On the Dynamics of Hedge Fund Risk Exposures

Andrew Patton and Tarun Ramadorai

Duke University and University of Oxford

June 2010

Hedge funds - some key features

- Hedge funds are a large and important segment of the money management industry.
    - Around U.S.$ 1.2 trillion under management in mid-2009 (down from around U.S.$ 1.5 trillion).
    - Their use of *leverage* means their impact in markets is greater than their assets under management.
- Unlike traditional fund managers, little is known in detail about the strategies employed by hedge funds. Anecdotally,
    - Strategies are *dynamic*, with fast turnover.
    - Involve long and short positions.
    - Often use relatively illiquid assets.

Hedge fund risk exposures

- Hedge funds are significantly exposed to systematic risk, proxied by indices of equity, bond and options returns.
  - Agarwal and Naik (2004, *RFS*); Fung and Hsieh (1997, 2001, 2004); Jagannathan et al. (2010, *JF*); many more.
- Growing interest in capturing how these risk exposures change through time and across market conditions:
  - Optimal changepoint regressions on factors (Bollen and Whaley, 2009, *JF*).
  - Factor loadings modelled as latent variables (Bollen and Whaley, 2009, Mamaysky, Spiegel and Zhang, 2007, *RFS*).

Contribution of this paper: Method

- We propose a new method for capturing time-varying hedge fund risk exposures, which exploits information from relatively high frequency conditioning variables.

1. The new method enables us to explicitly determine *which* variables drive movements in risk exposures, via a Ferson-Schadt (1996, *JF*) style model.
   - We search across a large set of possible variables, and account for this search via the bootstrap, as in White (2000, *Econometrica*).
2. We adapt Ferson-Schadt to enable us to capture *intra-monthly* variation in risk exposures, using only monthly returns on hedge funds.
   - A simulation study and an application to daily hedge fund index returns confirms that our method works well in practice.

Contribution of this paper: Empirical

- We use data on 14,194 hedge funds and funds-of-funds from a consolidated hedge fund data set (HFR, TASS, CISDM, BarclayHedge, Morningstar) from 1994 to 2009.

1. We compare our technique with the changepoints approach of Bollen and Whaley (2009, *JF*):
    1. Our method shifts the cross-sectional distribution of R^2 statistics to the right by $\sim 6\%$.
    2. We also find that a 'hybrid' model (changepoints plus interactions) does better than either model alone.
2. We find that modelling intra-month exposures is important:
    1. Using only monthly conditioning variables leads to performance close to that of the changepoints model.
    2. Including daily conditioning information increases (by $\sim 60\%$) the number of funds with significant time variation in exposures.

A model with monthly changes in risk exposures

- A monthly model (a la Ferson and Schadt, 1996, *JF*):

$$\begin{aligned} r_{it} &= \alpha_i + \beta_{it} f_t + \varepsilon_{it} \\ \text{where} \quad \beta_{it} &= \beta_i + \gamma_i Z_{t-1} \end{aligned}$$

  Risk exposures are driven by the variable Z.

- Search across Z variables for best description of hedge fund returns.
- Substituting second equation into first we obtain:

$$r_{it} = \alpha_i + \beta_i f_t + \gamma_i f_t Z_{t-1} + \varepsilon_{it}$$

  This basic interaction model can be estimated using OLS.

- But hedge fund factor exposures vary at higher frequencies, too...

A model with *daily* changes in risk exposures

- Our hedge fund returns data are only available monthly, but consider the following model for *daily* hedge fund returns:

$$r_{id}^* = \alpha_i + \beta_{id} f_d^* + \varepsilon_{id}^*$$

- Again, betas change with Z. Z_d^* denotes Z measured at the daily frequency, and Z_d at the monthly frequency (Z_d constant within each month; jumps to a new level at the start of each month):

$$\begin{aligned} \beta_{id} &= \beta_i + \gamma_i Z_{d-1} + \delta_i Z_{d-1}^* \\ \text{so} \quad r_{id}^* &= \alpha_i + \beta_i f_d^* + \gamma_i f_d^* Z_{d-1} + \delta_i f_d^* Z_{d-1}^* + \varepsilon_{id}^* \end{aligned}$$

- Next, we aggregate returns up to the monthly frequency:

$$\begin{aligned} r_{it} &\equiv \sum_{d \in \mathcal{M}(t)} r_{id}^* \\ r_{it} &= \alpha_i n_t + \beta_i f_t + \gamma_i f_t Z_{t-1} + \delta_i \sum_{d \in \mathcal{M}(t)} f_d^* Z_{d-1}^* + \varepsilon_{it} \end{aligned}$$

A model with daily changes in risk exposures, cont'd

- Our model for monthly hedge fund returns:

$$r_{it} = \alpha_i n_t + \beta_i f_t + \gamma_i f_t Z_{t-1} + \delta_i \sum_{d \in \mathcal{M}(t)} f_d^* Z_{d-1}^* + \varepsilon_{it}$$

1. The first two terms are the usual constant-parameter factor model
2. The third term is the familiar Ferson-Schadt style term
3. The fourth term is new: it uses a monthly aggregate of daily returns to capture daily changes in hedge fund betas

- Assuming ε_{id} is serially uncorrelated and uncorrelated with the RHS variables in the daily model for all (d, s) we can estimate the model using standard OLS.
    - An important assumption, studied below.

Factors, Interactions and Changepoints

- Need an initial low-dimensional static factor model per fund, accounting for dynamics expands it.
  - Like BW(2009), we search across Fung-Hsieh seven factors (SP500, SMB, BAAMTSY, TCM10Y, PTFs) for best 2 factors per fund.

- Best interaction variable per fund picked from a set of 19 (below). Need to account for search:
  - White's (2000) 'Reality check' for data mining, using stationary bootstrap of Politis and Romano (1994).

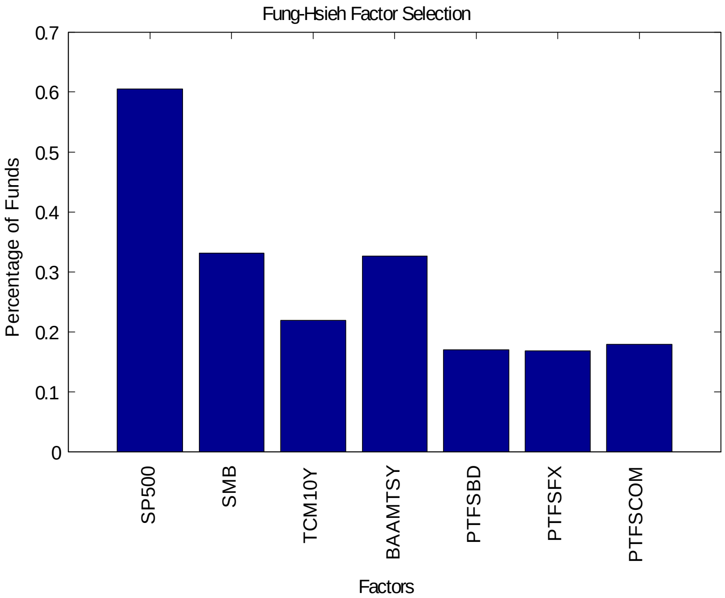
- Changepoint regressions implemented on the same static factor model.
  - One break assumed. Again need to account for search:
  - Andrews, Lee and Ploberger (1996) *supF* statistic, stationary bootstrap of Politis and Romano (1994).

Summary statistics

	Returns	Unsmoothed Returns	AUM ($MM)	Management Fee	Incentive Fee	Lockup (Days)	Redemption Notice (Days)
25th Prctile	-0.700	-0.841	9.400	1.000	10.000	0.000	10.000
50th Prctile	0.720	0.710	32.000	1.500	20.000	0.000	30.000
75th Prctile	2.230	2.363	106.756	2.000	20.000	90.000	45.000
Mean	0.845	0.847	166.714	1.484	15.162	94.176	33.913

	<36 Months	>=36 , <60	>=60
Length(Return History)	17.423	31.062	51.515

First-stage factor selection



Conditioning variables

- Variables (daily data preferred) in four categories: Funding and leverage, Liquidity, Volatility and Performance..

  1. **Funding and Leverage**: ΔLevel, ΔSlope and ΔCurvature, Default spread (BAA minus 10yr Tsy), TED spread (3M LIBOR minus 3M T-bill), USD/JPY carry trade return (change in the spot rate less the interest differential).
  2. **Liquidity**: NYSE Turnover, Amivest.
  3. **Volatility**: VIX, Realized Volatility (RV) on the S&P 500 index, RV on the USD/JPY exchange rate, Variance risk premium on the S&P 500 index.
  4. **Performance**: S&P 500, NASDAQ, SMB, HML and UMD returns, Fund's own lagged returns (one-month, one-quarter).

- Use an EWMA to detrend the persistent variables, and use the surprise component.

The accuracy of the proposed method

- Before estimating our proposed model, we first test whether it is likely to be accurate. We do so in two ways:

1. **Using daily hedge fund index returns:** a limited amount of *daily* data on hedge fund index returns is currently available – we use that to check how close our method (using only monthly returns) comes to what would be obtained using daily data.
2. **Using simulations:** we use a simulation study, calibrated to match our real data, to check the accuracy and sensitivity of the method to various parameters.

Using daily hedge fund index returns

- HFR indices are available at the daily frequency starting in April 2003, for 12 hedge fund styles.
- We estimate the *daily* version of our model on these returns.
- Then compare resulting estimates with those from our method applied to the monthly returns on these indices:

$$\begin{aligned}\text{Daily } r^*_{id} &= \alpha_i + \beta_{i1} f^*_{1d} + \beta_{i2} f^*_{2d} + \gamma_{i1} f^*_{1d} Z_{d-1} \\ &\quad + \gamma_{i2} f^*_{2d} Z_{d-1} + \delta_{i1} f^*_{1d} Z^*_{d-1} + \delta_{i2} f^*_{2d} Z^*_{d-1} + \varepsilon^*_{id},\end{aligned}$$

$$\begin{aligned}\text{Monthly } r_{it} &= \alpha_i n_t + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \gamma_{i1} f_{1t} Z_{t-1} + \gamma_{i2} f_{2t} Z_{t-1} \\ &\quad + \delta_{i1} \sum_{d \in \mathcal{M}(t)} f^*_{1d} Z^*_{d-1} + \delta_{i2} \sum_{d \in \mathcal{M}(t)} f^*_{2d} Z^*_{d-1} + \varepsilon_{it}.\end{aligned}$$

Using daily index returns - parameter estimates

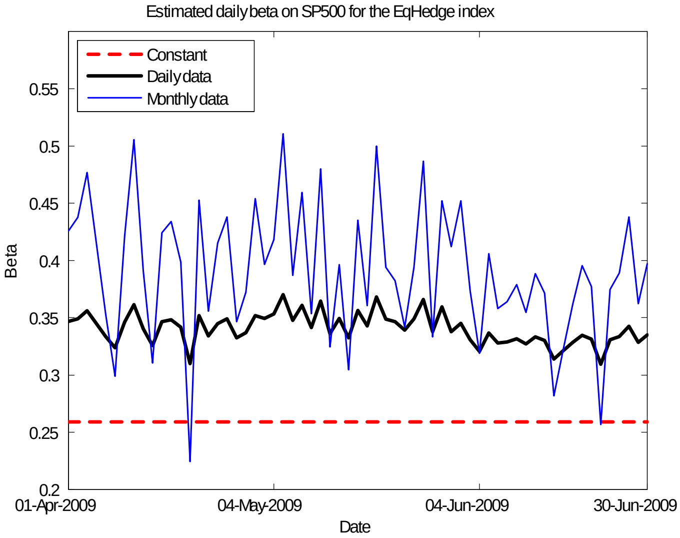
	Equity Hedge		Directional	
	Daily	Monthly	Daily	Monthly
Alpha	1.397	3.106	3.702	5.068
t-stat	0.643	1.508	1.113	1.613
Beta1	0.301	0.321	0.264	0.357
t-stat	38.426	6.374	29.344	3.940
Beta2	-1.832	-1.829	-2.590	-1.920
t-stat	-6.417	-2.613	-6.789	-1.468
Gamma1	0.006	0.009	0.086	0.272
t-stat	8.053	1.526	7.839	2.395
Gamma2	-0.007	0.025	-0.352	1.209
t-stat	-0.228	0.223	-0.759	0.641
Delta1	0.006	0.034	0.053	-0.164
t-stat	2.682	2.848	0.878	-0.570
Delta2	0.216	0.340	-14.831	-42.915
t-stat	3.053	0.444	-5.305	-1.961

Using daily index returns - model results

	Eq. Hedge	Macro	Dir. Traders	Merger Arb.	Rel. Value
Joint sig.(interactions)					
Boot p-val - daily	0.000	0.724	0.000	0.000	0.000
Boot p-val - monthly	0.010	0.692	0.044	0.633	0.004
Corr.(true,estimated)					
Corr $\left[\hat{\beta}^*_{1d}, \hat{\beta}_{1d}\right]$	0.887	-0.404	0.974	0.981	0.785
Corr $\left[\hat{\beta}^*_{2d}, \hat{\beta}_{2d}\right]$	0.936	-0.994	0.963	0.630	-0.770

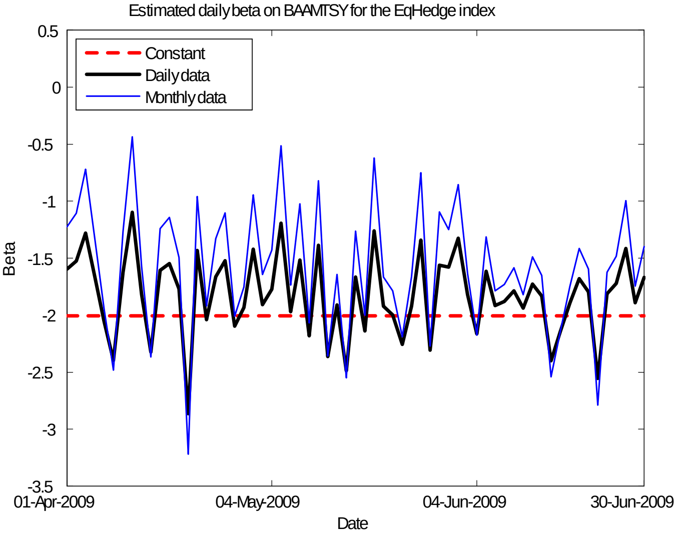
Daily betas for Equity Hedge Index

2009Q2, SP500 factor, SP500 conditioning variable



Daily betas for Equity Hedge Index

2009Q2, BAAMTSY factor, SP500 conditioning variable



A small simulation study: Design

- We next consider a small simulation study of the accuracy of the proposed method.

1. Assume that daily hedge fund returns are driven by a one-factor model, with time-varying factor loadings:

$$r_d^* = \alpha + \beta f_d^* + \gamma f_d^* Z_{d-1} + \delta f_d^* Z_{d-1}^* + \varepsilon_{R,d}^*, \quad d = 1, 2, ..., 22 \times T,$$

2. Assume that the conditioning variable follows an AR(1):

$$Z_d^* = \phi_Z Z_{d-1}^* + \varepsilon_{Z,d}^*$$

3. We also allow the factor return to be persistent:

$$f_d^* = \mu_F + \phi_F \left(f_{d-1}^* - \mu_F \right) + \varepsilon_{F,d}^*$$

A small simulation study: Parameter values

- We calibrate the parameters of the model to the results obtained from estimation using daily HFR equity hedge index returns:

$$\alpha = 2/(22 \times 12),\ \beta = 0.4,\ \gamma = 0.002,\ \delta = -0.004$$
$$\mu_F = 10/\left(22 \times 12\right),\ \sigma_F = 20/\sqrt{22 \times 12},\ \sigma_Z = 10,\ \sigma_{\varepsilon R} = \sqrt{0.1}$$

- All innovations are normally distributed, with:

$$Corr\left[\varepsilon^*_{R,d}, \varepsilon^*_{Z,d}\right] = Corr\left[\varepsilon^*_{R,d}, \varepsilon^*_{F,d}\right] = 0$$
$$Corr\left[\varepsilon^*_{Z,d}, \varepsilon^*_{F,d}\right] \equiv \rho_{FZ} \in \{\ 0\ ,\ 0.5\ \}$$

- Key parameters:

$$\phi_Z \in \{\ 0\ ,\ 0.5\ ,\ 0.9\ \}$$
$$\phi_F \in \{\ -0.2\ ,\ 0\ ,\ 0.2\ \}$$
$$T \in \{\ 24\ ,\ 60\ ,\ 120\ \}$$

A small simulation study: Results

	True values	Base case	Short sample	High ϕ_Z	Corr ρ_{FZ}	ϕ_F ρ_{FZ}
T		60	24	60	60	60
ρ_{FZ}		0.0	0.0	0.0	0.5	0.5
ϕ_Z		0.5	0.5	0.9	0.5	0.5
ϕ_F		0.0	0.0	0.0	0.0	0.2
Mean α	0.76	0.72	0.83	0.77	0.81	0.72
Mean β	0.40	0.40	0.40	0.40	0.40	0.40
Mean γ	0.20	0.20	0.20	0.20	0.20	0.20
Mean δ	-0.40	-0.39	-0.40	-0.40	-0.41	-0.39
St dev α		0.09	0.15	0.09	0.19	0.19
St dev β		0.04	0.06	0.04	0.04	0.03
St dev γ		0.01	0.01	0.00	0.01	0.00
St dev δ		0.04	0.06	0.05	0.03	0.03

Empirical analysis of individual hedge funds

- We now turn to the analysis individual hedge fund data
- We first show the conditioning variables that are most often selected, across funds.
- We then compare our model to an optimal changepoint regression and to a model that only uses monthly conditioning information.

Top 10 conditioning variables

Yellow: leverage, Green: performance, Pink: volatility

	Funds for which variable is *significant*			
Variable	Number	Base R2	Best R2	Best/Base
d(Level)	465	0.383	0.624	1.631
Nasdaq Return	249	0.400	0.650	1.626
USD/JPY Carry Trade Return	243	0.397	0.666	1.677
d(Slope)	195	0.435	0.692	1.591
Fund Performance (3 months)	190	0.397	0.620	1.561
S & P 500 Return	186	0.397	0.644	1.622
TED Spread	186	0.358	0.625	1.748
Variance Risk Premium	186	0.407	0.663	1.629
BAAMTSY	169	0.370	0.628	1.699
Fund Performance (1 month)	168	0.348	0.633	1.817
Total/Average	3321	0.392	0.646	1.666

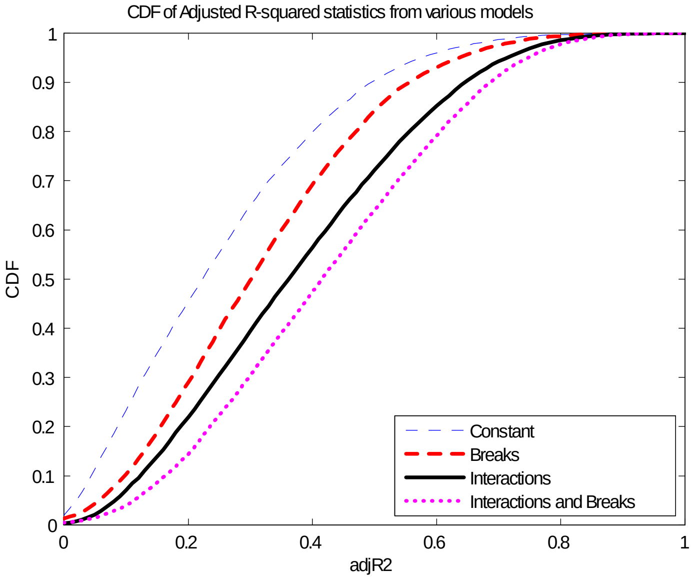
Economic drivers of time-varying factor exposures

- Policy-makers care about HF leverage, but measurement is hard.
  - Lo and Khandani (2008) show that deleveraging by L/S equity HFs could have caused "quant meltdown" of August 2007.
  - Liang (1999) shows that merger arbitrage funds benefit from leverage (we find that merger arb funds most frequently select leverage).
- Our results: Variations in costs of leverage significantly affect HF factor exposures.
  - One application: Can measure how *aggregate* HF factor exposures vary with costs of leverage.
- Daily and monthly performance interactions also selected frequently.
  - Suggests that benchmarking may be very important.
  - Also, past fund performance affects risk-exposures, an important fact for investors and policymakers...

Comparison of goodness-of-fit

- Our method yields economically interpretable variation in factor loadings, but how good is it at capturing return variation?
  - Bollen and Whaley (2009) show that the optimal changepoint regression is preferred to the latent variable approach.
  - Use this as our benchmark.
- First, compute the R^2 for all funds using best changepoint; do the same for best interaction.
  - Plot the CDFs of the two sets of R-squared statistics
  - Also look at 'hybrid' models.

Changepoints vs. Conditioning Variables vs. Hybrid



Significance of time-varying risk exposures

Proportion of 14,194 funds that reject null of constant risk exposures at 0.10 level

Strategy	Number of Funds	Changepoints	Interactions	Changepoints and Interactions	Interactions, but only Monthly
Security Selection	2942	0.229	0.200	0.263	0.131
Macro	885	0.215	0.116	0.179	0.084
Relative Value	146	0.144	0.240	0.322	0.158
Directional Traders	1813	0.240	0.266	0.326	0.189
Fund of Funds	3309	0.210	0.344	0.424	0.200
Multi-Process	1775	0.226	0.246	0.326	0.148
Emerging	478	0.186	0.176	0.232	0.117
Fixed Income	805	0.195	0.278	0.350	0.175
CTA	1981	0.187	0.110	0.154	0.083
Other	43	0.140	0.186	0.419	0.093
All	14194	0.214	0.234	0.301	0.149

Implications for performance measurement

- In the original Ferson-Schadt (1996) study, the goal was conditional performance evaluation. The use of $t-1$ interaction variables yields a managed strategy implementable with public information; can be used to benchmark the portfolio manager.
- We find that hedge fund performance looks slightly better on average using our method: average alpha increases by $\sim 0.5\%$ per annum relative to the static model.
- However there are big changes in alpha for any individual fund: the mean *absolute* difference in alpha is large, between 3.2% and 4.6% per annum.

Differences in alpha b/w constant and interaction models

	Alpha from Static Model – Alpha from Interactions Model	Absolute Value of Difference in Alphas	Rank Correlation
All Funds			
Mean	-0.034	3.225	0.860
t-stat	-0.594	63.987	201.180
Funds w/Significant Time Variation			
Average	-0.502	4.588	0.736
t-stat	-3.119	32.746	62.580

Robustness checks

- We considered a variety of robustness checks:
  1. Sub-samples: 1994-2001 vs. 2002-2009
  2. History length: $[24, 36]$, $(36, 60]$, $(60, 186]$ months of data
  3. Assets under management: low, mid and high terciles

- Our conclusions are generally robust to these variations
  - Our method works better in latter sub-period, and better for funds with shorter samples.

Summary and conclusions

- We present a new method to model changes in hedge fund risk exposures, using information from variables measured at a higher frequency than hedge fund returns.
  - Simulations and daily index returns confirm that the proposed method works well.
  - We find significant evidence of higher-frequency changes in beta: around 25% of funds exhibit daily changes in beta (relative to 15% using only monthly data)
  - The proposed method beats existing alternative approaches in terms of fit, and also provides economic interpretability to the changes in exposures.
- Our approach may be useful for other managed investment performance evaluation applications (mutual funds, private equity) where "interim trading" is a concern.