On the Dynamics of Hedge Fund Risk Exposures

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Hedge funds are a large and important segment of the money management industry. Around U.S.$ 1.2 trillion under management in mid-2009 (down from around U.S.$ 1.5 trillion). Their use of leverage means their impact in markets is greater than their assets under management. Unlike traditional fund managers, little is known in detail about the strategies employed by hedge funds. Anecdotally, strategies are dynamic, with fast turnover. Involve long and short positions. Often use relatively illiquid assets.
Hedge fund risk exposures

- Hedge funds are significantly exposed to systematic risk, proxied by indices of equity, bond and options returns.

- Growing interest in capturing how these risk exposures change through time and across market conditions:
  - Optimal changepoint regressions on factors (Bollen and Whaley, 2009, *JF*).
  - Factor loadings modelled as latent variables (Bollen and Whaley, 2009, Mamaysky, Spiegel and Zhang, 2007, *RFS*).
Contribution of this paper: Method

- We propose a new method for capturing time-varying hedge fund risk exposures, which exploits information from relatively high frequency conditioning variables.

1. The new method enables us to explicitly determine *which* variables drive movements in risk exposures, via a Ferson-Schadt (1996, *JF*) style model.
   - We search across a large set of possible variables, and account for this search via the bootstrap, as in White (2000, *Econometrica*).

2. We adapt Ferson-Schadt to enable us to capture *intra-monthly* variation in risk exposures, using only monthly returns on hedge funds.
   - A simulation study and an application to daily hedge fund index returns confirms that our method works well in practice.
Contribution of this paper: Empirical

- We use data on 14,194 hedge funds and funds-of-funds from a consolidated hedge fund data set (HFR, TASS, CISDM, BarclayHedge, Morningstar) from 1994 to 2009.

1. We compare our technique with the changepoints approach of Bollen and Whaley (2009, *JF*):
   - Our method shifts the cross-sectional distribution of $R^2$ statistics to the right by $\sim 6\%$.
   - We also find that a ‘hybrid’ model (changepoints plus interactions) does better than either model alone.

2. We find that modelling intra-month exposures is important:
   - Using only monthly conditioning variables leads to performance close to that of the changepoints model.
   - Including daily conditioning information increases (by $\sim 60\%$) the number of funds with significant time variation in exposures.
A model with monthly changes in risk exposures

- A monthly model (a la Ferson and Schadt, 1996, *JF*):

  \[
  r_{it} = \alpha_i + \beta_{it} f_t + \varepsilon_{it}
  \]

  where \( \beta_{it} = \beta_i + \gamma_i Z_{t-1} \)

  Risk exposures are driven by the variable \( Z \).

- Search across \( Z \) variables for best description of hedge fund returns.

- Substituting second equation into first we obtain:

  \[
  r_{it} = \alpha_i + \beta_i f_t + \gamma_i f_t Z_{t-1} + \varepsilon_{it}
  \]

  This basic interaction model can be estimated using OLS.

- But hedge fund factor exposures vary at higher frequencies, too...
A model with *daily* changes in risk exposures

- Our hedge fund returns data are only available monthly, but consider the following model for *daily* hedge fund returns:

\[ r_{id}^* = \alpha_i + \beta_{id} f_d^* + \epsilon_{id} \]

- Again, betas change with \( Z \). \( Z_d^* \) denotes \( Z \) measured at the daily frequency, and \( Z_d \) at the monthly frequency (\( Z_d \) constant within each month; jumps to a new level at the start of each month):

\[
\begin{align*}
\beta_{id} &= \beta_i + \gamma_i Z_{d-1} + \delta_i Z_d^* \\
\text{so} \quad r_{id}^* &= \alpha_i + \beta_i f_d^* + \gamma_i f_d^* Z_{d-1} + \delta_i f_d^* Z_d^* + \epsilon_{id}
\end{align*}
\]

- Next, we aggregate returns up to the monthly frequency:

\[
\begin{align*}
\sum_{d \in \mathcal{M}(t)} r_{id}^* \\
\sum_{d \in \mathcal{M}(t)} f_d^* Z_d^* + \epsilon_{it}
\end{align*}
\]

\[
r_{it} = \alpha_i n_t + \beta_i f_t + \gamma_i f_t Z_t - 1 + \delta_i \sum_{d \in \mathcal{M}(t)} f_d^* Z_d^* + \epsilon_{it}
\]
A model with daily changes in risk exposures, cont’d

- Our model for monthly hedge fund returns:

\[ r_{it} = \alpha_i n_t + \beta_i f_t + \gamma_i f_t Z_{t-1} + \delta_i \sum_{d \in \mathcal{M}(t)} f_d^* Z_{d-1}^* + \epsilon_{it} \]

1. The first two terms are the usual constant-parameter factor model
2. The third term is the familiar Ferson-Schadt style term
3. The fourth term is new: it uses a monthly aggregate of daily returns to capture daily changes in hedge fund betas

- Assuming \( \epsilon_{id} \) is serially uncorrelated and uncorrelated with the RHS variables in the daily model for all \((d, s)\) we can estimate the model using standard OLS.
  - An important assumption, studied below.
Factors, Interactions and Changepoints

- Need an initial low-dimensional static factor model per fund, accounting for dynamics expands it.
  - Like BW(2009), we search across Fung-Hsieh seven factors (SP500, SMB, BAAMTSY, TCM10Y, PTFs) for best 2 factors per fund.

- Best interaction variable per fund picked from a set of 19 (below). Need to account for search:

- Changepoint regressions implemented on the same static factor model.
  - One break assumed. Again need to account for search:
## Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Returns</th>
<th>Unsmoothed Returns</th>
<th>AUM ($MM)</th>
<th>Management Fee</th>
<th>Incentive Fee</th>
<th>Lockup (Days)</th>
<th>Redemption Notice (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25th Prctile</td>
<td>-0.700</td>
<td>-0.841</td>
<td>9.400</td>
<td>1.000</td>
<td>10.000</td>
<td>0.000</td>
<td>10.000</td>
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<td>50th Prctile</td>
<td>0.720</td>
<td>0.710</td>
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<td>1.500</td>
<td>20.000</td>
<td>0.000</td>
<td>30.000</td>
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<tr>
<td>75th Prctile</td>
<td>2.230</td>
<td>2.363</td>
<td>106.756</td>
<td>2.000</td>
<td>20.000</td>
<td>90.000</td>
<td>45.000</td>
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<tr>
<td>Mean</td>
<td>0.845</td>
<td>0.847</td>
<td>166.714</td>
<td>1.484</td>
<td>15.162</td>
<td>94.176</td>
<td>33.913</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>&lt;36 Months</th>
<th>&gt;=36, &lt;60</th>
<th>&gt;=60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length(Return History)</td>
<td>17.423</td>
<td>31.062</td>
<td>51.515</td>
</tr>
</tbody>
</table>
First-stage factor selection

![Fung-Hsieh Factor Selection](chart.png)

- SP500
- SMB
- TCM10Y
- BAAMTSY
- PTFSBD
- PTFSFX
- PTFSCOM
Variables (daily data preferred) in four categories: Funding and leverage, Liquidity, Volatility and Performance.

1. **Funding and Leverage**: \( \Delta \text{Level}, \Delta \text{Slope} \) and \( \Delta \text{Curvature} \), Default spread (BAA minus 10yr Tsy), TED spread (3M LIBOR minus 3M T-bill), USD/JPY carry trade return (change in the spot rate less the interest differential).

2. **Liquidity**: NYSE Turnover, Amivest.


4. **Performance**: S&P 500, NASDAQ, SMB, HML and UMD returns, Fund’s own lagged returns (one-month, one-quarter).

- Use an EWMA to detrend the persistent variables, and use the surprise component.
Before estimating our proposed model, we first test whether it is likely to be accurate. We do so in two ways:

1. **Using daily hedge fund index returns:** A limited amount of daily data on hedge fund index returns is currently available – we use that to check how close our method (using only monthly returns) comes to what would be obtained using daily data.

2. **Using simulations:** We use a simulation study, calibrated to match our real data, to check the accuracy and sensitivity of the method to various parameters.
HFR indices are available at the daily frequency starting in April 2003, for 12 hedge fund styles.

We estimate the *daily* version of our model on these returns.

Then compare resulting estimates with those from our method applied to the monthly returns on these indices:

\[
\text{Daily } r_{id}^* = \alpha_i + \beta_{i1} f_{1d}^* + \beta_{i2} f_{2d}^* + \gamma_{i1} f_{1d}^* Z_{d-1} + \gamma_{i2} f_{2d}^* Z_{d-1}^* + \delta_{i1} f_{1d}^* Z_{d-1} + \delta_{i2} f_{2d}^* Z_{d-1}^* + \varepsilon_{id}, \\
\text{Monthly } r_{it} = \alpha_i n_t + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \gamma_{i1} f_{1t} Z_{t-1} + \gamma_{i2} f_{2t} Z_{t-1} + \delta_{i1} \sum_{d \in M(t)} f_{1d}^* Z_{d-1} + \delta_{i2} \sum_{d \in M(t)} f_{2d}^* Z_{d-1}^* + \varepsilon_{it}.
\]
Using daily index returns - parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>Equity Hedge</th>
<th></th>
<th>Directional</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Daily</td>
<td>Monthly</td>
<td>Daily</td>
<td>Monthly</td>
</tr>
<tr>
<td>Alpha</td>
<td>1.397</td>
<td>3.106</td>
<td>3.702</td>
<td>5.068</td>
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<tr>
<td>t-stat</td>
<td>0.643</td>
<td>1.508</td>
<td>1.113</td>
<td>1.613</td>
</tr>
<tr>
<td>Beta1</td>
<td>0.301</td>
<td>0.321</td>
<td>0.264</td>
<td>0.357</td>
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<tr>
<td>t-stat</td>
<td>38.426</td>
<td>6.374</td>
<td>29.344</td>
<td>3.940</td>
</tr>
<tr>
<td>Beta2</td>
<td>-1.832</td>
<td>-1.829</td>
<td>-2.590</td>
<td>-1.920</td>
</tr>
<tr>
<td>t-stat</td>
<td>-6.417</td>
<td>-2.613</td>
<td>-6.789</td>
<td>-1.468</td>
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<tr>
<td>Gamma1</td>
<td>0.006</td>
<td>0.009</td>
<td>0.086</td>
<td>0.272</td>
</tr>
<tr>
<td>t-stat</td>
<td>8.053</td>
<td>1.526</td>
<td>7.839</td>
<td>2.395</td>
</tr>
<tr>
<td>Gamma2</td>
<td>-0.007</td>
<td>0.025</td>
<td>-0.352</td>
<td>1.209</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.228</td>
<td>0.223</td>
<td>-0.759</td>
<td>0.641</td>
</tr>
<tr>
<td>Delta1</td>
<td>0.006</td>
<td>0.034</td>
<td>0.053</td>
<td>-0.164</td>
</tr>
<tr>
<td>t-stat</td>
<td>2.682</td>
<td>2.848</td>
<td>0.878</td>
<td>-0.570</td>
</tr>
<tr>
<td>Delta2</td>
<td>0.216</td>
<td>0.340</td>
<td>-14.831</td>
<td>-42.915</td>
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<tr>
<td>t-stat</td>
<td>3.053</td>
<td>0.444</td>
<td>-5.305</td>
<td>-1.961</td>
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</tbody>
</table>
Using daily index returns - model results

<table>
<thead>
<tr>
<th></th>
<th>Eq. Hedge</th>
<th>Macro</th>
<th>Dir. Traders</th>
<th>Merger Arb.</th>
<th>Rel. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Joint sig. (interactions)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boot p-val - daily</td>
<td>0.000</td>
<td>0.724</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Boot p-val - monthly</td>
<td>0.010</td>
<td>0.692</td>
<td>0.044</td>
<td>0.633</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>Corr. (true,estimated)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr $\hat{\beta}^*<em>1, \hat{\beta}</em>{1d}$</td>
<td>0.887</td>
<td>-0.404</td>
<td>0.974</td>
<td>0.981</td>
<td>0.785</td>
</tr>
<tr>
<td>Corr $\hat{\beta}^*<em>2, \hat{\beta}</em>{2d}$</td>
<td>0.936</td>
<td>-0.994</td>
<td>0.963</td>
<td>0.630</td>
<td>-0.770</td>
</tr>
</tbody>
</table>
Daily betas for Equity Hedge Index
2009Q2, SP500 factor, SP500 conditioning variable

Estimated daily beta on SP500 for the EqHedge index

Date
Beta

01-Apr-2009
04-May-2009
04-Jun-2009
30-Jun-2009

0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55

Constant
Daily data
Monthly data

01-Apr-2009
04-May-2009
04-Jun-2009
30-Jun-2009
Daily betas for Equity Hedge Index
2009Q2, BAAMTSY factor, SP500 conditioning variable

Estimated daily beta on BAAMTSY for the EqHedge index

Date
Beta
-3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5

Constant
Daily data
Monthly data
A small simulation study: Design

- We next consider a small simulation study of the accuracy of the proposed method.

1. Assume that daily hedge fund returns are driven by a one-factor model, with time-varying factor loadings:

   \[ r_d^* = \alpha + \beta f_d^* + \gamma f_d^* Z_{d-1} + \delta f_d^* Z_{d-1}^* + \epsilon_{R,d}^*, \quad d = 1, 2, \ldots, 22 \times T, \]

2. Assume that the conditioning variable follows an AR(1):

   \[ Z_d^* = \phi_Z Z_{d-1}^* + \epsilon_{Z,d}^* \]

3. We also allow the factor return to be persistent:

   \[ f_d^* = \mu_F + \phi_F (f_{d-1}^* - \mu_F) + \epsilon_{F,d}^* \]
We calibrate the parameters of the model to the results obtained from estimation using daily HFR equity hedge index returns:

\[
\begin{align*}
\alpha &= \frac{2}{(22 \times 12)}, \quad \beta = 0.4, \quad \gamma = 0.002, \quad \delta = -0.004 \\
\mu_F &= \frac{10}{(22 \times 12)}, \quad \sigma_F = \frac{20}{\sqrt{22 \times 12}}, \quad \sigma_Z = 10, \quad \sigma_{\varepsilon R} = \sqrt{0.1}
\end{align*}
\]

All innovations are normally distributed, with:

\[
\begin{align*}
\text{Corr} \left[ \varepsilon_{R,d}, \varepsilon_{Z,d} \right] &= \text{Corr} \left[ \varepsilon_{R,d}, \varepsilon_{F,d} \right] = 0 \\
\text{Corr} \left[ \varepsilon_{Z,d}, \varepsilon_{F,d} \right] &\equiv \rho_{FZ} \in \{ 0, 0.5 \}
\end{align*}
\]

Key parameters:

\[
\begin{align*}
\phi_Z &\in \{ 0, 0.5, 0.9 \} \\
\phi_F &\in \{ -0.2, 0, 0.2 \} \\
T &\in \{ 24, 60, 120 \}
\end{align*}
\]
A small simulation study: Results

<table>
<thead>
<tr>
<th></th>
<th>True values</th>
<th>Base case</th>
<th>Short sample</th>
<th>High</th>
<th>Corr</th>
<th>ϕ_F</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>60</td>
<td>24</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>ρ_{FZ}</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>ϕ_Z</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>ϕ_F</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean α</td>
<td>0.76</td>
<td>0.72</td>
<td>0.83</td>
<td>0.77</td>
<td>0.81</td>
<td>0.72</td>
</tr>
<tr>
<td>Mean β</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Mean γ</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Mean δ</td>
<td>-0.40</td>
<td>-0.39</td>
<td>-0.40</td>
<td>-0.40</td>
<td>-0.41</td>
<td>-0.39</td>
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<tr>
<td>St dev α</td>
<td>0.09</td>
<td>0.15</td>
<td>0.09</td>
<td>0.19</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>St dev β</td>
<td>0.04</td>
<td>0.06</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>St dev γ</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>St dev δ</td>
<td>0.04</td>
<td>0.06</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>
Empirical analysis of individual hedge funds

- We now turn to the analysis individual hedge fund data

- We first show the conditioning variables that are most often selected, across funds.

- We then compare our model to an optimal changepoint regression and to a model that only uses monthly conditioning information.
## Top 10 conditioning variables

Yellow: leverage, Green: performance, Pink: volatility

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number</th>
<th>Base R2</th>
<th>Best R2</th>
<th>Best/Base</th>
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</thead>
<tbody>
<tr>
<td>d(Level)</td>
<td>465</td>
<td>0.383</td>
<td>0.624</td>
<td>1.631</td>
</tr>
<tr>
<td>Nasdaq Return</td>
<td>249</td>
<td>0.400</td>
<td>0.650</td>
<td>1.626</td>
</tr>
<tr>
<td>USD/JPY Carry Trade Return</td>
<td>243</td>
<td>0.397</td>
<td>0.666</td>
<td>1.677</td>
</tr>
<tr>
<td>d(Slope)</td>
<td>195</td>
<td>0.435</td>
<td>0.692</td>
<td>1.591</td>
</tr>
<tr>
<td>Fund Performance (3 months)</td>
<td>190</td>
<td>0.397</td>
<td>0.620</td>
<td>1.561</td>
</tr>
<tr>
<td>S &amp; P 500 Return</td>
<td>186</td>
<td>0.397</td>
<td>0.644</td>
<td>1.622</td>
</tr>
<tr>
<td>TED Spread</td>
<td>186</td>
<td>0.358</td>
<td>0.625</td>
<td>1.748</td>
</tr>
<tr>
<td>Variance Risk Premium</td>
<td>186</td>
<td>0.407</td>
<td>0.663</td>
<td>1.629</td>
</tr>
<tr>
<td>BAAMTSY</td>
<td>169</td>
<td>0.370</td>
<td>0.628</td>
<td>1.699</td>
</tr>
<tr>
<td>Fund Performance (1 month)</td>
<td>168</td>
<td>0.348</td>
<td>0.633</td>
<td>1.817</td>
</tr>
<tr>
<td>Total/Average</td>
<td>3321</td>
<td>0.392</td>
<td>0.646</td>
<td>1.666</td>
</tr>
</tbody>
</table>
Economic drivers of time-varying factor exposures

- Policy-makers care about HF leverage, but measurement is hard.
  - Lo and Khandani (2008) show that deleveraging by L/S equity HFs could have caused “quant meltdown” of August 2007.
  - Liang (1999) shows that merger arbitrage funds benefit from leverage (we find that merger arb funds most frequently select leverage).

- Our results: Variations in costs of leverage significantly affect HF factor exposures.
  - One application: Can measure how aggregate HF factor exposures vary with costs of leverage.

- Daily and monthly performance interactions also selected frequently.
  - Suggests that benchmarking may be very important.
  - Also, past fund performance affects risk-exposures, an important fact for investors and policymakers...
Comparison of goodness-of-fit

- Our method yields economically interpretable variation in factor loadings, but how good is it at capturing return variation?
  - Bollen and Whaley (2009) show that the optimal changepoint regression is preferred to the latent variable approach.
  - Use this as our benchmark.

First, compute the $R^2$ for all funds using best changepoint; do the same for best interaction.

- Plot the CDFs of the two sets of R-squared statistics
- Also look at ‘hybrid’ models.
## Significance of time-varying risk exposures

Proportion of 14,194 funds that reject null of constant risk exposures at 0.10 level

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Number of Funds</th>
<th>Changepoints</th>
<th>Interactions</th>
<th>Changepoints and Interactions</th>
<th>Interactions, but only Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security Selection</td>
<td>2942</td>
<td>0.229</td>
<td>0.200</td>
<td>0.263</td>
<td>0.131</td>
</tr>
<tr>
<td>Macro</td>
<td>885</td>
<td>0.215</td>
<td>0.116</td>
<td>0.179</td>
<td>0.084</td>
</tr>
<tr>
<td>Relative Value</td>
<td>146</td>
<td>0.144</td>
<td>0.240</td>
<td>0.322</td>
<td>0.158</td>
</tr>
<tr>
<td>Directional Traders</td>
<td>1813</td>
<td>0.240</td>
<td>0.266</td>
<td>0.326</td>
<td>0.189</td>
</tr>
<tr>
<td>Fund of Funds</td>
<td>3309</td>
<td>0.210</td>
<td>0.344</td>
<td>0.424</td>
<td>0.200</td>
</tr>
<tr>
<td>Multi-Process</td>
<td>1775</td>
<td>0.226</td>
<td>0.246</td>
<td>0.326</td>
<td>0.148</td>
</tr>
<tr>
<td>Emerging</td>
<td>478</td>
<td>0.186</td>
<td>0.176</td>
<td>0.232</td>
<td>0.117</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>805</td>
<td>0.195</td>
<td>0.278</td>
<td>0.350</td>
<td>0.175</td>
</tr>
<tr>
<td>CTA</td>
<td>1981</td>
<td>0.187</td>
<td>0.110</td>
<td>0.154</td>
<td>0.083</td>
</tr>
<tr>
<td>Other</td>
<td>43</td>
<td>0.140</td>
<td>0.186</td>
<td>0.419</td>
<td>0.093</td>
</tr>
<tr>
<td>All</td>
<td>14194</td>
<td>0.214</td>
<td>0.234</td>
<td>0.301</td>
<td>0.149</td>
</tr>
</tbody>
</table>
In the original Ferson-Schadt (1996) study, the goal was conditional performance evaluation. The use of $t - 1$ interaction variables yields a managed strategy implementable with public information; can be used to benchmark the portfolio manager.

We find that hedge fund performance looks slightly better on average using our method: average alpha increases by $\sim 0.5\%$ per annum relative to the static model.

However there are big changes in alpha for any individual fund: the mean *absolute* difference in alpha is large, between 3.2% and 4.6% per annum.
Differences in alpha b/w constant and interaction models

<table>
<thead>
<tr>
<th></th>
<th>Alpha from Static Model – Alpha from Interactions Model</th>
<th>Absolute Value of Difference in Alphas</th>
<th>Rank Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Funds</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.034</td>
<td>3.225</td>
<td>0.860</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.594</td>
<td>63.987</td>
<td>201.180</td>
</tr>
<tr>
<td><strong>Funds w/Significant Time Variation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>-0.502</td>
<td>4.588</td>
<td>0.736</td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.119</td>
<td>32.746</td>
<td>62.580</td>
</tr>
</tbody>
</table>
Robustness checks

- We considered a variety of robustness checks:
  - History length: [24, 36], (36, 60], (60, 186] months of data
  - Assets under management: low, mid and high terciles

- Our conclusions are generally robust to these variations
  - Our method works better in latter sub-period, and better for funds with shorter samples.
Summary and conclusions

- We present a new method to model changes in hedge fund risk exposures, using information from variables measured at a higher frequency than hedge fund returns.

- Simulations and daily index returns confirm that the proposed method works well.

- We find significant evidence of higher-frequency changes in beta: around 25% of funds exhibit daily changes in beta (relative to 15% using only monthly data)

- The proposed method beats existing alternative approaches in terms of fit, and also provides economic interpretability to the changes in exposures.

- Our approach may be useful for other managed investment performance evaluation applications (mutual funds, private equity) where “interim trading” is a concern.