Learning in Real Time: Theory and Empirical Evidence from the Term Structure of Survey Forecasts

Andrew Patton and Allan Timmermann

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Motivation

- Uncertainty about macroeconomic variables such as GDP growth and inflation enters into the decision-making processes of governments, firms and individuals.
  - welfare implications of macroeconomic volatility, see Ramey and Ramey (1995, AER)
  - irreversibility and lags in investment decisions, see Kydland and Prescott (1982, Econometrica)
  - determination of asset prices, see Andersen et al. (2003, AER)
  - volatility and volume in asset markets, see Beber and Brandt (2006)
  - responsiveness of investment to information, see Bloom et al. (2007, REStud)

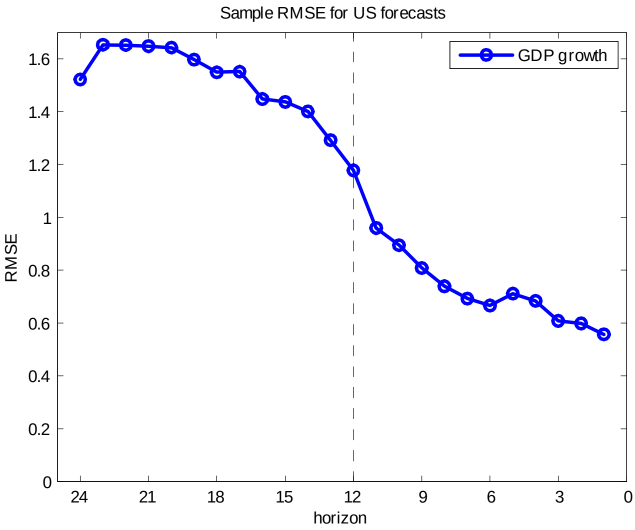
Contributions of this paper

1. We use a rich data set containing survey forecasts of annual GDP growth and inflation, with forecast horizons ranging from 1 to 24 months, to shed light on:
    1. How quickly agents learn about these GDP growth and inflation
    2. What factors affect the accuracy of their forecasts
    3. What factors might be behind the observed amount of disagreement between individual forecasters
2. We analyse this data via a simple but flexible theoretical framework for studying panels of forecasts containing multiple horizons.
    1. This framework is designed to model both the consensus forecast and the dispersion amongst forecasts, across many horizons.

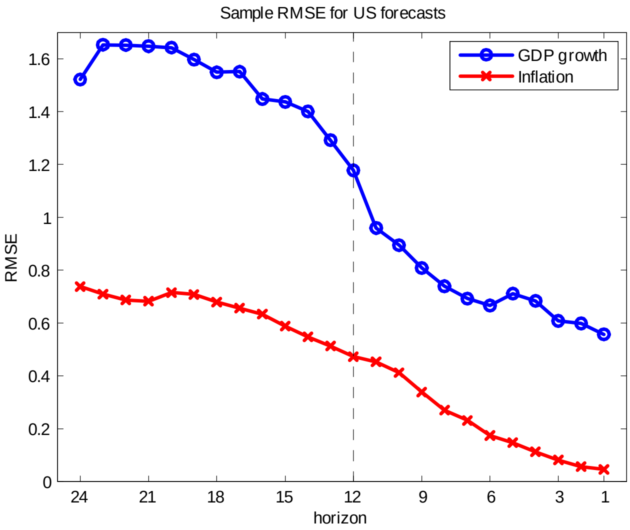
Detailed description of the data

- We have $T = 14$ years of data (1991-2004) from Consensus Economics for real GDP growth and CPI inflation.
- In each year we have forecasts for $H = 24$ different forecast horizons.
- We have consensus forecasts and the cross-sectional dispersion of individual forecasts, but not the individual forecasts themselves.
- The realization of the target variable is the value reported by the IMF in the September of the following year.
  - *Our* measure of the target variable may be subject to measurement error - we do not explicitly model this.

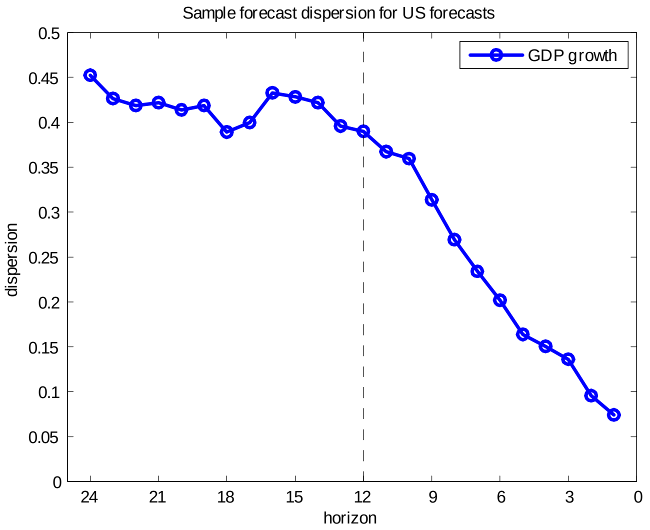
Sample root mean squared errors as a function of the forecast horizon: GDP growth



Sample root mean squared errors as a function of the forecast horizon: GDP growth and Inflation



Cross-sectional forecast dispersion as a function of the forecast horizon: GDP growth



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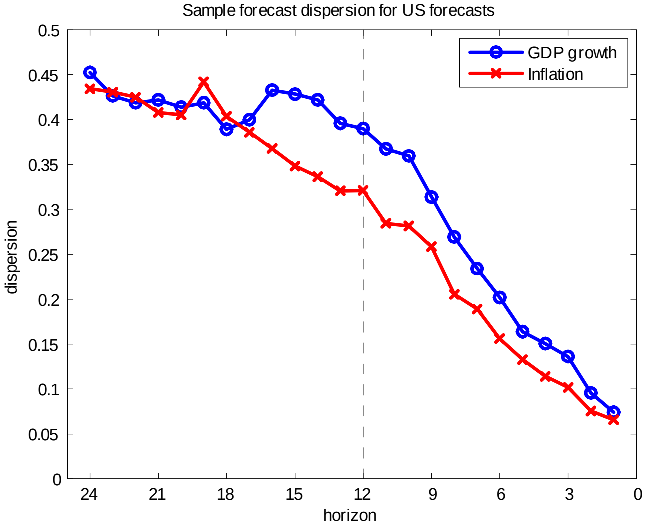


Outline of the talk

1. Introduction and brief description of the data
2. **A theoretical model for the term structure of RMSE**
    1. Theoretical predictions
    2. Estimation by generalised method of moments
    3. Estimating the persistent components from GDP growth and inflation from the survey forecasts
3. A theoretical model for the term structure of forecast dispersion
    1. Theoretical predictions
    2. Joint estimation by simulated method of moments
    3. A simple model for time-varying dispersion
4. Summary and conclusions

First: Are the survey forecasts unbiased?

- Our use of the Consensus Economics survey forecasts in this study relies on the assumption that

$$\hat{z}_{t,t-h} = E\left[z_t | \tilde{\mathcal{F}}_{t-h}\right]$$

  i.e., that the forecasts are optimal under quadratic loss, $L(z, \hat{z}) = (z - \hat{z})^2$

- We ran a battery of tests of forecast optimality: testing for bias, Mincer-Zarnowitz regressions, and testing for weakly increasing MSE as a function of the forecast horizon.

- Almost all of these tests failed to reject the null of optimality (unsurprising with $T = 14$) and so we proceed as though the condition is satisfied.

Bias and MZ tests of the consensus forecast

Horizon	Bias		MZ p-values	
	GDP growth	Inflation	GDP growth	Inflation
1	0.11	0.01	0.99	1.00
2	0.07	0.02	0.99	1.00
3	0.03	0.04^{**}	0.99	1.00
6	0.05	0.08^{*}	0.99	1.00
9	-0.06	0.01	0.98	0.99
12	-0.34	0.03	0.95	0.96
18	-0.06	0.29	0.99	0.98
24	-0.09	0.37^{*}	0.98	0.98

Learning and uncertainty in our model

- We assume that forecasters know the DGP *and* its parameters, thus ruling out estimation error and learning as sources of forecast errors and dispersion.
  - The primary source of uncertainty in the model concerns the future value of the variable of interest
  - When we extend to allow for measurement error, uncertainty exists also for current and past values of the variable of interest
- Modelling the forecasters' learning about the DGP and/or its parameters requires a long time series, whereas our sample is just $T = 14$ years.

Notation

$$y_t \quad : \quad \text{monthly value of variable of interest}$$

$$z_t \quad = \quad \sum_{j=0}^{11} y_{t-j} : \ \text{annual value of variable of interest}$$

$$\hat{z}_{t,t-h} \quad : \quad \text{the } \textit{consensus} \text{ forecast of } z_t \text{ made at time } t-h$$

$$\tilde{\mathcal{F}}_{t-h} \quad : \quad \text{information available at time } t-h$$

$$e_{t,t-h} \quad = \quad z_t - \hat{z}_{t,t-h} : \ \text{consensus forecast error}$$

The benchmark model

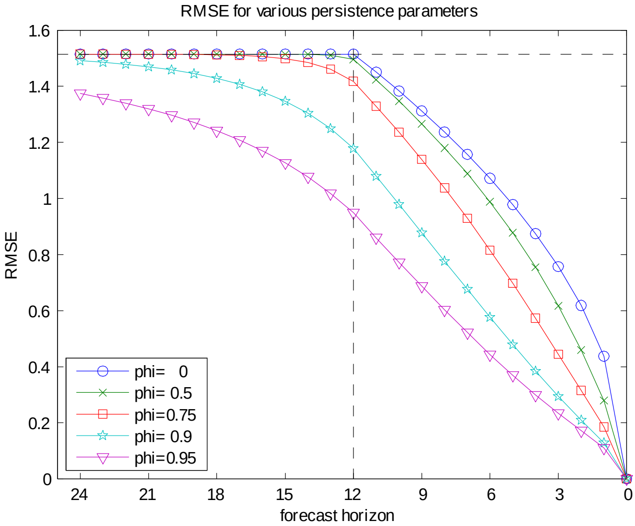
$$
\begin{aligned}
z_t &= \sum_{j=0}^{11} y_{t-j} \\
y_t &= x_t + u_t \\
x_t &= \phi x_{t-1} + \varepsilon_t, \qquad |\phi| < 1 \\
\begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} &\sim iid \left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix}\right) \\
\hat{z}_{t,t-h} &= E\left[z_t | \mathcal{F}_{t-h}\right] \\
\mathcal{F}_{t-h} &= \sigma\left(\left\{x_s, y_s\right\}_{s=1}^{t-h}\right)
\end{aligned}
$$

Optimal forecasts and MSE term structures

Prop 1 (ii): The mean squared error of the optimal forecast as a function of the forecast horizon is:

$$E\left[e_{t,t-h}^2\right] = \begin{cases} h\sigma_u^2 + \frac{1}{(1-\phi)^2}\left(h - 2\frac{\phi\left(1-\phi^h\right)}{1-\phi} + \frac{\phi^2\left(1-\phi^{2h}\right)}{1-\phi^2}\right)\ \sigma_\varepsilon^2, \\ \text{for } 1 \leq h < 12 \\ \\ 12\sigma_u^2 + \frac{1}{(1-\phi)^2}\left(12 - 2\frac{\phi\left(1-\phi^{12}\right)}{1-\phi} + \frac{\phi^2\left(1-\phi^{24}\right)}{1-\phi^2}\right)\sigma_\varepsilon^2 \\ + \frac{\phi^2\left(1-\phi^{12}\right)^2\left(1-\phi^{2h-24}\right)}{(1-\phi)^3(1+\phi)}\sigma_\varepsilon^2, \quad \text{for } h \geq 12 \end{cases}$$

RMSE term structures under the benchmark model, for various levels of persistence.



Measurement errors - a Kalman filter approach

- Our benchmark model assumed that both the predictable and unpredictable components of the target variable are perfectly observed by the forecasters.

- This implies that the squared forecast errors converge to zero as the horizon shrinks - this does not match the data.

- We need to extend the model to allow for imperfect observation of the variable(s) of interest.

  - A more realistic framework would allow both components to be measured with noise. We use the Kalman filter to handle such an approach.

A state-space model

- The state equation for this model is unchanged:

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix}$$

$$\begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} \sim iid \left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix} \right)$$

- The measurement equation is assumed to be:

$$\begin{bmatrix} \tilde{y}_t \\ \tilde{x}_t \end{bmatrix} = \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} \eta_t \\ \psi_t \end{bmatrix}$$

$$\begin{bmatrix} \eta_t \\ \psi_t \end{bmatrix} \sim iid \left(0, \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\psi^2 \end{bmatrix} \right)$$

- This is a standard state-space form and can be studied using textbook methods (see Harvey 1989 or Hamilton 1994).

Term structure of RMSE for various measurement errors

- The mean squared error of the optimal forecast in this state-space framework can be obtained analytically:
  - For $h \geq 12$ it is simply a function of the forecast errors for horizons of different lengths
  - For $h < 12$ it is a (complicated) function of forecast, "nowcast" and "backcast" errors
- While these expressions can be obtained in closed-form, they are hard to interpret directly (see technical appendix of the paper)

Term structure of RMSE for various measurement errors

Sig-psi = +infinity, sig-eta = k*sig-u for various k

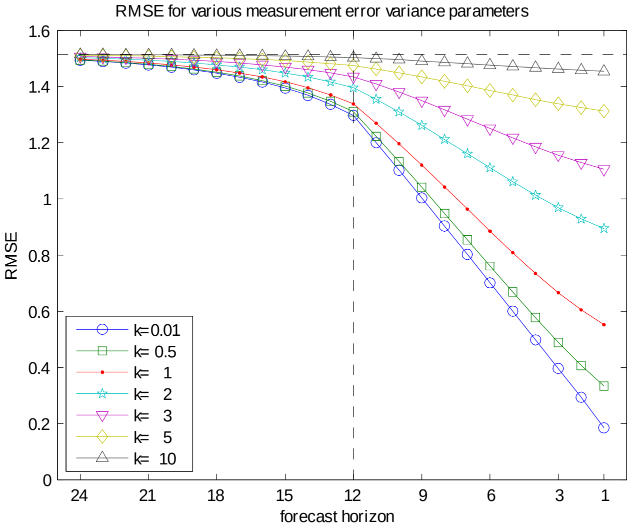


Figure:

Estimation method

- Our vector of unknown parameters for the model of the "term structure" of mean squared consensus errors is $\theta = \left[\sigma_u, \sigma_\varepsilon, \phi, \sigma_\eta, \sigma_\psi\right]'$.

- We estimate these parameters by GMM:

$$\hat{\theta}_T \equiv \arg\min_{\theta \in \Theta} \; g_T(\theta)' W_T g_T(\theta)$$

$$\text{where} \quad g_T(\theta) \equiv \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} e^2_{t,t-1} - MSE_1(\theta) \\ \vdots \\ e^2_{t,t-24} - MSE_{24}(\theta) \end{bmatrix}$$

where $MSE_h(\theta)$ is the model-implied mean-squared error for horizon h.

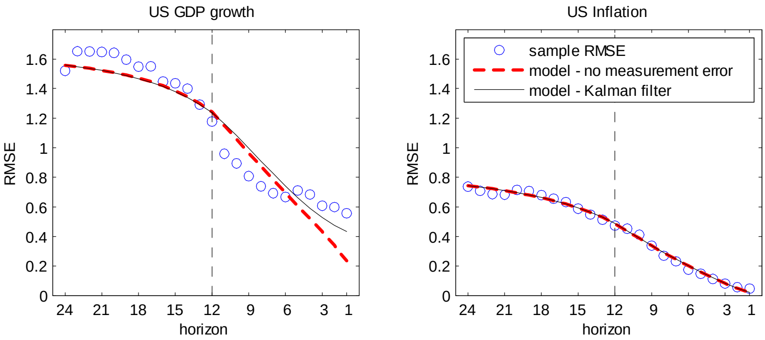
Estimation method - details

- The weight matrix used is the identity matrix: less efficient, but maintains focus on the entire term structure.
- The model-implied covariance matrix of the moments, obtained via simulation of 10,000 non-overlapping years of data, is used to compute standard errors and the test of over-identifying restrictions.
- After some experimentation we fixed $\sigma_\eta = 2\sigma_u$ and set $\sigma_\psi \rightarrow \infty$ to improve the identification of the model.
- We initially used all 24 horizons for estimation, but in light of finite-sample studies of GMM estimators, we settled on estimating the model with just six forecast horizons: $h = 1, 3, 6, 12, 18, 24$.

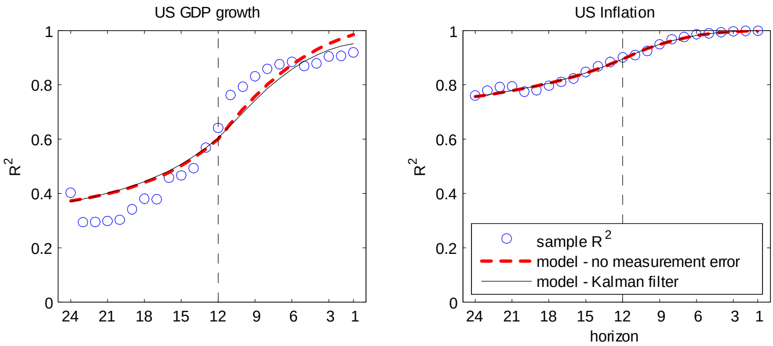
GMM parameter estimates: consensus model

	σ_u	σ_ε	ϕ	J p-val
GDP growth	0.06 (0.01)	0.05 (0.01)	0.94 (0.03)	0.40
Inflation	0.00 (−−)	0.02 (0.01)	0.95 (0.05)	0.94

Term structures of RMSE for GDP growth and inflation.



Term structures of R2 for GDP growth and inflation.



Extracting the components of GDP and Inflation

- Our simple framework for the monthly data generating process:

$$\xi_t \equiv \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \phi \\ 0 & \phi \end{bmatrix}}_{\equiv F} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_t + \varepsilon_t \\ \varepsilon_t \end{bmatrix}$$

$$\mathbf{w}_t \equiv \sum_{j=0}^{11} \xi_{t-j} = \begin{bmatrix} \sum_{j=0}^{11} y_{t-j} \\ \sum_{j=0}^{11} x_{t-j} \end{bmatrix}$$

Extracting the components of GDP and Inflation

- Optimal forecasts in this case are:

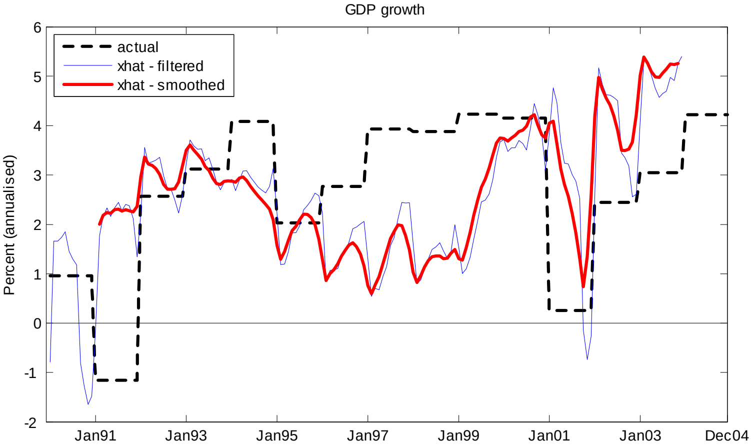
$$E\left[\mathbf{w}_t|\tilde{\mathcal{F}}_{t-h}\right] = \left(\sum_{j=0}^{11} F^{h-j}\right) E\left[\boldsymbol{\xi}_{t-h}|\tilde{\mathcal{F}}_{t-h}\right], \quad h \geq 12$$

$$E\left[\mathbf{w}_t|\tilde{\mathcal{F}}_{t-h}\right] = \sum_{j=h+1}^{11} E\left[\boldsymbol{\xi}_{t-j}|\tilde{\mathcal{F}}_{t-h}\right] + \left(\sum_{j=0}^{h} F^{h-j}\right) E\left[\boldsymbol{\xi}_{t-h}|\tilde{\mathcal{F}}_{t-h}\right],$$

- So optimal forecasts are a function of the "nowcast" and "backcast" values of w_t. Given an estimate of the parameters defining the DGP, it is possible to extract filtered and smoothed estimates of the persistent (x_t) and transitory (u_t) components of the variable of interest.

Forecasters' implied persistent component of GDP growth

Recall y[t] = x[t] + u[t] (total = persistent + transitory)



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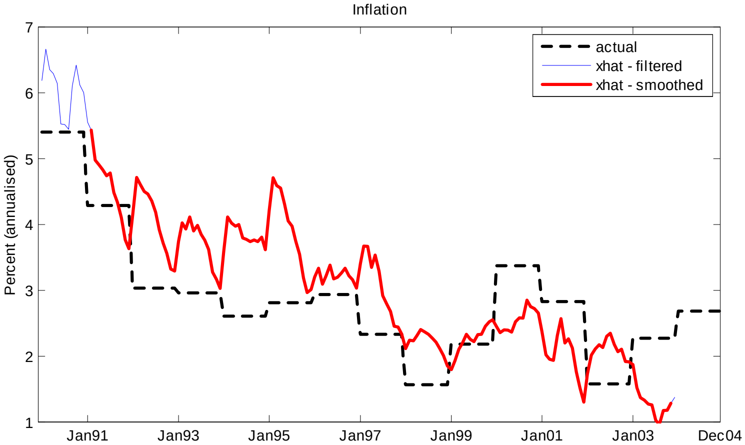

Summary of findings for the RMSE term structure

- A simple model based on a persistent and a transitory decomposition of the target variable fits the data well

  1. A simple AR(1) model for the persistent component was sufficient, though our framework could be extended to handle more general AR(p) or multiple component AR(p_i) models
  2. Allowing for measurement error is important for GDP growth, though not for inflation. This is consistent with the "real time data" literature in macroeconomics, see Croushore and Stark (2001) for example.
  3. No "seasonal" component was needed (quarterly releases of figures, monthly announcements, etc): information appears to be smoothly incorporated into consensus forecasts.

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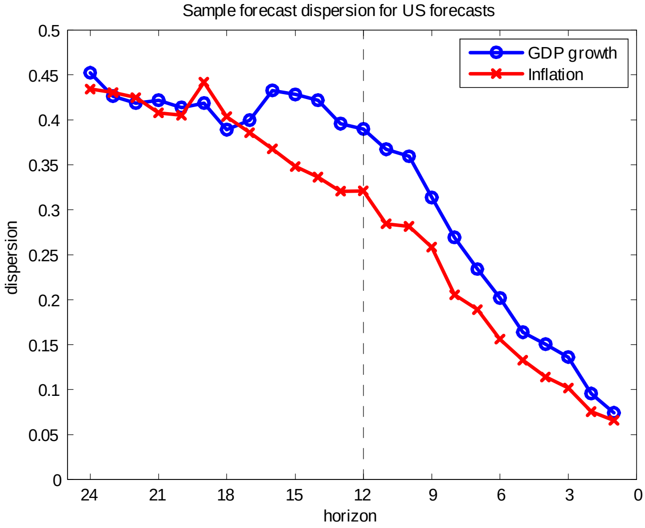
Dispersion among forecasters

- Our second variable of interest is the degree to which agents disagree about the expected value of the target variable. We measure this by

$$d^2_{t,t-h} \equiv \frac{1}{N_{t,t-h}} \sum_{i=1}^{N_{t,t-h}} \left(\hat{z}_{i,t,t-h} - \bar{z}_{t,t-h}\right)^2$$

Cross-sectional forecast dispersion as a function of the forecast horizon: GDP growth and Inflation



A model for dispersion

- The first source of dispersion is differences in signals:

$$\begin{aligned} \tilde{y}_{i,t} &= y_t + \eta_t + \nu_{i,t} \\ \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix} &\sim iid\left(0, \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\nu^2 \end{bmatrix}\right) \end{aligned}$$

But as the forecast survey results are published (with a lag) we also assume that the forecaster's time t information set includes:

$$\tilde{y}_{t-1} = y_{t-1} + \eta_{t-1}$$

From these two measurement variables, the individual forecaster computes the optimal forecast using the Kalman filter:

$$\hat{z}^*_{i,t,t-h} \equiv E\left[z_t | \tilde{\mathcal{F}}_{i,t-h}\right]$$

Differences in signals & dispersion as h grows

- Allowing for differences in signals is a natural starting point for capturing dispersion, but it has an important counter-factual implication:

- As $h \to \infty$, the value of any signal for predicting the target variable goes to zero, and so all forecasts converge to the unconditional mean:

$$\hat{z}^{*}_{i,t,t-h} \equiv E\left[z_t | \tilde{\mathcal{F}}_{i,t-h}\right] \to E\left[z_t\right] \quad \text{as } h \to \infty$$

$$\text{so } d^2_{t,t-h} \equiv \frac{1}{N_{t,t-h}} \sum_{i=1}^{N_{t,t-h}} \left(\hat{z}_{i,t,t-h} - \bar{z}_{t,t-h}\right)^2 \to 0 \ \text{ as } h \to \infty$$

- Yet we saw that $d^2_{t,t-h} \to \bar{\delta} > 0$ as $h \to \infty$ for both variables in our sample. So there must be another source of dispersion.

Differences in beliefs about long-run values

- One simple way of allowing for dispersion at long horizons is to allow the forecasters to have differing beliefs about the long-run average values of GDP growth and inflation.
- This approach is a special case of allowing for different subjective probability densities across forecasters, see Pesaran and Weale (2005).
- These differences can be motivated in a number of ways:
  - Different models for equilibrium rates of GDP growth and inflation

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    - Different samples of data, based on different beliefs about previous structural breaks, etc.

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  - Different models for equilibrium rates of GDP growth and inflation
  - Different samples of data, based on different beliefs about previous structural breaks, etc.
  - Different Bayesian priors, which affect the forecasts issued.
  - The outcome of a "game" played between forecasters, see Laster, *et al.* (1999) or Ottaviani and Sørensen (2006) for example

A model for differences in beliefs

- Forecaster i's prior belief about the average value of z_t is denoted μ_i.
- We assume that forecaster i "shrinks" the Kalman filter forecast towards his prior belief about the unconditional mean of z_t. The degree of shrinkage is governed by the parameter $\kappa^2 \geq 0$

$$\begin{aligned}
\hat{z}_{i,t-h,t} &= \omega_h \mu_i + (1-\omega_h) E\left[z_t | \tilde{\mathcal{F}}_{i,t-h}\right] \\
\text{where} \quad \omega_h &= \frac{E\left[e^{*2}_{i,t,t-h}\right]}{\kappa^2 + E\left[e^{*2}_{i,t,t-h}\right]} \\
e^{*}_{i,t-h,t} &\equiv z_t - E\left[z_t | \tilde{\mathcal{F}}_{i,t-h}\right]
\end{aligned}$$

The degree of "shrinkage"

- The weights, ω_h, placed on the prior vary across h in a manner consistent with standard forecast combinations: as the Kalman filter forecast becomes more accurate the weight attached to that forecast increases.
- For short horizons, $h \longrightarrow 0$, the weight attached to the prior falls, while for long horizons the weight attached to the prior grows.
- For analytical tractability, and for better finite sample identification of κ^2, we impose that κ^2 is constant across all forecasters.

Model-implied dispersion

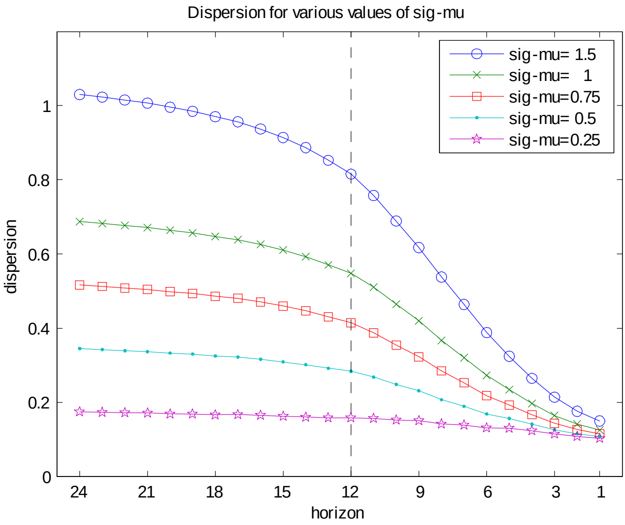
- We normalise $\bar{\mu} = 0$ since we cannot separately identify $\bar{\mu}$ and σ_μ^2 from our data. This is reasonable if the number of "optimistic" forecasters is approximately equal to the number of "pessimistic" forecasters.
- Our model for dispersion is the unconditional expectation of $d_{t,t-h}^2$. We allow for a heteroskedastic residual term for our model of dispersion, with variance related to the level of the dispersion.

$$d_{t,t-h}^2 = \delta_h^2(\theta) \cdot \lambda_{t,t-h}$$

$$\text{where} \qquad \begin{aligned} \delta_h^2(\theta) &\equiv E\left[d_{t,t-h}^2\right] \\ E\left[\lambda_{t,t-h}\right] &= 1 \\ V\left[\lambda_{t,t-h}\right] &= \sigma_\lambda^2. \end{aligned}$$

Model-implied dispersion as a function of Sig-mu

With Sig-nu=2*Sig-u, kappa=0.5*Sig-z, Sig-eta = 2*Sig-u.



Estimation method

- Our vector of parameters for the model of the mean squared consensus errors and dispersion is $\theta = \left[\sigma_u, \sigma_\varepsilon, \phi, \sigma_\nu, \sigma_\mu, \kappa, \sigma_\lambda\right]'$. We estimate these parameters by SMM:

$$\hat{\theta}_T \equiv \arg\min_{\theta \in \Theta} g_T(\theta)' W_T g_T(\theta)$$

$$\text{where} \quad g_T(\theta) \equiv \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} e_{t,t-1}^2 - MSE_1(\theta) \\ \vdots \\ d_{t,t-1}^2 - \delta_1^2(\theta) \\ \vdots \\ \left(d_{t,t-1}^2 - \delta_1^2(\theta)\right)^2 - \sigma_{\delta,h}^2(\theta) \\ \vdots \end{bmatrix}$$

where $MSE_h(\theta)$, $\delta_h^2(\theta)$ and $\sigma_{\delta,h}^2(\theta)$ are the model-implied MSE, dispersion, and dispersion variance.

Estimation method - details

- The value of $\delta_h^2(\theta)$ cannot be obtained in closed-form; we simulate $N = 30$ individual forecasters for $T = 600$ months of non-overlapping data.

- The weight matrix used is again the identity matrix, so all horizons of RMSE and dispersion (and dispersion variance) get equal weight in the estimation.

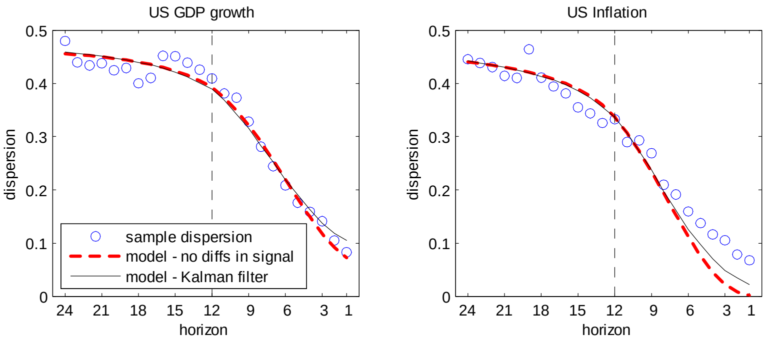
- Standard errors and the test of over-identifying restrictions are again based on the model-implied covariance matrix of the moments.

- We again estimated the model with just six forecast horizons: $h = 1, 3, 6, 12, 18, 24$.

GMM parameter estimates: consensus and dispersion model

	σ_u	σ_ε	ϕ	σ_ν	σ_μ	κ	J p-val
GDP growth	0.06 (0.01)	0.05 (0.01)	0.94 (0.03)	0.69 (1.00)	0.67 (0.39)	1.41 (0.94)	0.86
Inflation	0.00 (−−)	0.02 (0.01)	0.95 (0.05)	0.05 (0.17)	0.51 (0.13)	0.49 (0.17)	0.00

Term structures of dispersion for GDP growth and inflation in the US.



Interpreting the results from the dispersion model

- The model for inflation dispersion fits well for $h \geq 12$, but for short horizons it systematically under-estimates dispersion
  - why is observed inflation dispersion is so high at short horizons?

	GDP growth			Inflation		
h	RMSE	DISP	*ratio*	RMSE	DISP	*ratio*
1	0.56	0.08	*0.15*	0.05	0.07	*1.45*
3	0.61	0.14	*0.23*	0.08	0.11	*1.29*
6	0.67	0.21	*0.31*	0.17	0.16	*0.91*
12	1.18	0.41	*0.35*	0.47	0.33	*0.70*
24	1.52	0.48	*0.32*	0.74	0.45	*0.60*

A model for time-varying dispersion

- There is a growing body of empirical and theoretical work on the relationship between uncertainty (somehow defined) and the economic environment.

- We use the default spread on corporate bonds for this purpose - this is known to be a strongly counter-cyclical variable.

- We model increased uncertainty as an increase in the differences in beliefs about long-run values

$$\log \sigma^2_{\mu,t} = \beta^{\mu}_0 + \beta^{\mu}_1 \log S_t$$

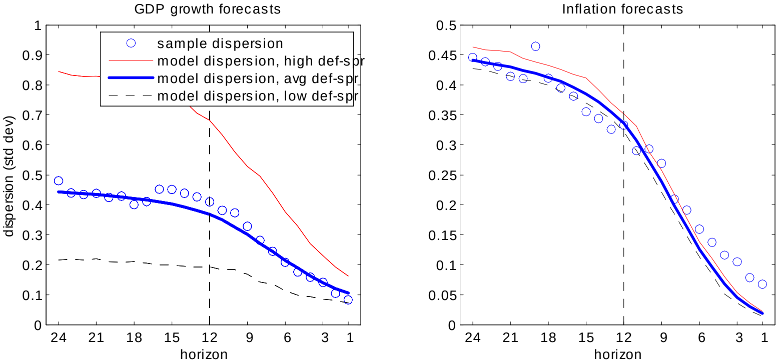
- A positive coefficient on $\log S_t$ will imply that higher default spreads coincide with greater disagreement about long-run values of the series, generating higher dispersion.

GMM parameter estimates: consensus and time-varying dispersion model

	σ_u	σ_ε	ϕ	σ_ν	κ	β_0^μ	β_1^μ	J p-val
GDP growth	0.06 (0.01)	0.05 (0.01)	0.94 (0.03)	0.69 (0.87)	1.41 (0.74)	-0.56 (1.10)	**3.08** (1.83)	0.91
Inflation	0.00 (−−)	0.02 (0.01)	0.95 (0.05)	0.05 (0.17)	0.49 (0.17)	-1.32 (0.55)	0.18 (2.37)	0.00

Term structures of dispersion for GDP growth and inflation in the US.

Conditional on the level of the default spread



Summary of results: the RMSE term structure

- Main finding: A simple model of GDP and inflation dynamics is sufficient to accurately describe the complete term structure of consensus mean squared forecast errors.
  - The estimated persistence parameters are consistent with estimates obtained using lower-frequency data.
  - Measurement error is an important part of the model for GDP growth, though not for inflation
  - The forecasters in this panel appear to have taken several years to adjust their forecasts to reflect the higher GDP growth and lower inflation figures during the 1990s

Summary of results: the dispersion term structure

- Differences in prior beliefs about long-run averages explain almost all of the observed dispersion for both GDP growth and inflation forecasts.
- Important differences in the properties of forecast dispersion for these two variables emerged:
  - The GDP forecast dispersion fit the data well, while the inflation dispersion model failed at short horizons
  - Inflation dispersion is high relative to the RMSE of the consensus forecast, while GDP dispersion is only a fraction of the RMSE
  - Dispersion in GDP forecasts varies counter-cyclically in a significant fashion, while inflation forecast dispersion appears unrelated to the business cycle

Extensions

1. Formally model learning by forecasters: allow them to update estimates of parameters rather than assume them known.
2. Consider the impact of other non-stationarities in the data: structural breaks in GDP variance or the level of inflation, for example.
3. Combine our panel of forecasts with samples of data on the target variables over a longer sample but at a lower frequency.

The form of the target variable

- Our analysis in the previous sections take the target variable, z_t, as the December-on-December change in the log-level of US real GDP or the Consumer Price Index

- In the Consensus Economics survey, however, the target variables are defined as:

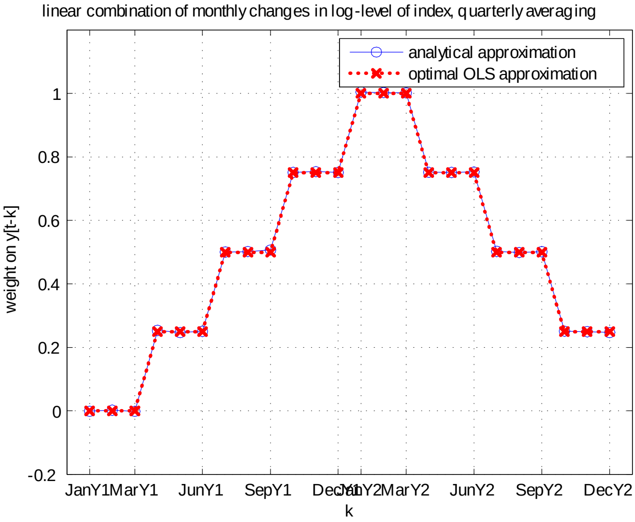
$$
\begin{aligned}
z_t^{GDP} &\equiv \frac{\bar{P}_t}{\bar{P}_{t-1}} - 1 \\
z_t^{INF} &\equiv \frac{\overline{\overline{P}}_t}{\overline{\overline{P}}_{t-1}} - 1, \\
\textit{where} \quad \bar{P}_t &\equiv \frac{1}{4}\sum_{j=0}^{3} P_{t-3j}^{GDP} \quad , \quad \text{and} \quad \overline{\overline{P}}_t \equiv \frac{1}{12}\sum_{j=0}^{11} P_{t-j}^{INF}
\end{aligned}
$$

The form of the target variable, cont'd

- For reasonable values of y_t, the variables z_t^{GDP} and z_t^{INF} can be shown to be accurately approximated as a linear combination of $(y_t, y_{t-1}, ..., y_{t-23})$:

$$
\begin{aligned}
z_t(\mathbf{w}) &\equiv \sum_{j=0}^{23} w_j y_{t-j} \\
w_j^{GDP} &= \begin{cases} \frac{1+\left\lfloor \frac{j}{3} \right\rfloor}{4}, & 0 \leq j \leq 11 \\ \frac{3-\left\lfloor \frac{j-12}{3} \right\rfloor}{4}, & 12 \leq j \leq 23 \end{cases}, \quad j = 0, 1, ..23 \\
w_j^{INF} &= \begin{cases} \frac{j+1}{12}, & 0 \leq j \leq 11 \\ \frac{23-j}{12}, & 12 \leq j \leq 23 \end{cases}, \quad j = 0, 1, .., 23 \\
w_j^{*} &= \begin{cases} 1, & 0 \leq j \leq 11 \\ 0, & 12 \leq j \leq 23 \end{cases}, \quad j = 0, 1, ..23
\end{aligned}
$$

The weights for GDP growth



The weights for inflation