from the fixed MA(4) process are listed in the first row of Table 6.³ Note, according to the model in <8> the conditional variance of the future spot rate entering the risk premium in <9>, i.e., $\text{Var}_t(s_{it+k})$, is equal to $\text{Var}_t(s_{it+k} - f_{it})$, which will be a linear function of h_{it} . For a discussion of some of the issues involved in forecasting from GARCH models, see Engle and Bollerslev (1986). The results basically confirm the findings in Domowitz and Hakkio (1985), who found only weak support for a simple univariate ARCH in a mean model for the same four currencies, including Japan, but estimated on monthly data for the earlier part of the floating rate period. Although the ARCH effects are much more pronounced with weekly data, the own conditional variance remains an insignificant determinant for the forward rate forecast errors for three of the four currencies. A similar conclusion was reached for foreign exchange futures data in a series of papers by McCurdy and Morgan (1987, 1988, 1989).

However, as noted by Domowitz and Hakkio (1985), it is certainly possible that by generalizing their univariate ARCH models to a multivariate framework, the conditional covariances between the currencies may have more explanatory power

TABLE 6. Lagrange multiplier tests.

i	UK	WG	SW	FR
h_{it}	0.869 (0.351)	0.023 (0.879)	0.083 (0.773)	0.106 (0.303)
h_{ijt}	12.183 (0.016)	1.326 (0.857)	3.003 (0.557)	0.245 (0.621)
$j = \text{UK}, \dots, \text{FR}$				
$s_{it-1} - f_{it-5}$	0.776 (0.378)	1.848 (0.174)	0.005 (0.944)	0.028 (0.867)
$s_{jt-1} - f_{jt-5}$	2.948 (0.567)	5.438 (0.245)	1.179 (0.882)	4.523 (0.340)
$j = \text{UK}, \dots, \text{FR}$				
Δf_{it}	36.572 (0.000)	11.324 (0.001)	21.257 (0.000)	13.490 (0.000)
Δf_{jt}	74.765 (0.000)	51.453 (0.000)	87.517 (0.000)	48.561 (0.000)
$j = \text{UK}, \dots, \text{FR}$				

Asymptotic p -values in parentheses.

than the own conditional variances, thus acting as proxies for the other components for the time varying risk premia in $\langle 9 \rangle$ or some alternative asset pricing paradigm. The second row of tests reported in Table 6 investigates this conjecture. Again, the results are somewhat disappointing. The four test statistics do not lend much support to the idea that the risk premium is a simple linear function of the corresponding covariances. Only for the UK is there some evidence that the conditional covariances explain anything in addition to the own conditional variances. These results are in line with the recent findings in Engel and Rodrigues (1989), who estimate an International Capital Asset Pricing Model (ICAPM) as in Frankel (1982), but allow the conditional covariance matrix to change through time as in the domestic CAPM reported in Bollerslev *et al.* (1988). However, as the Engel and Rodrigues (1989) model is estimated on monthly data only relatively minor ARCH effects are present and no significant mean covariance tradeoff is discovered. On the other hand, the results in Lastrapes (1987), based on weekly data and an indirect three-step procedure suggests that the conditional covariances may help in explaining the forward bias for some currencies. For the weekly sample and the model estimated here, we do not find such a relationship to be particularly strong.

Many previous studies have tested for the presence of a time varying risk premium by including the lagged forward rate forecast error in a regression type framework; see, for instance, Hansen and Hodrick (1980, 1983), Gregory and McCurdy (1984), and Korajczyk (1985). LM tests for the significance of $s_{it-1} - f_{it-5}$ in the present context are reported in the third row of Table 6. None of these test statistics was significant at any reasonable level of significance. Including cross country values of the lagged premium as in the fourth row does not alter that conclusion. However, Table 6 also reports LM tests for the presence in the conditional mean of the change in the logarithm of the contemporaneous forward rate for each currency and the logarithm of the forward rate for other currencies. The use of the change in the forward rates is similar to the use of the forward premium as used by a number of other authors including Geweke and Feige (1979), Hansen and Hodrick (1983), Hsieh (1984), and Hodrick and Srivastava (1984) among others. Based on these tests we easily reject the hypothesis of no time varying risk premium for all the four countries.

Along those lines it is interesting to note that any tests for the presence of the squared forward rate changes in the conditional variances are likely to come out significant as well. Indeed the LM tests for the inclusion of $(\Delta f_{it})^2$ and also $(\Delta f_{jt})^2$, $j = \text{UK}, \dots, \text{FR}$ in each of the four conditional variances takes the values 10.566, 5.738, 8.884, and 13.943 respectively which is significant at the 0.05 per cent level in the asymptotic χ^2_4 distribution for the UK and France. See also Giovannini and Jorion (1987) and Hodrick (1989), who investigate the explanatory power of the squared interest rate differentials for time varying variances in the foreign exchange market. This highlights the importance of simultaneously arriving at a specification for both the time varying risk premium and the time varying conditional covariance matrix. The multivariate GARCH model developed here and the test statistics discussed above should prove helpful in that line of research.

IV. Conclusion

This paper has considered the specification of a model to represent the vector

process defined by the weekly forecast errors between future spot exchange rates and current forward rates. The process appears to be well described by a vector MA with coefficients implied by the martingale property of daily spot rates along with a multivariate GARCH model for the conditional covariance matrix. The simplifying assumption of constant conditional correlations seems empirically reasonable as a method for reducing the number of GARCH parameters to be estimated. Furthermore, the implied vector MA process is seen to account for a large part of the overall variation in the forward rate forecast errors.

Tests for the model using the elements of the conditional covariance matrix to proxy a time varying risk premium fails to find much of a significant relationship. However, the first difference of the forward rate comes out highly significant in tests for a time varying risk premium. Based on these results it appears that the violations of forward market efficiency found here, and by several other studies, are either due to inefficient processing of information by market participants, so that marked deviations from rationality occur, or alternatively that further theoretical models to explain time varying risk premia are required. To this extent we regard our analysis to be important initial evidence on the magnitude of the time varying risk premia and ARCH effects in forward foreign exchange markets that such theoretical models must be able to explain.

Notes

1. While this generally matches the forward rate with the spot rate in the future that would be used to cover an open forward position, the alignment could be one or two days off around the beginning of a new month; see Riehl and Rodriguez (1977).
2. Note, strictly speaking, the proper time subscripts would be s_{t-4i} , e_{t+4i-j} and e_{t-4i} , but here and in the rest of this paper we shall use the simpler notation in <7>.
3. Note, due to the difference in timing of s_{it} , f_{it} ; and e_{it} or rather e_{it+1} is unknown at time t . However, the corresponding conditional variance, h_{it} , is measurable with respect to ψ_t .

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