Abstract

Theoretical models of strategic investment often assume that information is incomplete, creating incentives for firms to signal information to deter entry or encourage exit. However, the simple one-shot nature of existing models limits our ability to test whether these models quantitatively or even qualitatively fit the data. We develop a dynamic model with persistent asymmetric information, where an incumbent has incentives to repeatedly signal information about its costs to a potential entrant. The model has a unique Markov Perfect Bayesian Equilibrium under a standard form of refinement and equilibrium strategies can be computed easily, making it well-suited for empirical work. We calibrate our model using parameters estimated from airline data to test whether it can explain why incumbent airlines cut prices substantially when Southwest becomes a potential entrant, but not an actual entrant, on a route. We show that our model can generate price declines of the observed size, and we also consider other evidence that these declines are motivated by deterrence.
1 Introduction

In markets where entry costs are significant, a dominant incumbent firm may have an incentive to take actions that deter entry by rivals. Since at least the influential work of Bain (1949), many theoretical models have explored possible examples of entry deterring strategies (Tirole (1994), Belleflamme and Peitz (2009)). However, although these models are widely taught, there is very little evidence that that these models can explain real world data, either qualitatively or quantitatively. One reason for this is that most of these models have a very simple two-period structure, which makes it unclear what one should expect to observe when an incumbent and potential entrants interact repeatedly as they will typically do in the real-world. The lack of both evidence and understanding of what would happen with repeated interaction are particularly striking for models where strategic behavior is driven by an asymmetry of information between incumbents and entrants, as in Milgrom and Roberts (1982)’s limit pricing model (MR hereafter). These weaknesses in the literature may have the practical effect of making it more difficult for the antitrust authorities to aggressively pursue cases against business practices that might be intended to prevent entry.

In this paper we develop a dynamic version of the MR model. In our model an incumbent monopolist’s costs, which are private information, are correlated over time, but not perfectly persistent. A long-lived potential entrant observes the price set by the incumbent and its quantity, and, as in the MR model, the incumbent can try to signal that it has a low cost by setting a low price. The imperfect persistence of the incumbent’s costs create an incentive for limit pricing in every period until entry occurs, at which point we assume (like MR) that the game becomes one of perfect information, where the duopolists observe each other’s costs. While the model is a dynamic game with persistent asymmetric information, it is actually quite tractable. In particular, we show, using results from the theoretical literature, that under some plausible conditions on the primitives of the model, there is a unique Markov Perfect Bayesian Equilibrium under refinement, where the incumbent’s pricing strategy perfectly reveals its costs in every period. As a result, the potential entrant’s beliefs on the equilibrium path have a very simple form and equilibrium pricing and entry strategies can be easily computed.

We exploit the model’s tractability to try to consider whether our model can explain the real-world phenomenon that the prices of incumbent airlines appear to decline when Southwest, the most well-known US low-cost carrier, becomes a potential entrant on an airport-pair route (meaning that it serves both endpoints without yet serving the route). Morrison (2001) and Goolsbee and Syverson (2008) document that these price declines can be very large: up to 20% of previous prices, and at least as large the additional falls which take places if and when Southwest actually enters the route.
We therefore want to know whether a suitably parameterized version of our model can generate prices declines of this size.

Rather than imposing our model on the data, we use a calibration approach, estimating the main parameters of our model from periods and markets where we would not expect this type of strategic behavior, and then testing what they predict for the markets where we believe the model may apply. Our preliminary results indicate that our model can indeed generate price declines of the magnitude observed in the data. We also provide some new evidence about how price declines vary with market size which, following the logic of Ellison and Ellison (2011), provides some additional evidence that these price declines might reflect entry deterring rather than, for example, entry accommodating strategies (for example, accommodating strategies aimed at increasing consumers investment in the incumbent’s frequent-flyer program).

Our paper is related and makes contributions to at least four distinct literatures. The first is the theoretical literature on models of limit pricing and, more generally, strategic investment under asymmetric information. The possibility of firms setting low prices to deter rivals from entering a market has its origins in the work of Bain (1949). The obvious question is how a pre-entry price could affect a potential entrant’s expectations of its post-entry profits. One approach was that potential entrants might view the incumbent as committed to the pre-entry price even if entry occurs (e.g., Gaskins (1971)), although it was not clear why the potential entrant’s expectation was reasonable, as the source of commitment was not specified and it appeared more plausible that incumbents would change their prices if entry occurred (Friedman (1979)).

Milgrom and Roberts (1982) addressed this problem by allowing for asymmetric information about the profitability of entry which meant that incumbents could use pre-entry prices to signal information to the potential entrant, which could affect its entry decision, even though the incumbents could change their prices post-entry. Another feature of these models is that, depending on the type of equilibrium played, limit pricing can enhance efficiency by leading to the same entry decisions that would be taken if information was complete and lowering prices prior to entry. While the majority of the signalling literature has focused on two-period models, which are limited in their ability to yield testable predictions for settings where firms set prices repeatedly, a much more recent, but less applied, literature has looked at games where both sender and receiver are long-lived and the sender can signal repeatedly. In some of these papers the type of the sender is fixed over time, so that signaling happens mainly in initial periods (Kaya (2009) and

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1 Although notions of the idea can be found in the even earlier writings of Kaldor (1935) and Clark (1940).

2 Other models of limit pricing around this time which also did not allow for rational potential entrants include Kamien and Schwartz (1971), Baron (1973), De Bondt (1976) and Lippman (1980).
Toxvaerd (2010)). Instead we follow Roddie (2012a) and Roddie (2012b) in assuming that the sender’s type evolves over time. In those papers Roddie computes some examples of a quantity-setting game played between duopolists, with the marginal cost of one of the firms being private information and serially correlated. He shows how the privately-informed firm can end up looking like a Stackelberg leader in equilibrium.

The second related literature is the limited empirical literature that has looked for evidence of strategic entry deterrence. Most traditional tests looked at identifying differences in the investment decisions of different types of firms (e.g., Lieberman (1987) study of excess capacity in chemical industries) or tried to identify indirect evidence of deterrence by looking for a correlation between an incumbent firm’s investment decisions and the subsequent entry decisions of competitors (e.g., Chevalier (1995) analysis of supermarkets), without necessarily proving that the investment decisions are motivated by deterrence.³ Smiley (1988) takes a different approach by conducting a survey of managers in order to identify which strategies they believe are used to deter entry. In this survey some managers do report using (limit) pricing to either prevent entry or limit the growth of rivals, although it is less commonly cited than alternatives such as engaging in excess advertising. Ellison and Ellison (2011) (EE, hereafter) develop and apply a different, but still qualitative, approach to testing for strategic behavior based on trying to identify non-monotonicities in the level of incumbent investment with respect to exogenous factors that affect the attractiveness of entry. The theoretical structure behind the EE test is consistent with asymmetric information models such as MR, and we apply it here finding some evidence in favor of a deterrence story, like EE and Dafny (2005).

However, we go beyond this qualitative test to consider whether our specific model can generate the size of the price changes observed in the airline data. In this regard we are closer to two structural empirical papers, Snider (2009) and Chicu (2012), that estimate dynamic oligopoly models with complete information (at least up to iid payoff shocks), in the spirit of Ericson and Pakes (1995), to try to quantify how far capacity investment is used to deter entry. We differ from these papers in considering a dynamic model with asymmetric information and deriving conditions under which the equilibrium that we look at is unique. We also approach our empirical exercise by calibrating our model rather than imposing our model, which is certainly a simplification of reality, on the data. We regard this as a sensible approach given our primary interest is in testing whether a dynamic limit pricing can explain the main patterns in the data.

³Salvo (2010) argues that Brazilian cement producers set prices at a level that prevents imports being profitable. However, this setting differs from standard models of strategic investment where a potential entrant has to pay a sunk cost to enter the market.
The third related literature has looked at the response of incumbents to potential entry in the airline industry. As mentioned above, Morrison (2001) and Goolsbee and Syverson (2008) identify what has sometimes been called the ‘Southwest effect’ where the emergence of Southwest as a potential, rather than actual, entrant on the route leads to a substantial decline in prices. Entry leads to further large price declines. In their analyses, however, the exact reason why prices fall is not identified. The two obvious alternatives are that incumbents cut prices in order to try to deter entry or they cut prices in order to affect the nature of competition if and when Southwest enters (i.e., as part of an entry accommodation strategy). In our paper we focus on a subset of markets, ones dominated by a single incumbent prior to Southwest becoming a potential entrant, where we believe that an entry deterrence story makes particular sense, and which could also be consistent with a model where a monopolist incumbent is assumed. While we focus on explaining price declines, Goetz and Shapiro (2012) argue that incumbent airlines also used strategic alliances in order to try to deter entry by low-cost carriers such as Southwest.

The fourth related literature is the general literature on the modeling of dynamic games. Almost all of the existing literature follows Ericson and Pakes (1995) in assuming that firms either observe all payoff-relevant variables or observe them up to some payoff shocks that are iid over time. Given these assumptions, there is no scope for firms to signal information about the future profitability of entry, or any need for potential entrants to form beliefs about their opponents’ types. This means that the effects of the strategic investment incentives that arise in the incomplete information models of MR and Fudenberg and Tirole (1986) cannot be studied in a fully dynamic setting. This is a serious limitation because it is not immediately obvious what these models predict when, as is typically the case in reality, firms set prices repeatedly. For example, if a firm wants to signal that it has low costs does it need to set low prices in every period, or can it ‘get away’ with only setting a low price in an initial period? In a recent paper, Fershtman and Pakes (2012) provide a general framework for modeling a dynamic game with persistent incomplete information but, because it can be difficult to keep track of players’ beliefs, they argue for replacing standard equilibrium concepts such as PBE with a concept, Experience Based Equilibrium, where beliefs are not specified. This alternative concept makes it very difficult to think about strategic incentives to signal information, as these should naturally operate by

\[ \text{In the theoretical literature, some models (e.g., Waldman (1983), McLean and Riordan (1989)) show that when there are several incumbents the incentive of any one incumbent to engage in strategic investment to deter entry is severely limited due the free-rider problem (deterrence is a public good for the incumbents). While this argument seems intuitive, several alternative models show that there are circumstances in which oligopolists can still engage in substantially entry deterring behavior (e.g., Gilbert and Vives (1986), Waldman (1987), Kirman and Masson (1986)). It would be interesting to extend our model to the case of several incumbents, although this could be difficult if incumbents are asymmetric.} \]
affecting beliefs. In contrast, we propose a simple dynamic model for studying the effects of signaling incentives on pricing using PBE as our equilibrium concept. We can do so because our model has an equilibrium where the incumbent’s choice perfectly reveals his cost (type) to the potential entrant, so that beliefs have a particularly simple form and it is not necessary to keep track of beliefs as a function of the complete history of the game. We can also show that, under a standard type of refinement, this equilibrium will be unique (or more precisely, behavior on the equilibrium path is uniquely determined), which also makes the model highly suitable for empirical work.

This paper is organized as follows. Section 2 lays out our model of dynamic limit pricing and characterizes the equilibrium. Section 3 presents the airline data and documents the incumbent price declines found in other papers focused on the impact of Southwest becoming a potential entrant. Section 4 provides both the EE-style evidence in favor of deterrence and the results of our calibration exercise. Section 5 concludes.

2 Model

In this section we develop our theoretical model of a dynamic entry deterrence game with serially correlated asymmetric information. We describe a Markov Perfect Bayesian equilibrium of the model where incumbents perfectly reveal their type each period and set out the conditions under which this equilibrium both exists and is unique under a dynamic version of the D1 refinement. We then explain how the equilibrium strategies can be computed, and present some examples based on simple, but standard, demand models.

We present the model in an abstract way, so that we can be clear about what features of our results are general. We conclude the section with a discussion of why we believe that is reasonable to try to apply the model to study the Southwest effect.

2.1 Set-Up

There is an finite sequence of discrete time periods, \( t = 1, \ldots, T \), two long-lived firms and a common discount factor of \( \beta \). We assume finite \( T \) so that we can apply backwards induction to prove certain properties of the model, but, when solving the model, we will consider large \( T \) and focus on the pricing

\(^5\)Fershtman and Pakes (2012) consider a model with a finite action space. This may also limit a player’s ability to signal its type, by requiring at least some pooling when there are more types than possible actions. In contrast we consider a game where the signaling firm chooses a continuous action (price).
strategies that will be almost stationary in the early part of the game.\footnote{We could follow Toxvaerd (musical chairs) who shows properties for an infinite horizon game by taking the $T \to \infty$ limit of finite horizon games for which properties can be shown using backwards induction.}

At the start of the game, firm $I$ is an incumbent, who is assumed to remain in the market forever, and firm $E$ is a long-lived potential entrant. It is assumed that $E$ will remain as a potential entrant until it enters, and that, once it enters, it will also remain in the market forever. The marginal costs of the firms, $c_{Et}$ and $c_{It}$ lie on compact intervals $C_E := [c_{E}, \overline{c}_E]$ and $C_I := [c_{I}, \overline{c}_I]$ and evolve, independently, according to Markov processes $\psi_I : c_{It-1} \rightarrow c_{It}$ and $\psi_E : c_{Et-1} \rightarrow c_{Et}$, with full support (i.e., costs can evolve to any point on the support in the next period), and we will denote the conditional pdfs $\psi_I(c_{It}|c_{It-1})$ and $\psi_E(c_{Et}|c_{Et-1})$. We will assume that these processes have two properties:

1. $\psi_I(c_{It}|c_{It-1})$ and $\psi_E(c_{Et}|c_{Et-1})$ are continuous and differentiable.

2. $\psi_I(c_{It}|c_{It-1})$ and $\psi_E(c_{Et}|c_{Et-1})$ are strictly increasing (i.e., a higher cost in one period implies that higher costs in the following period are more likely). Specifically we will require that for all $c_{jt-1}$ there is some $x'(c_{jt-1})$ such that $\frac{\partial \psi_I(c_{jt}|c_{jt-1})}{\partial c_{jt-1}}|_{c_{jt}=x'} = 0$ and $\frac{\partial \psi_I(c_{jt}|c_{jt-1})}{\partial c_{jt-1}} < 0$ for all $c_{jt} < x'$ and $\frac{\partial \psi_I(c_{jt}|c_{jt-1})}{\partial c_{jt-1}} > 0$ for all $c_{jt} > x'$. Obviously it will also be the case that $\int_{c_{jt}}^{\overline{c}_j} \frac{\partial \psi_I(c_{jt}|c_{jt-1})}{\partial c_{jt-1}} dc_{jt} = 0$.

To enter in period $t$, $E$ has to pay a private-information sunk entry cost $\kappa_t$, which is an iid draw from a time-invariant distribution $G$ (density $g$) on $K = [0, \overline{k}]$. Demand is assumed to be common knowledge and fixed, although it would be straightforward to extend the model to allow for time-varying demand as long as it is observed by both firms.

### 2.1.1 Pre-Entry Game

Before $E$ has entered, so $I$ is a monopolist, $E$ does not observe $c_{It}$. $E$ does observe the whole history of the game to that point. The timing of the game in each of these periods is as follows:

1. $I$ and $E$ observe $c_{Et}$ (the entrant’s marginal cost).

2. $E$ observes $\kappa_t$.

3. $I$ sets a price $p_{It}$, and receives profit

$$\pi^M_I(p_{It}, c_{It}) = q^M(p_{It})(p_{It} - c_{It}) \quad (1)$$

where $q^M(p_{It})$ is the downward-sloping demand function of a monopolist. The incumbent can choose a price from the compact interval $[\underline{p}, \overline{p}]$ where $\overline{p}$ is above the static monopoly price that
an incumbent with cost $c_I$ would choose. $p$ is low enough such that in equilibrium no incumbent would want to choose it, whatever effects this had on the beliefs of the potential entrant.\(^7\) We will assume that $\pi_I^M(p_{It}, c_{It})$ has a unique maximum, is in price is concave at this maximum and is strictly quasi-concave for all prices $[p, \bar{p}]$. This will be true for standard demand functions such as linear, logit and nested logit.

4. $E$ observes $p_{It}$.

5. $E$ decides whether to enter. If so, it is active at the start of the following period.

6. Marginal costs of both firms evolve independently according to $\psi_I$ and $\psi_E$.

2.1.2 Post-Entry Game

We assume that once $E$ has entered both firms observe both marginal costs, so there is no scope for further signalling, and that they receive static equilibrium flow profits $\pi^D_I(c_{It}, c_{Et})$ and $\pi^D_E(c_{Et}, c_{It})$, where the different functions allow for fixed quality differences between the firms. The firms’ equilibrium outputs are $q^D_I(c_{It}, c_{Et})$ and $q^D_E(c_{Et}, c_{It})$. The choice variables of the firms in the static duopoly game are $a_{It}$ and $a_{Et}$. We make a number of assumptions on the duopoly game:

1. $\pi^D_I(c_{It}, c_{Et}), \pi^D_E(c_{Et}, c_{It}) \geq 0$ for all $c_{It}, c_{Et}$, which rationalizes why neither firms exists.

2. $\pi^D_I(c_{It}, c_{Et})$ and $\pi^D_E(c_{Et}, c_{It})$ are continuous and differentiable in both arguments.

3. $q^D_I(c_{It}, c_{Et}) - q^M_{It}(c_{It}) - \frac{\partial \pi^D_I(c_{It}, c_{Et})}{\partial a_{Et}} \frac{\partial a_{Et}}{\partial c_{It}} < 0$ for all $(c_{It}, c_{Et})$. Note that $\frac{\partial \pi^D_I(c_{It}, c_{Et})}{\partial a_{Et}} \frac{\partial a_{Et}}{\partial c_{It}}$ will be positive if there is price-setting in the duopoly game, which makes this the natural case to think about.\(^8\)

2.1.3 Comparison with Milgrom and Roberts (1982, MR)

MR provide the classic two-period model of limit pricing in which one firm (our $I$) is a monopolist in the first period, and a second firm ($E$) can observe its price or output before deciding whether to enter in the second period. In the model on which they focus, the marginal costs of both firms are private-information in the first period while $E$’s entry cost is commonly known. As in our model

\(^7\)Potentially this formulation may create a problem because, for a given cost and demand formulation we might require $p < 0$ in order to insure that all types are able to separate. In this sense, it might be more palatable to assume that the incumbent chooses a quantity on $[0, \bar{q}]$ where $\bar{q}$ is sufficiently high. All of our theoretical results hold when the monopolist sets a quantity, but we choose not to present our model in this way because it is more natural to assume price-setting given that we want to talk about limit pricing.

\(^8\)Note that this condition is similar to one identified by ?, p. 370, for the MR model.
the marginal costs of both firms, which in their case do not vary over time, are publicly revealed if $E$ enters, so that, with entry, second period competition is static Cournot or Bertrand. $I$ can attempt to deter entry by using its first-period quantity/price choice to signal its marginal cost to $E$.

Our cost assumptions differ in two respects. First, in our model the incumbent’s marginal costs can evolve over time, specifically in a positively serially correlated way. We need correlation (which takes an extreme form in the MR model) in order for the monopolist to have something that will affect an entrant’s profit to signal. On the other hand, we need the incumbent’s marginal costs to be able to change from period-to-period in order to generate an incentive for the incumbent to engage in limit-pricing in every period before entry occurs. As a comparison, Kaya (2009) and Toxvaerd (2010) consider dynamic models where the incumbent’s type is fixed over time, where the natural equilibrium to consider is one where the incumbent only signals in initial periods.

Second, we assume that the entrant’s marginal costs, which we allow to be serially correlated, are observed, but that its entry costs, which are iid draws, are not. What is important for our model is that we want to focus on learning by only one player ($E$) and we need that entry to be probabilistic from the perspective of the incumbent. In these respects we are just like MR. In contrast, if we assumed that $E$ had a serially correlated unobserved state variable (either marginal costs or entry costs) then the incumbent would be able to draw inferences about that variable from $E$’s entry decisions, leading to a two-way model of learning. This would significantly complicate the model, and the equilibrium could not be fully revealing as a failure to enter could only imperfectly reveal $E$’s costs. Therefore to get what we want we could assume that either entry costs are observed with $c_{Et}$ is unobserved and iid over time, or that entry costs are unobserved and iid while marginal costs are observed and possibly serially correlated. We chose the second option for two reasons: first, in the entry literature it is quite standard to treat entry cost draws as being private information and independent; and, second, it seems natural to allow for serial correlation in the entrant’s marginal costs given our assumptions on $I$’s marginal costs and the evidence of our data described in Section 3. We will also argue below that Southwest’s business model, which involves a simpler network structure than legacy carriers, adds some plausibility to the idea that Southwest’s operating costs might be more transparent than those of other carriers.
2.2 Equilibrium

Unique Nash equilibrium behavior post-entry, where two firms play a complete information duopoly game, is assumed. We are therefore interested in characterizing play before $E$ enters. Our basic equilibrium concept is that of Markov Perfect Bayesian Nash equilibrium (MPBE) (Roddie (2012a), Toxvaerd (2008)). The definition of a MPBE requires, for each period:

- a pricing strategy for $I$, as a function of both firms’ marginal costs ($\varsigma_{It} : (c_{It}, c_{Et}) \rightarrow p_{It}$).
- an entry rule for $E$, as a function of its beliefs about $I$’s marginal cost, its own marginal costs and its own entry cost draw, $\sigma_{Et} : (\hat{c}_{It}, c_{Et}, \kappa_t) \rightarrow \{\text{Enter}, \text{Stay Out}\}$; and
- a specification of $E$’s beliefs about $I$’s marginal costs given all possible histories of the game.

In an equilibrium, for all possible $c_E$ and $c_I$, $E$’s entry rule should be optimal given its beliefs, its beliefs should be consistent with $I$’s strategy and the evolution of marginal costs, and $I$’s pricing rule will be optimal given what $E$ will infer from $I$’s price and how $E$ will react based on these inferences.

**Fully Separating Riley Equilibrium (FSRE).** We will consider an equilibrium where $I$’s price perfectly reveals its current marginal cost, i.e., there is full separation, and where signalling is achieved at minimum cost to $I$ subject to the incentive compatibility constraints being satisfied. In the signalling literature this is known as the Riley equilibrium (Riley (1979)).

The following proposition contains our main theoretical results

**Proposition 1** Consider the following strategies and on-the-equilibrium-path beliefs: (i) $E$’s entry strategy will be to enter if entry costs $\kappa_t$ are lower than a threshold $\kappa^*_t(\hat{c}_{It}, c_{Et})$, where $\hat{c}_{It}$ is $E$’s (point) belief about $I$’s marginal cost

$$\kappa^*_t(\hat{c}_{It}, c_{Et}) = E_t(\phi^E_{t+1}|\hat{c}_{It}, c_{Et}) - E_t(V^E_{t+1}|\hat{c}_{It}, c_{Et})$$

where $V^E_{t+1}$ is $E$’s value to being a potential entrant in period $t + 1$ (i.e., if it does not enter now)\(^{10}\) given equilibrium behavior at $t + 1$

$$V^E_{t+1} = \max\{\beta E_{t+1}(V^E_{t+2}|\hat{c}_{It+1}, c_{Et+1}), E_{t+1}(\phi^E_{t+2}|\hat{c}_{It+1}, c_{Et+1}) - \kappa_{t+1}\}$$

\(^{9}\)Existence and uniqueness will depend on the particular form of demand assumed, and will hold for the forms we consider (linear, logit, nested logit with single product firms and linear marginal cost).

\(^{10}\)The value is calculated at point 2 in the within-period timeline described above (i.e., when $c_{Et+1}$ and $\kappa_{t+1}$ are known).
and $\phi_{t+1}^E$ is $E$'s value to being a duopolist in period $t+1$ (which assumes it has entered prior to $t+1$).\(^{11}\) The threshold $\kappa_t^e(c_{It}, c_{Et})$ is strictly decreasing in $c_{Et}$ and strictly increasing in $\widehat{c}_{It}$; (ii) $I$'s pricing strategy $\varsigma_{It} : (c_{It}, c_{Et}) \rightarrow p_{It}^*$ will be the solution to a differential equation

\begin{equation}
\frac{\partial p_{It}^*}{\partial c_{It}} = \beta [g(.) \frac{\partial \kappa_t^e(c_{It}, c_{Et})}{\partial c_{It}}] \left[ \mathbb{E}_t[V_{t+1}^I|c_{It}, c_{Et}] - \mathbb{E}_t[\phi_{t+1}^I|c_{It}, c_{Et}] \right] \left[ q^M(p_{It}) + \frac{\partial q^M(p_{It})}{\partial p_{It}}(p_{It} - c_{It}) \right] \tag{2}
\end{equation}

and an upper boundary condition $p_{It}^*(\widehat{c}_{It}) = p^{\text{static monopoly}}(\widehat{c}_{It})$. $\mathbb{E}_t[V_{t+1}^I|c_{It}, c_{Et}]$ is $I$'s expected value of being a monopolist at the start of period $t+1$ given current costs and equilibrium behavior at $t+1$.\(^{12}\) $\mathbb{E}_t[\phi_{t+1}^I|c_{It}, c_{Et}]$ is its expected value of being a duopolist in period $t+1$;\(^{13}\) (iii) $E$'s on-the-equilibrium path beliefs: observing an output $q_{It}$, $E$ believes that $I$'s marginal cost is $\varsigma_{It}^{-1}(p_{It}, c_{Et})$. These are the unique MPBE strategies and equilibrium-path beliefs consistent with a recursive application of the D1 refinement. For completeness we assume that if $E$ observes a price which is not in the range of $\varsigma_{It}(c_{It}, c_{Et})$ it believes that the incumbent has marginal cost $\overline{c}_{It}$.

\textbf{Proof.} See Appendix. \(\blacksquare\)

We show existence and uniqueness by applying existing results from the theoretical literature on signaling models. The first result that we use is from Mailath and von Thadden (forthcoming) conditions on signaling payoffs\(^{14}\) that lead to a unique separating equilibrium, where the signaler’s strategy is determined by a differential equation and a boundary condition.\(^{15}\) The key conditions are type monotonicity (a given price reduction is always more expensive for an incumbent with higher marginal costs), belief monotonicity (the incumbent always benefits when the entrant believes that he has lower $c_{It}$, which reflects the monotonicity of the entrant’s entry rule) and a single-crossing condition (implies that a lower cost incumbent is always willing to cut price slightly more in order to differentiate itself from higher cost incumbents). Our contribution is to show that our assumptions on the primitives of the model are \textit{sufficient} for $I$'s signaling payoff function to satisfy these conditions in every period by applying backwards induction. Our conditions are not necessary so that our equilibrium may exist more generally.

Mailath and von Thadden’s results do not rule out the possible existence of pooling equilibria. To do so we apply the D1 refinement (Cho and Sobel (1990), Ramey (1996)), which is a restriction on the

\(^{11}\)The value is defined at the point where it knows $c_{It+1}$ and $c_{Et+1}$.

\(^{12}\)This value is defined after point 1 in the timeline, when $I$ knows $c_{Et+1}$.

\(^{13}\)This value is calculated at the point where $c_{It+1}$ and $c_{Et+1}$ are known.

\(^{14}\)The signaling payoff function can be written as $\Pi_t(c_{It}, c_{It}', p_{It}, c_{Et})$ where $\widehat{c}_{It}$ is $E$’s point-belief about the incumbent’s marginal cost when taking its period $t$ entry decision. An alternative way of writing the payoff function that is used when ruling out pooling equilibria is $\Pi_t(c_{It}, k_{Et}, p_{It}, c_{Et})$ where $k_{Et}$ is the threshold used by the potential entrant.

\(^{15}\)We use Theorem 1 of \?, which is effectively a re-statement of a result from Mailath (1987).
possible inferences that \( E \) could make when observing off-the-equilibrium-path actions. Specifically, in a one-shot signaling game, D1 restricts how off-the-equilibrium path beliefs are interpreted, and it requires a receiver to place zero posterior weight on a signaler having a type \( \theta_1 \) if there is another type \( \theta_2 \) who would have a strictly greater incentive to deviate for any set of post-signal beliefs that would give \( \theta_1 \) an incentive to deviate. Theorem 3 of Ramey (1996) shows that as long as a pool does not involve signalers choosing (in our setting) the lowest possible prices, pooling equilibria can be eliminated when the signaling payoff function satisfies a single-crossing condition.

Applying D1 in a setting with repeated signaling is potentially complicated by the possibility that an off-the-equilibrium path signal in one period could change how the receiver interprets signals in future periods. Here we follow Roddie (2012a) in applying a recursive version of D1, where we work backwards through the game, applying the refinement in each period, assuming equilibrium behavior and inferences in future periods.

### 2.3 Examples and Comparative Statics

![Graphs showing price vs cost for small and large market sizes.]

### 2.4 Application of the Model to the Understanding the Reactions of the Incumbent Airlines to Potential Entry by Southwest

We want to test whether our model can potentially explain why incumbent carriers cut prices substantially when faced by the threat of potential entry by Southwest. To make any such argument plausible we need to explain why our model and its assumptions may be a good match for a relevant set of airline markets.

In our data, there are a wide range of markets, many of which have multiple incumbent carriers.
Our model assumes a monopolist incumbent, so in our empirical work we focus on a subset of markets where there is a single dominant incumbent before Southwest enters, leaving the question of whether a signaling model with multiple firms could also explain why prices fall in these markets as well as an open question for further research.\textsuperscript{16} In practice, there are more potential entrants into these markets that just Southwest, but given Southwest’s position as the most successful low-cost carrier in the US, its rapid growth since the 1980s and the large effect that it had on prices post-entry, it is plausible that it is the potential entrant that incumbent carriers would be most interested in trying to deter. Its potential entry) has on prices. We also restrict ourselves to focusing on markets where both the incumbent and Southwest remain active once entry occurs, consistent with another assumption of our model. Southwest also provides a good example in this regard given that the exit of Southwest from a route-market is relatively rare.

Our model also makes strong assumptions that the incumbent’s marginal costs are not observed, at least prior to entry, whereas those of Southwest are publicly observed. The private-information assumption may seem problematic given that some input prices, such as fuel costs, are easily observed and all firms have access to relatively high-quality data on fares and passengers flows, while Southwest’s success indicates that it has a good understanding of the economics of the industry. Here it is important to make a distinction between different elements of a firm’s marginal costs. While the formal model that we laid out above did not allow for an observed, serially-correlated component of these costs (e.g., fuel prices), we could easily extend our model, at the cost of additional notation, to allow for this. All that we need is that some component of the incumbent’s costs are serially correlated and unobserved. Here we have in mind the fact that the appropriate marginal or average costs for a legacy carrier that operates a hub-and-spoke network on a particular segment will depend on how passengers traveling on this segment flow onto the rest of its network, together with costs of capacity, congestion and any economics of density at the carrier’s hub. The difficulties of measuring these costs are illustrated by the disagreements raised in legal cases concerning alleged predatory pricing in the airline industry (e.g., Edlin and Farrell (2004)), when the internal accounting data on the airlines concerned was also available. Therefore it seems plausible that there is going to be some component of costs that an incumbent carrier might be able to signal.\textsuperscript{17} At the same time, the fact that Southwest operates a primarily point-to-point network with no official hubs and a common aircraft means that its costs may

\textsuperscript{16}As noted in the Introduction (footnote 4), strategic investment incentives are not necessarily weaker when there are several incumbents.

\textsuperscript{17}Note that the equilibrium in our model would not be affected if we allowed for the incumbent’s marginal costs in period \( t \) to be revealed once the potential entrant has made its entry decision. This provides a further rationalization given that publicly-available airline data is only released with a two-quarter delay.
be easier for an incumbent to predict accurately. It is probably harder to rationalize the assumption that Southwest has complete information on the incumbent’s costs once it enters. In general it seems plausible that participation in a market provides a firm with greater information about its rival than it could have had pre-entry. This assumption is primarily for simplicity and we could still have a pre-entry equilibrium qualitatively similar to the one we look at if the post-entry game involved incomplete information as long as the post-entry profit functions still satisfied the appropriate conditions.

3 Data

Our aim is to evaluate the model’s ability to match observed pricing patterns of incumbent airlines once Southwest becomes a potential entrant. To do so we gather data from data two main sources. The first is the U.S. Department of Transportation’s Origin-Destination Survey of Airline Passenger Traffic (also known as DB1B). The DB1B data represent a 10 percent quarterly sample of all domestic tickets in each quarter. This will be the main data set that we will use to construct measures of the prices that airlines charge and the number of passengers they carry. We will also use another data source from U.S. Department of Transportation called the T-100 database, which contains monthly measures of the number of passengers and flights at the carrier-segment level. This latter database will be useful in defining whether Southwest is a potential entrant, as described below.

We define a market to be a non-directional airport pair, as do Goolsbee and Syverson (2008), and a period will be a quarter (the frequency of the DB1B data). We focus on markets for which Southwest at some point became a potential entrant. Following convention in the literature (e.g. Goolsbee and Syverson (2008), Berry (1992)), we say that Southwest is a potential entrant into, but has not yet entered, the market when it flies passengers in and out of each of the market’s endpoints, but does not serve the market. We date the quarter in which Southwest becomes a potential entrant into a market as being when, according to the T-100 data, Southwest flies at least one flight into or out of each of the market’s endpoints. In accordance with our model, we select markets that are dominated by a single incumbent prior to Southwest becoming a potential entrant. To do so, we use the DB1B data to construct, for each market, the number of passengers that each firm carries round-trip, directly and indirectly. We count oneway tickets as being one half of a round-trip ticket. Our baseline sample consists of those markets for which, when Southwest becomes a potential entrant, there is an incumbent carrier that carries at least 80% of the direct traffic and at least 50% of total traffic.

\footnote{An alternative approach is to define markets as city pairs.}
We are left with 139 markets. We say that Southwest has entered a market when it flies at least 50 direct passengers in the DB1B data between the market’s endpoints.\textsuperscript{19} By this definition, Southwest enters 46 of our sample markets. We drop quarters following Southwest’s exit from a market it has entered, or when it ceases to be a potential entrant into a market that it has not yet entered. We calculate market size as the predicted number of passengers from poisson regression of the number of total passengers traveling on a route on a third order polynomial in the (log of) the one-year-lagged number of passengers traveling into and out of each endpoint and the distance between the endpoints as well as separate controls for whether the route is particularly long (greater than 2300 miles round trip), which is included on its own and also is interacted with separate market dummy variables and multiply the prediction for our data by 3.5. Benkard, Bodoh-Creed, and Lazarev (2010) use a similar approach.

Table 1 summarizes this sample of data. The table divides the data into three time periods. Period 1 are the quarters before Southwest becomes a potential entrant. Period 2 are the quarters beginning when Southwest becomes a potential entrant up until the last quarter before Southwest enters the market, if it does. Period 3 are the quarters starting with, and following, Southwest’s entrance into a market. The table shows three interesting facts about the data. First, the average price charged by the dominant incumbent is lower upon Southwest becoming a potential entrant (Period 2 versus Period 1). Second, if Southwest does enter a market, the average incumbent price is lowered further (Period 3 versus Period 2). Third, even with this lower price, the average incumbent fare is still greater than the average fare charged by Southwest.

To more fully explore the basic patterns in Table 1, we replicate the main empirical framework of Goolsbee and Syverson (2008) using our sample of markets dominated by a single incumbent (Goolsbee and Syverson looked across markets that varied in the number of incumbents). To measure the impact of Southwest becoming a potential entrant into a dominant incumbent’s market, we use a standard event study type of approach by running the following regression:

\[
\ln(p_{j,m,t}) = \gamma_m + \tau_t + \alpha X_{j,m,t} + \sum_{\tau=-8}^{8} \beta_{\tau} SWPE_{m,t_{0}+\tau} + \sum_{\tau=0}^{3} \beta_{\tau} SWPE_{m,t_{e}+\tau} + \varepsilon_{j,m,t}
\]  

\textsuperscript{19}\text{Beginning in the first quarter of 2005, Southwest entered into a series of increasingly expansive code sharing agreements with ATA and ended up buying ATA at the end of 2008 after it had gone bankrupt a few months earlier. For those markets and quarters that Southwest had a code sharing agreement with ATA, we consider Southwest to be a potential entrant if ATA meets the criteria outlined above for being a potential entrant.}
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Period 1: $t &lt; t_0$</th>
<th></th>
<th>Period 2: $t_0 \leq t &lt; t_e$</th>
<th></th>
<th>Period 3: $t_e \leq t$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Incumbent Fare</td>
<td>541.79</td>
<td>171.20</td>
<td>445.01</td>
<td>125.66</td>
<td>280.30</td>
<td>80.61</td>
</tr>
<tr>
<td>Incumbent Market Share</td>
<td>0.230</td>
<td>0.107</td>
<td>0.242</td>
<td>0.108</td>
<td>0.160</td>
<td>0.054</td>
</tr>
<tr>
<td>Southwest Fare</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>225.42</td>
<td>66.75</td>
</tr>
<tr>
<td>Southwest Market Share</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.155</td>
<td>0.079</td>
</tr>
<tr>
<td>Number Markets</td>
<td>139</td>
<td>139</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fare is the average, across firm and quarter (within a period) passenger weighted average price paid per ticket, measured in 2009 $. Passengers are the average, across firm and quarter (within a period) round-trip passengers carried during a quarter. $t_0$ denotes the quarter in which Southwest becomes a potential entrant. $t_e$ denotes the quarter in which Southwest becomes a potential entrant. Period 1 is the quarters prior to Southwest becoming a potential entrant. Period 2 are the quarters beginning when Southwest becomes a potential entrant up until the last quarter before Southwest enters the market, if it does. Period 3 are the quarters starting with, and following, Southwest’s entrance into a market.

where $p_{j,m,t}$ is the average fare charged by carrier $j$ in market $m$ in quarter $t$, $\gamma_m$ are market fixed effects, $\tau_t$ are quarter fixed effects, $X$ includes the number of potential entrants into the market and the number of other firms currently serving the market (besides the dominant incumbent), $SWPE_{m,t_0+\tau}$ is an indicator for Southwest being a potential entrant into market $m$ at quarter $t_0 + \tau$ and $SWE_{m,t_e+\tau}$ is an indicator for Southwest having entered market $m$ at quarter $t_e + \tau$. So with a slight abuse of notation we let $t_0$ indicate the quarter in which Southwest becomes a potential entrant and $t_e$ denote the quarter in which Southwest becomes a potential entrant. Our main interest is in the $\beta_e$ on these last two indicators, which measure the effect of Southwest becoming a potential entrant, and entrant on a dominant incumbent’s fare. As in Goolsbee and Syverson, these $\beta$ coefficients are mutually exclusive, so they are not additive, and are relative to a base period of the logged average fare over the quarters prior to Southwest becoming a potential entrant.

Table 2 displays the regression results for estimating Equation 3. Standard errors are clustered at the market-carrier level to allow for correlation in the error terms over time within a carrier-market combination. As in Goolsbee and Syverson, the observations are weighted by the average number of passengers flying with the carrier on this market over the sample so that we can aggregate responses to Southwest. Focusing on the estimates of the $\beta$ coefficients, we begin including dummy variables for the eighth quarter prior to Southwest becoming a potential entrant, and so all effects will be relative to the average price set by the incumbent prior to two years before Southwest became a potential entrant.

\footnote{These of course vary across markets.}
entrant. Incumbent fares begin falling a bit before Southwest becomes a potential entrant into the market.\footnote{This is not terribly surprising as the incumbent may anticipate Southwest becoming a potential entrant, and respond to it, prior to Southwest actually starting service out of each endpoint. Some ways that the incumbent may anticipate this is by observing or learning that Southwest is negotiating landing slots, gate leases, labor contracts as well as commencing advertising and sell tickets, all of which are necessary activities for Southwest to engage in prior to starting service in an airport. Goolsbee and Syverson find similar trends in price before Southwest becomes a potential entrant.} By the time Southwest becomes a potential entrant, the incumbent has lowered its price by 10\% relative to its price two years before Southwest becoming a potential entrant. We separately estimate the impact of Southwest being a potential entrant for the following five quarters, and then estimate the average effect for quarters six through 12 after Southwest becomes a potential entrant and the average effect for all quarters from the 13th quarter on after Southwest becomes a potential entrant. Prices continue to fall during the period in which Southwest is a potential entrant but has not yet begun serving the market. Goolsbee and Syverson find somewhat larger price declines (about 17\%) based on a different sample of markets in which there may be no dominant carrier, but the trends of price declines that we find are very similar to theirs both before and after Southwest’s entry. For those markets where Southwest enters, there is an additional competitive effect on prices. For the quarter in which Southwest enters, the incumbent’s price is 24.6\% lower than the reference period. We separately estimate the effect of Southwest’s entry for the next five quarters, and as for the period in which it is a potential entrant, we estimate the average effect of Southwest’s entry on incumbent’s price for quarters six through twelve following its entry, and from quarter thirteen on following its entry. These price declines are also similar to those in Goolsbee and Syverson.

The pricing patterns of incumbent airlines once faced by the threat of potential entry by Southwest that we have documented in this section are new only in that we document them for markets in which there is a dominant carrier. We now turn to our main contribution which is exploring the extent to which the price declines correspond to a strategic motive on the part of the incumbent, and in particular, the extent to which a model of dynamic limit pricing can rationalize this price declines.

## 4 Empirical Results

We begin by testing whether incumbent price declines appear to be motivated by strategic entry deterring motives.
Table 2: Incumbent Responses to the Threat of Entry

<table>
<thead>
<tr>
<th>β Estimates</th>
<th>Before WN is PE:</th>
<th>WN is PE:</th>
<th>WN is E:</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₀ - 8</td>
<td>0.023 (0.017)</td>
<td>-0.100*** (0.033)</td>
<td>-0.280*** (0.064)</td>
</tr>
<tr>
<td>t₀ - 7</td>
<td>-0.007 (0.019)</td>
<td>-0.108*** (0.036)</td>
<td>-0.332*** (0.071)</td>
</tr>
<tr>
<td>t₀ - 6</td>
<td>-0.044** (0.022)</td>
<td>-0.189*** (0.039)</td>
<td>-0.333*** (0.067)</td>
</tr>
<tr>
<td>t₀ - 5</td>
<td>-0.057** (0.023)</td>
<td>-0.179*** (0.038)</td>
<td>-0.393*** (0.065)</td>
</tr>
<tr>
<td>t₀ - 4</td>
<td>-0.022 (0.022)</td>
<td>-0.225*** (0.044)</td>
<td>-0.403*** (0.062)</td>
</tr>
<tr>
<td>t₀ - 3</td>
<td>-0.032 (0.026)</td>
<td>-0.239*** (0.044)</td>
<td>-0.466*** (0.067)</td>
</tr>
<tr>
<td>t₀ - 2</td>
<td>-0.066*** (0.025)</td>
<td>-0.279*** (0.046)</td>
<td>-0.418*** (0.065)</td>
</tr>
<tr>
<td>t₀ - 1</td>
<td>-0.069*** (0.029)</td>
<td>-0.236*** (0.048)</td>
<td>-0.319*** (0.070)</td>
</tr>
</tbody>
</table>

Market-Carrier Fixed Effects: Yes
Quarter Fixed Effects: Yes

N 4,549
R² 0.851

The dependent variable is the logged passenger-weighted average fare. Standard errors are in parentheses and are clustered by route-carrier. *** denotes significance at the 1% level, ** at 5% and * at 10%.
4.1 A Test of Strategic Entry Deterrence

Empirically identifying strategic investment requires an assessment of the ex ante threat of entry and assumptions on the counterfactual that would have been observed in the absence of a strategic incentive. At a general level we might expect incumbent investment to move monotonically with measures of market attractiveness when decisions are devoid of strategic intent. However, when firms internalize their effect on potential entrants it is markets of intermediate attractiveness that are likely to be the most amenable to strategic investment. The source for this intuition is Ellison and Ellison (2011) who point out that in unattractive markets incumbents are sheltered from entry, whereas in highly attractive markets entry is inevitable with the corollary that both these market types are absent of strategic incentives. Ellison and Ellison therefore develop and implement a test that utilizes this insight.

In our model, strategic entry deterrence takes the form of limit pricing. As made clear in Section 2, the incumbent firm has the greatest incentive to engage in limit pricing when it is uncertain whether Southwest will enter. In markets that are too unprofitable for Southwest to enter, or which are surely profitable even given the incumbent’s presence, there is no incentive for the dominant carrier to forgo profits today by setting a suboptimal static price in hopes of dissuading Southwest from entering a market. We therefore perform a similar test similar in spirit to those in Ellison and Ellison (2011) and Dafny (2005). Specifically, we investigate the change in an incumbent’s price after Southwest becomes a potential entrant as a function of the market’s population by estimating the following regression:

\[ p_{j,m,t} = \gamma_m + \tau_t + \alpha X_{m,t} + \sum_{s=1}^{5} \beta_s I[ \text{avgpop}_m \in \text{Quintile}_s] \times SWPE_{m,t} + \varepsilon_{j,m,t} \]  

(4)

where the \( \tau \) and \( \gamma \) parameters are quarter and market fixed effects, \( X \) includes the number of potential entrants into the market and the number of other firms currently serving the market (besides the dominant incumbent) and \( I[ \text{avgpop}_m \in \text{Quintile}_s] \) indicates that the geometric average population of the endpoints of market \( m \) place the market in the \( s \)th quintile across all markets.\(^{22}\) We calculate robust standard errors and again weight each observation by the average number of passengers carried by the incumbent per market.

Our interest is in the estimates of the five \( \beta_s \) parameters, which estimate the change in average price charged by an incumbent after Southwest becomes a potential entrant, but before it enters, for each population quintile. Figure ?? shows estimates of these parameters with 95% confidence intervals. The

\(^{22}\)The quintiles are defined by the following thresholds of population: 1,608,562, 2,421,354, 3,600,733, 5,503,308.
figure displays a u-shape for the parameter estimates: the greatest price declines are for those markets with populations in the third and fourth quintiles. For the first, second and fifth quintiles, that is very small or very large markets, the incumbents’ prices hardly decline, and in the case of the fifth quintile, we cannot reject the null hypothesis that prices are unchanged following Southwest becoming a potential entrant. This is in sharp contrast to the third and fourth quintiles, where the incumbents’ price falls on average by $43.45 and $48.02, respectively (these represent approximately 9% and 10% price declines, respectively). These results are in line with our model’s predictions about the kinds of markets for which we would expect to see an incumbent engaging in entry deterring behavior, and are in line with the intuition and test in Ellison and Ellison (2011) and Dafny (2005). While our ultimate goal will be investigating how well a dynamic model of limit pricing can match these price declines, we note that just in terms of documenting strategic behavior on the part of incumbents, these findings present the clearest such evidence to date, as the empirical results in Ellison and Ellison (2011) and Dafny (2005) are weak and often statistically insignificant.

4.2 Can Limit Pricing Explain Incumbents’ Behavior?

In this section we aim to calibrate a model of dynamic limit pricing to see how well it can explain the observed incumbent price declines documented above. To see how our model of dynamic limit pricing predicts incumbents to behave, we need to estimate the demand for air travel and carriers’ marginal and fixed costs. We begin by explaining how we estimate demand and marginal costs, and later address carrier fixed costs.

Because our goal is to use parameter estimates to calibrate a model of limit pricing, we wish to avoid imposing the model during the estimation of these parameters. Therefore, we wish to identify a set of markets in which we believe that incumbents are not engaging in limit pricing. The nature of the problem identifies two kinds of markets, and two time periods, that match this criteria. Incumbents should not be engaging in limit pricing before Southwest becomes a potential entrant, or after it enters (if it does), and following our model predictions and the intuition in Ellison and Ellison (2011), as well as the evidence above, incumbents do not limit price in very small, or very large markets. Therefore, we will estimate demand and marginal cost parameters using markets in quintiles one, two and five, and quarters that are in Period 1 and Period 3.23

23One might also think of using quarters in Period 2 for these markets, or using quarters in Periods 1 and 3 for markets in quintiles three and four, but our baseline results do not do so.
Demand

We consider a simple one-level nested logit discrete choice model of the demand for air travel where the nests are ‘fly’ and ‘no fly’. Consumer \(i\)’s utility from choosing a carrier \(j\) to fly on market \(m\) in quarter \(t\) is:

\[
\begin{align*}
    u_{i,j,m,t} = & \mu_j + \tau_1 T_t + \tau_2 Q_2 t + \tau_3 Q_3 t + \tau_4 Q_4 t + \gamma X_{j,m,t} - \alpha p_{j,m,t} + \xi_{j,m,t} + \zeta_{FLY} + \xi_{FLY} + (1 - \lambda) \varepsilon_{i,j,m,t}
\end{align*}
\]

where \(\mu_j\) is firm \(j\)’s mean quality, \(T_t\) is a time trend, \(Q_2\), \(Q_3\) and \(Q_4\) are quarter dummies, \(X_{j,m,t}\) includes the round-trip distance, and distance squared, between market \(m\)’s endpoints and an indicator for whether market \(m\) is a hub for carrier \(j\), \(p_{j,m,t}\) is the passenger weighted average round trip fare for firm \(j\) on market \(m\) in quarter \(t\), \(\xi_{j,m,t}\) is the unobserved (to the econometrician) characteristic, \(\lambda\) is the nesting parameter which varies between 0 and 1, and \(\varepsilon_{i,j,m,t}\) is the standard logit error that is i.i.d. over time, markets and consumers. The distribution of \(\varepsilon_{FLY} + (1 - \lambda) \varepsilon_{i,j,m,t}\) is assumed to be such that the probability consumers choose a particular option follows the standard nested logit formulae Cardell (1991). We normalize the utility of the outside good to be zero, \(u_{i,0,m,t} = 0\). Consumers choose their most preferred option which leads to the following familiar expression for the relative (to the outside good) market share for firm \(j\) in market \(m\) at time \(t\):

\[
\ln \left( \frac{s_{j,m,t}}{s_{0,m,t}} \right) = \mu_j + \tau_1 T_t + \tau_2 Q_2 t + \tau_3 Q_3 t + \tau_4 Q_4 t + \gamma X_{j,m,t} - \alpha p_{j,m,t} + \lambda \xi_{j,m,t|FLY} + \xi_{j,m,t}
\]

where \(\xi_{j,m,t|FLY}\) is firm \(j\)’s market share conditional on flying and \(s_{j,m,t}\) is firm \(j\)’s market share. Equation 6 can be estimated using standard OLS. The obvious concerns regard the endogeneity of \(p_{j,m,t}\) and \(\xi_{j,m,t|FLY}\). We instrument for price using a measure of each carrier’s average price paid for jet fuel, lagged one quarter, and this fuel price interacted with distance and distance squared. The price of jet fuel is a natural cost shifter, and there is evidence that the effect of an increase in the price of jet fuel on the marginal cost of a trip is non-monotonic in distance, because the type of aircraft (which differ according to fuel efficiency) used will vary with distance. We instrument for a firm’s share of the nest using an indicator for whether Southwest serves the market if firm \(j\) is not Southwest, and whether the market is a hub for the incumbent if firm \(j\) is Southwest.

Table ?? presents estimates of our model of demand, both ignoring and controlling for endogeneity. Comparing the estimates of \(\alpha\) across the columns, our instruments have the expected effect of increasing the estimated elasticity of demand. At the observed prices and market shares, we estimate the average elasticity of demand to be 1.93, which is line with previous studies of this industry (e.g. Berry and
Jia (2010), who use a different sample of markets). After instrumenting, Southwest’s estimated mean quality is in line with those of the major legacy carriers. Other demand parameters also have the expected effect. For example, we find that demand increases in distance up to a little over 1,300 miles (one-way) and then declines. Berry and Jia (2010) estimate this decline to begin at around 1,600 miles. We also find that consumers prefer to choose an airline for whom at least one endpoint of the market is a hub. This too is in line with previous research (e.g., Borenstein (1989), Berry (1992)) that normally cites amenities like more convenient gate access or frequent flyer programs as sources of these preferences.

Marginal Costs

Using the demand estimates in Table ?? we compute firm-market-quarter specific marginal costs using firms’ first order conditions associated with differentiated product static price competition. Here again we see the importance of using markets and quarters in which one does not think limit pricing is going on because otherwise the standard first order condition for equilibrium pricing would not hold. Given our nested logit model (see for example Berry (1994))

\[ \hat{mc}_{j,m,t} = p_{j,m,t} - \frac{1-\lambda}{\alpha} / \left[ \left( 1 - \hat{\lambda} \sigma_{j,m,t|FLY} \right) (1 - \hat{\lambda}) s_{j,m,t} \right]. \]

We then regress these marginal costs in \( t \) on margin costs in period \( t - 1 \), controlling for airline and market fixed effects. This gives an estimate \( \hat{\rho} = 0.733 \) (s.e. 0.015).

Dynamic Limit Pricing Predictions

5 Conclusion

This paper develops a dynamic model of limit pricing where, because of evolving costs and uncertainty about the entry cost draws of a potential entrant, there is an incentive for an incumbent to signal its costs every period using prices in order to try to deter entry. The model is well-suited to empirical work as there is a unique PBE under a standard refinement, and equilibrium strategies are relatively easy to calculate. We aim to test whether our model can explain the stylized fact that incumbent airlines cut prices quite markedly when faced with the possibility of entry by Southwest. Currently, we provide quite strong evidence of strategic pricing behavior on the routes in our sample using the framework proposed by Ellison and Ellison (2011).

While the work presented is preliminary and we hope to generalize the results in several ways, it is worth noting that some features may be difficult to relax. For example, if the entry cost or marginal cost of the potential entrant is serially correlated then the incumbent will also face an inference problem,
but it will have to make an inference from a discrete action. This would require keeping track of belief distributions, so that solving the game would be much more complicated. On the other hand, allowing for richer post-entry cost dynamics seems quite possible as long as appropriate conditions on costs can be developed so that the conditions on pre-entry value functions are satisfied.
References


A Proof of the Conditions Required for Existence and Uniqueness of the FSRE

In this Appendix, we prove that our model satisfies the conditions required for existence and uniqueness of the FSRE equilibrium.

We will make frequent use of the following result:

Lemma 2 Suppose

\[ k = \int_\pi^\tau \int_y^x f(x, y) \frac{\partial g(x \mid w)}{\partial w} h(y) dy \, dx \]

If (i) \( \int_x^x \frac{\partial g(x \mid w)}{\partial w} \, dx = 0 \), (ii) for a given value of \( w \) \( \exists x' \in (x, \pi) \) such that \( \frac{\partial g(x' \mid w)}{\partial w} = 0 \), \( \frac{\partial g(x \mid w)}{\partial w} < 0 \) \( \forall x < x' \) and \( \frac{\partial g(x' \mid w)}{\partial w} > 0 \) for \( \forall x > x' \), (iii) \( f(x, y) > 0 \) and \( \frac{\partial f(x, y)}{\partial x} > 0 \), (iv) \( h(y) > 0 \) and \( \int_x^\pi h(y) dy = 1 \) then \( k > 0 \). If (i), (ii), (iv) and \( f(x, y) > 0 \) and \( \frac{\partial f(x, y)}{\partial x} < 0 \) then \( k < 0 \).

Proof.

\[
k = \int_\pi^\tau \int_y^x f(x, y) \frac{\partial g(x \mid w)}{\partial w} h(y) dy \, dx
\]

\[
= \int_\pi^x \int_y^x f(x, y) \frac{\partial g(x \mid w)}{\partial w} h(y) dy \, dx + \int_\pi^{x'} \int_y^\tau f(x, y) \frac{\partial g(x \mid w)}{\partial w} h(y) dy \, dx
\]

\[
> \int_\pi^{x'} f(x', y) \left\{ \int_x^{x'} \frac{\partial g(x \mid w)}{\partial w} \, dx + \int_x^{x'} \frac{\partial g(x \mid w)}{\partial w} \, dx \right\} h(y) dy = 0 \text{ if } \frac{\partial f(x, y)}{\partial x} > 0
\]

or \[
< \int_\pi^{x'} f(x', y) \left\{ \int_x^{x'} \frac{\partial g(x \mid w)}{\partial w} \, dx + \int_x^{x'} \frac{\partial g(x \mid w)}{\partial w} \, dx \right\} h(y) dy = 0 \text{ if } \frac{\partial f(x, y)}{\partial x} < 0
\]
A.1 Existence and Monotonicity of the Potential Entrant’s Threshold Rule

Proposition 3 Assume that E has the point belief that I’s current marginal cost is \(\hat{c}_I \in [c_I, \bar{c}_I]\). E’s entry strategy will be to enter if entry costs \(\kappa\) are lower than a uniquely defined period-specific threshold \(\kappa_t^*(\hat{c}_I, c_{Et})\). The threshold is defined by

\[
\kappa^* = \mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}] - \mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}]
\]

where \(V_{t+1}^E\) is E’s value to being a potential entrant in period \(t + 1\) (i.e., if it does not enter now) and \(\phi_{t+1}^E\) is E’s value to being a duopolist in period \(t + 1\) (which assumes it has entered prior to \(t + 1\)).

\[
V_{t+1}^E = \max\{\beta\mathbb{E}_{t+1}[V_{t+2}^E|\hat{c}_{I+1}, c_{Et+1}], \mathbb{E}_{t+1}[\phi_{t+2}|\hat{c}_{I+1}, c_{Et+1}] - \kappa_{t+1}\}
\]

The threshold \(\kappa^*\) is strictly decreasing in \(c_{Et}\) and strictly increasing in \(\hat{c}_I\).

**Proof.** The following conditions, implied by the assumptions of our model, are sufficient for this proposition to hold:

(i) \(\mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}] - \mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}] > \kappa = 0\) for all \((\hat{c}_I, c_{Et})\) (there is always some possibility of entry).

(ii) \(\mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}] - \mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}] < \bar{\kappa}\) for all \((\hat{c}_I, c_{Et})\) (there is always some possibility of no entry).

(iii) \(\mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}]\) and \(\mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}]\) are continuous and differentiable in both arguments. For \(t < T\), \(\mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}]\) is strictly increasing in \(\hat{c}_I\) and decreasing in \(c_{Et}\). For \(t < T - 1\), \(\mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}]\) is strictly increasing in \(\hat{c}_I\) and decreasing in \(c_{Et}\). For \(t = T - 1\), \(\mathbb{E}_{T-1}[V_T^E|\hat{c}_I, c_{Et}] = 0\).

(iv) \(\frac{\partial \mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}]}{\partial c_{Et}} > \frac{\partial \mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}]}{\partial c_{Et}}\) and \(\frac{\partial \mathbb{E}_t[V_{t+1}^E|\hat{c}_I, c_{Et}]}{\partial \hat{c}_I} > \frac{\partial \mathbb{E}_t[\phi_{t+1}^E|\hat{c}_I, c_{Et}]}{\partial \hat{c}_I}\).

Under these conditions the geometry of value functions can be represented in the following diagrams, where the gap between the expected valuations as a duopolist and a potential entrant defines the threshold level of entry cost \(\kappa^*\) required to induce entry.
We now show that conditions (i), (iii) and (iv) hold in our model. (ii) requires a minimum upper bound on \( \kappa \) (specifically we need \( \mathbb{E}_{t}[\phi_{t+1} | \bar{c_{t}}, \bar{c_E}] - \mathbb{E}_{t}[V_{t+1} | \bar{c_{t}}, \bar{c_E}] < \kappa \), as the value of being in the market already is maximized when the entrant has the largest possible marginal cost advantage).

In period \( T - 1 \):

(i) \( \mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}}, c_{ET}] = \mathbb{E}_{T-1}[\pi^{D}_{E,T}\bar{c_{IT-1}}, c_{ET-1}] > \mathbb{E}_{T}(V^{E}_{T}\bar{c_{IT-1}}, c_{ET}) = 0 \) as all \( \pi^{D}_{E} > 0 \) by assumption and there is no value to being a potential entrant in the last period.

(iii) 
\[
\mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}}, c_{ET}] = \int_{\bar{c_{IT}}}^{\bar{c_{ET}}} \int_{\bar{c_{ET}}}^{\bar{c_{ET}}} \pi^{D}_{E}(c_{ET}, c_{IT})\psi_{I}(c_{IT}|\bar{c_{IT-1}})\psi_{E}(c_{ET}|c_{ET-1}) dc_{ET} dc_{IT}
\]
Continuity and differentiability of \( \mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}}, c_{ET}] \) follows from continuity and differentiability of \( \pi^{D}_{E}(c_{IT}, c_{ET}) \) and the continuity and differentiability of the Markov transition functions \( \psi_{I}(c_{IT}|\bar{c_{IT-1}}) \) and \( \psi_{E}(c_{ET}|c_{ET-1}) \) with respect to their conditioning variables.

(iv) 
\[
\frac{\partial \mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}, c_{ET-1}}]}{\partial c_{IT-1}} = \frac{\partial \mathbb{E}_{T-1}[\pi^{D}_{E}\bar{c_{IT-1}, c_{ET-1}}]}{\partial c_{IT-1}} = \int_{\bar{c_{IT}}}^{\bar{c_{ET}}} \int_{\bar{c_{ET}}}^{\bar{c_{ET}}} \pi^{D}_{E}(c_{ET}, c_{IT})\frac{\partial \psi_{I}(c_{IT}|\bar{c_{IT-1}})}{\partial c_{IT-1}}\psi_{E}(c_{ET}|c_{ET-1}) dc_{ET} dc_{IT}.
\]

Application of Lemma 1 implies that \( \frac{\partial \mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}, c_{ET-1}}]}{\partial c_{IT-1}} > 0 \). Also, \( \frac{\partial \mathbb{E}_{T-1}[\phi^{E}_{T}\bar{c_{IT-1}, c_{ET-1}}]}{\partial c_{ET-1}} > 0 \).
Application of the same lemma shows that \( \frac{\partial \mathbb{E}_{T-1}[V^E_T|c_{IT-1},c_{ET-1}]}{\partial c_{ET-1}} < \frac{\partial \mathbb{E}_{T-1}[V^E_T|c_{IT-1},c_{ET-1}]}{\partial c_{ET-1}} = 0 \) as \( \pi^D_E(c_{ET},c_I) \) is decreasing in \( c_{ET} \), the entrant’s own marginal cost.

We can also show that in period \( T - 2 \), \( \frac{\partial \mathbb{E}_{T-2}[V^E_T|c_{IT-2},c_{ET-2}]}{\partial c_{IT-2}} > 0 \),

\[
\mathbb{E}_{T-2}[V^E_T|c_{IT-2},c_{ET-2}] = \int \int \int \max\{\beta \mathbb{E}_{T-1}[V^E_T|c_{IT-1},c_{ET-1}], \beta \mathbb{E}_{T-1}[\phi_T|c_{IT-1},c_{ET-1}] - \kappa_{T-1}\} \cdots \\
\psi_I(c_{IT-1}|c_{IT-2}) \psi_E(c_{ET-1}|c_{ET-2}) g(\kappa_{T-1}) dc_{IT-1} dc_{ET-1} d\kappa_{T-1} \\
= \int \int \int \max\{0, \beta \mathbb{E}_{T-1}[\pi^D_E|c_{IT-1},c_{ET-1}] - \kappa_{T-1}\} \cdots \\
\psi_I(c_{IT-1}|c_{IT-2}) \psi_E(c_{ET-1}|c_{ET-2}) g(\kappa_{T-1}) dc_{IT-1} dc_{ET-1} d\kappa_{T-1}
\]

Therefore

\[
\frac{\partial \mathbb{E}_{T-2}[V^E_T|c_{IT-2},c_{ET-2}]}{\partial c_{IT-2}} = \int \int \int \left\{ \int \kappa \{\beta \mathbb{E}_{T-1}[\pi^D_E|c_{IT-1},c_{ET-1}] - \kappa_{T-1}\} d\kappa_{T-1} \right\} \cdots \\
\frac{\partial \psi_I(c_{IT-1}|c_{IT-2})}{\partial c_{IT-2}} \psi_E(c_{ET-1}|c_{ET-2}) g(\kappa_{T-1}) dc_{IT-1} dc_{ET-1} > 0
\]

by the application of Lemma 1 as \( \int \kappa \{\beta \mathbb{E}_{T-1}[\pi^D_E|c_{IT-1},c_{ET-1}] - \kappa_{T-1}\} d\kappa_{T-1} > 0 \) and is increasing in \( c_{IT-2} \). \( \frac{\partial \mathbb{E}_{T-2}[V^E_T|c_{IT-2},c_{ET-2}]}{\partial c_{IT-2}} < 0 \) follows from application of the same lemma. Continuity and differentiability of \( \mathbb{E}_{T-2}[V^E_T|c_{IT-2},c_{ET-2}] \) follows from the continuity of \( \mathbb{E}_{T-1}[\pi^D_E|c_{IT-1},c_{ET-1}] \) and continuity and differentiability of \( \psi_I(c_{IT-1}|c_{IT-2}) \) and \( \psi_E(c_{ET-1}|c_{ET-2}) \).

Now consider a period \( t < T - 1 \). Suppose that the conditions hold at \( t \) (period \( t \) expectations). Show that this implies that they hold at \( t - 1 \).

(i)

\[
V_t^E(c_{IT},c_{ET}) = \max\{\beta \mathbb{E}_t[V^E_{t+1}|c_{IT-1},c_{ET-1}], \mathbb{E}_t[\phi^E_{t+1}|c_{IH},c_{ET}] - \kappa_t\} \\
\phi^E_t(c_{IH},c_{ET}) = \pi^D_E(c_{IH},c_{ET}) + \mathbb{E}_t[\phi^E_{t+1}|c_{IH},c_{ET}] \\
\phi^E_t(c_{IH},c_{ET}) - V_t^E(c_{IH},c_{ET}) = \pi^D_E(c_{IH},c_{ET}) + \min\{\kappa_t, \beta \mathbb{E}_t[\phi_{t+1}|c_{IH},c_{ET}] - \beta \mathbb{E}_t[V^E_{t+1}|c_{IH},c_{ET}]\}
\]
As \( \kappa_t \geq \kappa = 0 \) and (by the induction hypothesis) \( \beta \mathbb{E}_t[\phi_{t+1}^E | \hat{c}_{H-1}, c_{E,t}] - \beta \mathbb{E}_t[V_{t+1}^E | \hat{c}_{H}, c_{E,t}] > 0, \phi_{t}^E - V_{t}^E > 0 \) for all \((\hat{c}_{H}, c_{E,t})\). Therefore \( \mathbb{E}_{t-1}[\phi_{t}^E - V_{t}^E | \hat{c}_{H-1}, c_{E,t-1}] > 0 \).

(iii)

\[
\mathbb{E}_{t-1}[\phi_{t}^E | \hat{c}_{H-1}, c_{E,t-1}] = \int \int \int \pi_t^D(c_{H,t}, c_{E,t}) \psi_I(c_{H} | \hat{c}_{H-1}) \psi_E(c_{E,t} | c_{E,t-1}) dc_H dc_{E,t} + \ldots
\]

\[
\beta \int \int \mathbb{E}_{t-1}[\phi_{t+1}^E | c_{H,t}, c_{E,t}] \psi_I(c_{H} | \hat{c}_{H-1}) \psi_E(c_{E,t} | c_{E,t-1}) dc_H dc_{E,t}
\]

\[
\mathbb{E}_{t-1}[V_{t}^E | \hat{c}_{H-1}, c_{E,t-1}] = \int \int \max \{ \beta \mathbb{E}_t[V_{t+1}^E | c_{H,t}, c_{E,t}], \beta \mathbb{E}_t[\phi_{t+1} | c_{H,t}, c_{E,t}] - \kappa_t \} \ldots
\]

\[
\psi_I(c_{H} | \hat{c}_{H-1}) \psi_E(c_{E,t} | c_{E,t-1}) g(\kappa_t) dc_H dc_{E,t} d\kappa_t
\]

and continuity and differentiability of \( \mathbb{E}_{t-1}[\phi_{t}^E | c_{H-1}, c_{E,t-1}] \) and \( \mathbb{E}_{t-1}[V_{t}^E | c_{H-1}, c_{E,t-1}] \) follow from continuity and differentiability of \( \pi_t^D(c_{H,t}, c_{E,t}) \) (assumed), \( \psi_I(c_{H} | c_{H-1}) \) and \( \psi_E(c_{E,t} | c_{E,t-1}) \) (assumed) and \( \mathbb{E}_t[V_{t+1}^E | c_{H,t}, c_{E,t}] \) and \( \beta \mathbb{E}_t[\phi_{t+1} | c_{H,t}, c_{E,t}] \) (induction hypothesis).

(iv)

\[
\mathbb{E}_{t-1}[\phi_{t}^E | \hat{c}_{H-1}, c_{E,t-1}] - \mathbb{E}_{t-1}[V_{t}^E | \hat{c}_{H-1}, c_{E,t-1}] = \int \int \int \pi_t^D(c_{H,t}, c_{E,t}) \psi_I(c_{H} | \hat{c}_{H-1}) \psi_E(c_{E,t} | c_{E,t-1}) dc_H dc_{E,t} + \ldots
\]

\[
+ \int \int \max \{ \kappa_t, \beta \mathbb{E}_t[\phi_{t+1} | c_{H,t}, c_{E,t}] - \beta \mathbb{E}_t[V_{t+1}^E | c_{H,t}, c_{E,t}] \} \ldots
\]

\[
\psi_I(c_{H} | \hat{c}_{H-1}) \psi_E(c_{E,t} | c_{E,t-1}) g(\kappa_t) dc_H dc_{E,t} d\kappa_t
\]

so that

\[
\frac{\partial \mathbb{E}_{t-1}[\phi_{t}^E | \hat{c}_{H-1}, c_{E,t-1}]}{\partial \hat{c}_{H-1}} - \frac{\partial \mathbb{E}_{t-1}[V_{t}^E | \hat{c}_{H-1}, c_{E,t-1}]}{\partial \hat{c}_{H-1}} = \int \int \int \pi_t^D(c_{H,t}, c_{E,t}) \frac{\partial \psi_I(c_{H} | \hat{c}_{H-1})}{\partial \hat{c}_{H-1}} \psi_E(c_{E,t} | c_{E,t-1}) dc_H dc_{E,t}
\]

\[
+ \int \int \max \{ \kappa_t, \beta \mathbb{E}_t[\phi_{t+1} | c_{H,t}, c_{E,t}] - \beta \mathbb{E}_t[V_{t+1}^E | c_{H,t}, c_{E,t}] \} \ldots
\]

\[
\frac{\partial \psi_I(c_{H} | \hat{c}_{H-1})}{\partial \hat{c}_{H-1}} \psi_E(c_{E,t} | c_{E,t-1}) g(\kappa_t) dc_H dc_{E,t} d\kappa_t
\]
Considering the terms in the second row: \( \kappa_t \geq 0 \), and from the induction hypotheses \( \beta E_t[\phi_{t+1}^I|c_H, c_{E_t}] - \beta E_t[V_{t+1}^E|c_H, c_{E_t}] > 0 \) and is increasing in \( c_H \). Application of Lemma 1 then implies that the second row is \( \geq 0 \).

Considering the term on the right-hand side of the first row: \( \pi_E^D(c_H, c_{E_t}) > 0 \) and is strictly increasing in \( c_H \). Application of Lemma 1 then implies that this term is \( > 0 \). Therefore,

\[
\frac{\partial E_{t-1}[\phi_{t-1}^I|c_{H_{t-1}, c_{E_{t-1}}}|]}{\partial c_{H_{t-1}}} - \frac{\partial E_{t-1}[V_{t-1}^E|c_{H_{t-1}, c_{E_{t-1}}}|]}{\partial c_{E_{t-1}}} > 0
\]

The proof that \( \frac{\partial E_{t-1}[\phi_{t-1}^I|c_{H_{t-1}, c_{E_{t-1}}}|]}{\partial c_{E_{t-1}}} < \frac{\partial E_{t-1}[V_{t-1}^E|c_{H_{t-1}, c_{E_{t-1}}}|]}{\partial c_{E_{t-1}}} < 0 \) follows exactly the same logic, using the facts that \( \frac{\partial E^D}{\partial c_H} < 0 \) and the induction hypothesis that \( \beta E_t[\phi_{t+1}^I|c_H, c_{E_t}] - \beta E_t[V_{t+1}^E|c_H, c_{E_t}] \) is decreasing in \( c_{E_t} \).

### A.2 Incumbent’s Payoff Function Satisfies the Conditions for Existence and Uniqueness of the FSRE

Based on Theorem 1 of Mailath and von Thadden (2013) (which in turn is based on Mailath (1987)), we have the following result.

**Proposition 4** Take the signalling payoff \( \Pi^H(c_H, \tilde{c}_H, p_H, c_{E_t}) \) where \( \tilde{c}_H \) is a (point) belief that \( E \) has about \( I \)’s marginal cost. If (i) \( \Pi^H(c_H, c_{E_t}, p_H, c_{E_t}) \) has a unique optimum in \( p_H \) and, for any \( p_H \in [\underline{p}, \overline{p}] \) where \( \Pi^H_{33}(c_H, c_{E_t}, p_H, c_{E_t}) > 0 \) \( \exists k > 0 \) such that \( |\Pi^H_3(c_H, c_{E_t}, p_H, c_{E_t})| > k \) for all \( (c_H, c_{E_t}) \); (ii) \( \Pi^H_3(c_H, \tilde{c}_H, p_H, c_{E_t}) \neq 0 \) for all \( c_H, \tilde{c}_H, p_H, c_{E_t} \); (iii) \( \Pi^H_2(c_H, \tilde{c}_H, p_H, c_{E_t}) \neq 0 \) for all \( c_H, \tilde{c}_H, p_H, c_{E_t} \); (iv) \( \Pi^H_4(c_H, \tilde{c}_H, p_H, c_{E_t}) \) is a monotone function of \( c_H \) for all \( \tilde{c}_H, c_{E_t} \); (v) \( \overline{p} \geq p^{\text{static monopoly}}(\overline{c}_I) \) and \( \Pi^H(c_H, \tilde{c}_H, \overline{p}, c_{E_t}) < \max \Pi^H(c_H, \overline{c}_H, \overline{p}, c_{E_t}) \) for all \( t, c_{E_t} \). Then, \( I \)’s period \( t \) unique separating pricing strategy is differentiable on the interior of \( [\underline{c}_I, \overline{c}_I] \) and satisfies the differential equation that

\[
\frac{\partial p^*_H}{\partial c_H} = -\frac{\Pi^H_2}{\Pi^H_3}
\]

and the boundary condition that \( p^*_H(\overline{c}_I) = p^{\text{static monopoly}}(\overline{c}_I) \).

We now prove that our game satisfies conditions (i)-(iv). Condition (v) is a condition that \( \overline{p} \) is so low that no firm would ever want to choose it.

**Proof.** Note that

\[
\Pi^H(c_H, \tilde{c}_H, p_H, c_{E_t}) = q^M(p_H)(p_H-c_H)+\beta \left[ (1-G(\kappa_t^*(\tilde{c}_H, c_{E_t})))\mathbb{E}_t[V_{t+1}^I|c_H, c_{E_t}] + G(\kappa_t^*(\tilde{c}_H, c_{E_t})))\mathbb{E}_t[\phi_{t+1}^I|c_H, c_{E_t}] \right]
\]
where \( \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] \) and \( \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}] \) are the expected value to being a monopolist and duopolist in period \( t + 1 \) given current marginal costs. Given \( E \)'s threshold rule \( \Pr(E \text{ not enter} | \hat{c}_{It}, c_{Et}) = (1 - G(\kappa^*_t(c_{It}, c_{Et}))) \).

(i) \( \Pi^H(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) \) only depend on \( p_{It} \) through the static monopoly profit function \( q^M(p_{It})(p_{It} - c_{It}) \). The profit function will hold satisfy the conditions if it is continuous, concave at the profit maximizing price and strictly quasi-concave in \( p_{It} \) on \([p, \bar{p}]\), as assumed.

(ii) \( \Pi^H_{13}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) \neq 0 \) for all \( c_{It}, \hat{c}_{It}, p_{It}, c_{Et} \).

\[
\Pi^H_{13}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) = -\frac{\partial q^M(p_{It})}{\partial p_{It}} \neq 0
\]
as long as monopoly demand is downward sloping on \([p, \bar{p}]\).

(iii) \( \Pi^H_{2}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) \neq 0 \) for all \( c_{It}, \hat{c}_{It}, p_{It}, c_{Et} \).

\[
\Pi^H_{2}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) = -\beta g(\kappa^*_t(\hat{c}_{It}, c_{Et})) \frac{\partial \kappa^*_t(\hat{c}_{It}, c_{Et})}{\partial c_{It}} \left\{ \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] - \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}] \right\}
\]

This will not be equal to zero if \( g(\kappa^*_t(\hat{c}_{It}, c_{Et})) > 0 \) (which depends on entry costs having sufficient support), \( \frac{\partial \kappa^*_t(\hat{c}_{It}, c_{Et})}{\partial c_{It}} > 0 \) for all \( (\hat{c}_{It}, c_{Et}) \), which follows from monotonicity of \( E \)'s entry threshold rule in perceived incumbent marginal cost, and \( \left\{ \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] - \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}] \right\} > 0 \) (expected continuation value as a monopolist greater than as a duopolist). This will be shown below.

(iv) \( \Pi^H_{34}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) \) is a monotone function of \( c_{It} \) for all \( \hat{c}_{It}, c_{Et} \).

\[
\Pi^H_{34}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) = \left[ q^M(p_{It}) + \frac{\partial q^M(p_{It})}{\partial p_{It}}(p_{It} - c_{It}) \right] x... \left(-\beta g(\kappa^*_t(\hat{c}_{It}, c_{Et})) \frac{\partial \kappa^*_t(\hat{c}_{It}, c_{Et})}{\partial c_{It}} \left\{ \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] - \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}] \right\} \right)^{-1}
\]
differentiate wrt \( c_{It} := \frac{\partial q^M(p_{It})}{\partial p_{It}}(p_{It} - c_{It}) \left[ q(p_{It}) + \frac{\partial q^M(p_{It})}{\partial p_{It}}(p_{It} - c_{It}) \right] \frac{\partial \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] - \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}]}{\partial c_{It}} \left( \beta g(\kappa^*_t(\hat{c}_{It}, c_{Et})) \frac{\partial \kappa^*_t(\hat{c}_{It}, c_{Et})}{\partial c_{It}} \right)^2 \]

Sufficient conditions for this expression to be < 0 (implying \( \Pi^H_{34}(c_{It}, \hat{c}_{It}, p_{It}, c_{Et}) \) monotonic in \( c_{It} \)) are that:

(i) \( V^I_{t+1}(c_{It+1}, c_{Et+1}) - \phi^I_{t+1}(c_{It+1}, c_{Et+1}) > 0 \) for all \( (c_{It+1}, c_{Et+1}) \), which implies that \( \left\{ \mathbb{E}_t[V^I_{t+1} | c_{It}, c_{Et}] - \mathbb{E}_t[\phi^I_{t+1} | c_{It}, c_{Et}] \right\} > 0 \) (by application of

(ii) \( \frac{\partial V^I_{t+1}(c_{It}, c_{Et})}{\partial c_{It}} \phi^I_{t+1}(c_{It}, c_{Et}) \) < 0 which implies \( \frac{\partial V^I_{t+1}(c_{It}, c_{Et})}{\partial c_{It}} \neq 0 \) by application of
Lemma 1\textsuperscript{24}, (iii) \( q(p_H) + \frac{\partial q^M(p_H)}{\partial p_I}(p_H - c_H) \geq 0 \) for all prices below the monopoly price (given quasi-concavity of the profit function), (iv) \( \frac{\partial c^I,CE_I(t)}{\partial p_I} > 0 \) and (v) \( \frac{\partial p^M(p_H)}{\partial p_I} < 0 \). We now show (i) and (ii) by backwards induction.

(i) \( V^I_{t+1}(c_H, c_{ET}) - \phi^I_{t+1}(c_H, c_{ET}) > 0 \).

In the final period, payoffs are simply static (no signalling) monopoly and duopoly profits, so, from our assumptions on \( \pi^D_I \), \( V^I_T - \phi^I_T > 0 \) for all \( c_{IT}, c_{ET} \).

Now consider any period \( t < T \). Assume that \( V^I_{t+1}(c_{IT+1}, c_{ET+1}) - \phi^I_{t+1}(c_{IT+1}, c_{ET+1}) > 0 \). We show that \( V^I_t(c_H, c_{ET}) - \phi^I_t(c_H, c_{ET}) > 0 \) is implied.

\[
V^I_t(c_H, c_{ET}) = \max_{p_H} q^M(p_H)(p_H - c_H) + \ldots \beta \left[ (1 - G(\kappa^*(\varsigma^{-1}(p_H), c_{ET}))) E_t[V^I_{t+1}|c_H, c_{ET}] + G(\kappa^*(\varsigma^{-1}(p_H), c_{ET})) E_t[\phi^I_{t+1}|c_H, c_{ET}] \right] \phi^I_t(c_H, c_{ET}) = \pi^D_I(c_H, c_{ET}) + \beta E_t[\phi^I_{t+1}|c_H, c_{ET}]
\]

Now, under the assumption that \( V^I_{t+1}(c_{IT+1}, c_{ET+1}) - \phi^I_{t+1}(c_{IT+1}, c_{ET+1}) > 0 \) for all \( c_{IT+1}, c_{ET+1} \),

\[
\beta \left[ (1 - G(\kappa^*(\varsigma^{-1}(p_H), c_{ET}))) E_t[V^I_{t+1}|c_H, c_{ET}] + G(\kappa^*(\varsigma^{-1}(p_H), c_{ET})) E_t[\phi^I_{t+1}|c_H, c_{ET}] \right] > \beta E_t[\phi^I_{t+1}|c_H, c_{ET}]
\]

for any \( p_H \) (including the static monopoly price). But, as \( q^M(p_H)(p_H - c_H) > \pi^D_I(c_H, c_{ET}) \) when the static monopoly price is chosen, it follows that \( V^I_t(c_H, c_{ET}) > \phi^I_t(c_H, c_{ET}) \) (when a possibly different price is chosen by the monopolist).

(ii) \( \frac{\partial \{V^I_t(c_H, c_{ET}) - \phi^I_t(c_H, c_{ET})\}}{\partial c_{IT}} < 0 \) for all \( (c_H, c_{ET}) \). Proof by backwards induction.

In the final period, a monopolist will produce the static monopoly output so that \( \frac{\partial V^I_T(c_{IT}, c_{ET})}{\partial c_{IT}} = -q^\text{static monopoly}(c_{IT}) \). For the duopolist, \( \frac{\partial \phi^I_T(c_{IT}, c_{ET})}{\partial c_{IT}} = -q^D_I(c_{IT}, c_{ET}) + \frac{\partial \pi^D I}{\partial a^D_E} \frac{\partial a^D_E}{\partial c_{IT}} \). Therefore

\[
\frac{\partial \{V^I_T(c_{IT}, c_{ET}) - \phi^I_T(c_{IT}, c_{ET})\}}{\partial c_{IT}} = q^D_I(c_{IT}, c_{ET}) - q^\text{static monopoly}(c_{IT}) - \frac{\partial \pi^D_I}{\partial a^D_E} \frac{\partial a^D_E}{\partial c_{IT}}
\]

Under our assumptions on the duopoly game, this expression will be negative as required.

\[
\frac{\partial E_{t-1}[V^I_{t+1}|c_{ET}]}{\partial c_{IT}} - \frac{\partial E_{t-1}[\phi^I_{t+1}|c_{ET}]}{\partial c_{IT}} = \int_{c_{ET}} \int_{c_{IT}} \{V^I_{t+1}(c_{IT+1}, c_{ET+1}) - \phi^I_{t+1}(c_{IT+1}, c_{ET+1})\} \frac{\partial \psi(c_{IT+1}|c_{IT})}{\partial c_{IT}} \psi(c_{ET+1}|c_{ET}) dc_{IT} dc_{ET}
\]
Consider a period $t < T$:

\[
\frac{\partial V^I(t, c_{Et})}{\partial c_{It}} = \frac{\partial \pi^M(p^*, c_{It}, c_{Et})}{\partial c_{It}} + \beta \frac{\partial E_t[\varphi^I_{t+1}c_{It}, c_{Et}]}{\partial c_{It}} - \ldots
\]

\[
\beta \frac{\partial \kappa^*(c_{It}, c_{Et})}{\partial c_{It}} g(\cdot) \left\{ E_t[V^I_{t+1}|c_{It}, c_{Et}] - E_t[\varphi^I_{t+1}|c_{It}, c_{Et}] \right\} + \ldots
\]

\[
\beta(1 - G(\kappa^*(c_{It}, c_{Et}))) \left[ \frac{\partial E_t[V^I_{t+1}|c_{It}, c_{Et}] - E_t[\varphi^I_{t+1}|c_{It}, c_{Et}]}{\partial c_{It}} \right]
\]

\[
\frac{\partial \pi^M(p^*, c_{It}, c_{Et})}{\partial c_{It}} = -q^M(p^*) + \frac{\partial^*c_{It}}{p^*} \left\{ q(p^*) + \frac{\partial^M(p^*)}{\partial p} (p^* - c_{It}) \right\}. \quad \text{But from the equilibrium strategy of the incumbent (recall that $V^I_{t}(c_{It}, c_{Et})$ is the value to being an incumbent in period $t$ assuming equilibrium play),}
\]

\[
\frac{\partial p^*}{\partial c_{It}} \left\{ q^M(p^*) + \frac{\partial^M(p^*)}{\partial p} (p^* - c_{It}) \right\} = \beta g(\cdot) \frac{\partial \kappa^*}{\partial c_{It}} \left[ E_t[V^I_{t+1}(c_{It}, c_{Et}) - E_t[\varphi^I_{t+1}(c_{It}, c_{Et}, t)] \right]
\]

so

\[
\frac{\partial V^I(t, c_{Et})}{\partial c_{It}} = -q^M(p^*) + \beta \frac{\partial E_t[\varphi^I_{t+1}|c_{It}, c_{Et}]}{\partial c_{It}} + \ldots
\]

\[
\beta(1 - G(\kappa^*(c_{It}, c_{Et}))) \left[ \frac{\partial E_t[V^I_{t+1}|c_{It}, c_{Et}] - E_t[\varphi^I_{t+1}|c_{It}, c_{Et}]}{\partial c_{It}} \right]
\]

and

\[
\frac{\partial \varphi^I_{t}(c_{It}, c_{Et})}{\partial c_{It}} = \frac{\partial \pi^D(c_{It}, c_{Et})}{\partial c_{It}} + \beta \frac{\partial E_t[\varphi^I_{t+1}|c_{It}, c_{Et}]}{\partial c_{It}}
\]

\[
= -q^D_{It}(c_{It}, c_{Et}) + \frac{\partial \pi^D}{\partial a^D_{Et}} \frac{\partial a^D_{Et}}{\partial c_{It}} + \beta \frac{\partial E_t[\varphi^I_{t+1}|c_{It}, c_{Et}]}{\partial c_{It}}
\]

Therefore,

\[
\frac{\partial V^I(t, c_{Et})}{\partial c_{It}} - \frac{\partial \varphi^I_{t}(c_{It}, c_{Et})}{\partial c_{It}} = q^D_{It}(c_{It}, c_{Et}) - q^M(p^*(c_{It}, c_{Et})) - \frac{\partial \pi^D}{\partial a^D_{Et}} \frac{\partial a^D_{Et}}{\partial c_{It}} + \ldots
\]

\[
\beta(1 - G(\kappa^*(c_{It}, c_{Et}))) \left[ \frac{\partial \{ E_t[V^I_{t+1}|c_{It}, c_{Et}] - E_t[\varphi^I_{t+1}|c_{It}, c_{Et}] \}}{\partial c_{It}} \right]
\]

We need this expression to be negative. If \( \frac{\partial \{ V^I_{t+1}(c_{It+1}, c_{Et+1}) - \varphi^I_{t+1}(c_{It+1}, c_{Et+1}) \}}{\partial c_{It+1}} < 0 \) (the induction hypothesis), then the term in square brackets is negative. Therefore a sufficient condition is that \( q^D_{It}(c_{It}, c_{Et}) - q^M(p^*(c_{It}, c_{Et})) - \frac{\partial \pi^D}{\partial a^D_{Et}} \frac{\partial a^D_{Et}}{\partial c_{It}} \) is negative. As the limit pricing output \( q^M(p^*(c_{It}, c_{Et})) \) will be greater than the static monopoly output, a sufficient condition for the whole expression to be negative
is simply the sufficient conditions that we had for period $T$, which we know are satisfied for our duopoly game.

Based on Theorem 3 of Ramey (1996), we have the following result.

**Proposition 5** Take the signalling payoff $\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})$ where $\kappa_{lt}$ is $E$’s threshold. If Conditions

(i) $\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El}) \neq 0$ for all $c_{lt}, \kappa_{lt}, p_{lt}, c_{El}$; (ii) $\frac{\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}{\Pi^H(c_{lt}, \kappa_{lt}, p_{lt'}, c_{El})}$ is a monotone function of $c_{lt}$ for all $\kappa_{lt}, c_{El}$; and (iii) $p_t \geq p_{\text{static monopoly}}(c_t)$ and $\Pi^H(c_{lt}, \Phi, p_{lt}, c_{El}) < \max \Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})$ for all $t, c_{El}$.

Then, an equilibrium satisfying the D1 refinement will be fully separating.

We can show that our game satisfies these conditions by essentially replicating the proofs from before.

**Proof.** (i) $\Pi^H_2(c_{lt}, \kappa_{lt}, p_{lt}, c_{El}) \neq 0$ for all $c_{lt}, \kappa_{lt}, p_{lt}, c_{El}$.

$$\Pi^H_2(c_{lt}, \kappa_{lt}, p_{lt}, c_{El}) = -\beta g(\kappa_{lt}) \left\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \right\}$$

This will not be equal to zero if $g(.) > 0$ (which depends on entry costs having sufficient support), and

$\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \} > 0$ (expected continuation value as a monopolist greater than as a duopolist). This property was shown above.

(ii) $\frac{\Pi^H_2(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}{\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}$ is a monotone function of $c_{lt}$ for all $\kappa_{lt}, c_{El}$.

$$\frac{\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}{\Pi^H_2(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})} = q^M(p_{lt}) + \frac{\partial q^M(p_{lt})}{\partial p_{lt}}(p_{lt} - c_{lt})$$

$$\left( -\beta g(\kappa_{lt}) \left\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \right\} \right)^{-1}$$

Differentiate wrt $c_{lt}$ :=

$$\frac{\partial g(\kappa_{lt})}{\partial p_{lt}} \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] + ...$$

$$\left[ q(p_{lt}) + \frac{\partial q^M(p_{lt})}{\partial p_{lt}}(p_{lt} - c_{lt}) \right] \frac{\partial \left( \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \right)}{\partial c_{lt}} \left( -\beta g(\kappa_{lt}) \left\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \right\} \right)^2$$

Sufficient conditions for this expression to be $< 0$ (implying $\frac{\Pi^H_2(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}{\Pi^H(c_{lt}, \kappa_{lt}, p_{lt}, c_{El})}$ monotonic in $c_{lt}$) are that:

(i) $V^I_{t+1}(c_{lt+1}, c_{El+1}) - \phi^I_{t+1}(c_{lt+1}, c_{El+1}) > 0$ for all $(c_{lt+1}, c_{El+1})$, implying $\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \} > 0$; (ii) $\frac{\partial \left( V^I_{t+1}(c_{lt+1}, c_{El}) - \phi^I_{t+1}(c_{lt+1}, c_{El}) \right)}{\partial c_{lt+1}} < 0$ which implies $\{ \mathbb{E}_t[V^I_{t+1}|c_{lt}, c_{El}] - \mathbb{E}_t[\phi^I_{t+1}|c_{lt}, c_{El}] \} < 0$ by application of Lemma 25, (iii) $q(p_{lt}) + \frac{\partial q^M(p_{lt})}{\partial p_{lt}}(p_{lt} - c_{lt}) \geq 0$ for all prices below the monopoly price (which will hold given that the static profit function is strictly quasi-concave). These properties were shown above.