# Realized Semibetas:
# Disentangling “good” and “bad” downside risks


**Tim Bollerslev**
*Duke University*

**Andrew J. Patton**
*Duke University*

**Rogier Quaedvlieg**
*Erasmus University Rotterdam*



2021 North American Summer Meeting
of the Econometric Society

# Motivation

- Do expected returns vary significantly with the degree of diversification offered during market downturns/upturns?
  - We measure this using semibetas, defined below.
- Can we better explain the cross-section of expected returns by drawing on the information in semibetas? Compare with:
  - Mean-semivariance framework of Hogan and Warren (1972, 1974, *JFQA*)
  - Downside beta framework of Ang Chen and Xing (2006, *RFS*)
  - Co-skewness and co-kurtosis pricing, see Kraus and Litzenberger (1976, *JF*), Harvey and Siddique (2000, *JF*), Langlois (2019, *JFE*) and others.

# Semicovariances and Semibetas

- Ignoring the mean, the covariance between a stock $(r)$ and the market $(f)$ is:

$$
\begin{aligned}
Cov(r,f) &= E[r \cdot f] \\
&= E[r \cdot f \cdot \mathbf{1}(r<0, f<0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f>0)] \\
&\quad + E[r \cdot f \cdot \mathbf{1}(r<0, f>0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f<0)]
\end{aligned}
$$

# Semicovariances and Semibetas

- Ignoring the mean, the covariance between a stock $(r)$ and the market $(f)$ is:

$$
\begin{aligned}
Cov(r,f) &= E[r \cdot f] \\
&= E[r \cdot f \cdot \mathbf{1}(r<0, f<0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f>0)] \\
&\quad + E[r \cdot f \cdot \mathbf{1}(r<0, f>0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f<0)] \\
&\equiv \mathcal{N} + \mathcal{P} + \mathcal{M}^{+} + \mathcal{M}^{-}
\end{aligned}
$$

# Semicovariances and Semibetas

- Ignoring the mean, the covariance between a stock $(r)$ and the market $(f)$ is:

$$
\begin{aligned}
Cov(r,f) &= E[r \cdot f] \\
&= E[r \cdot f \cdot \mathbf{1}(r<0, f<0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f>0)] \\
&\quad + E[r \cdot f \cdot \mathbf{1}(r<0, f>0)] + E[r \cdot f \cdot \mathbf{1}(r>0, f<0)] \\
&\equiv \mathcal{N} + \mathcal{P} + \mathcal{M}^{+} + \mathcal{M}^{-}
\end{aligned}
$$

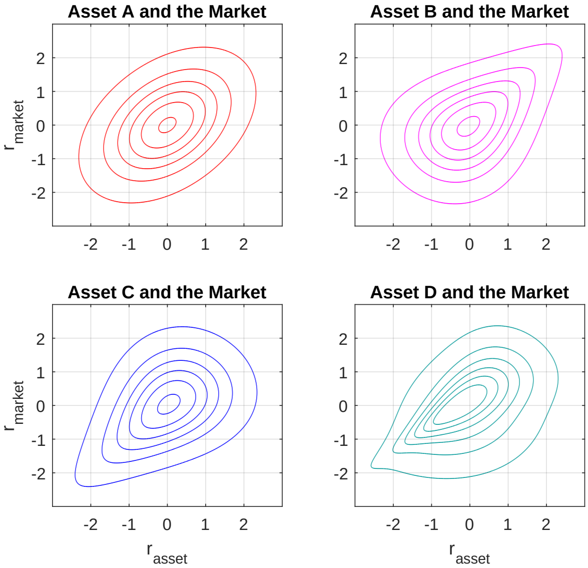
- Scaling by $Var(f)$, we obtain an exact decomposition of beta into semibetas:

$$
\begin{aligned}
\beta &= \frac{Cov(r,f)}{Var(f)} \\
&= \frac{\mathcal{N} + \mathcal{P} + \mathcal{M}^{+} + \mathcal{M}^{-}}{Var(f)} \\
&\equiv \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}+} - \beta^{\mathcal{M}-}
\end{aligned}
$$

  - As $\mathcal{M} < 0$, we switch the sign when defining $\beta^{\mathcal{M}}$ to ease interpretation.

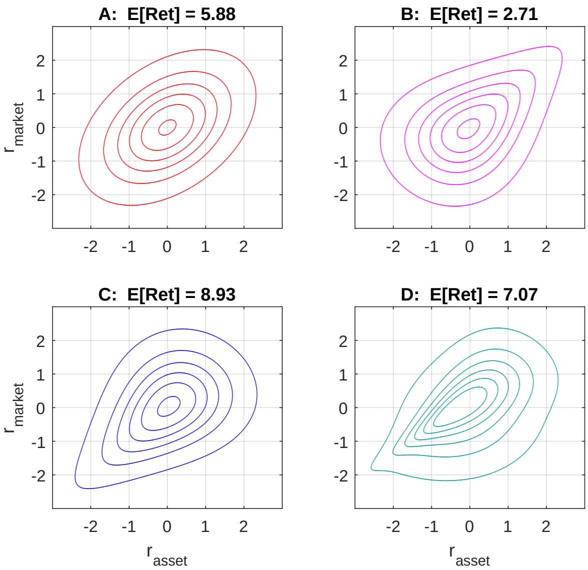
# Which asset would you prefer?

A: Gaussian, B: Correlated booms, C: Correlated crashes, D: Less correlated crashes



# Which asset would you prefer?

## A: Gaussian, B: Correlated booms, C: Correlated crashes, D: Less correlated crashes



# Outline

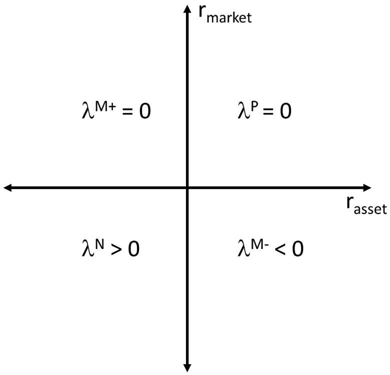
- Mean-semivariance pricing
- Realized semibetas
- Cross-sectional pricing
- Betting on semibetas

# Mean-Semivariance Pricing

- Investors only care about downside return variation
  - Only the covariation with negative aggregate market returns should be priced
  - Standard CAPM and single beta security market line too simplistic
  - Traces back to early work by Roy (1952, *Ecta*), Markowitz (1959, *book*), Mao (1970, *JF*), Hogan and Warren (1972, 1974, *JFQA*), Bawa and Lindenberg (1977, *JFE*), Fishburn (1977, *AER*)
  - May also be justified by prosect theory and the notion of loss aversions proposed by Kahneman and Tversky (1979, *Ecta*)

# Mean-semivariance pricing

- Hogan and Warren, 1974, *JFQA*:




- Naturally suggests measuring risk using semibetas:

$$\beta \equiv \frac{Cov(r,f)}{Var(f)} = \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}^+} - \beta^{\mathcal{M}^-}$$

# Realized semibetas

- True semibetas are latent:

$$\beta_{t,i}^{\mathcal{N}} \equiv \frac{\mathcal{N}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{P}} \equiv \frac{\mathcal{P}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^+} \equiv \frac{-\mathcal{M}_{t,i}^{+}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^-} \equiv \frac{-\mathcal{M}_{t,i}^{-}}{\mathcal{V}_t}$$

# Realized semibetas

- True semibetas are latent:

$$\beta_{t,i}^{\mathcal{N}} \equiv \frac{\mathcal{N}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{P}} \equiv \frac{\mathcal{P}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^+} \equiv \frac{-\mathcal{M}_{t,i}^{+}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^-} \equiv \frac{-\mathcal{M}_{t,i}^{-}}{\mathcal{V}_t}$$

- But using results in Bollerslev, Li, Patton and Quaedvlieg (2020, *Econometrica*), these can be consistently ($m \to \infty$) estimated by realized semibetas:

$$\widehat{\beta}_{t,i}^{\mathcal{N}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}} \qquad \widehat{\beta}_{t,i}^{\mathcal{P}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

$$\widehat{\beta}_{t,i}^{\mathcal{M}^-} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}} \qquad \widehat{\beta}_{t,i}^{\mathcal{M}^+} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

where $r_{t,k,i}^{+} \equiv \max(r_{t,k,i}, 0)$ and $r_{t,k,i}^{-} \equiv \min(r_{t,k,i}, 0)$

# Realized semibetas

- True semibetas are latent:

$$\beta_{t,i}^{\mathcal{N}} \equiv \frac{\mathcal{N}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{P}} \equiv \frac{\mathcal{P}_{t,i}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^+} \equiv \frac{-\mathcal{M}_{t,i}^{+}}{\mathcal{V}_t} \qquad \beta_{t,i}^{\mathcal{M}^-} \equiv \frac{-\mathcal{M}_{t,i}^{-}}{\mathcal{V}_t}$$

- But using results in Bollerslev, Li, Patton and Quaedvlieg (2020, *Econometrica*), these can be consistently ($m \to \infty$) estimated by realized semibetas:

$$\widehat{\beta}_{t,i}^{\mathcal{N}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}} \qquad \widehat{\beta}_{t,i}^{\mathcal{P}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

$$\widehat{\beta}_{t,i}^{\mathcal{M}^-} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}} \qquad \widehat{\beta}_{t,i}^{\mathcal{M}^+} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

  where $r_{t,k,i}^{+} \equiv \max(r_{t,k,i}, 0)$ and $r_{t,k,i}^{-} \equiv \min(r_{t,k,i}, 0)$

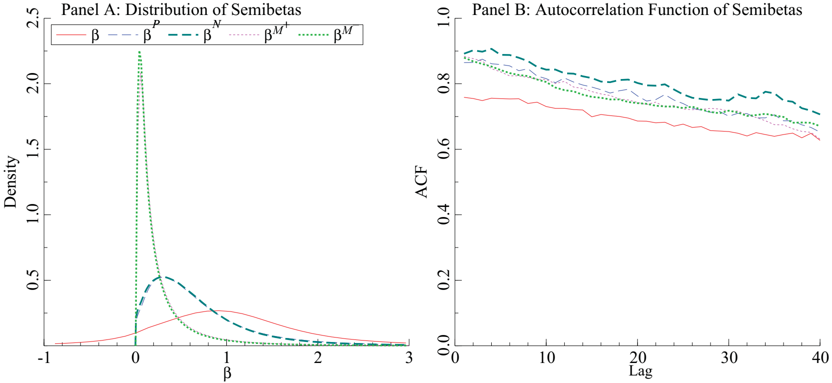
  - By construction $\widehat{\beta}_{t,i} = \widehat{\beta}_{t,i}^{\mathcal{N}} + \widehat{\beta}_{t,i}^{\mathcal{P}} - \widehat{\beta}_{t,i}^{\mathcal{M}^+} - \widehat{\beta}_{t,i}^{\mathcal{M}^-}$, for all $m$

# Realized semibetas

- High-frequency intraday data from TAQ
- S&P 500 constituent stocks, 1993–2014
  - $T =$ 5,541 trading days
  - 1,049 unique securities ($\bar{N} = 722$)
  - 15-minute returns excluding overnight (so $m = 26$)
- Total daily returns based on CRSP to account for overnight and dividends

# Realized semibetas

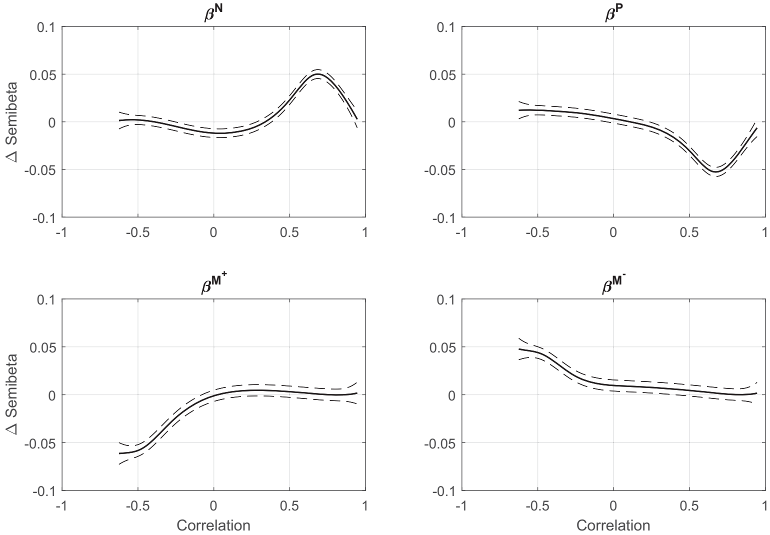
- Unconditional distributions and autocorrelations:




  - Distributions of two (dis)concordant semibetas almost indistinguishable
  - Semibetas more strongly autocorrelated than conventional betas

# Realized semibetas

- Average deviations from implied Gaussian values as a function of $\rho$
  - $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ typically higher; $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ typically lower



# Cross-sectional pricing

- Standard Fama-MacBeth approach, with cross-sectional regressions:

$$r_{t+1,i} \;=\; \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}\hat{\beta}^{\mathcal{N}}_{t,i} + \lambda^{\mathcal{P}}_{t+1}\hat{\beta}^{\mathcal{P}}_{t,i} + \lambda^{\mathcal{M}^+}_{t+1}\,\hat{\beta}^{\mathcal{M}^+}_{t,i} + \lambda^{\mathcal{M}^-}_{t+1}\,\hat{\beta}^{\mathcal{M}^-}_{t,i} + \;\epsilon_{t+1,i}$$

# Cross-sectional pricing

- Standard Fama-MacBeth approach, with cross-sectional regressions:

$$r_{t+1,i} = \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}\hat{\beta}^{\mathcal{N}}_{t,i} + \lambda^{\mathcal{P}}_{t+1}\hat{\beta}^{\mathcal{P}}_{t,i} + \lambda^{\mathcal{M}^+}_{t+1}\hat{\beta}^{\mathcal{M}^+}_{t,i} + \lambda^{\mathcal{M}^-}_{t+1}\hat{\beta}^{\mathcal{M}^-}_{t,i} + \epsilon_{t+1,i}$$

  - Risk premium estimates, $\hat{\lambda}^j = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\lambda}^j_t$ :

| $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $R^2$ |
|---|---|---|---|---|---|
| 4.58 | | | | | 2.70 |
| 3.04 | | | | | |
| | 22.54 | -1.58 | -4.29 | -8.48 | 5.43 |
| | 5.62 | -0.52 | -0.86 | -2.02 | |

  - Stocks with higher $\beta^{\mathcal{N}}$ earn significantly higher risk premium
  - Stocks with higher $\beta^{\mathcal{M}^-}$ earn significantly lower risk premium

# Cross-sectional pricing

- Standard Fama-MacBeth approach, with cross-sectional regressions:

$$r_{t+1,i} = \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}\hat{\beta}^{\mathcal{N}}_{t,i} + \lambda^{\mathcal{P}}_{t+1}\hat{\beta}^{\mathcal{P}}_{t,i} + \lambda^{\mathcal{M}^+}_{t+1}\hat{\beta}^{\mathcal{M}^+}_{t,i} + \lambda^{\mathcal{M}^-}_{t+1}\hat{\beta}^{\mathcal{M}^-}_{t,i} + \epsilon_{t+1,i}$$

  - Risk premium estimates, $\hat{\lambda}^j = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\lambda}^j_t$ :

| $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $R^2$ |
|---|---|---|---|---|---|
| 4.58 | | | | | 2.70 |
| 3.04 | | | | | |
| | 22.54 | -1.58 | -4.29 | -8.48 | 5.43 |
| | 5.62 | -0.52 | -0.86 | -2.02 | |

  - Stocks with higher $\beta^{\mathcal{N}}$ earn significantly higher risk premium
  - Stocks with higher $\beta^{\mathcal{M}^-}$ earn significantly lower risk premium
  - $H^{CAPM}_t : \lambda^{\mathcal{N}}_t = \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t = -\lambda^{\mathcal{M}^-}_t$
    - Rejected at 5% level for 68% of 5,541 days in sample

# Short-sales constraints

- In a frictionless market without any short-sales constraints:

$$H_t^{SYM} : \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}$$

# Short-sales constraints

- In a frictionless market without any short-sales constraints:

$$H_t^{SYM}: \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}$$

  - Rejected at 5% level for 58% of 5,541 days in sample

# Short-sales constraints

- In a frictionless market without any short-sales constraints:

$$H_t^{SYM}: \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}$$

  - Rejected at 5% level for 58% of 5,541 days in sample

- D'Avolio (2002, *JFE*) and Henderson, Jostova and Philipov (2019, *wp*) indicate that almost *all* of the S&P 500 stocks can easily and cheaply be borrowed

  - Diffs in risk premia cannot be attributed to "hard" short-selling constraints

# Short-sales constraints

- In a frictionless market without any short-sales constraints:

$$H_t^{SYM}: \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}$$

  - Rejected at 5% level for 58% of 5,541 days in sample

- D'Avolio (2002, *JFE*) and Henderson, Jostova and Philipov (2019, *wp*) indicate that almost *all* of the S&P 500 stocks can easily and cheaply be borrowed
  - Diffs in risk premia cannot be attributed to "hard" short-selling constraints

- Legal and institutional constraints impede many individual and institutional investors from short-selling
  - Creates limits to arbitrage as in Pontiff (1996, *QJE*) and Schleifer and Vishny (1997, *JF*), and related arbitrage risks (Hong and Sraer, 16, *JF*)

# Arbitrage risk

- We adopt two proxies for the *arbitrage risk* of a stock

# Arbitrage risk

- We adopt two proxies for the *arbitrage risk* of a stock
  - Idiosyncratic volatility (Pontiff, 96, *QJE*; Stambaugh, Yu and Yuan, 15, *JF*)
  - Turnover (Harris and Raviv, 93, *RFS*; Blume, Easley and O'Hara, 94, *JF*)

# Arbitrage risk

- We adopt two proxies for the *arbitrage risk* of a stock
  - Idiosyncratic volatility (Pontiff, 96, *QJE*; Stambaugh, Yu and Yuan, 15, *JF*)
  - Turnover (Harris and Raviv, 93, *RFS*; Blume, Easley and O'Hara, 94, *JF*)
- We apply a linear rotation to our model to make it easier to observe whether there is a violation of the "symmetric" pricing restrictions:

$$r_i = \lambda_0 + \lambda^{\mathcal{N}}(\beta_i^{\mathcal{N}} - \beta_i^{\mathcal{M}^-}) + \lambda^{\mathcal{P}}(\beta_i^{\mathcal{P}} - \beta_i^{\mathcal{M}^+}) + \delta^{\mathcal{M}+}\beta_i^{\mathcal{M}^+} + \delta^{\mathcal{M}-}\beta_i^{\mathcal{M}^-} + \epsilon_{t+1}$$

- If the symmetric pricing restrictions hold, then $\delta^{\mathcal{M}+} = \delta^{\mathcal{M}-} = 0$

# Arbitrage risk and semibeta pricing

| | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\delta^{\mathcal{M}-}$ | $\delta^{\mathcal{M}+}$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\delta^{\mathcal{M}-}$ | $\delta^{\mathcal{M}+}$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| | Panel A: Full-Sample Estimates | | | | | | | | |
| | 22.54 | -1.58 | 12.16 | -5.81 | | | | | 5.43 |
| | 5.62 | -0.52 | 2.22 | -0.93 | | | | | |
| | Panel B: Sorting on Arbitrage Risk | | | | | | | | |
| | Below Median | | | | Above Median | | | | |
| IVOL | 22.29 | -0.10 | 3.99 | -3.41 | 17.43 | -4.63 | 34.81 | 10.19 | 6.62 |
| | 5.45 | -0.03 | 0.72 | -0.52 | 4.03 | -1.52 | 4.73 | 1.33 | |
| TO | 22.23 | -12.64 | -2.10 | -9.54 | 19.57 | 1.69 | 8.06 | -5.13 | 6.93 |
| | 5.42 | -4.40 | -0.34 | -1.37 | 4.66 | 0.50 | 2.27 | -0.76 | |

- Differences in semibeta risk premia are driven by stocks that are more difficult to short (higher IVOL) or are more difficult to value (higher turnover).

# Including control variables

- Including some “standard,” lower frequency, control variables:

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | ME | BM | MOM | REV | IVOL | ILLIQ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 22.54 | -1.58 | -4.29 | -8.48 | | | | | | | 5.43 |
| 5.62 | -0.52 | -0.86 | -2.02 | | | | | | | |
| 22.47 | -5.67 | -2.90 | -12.20 | -2.23 | -1.77 | 0.11 | | | | 8.23 |
| 5.75 | -2.02 | -0.65 | -3.14 | -3.83 | -1.95 | 3.47 | | | | |
| 20.36 | -2.91 | 1.68 | -6.15 | -7.42 | -1.65 | 0.09 | -0.55 | -3.07 | -4.88 | 10.32 |
| 5.44 | -1.08 | 0.41 | -1.68 | -7.71 | -1.87 | 2.55 | -5.82 | -3.56 | -6.42 | |

  - Average $R^2$s increase
  - Still the case that:
    - Stocks with higher $\beta^{\mathcal{N}}$ earn higher returns
    - Stocks with higher $\beta^{\mathcal{M}^-}$ earn lower returns

# Comparison with up- and down-side betas

- Up- and down-side betas (Ang, Chen and Xing, 2006, *RFS*):

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $\beta^+$ | $\beta^-$ | $R^2$ |
|---|---|---|---|---|---|---|
| 22.54 | -1.58 | -4.29 | -8.48 | | | 5.43 |
| 5.62 | -0.52 | -0.86 | -2.02 | | | |
| | | | | -1.17 | 6.88 | 3.70 |
| | | | | -1.11 | 5.54 | |
| 17.31 | -8.10 | -12.66 | -3.86 | -2.40 | 7.90 | 6.61 |
| 3.86 | -0.13 | 0.03 | -1.79 | -1.67 | 0.82 | |

where $\hat{\beta}^+_{t,i} \equiv \frac{\sum_{k=1}^m r_{t,k,i} f^+_{t,k}}{\sum_{k=1}^m (f^+_{t,k})^2}$ and $\hat{\beta}^-_{t,i} \equiv \frac{\sum_{k=1}^m r_{t,k,i} f^-_{t,k}}{\sum_{k=1}^m (f^-_{t,k})^2}$

- $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain significant in combined regression

# Comparison with up- and down-side betas

- Up- and down-side betas (Ang, Chen and Xing, 2006, *RFS*):

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $\beta^+$ | $\beta^-$ | $R^2$ |
|---|---|---|---|---|---|---|
| 22.54 | -1.58 | -4.29 | -8.48 | | | 5.43 |
| 5.62 | -0.52 | -0.86 | -2.02 | | | |
| | | | | -1.17 | 6.88 | 3.70 |
| | | | | -1.11 | 5.54 | |
| 17.31 | -8.10 | -12.66 | -3.86 | -2.40 | 7.90 | 6.61 |
| 3.86 | -0.13 | 0.03 | -1.79 | -1.67 | 0.82 | |

Note: $\hat{\beta}^+_{t,i} = (\widehat{\beta}^{\mathcal{P}}_{t,i} - \widehat{\beta}^{\mathcal{M}^+}_{t,i}) \frac{\sum_{k=1}^m f^2_{t,k}}{\sum_{k=1}^m (f^+_{t,k})^2}$ and $\hat{\beta}^-_{t,i} = (\widehat{\beta}^{\mathcal{N}}_{t,i} - \widehat{\beta}^{\mathcal{M}^-}_{t,i}) \frac{\sum_{k=1}^m f^2_{t,k}}{\sum_{k=1}^m (f^-_{t,k})^2}$

- Pricing implications coincide if $H^{SYM}_t : \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t \cap \lambda^{\mathcal{N}}_t = -\lambda^{\mathcal{M}^-}_t$
  - Rejected at 5% level for 58% of 5,541 days in sample

# Comparison with co-skewness and co-kurtosis

- Coskewness and Cokurtosis (Harvey and Siddique, 2000, *JF*):

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | CSK | CKT | $R^2$ |
|---|---|---|---|---|---|---|
| 22.54 | -1.58 | -4.29 | -8.48 | | | 5.43 |
| 5.62 | -0.52 | -0.86 | -2.02 | | | |
| | | | | -4.40 | 0.81 | 1.52 |
| | | | | -1.55 | 0.76 | |
| 30.92 | -3.79 | -3.89 | -16.33 | 10.09 | -3.59 | 6.26 |
| 6.20 | -1.12 | -0.76 | -3.69 | 2.66 | -3.22 | |

$$CSK_{t,i} \propto \frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i} - \bar{r}_{t,i})(f_{t,k} - \bar{f}_t)^2$$

$$CKT_{t,i} \propto \frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i} - \bar{r}_{t,i})(f_{t,k} - \bar{f}_t)^3$$

- $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain significant; additional information about the tails captured by CSK and CKT

# High frequency data; longer investment horizons

| $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | ME | BM | MOM | REV | IVOL | ILLIQ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Weekly: | | | | | | | | | | | |
| 4.69 | | | | | | | | | | | 2.37 |
| 3.72 | | | | | | | | | | | |
| | 14.58 | 0.96 | 5.20 | -13.80 | | | | | | | 5.07 |
| | 5.90 | 0.50 | 1.87 | -3.71 | | | | | | | |
| | 10.85 | -0.52 | 3.67 | -13.58 | -6.36 | -1.94 | 0.08 | -0.35 | -1.06 | -3.63 | 10.83 |
| | 5.92 | -0.38 | 1.76 | -5.03 | -7.50 | -2.30 | 2.54 | -4.39 | -1.40 | -5.77 | |
| Monthly: | | | | | | | | | | | |
| 2.93 | | | | | | | | | | | 1.90 |
| 2.71 | | | | | | | | | | | |
| | 8.70 | 1.67 | 3.51 | -3.36 | | | | | | | 4.45 |
| | 4.39 | 1.15 | 1.55 | -1.30 | | | | | | | |
| | 4.42 | -0.57 | -0.65 | -6.41 | -4.89 | -1.86 | 0.08 | -0.25 | 0.98 | -2.27 | 10.79 |
| | 3.55 | -0.58 | -0.44 | -3.34 | -7.06 | -2.58 | 2.64 | -4.15 | 1.56 | -4.60 | |

- $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain significant at weekly and monthly horizons; estimated premiums decrease with horizon as predictability diminishes

# Lower frequency data

- Now we consider monthly semibetas based on daily data
  - All CRSP common stocks (with codes 10 and 11 and price>$5)
    - $\bar{N} = 390$ stocks per period (unbalanced panel)
  - Longer January 1963 to December 2017 sample
    - $T = 660$ months
  - Fewer "high frequency" observations per period
    - $m \approx 21$ observations

# Lower frequency data

- Consider monthly semibetas and monthly Fama-MacBeth regressions:

| $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | ME | BM | MOM | REV | IVOL | ILLIQ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.10 | | | | | | | | | | | 2.36 |
| 3.77 | | | | | | | | | | | |
| | 10.43 | 1.40 | 4.15 | -6.42 | | | | | | | 5.22 |
| | 4.46 | 0.87 | 1.15 | -2.03 | | | | | | | |
| | 8.66 | -0.66 | 5.60 | -14.09 | -2.55 | -0.47 | 0.06 | | | | 10.70 |
| | 3.56 | -0.43 | 1.42 | -3.72 | -4.93 | -0.40 | 2.14 | | | | |
| | 6.59 | -1.90 | 6.33 | -15.59 | -2.08 | -0.75 | 0.07 | -0.12 | -1.60 | 2.40 | 13.38 |
| | 2.85 | -1.06 | 1.50 | -3.82 | -4.22 | -0.66 | 2.61 | -1.97 | -1.48 | 2.44 | |

  - Premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain significant

# Lower frequency data

- Consider monthly semibetas and monthly Fama-MacBeth regressions:

| $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | ME | BM | MOM | REV | IVOL | ILLIQ | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.10 | | | | | | | | | | | 2.36 |
| 3.77 | | | | | | | | | | | |
| | 10.43 | 1.40 | 4.15 | -6.42 | | | | | | | 5.22 |
| | 4.46 | 0.87 | 1.15 | -2.03 | | | | | | | |
| | 8.66 | -0.66 | 5.60 | -14.09 | -2.55 | -0.47 | 0.06 | | | | 10.70 |
| | 3.56 | -0.43 | 1.42 | -3.72 | -4.93 | -0.40 | 2.14 | | | | |
| | 6.59 | -1.90 | 6.33 | -15.59 | -2.08 | -0.75 | 0.07 | -0.12 | -1.60 | 2.40 | 13.38 |
| | 2.85 | -1.06 | 1.50 | -3.82 | -4.22 | -0.66 | 2.61 | -1.97 | -1.48 | 2.44 | |

- Premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain significant
- $H_t^{CAPM}: \lambda_t^{\mathcal{N}} = \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+} = -\lambda_t^{\mathcal{M}^-}$
  - Rejected at 5% level for 45% of 659 months in sample

# Comparison with other measures

- Monthly semibetas and other measures:

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $\beta^+$ | $\beta^-$ | CSK | CKT | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| 10.43 | 1.40 | 4.15 | -6.42 | | | | | 5.22 |
| 4.46 | 0.87 | 1.15 | -2.03 | | | | | |
| | | | | 1.06 | 3.16 | | | 3.42 |
| | | | | 1.61 | 3.74 | | | |
| 12.37 | 2.90 | 2.41 | -7.56 | -6.64 | -0.90 | | | 5.57 |
| 4.97 | 1.03 | 1.20 | -2.48 | -0.95 | -0.28 | | | |
| | | | | | | 5.00 | 1.98 | 1.69 |
| | | | | | | 2.81 | 2.57 | |
| 18.11 | -2.27 | 2.87 | -12.09 | | | 12.10 | -2.80 | 6.49 |
| 4.98 | -1.04 | 0.81 | -3.40 | | | 4.26 | -3.43 | |

  - Premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ always significant

# Comparison with other measures

- Monthly semibetas and other measures:

| $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{P}}$ | $\beta^{\mathcal{M}^+}$ | $\beta^{\mathcal{M}^-}$ | $\beta^+$ | $\beta^-$ | CSK | CKT | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| 10.43 | 1.40 | 4.15 | -6.42 | | | | | 5.22 |
| 4.46 | 0.87 | 1.15 | -2.03 | | | | | |
| | | | | 1.06 | 3.16 | | | 3.42 |
| | | | | 1.61 | 3.74 | | | |
| 12.37 | 2.90 | 2.41 | -7.56 | -6.64 | -0.90 | | | 5.57 |
| 4.97 | 1.03 | 1.20 | -2.48 | -0.95 | -0.28 | | | |
| | | | | | | 5.00 | 1.98 | 1.69 |
| | | | | | | 2.81 | 2.57 | |
| 18.11 | -2.27 | 2.87 | -12.09 | | | 12.10 | -2.80 | 6.49 |
| 4.98 | -1.04 | 0.81 | -3.40 | | | 4.26 | -3.43 | |

  - Premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ always significant
  - $H_t^{SYM}: \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}$
    - Rejected at 5% level for 34% of 659 months in sample

# Betting on semibetas

- We next consider long/short portfolios based on semibetas

- Conventional CAPM security market line "too flat"
  - Friend and Blume (1970, *AER*), Black, Jensen and Scholes (1972, *Book*), Kandel (1984, *JFE*), Shanken (1985, *JFE*), Fama and French (1992, *JF*), Fama and French (2006, *JF*)

- Recent literature on betting against beta (BAB)
  - Frazzini and Pedersen (2014, *JFE*), Cederburg and O'Doherty (2016, *JF*), Bali, Brown, Murray and Tang (2017, *JFQA*), Novy-Marx and Velikov (2018, *wp*), Schneider, Wagner and Zechner (2020, *JF*)

# Betting on semibetas

- We will look at:
  - Betting on $\beta$ (standard case)
  - Betting on $\beta^{\mathcal{N}}$
  - Betting against $\beta^{\mathcal{M}^-}$
  - Betting on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ (“Semi $\beta$”)

# Betting on semibetas

- We will look at:
  - Betting on $\beta$ (standard case)
  - Betting on $\beta^{\mathcal{N}}$
  - Betting against $\beta^{\mathcal{M}^-}$
  - Betting on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ (“Semi $\beta$”)
- Value-weighted long/short positions in high/low quintile of S&P 500 stocks
  - Avoids small and difficult to short micro-cap stocks, and the use of rank-weighted portfolios

# Betting on semibetas

- We will look at:
  - Betting on $\beta$ (standard case)
  - Betting on $\beta^{\mathcal{N}}$
  - Betting against $\beta^{\mathcal{M}^-}$
  - Betting on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ ("Semi $\beta$")
- Value-weighted long/short positions in high/low quintile of S&P 500 stocks
  - Avoids small and difficult to short micro-cap stocks, and the use of rank-weighted portfolios
- Sharpe ratios, alphas and factor loadings:
  - Four-factor Fama-French-Carhart model: MKT, SML, HML, MOM
  - Five-factor Fama-French model: MKT, SML, HML, RMW (profitability: robust minus weak), CMA (investment: conservative minus aggressive)

# Betting on semibetas

- Daily rebalancing:

| | $\beta$ | | Semi $\beta$ | | $\beta^{\mathcal{N}}$ | | $\beta^{\mathcal{M}^-}$ | |
|---|---|---|---|---|---|---|---|---|
| Avg ret | 4.98 | | 9.76 | | 10.98 | | 7.66 | |
| Std dev | 16.57 | | 9.30 | | 16.89 | | 8.00 | |
| Sharpe | 0.30 | | 1.05 | | 0.65 | | 0.96 | |
| $\alpha$ | 2.19 | 4.17 | 8.35 | 9.68 | 8.05 | 10.62 | 7.76 | 7.88 |
| | 0.87 | 1.77 | 6.32 | 7.38 | 3.36 | 4.58 | 4.17 | 4.19 |
| $\beta_{MKT}$ | 0.59 | 0.51 | 0.30 | 0.25 | 0.61 | 0.52 | -0.02 | -0.03 |
| | 67.61 | 55.78 | 64.70 | 47.90 | 73.15 | 56.89 | -2.34 | -3.44 |
| $\beta_{SMB}$ | 0.30 | 0.12 | 0.30 | 0.21 | 0.40 | 0.22 | 0.20 | 0.20 |
| | 18.10 | 7.36 | 33.91 | 22.61 | 25.00 | 13.49 | 16.08 | 14.98 |
| $\beta_{HML}$ | -0.02 | 0.18 | -0.01 | 0.11 | -0.08 | 0.13 | 0.05 | 0.08 |
| | -1.24 | 10.58 | -1.61 | 11.10 | -4.75 | 7.57 | 3.82 | 6.08 |
| $\beta_{MOM}$ | -0.24 | | -0.14 | | -0.22 | | -0.07 | |
| | -19.53 | | -22.46 | | -19.01 | | -7.31 | |
| $\beta_{RMW}$ | | -0.50 | | -0.28 | | -0.53 | | -0.04 |
| | | -22.15 | | -22.56 | | -23.70 | | -2.28 |
| $\beta_{CMA}$ | | -0.35 | | -0.28 | | -0.44 | | -0.13 |
| | | -13.21 | | -19.09 | | -16.74 | | -5.95 |
| $R^2$ | 58.15 | 60.26 | 55.92 | 59.55 | 60.59 | 64.38 | 6.72 | 7.42 |

# Betting on semibetas

- Weekly rebalancing:

| | $\beta$ | | Semi $\beta$ | | $\beta^{\mathcal{N}}$ | | $\beta^{\mathcal{M}^-}$ | |
|---|---|---|---|---|---|---|---|---|
| Avg ret | 1.18 | | 5.47 | | 7.76 | | 2.40 | |
| Std dev | 15.30 | | 8.64 | | 15.50 | | 7.52 | |
| Sharpe | 0.08 | | 0.63 | | 0.50 | | 0.32 | |
| $\alpha$ | -1.11 | 0.71 | 4.37 | 5.66 | 5.54 | 7.71 | 2.40 | 2.84 |
| | -0.46 | 0.31 | 3.45 | 4.35 | 2.52 | 3.46 | 1.46 | 1.72 |
| $\beta_{MKT}$ | 0.52 | 0.44 | 0.25 | 0.20 | 0.51 | 0.43 | -0.02 | -0.04 |
| | 60.65 | 48.93 | 55.60 | 38.70 | 66.56 | 49.31 | -3.40 | -5.54 |
| $\beta_{SMB}$ | 0.30 | 0.13 | 0.30 | 0.22 | 0.39 | 0.23 | 0.21 | 0.21 |
| | 18.43 | 7.83 | 35.58 | 23.57 | 26.64 | 14.41 | 19.12 | 17.76 |
| $\beta_{HML}$ | -0.08 | 0.08 | -0.06 | 0.02 | -0.12 | 0.05 | 0.00 | -0.01 |
| | -4.68 | 4.54 | -7.02 | 2.26 | -8.05 | 2.96 | -0.10 | -0.55 |
| $\beta_{MOM}$ | -0.20 | | -0.11 | | -0.20 | | -0.01 | |
| | -17.14 | | -17.93 | | -19.12 | | -1.85 | |
| $\beta_{RMW}$ | | -0.47 | | -0.26 | | -0.48 | | -0.05 |
| | | -21.38 | | -21.18 | | -22.57 | | -3.05 |
| $\beta_{CMA}$ | | -0.26 | | -0.22 | | -0.35 | | -0.08 |
| | | -9.80 | | -14.89 | | -14.07 | | -4.48 |
| $R^2$ | 51.93 | 54.10 | 47.31 | 51.03 | 53.19 | 56.27 | 7.21 | 8.44 |

# Betting on semibetas

- Monthly rebalancing:

| | $\beta$ | | Semi $\beta$ | | $\beta^{\mathcal{N}}$ | | $\beta^{\mathcal{M}^-}$ | |
|---|---|---|---|---|---|---|---|---|
| Avg ret | -0.64 | | 4.04 | | 3.10 | | 4.33 | |
| Std dev | 14.43 | | 8.39 | | 14.72 | | 7.18 | |
| Sharpe | -0.04 | | 0.48 | | 0.21 | | 0.60 | |
| $\alpha$ | -2.48 | -1.34 | 2.96 | 4.22 | 1.17 | 2.85 | 4.09 | 4.94 |
| | -1.17 | -0.64 | 2.30 | 3.24 | 0.55 | 1.34 | 2.83 | 3.42 |
| $\beta_{MKT}$ | 0.46 | 0.41 | 0.23 | 0.19 | 0.46 | 0.39 | 0.00 | -0.02 |
| | 62.38 | 50.99 | 51.68 | 36.54 | 62.55 | 47.54 | 0.90 | -4.01 |
| $\beta_{SMB}$ | 0.26 | 0.11 | 0.31 | 0.23 | 0.38 | 0.23 | 0.24 | 0.23 |
| | 18.67 | 7.27 | 36.10 | 24.59 | 26.93 | 14.85 | 25.18 | 22.62 |
| $\beta_{HML}$ | -0.08 | 0.05 | -0.06 | 0.01 | -0.08 | 0.08 | -0.04 | -0.06 |
| | -5.79 | 3.22 | -6.46 | 1.06 | -5.48 | 5.15 | -3.64 | -5.76 |
| $\beta_{MOM}$ | -0.21 | | -0.10 | | -0.22 | | 0.02 | |
| | -20.65 | | -16.43 | | -21.44 | | 2.22 | |
| $\beta_{RMW}$ | | -0.42 | | -0.26 | | -0.45 | | -0.08 |
| | | -21.18 | | -21.10 | | -21.89 | | -5.86 |
| $\beta_{CMA}$ | | -0.17 | | -0.19 | | -0.30 | | -0.08 |
| | | -7.03 | | -12.88 | | -12.57 | | -4.91 |
| $R^2$ | 49.21 | 49.52 | 46.76 | 50.29 | 50.83 | 52.42 | 11.71 | 13.32 |

# Betting on semibetas

- What about transaction costs?

- Many practical procedures to help mitigate the cost of trading
  - Bertsiman and Lo (1998, *JFinMkts*), Engle and Ferstenberg (2007, *JPorMgmt*), Obizhaeva and Wang (2013, *JFinMkts*)
  - Dependent on specific trading strategies and settings

# Betting on semibetas

- What about transaction costs?

- Many practical procedures to help mitigate the cost of trading
  - Bertsiman and Lo (1998, *JFinMkts*), Engle and Ferstenberg (2007, *JPorMgmt*), Obizhaeva and Wang (2013, *JFinMkts*)
  - Dependent on specific trading strategies and settings

- Trading only partially towards the "target" (Garleanu and Pedersen, 2013, *JF*):

$$\omega_t = \lambda \omega_{t-1} + (1-\lambda)\omega_t^{Target}, \qquad \lambda = 0.95$$

  - We will assume transaction costs proportional to the turnover of the long and short positions
  - We will consider round-trip costs of 0.5% and 1.0% (Novy-Marx and Velikov, 2016, *RFS*)

# Betting on semibetas: T-costs and partial adjustment

- Alphas remain economically large and statistically significant:

| T-cost | 0% | | 0% | | 0.5% | | 1.0% | |
|---|---|---|---|---|---|---|---|---|
| Adjustment | Full | | Partial | | Partial | | Partial | |
| Avg ret | 4.04 | | 4.62 | | 4.32 | | 4.02 | |
| Std dev | 8.39 | | 7.77 | | 7.77 | | 7.77 | |
| Sharpe | 0.48 | | 0.59 | | 0.56 | | 0.52 | |
| $\alpha$ | 2.96 | 4.22 | 3.09 | 5.31 | 2.79 | 5.01 | 2.49 | 4.71 |
| | 2.30 | 3.24 | 3.05 | 5.33 | 2.76 | 5.03 | 2.46 | 4.72 |
| $\beta_{MKT}$ | 0.23 | 0.19 | 0.25 | 0.17 | 0.25 | 0.17 | 0.25 | 0.17 |
| | 51.68 | 36.54 | 69.25 | 44.87 | 69.23 | 44.85 | 69.18 | 44.81 |
| $\beta_{SMB}$ | 0.31 | 0.23 | 0.27 | 0.20 | 0.27 | 0.20 | 0.27 | 0.20 |
| | 36.10 | 24.59 | 40.61 | 27.95 | 40.58 | 27.93 | 40.54 | 27.89 |
| $\beta_{HML}$ | -0.06 | 0.01 | -0.13 | -0.11 | -0.13 | -0.11 | -0.13 | -0.11 |
| | -6.46 | 1.06 | -18.72 | -14.51 | -18.69 | -14.49 | -18.66 | -14.46 |
| $\beta_{MOM}$ | -0.10 | | 0.01 | | 0.01 | | 0.01 | |
| | -16.43 | | 2.59 | | 2.59 | | 2.59 | |
| $\beta_{RMW}$ | | -0.26 | | -0.26 | | -0.26 | | -0.26 |
| | | -21.10 | | -27.01 | | -27.01 | | -27.00 |
| $\beta_{CMA}$ | | -0.19 | | -0.20 | | -0.20 | | -0.20 |
| | | -12.88 | | -18.16 | | -18.15 | | -18.13 |
| $R^2$ | 46.76 | 50.29 | 52.20 | 58.90 | 52.19 | 58.90 | 52.16 | 58.87 |

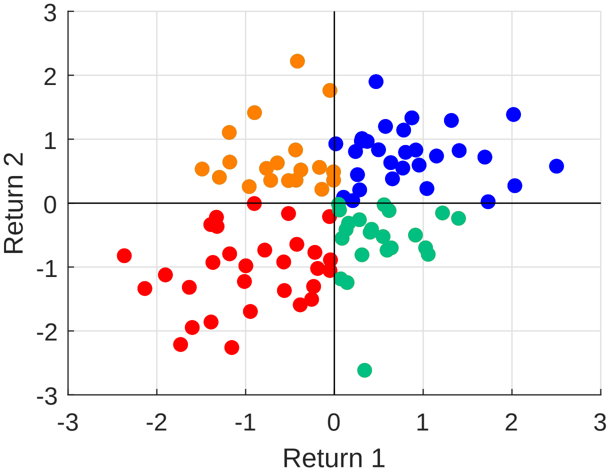
# Summary

- We propose the use of semibetas to gain richer information about the diversification benefits offered by various assets
  - Semibetas offer an exact decomposition: $\beta = \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}+} - \beta^{\mathcal{M}-}$
  - We estimate realized semibetas using high frequency data
- We show that semibetas are better able to explain the cross-section of expected returns than existing alternatives
  - As expected, much better than the CAPM, and also significantly better than up- and down-side betas
  - We find that the risk premium associated with $\beta^{\mathcal{N}}$ is around 23%, with $\beta^{\mathcal{M}-}$ around -9%, and is not different from zero for the other semibetas.
- We find that a long-short portfolio based on semibetas generates large and significant alphas
  - Sharpe ratios are double that of the market; alphas are 8-9%
  - Don't bet on or against beta, bet on *and* against the "right" semibetas

# Appendix

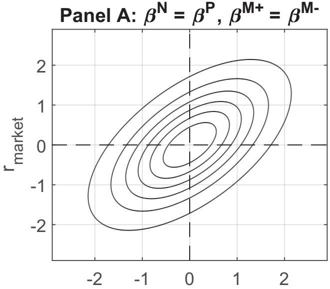
# Computing semibetas

## There's nothing to it (once you've got the data ready)



# Mean-semivariance pricing

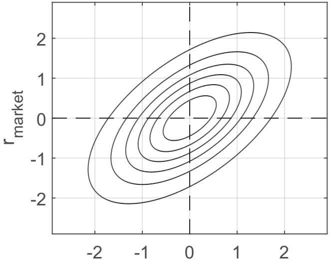
- Jointly normally distributed market and asset return:



# Mean-semivariance pricing

- Jointly normally distributed market and asset return:

Panel A: $\beta^N = \beta^P$, $\beta^{M+} = \beta^{M-}$



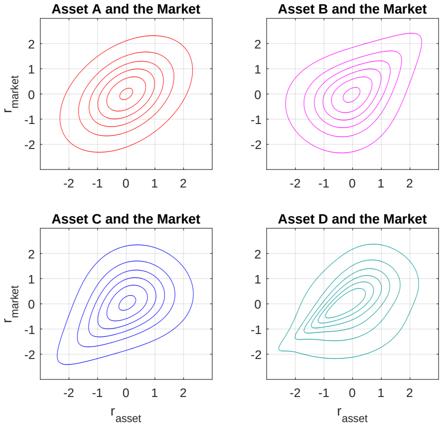

- No new information over traditional CAPM $\beta$:

$$\beta^{\mathcal{N}} = \beta^{\mathcal{P}} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} + \beta \arccos\left(-\frac{\sigma_f}{\sigma_r}\beta\right)\right)$$

$$\beta^{\mathcal{M}^-} = \beta^{\mathcal{M}^+} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} - \beta \arccos\left(\frac{\sigma_f}{\sigma_r}\beta\right)\right)$$

# Mean-semivariance pricing

- Jointly non-normally distributed market and asset returns:




- Identical CAPM betas, but very different semibetas and expected returns:
  - $E(r^B) < E(r^A) < E(r^D) < E(r^C)$

# Betting on the competition

| | $\beta^-$ - $\beta^+$ | | $\beta^-$ | | $\beta^+$ | | CKT - CSK | | CSK | | CKT | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Avg ret | 2.96 | | 7.53 | | -3.61 | | -0.16 | | -1.21 | | 0.54 | |
| Std dev | 5.45 | | 15.56 | | 14.67 | | 6.30 | | 7.70 | | 9.55 | |
| Sharpe | 0.54 | | 0.48 | | -0.25 | | -0.03 | | -0.16 | | 0.06 | |
| $\alpha$ | 2.56 | 2.93 | 4.78 | 6.80 | -1.67 | -2.94 | -0.60 | -0.03 | -1.44 | -1.27 | -0.11 | 0.86 |
| | 2.10 | 2.42 | 1.90 | 2.76 | -0.69 | -1.28 | -0.49 | -0.02 | -0.86 | -0.76 | -0.06 | 0.47 |
| $\beta_{MKT}$ | 0.04 | 0.03 | 0.55 | 0.47 | -0.47 | -0.41 | 0.13 | 0.11 | 0.02 | 0.01 | 0.24 | 0.21 |
| | 9.32 | 6.38 | 61.82 | 48.63 | -54.89 | -45.37 | 30.25 | 23.49 | 2.60 | 1.94 | 36.63 | 28.96 |
| $\beta_{SMB}$ | 0.01 | 0.01 | 0.27 | 0.12 | -0.25 | -0.10 | -0.02 | -0.06 | 0.02 | 0.03 | -0.06 | -0.15 |
| | 1.01 | 1.43 | 16.31 | 7.10 | -15.85 | -6.02 | -2.58 | -7.10 | 1.87 | 2.11 | -4.96 | -11.23 |
| $\beta_{HML}$ | -0.02 | -0.04 | -0.04 | 0.13 | -0.01 | -0.20 | -0.09 | -0.06 | -0.03 | -0.06 | -0.16 | -0.06 |
| | -2.53 | -4.09 | -2.07 | 7.05 | -0.38 | -11.98 | -11.15 | -6.98 | -2.45 | -4.85 | -12.20 | -4.72 |
| $\beta_{MOM}$ | 0.04 | | -0.17 | | 0.25 | | -0.01 | | 0.04 | | -0.06 | |
| | 6.29 | | -14.09 | | 21.19 | | -2.51 | | 4.43 | | -7.07 | |
| $\beta_{RMW}$ | | 0.00 | | -0.43 | | 0.42 | | -0.10 | | 0.00 | | -0.21 |
| | | -0.30 | | -18.06 | | 19.01 | | -9.15 | | 0.26 | | -12.22 |
| $\beta_{CMA}$ | | -0.01 | | -0.32 | | 0.30 | | -0.04 | | 0.03 | | -0.10 |
| | | -1.04 | | -11.57 | | 11.42 | | -2.70 | | 1.69 | | -5.05 |
| $R^2$ | 2.67 | 1.93 | 52.93 | 55.64 | 50.17 | 50.32 | 17.43 | 18.91 | 0.89 | 0.58 | 27.91 | 30.36 |

- Lower Sharpe ratios, and smaller, less significant alphas

# Betting on semibetas: Conditional alphas

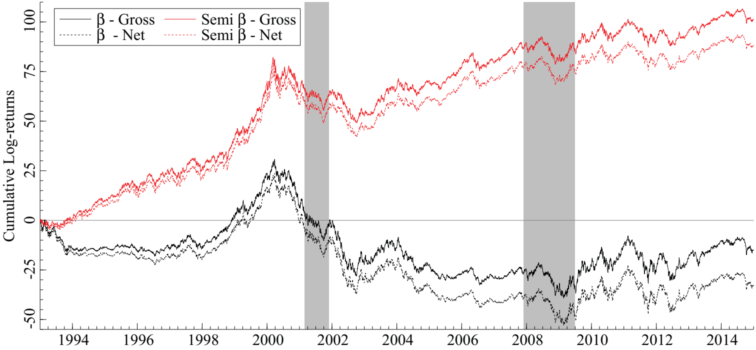
- Time varying conditional betas may bias unconditional alphas
  - Jagannathan and Wang (1996, *JF*), Lewellen and Nagel (2006, *JFE*)
- Conditional alphas following Cederburgh and O'Doherty (2016, *JF*):

| | FFC4 | | | | FF5 | | | |
|---|---|---|---|---|---|---|---|---|
| | $\beta$ | Semi $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{M}^-}$ | $\beta$ | Semi $\beta$ | $\beta^{\mathcal{N}}$ | $\beta^{\mathcal{M}^-}$ |
| I | 3.01 | 6.35 | 7.48 | 2.00 | 3.25 | 8.07 | 9.61 | 4.83 |
| | 1.02 | 2.65 | 2.42 | 1.05 | 0.85 | 3.08 | 2.60 | 1.95 |
| II | 3.87 | 7.72 | 10.09 | 4.25 | 3.18 | 7.59 | 9.32 | 4.91 |
| | 1.27 | 3.56 | 3.38 | 1.86 | 1.01 | 4.15 | 2.90 | 2.36 |
| III | 3.58 | 6.75 | 8.57 | 4.21 | 2.87 | 7.57 | 8.88 | 4.91 |
| | 1.24 | 3.34 | 3.28 | 2.18 | 0.92 | 3.77 | 3.23 | 2.13 |

  - Same qualitative findings for all sets of instruments I, II and III
  - Conditional alphas for semi $\beta$ strategy remain large and significant

# Betting on Semibetas

- Timing of returns:




  - Semibeta strategy generally performs well
  - Consistent with "betting against beta" (Frazzini and Pedersen, 2014, *JFE*), conventional beta strategy generally performs poorly