Competition and Dynamic Pricing in a Perishable Goods Market

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VERY PRELIMINARY

Abstract

We estimate a model of entry, exit and pricing decisions in an online market for event tickets where there are many competing sellers and prices change frequently. We use the estimates from our model to analyze the optimality of the pricing policy used by the largest seller (broker) in the market. We show that the broker’s pricing policies substantially affect the prices set by his competitors. When we compare the broker’s pricing policy with the prices that our model predicts are optimal we find that the broker sets approximately correct prices close to the game, when his pricing problem resembles a static one, but that he might be able to gain from using different pricing rules and updating prices more frequently further from the game.

1 Introduction

Sellers of perishable goods, such as airlines, ticket brokers, concert organizers and retailers of fashion and seasonal items, have to sell inventory within a fixed time horizon. These firms increasingly use dynamic pricing (DP) strategies, where they change prices as a function of both inventory and the time remaining, as technology makes it cheaper to change prices, track inventory and model consumer behavior. Managers often identify these types of revenue management strategies as being very valuable. For example, Robert Crandall, the former CEO of American Airlines, has been widely
quoted as describing them as “the single most important technical development in transportation management since we entered the era of airline deregulation in 1979”.

The theoretical basis of DP strategies is now well-established, principally in cases where there is a single seller (see references below). However, there are few empirically-tractable models that can be used to guide managers’ pricing decisions, especially in settings where (a) products are differentiated; (b) there is significant competition from other sellers; and (c) it is possible that the price that the manager sets will affect the prices that competitors set in the future. These features are present in all of the examples of perishable goods markets given above. In this paper, we introduce an empirical model that can be used for this purpose, and we show how it can be estimated using the type of data that a seller will typically have available. We use our model to analyze whether the pricing strategy that a seller currently uses is approximately optimal.

Our empirical setting is an online secondary ticket market (Stubhub) for sports event tickets. We use data for 15 home games played by a single major league sports team in 2010, where an anonymous large broker, who provided us with its sales data and whose explicit objective is revenue-maximization, accounted for a significant share of the market. In this market the broker faces competition from a large number of smaller sellers, many of whom are season ticket holders who do not want to attend a particular game. There are three features of the data that make the broker’s dynamic price-setting problem an interesting one to study, both methodologically and substantively.

First, the broker cuts prices over time, as illustrated in Figure 1, where prices are relative to the face value of the ticket. The general pattern of falling prices has been documented using different data from secondary ticket markets by Sweeting (2012). The ability to reduce prices as the game approaches if tickets remain unsold makes it optimal to set higher prices further from the game, rationalizing a price-cutting pattern.

[FIGURE 1 ABOUT HERE]

Second, most of the broker’s sales are made in the last few days before the game when the broker’s prices are lowest, as shown in Figure 2. This raises the question of whether it is optimal for the

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1Smith, Leimkuhler, and Darrow (1992) estimate that yield management increased AA’s annual revenues by $500 million. The San Francisco Giants implemented dynamic pricing for parts of their stadium in 2010 and estimated that it would increase their revenues by $5m per year and the Giants’ ticketing manager described DP as “changing the ticket world” (taken from an article by Adam Satarino in Bloomberg Businessweek, May 20 2010, accessed July 19, 2011).
seller to delay so long before cutting prices, or whether he would be better off setting a slightly lower price further from the game in order to increase his probability of sale when competitors prices tend to be higher. It is not clear *ex-ante* whether the broker’s current pricing rules achieve the trade-off between higher prices and lower probabilities of sale optimally.

[FIGURE 2 ABOUT HERE]

Third, competition and price-setting work in an interesting way which is also found in lots of other markets but which has been ignored in the existing literature. Different sellers compete with each other, but they know that they update their prices only infrequently and, importantly, not at the same time. Therefore, when a seller sets his price today he knows that other sellers will treat this price as given when setting their prices in the future. We show below that non-brokers cut prices in response to the broker’s price cutting policy, which may reduce the broker’s profits, and our results suggest that the broker may respond sub-optimally to the prices set by non-brokers. From a modeling perspective, the existing literature has either modeled monopolists or a fixed number of competing firms who set prices simultaneously. Providing an empirical framework to model this type of stochastic pricing decisions is an important contribution of this paper, which may be relevant for thinking about a wide range of markets.

Central to our paper is an estimable continuous time model of the market where potential buyers, sellers and opportunities to change prices arrive stochastically. The optimal dynamic pricing policy for our broker depends qualitatively on several factors that are clearly related to particular parameters of the model: the arrival rate and preferences of consumers, and particularly their price sensitivity; the probability with which the broker can change prices and the number of listings that he has to sell; and, how the distribution of competitors’ prices can be expected to respond to the price that the broker sets. We provide a quantitative assessment of the broker’s optimal policy by estimating the parameters of the model and then performing counterfactual simulations. We find that, on average, the broker’s prices close to the game are fairly close to optimal but that there are potential advantages to using different pricing rules further from the gain and from updating prices more frequently.

Our paper makes at least three contributions. First, we provide the first empirical framework for calculating optimal dynamic pricing policies in a setting with competing sellers, and we provide an assessment of a pricing policy that is used in this type of setting. Our framework could be applied
to other markets where a seller has some ability to ‘move the market’ by affecting the distribution of 
prices that competitors set in the future. Second, we show how our model can be estimated using 
data that may be widely available to sellers. For example, we assume only the availability of trans-
action data from one seller, not competing sellers in the market. Our methods build on techniques 
recently developed for estimating continuous time games in the economics literature (Arcidiacono, 
Bayer, Blevins, and Ellickson (2010)). Finally, we can use the estimated model to highlight which 
parameters are key for determining the optimal pricing policy. We do this using counterfactual 
policies, where we vary the parameters from their estimated values.

We conclude this introduction by describing the most closely-related theoretical and empirical 
literature. Section 2 describes our data, and highlights the stylized facts that motivate our analysis. 
Section 3 presents the model and Section 4 describes how it is estimated. Section 5 presents the 
estimates and Section 6 the results of our counterfactuals. Section 7 concludes.

1.1 Literature Review

The theoretical literature on dynamic pricing, reviewed in Elmaghraby and Keskinocak (2003), dates 
back to Kincaid and Darling (1963). In this paper, and in later papers such as Gallego and Van Ryzin 
(1994), Bitran and Mondschein (1997) and McAfee and Velde (2008) a monopolist seller of a per-
ishable good, with a fixed initial inventory, faces consumers who arrive stochastically, have to buy 
at once or exit the market and have unit demand with valuations drawn from a time-invariant dis-
tribution. In Gallego and Van Ryzin (1994) and McAfee and Velde (2008) the seller is assumed 
to update its price continuously, whereas Bitran and Mondschein (1997) assume that he updates it 
infrequently using a policy of periodic price reviews. With continuous updating, the optimal price is 
equal to the opportunity cost of a sale plus a mark-up, which depends on the shape of the valuation 
distribution (equivalently the flow demand curve). This opportunity cost will go down over time 
if sales are not made as future selling opportunities disappear. On the other hand, when a sale is 
made opportunity costs will increase. McAfee and Te Velde (2006) show that on average the seller 
tends to cut prices, if he has any units left, at the end of the time horizon.

Many of the assumptions made in these papers have been relaxed in the subsequent theoretical 
literature. Zhao and Zheng (2000a) allow for time-varying elasticities of demand which can change 
how optimal prices tend to evolve. For example, if demand becomes more elastic closer to the
game, which may happen if the type of consumer who visits the market changes, it will be optimal to set lower prices. We allow for this type of variation when we estimate our model. We also allow for competition between a small number of sellers. In the existing literature competition has been handled in a particular stylized way. For example, Gallego and Hu (2007) and Xu and Hopp (2006) provide methods for solving continuous time models with a fixed number of ex-ante symmetric firms. Lin and Sibdari (2009) solve a discrete time game where logit differentiated duopolists set prices each period, and they show how it can be optimal to raise its price in order to make it more likely that its competitor sells out and leaves the market. All of these models neglect the possibility that new sellers may enter, which is a clear feature of our data and allowed for in our model, and they also assume that sellers adjust prices simultaneously. This can mean either instantaneously in continuous time or every period in discrete time. Our model assumes that opportunities to change prices only arise stochastically, which is a realistic assumption for online markets where firms may only review their prices infrequently. If prices are strategic complements, the stochastic nature of price setting might increase incentives to raise prices further from the game. One feature which is not included in our model, which has been included in some of the recent theoretical literature, is strategic consumers, who can respond to higher prices by waiting in the market. This can make demand a long time before the game more elastic, leading to lower prices. Sweeting (2012) provides evidence against this type of strategic consumer behavior being of first-order importance in secondary market for event tickets.

The empirical dynamic pricing literature has focused on two questions. The first question is which existing theories of dynamic pricing explain how sellers price, assuming that they behave optimally (McAfee and Te Velde (2006), Puller, Sengupta, and Wiggins (2009), Sweeting (2012)). The second question is whether firms set prices optimally, assuming that a particular model is correct and this is also the question in our paper. The existing studies of this question, such as Heching, Gallego, and van Ryzin (2002), Vulcano, van Ryzin, and Chaar (2010), Caro and Gallien (2010), Soysal and Krishnamurthi (2007) and Li, Granados, and Netessine (2011), focus on a single seller, although Vulcano, van Ryzin, and Chaar (2010) allow for consumers to substitute between multiple products sold by the same seller, in that case a fashion retailer. In our paper, we want to investigate how competition between sellers - in particular a large broker and many smaller sellers - should affect pricing. There are two elements to this competition effect: first, as other sellers cut prices,
it may be optimal for the broker to set a lower price, and the importance of this may vary with the
time until the game; and, second, the broker might strategically want to influence how other sellers
prices evolve as suggested above. In doing so, we assume that consumers are not strategic (i.e.,
they have to buy as soon as they arrive or exit the market), an assumption that has been relaxed
in Soysal and Krishnamurthi (2007) and Li, Granados, and Netessine (2011), although including
strategic consumers in the model would be an interesting direction for future research.\footnote{As
mentioned before, the results in Sweeting (2012) suggest that strategic consumer behavior is not too important
in secondary ticket markets, possibly because the nominal gains to optimal timing decisions are quite small. Li,
Granados, and Netessine (2011) find that the proportion of strategic consumers in most markets for airline tickets,
where the nominal gains are potentially larger because prices are higher, is not too large.}

Solving for equilibrium strategies in a dynamic pricing model with many, possibly asymmetric,
competing sellers is beyond the current literature. In this paper we therefore take a simpler approach
to modeling seller behavior, based on the estimation of policy functions as functions of the state vari-
ables. This approach has been widely used in the literature estimating dynamic games in economics
(Bajari, Benkard, and Levin (2007), Ryan (forthcoming)), although applying it here requires us to
innovate because our data involves repeated snapshots of a market that evolves continuously. We
use the estimated functions describing how other sellers’ prices evolve, to conduct counterfactuals,
as in Benkard, Bodoh-Creed, and Lazarev (2009). This makes the strong assumption that a change
in the broker’s policy would not change the entry, exit and pricing \textit{policies} of other sellers, although
it may change the exact prices that they set. There are some ways to test this assumption and we
will do so in future work.

\section{Data}

\subsection{Sources}

Our data comes from three sources: a professional ticket broker, Stubhub.com and the official website
of the major league sports team whose games we use in our analysis. The broker provided us with
its transaction data for games played by the team from December 2009 to April 2010. This data
includes details of the exact seats (section, row, seat number) sold, the sale price, the time at which
the broker registered the sale in its internal system and the distribution channel through which the
sale took place. It also lists the tickets that the broker had that were not eventually sold, although
for this team the broker sells the vast majority of his listings. In this paper we use data from 14 games and we only use data on listing and sales on Stubhub, which was the outlet for most of the broker’s listings as well as being the largest online secondary market for event tickets.

The Stubhub data was collected using a web scraper from the ‘buy page’ for each game. Our aim was to collect a snapshot of all tickets available on Stubhub about once every three hours, although in practice problems with the web scraper caused by Stubhub changing the presentation of its website, meant that the dataset is not complete, which is one reason why we pool games in estimation. This data contains the section, row, number of seats available and list price for each listing posted. The seller can also indicate whether a smaller number of tickets can be purchased, and sellers typically do this when they are trying to sell three or more seats. Stubhub does not show information about the seller as it provides a guarantee that someone buying from its site will receive tickets at least as good as those purchased. It also charges sizable commissions to both buyers (10%) and sellers (15%), and sets the shipping fees that have to be paid.

To estimate demand for listings on Stubhub, we need to match the listing data on Stubhub with the broker data. The aim of this matching process is to both identify which tickets on Stubhub belong to the broker and the prices that he set prior to sale, and to identify when broker sales occur and what was the competition that the broker’s listings faced at the time of sale. The matching process is described in detail in the Appendix. Finally, the team’s website was used to obtain the single game price (face value) of tickets in every section. During our sample, the team had a successful season, with realized attendances of at least 96% of stadium capacity for all of the 14 games that we use in estimation.

2.2 Summary Statistics

Table 1 shows how many downloads of data from Stubhub that we have for each game as the game approaches. The left-hand column shows the number of number of days prior to the game (so the top line is the day of the game), and each entry gives the number of downloads of data that we have. For example, when the number is 8, we have one download on average every three hours. For the first games in our sample, we lack data a long time before the game simply because these games are in January and we only started collecting data in December. For the later games, we have a fairly complete dataset for the month prior to the game, and when we interpret our results we will focus
on this final month which is when the market is busiest and the most interesting price dynamics occur (Sweeting (2012)) shows that this is also true in the market for MLB tickets, using data for all teams).

[TABLE 1 ABOUT HERE]

Table 2 shows summary statistics for the broker’s and competitors listings on Stubhub, both in the full sample and in the set of game-sections that we use in estimation. The main criteria for selecting these listings is that these are game-sections where the broker has at most one listing (e.g., one group of 6 seats in the same row). On average, competitors set prices that are close to face value whereas the broker sets lower prices. This pattern is consistent - in an optimal pricing framework - with the broker having no value to unsold tickets, whereas other sellers may be willing to attend the game themselves. This pattern is true for both the full sample and the estimation sample, although more expensive sections are over-represented in the estimation sample. The broker never has seats in the front row of a section, whereas about 7% of competitors’ listings are for front-row seats. These listings may well be posted by season-ticket holders who are more likely to own front-row seats. The average number of seats in a broker’s listing is 4.27, with a range from 1 to 23. However, 25% of these listings had two seats, 32.1% had four seats and 26% had five or six seats, with 90% having six seats or less. When we estimate the model we only use those observations where the broker has no more than six seats to sell.

[TABLE 2 ABOUT HERE]

Table 3 shows many seats the broker sells when a transaction takes place. The most common transaction involves a pair of seats, often from a larger listing. As we illustrate below, this can have an important impact on the optimal profile of prices as the optimal price increases when a sale of a subset of seats from a listing takes place. At the moment when we estimate the dynamic model we assume that all buyers who arrive in the market only want two seats, although when we estimate the demand parameters we are more flexible. Going forward we will try to incorporate the arrival of buyers wanting more than two seats into the model. A comparison of the number of observations for the broker and competitors in the estimation sample indicates that the broker has about 20% of listings in this sample.

[TABLE 3 ABOUT HERE]
2.3 Stylized Facts and Current Broker Pricing Behavior

We now describe how prices evolve over time, and what are the key facts about how the broker currently sells tickets that motivate our interest in his optimal pricing policy.

As prices and games are heterogeneous, we examine the dynamics of prices using a regression model

\[ p_{it} = X_{it} \beta + D_t \alpha + FE_i + \varepsilon_{it} \]

where \( X_{it} \) are observed characteristics (e.g., the number of seats, the number of the row and measures of the performance of the home and away teams that may affect demand), \( D_t \) are dummies measuring the number of days prior to the game and \( FE_i \) are game-face value fixed effects. Prices (\( p_{it} \)) are the Stubhub list price divided by the face value of the ticket, although using the log of the nominal price gives broadly similar implications.

[TABLE 4 ABOUT HERE]

The coefficients are shown in Table 4, using different subsets of the data. In columns (1)-(4) observations are listings. The specifications differ in whether the sample comprises the broker’s listings, competitors’ listings, and whether or not the observations are from game-sections included in the estimation sample. Here we see that based on all of the data, non-brokers tend to maintain fairly constant prices just above face value (add together the price 0-2 days before the game and the relevant time coefficient) until the last few days before the game. Figure 3 also shows this average price path. On the other hand, the broker cuts prices more smoothly as a game approaches, dropping prices by 20% of face value in the last five days before the game and an additional 30% of face value in the 10 days before that. As can be seen in the Figure, the broker’s average list prices are always below those of the average non-broker.

[FIGURE 3 ABOUT HERE]

However, despite always having lower prices, most of the broker’s sales are concentrated immediately before the game. Figure 4 shows how the sales rate of the broker evolves as a game approaches, where this rate is defined as the number of sales that the broker makes during the day divided by the number of listings that the broker has at the start of the day. This rate can be greater than
because a single listing can result in more than one transaction if multiple subsets of the tickets listed are sold. The sales rate climbs steeply in the last five days before the game. This could be caused by the broker’s falling price, a decline in the number of competitor listings, which can be seen in Figure 5, or an increase in the number of buyers in the market. If the last two factors drive the increase in demand, this might suggest that the broker could raise his revenues by not cutting prices so dramatically. On the other hand, if it is the broker’s lower price that increases his demand, this may suggest that the broker would do better by setting a slightly lower price earlier before the game to increase the likelihood that he sells. Our model helps us to distinguish what drives the increase in sale, and therefore to establish what the optimal pricing policy should be.

A comparison of columns (1) and (3) in Table 4 also highlights another potentially important factor in a seller’s pricing decision. Using the full sample, which mainly consists of sections where the broker is not trying to sell tickets, competitors’ maintain fairly constant prices until quite close to the game. On the other hand, when we use the estimation sample, which only includes sections where the broker has tickets and is cutting prices, we observe that non-brokers are cutting prices too. Assuming that it is the price cutting behavior of the broker that causes the non-brokers to cut their prices (which should be true as long as the sections in which the broker has tickets are random, as all of these specifications include game-face value fixed effects), this suggests that if market demand is low in the weeks prior to the game, the broker may be hurting himself when he cuts prices by causing his competitors’ prices to fall, thereby reducing the price that he can set when the market is very active close to the game.

3 Model

Calculating the broker’s optimal pricing rule requires a model that predicts (i) the probability that the broker will sell as a function of the broker’s price and the price of competitors; and (ii) how competitors’ prices will evolve as a function of the broker’s price. We now describe the parsimonious, continuous time model that provides us with these predictions.

In outline, the model works in the following way: at any point in time the broker and his competitors have given numbers of tickets and particular prices for their listings. A number of stochastic
events, with Poisson arrivals, can happen. These events are: (i) a potential buyer may arrive in the market, in which case she can choose to buy one of the available listings or exit the market forever. We therefore assume away the possibility that potential buyers can strategically delay purchasing, so that a buyer has a static, rather than a dynamic, problem to solve; (ii) the broker may change his price; (iii) a competitor may enter the market, exit the market (without being sold, e.g., the seller decides to go to the game himself) or change his price. We model a specific arrival rate for each of these events and we model, as a stochastic process, what happens when one of these events occurs.

To explain the model we begin by specifying the static, logit preferences of a buyer. This is useful as the state space of the dynamic model is defined using these preferences, following ideas in Nevo and Rossi (2008). We then explain how we parametrize the arrival rates and the evolution of the state variables. We model markets at the game-section level, and, in estimation, we use game-sections where the broker has only one set of tickets to sell (e.g., a group of 6 seats in row 8). In some ways this is too narrow, as consumers are likely to substitute across similar sections, but this approach means that we only have to deal with the broker setting a single price, which is easier in the counterfactuals. In future iterations, we hope to be more general. When estimating the dynamic part of the model we also assume that buyers are only interested in buying a pair of seats (recall, 54% of purchases are of two seats), although we are more flexible when estimating preferences.

### 3.1 Buyer Demand

We follow most of the literature on consumer demand by assuming that a buyer \( i \) arriving in the market will choose the listing that maximizes his utility, where his utility for listing \( j \) is given by a linear function:

\[
    u_{ij} = x_j \beta - \alpha p_j + \varepsilon_{ij} = \delta_j + \varepsilon_{ij}
\]

where \( x_j \) are listing \( j \)'s characteristics (specifically, the row), \( p_j \) is the price (measured relative to face value) and \( \varepsilon_{ij} \) is a Type I extreme value (logit) error that independently and identically distributed across listings. The \( \delta_j \)s are known as “mean utilities”. When faced by a set of \( K \) listings in a market,
the probability that consumer buys listing $j$ is

$$Pr(i \text{ chooses } j) = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{K} \exp(\delta_k)}$$

where the 1 in the denominator reflects the fact that the consumer may choose not to purchase any tickets. We assume that a consumer who wants to buy two seats chooses from the set of listings that have at least two seats (this implicitly assumes that all listings with more than two seats allow only two seats to be bought, which is true in 85% of cases, and in 95% of cases with four or more seats), whereas a consumer who wants to buy four seats chooses from the set of listings with at least four seats, and so on. Apart from through this effect on the choice set, the number of seats in a listing is assumed to have no effect on choice probabilities.\(^3\)

### 3.2 State Space

Dealing with a large state space, containing information on the prices, rows and number of seats being sold in each listing is infeasible (at least using our current estimation methodology). Therefore we need to reduce the number of state variables, and to do so we base our model on the $\delta$ (mean utility) terms implied by preferences. Specifically, the state of a particular game-section is defined by the number of seats that the broker has left to sell (2, 4 or 6), $\delta_B$ of the broker’s listing and the value of $\delta_{NB}$ for non-brokers’ listings where

$$\delta_{NB} = \log \left( \sum_{k \in NB} \exp(\hat{\delta_k}) \right).$$

For estimation, we discretize the values of the $\delta_B$ and $\delta_{NB}$ state variables. Specifically we allow for 23 bins of $\delta_B$, including an absorbing state associated with all of the broker’s tickets having been sold, and 23 bins for $\delta_{NB}$, including a state where no non-broker tickets are available. The boundaries of the bins are chosen so that roughly equal number of observations are in each bin.

\(^3\)For example, this rules out the possibility that someone who wants to buy two seats might prefer to buy two out of four seats in order to increase the probability that they will have an empty seat next to them.
3.3 State Transition Probabilities

In a continuous time, the probability of a transition from one state to another is determined by the arrival rate of the relevant event and the conditional probability of that particular transition given that an event arrives. For example, the probability that the number of broker’s tickets falls by two is determined by the arrival rate of a buyer and the probability that an arriving buyer purchases from the broker’s listing. Similarly, the probability that \( \delta_B \) changes from one value to another is determined by the arrival rate of a move where the broker can change his price and the probability that the price change takes the value of \( \delta_B \) to its new level. In continuous time we can treat these events as happening individually, as the probability that two events happen at the same instant is zero. I now describe each of these stochastic processes in turn.

The modeling of state transitions statistical processes is quite similar to approaches that have recently been taken to the estimation of dynamic games where players are assumed to use Markov Perfect equilibrium strategies (Ryan (forthcoming); Benkard, Bodoh-Creed, and Lazarev (2009); Bajari, Benkard, and Levin (2007)). However, although we assuming that transitions are functions of the previous state, we do not assume that either the broker or his competitors set prices in an optimal way. This reflects the fact that our question pre-supposes that even the largest seller may not set prices optimally.

3.3.1 Buyer Arrival and Purchase

Two broker tickets are sold if a buyer arrives and chooses to buy from the broker’s listing. The Poisson arrival rate for a buyer is given by \( \lambda^{BUYER} \) and the probability that an arriving buyer purchases two tickets from the broker is

\[
\frac{\exp(\delta_B)}{1 + \exp(\delta_B) + (I_{NB} \neq 0)\exp(\delta_{NB})}
\]

where \((I_{NB} \neq 0)\) means that there is a non-broker listing present. The arrival rate for buyers is allowed to vary as a game approaches, so that we can capture the market becoming more active over time. If a broker ticket is bought, the number of remaining broker tickets changes mechanically or, if the final broker tickets are bought, the state changes to the absorbing one where there are no broker tickets left.

Of course, it may be the case that the buyer arrives and buys non-broker tickets which may cause...
the value of $\delta_{NB}$ to change. The probability that a non-broker listing is sold is

$$\frac{\exp(\delta_{NB})}{1 + \exp(\delta_B) + (I_{NB} \neq 0)\exp(\delta_{NB})}$$

and we model the evolution of $\delta_{NB}$ if this happens using a transition matrix $P^{NB}$ which allows for the possibility for $\delta_{NB}$ to evolve to any lower value or stay the same. If the consumer chooses to buy no listing, he exits the market without the state changing.

3.3.2 Broker Price Change

We assume that opportunities for a price change arrive stochastically. The Poisson arrival rate for a broker change is $\lambda^{BROKER}$ and, conditional on a move arriving, $\delta_B$ is assumed to evolve according to a stochastic process:

$$\delta^{a'}_B = \alpha_0 + \alpha_1\delta^a_B + \alpha_2(I_{NB} \neq 0) \cdot \delta^b_{NB} + \alpha_3(1 - (I_{NB} \neq 0)) + \xi_B$$  \hspace{1cm} (3)

where $\xi_B$ is assumed to be normally distributed with mean 0 and variance $\sigma^2_1$. Given the discrete nature of the state space this implies that the probability of the next value of $\delta_B$ being $\delta^{a'}_B$ is

$$\frac{\phi\left(\frac{\delta^{a'}_B - \alpha_0 - \alpha_1\delta^a_B - \alpha_2(I_{NB} \neq 0) \cdot \delta^b_{NB} - \alpha_3(1 - (I_{NB} \neq 0))}{\sigma_1}\right)}{\sum_{a''} \phi\left(\frac{\delta^{a''}_B - \alpha_0 - \alpha_1\delta^a_B - \alpha_2(I_{NB} \neq 0) \cdot \delta^b_{NB} - \alpha_3(1 - (I_{NB} \neq 0))}{\sigma_1}\right)}$$  \hspace{1cm} (4)

where $\phi(\cdot)$ is the standard normal probability density function and the $\alpha$s and $\sigma_1$ are parameters to estimate.

3.3.3 Non-Broker State Change

$\delta_{NB}$ may change if more sellers enter the market, sellers leave the market or an existing seller changes his price. As we have aggregated non-broker listings into a single variable, we cannot model individual decisions, but we do model three process by which $\delta_{NB}$ may change.

The first process, which one can think of as “non-broker arrival” allows for the possibility that non-broker listings will enter the market when there were none previously. The Poisson arrival rate
is $\lambda^{NB,IN}$, and the probability that the state $(n_B, \delta_B^a, I_{NB} = 0)$ moves to state $(n_B, \delta_B^a, \delta_{NB}^b)$ is
\[
\frac{\exp(\eta_1 \delta_B^a + \eta_2 \delta_{NB}^b + \eta_3 \delta_B^a \delta_{NB}^b)}{1 + \sum_{\delta_{NB}^b \neq 0} \exp(\eta_1 \delta_B^a + \eta_2 \delta_{NB}^b + \eta_3 \delta_B^a \delta_{NB}^b)} \tag{5}
\]
where the $\eta$s are parameters to estimate. Note that this function allows the new value of $\delta_{NB}$ to depend on the value of $\delta_B$ as we would expect it to do if competitors set lower prices when the broker has a lower price. We could enrich this specification to also depend on the number of tickets that the broker has left.

The second process, which one can think of as “non-broker exit” allows for the possibility that non-broker listings will exit the market without being sold. The Poisson arrival rate is $\lambda^{NB,OUT}$ and the probability that the state changes from $(n_B, \delta_B^a, \delta_{NB}^b)$ to $(n_B, \delta_B^a, I_{NB} = 0)$ is
\[
\frac{\exp(\kappa_1 \delta_B^a + \kappa_2 \delta_{NB}^b + \kappa_3 \delta_B^a \delta_{NB}^b)}{1 + \exp(\kappa_1 \delta_B^a + \kappa_2 \delta_{NB}^b + \kappa_3 \delta_B^a \delta_{NB}^b)} \tag{6}
\]
where the $\kappa$s are parameters to be estimated.

The third process is a change in $\delta_{NB}$ which does not involve the $I_{NB} = 0$ state. One can think of this a non-broker price change although it could also represent the entry or exit of a subset of non-broker listings. The Poisson arrival rate of this event is $\lambda^{NB,CHANGE}$, and we assume that $\delta_{NB}$ would evolve according to an stochastic process where
\[
\delta_{NB}' = \tau_0 + \tau_1 \delta_B^a + \tau_2 \delta_{NB}^b + \xi_2 \tag{7}
\]
where $\xi_2$ is assumed to be normally distributed with mean 0 and variance $\sigma_2^2$. The corresponding conditional probability can be expressed in a form similar to (4).

4 Estimation

There are two steps in our estimation. In the first stage we estimate the preference parameters, in order to define the value of the state variables. In the second stage we estimate the transition functions.
4.1 Preference Parameters

As discussed above, the probability that a consumer who arrives in the market purchases a ticket listed by the broker is

\[ Pr(i \text{ chooses } j) = \frac{\exp(\delta_j)}{1 + \sum_{k=1}^{K} \exp(\delta_k)} \]  

(8)

where

\[ \delta_j = x_j \beta - \alpha p_j \]  

(9)

To estimate the parameters \( \beta \) and \( \alpha \) we have to take into account that no buyers may arrive. For clarity, suppose that we are interested in whether two seats are sold from a listing \( j \) that has four seats between two particular downloads. We parametrize this probability in the following way

\[ Pr(\text{two tickets purchased from } j) = \frac{\exp(W_g \gamma)}{1 + \exp(W_g \gamma)} \frac{\exp(x_j \beta - \alpha p_j)}{1 + \sum_{k=1}^{K} \exp(x_k \beta - \alpha p_k)} \]  

(10)

where the first term is used to represent in a reduced-form way the probability that a buyer who wants two seats arrives. \( W \) contains dummies for the number of tickets being purchased (e.g., constant, 4 seats or 6 seats), dummies for 0 to 5, 6 to 10, 11 to 20 and more than 21 days before the game, to capture differences in the arrival rates, and weekend, working day time dummy and nighttime dummies to capture the possibility that arrival rates may also differ during the day. For a two seat purchase the sum over listings in the denominator of the second term would be over all listings with at least two seats. For a four seat listing it would be over listings with at least four seats. Estimation is done using Maximum Likelihood.

We note that this specification is not ideal. In particular, it assumes that at most one customer turns up between downloads and that the set of listings available to that customer is the set of listings available at the first download. These assumptions are not too unreasonable given that we are using downloads that are three hours apart but it could be improved upon.

4.2 State Space Definition

As described above, we use the estimated preference parameters to calculate the values of \( \delta_B \) and \( \delta_{NB} \) that define the state space. The bins that we use for these values are shown in Figure 6.

[FIGURE 6 ABOUT HERE]
4.3 Estimation of the Continuous Time Processes

Our data comes in the form of snapshots of the state of the Stubhub market every few hours, plus indicators for when a sale from a broker listing is made between the downloads. Our model of how the market evolves is in continuous time, so that any number of events may happen between downloads. We therefore need to use our model to calculate the probability that the market will be in each possible state at the next download given the current state. The math of continuous time Markov processes makes this straightforward to do.

The first step is to construct the “intensity matrices” \( Q \), dimension 1,519 x 1,519 where 1,519 is the total number of states) for each event, which summarizes the finite Markov jump process. Entry \( Q_{m,n} \) \((m \neq n)\) equals the Poisson arrival rate of the event multiplied by the conditional probability of moving from state \( m \) to state \( n \) given that the relevant event arrives. The diagonal elements of the intensity matrix is the minus sum of the other elements in the row so that the sum of the elements in a row is 0. For example, the intensity matrix for the event associated with a buyer arriving, \( Q^{BUYER} \), has the elements corresponding to the broker sale in state \( (n_B, \delta_B^a, \delta_B^b) \) equal to arrival rate \( \lambda^{BUYER} \) multiplied by the probabilities associated with the broker’s tickets being bought so that \( n_B \) falls by two, or non-broker tickets being purchased so that \( \delta_{NB} \) changes according to matrix \( P^{NB} \). Off-diagonal elements associated with the values of \( n_B \) or \( \delta_{NB} \) increasing would be equal to zero. Intensity matrices for broker price changes \( Q^{BROKER} \), non-broker entry \( Q^{NB,IN} \), non-broker exit \( Q^{NB,OUT} \) and non-broker price change \( Q^{NB,CHANGE} \) are defined similarly.

The “aggregate intensity matrix” is calculated by summing up the intensity matrices, i.e. \( Q = Q^{BUYER} + Q^{BROKER} + Q^{NB,IN} + Q^{NB,OUT} + Q^{NB,CHANGE} \). The transition matrix \( P(t) \), which reflects the probability of transitioning from one state to the other after a time of length \( t \) (via any combination of state changes), can be found as the unique solution to the system of ordinary differential equations

\[
P'(t) = P(t)Q \\
P(0) = I
\]

where \( I \) is the identity matrix of the same size as \( P(t) \). Solving the above system of equations gives \( P(t) = e^{tQ} \), a matrix exponential which is calculated using EXPOKIT in MATLAB. With the
transition matrix, we are able to find the corresponding likelihood for each observation and estimate the transition parameters using MLE. The log-likelihood function for transition parameters \((\theta_2)\) is

\[
L(\theta_2) = \sum_g \sum_{j \in g} \log P(t_j, \Theta)(s_{j-1}, s_j)
\]  

(12)

where \(s_j\) is the state index for observation \(j\), \(\Theta\) are all of the parameters, \(g\) denotes the game, \(j - 1\) denotes the previous download and \(t_j\) is the time between the downloads of observation \(j - 1\) and \(j\)

5 Parameter Estimates

We now discuss the parameter estimates, with the counterfactuals presented in the next section.

[TABLE 5 ABOUT HERE]

Table 5 shows the first stage estimates. Higher prices significantly reduce the probability that a listing is sold, with an average own price elasticity, conditional on the arrival of a buyer, of -4.44, which is the range usually considered for consumer markets although it is below the elasticities sometimes found for undifferentiated products sold via price search engines on the internet (Ellison and Ellison (2009)).

We also tried interacting the price coefficient with the number of days to go, to see if there was evidence consistent with the type of consumer who is in the market changing over time. However, in most of these specifications there was no clear pattern to the interaction coefficients and they were imprecisely estimated. The coefficient on row is negative, and significant at the 5% level, implying (sensibly) that consumers value rows that are further back less. We have tried to incorporate additional terms to capture whether front row seats are much more attractive, but the coefficient on this term was never precisely estimated. This partly reflects the fact that the broker never has front row seats in our sample. The coefficients on the \(W\) variables are also sensible, consistent with the market being more active close to the game, two seat buyers being the most common and the market

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4Ellison and Ellison look at unbranded memory chips and other computer components sold by small firms that are listed on Pricewatch.com. For the lowest quality chips estimated elasticities are around -25, which is exceptionally high, whereas for medium and high quality chips, for which it is harder to compare prices, elasticities are in the range of 4 to 7. Obviously if there is some endogeneity of price (caused, by, for example, positive demand shocks) we may underestimate the elasticity of demand resulting in optimal prices that are too high. Future versions will include more fixed effects to see if we can get this elasticity to increase.
being more active during the workday and at night than during the early morning (12 am to 6 am is the excluded category).

The second stage estimates of the state transitions are in Table 6. Before we discuss the results, it is worth commenting that one potential problem when estimating such a large number of parameters is that maximum likelihood may not find a global maximum. However, when we re-started the estimation from quite different sets of parameters we got almost identical coefficients, indicating that this is not a significant problem in our setting.

TABLE 6 ABOUT HERE

The signs and size of most of the coefficients are intuitive. Beginning with the arrival rates ($\lambda$), we see that all the arrival rates increase as a game approaches, consistent that there are both more consumers in the market and that sellers are more likely to change their prices as the moment when the tickets “perish” approaches. The high rate of non-broker exit in the last five days is also driven by Stubhub’s rule that hard copies of tickets can only be sold in the last three days if they are provided to Stubhub. This constraint does not affect the broker, but is likely to impact small sellers.

The transition coefficients imply several intuitive effects. A higher value of $\delta_B$, for example a lower broker price, implies that $\delta_{NB}$ is more likely to increase, i.e., that non-brokers are likely to lower their prices, and that if non-broker listings enter they will do so at higher $\delta_{NB}$. These effects should create an incentive for the broker to keep his price high when the market is inactive. On the other hand, there are effects that offset this incentive. For example, a lower broker price makes it more likely that non-broker listings will exit, raising the broker’s future demand, and creating an incentive for the broker to lower his price early on in order to drive other listings out. $\delta_{NB}$ also affects the broker’s pricing rule in an intuitive way, with lower non-broker prices making it more likely that the broker will set a lower price.

6 Counterfactuals: Optimal Prices

This section describes the counterfactuals that we have run, calculating the broker’s optimal price under various scenarios. Our focus at the moment is to compare the level and time profile of the broker’s current prices with those which our model predicts should be optimal. We can also investigate
how the broker’s profits and optimal policies vary with the model’s parameters. This can help to tell us more about the economics of the pricing problem, but it can also provide useful advice to the broker. For example, we can consider how valuable it is to change price more frequently.

It is important to note that when we consider the broker changing his price we allow for non-brokers to change their prices, or possibly enter or exit the market, according to these transitions. However, we assume that the value of the parameters of these functions would not change, i.e., we do not try to re-compute the full equilibrium of the pricing model. Our approach to performing counterfactuals is therefore similar to the one used in recent papers such as Benkard, Bodoh-Creed, and Lazarev (2009).

6.1 Calculations in a Base Case

We now explain how the broker’s optimal price is calculated using the simplest example where the broker has two tickets to sell and his listing’s \( \text{ROW}=10 \). We assume that the broker can change his price once each day, at the start of the day. For any value of this price, call it \( p_B \), we calculate expected revenues in the following steps, starting one day before the game when the price set is \( p^{T-1}_B \) and the initial non-broker state is \( \delta^{T-1}_{NB} \):

(i) calculate the value of \( \delta_B \) associated with \( p^{T-1}_B \) and \( \text{ROW}=10 \);
(ii) calculate the probability that the listing is sold within the next 24 hours using the appropriate transition matrix, \( P(24) \) and the initial state;
(iii) calculate the value of the expected payoff to \( p^{T-1}_B \) where the payoff is equal to

\[
p^{T-1}_B \Pr(\text{sale} \mid \delta_B(p^{T-1}_B), \delta^{T-1}_{NB}, \Theta)
\]

which implicitly assumes that the broker has no value from unsold tickets and \( \Theta \) are all of the parameters estimated above;
(iv) for each state \( \delta_{NB} \) find the optimal price, i.e., the price that maximizes the payoff. The associated payoff is the “value” \( V^{T-1}(\delta_{NB}) \) to being in state \( \delta^{T-1}_{NB} \) with one day remaining;
(v) repeat the calculations in (i)-(iv) for the previous day \( (T-2) \) where the expected payoff is
so the broker takes into account how the price that he sets at $T - 2$ may influence the transition of $\delta_{NB}$, which he cares about if he does not sell;

(vi) repeat (v) for $T - 3$ and so until $T - 30$.

This calculation assumes that the broker started with only two tickets to sell, so that the game ends as soon as a single sale is made. However, the logic extends quite naturally when we consider the broker having more tickets to sell. Suppose, for example, that the broker has 4 seats to sell two days before the game. In this case, he needs to take into account that, for a given price, he may sell all of his tickets today (to two different buyers given his assumptions), or he may sell two tickets, and so be left with two tickets to sell on the final day or he may sell no tickets and have four to sell on the final day. This simply involves adding additional terms to the calculation in 14.

### 6.2 Results

#### 6.2.1 Base Case

Tables 7(a)-(c) show the optimal prices for each (day-to-game, non-broker state, number of tickets remaining) combination where we allow for the broker to start with up to six tickets. The top line of each table shows the average price for that day, averaging across non-broker states. Figure 7 compares these 'average state' paths to the evolution of actual prices (based on the regression results in Table 4 where we assume that the broker’s listings are in row 10).  

At least four features of these results deserve discussion.

(i) the broker sets prices that are of approximately the correct level immediately before the game, which is when the market is most active. At this point, the pricing problem is fairly close to being a static one (as future opportunities to sell are limited), and finding that the broker’s pricing decisions are consistent with our model when the pricing problem is static provides some indirect evidence that our demand estimates, which determine the optimal static pricing policy, are approximately correct.  

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5We have also simulated what the average price would be given how the distribution of non-broker states can vary over time, and given the ability of the broker to influence how the non-broker state evolves. The differences in the average optimal prices are small, so we report average prices here.
It is also worth noting that the average level of prices around one month before the game is also approximately correct for listings with 2 or 4 seats.

(ii) immediately before the game the optimal price are sensitive to the level of competition $\delta_{NB}$, as can be seen in Table 7, with stronger competition implying a lower optimal price, as one would expect in a static demand model. However, this sensitivity declines as one moves further from the game, so that more than 10 days before the game the optimal price is almost invariant to the non-broker state.$^6$ This reflects the fact that the broker knows that he is less likely to sell further from the game and that it is possible that the non-broker state will improve over time, and that by allowing low priced non-broker tickets to be sold, by setting a high price, the broker may be able to encourage this.

We can compare these predictions with how the broker currently prices. In Table 8 we repeat the regression reported in column (2) of Table 4, which used broker listings, adding two variables. The first one measures the minimum price of non-broker listings in the game-section and the second is a dummy variable that is equal to one when there are no non-broker listings. In second column of Table 8 these variables are interacted with a count of many days there are until the game to test whether the broker’s prices are more or less sensitive to these measures of competition as a game approaches.

The positive coefficients on these variables in the first column indicate that the broker sets higher prices when he faces less competition. The effects are large relative to what an optimal pricing strategy would predict. For example, the 25th and 75th percentiles of the minimum price variable are 0.54 and 0.80, so that moving between these two values would predict that the broker’s relative price should increase by 0.08. Optimal prices only display anything comparable to this sensitivity in the last couple of days before the game, and not on average. Similarly no non-broker competition implies a price increase of almost 0.25, which is a much larger effect than we see for optimal prices.

In the second column these additional variables are interacted with the number of days to go before the game. The coefficients on the main effects are zero, indicating insignificant differences immediately before the game, whereas optimal pricing implies large differences. In contrast, the

$^6$We note that the highest $\delta_{NB}$ is an obvious exception to this statement and we are in the process of figuring out why.
positive coefficients on the interactions indicate that the prices are sensitive further from the game, and once again this is the opposite of what the optimal pricing results predict.

(iii) the optimal pricing policy involves setting lower prices when there are more tickets to sell as multiple purchases will be required to clear the inventory. This reflects a standard feature of dynamic pricing models (e.g., McAfee and Te Velde (2006)) where the opportunity cost of sale of a given seat decreases with the number of seats left to sell, which causes the optimal price to fall. In contrast, the broker sets similar current prices for listings with 2, 4 or 6 seats. This suggests that there may be gains (which we could quantify) to raising prices are subsets of seats from a listing are sold. One caveat here is that our assumption that buyers only ever want to purchase two seats probably leads to larger differences in optimal prices. In future revisions we will allow for some buyers to want four seats, and it will be interesting to see how much the differences in optimal prices fall;

(iv) while actual and optimal prices both fall as a game approaches the shape of the declines is not the same. Optimal prices have a concave time profile, which is exactly dynamic pricing models usually predict even in the absence of competition and intertemporal changes in market activity (see, for example, the simulations in McAfee and Te Velde (2006)). Adding these factors should make a concave (i.e., quickening pace of decline with initially fairly flat prices) even more optimal, as the optimizing seller will place more value on trying to keep competitors’ prices high (by not cutting his own price) and less weight on trying to sell tickets early on by cutting prices because buyers are scarce. In contrast, the broker’s actual prices decline fairly steadily starting about two weeks before the game, when we estimate that there are still relatively few consumers in the market, so that even with the price cut the probability of sale is low. We also fail to explain the small jump in the broker’s prices that is observed about two weeks before the game. This is hard to explain in our model without introducing some additional element such as a change in the price elasticity of buyers’ who enter the market during this period of time.  

7Interestingly when we estimated a more flexible demand model that allow for the price coefficient to vary with the number of days until the game we did estimate that demand was less elastic with two weeks ago than either more than 20 days or less than 10 days before the game, although the difference in the coefficients was not statistically significant.
6.2.2 Frequency of Price Setting

In future revisions we will vary many of the parameters to investigate how they affect optimal pricing and the relationship between optimal prices and the prices that we observe in the data. Here we investigate the role of how frequently the broker sets prices, holding the remaining parameters fixed.

The base case assumed that the broker updates prices once per day which is more frequently than we estimate happens in the data. To be precise our estimates imply that the probability that the broker updates his prices at least once during the day is 0.73 for 0-5 days before the game, 0.41 5-10 days before, 0.26 11-20 days before and 0.21 more than 21 days before the game. Figure 8 shows how optimal prices change when we assume that the broker updates with these probabilities, with the $\lambda_B$ prices indicating the price that the broker should set if he gets the opportunity to change his price, knowing that he may not be able to change his price in the future. The key finding is that the when price changes are less frequent the broker should set lower prices because there is some probability that the broker will be stuck with his earlier prices when the (unconstrained) optimal price falls. On the other hand, updating close to the game is sufficiently likely that the differences in optimal prices in the last few days are actually quite small.

7 Conclusion

In this paper we provide an empirical framework for analyzing settings where a seller faces both a dynamic pricing problem and significant competition, and is able to influence how his competition evolves when he sets his price. We estimate our model using data from a large broker who sells event tickets on Stubhub. In the data we see that the prices that the broker sets appear to significantly affect the prices of his competitors, as well as affecting the probability with which his tickets are sold. We find evidence that the broker’s current pricing rules are optimal in some respects but not in others. For example, close to the game, when the market is most active, the broker sets prices that are approximately the right level to maximize revenues. On the other hand, the broker’s prices further from the game are (i) too insensitive to the number of tickets that the broker has left; (ii) too sensitive to the current level of competition that the broker faces; and (iii) update less frequently that would be optimal. In future revisions we flesh out these differences in more detail.
8 References


A Identifying Broker Listings in the StubHub Data

In this section, we discuss how we identified broker’s listings in the StubHub data. From the broker we have a record of sales transactions. This record indicates the number of seats sold, transaction time, the game, section, row, and number of seats, and the platform on which the tickets were sold (usually StubHub). From StubHub we have a record of all the tickets listed (by anyone, not just the broker) for each game-section-row at various points in time. The data was gathered from the StubHub website approximately every 3 hours from December 2009 to April 2010.

Since the listings on StubHub are anonymous, we need to identify broker’s listings in the StubHub data based on the available information. To accomplish this we first generated a variable called BrokerTrans, which is equal to one for a listing when tickets disappear from that listing at the next download and the timing and number of seats match with a sale recorded in the broker data. Then, we use logical tests in Stata to create categories for the StubHub listings. Below is a description for each category. Each description includes an explanation of the logical test performed and the rationale behind the category.

1. BrokerTicket1: This variable identifies StubHub listings which are the only ones in a game-section-download that match the number of seats held by the broker at the time of the listing. The variable equals one if the number of seats match, and zero otherwise. By definition, the listings with BrokerTrans equal to one must have BrokerTicket1 equal to one as well.

2. BrokerTicket2: This variable works backward by recognizing when a broker’s transaction has occurred, and then identifying the listings leading up to the transaction. If we find a listing for a game-section-row the only one that matches the broker data and BrokerTicket1 equals zero, then BrokerTicket2 equals one for this listing and for listings with the same number of seats when they appear at earlier download times. BrokerTicket2 is less restrictive than BrokerTicket1 in that it does not require the number of seats on the listing to match the number held by the broker. This accounts for the possibility that the broker did not list all tickets together simultaneously on StubHub.

3. ProbableBrokerTicket1: This variable follows BrokerTicket1 except it identifies multiple listings from the same download time with the same number of seats. For instance, if the broker has
four seats for a particular game-section-row and there are two listings of four seats on StubHub, 
BrokerTicket1 equals zero for both listings since it cannot distinguish between the two identical 
listings. In this case ProbableBrokerTicket1 equals one for both listings.

4. ProbableBrokerTicket2: This variable follows BrokerTicket1, but it allows for the possibility 
that the order of transactions from the broker data is different from the order on StubHub.

5. ProbableBrokerTicket3: This variable is equal to one whenever the number of tickets held by 
the broker for a particular game-section-row is greater than the total number of tickets listed 
for that game-section-row on StubHub and the listing has BrokerTicket2 equal to zero.

6. PossibleBrokerTicket1: This variable identifies tickets that are continuously listed except for a 
single gap of 1 to 4 downloads (i.e. for a period of time the tickets disappear but then reappear 
with the same price and the same number of seats). This test is only applied to listings where 
BrokerTicket1 or BrokerTicket2 equals one later.

7. PossibleBrokerTicket2: This variable is similar to PossibleBrokerTicket1 except the listing 
price must vary across the gap (number of seats must remain the same).

8. PossibleBrokerTicket3: This variable is the same as PossibleBrokerTicket1 except the gap 
length is longer (5 or more downloads).

9. PossibleBrokerTicket4: The same as PossibleBrokerTicket2 except the gap length is longer (5 
or more downloads).

Note that due to technical reasons, some broker transactions were not identified in the StubHub data. 
For example, a broker transaction which happened when our download system was down would not 
be found in the StubHub data. Also, the broker recorded transactions taking place on holidays in 
the morning of the business days right after the holidays. This made the transaction time from the 
broker not consistent with the time when the listings disappeared from StubHub during the holidays. 
Table A1 summarizes the results of the matching process.