Supplementary Appendix to:
Stock Return and Cash Flow Predictability:
The Role of Volatility Risk

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A Model Solution

Our basic solution method for the model is adopted from Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2007), and Drechsler and Yaron (2011). To begin, we follow Campbell and Shiller (1988) and solve for the return on consumption by log-linearizing \( r_{c,t+1} \) around the unconditional mean of the wealth-consumption ratio \( \nu_t \),

\[
r_{c,t+1} \approx \kappa_0 + \kappa_1 \nu_{t+1} - \nu_t + \Delta c_{t+1}, \tag{A.1}
\]

where \( \kappa_1 = \frac{\exp(E(\nu))}{1+\exp(E(\nu))} \), and \( \kappa_0 = \log[1+\exp(E(\nu))] - \kappa_1 E(\nu) \). We then conjecture a solution for \( \nu_t \) as a linear function of the state vector \( Y_t \),

\[

\nu_t = A_0 + A' Y_t, \tag{A.2}
\]

where \( A_0 \) is a scalar, and \( A = (0, A_x, A_r, A_q, 0) \) refer to the pricing coefficients. Next, by substituting \( \nu_t \) and \( \nu_{t+1} \) into equation (A.1), both \( r_{c,t+1} \) and the stochastic discount factor \( m_{t+1} \) defined in the main text may be expressed as linear functions of the state vector,

\[
m_{t+1} = \mu_m - (\gamma e_1' + (1-\theta) \kappa_1 A') Y_{t+1} - (\theta - 1) A' Y_t, \tag{A.3}
\]

\[
r_{c,t+1} = \mu_r + (e_1' + \kappa_1 A') Y_{t+1} - A' Y_t. \tag{A.4}
\]

Going one step further, it follows that the innovations to the pricing kernel and the return on the wealth claim may be expressed as,

\[
m_{t+1} - E_t(m_{t+1}) = -\Lambda' HG_t z_{t+1}, \tag{A.5}
\]

\[
r_{c,t+1} - E_t(r_{c,t+1}) = \Lambda_c' HG_t z_{t+1}, \tag{A.6}
\]

where \( \Lambda \) denotes the price of risk for the factor shocks,

\[

\Lambda = \gamma e_1 + \kappa_1 (1-\theta) A,
\]

for \( e_1 \equiv [1,0,0,0,0] \), and \( \Lambda_c = e_1 + \kappa_1 A \). The magnitude and sign of \( \Lambda \) are determined by the preference parameter \( \theta \) and the pricing coefficient vector \( A \). If investors prefer early resolution of uncertainty, i.e., \( \gamma > \phi^{-1} \), \( \Lambda \) reveals the sensitivity of the market prices for the different shocks to higher order consumption dynamics. When \( \gamma = \phi^{-1} \) (CRRA case), \( \Lambda \) collapses to \( \gamma e_1 \), and only transient shocks to consumption growth level \( z_{g,t+1} \) are priced.

Since the no-arbitrage condition must hold regardless of the realization of the state vector \( Y_t \), it is possible to solve for \( A \) by imposing the Euler equation,

\[

0 = \mu_m + \mu_r + [(-\Lambda + \Lambda_c)' F - \theta A'] Y_t + \frac{1}{2} (-\Lambda + \Lambda_c)' HG_t G_t' H' (-\Lambda + \Lambda_c). \tag{A.7}
\]
This in turn implies that

$$\theta A_{|i|} + (\tilde{\Lambda}_i F)_{|i|} = \frac{1}{2} (1_{|i|=3} \sum_{j=1,5} (\tilde{\Lambda}_j h_j)^2 + 1_{|i|=4} \sum_{j=2,3,4} (\tilde{\Lambda}_j h_j)^2),$$  

(A.8)

$$0 = \mu_m + \mu_e,$$  

(A.9)

where \(\tilde{\Lambda}_e = -\Lambda_e + \Lambda = (\gamma - 1)e_1 - \kappa_\theta A, \ i\) refers to the \(i^{th}\) element of vector, and \(1_{|i|=n}\) is an indicator function. The solutions are,

$$A_x = -\frac{\gamma - 1}{\theta (1 - \kappa_\rho x)},$$  

(A.10)

$$A_\sigma = \frac{(\gamma - 1)^2}{2\theta (1 - \kappa_1 \rho_\sigma)},$$  

(A.11)

while \(A_q\) solves the equation \(\frac{1}{2} a_q \theta^2 A^2 + (b_q + (1 - \kappa_1 \rho_q))(-\theta A_q) + \frac{1}{2} c_q = 0\), where

\[
\begin{align*}
a_q &= \kappa_1^2 (\varphi_x^2 s_{q,x} + s_{q,\sigma}^2 + \varphi_q^2) > 0, \\
b_q &= \kappa_1^2 \left(\varphi_x^2 (A_x \theta - A_\sigma \theta s_{q,x}) s_{q,x} - A_\sigma \theta s_{q,\sigma}\right), \\
c_q &= \kappa_1^2 \left(\varphi_x^2 (-A_x \theta - A_\sigma \theta s_{q,x}) s_{q,x} + A_\sigma \theta^2\right) > 0.
\end{align*}
\]

Since \(a_q > 0\) and \(c_q > 0\), the two roots are either negative or positive. We choose the larger root for \(-\theta A_q\) if \(b_q + (1 - \kappa_1 \rho_q) > 0\), or the smaller root if \(b_q + (1 - \kappa_1 \rho_q) < 0\). In both cases \(A_q\) reduces to zero when \(s_{q,x}, s_{q,\sigma}\) and \(\varphi_q\) are zero. Even though no closed-form expressions for \(A\) are available when we consider \(\kappa_0\) and \(\kappa_1\) as endogenous, the system of equations is still solvable. As shown in equation (A.8), \(A\) depends on \(\kappa_1, \mu, F, H\), as well as the preference parameters. Considering the definitions of \(\kappa_1\) and \(\kappa_0, \kappa_1\) and \(A\) are the only unknowns in the constant term in the Euler equation, so that \(\kappa_1\) may be solved endogenously together with \(A\). Finally, \(\kappa_0\) and \(A_0\) can be expressed as functions of \(A\) and \(\kappa_1\). For detailed numerical solutions, see Drechsler and Yaron (2011) Appendix A.1 and A.2.

Applying a similar conjecture-evaluation type method, it is possible to solve for the aggregate market return \(r_{t,t+1}\). Denote the price-dividend ratio by \(w_t\), and consider the conjecture solution \(w_t = A_{d,0} + A'_d Y\). Log-linearize \(r_{t,t+1}\) around the unconditional mean of the price-dividend ratio yields,

$$r_{t,t+1} \approx \kappa_{d,0} + \kappa_{d,1} w_{t+1} - w_t + \Delta d_{t+1}.$$

(A.12)

Substituting out \(w_t\) and \(w_{t+1}\) in the above equation, the return on the market may be rewritten as,

$$r_{t,t+1} = \mu_d + (e' + \kappa_{d,1} A'_d) Y_{t+1} - A'_d Y_t,$$  

(A.13)

where \(A_d = e + \kappa_{d,1} A_d\) and \(A_d = [0, A_{d,x}, A_{d,\sigma}, A_{d,q}, A_{d,d}]'\) is a vector of pricing coefficients.
Using the same solution method as the one previously used for \( A \), it follows by the no-arbitrage condition, 
\[
0 = \mu_m + \mu_\sigma + [(-\Lambda + \Lambda_d)^\prime F - (\theta - 1)A^\prime - A^\prime_\gamma]Y_t + 0.5(-\Lambda + \Lambda_d)^\prime HG_\gamma G_\gamma^\prime(-\Lambda + \Lambda_d),
\]
(A.14) which implies that
\[
(\theta - 1)A_{d,|i|} + A_{d,|i|} + (\tilde{\Lambda}_d^\gamma F)_{|i|} = 0.5[1_{j=3} \sum_{j=1,5} (\tilde{\Lambda}_d^\gamma h_j)^2 + 1_{i=4} \sum_{j=2,3,4} (\tilde{\Lambda}_d^\gamma h_j)^2],
\]
(A.15)
\[
0 = \mu_m + \mu_\sigma,
\]
(A.16)
where \( \tilde{\Lambda}_d = -\Lambda_d + \Lambda = \gamma e_1 - e_3 + \kappa_1(1 - \theta)A - \kappa_{d,1}A_d \).

The solution for \( A_d \) may therefore be expressed as,
\[
A_{d,d} = \frac{\rho_{d}}{1 - \kappa_{d,1}\rho_{d}}
\]
(A.17)
\[
\frac{1 - \kappa_1\rho_x}{1 - \kappa_{d,1}\rho_x}(1 - \theta)A_x - A_{d,x} = -\frac{-\gamma + \phi_{dx}(1 + \kappa_{d,1}A_{d,d})}{1 - \kappa_{d,1}\rho_x}
\]
(A.18)
\[
\frac{1 - \kappa_1\rho_{\sigma}}{1 - \kappa_{d,1}\rho_{\sigma}}(1 - \theta)A_{\sigma} - A_{d,\sigma} = -\frac{1}{2}\frac{\gamma^2 + \varphi_\sigma^2(1 + \kappa_{d,1}A_{d,d})^2}{1 - \kappa_{d,1}\rho_{\sigma}} < 0
\]
(A.19)
\[
\frac{1 - \kappa_1\rho_{q}}{1 - \kappa_{d,1}\rho_{q}}(1 - \theta)A_q - A_{d,q} = -\frac{1}{2}\frac{a_{d,q} (1 - \theta)A_q - \kappa_{d,1}A_{d,q})^2 + 2b_{d,q} (1 - \theta)A_q - \kappa_{d,1}A_{d,q}) + c_{d,d}}{1 - \kappa_{d,1}\rho_{q}}
\]
(A.20)
where
\[
a_{d,q} = a_q = \left(\varphi_\sigma^2 s_{q,x} + s_{q,\sigma}^2 + \varphi_q^2\right) > 0
\]
\[
b_{d,q} = \left(\varphi_\sigma^2 (1 - \theta)A_x - \kappa_{d,1}A_{d,x} + (1 - \theta)A_{\sigma} - \kappa_{d,1}A_{d,\sigma}) s_{q,x} + \frac{-1}{1 - \kappa_{d,1}\rho_{q}} s_{d,x}\right) s_{d,x}
\]
\[
\left. + \left(\kappa_1(1 - \theta)A_{\sigma} - \kappa_{d,1}A_{d,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_{d}} s_{d,\sigma}\right) s_{q,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_{d}} s_{d,q} \right)^2
\]
\[
c_{d,q} = \left(\varphi_\sigma^2 (1 - \theta)A_x - \kappa_{d,1}A_{d,x} + (1 - \theta)A_{\sigma} - \kappa_{d,1}A_{d,\sigma}) s_{q,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_{q}} s_{d,x}\right)^2
\]
\[
\left. + \left(\kappa_1(1 - \theta)A_{\sigma} - \kappa_{d,1}A_{d,\sigma} + \frac{-1}{1 - \kappa_{d,1}\rho_{d}} s_{d,\sigma}\right)^2 + \frac{-1}{1 - \kappa_{d,1}\rho_{q}} s_{d,q} \right)^2
\]
\[
\varphi_q^2 > 0
\]
In other words, \( A_{d,q} \) solves the equation (A.20) and we choose the root with smaller absolute value.

Both \( A_q \) and \( A_{d,q} \) reduce to zero when \( s_{q,x}, s_{q,\sigma} \) and \( \varphi_q \) are all zero. We will discuss the sign of \( A_{d,q} \) later on in the parameter implication section. We can explicitly express \( A_{d,x} \) and \( A_{d,\sigma} \) as
\[
A_{d,x} = \frac{(1 - \gamma)/\theta - 1 + \phi_{dx}/(1 - \kappa_{d,1}\rho_d)}{1 - \kappa_{d,1}\rho_x} = -\psi^{-1} + \phi_{dx}/(1 - \kappa_{d,1}\rho_d)
\]
(A.21)
\[
A_{d,\sigma} = \frac{(\gamma - 1)^2 + 2\theta\gamma + \theta(\varphi_\sigma^2(1 - \kappa_{d,1}\rho_d)^2 - 1)}{2\theta(1 - \kappa_{d,1}\rho_{\sigma})}.
\]
(A.22)
B Variance Risk Premium

In order to determine the factor structure for the variance risk premium, we first need to solve for
the second order moment of the return $r_{t,t+1}$. It follows from above that $r_{t,t+1} - E_t(r_{t,t+1}) = \Lambda_d^t H G_t z_{t+1}$, so that the conditional variance of the return is affine in $\sigma^2_t$ and $q_t$,

$$Var_t(r_{t,t+1}) = \sum_{j=1,5} \Lambda_d^t h_j h_j' \Lambda_d \sigma^2_t + \sum_{j=2,3,4} \Lambda_d^t h_j h_j' \Lambda_d q_t$$

$$= (1 + \kappa_d A_d, d)^2 \varphi_d^2 \sigma^2_t + \sum_{j=2,3,4} \Lambda_d^t h_j h_j' \Lambda_d q_t. \quad (B.23)$$

The first term is associated with the volatility of cash flow shocks, and the second term represents
the consumption uncertainty. Accordingly, the equity risk premium may be expressed as,

$$\log(E_t R_{t,t+1}) - r_{f,t} = E_t(m_{t+1}) + \frac{1}{2} Var_t(r_{t,t+1}) - r_{f,t}$$

$$= - Cov_t(m_{t+1}, r_{t+1}) = \sum_{j=2,3,4} \Lambda_d^t h_j h_j' \Lambda_d q_t \quad (B.24)$$

The first equality comes from the normality distribution of $r_{t,t+1} \equiv \log(R_{t,t+1})$, the second equality
comes from the no arbitrage condition and $r_{f,t} = \log(E_t m_{t+1} + \frac{1}{2} Var_t(m_{t+1}))$. The expectations of $Var_t(r_{t,t+1})$ under the physical and risk-neutral probability measures are,

$$E_t(Var_t(r_{d,t+2})) = \sum_{j=1,5} \Lambda_d^t h_j h_j' \Lambda_d (\mu_d + \rho_d \sigma^2_t) + \sum_{j=2,3,4} \Lambda_d^t h_j h_j' \Lambda_d (\mu_q + \rho_q q_t), \quad (B.25)$$

$$E_t^Q(Var_t(r_{d,t+2})) = \sum_{j=1,5} \Lambda_d^t h_j h_j' \Lambda_d (\mu_d + \rho_d \sigma^2_t + s_{q,1} q_t) + \sum_{j=2,3,4} \Lambda_d^t h_j h_j' \Lambda_d (\mu_q + \rho_q q_t + s_{q,2} q_t). \quad (B.26)$$

Under the risk neutral measure, we reweight probabilities according to the pricing kernel $\frac{e^{r_{t+1}}}{E_t e^{r_{t+1}}}$. If investor prefers early resolution of uncertainty, the shocks $z_{t+1}$’s conditional mean shifts away from zero. And this shift can be expressed as the conditional covariance between the state vector and SDF $m_{t,t+1}$,

$$s_{q,1} q_t = Cov_t(\epsilon_{3} H G_t z_{t+1}, -\Lambda^t H G_t z_{t+1})$$

$$= -(\varphi_3 s_{\epsilon_{q,1}} h'_2 + h'_3) \Lambda q_t, \quad (B.27)$$

$$s_{q,2} q_t = Cov_t(\epsilon_{3} H G_t z_{t+1}, -\Lambda^t H G_t z_{t+1})$$

$$= -(\varphi_3 s_{\epsilon_{q,1}} h'_2 + s_{q,2} h'_3 + \varphi_q h'_3) \Lambda q_t. \quad (B.28)$$
where,

\[ s_{q,1} = -\kappa_1(1 - \theta)(A_x\varphi_x^2s_{\sigma, x} + A_\sigma(\varphi_x^2s_{\sigma, x}^2 + 1) + A_q(\varphi_x^2s_{\sigma, x}s_{q, x} + s_{q, \sigma})) \]

\[ = -\kappa_1(1 - \theta)(\varphi_x^2s_{\sigma, x} + s_{q, \sigma}) \left( A_x\varphi_x^2s_{\sigma, x} + A_\sigma(\varphi_x^2s_{\sigma, x}^2 + 1) + A_q(\varphi_x^2s_{\sigma, x}s_{q, x} + s_{q, \sigma}) \right) \]

\[ s_{q,2} = -\kappa_1(1 - \theta)(A_x\varphi_x^2s_{q, x} + A_\sigma(\varphi_x^2s_{\sigma, x}s_{q, x} + s_{q, \sigma}) + A_q(\varphi_x^2s_{q, x}^2 + s_{q, \sigma}^2 + \varphi_q^2)) \]

\[ = -1/\kappa_1(1 - \theta)a_q \left( -b_q^{-1} + A_q \right) \]

By definition, \( s_{q,1} \) and \( s_{q,2} \) represent the market prices of shocks to \( \sigma_t^2 \) and \( q_t \), respectively. Thus, the variance risk premium is naturally defined by,

\[ VRP_t = E_r^Q(\text{Var}_{t+1}(r_{d, t+2})) - E_r(\text{Var}_{t+1}(r_{d, t+2})) = (\sum_{j=1,5}^n \lambda_j^d \lambda_j^d q_{t+1, d}) + (\sum_{j=2,3,4}^n \lambda_j^d \lambda_j^d q_{t+2, d})q_t. \] (B.29)

In the main text, we will refer to the expected return variation and the variance risk premium as,

\[ ERV_t = \frac{Q_{t,1}}{\rho_{q, \sigma}}(\mu_{\sigma} + \rho_{\sigma, q}\sigma_t^2) + \frac{Q_{t,2}}{\rho_q}(\mu_q + \rho_q q_t), \]

\[ VRP_t = Q_{2,2}q_t \]

for short, where

\[ Q_{1,1} = \sum_{j=1,5}^n \lambda_j^d \lambda_j^d q_{t+1, d} > 0, \] (B.30)

\[ Q_{1,2} = \sum_{j=2,3,4}^n \lambda_j^d \lambda_j^d q_{t+2, d}q_t > 0, \] (B.31)

\[ Q_{2,2} = \frac{Q_{1,1}}{\rho_{q, \sigma}} s_{q, 1} + \frac{Q_{1,2}}{\rho_q} s_{q, 2}. \] (B.32)

In order to determine the signs of \( A_{d, x} \), \( A_{d, \sigma} \) and \( A_{d, q} \), it is informative to write out the formula in terms of the estimated \( B \) and \( \hat{p} \) matrices,

\[ \frac{\phi_{d, x}}{-A_{d, x}} = \hat{p}_{3, d}, \quad \frac{A_{d, \sigma}}{Q_{1,1}} = B_{4, 1}, \quad \frac{Q_{1,2}}{Q_{2,2}} = -B_{1, 2}, \quad \frac{A_{d, q}}{Q_{2,2}} = B_{4, 2} - B_{1, 2}B_{4, 1}. \] (B.33)

Since \( \hat{p}_{3, d} < 0, \phi_{d, x} \) and \( A_{d, x} \) must have the same signs. Thus, by definition \( Q_{1,1} > 0 \) and \( Q_{1,2} > 0 \), which together with the estimates for \( B_{4, 1} = -0.60 < 0 \) and \( B_{1, 2} = -0.02 < 0 \), imply that \( A_{d, \sigma} < 0 \) and \( Q_{2,2} > 0 \). Consequently \( A_{d, q} = Q_{2,2}(B_{4, 2} - B_{1, 2}B_{4, 1}) = -1.45Q_{2,2} < 0. \)
C Alternative Setups

C.1 Separate Volatility Processes

We will consider the following alternative setup for $G_t$ and $H$, with $F$ unchanged,

$$G_t = \begin{pmatrix} \sigma_t & 0 & 0 & 0 & 0 \\ 0 & \sqrt{q_t} & 0 & 0 & 0 \\ 0 & 0 & \sigma_t & 0 & 0 \\ 0 & 0 & 0 & \sqrt{q_t} & 0 \\ 0 & 0 & 0 & 0 & \sigma_t \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & s_{\sigma,x} & \varphi_{\sigma} & 0 & 0 \\ 0 & s_{q,x} & 0 & \varphi_{q} & 0 \\ 0 & s_{d,x} & \varphi_{s\sigma} & \varphi_{q}\sigma_{d} & \varphi_{d} \end{pmatrix}$$ \hspace{1cm} (C.34)

This setup is related to Bansal and Shaliastovich (2013), where the volatilities of $x_t$ and $\sigma_t^2$ are modeled as two separate processes.

For simplicity, we use the same general notation as in the main setup for $A$, $\Lambda$, $\Lambda_c$ and $\Lambda_d$. However, the solutions for the pricing coefficients are obviously different from the main setup, except for $A_{d,d} = \frac{\rho d}{1 - \delta d}$.

$$\theta A_{[i]} + (\tilde{\Lambda}_t F)_{[i]} = 0.5[1 + \sum_{j=1,3} (\tilde{\Lambda}_t h_j)^2 + 1 + \sum_{j=2,4} (\tilde{\Lambda}_t h_j)^2]$$ \hspace{1cm} (C.35)

$$(\theta - 1) A_{[i]} + A_{d,[i]} + (\tilde{\Lambda}_d F)_{[i]} = 0.5[1 + \sum_{j=1,3} (\tilde{\Lambda}_d h_j)^2 + 1 + \sum_{j=2,4} (\tilde{\Lambda}_d h_j)^2]$$ \hspace{1cm} (C.36)

Since $r_{t+1} - E_t(r_{t+1}) = \Lambda'_d HG_t z_{t+1}$, the conditional variance of the return is again affine,

$$Var_t(r_{t+1}) = \sum_{j=1,3,5} \Lambda'_t h_j h_j' \Lambda_d \sigma_t^2 + \sum_{j=2,4} \Lambda'_t h_j h_j' \Lambda_d q_t$$ \hspace{1cm} (C.37)

The expectations of $Var_t(r_{t+1})$ under the physical and risk-neutral probability measures may further be expressed as,

$$E_t(Var_t(r_{d,t+2})) = \sum_{j=1,3,5} \Lambda'_t h_j h_j' \Lambda_d (\mu_r + \rho_r \sigma_t^2) + \sum_{j=2,4} \Lambda'_t h_j h_j' \Lambda_d (\mu_q + \rho_q q_t)$$ \hspace{1cm} (C.38)

$$E_t^0(Var_t(r_{d,t+2})) = \sum_{j=1,3,5} \Lambda'_t h_j h_j' \Lambda_d (\mu_r + \rho_r \sigma_t^2 + s_{r,1} q_t + s_{r,1} \sigma_t^2) + \sum_{j=2,4} \Lambda'_t h_j h_j' \Lambda_d (\mu_q + \rho_q q_t + s_{q,2} q_t)$$ \hspace{1cm} (C.39)

If investors prefer early resolution of uncertainty, the conditional means of the $z_{t+1}$ shocks shift away from zero under the risk-neutral measure,

$$s_{r,1} \sigma_t^2 + s_{q,1} q_t = Cov_t(e'_t HG_t z_{t+1}, -\Lambda' HG_t z_{t+1})$$

$$= -\varphi_{s} h'_3 \Lambda \sigma_t^2 - (\varphi_{s} x s_{\sigma} h'_2) \Lambda q_t,$$ \hspace{1cm} (C.40)

$$s_{q,2} q_t = Cov_t(e'_q HG_t z_{t+1}, -\Lambda' HG_t z_{t+1})$$

$$= - (\varphi_{r} s_{q} h'_2 + \varphi_{r} h'_4) \Lambda q_t.$$ \hspace{1cm} (C.41)
Defining the variance risk premium as before,

\[ VRP_t = E_t^Q(Var_{t+1}(r_{d,t+2})) - E_t(Var_{t+1}(r_{d,t+2})) \]

\[ = \sum_{j=1,3,5} \Lambda_d^j h_j^i h_j^j \Lambda_d(s_{r,1} \sigma_t^2 + s_{q,1} q_t) + \sum_{j=2,4} \Lambda_d^j h_j^i h_j^j \Lambda_d s_{q,2} q_t, \quad (C.42) \]

we may express the expected return variation and premium in short-hand form as,

\[ ERV_t = \frac{Q_{1,1}}{\rho_{\sigma}} (\mu_{\sigma} + \rho_{\sigma} \sigma_t^2) + \frac{Q_{1,2}}{\rho_q} (\mu_q + \rho_q q_t), \]

\[ VRP_t = Q_{2,1} \sigma_t^2 + Q_{2,2} q_t, \]

where

\[ Q_{1,1} = \sum_{j=1,3,5} \Lambda_d^j h_j^i h_j^j \Lambda_d \rho_{\sigma} > 0 \]

\[ Q_{1,2} = \sum_{j=2,4} \Lambda_d^j h_j^i h_j^j \Lambda_d \rho_q > 0 \]

\[ Q_{2,1} = \frac{Q_{1,1}}{\rho_{\sigma}} s_{r,1} + \frac{Q_{2,2}}{\rho_q} \]

\[ Q_{2,2} = \frac{Q_{1,2}}{\rho_q} s_{q,1} + \frac{Q_{2,2}}{\rho_q} s_{q,2}. \]

### C.2 Long-Run Stochastic Volatility

We will consider the following alternative setup for \( G_t, H, \) and \( F, \)

\[
F = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & \rho_{\sigma} & 0 & 0 & 0 \\
0 & 0 & \rho_{\sigma} & 1 & 0 \\
0 & 0 & 0 & \rho_q & 0 \\
0 & \phi_{d,\sigma} & 0 & 0 & \rho_d \\
\end{pmatrix}
G_t = \begin{pmatrix}
\sigma_t & 0 & 0 & 0 & 0 \\
0 & \sigma_t & 0 & 0 & 0 \\
0 & 0 & \sigma_t & 0 & 0 \\
0 & 0 & 0 & \sqrt{q_t} & 0 \\
0 & 0 & 0 & 0 & \sigma_t \\
\end{pmatrix}
H = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \varphi_{\sigma} & 0 & 0 & 0 \\
0 & \varphi_{\sigma} & \varphi_{\sigma} & 0 & 0 \\
0 & \varphi_{q,x} & \varphi_{q,z} & \varphi_{q} & 0 \\
0 & \varphi_{q} & \varphi_{q} & \varphi_{q} & \varphi_{q} \\
\end{pmatrix}
\]

This setup is motivated by the model analyzed by Branger and Völkert (2012), among others, allowing for a time-varying mean of the consumption variance \( \sigma_t^2. \)

Again, for simplicity we will use the same general notation as in the main setup for \( A, \Lambda, \Lambda_c \) and \( \Lambda_d. \) The solution for \( A_{d,d} = \frac{\rho_d}{1 - \delta_{d,1} \rho_d} \) remains the same, but the other the pricing coefficients now take the form,

\[ \theta A_{i|i} + (\tilde{\Lambda}_d^j F)_{i|i} = \frac{1}{2} \left[ \sum_{j=1,3,5} (\tilde{\Lambda}_d^j h_j)^2 + \sum_{j=4} (\tilde{\Lambda}_d^j h_j)^2 \right], \quad (C.44) \]

\[ (\theta - 1) A_{i|i} + A_{d,\sigma} + (\tilde{\Lambda}_d^j F)_{i|i} = \frac{1}{2} \left[ \sum_{j=1,3,5} (\tilde{\Lambda}_d^j h_j)^2 + \sum_{j=4} (\tilde{\Lambda}_d^j h_j)^2 \right]. \quad (C.45) \]

As before, \( r_{t,t+1} - E_t(r_{t,t+1}) = \Lambda_d^j h_j^i \Lambda_d z_{t+1}, \) so that the conditional variance of the return may be expressed as,

\[ Var_t(r_{t,t+1}) = \sum_{j=1,2,3,5} \Lambda_d^j h_j^i h_j^j \Lambda_d \sigma_t^2 + \sum_{j=4} \Lambda_d^j h_j^i h_j^j \Lambda_d q_t. \quad (C.46) \]
The expectation of \( \text{Var}(r_{t+1}) \) under the physical and risk-neutral probability measures are,
\[
E_t(\text{Var}_{t+1}(r_{d,t+2})) = \sum_{j=1,2,3,5} \Lambda_d^j h_j^i \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + q_t) + \sum_{j=4} \Lambda_d^j h_j^i \Lambda_d (\mu_q + \rho_q q_t),
\]
\[
E_t^Q(\text{Var}_{t+1}(r_{d,t+2})) = \sum_{j=1,2,3,5} \Lambda_d^j h_j^i \Lambda_d (\mu_\sigma + \rho_\sigma \sigma_t^2 + q_t + s_{\sigma,1} \sigma_t^2) + \sum_{j=4} \Lambda_d^j h_j^i \Lambda_d (\mu_q + \rho_q q_t + s_{\sigma,2} \sigma_t^2 + s_{q,2} q_t).
\]

The shifts in the conditional means of the \( z_{t+1} \) shocks under the risk-neutral measure become,
\[
s_{\sigma,1} \sigma_t^2 = \text{Cov}(e_t'H G_t z_{t+1}, -\Lambda^t H G_t z_{t+1}) = -\varphi_{\sigma} h_t' \Lambda \sigma_t^2 - (\varphi_{s,\sigma} h_t') \Lambda \sigma_t^2,
\]
\[
s_{\sigma,2} \sigma_t^2 + s_{q,2} q_t = \text{Cov}(e_t'H G_t z_{t+1}, -\Lambda^t H G_t z_{t+1}) = -(\varphi_{s,q} h_t') \Lambda \sigma_t^2 - (\varphi_{q} h_t') \Lambda q_t.
\]

As before, the expected return variation and variance risk premium, may be conveniently expressed as,
\[
ERV_t = \frac{Q_{1,1}}{\rho_\sigma} (\mu_\sigma + \rho_\sigma \sigma_t^2) + \frac{Q_{1,2}}{\rho_q} (\mu_q + \rho_q q_t),
\]
\[
VRP_t = Q_{2,1} \sigma_t^2 + Q_{2,2} q_t,
\]

where
\[
Q_{1,1} = \sum_{j=1,2,3,5} \Lambda_d^j h_j^i \Lambda_d \rho_\sigma > 0 \quad Q_{1,2} = \sum_{j=4} \Lambda_d^j h_j^i \Lambda_d \rho_q > 0
\]
\[
Q_{2,1} = \frac{Q_{1,1}}{\rho_\sigma} s_{\sigma,1} + \frac{Q_{1,2}}{\rho_q} s_{\sigma,2} \quad Q_{2,2} = \frac{Q_{1,2}}{\rho_q} s_{q,2}.
\]
D Detailed Derivations for Section 2.2

Substituting \( f_t \) by \( Q^{-1}(X_t - \mu_X) \) in the basic relation \( f_{t+1} = \mu + \rho f_t + S \epsilon_{t+1} \), it follows that

\[
Q^{-1}X_{t+1} = \mu + Q^{-1}\mu_X - \rho Q^{-1}\mu_X + \rho Q^{-1}X_t + S \epsilon_{t+1}.
\]  
(D.49)

Normalizing each element of \( Q^{-1}X_{t+1} \) by the corresponding diagonal element of \( Q^{-1} \), the model may be rewritten as,

\[
BX_{t+1} = \bar{\mu} + \bar{\rho}BX_t + \bar{S} \bar{\epsilon}_{t+1},
\]

where

\[
B \equiv \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot Q^{-1}.
\]

To match with equation (D.49),

\[
\bar{\mu} = \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot (\mu - \rho Q^{-1}\mu_X),
\]

and

\[
\bar{\rho} = \left[ \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot (\rho Q^{-1}) \right] B^{-1}
= \left[ \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot (Q^{-1} \text{diag}(\rho) + (\rho - \text{diag}(\rho))Q^{-1}) \right] B^{-1}
= \left[ \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot (Q^{-1} \odot (\text{vec}(\text{diag}(\rho)) \otimes I_{1 \times 4}) + (\rho - \text{diag}(\rho))Q^{-1}) \right] B^{-1}
= \left[ B \odot (\text{vec}(\text{diag}(\rho)) \otimes I_{1 \times 4} + \frac{\rho - \text{diag}(\rho)}{-A_{d,x}}) B \right] B^{-1}
= \rho + \frac{\rho - \text{diag}(\rho)}{-A_{d,x}}
\]  
(D.50)

or

Defining \( \bar{\epsilon}_{t+1} \) as

\[
\bar{\epsilon}_{t+1} \equiv \frac{1}{\text{diag}(Q^{-1})} \odot \epsilon_{t+1},
\]

it follows again from equation (D.49) that

\[
\bar{S} = \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right) \odot S \odot \left( \frac{1}{\text{diag}(Q^{-1})} \otimes I_{1 \times 4} \right).
\]

Based on the formula for \( Q \) in the main text, the inverse \( Q^{-1} \) and \( \frac{1}{\text{diag}(Q^{-1})} \) may be expressed as,

\[
Q^{-1} = \begin{pmatrix}
\frac{1}{Q_{1,1}} & -\frac{Q_{1,2}}{Q_{1,1}Q_{2,2}} & 0 & 0 \\
0 & \frac{1}{Q_{2,2}} & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{A_{d,x}}{Q_{1,1}A_{d,x}} & -\frac{Q_{1,1}A_{d,x} + Q_{1,2}A_{d,x}}{Q_{1,1}Q_{2,2}A_{d,x}} & \frac{A_{d,x}}{A_{d,x}} & -\frac{1}{A_{d,x}}
\end{pmatrix}
\]
\[
\frac{1}{\text{diag}(Q^{-1})} = \begin{pmatrix}
Q_{1,1} \\
Q_{2,2} \\
1 \\
-\frac{A_{d,x}}{A_{d,x}}
\end{pmatrix}
\]

9
Combining the expressions for $\rho$ and $S$, it therefore follows that

$$B = \begin{pmatrix} 1 & -\frac{Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{A_{d,}\varphi}{Q_{2,2} - Q_{1,2}} & \frac{A_{d,}\varphi}{Q_{1,2}} \frac{Q_{1,2}}{Q_{1,2} - Q_{2,2}} & 0 & 0 \\ \end{pmatrix} \quad \tilde{\rho} = \begin{pmatrix} \rho_{\varphi} & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\rho_d}{A_{d,}} \\ 0 & 0 & 0 & \rho_x \\ \end{pmatrix}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1}}{A_{d,}} S_{r,\varphi} \\ 0 & 1 & 0 & \frac{Q_{1,2}}{A_{d,}} S_{q,\varphi} \\ \frac{1}{Q_{2,2}} S_{d,\varphi} & \frac{1}{Q_{2,2}} S_{d,q} & 1 & \frac{1}{A_{d,}} S_{d,x} \\ 0 & 0 & 0 & 1 \\ \end{pmatrix} \quad \tilde{\epsilon}_{t+1} = \begin{pmatrix} Q_{1,1} & Q_{1,2} & 0 & 0 \\ Q_{2,1} & Q_{2,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -A_{d,\varphi} & -A_{d,q} & -A_{d,d} & -A_{d,x} \\ \end{pmatrix} \odot \epsilon_{t+1}.$$  

**D.1 Separate Volatility Dynamics**

In the alternative setup with separate volatility dynamic, $\rho$, $\epsilon_{t+1}$ and $S$ may be expressed as,

$$\rho = \begin{pmatrix} \rho_{\varphi} & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \phi_{dx} \\ 0 & 0 & 0 & \rho_x \\ \end{pmatrix} \quad \epsilon_{t+1} = \begin{pmatrix} \varphi_{\varphi} \sqrt{Q_{1,1}} z_{\varphi,t+1} \\ \varphi_q \sqrt{Q_{2,2}} z_{q,t+1} \\ \varphi_{dx} \sqrt{Q_{d,2}} z_{d,t+1} \\ \sqrt{Q_{x,2}} z_{x,t+1} \\ \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 & 0 & S_{r,\varphi} \\ 0 & 1 & 0 & S_{q,\varphi} \\ S_{d,\varphi} & S_{d,q} & 1 & S_{d,x} \\ 0 & 0 & 0 & 1 \\ \end{pmatrix}$$  

(D.51)

Consequently,

$$Q^{-1} = \begin{pmatrix} \frac{Q_{1,2}}{Q_{2,2} - Q_{1,2}} & -\frac{Q_{1,1}}{Q_{2,2} - Q_{1,2}} & 0 & 0 \\ \frac{Q_{1,2}}{Q_{2,2} - Q_{1,2}} & -\frac{Q_{1,1}}{Q_{2,2} - Q_{1,2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{Q_{2,1} A_{d,\varphi} - Q_{2,2} A_{d,q}}{(Q_{1,2} - Q_{1,2} - Q_{2,2}) A_{d,}} & -\frac{Q_{2,1} A_{d,\varphi} + Q_{1,2} A_{d,q}}{(Q_{1,2} - Q_{1,2} - Q_{2,2}) A_{d,}} & -\frac{A_{d,d}}{A_{d,}} & -\frac{1}{A_{d,}} \\ \end{pmatrix} \quad \frac{1}{\text{diag}(Q^{-1})} = \begin{pmatrix} Q_{1,1} & Q_{1,2} & Q_{1,2} & Q_{1,2} \\ Q_{2,1} & Q_{2,2} & Q_{2,2} & Q_{2,2} \\ 0 & 0 & 1 & 0 \\ -A_{d,\varphi} & -A_{d,q} & -A_{d,d} & -A_{d,x} \\ \end{pmatrix}$$

Combining these expressions, it follows that

$$B = \begin{pmatrix} 1 & -\frac{Q_{1,2}}{Q_{2,2}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-Q_{2,1} A_{d,\varphi} + Q_{1,2} A_{d,q}}{Q_{1,2} - Q_{1,2} - Q_{2,2}} & \frac{Q_{1,2} A_{d,\varphi} - Q_{1,2} A_{d,q}}{Q_{1,2} - Q_{1,2} - Q_{2,2}} & 0 & 0 \\ \end{pmatrix} \quad \tilde{\rho} = \begin{pmatrix} \rho_{\varphi} & 0 & 0 & 0 \\ 0 & \rho_q & 0 & 0 \\ 0 & 0 & \rho_d & \frac{\rho_d}{A_{d,}} \\ 0 & 0 & 0 & \rho_x \\ \end{pmatrix}$$

$$\tilde{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{Q_{1,1} Q_{2,2} - Q_{1,2} Q_{2,1}}{A_{d,}} S_{r,\varphi} \\ 0 & 1 & 0 & \frac{Q_{1,1} Q_{2,2} - Q_{1,2} Q_{2,1}}{A_{d,}} S_{q,\varphi} \\ 0 & 0 & 0 & \frac{1}{A_{d,}} S_{d,x} \\ \end{pmatrix} \quad \tilde{\epsilon}_{t+1} = \begin{pmatrix} Q_{1,1} & Q_{1,2} & Q_{1,2} & Q_{1,2} \\ Q_{2,1} & Q_{2,2} & Q_{2,2} & Q_{2,2} \\ 0 & 0 & 1 & 0 \\ -A_{d,\varphi} & -A_{d,q} & -A_{d,d} & -A_{d,x} \\ \end{pmatrix} \odot \epsilon_{t+1}.$$
D.2 Stochastic Volatility in the Long-Run

In the alternative setup with stochastic volatility in the long-run drift, $\rho$, $\epsilon_{t+1}$ and $S$ may be expressed as,

$$
\rho = \begin{pmatrix}
\rho_{\epsilon} & 1 & 0 \\
0 & \rho_q & 0 \\
0 & 0 & \rho_d
\end{pmatrix} \quad \epsilon_{t+1} = \begin{pmatrix}
\varphi_{\epsilon} \sqrt{\sigma_{\epsilon,t+1}} \\
\varphi_q \sqrt{\sigma_q,t+1} \\
\varphi_d \sqrt{\sigma_d,t+1}
\end{pmatrix} \quad S = \begin{pmatrix}
1 & 0 & 0 & s_{\epsilon,x} \\
0 & 1 & 0 & s_{q,x} \\
0 & s_{d,\epsilon} & s_{d,q} & s_{d,x}
\end{pmatrix}
$$

(D.52)

$$
X_t = \mu_X + Qf_t \quad Q = \begin{pmatrix}
Q_{1,1} & Q_{1,2} & 0 & 0 \\
Q_{2,1} & Q_{2,2} & 0 & 0 \\
0 & 0 & 1 & 0 \\
-A_{d,\epsilon} & -A_{d,q} & -A_{d,d} & -A_{d,x}
\end{pmatrix}
$$

Consequently,

$$
Q^{-1} = \begin{pmatrix}
\frac{Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{-Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & 0 & 0 \\
\frac{-Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & 0 & 0 \\
0 & 0 & \frac{-A_{d,d}}{A_{d,\epsilon}} & 1 \\
\frac{Q_{1,2}A_{d,\epsilon} - Q_{1,2}A_{d,d}}{(Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2})A_{d,\epsilon}} & \frac{-Q_{1,2}A_{d,\epsilon} + Q_{1,2}A_{d,d}}{(Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2})A_{d,\epsilon}} & \frac{-A_{d,d}}{A_{d,\epsilon}} & -\frac{1}{A_{d,\epsilon}}
\end{pmatrix}
$$

Combining these expressions, it follows that

$$
B = \begin{pmatrix}
1 & -\frac{Q_{1,2}}{Q_{1,1}} & 0 & 0 \\
\frac{-Q_{1,1}}{Q_{1,1}} & 1 & 0 & 0 \\
0 & 0 & \frac{-A_{d,d} + Q_{1,2}A_{d,\epsilon}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & 1 \\
\frac{-Q_{1,1}A_{d,\epsilon} + Q_{1,2}A_{d,d}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{Q_{1,1}A_{d,\epsilon} - Q_{1,2}A_{d,d}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{-A_{d,d}}{A_{d,\epsilon}} & 1
\end{pmatrix} \quad \tilde{\rho} = \begin{pmatrix}
\rho_{\epsilon} & \frac{Q_{1,1}}{Q_{1,2}} & 0 & 0 \\
0 & \rho_q & 0 & 0 \\
0 & 0 & \rho_d & \frac{\phi_{dx}}{-A_{d,\epsilon}} \\
0 & 0 & 0 & \rho_x
\end{pmatrix}
$$

$$
\tilde{S} = \begin{pmatrix}
\frac{Q_{2,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{2,1}} & \frac{Q_{1,1}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{2,1}} & \frac{1}{A_{d,\epsilon}} & \frac{s_{\epsilon,x}}{s_{\epsilon,x}} \\
\frac{1}{A_{d,\epsilon}} & \frac{1}{A_{d,\epsilon}} & \frac{s_{q,x}}{s_{q,x}} & \frac{1}{A_{d,\epsilon}} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \quad \tilde{\epsilon}_{t+1} = \begin{pmatrix}
\frac{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,1}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & 1 & 0 \\
\frac{Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & \frac{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}}{Q_{1,1}Q_{2,1} - Q_{1,2}Q_{1,2}} & 1 & 0 \\
0 & 0 & 1 & 0 \\
-A_{d,\epsilon} & -A_{d,\epsilon} & -A_{d,\epsilon} & -A_{d,\epsilon}
\end{pmatrix} \bigcirc \epsilon_{t+1}.
$$
Table D.1 Structural Factor GARCH Estimates—Separate Volatility Dynamics

The table reports the “structural” factor GARCH estimates for the alternative setup with separate volatility dynamics described in Sections C.1 and D.1, with the three restrictions: $A_{d}d_{t} = \phi_{d}d_{t} + \Gamma_{d}d_{t-1}$, $\Gamma_{d}d_{t} = \Gamma_{3}d_{t-3} = 0$. The resulting $J$-test with 7 degrees of freedom for the GMM-based estimation equals 26.31, corresponding to a p-value 0.0004.

<table>
<thead>
<tr>
<th>$B$</th>
<th>$ERV_{t+1}$</th>
<th>$VRP_{t+1}$</th>
<th>$\Delta d_{t+1}$</th>
<th>$d_{t+1}/p_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ERV_{t+1}$</td>
<td>1</td>
<td>-0.490 (0.117)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$VRP_{t+1}$</td>
<td>-0.022 (0.030)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$d_{t+1}/p_{t+1}$</td>
<td>-0.110(0.141)</td>
<td>-1.595 (0.063)</td>
<td>-0.158</td>
<td>1</td>
</tr>
</tbody>
</table>

Table D.2 Structural Model Implications—Separate Volatility Dynamics

The table reports the contemporaneous matrix $\Phi_{0}$, the reduced form matrix $\Phi$, and the return equation, implied by the alternative “structural” factor GARCH model defined in Sections C.1 and D.1.

<table>
<thead>
<tr>
<th>$\Phi_{0} \equiv B^{-1}S$</th>
<th>$ERV_{t+1}$</th>
<th>$VRP_{t+1}$</th>
<th>$\Delta d_{t+1}$</th>
<th>$d_{t+1}/p_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ERV_{t+1}$</td>
<td>1.011 (0.015)</td>
<td>0.496 (0.118)</td>
<td>0</td>
<td>0.198 (0.049)</td>
</tr>
<tr>
<td>$VRP_{t+1}$</td>
<td>0.022 (0.030)</td>
<td>1</td>
<td>0</td>
<td>-0.241 (0.017)</td>
</tr>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>-0.387 (0.080)</td>
<td>-0.134 (0.160)</td>
<td>1</td>
<td>0.095 (0.034)</td>
</tr>
<tr>
<td>$d_{t+1}/p_{t+1}$</td>
<td>0.085 (0.134)</td>
<td>1.646 (0.079)</td>
<td>0.158 (0.025)</td>
<td>0.652 (0.047)</td>
</tr>
</tbody>
</table>

$\Phi \equiv B^{-1} \Phi_{0}$

<table>
<thead>
<tr>
<th>$\Phi \equiv B^{-1} \Phi_{0}$</th>
<th>$ERV_{t+1}$</th>
<th>$VRP_{t+1}$</th>
<th>$\Delta d_{t+1}$</th>
<th>$d_{t+1}/p_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ERV_{t+1}$</td>
<td>0.013 (0.003)</td>
<td>0.833 (0.091)</td>
<td>-0.255 (0.075)</td>
<td>0</td>
</tr>
<tr>
<td>$VRP_{t+1}$</td>
<td>0.008 (0.001)</td>
<td>0.012 (0.016)</td>
<td>0.306 (0.070)</td>
<td>0.000 (0.000)</td>
</tr>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>-0.002 (0.016)</td>
<td>0.000 (0.000)</td>
<td>0.002 (0.006)</td>
<td>-0.187 (0.035)</td>
</tr>
<tr>
<td>$d_{t+1}/p_{t+1}$</td>
<td>-0.066 (0.030)</td>
<td>0.002 (0.045)</td>
<td>-1.103 (0.130)</td>
<td>-0.185 (0.035)</td>
</tr>
</tbody>
</table>

GMM Implied Return Equation

<table>
<thead>
<tr>
<th>$r_{t+1}$</th>
<th>$ERV_{t+1}$</th>
<th>$VRP_{t+1}$</th>
<th>$\Delta d_{t}$</th>
<th>$d_{t}/p_{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{t+1}$</td>
<td>0.062 (0.029)</td>
<td>-0.002 (0.044)</td>
<td>1.074 (0.127)</td>
<td>-0.007 (0.002)</td>
</tr>
</tbody>
</table>

Stru-shocks

<table>
<thead>
<tr>
<th>$\varepsilon_{x_{t+1}}$</th>
<th>$\varepsilon_{p_{t+1}}$</th>
<th>$\varepsilon_{d_{t+1}}$</th>
<th>$\varepsilon_{d_{t+1}}^{*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.470 (0.213)</td>
<td>-1.734 (0.178)</td>
<td>0.846 (0.041)</td>
<td>-0.539 (0.036)</td>
</tr>
</tbody>
</table>
Table D.3 Structural Factor GARCH Estimates—Long-Run Stochastic Volatility

The table reports the “structural” factor GARCH estimates for the alternative setup with long-run stochastic volatility described in Sections C.2 and D.2, with the two restrictions: \( \Delta d_{t,i,j} = \frac{\rho d_{t,i}}{1 - \rho d_{t,i,j}} \) and \( \Gamma_3 = 0 \). The resulting \( J \)-test with 6 degrees-of-freedom for the GMM-based estimation equals 37.02, corresponding to a p-value 0.0000.

<table>
<thead>
<tr>
<th>( B )</th>
<th>( ERV_{t+1} )</th>
<th>( VRP_{t+1} )</th>
<th>( \Delta d_{t,i} )</th>
<th>( d_{t+i}/p_{t+i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>1</td>
<td>0.000 (0.189)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>0.120 (0.039)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.016 (0.086)</td>
<td>-2.007 (0.156)</td>
<td>-0.249</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \hat{\rho} \] constant | \( ERV_t \) | \( VRP_t \) | \( \Delta d_t \) | \( d_t/p_t \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>0.003 (0.002)</td>
<td>1.000 (0.077)</td>
<td>-0.070 (0.255)</td>
<td>0</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>0.006 (0.001)</td>
<td>0.609 (0.079)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>-0.000 (0.014)</td>
<td>0</td>
<td>-0.329 (0.040)</td>
<td>0.001 (0.004)</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.075 (0.028)</td>
<td>0</td>
<td>0.982 (0.007)</td>
<td></td>
</tr>
</tbody>
</table>

\( \hat{\rho} \) constant | \( \epsilon_{ERV} \) | \( \epsilon_{VRP} \) | \( \epsilon_{\Delta d} \) | \( \epsilon_{d_t} \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>0.001 (0.000)</td>
<td>0.993 (0.089)</td>
<td>0.332 (0.036)</td>
<td></td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>0.000 (0.000)</td>
<td>0.322 (0.147)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>-0.524 (0.061)</td>
<td>0.454 (0.095)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.356 (0.052)</td>
<td>0.160 (0.041)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \hat{\sigma} \] constant | \( \hat{\tau} \) | \( \hat{\gamma} \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>0.001 (0.000)</td>
<td>0.756 (0.062)</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>0.000 (0.000)</td>
<td>0.454 (0.095)</td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>-0.524 (0.061)</td>
<td>0.454 (0.095)</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.356 (0.052)</td>
<td>0.766 (0.061)</td>
</tr>
</tbody>
</table>

Table D.4 Structural Model Implications—Long-Run Stochastic Volatility

The table reports the contemporaneous matrix \( \Phi_0 \), the reduced form matrix \( \Phi \), and the return equation, implied by the alternative “structural” factor GARCH model in Sections C.2 and D.2.

<table>
<thead>
<tr>
<th>( \Phi_0 )</th>
<th>( \Phi )</th>
<th>( \text{GMM Implied Return Equation} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>1.000 (0.023)</td>
<td>-0.000 (0.189)</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>-0.120 (0.037)</td>
<td>1</td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>-0.524 (0.061)</td>
<td>-0.069 (0.147)</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.356 (0.052)</td>
<td>1.990 (0.158)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \Phi )</th>
<th>( \text{GMM Implied Return Equation} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ERV_{t+1} )</td>
<td>0.003 (0.002)</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>0.005 (0.001)</td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>-0.000 (0.014)</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.065 (0.029)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( r_{t+1} )</th>
<th>( \text{stru-shocks} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{ERV} )</td>
<td>( \epsilon_{VRP} )</td>
</tr>
<tr>
<td>( ERV_{t+1} )</td>
<td>0.063 (0.027)</td>
</tr>
<tr>
<td>( VRP_{t+1} )</td>
<td>-0.178 (0.124)</td>
</tr>
<tr>
<td>( \Delta d_{t,i} )</td>
<td>0.758 (0.050)</td>
</tr>
<tr>
<td>( d_{t+i}/p_{t+i} )</td>
<td>-0.491 (0.035)</td>
</tr>
</tbody>
</table>
References


