When Should Sellers Use Auctions?

Preliminary and incomplete. Contains color figures.

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Abstract

Based on the advice of economists, firms and government agencies frequently use simultaneous sealed bid or open outcry auctions to sell goods or procure services. We compare the performance of simultaneous entry second price auctions with an alternative sequential entry and bidding mechanism in an environment with independent private values and potentially asymmetric buyers receive partially informative signals about their values prior to taking a costly entry decision. The signals result in a selective entry process where bidders with higher values are more likely to enter. In a setting with low entry costs and no selective entry, Bulow and Klemperer (2009) show that a seller’s expected revenues are almost always higher using auctions. Using standard equilibrium refinements to find a unique equilibrium of the sequential mechanism, we find that sellers should generally prefer the sequential mechanism and that the differences in expected revenues can be large. We illustrate our findings with parameters estimated from simultaneous entry, open outcry US Forest Service timber auctions in California. We predict that using the sequential mechanism would raise the USFS’s revenues by between 3% and 37% depending on the details of the sale, which is many times larger than the gain to setting an optimal reserve price within the standard auction.

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1 Introduction

Most economists believe that auctions have desirable efficiency and/or revenue maximization properties, and based on their advice, auctions have been used by many public authorities, private firms and universities to sell goods or procure services (Maskin (2004)). Beliefs in the superiority of auctions are well-founded in stylized settings: for example, in symmetric IPV environments with fixed numbers of bidders, a second price (Vickrey) auction efficiently allocates a good to the buyer with the highest value, while an optimal reserve price can maximize the revenue of the seller while maintaining efficiency if the good is sold. When bidder entry is endogenous, comparisons are more complicated but simple models still predict that auctions perform well. For example, Bulow and Klemperer (2009) (BK hereafter) compare the performance of a second-price auction and a sequential mechanism in an environment where symmetric potential buyers are uninformed about their values before deciding to enter. BK show that auctions generally give higher expected revenues, even if they are less efficient. In fact, the superiority of standard auctions has typically been challenged only in settings where additional concerns such as item complexity (Asker and Cantillon (2008)), budget constraints (Che and Gale (1996), bankruptcy (Zheng (2001)) or renegotiation (Bajari, McMillan, and Tadelis (2009)) are important.

This paper re-examines the efficiency and revenue performance of a standard second-price auction with a more general model of entry. We allow for entry to be imperfectly selective, in the sense that potential bidders with higher values may be more likely to enter, and we also consider the possibility that potential bidders are asymmetric. Otherwise, we maintain the assumptions of the standard auction model (single good, independent private values, no quality, credit or renegotiation concerns). We allow for a selective entry process using a two-stage entry model, where each firm receives a private information, noisy signal about its value before taking a costly entry decision, after which the firm learns its true value. A firm’s equilibrium entry strategy involves entering if its signal is high enough, so firms with higher values will be more likely to enter. This contrasts with a non-selective entry model (e.g., Levin and Smith (1994)’s model of simultaneous entry auctions and BK’s model) where firms only know the common distribution from which values are drawn before they decide to pay the entry cost, although, like in our model, entrants find out their values once the entry cost is paid.

We view selective entry as a plausible description of many auction settings (the sale of timber or oil and gas leases, government procurement contracts, firm takeovers) where all potential buyers are likely to have some idea of how they value the product, but must undertake research in order to learn their exact value. For example, in the case of timber auctions, potential buyers, such as sawmills or loggers, may place different values on different types of wood. These firms are likely to have some idea of the types of wood on a particular tract from their previous work in the forest, but bidders perform ‘cruises’ of tracts to establish the volumes of each species, as well as the likely

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1Selective entry contrasts with standard assumptions in the empirical entry literature (e.g., Berry (1992)) where entrants may differ from non-entering potential entrants in their fixed costs or entry costs, but not in characteristics such as marginal costs or product quality that affect their competitiveness or the profits of other firms once they enter.
costs of clearing the tract, before deciding how much to bid.

We use our selective entry model to compare the performance of simultaneous-entry second price auctions and BK’s sequential mechanism. In this mechanism, potential buyers are approached in turn. If a potential buyer enters, paying the same entry cost as in the auction, it learns its value. An incumbent potential buyer can name a price that it is willing to pay in order to try to deter further entry. If another firm enters then the two active firms bid against themselves in a knockout English auction which identifies the firm with the highest value, after which the firm with the lower value exits and the winner can, once again, submit a bid above the exit price to try to deter future entry. The incumbent at the end of the game pays the standing price. BK’s result in the model with no selective entry and no asymmetries between potential entrants, is that the seller’s expected revenues are almost always higher in the second price auction, although the sequential mechanism maximizes efficiency (the expected value of the winner less total entry costs).

Our main result is that for a wide range of plausible parameters, the sequential mechanism gives the seller higher expected revenues and the differences in expected revenues can be quite large. We show how the degree of selection, the level of entry costs and the degree to which bidders are asymmetric affect the relative performance of the two mechanisms. We illustrate the practical importance of our results using parameters estimated with data from simultaneous entry open outcry US Forest Service timber auctions. There we find large asymmetries between the value distribution of sawmills and loggers, and a moderately selective entry process. Based on a representative auction in our data, we estimate that the USFS’s expected revenues would be between 3% and 37% higher (depending on the details of the sale) using the sequential mechanism rather than the auction format that was actually used. These revenue difference are much larger than the gains to setting an optimal reserve prices in the auction. Of course, it may be that there are benefits to using simultaneous-entry auctions, such as transparency, which we do not consider. However, our results suggest that these benefits must be quite large to rationalize current practice.

There are at least two fundamental reasons why selective entry tends to lead to the sequential mechanism performing better. First, with no selection, entry ceases in the sequential mechanism once a firm enter that has a high enough value, which it can signal by posting a deterring bid, so that later entrants will not enter even if they have high values. In contrast, with selection, there is always some probability that a later firm with a value above the incumbent will enter. This is true because (i) a potential entrant may get a very favorable signal about its value; and (ii) in the equilibrium we consider, which is unique under standard refinements, (almost all) incumbents bid in a way which perfectly reveals their value to later competitors. These features make the sequential mechanism particularly efficient when there is selection, and the seller only has to capture some of this additional surplus to be better off. Second, firms may have to bid more aggressively to maintain the separating equilibrium. In contrast, when there is no selection, the unique sequential equilibrium of the model under standard refinements involves all incumbents with values above some threshold submitting the same deterring bid (pooling), and this deterring bid may be relatively low.

Two comments about the nature of our results are appropriate. First, we do not try to find the
seller’s optimal mechanism. Instead, in the same spirit as BK, we want to compare mechanisms that sellers may plausibly use (BK provide examples of sales formats that are similar to the sequential mechanism). However, results from the optimal mechanism design literature for the extreme cases where buyers either receive no signals before entering or know their values for sure before paying the entry cost, suggest that the optimal mechanism would be sequential, even if it would also have some less plausible features such as additional entry fees and bid-dependent payments to all firms that decide to enter. These features seem unlikely to be implemented in practice and setting them appropriately would also require the seller to have accurate knowledge of the parameters of the model. In contrast, the sequential mechanism considered here imposes the same informational demands on the seller as running the auction, except that an exhaustive set of potential buyers must be identified. One can therefore view our empirical results as providing a lower bound on how much better the USFS, or similar sellers, could do by using well-designed sequential mechanisms.

Second, while we characterize the equilibria we consider for each mechanism and show that they are unique under refinements, our revenue comparisons are computational rather than analytical. This reflects the fact that we are relaxing two assumptions (no selection and symmetry) that make algebraic analysis intractable. We view one contribution of our paper as showing that simplifications that have been made for tractability may give misleading impressions about how well auctions actually work.

The paper proceeds as follows. Section 2 introduces the models of each mechanism and characterizes the equilibria that we examine. Section 3 compares expected revenues from the two mechanisms for wide ranges of parameters, and provides some intuitions for when the sequential mechanism performs better than the auction. Section 4 describes the empirical setting of USFS timber auctions and explains how we estimate our model. Section 5 presents the parameter estimates and some initial comparisons of the revenue performance of each mechanism. Section 6 concludes.

2 Model

We now describe our model of firms’ values and their signals, before describing the mechanisms that we are going to compare and their associated equilibria.

2.1 A General Entry Model with Selection

Suppose that a seller has one unit of a good to sell, and that the seller gets a payoff of zero if the good is unsold. There is a set of potential buyers who may be one of \( \tau = 1, \ldots, \bar{\tau} \) types, with \( N_\tau \) of type \( \tau \). Buyers have independent private values (IPV) which can lie on \([0, V]\), distributed according

\[2\] Cremer, Spiegel, and Zheng (2009) characterize the mechanism design problem when uninformed potential buyers pay the entry cost. In that case, the sequential mechanism implements a sequential search rule where buyers are engaged in turn, with entry fees being charged and various reimbursements being made to some bidders. McAfee and McMillan (1988) characterize the design problem when potential buyers know their values but there is a cost to engaging each additional potential buyer. The optimal mechanism once again implements a sequential search amongst buyers, which ceases once a buyer with a high enough value is found. [NOTE: in future versions we will calculate how close our model comes to the optimal mechanism for these special cases].
to $F^V_{\tau}(V)$. $F^V_{\tau}$ is continuous and differentiable for all types. In this paper we will typically assume that the density of $V$ is proportional to the log-normal distribution on $[0, \overline{V}]$, and that $\overline{V}$ is high, so that the density of values at $\overline{V}$ is very small.\footnote{To be precise, $f^V_{\tau}(v|\theta) = \frac{g(v|\theta)}{\int g(x|\theta)dx}$ where $g(v|\theta)$ is the pdf of the log-normal distribution.} Before participating in any mechanism, a potential buyer must pay an entry cost $K_{\tau}$ (and we assume that a firm cannot choose not to pay $K$). Once it pays $K_{\tau}$, a potential buyer learns its value. However, prior to deciding whether to enter, a bidder receives a private information signal about his value. We focus on the case where the signal of potential buyer $i$ of type $\tau$ is determined by

$$S_{i\tau} = V_{i\tau} A_{i\tau} \text{ where } A = e^{\varepsilon_{i\tau}}, \varepsilon_{i\tau} \sim N(0, \sigma^2_{\varepsilon_{i\tau}})$$

In this model the variance of the $\varepsilon$s controls how much potential buyers know about their values before deciding whether to enter. As $\sigma^2_{\varepsilon_{i\tau}} \to \infty$, the model will tend towards the informational assumptions of the Levin and Smith (1994) model (signals are uninformative, LS model), while as $\sigma^2_{\varepsilon_{i\tau}} \to 0$ it tends towards the informational assumptions of the Samuelson (1985) (S) model where firms know their values prior to paying an entry cost (which is therefore interpreted as a bid preparation or attendance cost). As buyers with higher values are more likely to be allocated the good in both of the mechanisms that we consider, entry will be less selective as $\sigma^2_{\varepsilon}$ increases. In general, intermediate values of $\sigma^2_{\varepsilon_{i\tau}}$, implying that buyers have some idea of their values but have to conduct costly research to find them out for sure, seem plausible for most empirical settings. Draws of $\varepsilon$ are iid across bidders, and, having received his signal, a potential buyer forms posterior beliefs about his valuation using Bayes Rule.

We note that there are three differences between this model and the model considered by BK. First, we assume that potential entrants get some type of signal about their value prior to entering whereas BK assume that potential buyers only know the common distribution of values (the Levin and Smith assumption). Second, we allow for asymmetries between buyer types. Both of these changes require us to use computational techniques to compare revenues across mechanisms. Third, we assume that there is a fixed and known set of potential entrants which could, of course, differ across auctions. BK’s model is more general allowing for some probability ($0 \leq \rho_j \leq 1$) of a $j^{th}$ potential entrant if there are $j-1$ potential entrants.

### 2.2 Mechanism 1: Simultaneous Entry Second Price Auction

The first mechanism we consider is a simultaneous entry second price or open outcry auction. The auctions that we observe in the data will have an open outcry format. We note that this is slightly different from the main auction model which BK study, which has sequential entry (but firms place bids simultaneously). However, as BK argue, a simultaneous entry auction model would give similar results, and in general the simultaneous entry model seems a more reasonable description of most auctions in the real world.\footnote{“No important result is affected if potential bidders make simultaneous, instead of sequential, entry decisions into the auction” (page 1560).} In this mechanism all potential buyers first simultaneously decide
whether to enter (pay $K_\tau$) the auction based on their signal, which is private information, the number of potential entrants of each type and the auction reserve price, which are assumed to be common knowledge to all potential buyers. Entrants then learn their values and submit bids. We assume that an open outcry auction would give the same outcome as an English button auction, so that the good would be awarded to the firm with the highest value at the price which would make the firm with the second-highest value drop out of the auction or the reserve price if the seller uses a reserve.\footnote{When we introduce our estimation strategy in Section 4, we extend our methodology to cover more general models of bidding in open outcry auctions which does not require all bidders to bid up to their true value. We are also currently working on extending the analysis to first price auctions.}

Strategies will consist of a rule for entering the auction, as a function of the potential buyer’s own signal, and a rule for bidding as a function of his value and (potentially) his signal. In the bidding stage of the game, it will be a dominant strategy equilibrium for bidders to submit their values (second price auction) or bid up to their values (open outcry auction).

An optimal entry strategy will involve entering if and only if the signal lies above some threshold, as shown in the following simple proposition. This threshold signal is implicitly defined by the zero profit condition where the bidder receiving it is indifferent between staying out and paying the entry cost and entering the auction.

**Proposition 1.** The optimal bidding strategy for an entrant is to bid its value. The optimal entry strategy is given by a signal threshold $s'_\tau$ for any type $\tau$ bidder such that it enters if and only if its signal $s > s'_\tau$.

**Proof.** Let $\mathcal{M}_{-i}$ be the set of entrants other than bidder $i$. Entrants submit a bid to maximize their expected profit conditional on entry:

$$
(v_i - E[\max\{v_{-i}, r\}] : v_j \leq b, \forall j \in \mathcal{M}_{-i}) \prod_{\mathcal{M}_{-i}} F(b|\tau)
$$

Standard arguments show that participants have the dominant strategy to bid their value regardless of the number of potential bidders or asymmetries among them. This strategy is equivalent in an ascending auction. Profits are increasing in a bidder’s value. Because signals and values are affiliated, a higher signal leads the potential entrant to raise his beliefs about his value and because signals are independent across bidders, it does not alter his beliefs about other bidders’ post entry competitiveness. Thus, for any signal at which the bidder enters, he would enter for any higher signal and for any signal at which he doesn’t he wouldn’t for any lower signal. Therefore an equilibrium entry rule follows the threshold rule. An equilibrium exists because any bidder’s reaction function is continuous in his and his opponents’ thresholds. \textit{Q.E.D.}

If $\tau = 1$ (one type), then there is a unique symmetric equilibrium, which is also true in the LS and S models, and as is common in the literature this is the equilibrium that we focus on.\footnote{There can also be asymmetric equilibria where ex-ante identical firms use different entry thresholds. However as the number of players grows it is natural for these equilibria to disappear, as the degree of competition facing each player becomes increasingly symmetric.} This is
established in the following proposition.

**Proposition 2.** With one type of potential entrant there will be a unique symmetric equilibrium.

*Proof.* Suppose that there are two symmetric equilibria, where potential entrants have cutoffs $s^*_1$ and $s^*_2$ respectively and $s^*_1 > s^*_2$. Consider a potential entrant $i$. For any $u_i$, $i$’s expected profits from entering will be increasing in $s^*_{-i}$, the cut-off used by all other players as for any set of $v_{-i}$s an increase in $s_{-i}$ reduces the probability that rivals will enter. From this it follows that $i$’s best response cutoff to $s^*_{-i}$ is decreasing in $s^*_{-i}$. If so, if $s^*_1$ is $i$’s best response to $s^*_1$, it cannot also be the case that $s^*_2$ is $i$’s best response to $s^*_2$, so that $s^*_1$ and $s^*_2$ cannot both be symmetric equilibrium thresholds.

With multiple types, there can be multiple equilibria in the entry game even when we assume that only “type-symmetric” equilibria (i.e., ones in which all firms of a particular type using the same strategy) are played. As explained in Roberts and Sweeting (2010), we choose to focus on an equilibrium where the type with higher mean values has a lower entry threshold (lower thresholds make entry more likely). This type of equilibrium is intuitively appealing and when firms’ reaction functions are S-shaped (reflecting a normal or log-normal value distribution) and types only differ in the location parameters of their value distributions (i.e., the scale parameter, signal noise variance and entry costs are the same) then there is a unique equilibrium of this form.\(^7\)

Some features of equilibrium outcomes in this model should be noted. First, when there are more potential entrants the entry thresholds tend to rise, reducing the probability of entry of each firm, although the expected number of entrants tends to rise as well. When signals are very imprecise, an increase in the number of potential entrants can actually reduce expected revenues, which is a well-known feature of the simultaneous entry model with no signals and symmetric potential entrants (Levin and Smith (1994)), once the number of potential entrants is large enough that firms mix over entering in the symmetric equilibrium. This is illustrated in Figure 1, which shows expected revenues in the auction when firms are symmetric, and have values distributed with density proportional to $LN(4.5, 0.2)$ on $[0, 200]$. Increasing the number of potential entrants from 4 to 8 only increases revenues when $\sigma_e$ is less than 2. Figure 1 also illustrates how setting an optimal reserve in the auction is typically an ineffective way of raising revenues when entry is endogenous. This reflects the fact that while a reserve can help to extract revenues from a given set of entrant bidders, it tends to decrease entry.\(^8\)

### 2.3 Mechanism 2: Sequential Mechanism of Bulow and Klemperer (2009)

The alternative mechanism we consider is BK’s sequential mechanism, which they suggest reflects an alternative to standard auctions that approximately describes what happens in practice in some

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\(^7\)As we mention in Section 4, we have estimated the model using a nested pseudo-likelihood procedure which does not require us to use an equilibrium selection rule. The parameter estimates in this case indicate that the difference in mean values between our two types (sawmills and logging companies) are so large that multiple equilibria cannot be supported.

\(^8\)Levin and Smith (1994) show that the optimal reserve is zero when there is no selection. The optimal reserve is not zero when there are signals, but even when signals are informative, the effects of an optimal reserve are small.
settings, such as the sale of a company.

The mechanism operates in the following way. Potential buyers are placed in some order (which does not depend on their signals, but may depend on types), and the seller approaches each potential buyer in turn. We will call what happens between the seller’s approach to one potential buyer and its approach to the next potential buyer a “round”. In the first round, the first potential buyer observes his signal and then decides whether to enter the mechanism and learn his value by paying $K_T$. If he enters he can choose to place a ‘jump bid’ $b_1$ above the reserve price, which we assume to be zero. Given entry, submitting a bid is costless.

In the second round the potential buyer observes his signal, the entry decision of the first buyer and his jump bid, and then decides whether to enter himself. If the first firm did not enter and the second firm does, then the second firm can place a bid in exactly the same way as the first firm would have been able to do had he entered. If both enter, the firms bid against each other in a knockout button auction with tiny increments until one firm drops out, in which case the drop-out firm can never return to the mechanism. The remaining firm then has an opportunity to submit an additional higher jump bid above the bid at which the other firm dropped out. If the second

Figure 1: Expected Revenues in a Simultaneous Entry Auction as a function of the Number of Entrants, Use of an Optimal Reserve and Signal Noise ($\sigma_\varepsilon$). Firms are symmetric with values distributed $LN(4.5, 0.2)$ on $[0, 200]$. Note that the values of $\sigma_\varepsilon$ are not evenly spaced, but instead provide more detail about models approximating no selection and perfect selection.
firm does not enter but the first firm did, then the first firm can either keep its initial bid or submit
a higher jump bid.

This procedure is then repeated for each remaining potential buyer, so that in each round there
is at most one incumbent bidder coming from the previous round and one potential entrant. The
complete history of the game (entry decisions and bids, but not signals) is observed by all players.
The history up to round \( n \) will be denoted \( \Gamma_n \). If a firm drops out or chooses not enter it is
assumed to be unable to re-enter at a later date. At the end of the game the good is allocated to
the remaining bidder at a price equal to the current bid.

A strategy in the sequential model will consist of an entry rule and a bidding rule as a function
of the round, the potential buyer’s signal and value (for bidding) and the observed history. When
a potential buyer is bidding against an active opponent the dominant strategy is to bid up to its
value, so that the firm with the lower value will drop out at a price equal to its value. This does
not depend on the fact that we have a selective entry model because values are known at that stage.
However, the strategies that firms use to determine their jump bids and entry decisions do depend
on selective entry. To place our equilibrium in context, we begin describing what happens when
there are no signals and symmetric firms, which are the assumptions made by BK.

2.3.1 Equilibrium with No Pre-Entry Signals

BK show that there is a unique perfect sequential equilibrium in entry and jump bidding strategies
where in all rounds before the last round an incumbent will, depending on its value, either submit
no jump bid (i.e., keep its bid equal to its level from the previous round or the level at which it
defeated the other firm in the current round) or will submit a jump bid which, in equilibrium, deters
all future entry. Assuming only one type of firm, all firms with values below some \( V^S \) will keep
the existing standing bid \( b' \), while all firms with values above \( V^S \) will submit a jump bid equal to
\( b'' \). \( V^S \) will be determined by the condition that future potential entrants should be indifferent to
entering when the incumbent firm’s value is above \( V^S \)

\[
\int_{V^S}^{V} \int_{V^S}^{V} (x - v') f^V(v') f^V(x) dv' dx - K = 0
\]

where this condition recognizes the fact that an entrant will only win if its value is above the
incumbent and that in this case it will defeat the incumbent at a price equal to its value and then
deter entry in all future periods. In the equilibrium BK consider \( V^S \) will be the same independent
of the round the game is in, or the history of the game to that date. The deterring bid \( b'' \) is
determined by the condition that the bidder with value \( V^S \) should be indifferent between deterring
future entry with a bid of \( b'' \) (which it will pay because it will win) and allowing entry to occur by
keeping the standing bid \( b' \) (it may not win, but if it does it will pay a lower price than \( b'' \)). \( b'' \) may
depend on \( b' \) (which will depend on the history of the game) and the number of rounds remaining.

Equilibrium outcomes with no signals are therefore characterized by entry ceasing completely as
soon as there is an entrant who has a high enough value. As BK show, this leads to a more socially
efficient outcome than an auction, because entry only occurs when the probability that an entrant will raise the total surplus is relatively large. However, because deterring bids may be relatively low and more entry would increase prices, this equilibrium generally produces lower revenues for sellers than the auction.

2.3.2 Equilibrium with Pre-Entry Signals

When potential buyers receive pre-entry signals, there are important changes to the nature of the equilibrium even if signals are not very precise. We begin by describing the equilibrium we consider, before explaining the refinements which lead us to focus on it. We also provide examples showing how equilibrium strategies affect outcomes and how the parameters affect equilibrium strategies.

The equilibrium we consider can be characterized as follows:

- a potential entrant enters if its signal is above some threshold $s^*$ which will depend on the round and its beliefs about the value of the incumbent if there is one. For example, the final potential entrant (round $N$) will enter if and only his signal is above $s^*_N$ where $s^*_N$ is defined by the following zero profit condition

\[
\int_b^V \int_x^V (x - v')h(v' | \Gamma_N)g^V(x|s^*_N)dv'dx - K = 0
\]

where $h(v' | \Gamma_N)$ is the density describing the potential entrant’s belief about the incumbent’s value given the history of the game, $g^V(x|s_N)$ is the potential entrant’s own posterior belief about its value given its signal and $b$ is the standing bid. When $V$ is high, there will be some probability of entry for almost all beliefs about the incumbent’s value. A similar inequality, with additional notation to reflect a firm’s expectations about whether future entry will happen and how it might affect the price paid, will define $s^*$ for earlier rounds. If a firm believes that the incumbent’s value is certainly more than $V - K$ then it not enter.

- any firm will bid up to its value in a knockout auction (this is a dominant strategy, just as in the game with no signals prior to entry);

- incumbents with values above the standing bid will place a jump bid at their first opportunity to do so; for values less than $V - K$, the jump bid will perfectly reveal the incumbent’s valuation, i.e., there will be a fully separating equilibrium for values less than $V - K$. The equilibrium bid function in this case will be determined by a first-order differential equation and the initial condition that a firm with value equal to the standing bid will keep the standing bid. As an example, consider the decision of a new incumbent in the penultimate round. Given a bid function $b(v)$, which will reveal its value to the potential entrant, the incumbent
has to decide which $v$’s bid he should submit

$$\max_{v'} \int_0^\infty \int_{s^*_{N}(v')} s_{N}(v') (v - b(v')) q(s|x) f^V(x) dsdx + \int_0^{b(v')} \int_{s^*_{N}(v')} s_{N}(v') (v - b(v')) q(s|x) f^V(x) dsdx$$

$$+ \int_{b(v')}^\infty \int_{s^*_{N}(v')} s_{N}(v') (v - x) q(s|x) f^V(x) dsdx$$

where $s^*_{N}(v')$ is the final potential entrant’s threshold for entry when it believes the incumbent has exact value $v'$ and $q(s|x)$ is the density of the potential entrant’s signal when its value is $x$. The first two terms reflect the incumbent’s expected profit when it keeps the final potential entrant out or the entrant comes in but has a value less than the standing bid, so that the incumbent pays its bid, and the second term reflects the incumbent’s expected profit when the final potential entrant enters and has a value above $b(v')$. Differentiating this objective function with respect to $v'$ and requiring that the first order condition is equal to zero when $v = v'$ (so that local incentive compatibility constraints are satisfied) gives the differential equation that defines the bid function. The lower boundary condition is provided by $b(b') = b$, i.e., incumbents with values less than or equal to the standing bid will submit the standing bid. In equilibrium, the lowest value an incumbent will ever have is the standing bid, because no firm should ever bid above its value. In the equilibrium we consider incumbents only choose to submit jump bids once: these bids reveal their value to all future players, so that in later rounds they do not raise the standing bid. The bidding problem in earlier rounds can be defined in a similar way with additional notation. Firms with values above $V - K$ will pool, submitting bids equal to $b(V - K)$. For high $V$, this will happen very rarely.

- when an incumbent submits a jump bid $b$ less than $b(V - K)$, a potential entrant’s posterior belief about the incumbent’s value will place all of the weight on $b^{-1}(b)$. For bids at $b(V - K)$, the entrant’s beliefs will be consistent with Bayes Rule.

### 2.3.3 Sketch of Equilibrium Refinement (preliminary)

So far we have described one equilibrium of the sequential model, where jump bids are fully revealing up to $V - K$, but there may be other equilibria. However, given our assumptions, we can show that our equilibrium is the only one consistent with the “D1 Refinement” which has been widely used in the theoretical literature on signaling models (Fudenberg and Tirole (1991)). To make our arguments as clear as possible, we consider the case with two potential entrants, which matches existing models in the signalling literature quite closely. We then consider the extension to the case with more firms. As the distribution of values has the same support for all types, adding more types has no effect on our arguments, so we assume there is only one type to reduce notation.

With two rounds, the equilibrium has the following form when the first firm enters and has a value on $[0, V - K]$: (1) the jump bid of an incumbent (first round entrant), $b_1^*(v_1)$ is a smooth, monotonically increasing and differentiable function in the firm’s value; (2) a second round potential entrant seeing $b_1$ places probability 1 on the first round entrant having value $b_1^{-1}(b_1)$; (3) the second
round potential entrant enters if he expects positive surplus given this belief about 1’s value and the signal about his own value; (4) the bid function is defined by solving a differential equation implied by the first-order condition requiring each incumbent being willing to submit the bid associated with his true value rather than locally deviating; and (5) a boundary condition where the lowest value incumbent submits the lowest possible bid (zero).

The refinements will depend on three properties of the game. \( \pi_v(b_1, s_2) \) is the expected profit of a first round incumbent, where \( v \) is the incumbent’s value, \( b_1 \) is his bid and \( s_2 \) is the entry threshold of the second round potential entrant. Given our assumptions and the dominant strategies in the knockout game and in the absence of entry, \( \pi_v(b, s_2) \) will be continuous and differentiable in both its arguments. The conditions are:

1. \( \pi_v(b, s_2) / \partial s_2 > 0 \);
2. \( \partial \pi_v(b, s_2) / \partial b / \partial \pi_v(b, s_2) / \partial s_2 \) is monotonic in \( v \) (the Spence-Mirrlees single crossing condition).
3. the second potential entrant’s optimal entry threshold (its strategy) is uniquely defined for any belief about the first potential entrant’s value, and the second potential entrant chooses a better action for the incumbent (no entry) when he believes that the incumbent’s value is higher;

We verify that these properties hold in the Appendix. Mailath (1987) shows that an equilibrium bid function can be found using the differential equation (i.e., only checking local deviations) and the boundary condition in a continuous type signaling model when the single crossing condition holds. Mailath (1987)’s results also imply that our equilibrium will be the unique separating sequential PBE, although his results do not rule out the existence of a pooling equilibrium on the \([0, \bar{V} - K]\) interval. However, Ramey (1996) (which extends the results in Cho and Sobel (1990) to the case of an unbounded action space and continuous types on an interval) shows that the three properties outlined above imply that only a separating equilibrium will satisfy the D1 refinement, so our equilibrium must be the only sequential equilibrium satisfying D1. As noted by Mailath (1987), this equilibrium will also be the separating equilibrium which is least costly to the first round potential entrant. This is helpful for us, because it implies that other equilibria would give even higher revenues to the seller.

The conditions also imply that, if an incumbent with value \( \bar{V} - K \) prefers \( b_1(\bar{V} - K) \), which will stop all future entry, to a lower bid then all incumbents with values above \( \bar{V} - K \) will prefer \( b_1(\bar{V} - K) \) to lower bids. But, firms with values above \( \bar{V} - K \) will also strictly prefer to bid \( b_1(\bar{V} - K) \) than any higher bid (for any beliefs of the potential entrant following a higher bid), because by bidding \( b_1(\bar{V} - K) \) the incumbent can get the good for sure at a lower price.

**Three (or More) Rounds** We now consider a model with three potential entrants (arguments for more rounds would follow directly from this case). We make a small simplification by restricting ourselves to equilibria where all potential entrants make the same inferences from a bid by an
incumbent and incumbents only make jump bids in the first round that they enter. The two period equilibrium discussed above would define strategies for the final two rounds, if the second period entrant enters and defeats any incumbent entrant from the first round (with an adjusted boundary condition to reflect the new standing bid). It therefore only remains to argue that there is a unique sequential equilibrium bid function, which is fully separating for values \([0, V - K]\), for a first round entrant. A first round entrant’s jump bid sends a signal to the second round potential entrant, and, if he is still an incumbent in the final round, which must be the case if he is to win, the final potential entrant. Conditional on the incumbent surviving the second round, the third round is just a repeat of another two round game. The first round entrant’s expected profit function is now \(\pi_v(b_1, s_2, s_3)\), and the following properties hold:

1. \(\frac{\partial \pi_v(b_1, s_2, s_3)}{\partial s_2} > 0\) and \(\frac{\partial \pi_v(b_1, s_2, s_3)}{\partial s_3} > 0\);  
2. \(\frac{\partial \pi_v(b_1, s_2, s_3)}{\partial b_1} / \frac{\partial \pi_v(b_1, s_2, s_3)}{\partial s_2}\) is monotonic in \(v\) and \(\frac{\partial \pi_v(b_1, s_2, s_3)}{\partial b_1} / \frac{\partial \pi_v(b_1, s_2, s_3)}{\partial s_3}\) is monotonic in \(v\);  
3. both potential entrants’ optimal entry threshold are uniquely defined for any belief about the first entrant’s value, and they choose actions that are better for the first entrant when they believe that his value is higher.

These conditions allow us to apply the D1 refinement to the signaling game between the incumbent making the jump bid and every subsequent potential entrant.

### 2.3.4 Simple Examples Illustrating How the Mechanism Works

To provide some additional clarity about how the mechanism works, given equilibrium strategies, Table 2.3.4 presents what happens in two games with 4 potential entrants (rounds), and one type of firm with values distributed proportional to \(LN(4.5, 0.2)\) on \([0, 200]\), \(K = 4\) and \(\sigma = 0.2\).

In both games, the first potential entrant enters if he receives a signal greater than 89.4. The signal thresholds in later rounds depend on the number of rounds remaining and the incumbent’s value. So, when the incumbent is the same as in the previous round, the threshold \(s^*\) falls (e.g., round 3 in the first game and round 4 in the second game) as the expected profits of an entrant if he beats the incumbent rise because he will face less competition in the future. On the other hand, \(s^*\) does not depend on the level of the standing bid given the incumbent’s value, because it has no effect on the entrant’s profits if he beats the incumbent in a knockout, because the standing bid must be below the incumbent’s value. The examples also show what happens to the standing bid in different cases. In round 2 of the first game, the incumbent does not face entry, so there is no change in the standing bid because incumbents do not place additional jump bids. There would also have been no change in the standing bid if the entrant had come in (e.g., if his signal was 100)
2.3.5 Effect of the Parameters on Equilibrium Strategies

To give some intuition for how equilibrium bid functions and entry probabilities are affected by the parameters, consider a new incumbent whose value is distributed with density proportional to $LN(4.5, 0.2)$ on $[0, 200]$ in the penultimate round of the game, and the standing bid (the value of the previous incumbent) is 80. We consider how the values of $\sigma, K$ and the location parameter of the value distribution for the final firm affect the bid function and the probability of entry in the subsequent final period.

In Figure 2 the precision of the signal ($\sigma$) varies. When the signal is very imprecise, both the bid function and the entry probability approximate step functions. If there were no signals (BK’s model) then there would be an exact step function in both the bid function and the probability of entry function. When signals are more precise, the slope of the bid function is determined by how much more entry an incumbent could deter by submitting a slightly higher bid - the more entrants who would be deterred, the steeper the bid function must be in equilibrium for firms to truthfully reveal their values when bidding. As a result, when signals are precise the bid function starts rising significantly above the standing bid for relatively low incumbent values (because with precise signals more low value entrants can be discouraged from entering by slightly higher bids), whereas for mean incumbent values (the mean of the distribution is 91, and the mean conditional on having a value more than the standing bid is 99.8), bid functions tend to be flatter (because, with precise signals high value entrants are unlikely to be deterred from entering by slightly higher

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<table>
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<tr>
<th>Round</th>
<th>Initial Standing Bid</th>
<th>Potential Entrant Value</th>
<th>Potential Entrant Signal</th>
<th>Post-Knockout Standing Bid</th>
<th>Post-Jump Bid Standing Bid</th>
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</table>

Seller’s Revenue 100.1, social surplus (winner’s value less total entry costs) 92.9

| Example 2 | | | | | |
| 1 | - | 71.3 | 54.6 | 89.4 | No | - | - |
| 2 | - | 109.3 | 137.7 | 83.9 | Yes | 0 | 92.2 |
| 3 | 92.2 | 99.1 | 122.5 | 121.0 | Yes | 99.1 | 99.1 |
| 4 | 99.1 | 117.8 | 93.0 | 119.4 | No | 99.1 | 99.1 |

Seller’s Revenue 99.1, social surplus (winner’s value less total entry costs) 91.1

because the entrant’s value was below the current bid, so the standing bid would not have risen in the knockout. In round 3 of the first game, the standing bid rises during the knockout, and the new incumbent places an additional jump bid. On the other hand, in round 3 of the second game, the standing bid rises during the knockout phase, but there is no additional jump bid because the old incumbent wins the knockout.
bids). The probabilities of entry for the firm in the final round are consistent with these arguments: when signals are precise, the probability of entry falls more smoothly with the incumbent’s value. Overall there tends to be more entry when signals are imprecise, which reflects the greater option value of entry to a firm that does not know its value (but does know the incumbent’s value). On the other hand, the probability of entry by a firm with a high value will tend to be higher when signals are more precise, and these entrants are more valuable to the seller.

In Figure 3, the level of $K$ (entry cost) varies. In this case the comparative statics are simple. When $K$ is higher, there is less of an entry threat, which reduces the incumbent’s value of submitting a higher bid to deter entry resulting in a flatter bid function. The probability of entry falls monotonically in $K$.

Figure 4 shows the effect when the value distribution of the final potential entrant has a lower location parameter ($K = 1, \sigma_\varepsilon = 0.1$). When it has a lower distribution, future entry is less likely because a weaker firm is less likely to beat the incumbent. This makes the incumbent’s bid function flatter, at least when the standing bid is 80, which is high relative to the mean value of the potential entrant when $\mu_2 = 4.1$.

3 Comparison of Expected Revenues

Before introducing specific parameters estimated from data for USFS timber auctions, we present a more general comparison of expected revenues and efficiency between the sequential mechanism
and the simultaneous entry auction. We see this general comparison as valuable, because it shows that our results are not going to be particularly sensitive to the parameters that we estimate, and they also provide guidance about when auctions should perform well in other settings.

We focus on how the performance of the mechanisms depends on the level of entry costs \((K)\) and the level of the precision of the signal. We measure the precision of the signal by a parameter, \(\alpha = \sigma^2 / \sigma^2 + \sigma^2_\varepsilon\), which approximately measures the weight a potential entrant puts on its prior when forming his posterior belief about its log(value).\(^{10}\) A higher value of \(\alpha\) means that signals are less precise: as \(\alpha\) approaches 1 the informational assumptions approach those of BK’s model. As a base case, we consider 4 symmetric firms whose values are distributed \(LN(4.5, 0.2)\). Figure 5 shows the results of comparing expected revenues from the sequential mechanism (with no reserve) and a simultaneous entry auction with no reserve, based on a grid of points in \((K, \alpha)\) space.\(^{11}\) Filled blue circles represent outcomes where the expected revenues from the sequential mechanism are higher by more than 4% (of auction revenues), while hollow blue circles are outcomes where they are higher but only by between 1 and 4%. Red circles represent cases where the auction gives higher revenues. Black crosses on the grid mark locations where the difference in revenues is less than 1%. As there is some simulation error in calculating expected revenues and some numerical approximations in solving the differential equations in the sequential game, we see these are outcomes as cases where any difference in the revenues is small and we may not be completely confident about their signs.

\(^{10}\)Ignoring the upper bound \(V\), a potential entrant’s posterior distribution for its log value will be \(N(\mu + (1 - \alpha) \log(s), \sigma^2 \alpha^2)\) where \(\mu\) is the location parameter for its prior and \(s\) is its signal.

\(^{11}\)Expected revenues are calculated using 200,000 simulations.
Figure 4: Penultimate Round Equilibrium Bid Functions for a New Incumbent and Associated Final Round Entry Probabilities: Asymmetric firms, $\sigma_\varepsilon = 0.1$, $K = 1$, initial standing bid $80$

(the 1% band is conservative). Note that the grid points are not uniformly distributed in $\alpha$ space: instead, we sampled more points for very low and very high values of $\alpha$ to see how revenues compare when we approximate models with no signals or perfectly informative signals.

The results indicate that for very low values of $K$, the difference in expected revenues is small. This should be expected as when entry costs are low it is very likely that the firms with the two highest values enter, so that the final price will be equal to the value of the second highest firm in either mechanism (firms in the sequential mechanism submit relatively low jump bids because deterrence is ineffective when the entry cost is small). Revenues are also similar when signals are quite informative and $K$ is not too large (if it is large, the sequential mechanism does better). There are two small regions of the parameter space where the auction produces higher expected revenues, although the revenue advantage of the auction is never particularly large (the maximum difference found is 3.3%). The area in the top-left corresponds to cases where entry costs are low and signals are uninformative. These points are consistent with BK’s theoretical results as their model assumes no signals and their assumptions restrict them to consider cases where it is guaranteed that a certain number of firms will enter the auction, so implicitly $K$ must be quite low. There is also a region where the auction dominates in the middle of the parameter space where signals are moderately informative and entry costs are moderately high. For high levels of $K$ the sequential mechanism clearly dominates, especially for very low or very high values of $\alpha$.

Figure 6 compares revenues when the seller-optimal reserve is set in the auction but the se-
As one would expect, there are more parameters where the auctions dominates in this case, but the changes are quite small, reflecting the fact noted above that reserves are fairly ineffective at raising revenues in auctions with endogenous entry.

To explain why the sequential mechanism sometimes outperforms the auction, it is useful to compare the equilibrium efficiency of each mechanism and to understand how aggressively firms have to bid in the sequential mechanism to reveal their values. As BK note, in a model without selective entry and low entry costs, the sequential mechanism is always more (socially) efficient, but the seller may extract lower revenues if there is too much deterrence (from its perspective) and deterring bids are too low.

Figure 7 (a) shows how the efficiency of the auction and the sequential mechanism compare as a function of $\alpha$ (the degree of selection) for $K = 1$ and $K = 5$ for our baseline parameters when there is no reserve (this increases the efficiency of the auction). Efficiency is measured by the expected value of the firm receiving the good less total entry costs. The figure shows two features of the model which appear to be true in general (considering lots of parameter values). First, when entry costs are higher the relative efficiency of the sequential mechanism is greater (recall, BK's assumptions constrain entry costs to be fairly small). This reflects the fact that high entry costs make the feature of the sequential mechanism that more firms tend to enter only when existing entrants have low values more socially valuable. In fact, with any selection, the final potential

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12The optimal reserve is found using a grid search, with 25 cent increments and simulated revenues.
entrant enters only if its expected value is greater than the value of the incumbent less entry costs, which is the efficient criterion for entry. Second, while a small increase in selection causes a small decline in the efficiency of the sequential mechanism when $\alpha$ is very high (little selection), in general more selective entry increases the efficiency advantage of the sequential mechanism. For example, when $K = 5$ the sequential mechanism is 7% more efficient. To raise revenues, the seller only has to be able to appropriate some of this increase in surplus using the sequential mechanism.

One reason why the sequential mechanism is more efficient with selection is that it is more likely the good is allocated to the firm with the highest value whatever its position in the order chosen by the seller. Figure 7 (b) shows the relative probability that the first and last firms in the sequential mechanism are allocated the good in equilibrium when the order is chosen randomly. If the good was always allocated to the firm with the highest value then these probabilities would be equal to 1, and it will also be equal to 1 in the simultaneous entry auction where the order is irrelevant. On the other hand, entry deterrence in the sequential mechanism tends to move the probability above 1, with early buyers more likely to receive the good. Counterbalancing this tendency is the
possibility that the last firm may be more likely to enter in the sequential mechanism because it faces no future competition, and so does not have to submit a jump bid. When $K = 1$ incumbents only deter entry when they have very high values so that the relative probabilities are close to 1, independent of the degree of selection. On the other hand, $K = 5$, the relative probability is still fairly close to 1 even for values of $\alpha$ such as 0.6 or 0.7 that imply only moderate levels of selection - on the other hand, when $\alpha$ is higher later firms are less likely to win which, all else equal, reduces efficiency.\textsuperscript{13}

While the relative efficiency of the sequential mechanism increases with the degree of selection, revenues are also affected by how aggressively firms bid. When there is selection, the level of bids is determined by the fact that bids must be sufficiently high that firms with lower values will not want to copy them. In particular, if the entry decisions of later potential entrants are likely to be sensitive to beliefs about the incumbent’s value (which is true when entry costs are moderately high and signals are informative), equilibrium bid functions can be quite steep functions of the incumbent’s value, and this can raise revenues. An interesting illustration of this point comes from

\textsuperscript{13}Whether the probability would be equal to 1 in the \textit{optimal} (sequential search) mechanism would depend on how the mechanism was structured. If the seller was able to truthfully elicit signals from the buyers before visiting them, then the relative probabilities for the initial (signal discovery) should equal 1. If on the other hand, the seller could only visit potential buyers one at a time then the relative probability would be above 1 for the optimal mechanism as well. Note also that the relative probability being above 1 does not imply that the first potential buyer has higher expected profits in equilibrium. In fact, the expected profits of the final potential buyer tend to be higher unless $\alpha$ is very high, while the expected profits of all earlier buyers are fairly similar.
comparing the equilibrium bid function in the sequential mechanism when there is no selection (BK’s assumption) with the equilibrium bid functions in our model when potential entrants receive signals but they are not very informative. In Figure 8, the bid function of the LS model is a step function, which increases at a value of 119 (the level of the incumbent’s value that deters all future entry). The bid functions with signals lie above this bid function for all incumbent values, which, holding entry decisions in the last round constant must tend to increase revenues.  

![Bid Functions](image)

Figure 8: Penultimate Round Bid Function for A New Incumbent : Symmetric firms values LN(4.5,0.2) on [0,200], K=1, Initial standing bid 80

Of course, the fact that the sequential mechanism can lead to less (but more efficient) entry than the auction can still hurt the seller, and this helps to explain the second region (intermediate values of $\alpha$, moderately high $K$) where the auction gives higher expected revenues. For example, when $K = 8$ and $\alpha = 0.4$, the probability that each firm enters the auction is 0.56, but the probabilities that the firms enter the sequential mechanism are only 0.33, 0.32, 0.30, 0.27 for each firm in the order respectively, and the probability that only one firm enters is 0.76. In this case, the single entrant does not bid aggressively enough to offset the loss of competition through reduced entry.

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14 As $\sigma_\epsilon$ is increased to very high values (such as 20 or 30) the equilibrium bid function falls so that it becomes very close to the equilibrium bid function without signals.
For example, with probability 0.144 the last firm is the only firm to enter the sequential mechanism and it wins at a price equal to zero.

Figure 9 compares revenues when there are 8 rather than 4 firms (with symmetry and no reserve price in the auction). With more firms, the auction only gives higher revenues when entry costs are really low and $\alpha$ is very high (non-selective entry), and, in particular, it never dominates when there is a reasonably selective entry process. This reflects the fact that, even with some selection, simultaneous entry decisions are relatively inefficient and this becomes more important when there are more firms, so that, at least when entry costs are not very small, the incremental effect of additional firms on revenues is larger in the sequential mechanism. Figure 10 illustrates this point: when $K = 5$ the efficiency advantage of the sequential mechanism is at least a couple of percentage points higher than with 4 firms.

Finally we consider the effect of introducing a small asymmetry in values. Two potential entrants have values drawn from a distribution with density proportional to $LN(4.5,0.2)$. In the
Relative Mechanism Efficiency

Relative Efficiency of Sequential Mechanism (Same = 1)

K=1
K=5

Figure 10: Comparison of Mechanism Efficiency: 8 Symmetric firms values LN(4.5,0.2) on [0,200]
 sequential mechanism these firms are approached first. The other two firms have values drawn from a distribution with density proportional to LN(4.4,0.2). Figure 11 shows the comparison when the auction has no optimal reserve price. Even with only a small asymmetry in values, expected revenues are significantly higher using the sequential mechanism for a much broader range of values than was the case with 4 symmetric firms. In particular, the sequential mechanism always does at least 1% better when entry is relatively selective and \( K \) is above the minimum value we consider (0.01).

One reason why the sequential mechanism does better when firms are asymmetric and higher mean value firms move first is that the weaker-type firms make their entry decisions knowing the value of any incumbent higher value firms. Relative to the simultaneous entry case, this tends to make them more likely to enter, which raises the probability that they win, and also induces the higher mean value entrants to bid more aggressively. For example suppose that \( K = 8 \) and \( \alpha = 0.4 \), a case which favored the auction with symmetric firms. In the simultaneous-entry auction the probability that each of the weaker firms enters is 0.1 and the probability that one of them wins is only 0.12. On the other hand, in the sequential mechanism the entry probabilities are 0.18 and

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15Our simulations show that approaching all of the high value firms first, followed by all of the low values firms is better than doing the exact opposite. However, we have not yet checked whether mixing the types in someway is better than either of these alternatives.

16In future revisions we will consider the performance of a first-price auction using optimal reserves for different types of bidder. In a model without selective entry, such auctions would do better than simple second-price auctions. Some initial simulations suggest that in our model first-price auctions will do better than second price auctions but not by much.
0.16 (even though they move last) and the probability that one of them wins is 0.30. This is much closer to the probability that one of the weaker firms will have the highest value (0.33 given our distributional assumptions).

4 Empirical Application

We now turn to our empirical application and describe the data, reduced form evidence of selection in these auctions and the method and assumptions we use to estimate a selective entry model using data from open outcry auctions. We are rather brief here since Roberts and Sweeting (2010) provides a more detailed discussion of these topics.

4.1 Data and Context

We focus on federal auctions of timberland in California.\textsuperscript{17} In these auctions the U.S. Forest Service (USFS) sells logging contracts to individual bidders who may or may not have manufacturing capabilities (mills and loggers, respectively). When the sale is announced, the USFS provides its own “cruise” report of the tract’s timber. It also announces a reserve price and bidders must submit

\textsuperscript{17}We are very grateful to Susan Athey, Jonathan Levin and Enrique Seira for sharing their data with us.
a bid of at least this amount to qualify for the auction. After the sale is announced, bidders perform their own private cruises of the tract to assess its value. These cruises can be informative about the tract’s volume, species make-up and timber quality. Finally, bidders must post a deposit of 10% of the appraised value of the tract in order to be eligible to participate in the auction.

We summarize the data in Table 1. Bids are given in $/mbf (1983 dollars). We see that bids submitted by loggers tend to be lower than those submitted by mills, consistent with the results in Athey, Levin, and Seira (forthcoming). We define entrants to be the set of bidders we observe at the auction even if they did not submit a bid above the reserve price. We count the number of potential entrants as those bidders who bid within 50 km of an auction over the next month. One way of assessing the appropriateness of this definition is that less than 2% of the bidders in any auction fail to bid in another auction within 50 km of this auction over the next month.

Fewer loggers than mills enter on average and they are also less likely to enter. Among the set of potential logger entrants, on average 34% enter, whereas on average 66% of potential mill entrants enter. Finally, 4% of tracts failed to sell because they received no bids.

As in our model above, we assume that bidders have independent private values. This assumption is also made in other work with similar timber auction data (see for example Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (forthcoming)). A bidder’s private information is primarily related to its own contracts to sell the harvest, inventories and private costs of harvesting and thus is mainly associated with its valuation only.

In our model we allow bidders to receive an imperfect signal about their value prior to paying an investment cost to fully learn their value. There are multiple reasons why it is likely that in these timber auctions bidders only have imperfect knowledge about their value prior to entry. First, each tract is unique and therefore even if a bidder has previously bid on apparently similar tracts, they must still account for heterogeneity not realized prior to further investigation. Second, the cruise estimates published by the USFS are viewed as useful but imprecise. For example, Figure 12 summarizes the government’s misestimates of the (i) amount of timber and (ii) distribution of timber species on any tract. The latter is measured by the USFS’s prediction of the share of the tract’s most populous species. Clearly, the USFS’s estimates are useful for bidders but far from perfect in allowing them to determine their values without performing their own cruise. Third, bidders must also devote time to planning and organizing their team of harvesters and lining up potential end users for any given tract. These are likely to be only a few of the necessary investments a bidder must make prior to learning its true value for a particular stand of timber. Finally, industry sources have told us that it would be extremely unlikely for a firm to bid without undertaking its own cruise. Therefore, a model which at least allows bidders to have a noisy signal about their

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18From our discussions with industry sources, it is very rare for firms to bid without doing their own cruise.
19Details on our cleaning of the data appear in Roberts and Sweeting (2010).
20These figures are based on a sample of “scaled” sales we have data on the timber that was actually cut by the winner. This is the same data used in Athey and Levin (2001).
21To interpret the second figure, 10 means that the share of the (supposedly) most prevalent species was estimated to be 10% higher than it actually was.
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<td>9.98</td>
<td>6.45</td>
<td>5</td>
<td>9</td>
<td>14</td>
<td>1</td>
<td>38</td>
<td>988</td>
</tr>
<tr>
<td>LOGGER</td>
<td>5.26</td>
<td>4.71</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>27</td>
<td>988</td>
</tr>
<tr>
<td>MILL</td>
<td>4.73</td>
<td>2.78</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>14</td>
<td>988</td>
</tr>
<tr>
<td>PREVIOUS MILLS (6 mos)</td>
<td>4.57</td>
<td>2.85</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>0</td>
<td>14</td>
<td>988</td>
</tr>
<tr>
<td>SPECIES HERFINDAHL</td>
<td>0.55</td>
<td>0.23</td>
<td>0.35</td>
<td>0.51</td>
<td>0.72</td>
<td>0.20</td>
<td>1.00</td>
<td>988</td>
</tr>
<tr>
<td>DENSITY (acres/mbf)</td>
<td>0.21</td>
<td>0.21</td>
<td>0.07</td>
<td>0.16</td>
<td>0.28</td>
<td>0.02</td>
<td>1.81</td>
<td>988</td>
</tr>
<tr>
<td>VOLUME (hundred mbf)</td>
<td>75.17</td>
<td>45.41</td>
<td>41.05</td>
<td>68.6</td>
<td>103.1</td>
<td>5</td>
<td>275.4</td>
<td>988</td>
</tr>
<tr>
<td>RESERVE ($/mbf)</td>
<td>37.95</td>
<td>32.36</td>
<td>16.38</td>
<td>27.3</td>
<td>47.89</td>
<td>2.04</td>
<td>221.87</td>
<td>988</td>
</tr>
<tr>
<td>SELL VALUE ($/mbf)</td>
<td>291.05</td>
<td>64.18</td>
<td>259.51</td>
<td>292.29</td>
<td>326.03</td>
<td>0</td>
<td>518.95</td>
<td>976</td>
</tr>
<tr>
<td>ROAD CONST ($/mbf)</td>
<td>12.23</td>
<td>14.33</td>
<td>1.06</td>
<td>7.49</td>
<td>17.76</td>
<td>0</td>
<td>91.55</td>
<td>976</td>
</tr>
<tr>
<td>LOG COSTS ($/mbf)</td>
<td>116.53</td>
<td>33.13</td>
<td>98.4</td>
<td>113.12</td>
<td>133.78</td>
<td>0</td>
<td>252.46</td>
<td>976</td>
</tr>
<tr>
<td>MFCT COSTS ($/mbf)</td>
<td>134.46</td>
<td>24.09</td>
<td>127.07</td>
<td>136.22</td>
<td>146.15</td>
<td>0</td>
<td>227.55</td>
<td>976</td>
</tr>
<tr>
<td>MISSING APPRAISAL</td>
<td>0.01</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>988</td>
</tr>
</tbody>
</table>

Table 1: Summary statistics for California ascending auctions from 1982-1989. We exclude SBA set asides, salvage sales, auctions with very high or low volume to acreage ratios and failed sales. For calculating when a logger wins, we focus only on auctions where the tract sold. We count the number of potential entrants as those bidders who bid within 50km of an auction over the next month. Statistics about the potential mill bidders over the previous 182 days that bid in the same forest district is also given by PREVIOUS MILLS. We note that 26% of all bids are losing bids at the reserve. SPECIES HERFINDAHL is the Herfindahl index for wood species concentration on the tract. SELL VALUE, ROAD CONST, LOG COSTS and MFCT COSTS are USFS appraisals of the value of the tract and the road building, logging and manufacturing costs of the tract, respectively.
value, but can still permit this signal to be fairly precise, seems warranted.

4.2 Evidence of Selection

In this section we provide evidence that actual bidders in an auction are not a random sample of potential entrants from the data. We do this by examining the impact of the number of potential entrants on submitted bids. The findings are in Table 2.

One test for selection is whether the average valuations of bidders rise as potential entry increases. Therefore, first column regresses the log of bids on auction covariates as well as the number of potential entrants. If we believe that submitted bids represent values, then the positive impact of potential entrants suggests that bidders are not a random sample from the population of potential entrants. One might be concerned about unobserved heterogeneity in this context so we instrument for the number of potential entrants with the number of potential bidders by the number of mills who bid in the same forest during the preceding six months. Since there is some concern about interpreting bids as values in these auctions since they are not exactly English Button Auctions, but rather open outcry auctions (see for example Haile and Tamer (2003)), the second column uses only winning bids in the regression. These are the bids most interpretable as values since the are close to the valuation of the second highest valuation bidder. Again, the positive impact of the number of potential entrants on values suggests that actual bidders are a selected set from the group of potential bidders.

Finally, we can test for evidence of selection by applying a Heckman selection model (Heckman (1976)). Given that potential competition affects a bidder’s decision to enter an auction, but not his value conditional on entry, we can specify entry as a flexible function of covariates, including potential competition, at the first stage, and exclude competition at the second stage bid regression. The third column in Table 2 presents the second stage of this approach. The positive estimate of the inverse Mills ratio provides further evidence of selection.

4.3 Estimation Using Importance Sampling

We estimate the model using the importance sampling approach suggested by Ackerberg (2009). In this likelihood based approach we must solve the entry and bidding game. Solving the game amounts to finding the equilibrium entry thresholds for any set of bidders, reserve price, auction covariates and parameters. These entry thresholds are pinned down by the following zero profit condition (here for a type $\tau$ bidder):

$$\int_0^\infty \left( vG_{\tau}^{-1}(v|R, N, s_{-\tau}, s_{\tau}, \theta) - \int_0^v u^\prime g_{\tau}^{-1}(u^\prime|R, N, s_{-\tau}, s_{\tau}, \theta)du^\prime \right) f_{\tau}(v|s_{-\tau}, s_{\tau}, \theta)dv - K = 0 \quad (1)$$

$^{22}$Detailed discussion of this instrument can be found in Haile (2001) and Roberts and Sweeting (2010).
$^{23}$Alternative estimation methods with this data appear in Roberts and Sweeting (2010).
Figure 12: Evaluating the quality of the government’s predictions.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log POT. ENTRANTS</td>
<td>0.221***</td>
<td>0.924***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.136)</td>
<td></td>
</tr>
<tr>
<td>LOGGER</td>
<td>-0.125***</td>
<td>-0.155**</td>
<td>-0.211***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.077)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>SPEC. HERF.</td>
<td>-0.381***</td>
<td>-0.360***</td>
<td>-0.351***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.125)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>VOLUME (mbf)</td>
<td>-0.001***</td>
<td>-0.020***</td>
<td>-0.001***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.006)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\hat{\lambda}$ IMR</td>
<td></td>
<td></td>
<td>0.136***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.046)</td>
</tr>
<tr>
<td>APRSL VARS</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>YR,QTR,CNTY DUMS</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>FIRST STAGE</td>
<td>0.075***</td>
<td>0.087***</td>
<td></td>
</tr>
<tr>
<td>IV COEFFICIENT</td>
<td>(0.005)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>SPECIFICATION</td>
<td>All Bids</td>
<td>Winning Bids</td>
<td>Heckman</td>
</tr>
</tbody>
</table>

Table 2: Evidence of Selection. Dependent variable is log of bid per volume. POTENTIAL ENTRANTS are the bidders who bid in this auction or in those within 50 km over the next month. Instrument for potential entrants is the number of unique mill bidders in a forest during the previous 6 months. The first column uses all bids and the second uses only winning bids. The third column displays the second stage results of the two step selection model. The first stage probit is of entry where the exogenous shifters are potential other mill and logger entrants, incorporated as a flexible polynomial. $\hat{\lambda}$ is the estimated inverse Mills ratio from the first stage. Each column summarizes the findings from Tables 5, 6 and 7 in Roberts and Sweeting (2010).

where $G_\tau^{-1}(v|R, N, s'_{\tau}, s''_\tau, \theta)$ and $g_\tau^{-1}(v|R, N, s'_{\tau}, s''_\tau, \theta)$ are the cdf and pdf of the highest value from other entering bidders (of both types) given their entry strategies, the number of potential entrants of each type ($N$), the reserve price ($R$) and the parameters ($\theta$), and $f'_\tau(v|s'_{\tau}, s''_\tau, \theta)$ is the pdf of the distribution of values for the type $\tau$ firm.

Given the equilibrium entry thresholds, we calculate the likelihood of the observed outcome of the auction. We assume that potential entrants who do not submit any bid at the auction did not enter (and so did not pay $K$). Because the auctions operate as open outcry auctions, rather than second-price sealed bid or button auctions, it is not entirely clear how the observed bids should be treated. For example, the highest bid submitted by a losing bidder may be below his true value. In this case we lack a well-defined model for what determines the bid that we do observe. We therefore proceed by assuming that, when the second highest observed bid is greater than the reserve price, that this bid represents the valuation of the second highest bidder. Losing bidders who attend the auction are assumed to have values between this second highest bid and the reserve price. In forming the likelihood we allow potential bidders to enter, learn their value is less than the reserve, and not bid.

---

24 Alternative assumptions could be made. For example, we might assume that the second highest bidder has a value less than the winning bid, or that the second highest bidder’s value is some explicit function of his bid and the winning bid. In practice, 96% of second highest bids are within 1% of the high bid, so that any of these alternative specifications should give similar results. We have computed some estimates using the winning bid as the second highest value and the coefficient estimates are indeed similar.
The benefit of the importance sampling approach is that we do not need to resolve the entry game for each auction at each value of the parameters. Instead, we assume that all of the parameters of the model have some parametric distribution across auctions, which can depend on observables. We solve each auction and calculate the likelihood of the observed decisions for many different simulation draws of the parameters (done using many processors) and then estimate the distribution of the parameters by reweighting the likelihoods for each simulated game. When solving the games we assume that the equilibrium played involves the type with the higher mean having a lower \( s' \), and to make sure that there is exactly one equilibrium of this kind we assume that the parameters \( \sigma^2_V, K \) and \( \sigma^2_\varepsilon \) are the same across potential entrants within an auction even if they are different across auctions. Roberts and Sweeting (2010) discuss the presence of multiple equilibria in this game and why, if we assume constant \( K \), \( \sigma^V \) and \( \sigma^\varepsilon \) across bidder types within an auction, this is the natural equilibrium to focus on. Moreover, that paper shows the estimates (a) are robust to methods which can potentially identify the equilibrium actually played in the data (nested pseudo-likelihood) and (b) cannot support multiple equilibria based ex post on analysis of best response functions.

Specifically we assume that

- \( V_{\text{milt}} \sim \log N(\mu_{\text{milt},a}, \sigma^2_{V,a}) \) where \( \mu_{\text{milt},a} \sim N(X_a \beta_1, \omega^2_{\mu,\text{milt}}) \)
- \( V_{\text{logger}} \sim \log N(\mu_{\text{milt},a} + \mu_{\text{difference},a}, \sigma^2_{V,a}) \) where \( \mu_{\text{difference},a} \sim \text{Truncated } N(X_a \beta_2, \omega^2_{\mu,\text{difference}}) \)
- \( \sigma^2_{V,a} \sim \text{Truncated } N(X_a \beta_3, \omega^2_{\sigma^2_V}) \)
- \( K_a \sim \text{Truncated } N(X_a \beta_4, \omega^2_K) \)
- \( \sigma^2_\varepsilon,a \sim \text{Truncated } N(X_a \beta_5, \omega^2_{\sigma^2_\varepsilon}) \)

where the \( X \)s are observable variables. The parameters to estimate are the parameters which characterize the distributions of the structural parameters i.e., the \( \beta \)s, \( \omega^2_{\mu,\text{milt}}, \omega^2_{\mu,\text{difference}}, \omega^2_{\sigma^2_V}, \omega^2_K \) and \( \omega^2_{\sigma^2_\varepsilon} \). We label the collection of these parameters \( \Omega \). The truncation points are chosen so that loggers always have weakly lower mean values than mills and both \( K \) and the variance parameters are positive.

For given parameters \( \Omega \) the likelihood function for the observed outcome in auction \( a \) is

\[
\int L_a(\theta) \phi(\theta | X_a, \Omega) d\theta
\]

where \( L_a(\theta) \) is the likelihood of the outcome for a given auction for a given set of structural parameters. Estimation becomes costly if whenever one alters \( \Omega \) the set of \( \theta \)s to be evaluated is also

---

\(^{25}\text{Hartmann (2006) and Hartmann and Nair (forthcoming) provide applications of these methods to consumer dynamic discrete problems. Bajari, Hong, and Ryan (forthcoming) use a related method to analyze entry into first price procurement auctions assuming the LS model. However, their method assumes a complete information entry game where firms get draws on entry costs that depend on who else enters the auction. We do not make this assumption.} \)
altered. To motivate the use of importance sampling this can be rewritten as

\[ \int L_a(\theta) \frac{\phi(\theta|X_a, \Omega)}{g(\theta|X_a)} g(\theta|X_a) d\theta \]

where \( g(\theta|X_a) \) is an importance sampling density where the support of \( \theta \) does not depend on \( \Omega \), which is true in our case because the truncation points are not functions of the parameters. The likelihood can be simulated using

\[ \frac{1}{S} \sum_s L_a(\theta_s) \frac{\phi(\theta_s|X_a, \Omega)}{g(\theta_s|X_a)} \]

where \( \theta_s \) are vectors of parameter draws from \( g(\theta_s|X_a) \). For \( g \) we use uniform distributions over a very wide area of the parameter space with 80,000 draws for each auction. During estimation the weights \( \frac{\phi(\theta_s|X_a, \Omega)}{g(\theta_s|X_a)} \) change as \( \Omega \) varies but \( L_a(\theta_s) \) does not have to be recalculated. This allows us to control for a large number of observable variables directly, rather than using an ad-hoc first stage. Standard errors are calculated using a non-parametric bootstrap where we resample games and their associated draws with replacement.

5 Empirical Results

5.1 Parameter Estimates

The estimates in Table 3 allow the USFS estimate of sale value and its estimate of logging costs to affect mill and logger values (where \( \mu_{\text{logger}} = \mu_{\text{mill}} + \mu_{\text{difference}} \)). These variables are consistently the most significant in regressions of reserve prices or winning bids on observables. The right hand column shows the median value of the parameters across the auctions in the sample, taking into account observable auction characteristics and the fact that some of the parameter distributions are truncated. The coefficients show that tracts with greater sale values and lower costs are more valuable as one would expect. Interestingly the difference in values between mills and loggers appears independent of these variables, although it does appear that there is both unobserved heterogeneity in values across auctions (the standard deviation of \( \mu_{\text{mill}} \)) and heterogeneity in the difference between mill and logger mean values across auctions (the standard deviation of \( \mu_{\text{difference}} \)).

The estimates for \( K \) and \( \sigma_e \) in Table 3 indicate that entry costs are low and signals are quite precise, leading to a lot of selection. For example, in a representative auction, the mean value for a typical entering mill is $85.75/mbf, while that of a marginal mill (one who observes \( s^*_{\text{mill}} \)) is $22.47/mbf. For loggers, the respective mean values are $46.29/mbf and $24.59/mbf.

To give a better sense of what the distributions of values look like, we present some examples in Figures 13-14. We plot the distributions for the results in Table 3. The value distributions for both types appear in Figure 13. With our parameter estimates, if we assume a set of potential bidders and reserve price, we can compare the entrants’ and marginals’ value distributions. For the case where the reserve and the number of potential mill and logger entrants are set to their respective
Table 3: Importance Sampling parameter estimates. Importance Sampling parameter estimates of parameters for lognormal parameterization when we allow non-entrants to have paid the entry cost. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>Estimate</th>
<th>SE</th>
<th>Estimate</th>
<th>SE</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-8.0103</td>
<td>0.07753</td>
<td>3.1557</td>
<td>0.0585</td>
<td>-1.2716</td>
<td>0.1699</td>
<td>-0.0027</td>
<td>0.0187</td>
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<td>Herfindahl</td>
<td>-0.0149</td>
<td>0.01263</td>
<td>-0.0055</td>
<td>0.0424</td>
<td>0.3554</td>
<td>0.0346</td>
<td>3.7715</td>
<td></td>
</tr>
<tr>
<td>Log(Sale Value)</td>
<td>1.0904</td>
<td>0.0137</td>
<td>0.0008</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Log(Logging Costs)</td>
<td>-1.4463</td>
<td>1.4704</td>
<td>-0.0004</td>
<td>0.0133</td>
<td>-0.0020</td>
<td>0.0112</td>
<td>0.0030</td>
<td>0.0290</td>
</tr>
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<td>Density</td>
<td>-0.0111</td>
<td>0.0696</td>
<td>0.0151</td>
<td>0.0948</td>
<td>0.5758</td>
<td>1.7636</td>
<td>-1.3451</td>
<td></td>
</tr>
<tr>
<td>Log(Volume)</td>
<td>0.0030</td>
<td>0.0299</td>
<td>0.0020</td>
<td>0.0143</td>
<td>0.0151</td>
<td>0.0143</td>
<td>0.0143</td>
<td>0.0143</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>-0.0088</td>
<td>0.0059</td>
<td>0.0056</td>
<td>0.0056</td>
<td>0.0642</td>
<td>0.0885</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Median Value</td>
<td>0.0013</td>
<td>0.0014</td>
<td>1.0014</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Mean Value</td>
<td>1.0014</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N : 888. Simulated Maximum Likelihood: -10,582.1
means of $37/mbf, four and five, this comparison for mills and loggers appears in Figure 14.

As predicted by the Signal model, there is a stark difference in the marginal and infra-marginal bidders for each type.

5.2 Counterfactual Results (Preliminary)

Table 5.1 compares expected revenues from the sequential mechanism and the simultaneous entry auction for a range of parameters and different numbers of firms. We compute expected revenues in the auction for an auction with no reserve, and for an auction with an optimal reserve - these are always close, and when we use observed reserves the results lie in between these estimates. The first line gives the results for the representative auction used in the constructing the figures above. The sequential mechanism is projected to give 8.3% higher revenues than a second price auction with an optimal reserve price, and for a tract of average size (7,500 mbf) the expected revenue difference would be $56,400. The other lines in the table compare revenues when we change either the number of firms or one of the parameters for the mean value of the loggers, the variance of the signal noise or entry costs (the changing parameters are in italics), reflecting the fact that our estimates imply that the coefficients will differ across sales. For simplicity, we keep the parameters for the mean value for mills and the standard deviation of values for both types the same as in the representative case, but we will vary them in future revisions. The cases we do consider indicate that using the sequential mechanism always raises the seller’s expected revenues, with especially large increases when there are more loggers (weaker firms), higher entry costs and more precise signals (more selective entry), which are the broad patterns that we would expect from the simulations in Section 3. In all cases, the revenue increases from using the sequential mechanism appear to be much larger than the returns to using a (single) reserve price in a second-price auction, even though understanding optimal reserve price policies for timber auctions has been the subject of significant interest in the literature (Paarsch (1997), Haile and Tamer (2003), Aradillas-Lopez, Gandhi, and Quint (2010)).

6 Conclusion

This paper compares the performance - from a seller’s perspective - of a standard simultaneous entry second price auction with an alternative sequential mechanism in an environment where firms with higher values may be more likely to enter. We see this model as a plausible description of many environments where auctions are currently used, and potential bidders have to perform costly research to find out their values prior to bidding. We find that the sequential mechanism can do significantly better, which is contrary to what one might expect in the light of Bulow and Klemperer (2009)’s results which were based on an entry model which allowed no selection. The revenue advantage of the sequential mechanism is typically larger when entry is more selective, especially when entry costs are low, when entry costs are high and when there are asymmetries
between bidders, and so far we have not found any examples where the auction gives significantly higher (e.g., 4% or more) expected revenues. We use our model to examine whether the US Forest Service would do better using the sequential mechanism than the type of auction it currently uses, and find that the gains in revenues are potentially quite large in many representative auctions, and this pattern is robust to choosing different values for the main parameters. The revenue gains from using a different mechanism are generally much larger than those from setting the optimal reserve within the existing auction format. Our current results may well understate the gains to changing the mechanism, as we have only considered one relatively simple alternative mechanism with no reserve. Introducing a reserve price within the sequential mechanism may be more effective than within the simultaneous entry auction as it would raise the starting point for jump bids and, when we simulate the current model, there is often only one entrant who currently wins at a low deterring bid, or, in the case of the final firm, with a bid of zero.

Of course it may be the case that in practice standard auctions have advantages, such as transparency, which our model ignores. It is certainly true that our mechanism would require, for example, a process for establishing the set of potential buyers (maybe by setting a very small fee for indicating a potential interest in bidding) and then a commitment by the seller to approach firms in some random order (at least within-type), so that particular firms are not favored (typically the final firm approached has the highest expected profits because it does not have to jump bid to deter future entry). However, it is not obvious to us that the administrative burden involved would always exceed the quite large gains in revenues which appear to be available from considering different mechanisms, and it seems likely that requiring all firms to send representatives to a room on a particular date to hold a simultaneous auction also imposes some cost on bidders. The informational requirements of the sequential mechanism on the seller are also relatively small: in contrast, the benefits of more complicated auction schemes - involving the provision of subsidies to particular types of bidder, for example - might be dependent on the seller knowing the value distribution, entry costs and signal noise of different types of firm.

Our research highlights one obvious direction for future theoretical research. We take Bulow and Klemperer’s sequential mechanism as an off-the-shelf, and reasonably straightforward, alternative to current practice. The fact that a sequential mechanism, and particularly one that involves firms’ values being sequentially revealed, performs well is consistent with papers in the optimal mechanism design literature (Cremer, Spiegel, and Zheng (2009) and McAfee and McMillan (1988)) that consider settings where participation by potential buyers is costly, and they either have perfect or no information about their values prior to entering, and find that mechanisms which implement a sequential search policy are optimal. However, there are currently no models which consider optimal mechanism design when buyers have partially informative signals before they participate, and it is these models which seem most likely to be relevant in practice.
Mill Parameters: MeanV = 3.7715; StdV = 1.0014; StdEps = 0.497; K = 0.0885
Logger Parameters: MeanV = 2.4264; StdV = 1.0014; StdEps = 0.497; K = 0.0885
Reserve = 37; Num Mills = 4; Num Loggers = 5; Sprime Mill = 18.8457; Sprime Logger = 29.2562

Figure 13: Comparing the value distributions for Mills and Loggers. Based on estimates in Table 3.
Figure 14: Comparing the value distributions for entrant and “marginal” bidders by type. Based on estimates in Table 3.
Table 4: Comparison of Expected Revenues from Simultaneous Entry Auctions and the Sequential Mechanism for Various Hypothetical Timber Sales. In all cases the location parameters for mill values is 3.771 and the scale parameters for the value distribution of both mills and loggers is 1.001. The first line shows the results for the representative auction discussed above. Italics indicate changes from this representative auction.
References


Conditions for Unique Sequential PBE Under the D1 Refinement

We now verify the three conditions required to show that our equilibrium is the unique separating equilibrium.

(1) $\frac{\partial^2 \pi_v(b, v_H)}{\partial v \partial b} > 0$. Increasing the signal threshold keeps out more second round potential entrants. The bidding behavior of those entrants who have signals above the threshold is unchanged and so all this does is increase the incumbent’s probability of winning and lowers the expected price paid.

(2) Single crossing: the Spence-Mirrlees single crossing condition is that $\frac{\partial^2 \pi_v(b, v_H)}{\partial v^2}$ is monotonic in $v$. Differentiating gives:

$$\frac{\partial^2 \pi}{\partial b \partial v} \left( \frac{\partial \pi}{\partial s_2} \right)^{-1} - \left( \frac{\partial^2 \pi}{\partial s^2} \right) \left( \frac{\partial \pi}{\partial s_2} \right)^{-2} \frac{\partial \pi}{\partial b}$$

(2)

and we need to show that this must be either always positive or always negative.

(a) $\frac{\partial^2 \pi_v(b, v_H)}{\partial b} < 0$: Increasing the bid is costly when it does not affect the second round potential entrant’s decision. In particular, it reduces a firm’s payoff when the second round firm does not enter or it enters and has a value less than $b$. If the potential entrant enters with a value above $b$ then changing $b$ has no effect.

(b) $\frac{\partial^2 \pi_v(b, v_H)}{\partial b \partial v} = 0$: Consider two types of first round bidders $v_H$ and $v_L$, $v_H > v_L$ each considering increasing their bid $b$ to $b + \varepsilon$. If the second bidder stays out then the cost to each first round type is the same, $\varepsilon$. We now show that if the second round bidder comes in, the cost is still the same to each type of first round bidder. Consider three cases. (i) $v_2 < b$. The cost to each type of first round bidder is $\varepsilon$ since each still wins but pays more. (ii) $v_2 > b + \varepsilon$. The cost to each type will be same and equal to zero since the final price in this case is $\min\{v_L, v_2\}$ for the low type and $\min\{v_H, v_2\}$ for the high type. Both are independent of the deterring bid. (iii) $b \leq v_2 \leq b + \varepsilon$. The first round bidder still wins, regardless of type, but now he has to pay more since before he would have won at a price of $v_2$ but now he wins at a price of $b + \varepsilon$, yielding the same cost of $b + \varepsilon - v_2$ to each type of first round bidder. Therefore, the cost of raising the deterring bid, all else constant, is independent of the first bidder’s value.

(c) $\left( \frac{\partial^2 \pi_v(b, v_H)}{\partial s_2 \partial v} \right) > 0$: To show that the benefit of increasing the signal entry threshold is greater the higher is the first bidder’s signal, we can show that the benefit of excluding any second bidder type $v_2$ is greater, the higher is the first bidder type, regardless of $v_2$. Consider the value of excluding a second round bidder whose value is $v_2$ for any two types of first round bidders $v_H$ and $v_L$, $v_H > v_L$ both using deterring bid $b$. If $v_2 \leq b$ there is no change in benefit from exclusion for either first bidder type. If $b < v_2$ there are three cases. (i) $v_2 \leq v_L < v_H$. In this case the benefit of excluding the second round bidder is $v_2 - b$ for each first round bidder type. (ii) $v_L < v_2 \leq v_H$. In this case

\footnote{There are three cases within this case. The first is when $v_2 > v_H > v_L$. Regardless of deterring bid, both first round types would lose and so increasing the deterring bid has no effect on their cost. The second is when $v_H > v_2 > v_L$. Here the low type was going to lose regardless, and so it has no effect on his cost. Here the high type was going to win but pay $v_2$ no matter what and so increasing the bid has no effect on his cost. The third is when $v_H > v_L > v_2$. In either case both types were going to win but have to pay $v_2$ and so increasing the bid had no effect on either types’ costs.}
the benefit of exclusion is $v_L - b$ for the low type and $v_2 - b$ for the high type. Since by assumption $v_2 > v_L$, the benefit of exclusion is greater for the higher type. (iii) $v_L < v_H < v_2$. In this case the benefit of exclusion is $v_L - b$ for the low type and $v_H - b$ for the high type and so the benefit is greater for the higher first bidder type. Therefore, the benefit of excluding more second round bidders is greater the higher is the first round bidder’s value.

(d) $\frac{\partial \pi}{\partial b} < 0$ and $\left( \frac{\partial \pi}{\partial s_2} \right)^{-2} > 0$ as shown above. So (2) is

$$\frac{\partial^2 \pi}{\partial b \partial v} \left( \frac{\partial \pi}{\partial s_2} \right)^{-1} - \left( \frac{\partial^2 \pi}{\partial s_2 \partial v} \right) \left( \frac{\partial \pi}{\partial s_2} \right)^{-2} \frac{\partial \pi}{\partial b} > 0$$

and the condition is satisfied.

(3) the second potential entrant’s optimal entry threshold is uniquely defined for any belief about the first potential entrant’s value, and the second potential entrant chooses a better action for the incumbent (no entry) when he believes the incumbent’s value is higher: this is true in our case, as the optimal $s_2$, will be a continuous function of the second period potential entrant’s beliefs about the incumbent’s value (reflecting the zero profit condition and the potential entrant’s beliefs about his own value as a function of the signal) and the second potential entrant will increase $s_2$ if he believes bidder 1’s type is higher because his expected profits are decreasing in bidder 1’s type for any signal bidder 2 receives.