ARCH AND GARCH MODELS


Bibliography


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Many time series display time-varying dispersion or uncertainty, in the sense that large (small) absolute innovations tend to be followed by other large (small) absolute innovations. A natural way to model this phenomenon is to allow the variance to change through time in response to current developments of the system. Specifically, let \( y_t \) denote the observable univariate discrete-time stochastic process of interest. Denote the corresponding innovation process by \( \varepsilon_t \), where \( \varepsilon_t \equiv y_t - E_t(\varepsilon_t | y_{t-1}) \) and \( E_t(\cdot | \cdot) \) refers to the expectation conditional on the past \( (y_{t-1}, \ldots) \). A general specification for the innovation process that takes account of the time-varying uncertainty would then be given by

\[
\varepsilon_t = z_t \sigma_t, \tag{1}
\]

where \( z_t \) is an i.i.d. mean-zero, unit-variance stochastic process, and \( \sigma_t \) represents the time-\( t \) latent volatility, i.e., \( E_t(\varepsilon_t^2) = \sigma_t^2 \). Model specifications in which \( \sigma_t \) in (1) depends non-trivially on the past innovations and/or some other latent variables are referred to as stochastic volatility (SV) models. The historically first, and often most convenient, SV representations are the autoregressive conditionally heteroscedastic (ARCH) models pioneered by Engle [21]. Formally, the ARCH class of models is defined by (1), with the additional restriction that \( \sigma_t \) must be measurable with respect to the past \( (y_{t-1}, \ldots) \) observable information set. Thus, in the ARCH class of models \( \sigma_t \) is predetermined as of time \( t-1 \).

VOLATILITY CLUSTERING

The ARCH model was originally introduced for modeling inflationary uncertainty, but has subsequently found especially wide use in the analysis of financial time series. To illustrate, consider the plots in Figs. 1 and 2 for the daily deutsche-mark–U.S. dollar (DM/US) exchange rate and the Standard and Poor's 500 composite stock-market index (S&P 500) from October 1, 1979, through September 30, 1993. It is evident from panel (a) of the figures that both series display the long-run swings or trending behavior that are characteristic of unit-root*, or (1), nonstationary processes. On the other hand, the two returns series, \( r_t = \log(P_t/P_{t-1}) \), in panel (b) appear to be covariance-stationary. However, the tendency for large (and for small) absolute returns to cluster in time is clear.

Many other economic and financial time series exhibit analogous volatility clustering features. This observation, together with the fact that modern theories of price determination typically rely on some form of a risk-reward

\[\text{Figure 1 Daily deutsche-mark–U.S. dollar exchange rate. Panel (a) displays daily observations on the DM/US$ exchange rate, } r_t, \text{ over the sample period October 1, 1979 through September 30, 1993. Panel (b) graphs the associated daily percentage appreciation of the U.S. dollar, calculated as } r_t = 100 \log(P_t/P_{t-1}). \text{ Panel (c) depicts the conditional standard-deviation estimates of the daily percentage appreciation rate for the U.S. dollar implied by each of the three volatility model estimates reported in Table 1.}\]

\[\text{Figure 2 Daily S&P 500 stock-market index. Panel (a) displays daily observations on the value of the S&P 500 stock-market index, } P_t, \text{ over the sample period October 1, 1979 through September 30, 1993. Panel (b) graphs the associated daily percentage appreciation of the S&P 500 stock index including dividends, calculated as } r_t = 100 \log(P_t/P_{t-1}). \text{ Panel (c) depicts the conditional standard-deviation estimates of the daily percentage appreciation rate for the S&P 500 stock-market index implied by each of the three volatility-model estimates reported in Table 2.}\]
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trading relationship, underlying the very widespread applications of the ARCH class of time series models in economics and finance over the past decade. Simply treating the temporal dependencies in $\sigma_t^2$ as a nuisance would be inconsistent with the trust of the pertinent theories. Similarly, when evaluating economic and financial time series forecasts it is equally important that the temporal variation in the forecast error uncertainty be taken into account.

The next section details some of the most important developments along these lines. For notational convenience, we shall assume that the $\{c_t\}$ process is directly observable. However, all of the main ideas extend directly to the empirically more relevant situation in which $\{c_t\}$ describes the time-varying innovation of an autoregressive process, $y_t$, as defined above. We shall restrict discussion to the univariate case; cross-multiplicative generalizations follow by straightforward analogy.

**GARCH**

The definition of the ARCH class of models in (1) is extremely general, and does not lend itself to empirical investigation without additional assumptions on the functional form, or smoothness, of $a_t$. Arguably, the two most successful parametrizations have been the generalized ARCH, or GARCH($p,q$), model of Bollerslev [7] and the exponential GARCH, or GARCH($\infty$), model of Nelson [46]. In the GARCH($p,q$) model, the conditional variance is parameterized as a distributed lag of past squared innovations and past conditional variances,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where $\omega$ is the return variance, $\delta_t = \epsilon_t^2 - \delta_t^2$, and $\beta_j > 0$. Since $E(\delta_t | \mathcal{F}_{t-1}) = 0$, the GARCH($p,q$) formulation in (3) is readily interpreted as an ARMA(max($p,q$), $q$) model for the squared innovation process $\{\delta_t^2\}$; see Mloj [43] and Bollerslev [9]. Thus, if the roots of $1 - \alpha(1) - \beta(1) = 0$ lie outside the unit circle, then the GARCH($p,q$) process for $\{\epsilon_t\}$ is covariance-stationary, and the unconditional variance equals $\sigma^2 = \omega / (1 - \alpha - \beta)$. Furthermore, standard ARMA-based identification and inference procedures may be directly applied to the process in (3), although the heteroskedasticity in the innovations, $\{\epsilon_t\}$, renders such an approach inefficient.

In analogy to the improved forecast accuracy obtained in traditional time-series analysis by utilizing the conditional as opposed to the unconditional mean of the process, ARCH models allow for similar improvements when modeling second moments. To illustrate, consider the $\theta$-step-ahead ($\theta > 2$) minimum square error forecast for the conditional variance $\sigma^2$ in the simple GARCH(1,1) model.

$$E(\epsilon_t^2 | \mathcal{F}_{t-1}) = \alpha \sigma_t^2 + \beta E(\delta_t^2 | \mathcal{F}_{t-1})$$

If the process is covariance-stationary, i.e. $\alpha + \beta < 1$, it follows that $E(\epsilon_t^2 | \mathcal{F}_{t-1}) = \alpha \sigma_t^2 + \beta E(\delta_t^2 | \mathcal{F}_{t-1}) = \sigma_t^2$. Thus, if the current conditional variance is large (small) relative to the unconditional variance, the multistep forecast is also predicted to be above (below) $\sigma_t^2$, but converges to $\sigma_t^2$ at an exponential rate as the forecast horizon lengths. Higher-order covariance-stationary models display more complicated decay patterns [3].

**EGARCH**

While the GARCH($p,q$) model conveniently captures the volatility clustering phenomenon, it does not allow for asymmetric effects in the evolution of the volatility process. In the EGARCH($p,q$) model of Nelson [46], the logarithm of the conditional variance is given as an ARMA($p,q$) model in both the absolute size and the sign of the lagged innovation.

The assumption of covariance stationarity has been questioned by numerous studies which find that the largest root in the estimated log polynomial $1 - \alpha(1) - \beta(1) = 0$ is statistically indistinguishable from unity. Motivated by this stylized fact, Engle and Bollerslev [22] proposed the so-called integrated GARCH, or IGARCH($p,q$), process, in which the autoregressive polynomial in (3) has one unit root, i.e. $1 - \alpha(1) - \beta(1) = (1 - \beta(1))$, where $\beta(1) > 0$ for $|\beta| < 1$. However, the notion of a unit root is intrinsically a linear concept, and considerable care should be exercised in interpreting persistence in nonlinear models. For example, from (4), the IGARCH(1,1) model with $\alpha_1 + \beta_1 = 1$ behaves like a random walk, or an $\mathcal{I}(1)$ process, for forecasting purposes. Nonetheless, by repeated substitution, the GARCH(1,1) model may be written as

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \left( \alpha \epsilon_{t-i}^2 + \beta \sigma_{t-i}^2 \right)$$

in which $\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \epsilon_{t-i}^2$, $\epsilon_t$, and $\sigma_{t-i}^2$, the $\mathcal{I}(1)$ process, is given by

$$\epsilon_t = \theta \epsilon_{t-1} + \mu_t, \quad E(\mu_t | \mathcal{F}_{t-1} = \mathcal{I}(1))$$

along with the normalization $\theta_0 = 1$. By definition, the news impact function $\gamma(t)$ satisfies $E(\mu_t | \mathcal{F}_{t-1}) = 0$. When actually estimating $$E^GAR$$CH models the canonical stability of the optimization procedure is often enhanced by approximating $\gamma(t)$ by a smooth function that is differentiable at zero. Bollerslev et al. [2] also propose a richer parametrization for this function that outweighs the influence of large absolute innovations. Note that the IGARCH model still predicts that large (negative) innovations follow other large innovations, but if $\theta < 0$ the effect is accentuated for negative $\epsilon_t$.

As follows from Nelson [46], this stylized feature of equity returns is often referred to as the "leverage effect."

**ALTERNATIF PARAMETRIZATIONS**

In addition to GARCH, IGARCH, and EGBACH, numerous alternative univariate parameterizations have been suggested. An incomplete list includes: ARCH-in-mean, or ARCH-M [25], which allows the conditional variance to enter directly into the equation for the conditional mean of the process; nonlinear augmented ARCH, or NAARCH [27]; structural ARCH, or STARCH [53]; qualitative threshold ARCH, or QARCH [21]; asymmetric power ARCH, or AP-ARCH [19]; switching ARCH, or SWARCH [31]; periodic GARCH, or PGARCH [14]; and functionally integrated GARCH, or FIGARCH [4].

Additionally, several authors have proposed the inclusion of various asymmetric terms in the conditional variance equation to better capture the aforementioned leverage effect; see, e.g., refs. [17, 26, 30].

**TIME-VARYING PARAMETER AND BILINEAR MODELS**

There is a close relation between ARCH models and the widely-used time-varying parameter
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ter class of models. To illustrate, consider the simple ARCH(2) model in (2), i.e., $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2$. This model is observationally equivalent to the process defined by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{\infty} \alpha_i \epsilon_{t-i}^2,$$

where $\alpha_0, \alpha_1, \ldots, \alpha_i$ are i.i.d. random variables with mean zero and variances $\alpha_0, \alpha_1^2, \ldots, \alpha_i^2$, respectively; see Tsay [54] and Bera et al. [5] for further discussion. Similarly, the class of bilinear time series models discussed by Granger and Anderson [32] provides an alternative approach for modeling nonlinearity; see Weiss [56] and Granger and Teräsvirta [33] for a more formal comparison of ARCH and bilinear models. However, while time-varying parameters or nonlinear models may conveniently allow for heteroscedasticity and/or nonlinear dependencies through a set of nuisance parameters, in applications to economics and finance the temporal dependencies in $\sigma_t$ are often of primary interest. ARCH models have a distinct advantage in such situations by directly parametrizing this conditional variance.

ESTIMATION AND INFERENCE

ARCH models are most commonly estimated via maximum likelihood. Let the density for the i.i.d. process $\epsilon_t$ be denoted by $f(\epsilon_t; \theta)$, where $\theta$ represents a vector of nuisance parameters. Since $\sigma_t$ is measurable with respect to the time-$(t-1)$ observable information set, it follows that a standard prediction-error decomposition argument that, apart from initial conditions, the log of likelihood function for $\sigma_t = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_t\}$ equals

$$\log L(\sigma_t; \theta) = \sum_{t=1}^{T} \log f(\epsilon_t; \theta) + \frac{1}{2} \log \sigma_t^2,$$

(7)

where $\epsilon_t$ denotes the vector of unknown parameters parameterized for $\sigma_t$. Under conditional normality, $f(\epsilon_t; \theta) = (2\pi)^{-\frac{T}{2}} \exp \left\{-\frac{1}{2} \epsilon_t^T \sigma_t^{-1} \epsilon_t \right\}.$

Thus, even with conditionally normal innovations, the unconditional distribution for $\epsilon_t$ is leptokurtotic even for relatively light-tailed distributions $\sigma_t$. The $t$-distribution in Bollerslev [8] and the generalized error distribution (GED) in Nelson [46], while English and González-Rivera [22] suggest a nonparametric approach. However, if the conditional variance is correctly specified, the normal quasi-score vector based on (7) and (8) is a martingale difference sequence whose variance at the true parameters, $E_{t-1} \left( \sum_{s=1}^{T} \sigma_s^2 \right)$, denoted $\Psi$, generally remains consistent, and asymptotically valid inference may be conducted using an estimate of a robustified version of the asymptotic covariance matrix, $E_{t-1} \left( \sum_{s=1}^{T} \sigma_s^2 \right)$, where $E_{t-1}$ is the Hessessian and the outer product of the gradients respectively [55]. A convenient form of $\Psi$ with first derivatives only is provided in Bollerslev and Wooldridge [15].

Many of the standard mainframe and PC computer-based packages now contain ARCH estimation procedures. These include E-VIEW, RATS, SAS, TSP, and a special set of time series libraries for the GAUSS computer language.

TESTING

Conditional moment (CM) based misspecification tests are easily implemented in the ARCH context via simple auxiliary regressions [50, 53, 57, 58]. Specifically, following Wooldridge [58], the moment condition

$$E_{t-1} \left( \left[ \sigma_{t-1}^2 - \sigma_t^2 \right] \epsilon_t \right) = 0$$

(9)

( evaluated at the true parameter $\sigma_t$ ) provides insight into the substructure of the autoregressive polynomial by the vector $\lambda_t$ of misspecification indicators. By selecting these indicators as appropriate functions of the time-$(t-1)$ information set, the test may be designed to have asymptotically optimal power against a specific alternative; e.g., the conditional variance specification may be tested for goodness of fit over subsamples by letting $\lambda_t$ be the relevant indicator function, or for asymmetric effects by letting $\lambda_t = \epsilon_t^+/(\epsilon_t^+ - \epsilon_t^- < 0)$, where $f(\cdot)$ denotes the indicator function for $\epsilon_t^+ < 0$. Lagrange-multiplier-type tests that explicitly recognize the one-sided nature of the alternative when testing for the presence of ARCH have been developed by Lee and King [40].

EMPIRICAL EXAMPLE

As previously discussed, the two time-series plots for the DM$/$ES exchange rate and the S&P 500 stock market index in Figs. 1 and 2 both show a clear tendency for large (and for small) absolute returns to cluster in time. This is borne out by the highly significant Ljung-Box [41] portmanteau tests for up to 20th-order serial correlation in the squared residuals from the estimated ARCH(1) models, denoted by $Q_{20}$ in panel (a) of Tables 1 and 2. To accommodate this effect for the DM$/$ES returns, Panel (b) of Table 1 reports the estimates from an AR(1)-GARCH(1,1) model. The estimated ARCH coefficients are overwhelmingly significant, and, judged by the Ljung-Box test, this simple model captures the serial dependence in the squared returns remarkably well. Note also that $\alpha_1 + \beta_1$ is close to unity, indicative of IGARCH-type behavior. Although the estimates for the corresponding AR(1)-EGARCH(1,1) model in panel (c) show that the asymmetry coefficient $\theta$ is significant at the 5% level, the fit of the EGARCH model is comparable to that of the GARCH specification. This is also evident from the plot of the estimated volatility processes in panel (c) of Fig. 1.

The results of the symmetric AR(1)-GARCH(2,2) specification for the S&P 500 series reported in Table 2 again suggest a very high degree of volatility persistence. The largest eigenvalue of the autoregressive polynomial in (3) equals $\frac{3}{2} (\alpha_1 + \beta_1) + (\alpha_1 + \beta_1)^2 + 4(\alpha_2 + \beta_2) < 0.984$, which corresponds to a half-life of 43.6, or approximately two months, indicating the large differences between the conventional standard errors reported in parentheses and their robust counterparts in square brackets highlight the importance of the robust inference procedures with conditionally normal innovations. The two independent robust standard errors for $\alpha_2$ and $\beta_2$ suggest that a GARCH(1,1) specification may be sufficient, although previous studies covering longer time spans have argued for higher-order models [27, 52]. This is consistent with the results for the GARCH(2,1) model reported in panel (c), where both lags of $(\epsilon_t, \epsilon_{t-1})$ and ln $\sigma_t^2$ are highly significant. Factorizing the autoregressive polynomial for ln $\sigma_t^2$, the two inverse roots equal 0.989 and 0.824. Also, the GARCH model points to potentially important asymmetric effects in the volatility process. In summary, the GARCH and EGARCH volatility estimates depicted in panel (c) of Fig. 2 both do a good job of tracking and identifying the very high and low volatility in the U.S. equity market.

FUTURE DEVELOPMENTS

We have provided a very partial introduction to the vast ARCH literature. In many applications a multivariate extension is called for (see refs. [13, 18, 10, 11, 24, 48] for various par-}

mometrics multivariate specifications. Important issues related to the temporal aggregation of ARCH models are addressed by Dixit and Nijman [20]. Rather than directly parametrizing the functional form for $\lambda_t$, (in), Gallant and Tauchen [29], and Gallant et al. [26] have developed flexible parametric techniques for analysis of data with ARCH features. Much recent research has focused on the estimation of stochastic volatility models, to which the importance of the process for $\sigma_t$ treated as a latent variable has [1, 36, 38]. For a more detailed discussion of all of these ideas, see the many surveys listed in the Bibliography below.

A conceptually important issue concerns the rationale behind the widespread empirical findings of IGARCH-type behavior, as exemplified by the two time series analyzed in this paper. An alternative possibility is provided by the continuous record asymptotics developed in a series of papers by Nelson [45, 47] and Nelson and Foust [19]. Specifically, suppose that the discrete sampled observed process is generated by a continuous-time diffusion, so that the
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Table 1 Daily Deutsche-Mark–U.S. Dollar Exchange-Rate Appreciation

<table>
<thead>
<tr>
<th>AR(1):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_t = -0.002 - 0.033 \cdot r_{t-1} + \epsilon_t )</td>
</tr>
<tr>
<td>(0.013) (0.017)</td>
</tr>
<tr>
<td>( \sigma_t^2 = 0.585 )</td>
</tr>
<tr>
<td>(0.014)</td>
</tr>
<tr>
<td>Logl = -404.33, ( b_0 = -0.25, \ b_1 = 5.88, \ Q_0 = 19.60, \ Q_0^* = 231.17 )</td>
</tr>
<tr>
<td>(b)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AR(1)–GARCH(1,1):</th>
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<tbody>
<tr>
<td>( r_t = -0.001 - 0.035 \cdot r_{t-1} + \epsilon_t )</td>
</tr>
<tr>
<td>(0.012) (0.018)</td>
</tr>
<tr>
<td>( \sigma_t^2 = 0.019 + 0.303 \cdot \sigma_{t-1}^2 + 0.879 \cdot \epsilon_{t-1}^2 )</td>
</tr>
<tr>
<td>(0.004) (0.001) (0.021)</td>
</tr>
<tr>
<td>Logl = -387.8, ( b_0 = -0.0, \ b_1 = 4.67, \ Q_0 = 32.48, \ Q_0^* = 22.45 )</td>
</tr>
<tr>
<td>(c)</td>
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<table>
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<tr>
<th>AR(1)–EGARCH(1,0):</th>
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<tbody>
<tr>
<td>( r_t = 0.008 - 0.034 \cdot r_{t-1} + \epsilon_t )</td>
</tr>
<tr>
<td>(0.012) (0.018)</td>
</tr>
<tr>
<td>( \ln \sigma_t^2 = -0.447 + 0.030 \cdot \ln \sigma_{t-1}^2 + 0.260 \cdot (1 - (\sqrt{1/C^2})) + 0.006 \cdot \ln(\epsilon_{t-1}^2) )</td>
</tr>
<tr>
<td>(0.081) (0.009) (0.019) (0.004)</td>
</tr>
<tr>
<td>Logl = -387.8, ( b_0 = -0.36, \ b_1 = 4.54, \ Q_0 = 33.61, \ Q_0^* = 24.22 )</td>
</tr>
</tbody>
</table>

Notes: All the model estimates are obtained under the assumption of conditional normality (i.e., \( \epsilon_t \sim e_i \sigma_t^2 \)), i.i.d. with zero mean. Conventional asymptotic standard errors based on the inverse of Fisher’s information matrix are given in parentheses, while the numbers in square brackets represent the corresponding robust standard errors as described in the text. The maximized value of the pseudo-log-likelihood function is denoted Logl. The skewness and kurtosis of the standardized residuals, \( b = \frac{\epsilon_t}{\sqrt{\sigma_t^2} \sigma_t^2} \), are given by \( b_0 \) and \( b_1 \), respectively. \( Q_0 \) and \( Q_0^* \) refer to the Ljung–Box portmanteau test for up to 20th-order serial correlation in \( y_t \) and \( \epsilon_t \), respectively.

A sample path for the latent instantaneous volatility process \( \sigma_t^2 \) is continuous almost surely. Then one can show that any consistent ARCH filter must approach an IGARCH model in the limit as the sampling frequency increases. The empirical implications of these theoretical results should not be carried too far, however. For instance, while daily GARCH(1,1) estimates typically suggest \( b_0 + b_1 = 1 \), on estimating GARCH models for financial returns at intraday frequencies, Andersen and Bollerslev [2] document large and systematic deviations from the theoretical predictions of approximate IGARCH behavior. This breakdown of the most popular ARCH parametrizations at the very high intraday frequencies has a parallel at the lowest frequencies. Recent evidence suggests that the exponential decay of volatility shocks in covariance-stationary GARCH and EGARCH parametrizations results in too high a dissipation rate at low frequencies, whereas the infinite persistence implied by IGARCH-type formulations is too restrictive. The fractionally integrated GARCH, or FIGARCH, class of models [4] explicitly recognizes this by allowing for a low hyperbolic rate of decay in the conditional variance function. However, a reconciliation of the empirical findings at the very high and low sampling frequencies within a single consistent modeling framework remains an important challenge for future work in the ARCH area.

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FINANCIAL STATISTICS IN HETEROGENEOUS-DISTRIBUTION TIME SERIES, NONSTATIONARY

TORRAS G. ANDERSEN TIM BOLLERSLEV

ARES PLOTS

A procedure suggested by Cook and Weisberg [1] and originally called "an animated plot for adding regression variables smoothly" was designed to show the impact of adding a set of predictors to a linear model by providing an "animated" plot. The term ARES is an acronym for "adding regressors smoothly" [1]. The basic idea is to display a smooth transition between the fit of a smaller model and that of a larger one. In the case of the general linear model, we could start with the fit of the subset model

\[ Y - X_0 \beta_0 + e \]  

and then smoothly add \( X_1 \) according to some control parameter \( A \in [0, 1] \) with \( A = 0 \) corresponding to (1) and \( A = 1 \) corresponding to the full model:

\[ Y - X_1 \beta_1 + X_1 \beta_1^\prime + e \] 

The procedure continues in plotting

\[ \hat{f}(A), \hat{e}(A) \]

where \( \hat{f}(A) \) and \( \hat{e}(A) \) are the corresponding residuals obtained when the control parameter is equal to \( A \). A similar device of plotting \( \hat{e}(A) \) and \( \hat{f}(A) \) was suggested by Pregibon [4], who calls it a traceplot or A-space.

Cook and Weisberg [2] extend ARES for generalized linear models [see, e.g., McCullagh and Nelder [3]] and provide details on the available software in the LISP-STAT code.

References


(GRAPHICAL REPRESENTATIONS OF DATA REGRESSION DIAGNOSTICS)

ASESSMENT OF PROBABILITIES

NOMINATE AND DESCRIPTIVE VIEWS

Central to this entry is a person, conveniently referred to as 'you', who is contemplating a set of propositions \( A, B, C, \ldots \). You are uncertain about some of them; that is, you do not know, in your current state of knowledge \( K \), whether they are true or false. (An alternative form of language is often employed, in which \( A, B, C, \ldots \) are events and their occurrence is in doubt for you. The linkage between the two forms is provided by propositions of the form '\( A \) has occurred'.) You are convinced that the only logical way to treat your uncertainty, when your knowledge is \( K \), is to assign to each proposition, or combination of propositions, a probability \( P(A|K) \) that \( A \) is true, given \( K \). This probability measures the strength of your belief in the truth of \( A \). The task for you is that of assessing your probabilities; that is, of providing numerical values. That task is the subject of this article but, before discussing it, some side issues need to be clarified.

When it is said that you think that uncertainty is properly described by probability, you do not merely contemplate assigning numbers lying between 0 and 1, the convexity rule. Rather, you wish to assign numbers that obey all three rules of the probability calculus; convexity, addition, and multiplication (see foundations of probability). This is often expressed in saying that your beliefs must cohere (see coherence). It is therefore clear that, in order to perform the assessment, you must understand the rules of probability and their implications. You must be familiar with the calculus, in a sense, you have set yourself a standard, of coherence, and wish to adhere to that standard, or norm. This is often called the normative view of probability. A surveyor uses the normative theory of Euclidean geometry.

In contrast to the normative is the descriptive view, which aims to provide a description of how people in general attempt to deal with their uncertainties. In other words, it studies how people assess the truth of propositions when they do not have the deep acquaintance with the probability calculus demanded of the normative approach, nor feel the necessity of using that calculus as the correct, logical tool. There are several types of people involved. At one extreme are children making the acquisition of uncertainty for the first time. At the other extreme are sophisticated people who employ different rules than those of the probability calculus; for example, the rules of fuzzy logic. This entry is not concerned with the descriptive concept. The seminal work on that subject is Kahneman et al. [3]. A recent collection of essays is that of Wright and Apton [8].

Knowledge gained in descriptive studies may be of value in the normative view. For example, the former have exposed a phenomenon called anchoring, where a subject, having assessed a probability in one state of knowledge, may remain unduly anchored to that value when additional knowledge is acquired, and not change sufficiently. The coherent subject will update probabilities by Bayes' rule of the probability calculus (see Bayes' theorem). Nevertheless, an appreciation of the dangers of anchoring may help in the avoidance of pitfalls in the assessment of the numerical values to use in the rule. In view of the central role played by Bayes' rule, the coherent approach is sometimes called Bayesian, at least in some contexts.

Tests on the probability calculus do not include material on the assignment of numerical values, just as tests on geometry do not discuss mensuration or surveying. Assessment is an adjunct to the calculus, as surveying is to Euclidean geometry. Yet the calculus loses a lot of its value without the numbers.

SUBJECTIVE PROBABILITY

In the context adopted here of your contemplating uncertain propositions or events and adhering to the probability calculus, the form of probability employed is usually termed subjective or personal. Your appreciation of uncertainty may be different from mine. Subjective probabilities have been around as long as probability, but it is only in the second half of the twentieth century that they have attained the force of