Time-Varying Systemic Risk: Evidence from a Dynamic Copula Model of CDS Spreads

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Motivation

- "**Systemic risk**" is broadly defined as the risk of a crash in a large number of firms. It is an "extreme event" in two directions:

  1. A **large loss** (ie, a left-tail realization for stock returns)
  2. Across a **large proportion** of firms under analysis

- There are a variety of methods for studying risk and dependence for small collections of assets, but a relative paucity of models of dependence for large collections of assets

  - There is a growing literature on models for **large covariance matrices** (eg, Engle and Kelly, 2008, Engle, Shephard and Sheppard, 2008, Hautsch, Kyj and Oomen, 2010)
  - We propose a new **high dimension copula-based model** that builds on this literature

Main contributions of this paper

1. A flexible, simple, class of **dynamic factor copula** models that can be applied in high dimensional problems.
   - Closed-form expression for these models not generally available, but analytical results on tail dependence available using EVT
   - A "variance targeting" method makes high dimension application feasible
2. An application to **CDS spreads on 100 US firms** with focus on systemic risk:
   - We find significant evidence of tail dependence, asymmetric dependence, and heterogeneous dependence.
   - We find that the risk of systemic distress has increased since the financial crisis

Outline

1. Introduction
2. **Dynamic high dimension copula models**
   - Factor copulas
   - "GAS" dynamics
   - Simulation results
3. Measuring time-varying systemic risk using CDS data
   - Description of CDS spread data
   - Models for the joint distribution of CDS spreads
   - Estimates of systemic risk
4. Conclusion

Copula-based models for economic dependence I

- **Time-varying, low dimension, copulas:**
  - *GARCH-type:* Patton (2006, IER), Jondeau and Rockinger (2006, JIMF), Creal, *et al.* (2011, JBES), Christoffersen, *et al.* (2012, RFS)
  - *Stochastic Vol-type:* Hafner and Manner (2012, JAE)
  - *Regime switching:* Rodriguez (2007, JEF), Okimoto (2008, JFQA), Garcia and Tsafack (2009, JBF)
- **High dimension, constant, copulas:**
  - Normal, Student's t, etc.
  - *Vine copulas:* Aas *et al.* (2007, IME), Kurowicka, and Joe (2011, book), Acar, *et al.* (2012, JMVA)
  - *Nested Archimedean:* Hofert and Scherer (2011, QF), Joe (1997, book), McNeil, *et al.* (2005, book)
  - *Factor copulas:* Oh and Patton (2014, JBES)

Copula-based models for economic dependence II

- **Time-varying and high (N≥10) dimension copulas:**

Authors	**N**	**Copula**	**Dynamics**	**Estim**
Zhang, *et al.* (2011, wp)	10	Skew *t*	GAS	ML
Christoffersen, *et al.* (2012, RFS)	33	Skew *t*	DCC	ML
Almeida, *et al.* (2012, wp)	30	Vine	SV	SML
Stöber and Czado (2012, wp)	10	Vine	RS	Bayes
Christoffersen, *et al.* (2013, wp)	233	Skew *t*	DCC	CML
This paper	**100**	**Factor**	**GAS**	**ML**

A simple factor copula model

- Consider a vector of n variables, $\mathbf{Y}$, with some joint distribution $\mathbf{F}^*$, marginal distributions F_i^*, and copula $\mathbf{C}^*$

$$[Y_1, ..., Y_N]' \equiv \mathbf{Y} \sim \mathbf{F}^* = \mathbf{C}^* (F_1^*, ..., F_N^*)$$

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- Oh and Patton (2015, *JBES*) propose a model for $\mathbf{C}^*$ as the copula $\mathbf{C}(\boldsymbol{\theta})$ implied by the following model:

$$\begin{aligned} \text{Let} \quad X_i &= \lambda_i Z + \varepsilon_i, \quad i = 1, 2, ..., N \\ Z &\sim F_z(\boldsymbol{\theta}), \quad \varepsilon_i \sim iid\ F_\varepsilon(\boldsymbol{\theta}), \quad Z \perp\!\!\!\perp \varepsilon_i \ \forall\ i \\ \text{So} \quad [X_1, ..., X_N]' &\equiv \mathbf{X} \sim \mathbf{F}_x(\boldsymbol{\theta}) = \mathbf{C}\left(G_1(\boldsymbol{\theta}), ..., G_N(\boldsymbol{\theta}); \boldsymbol{\theta} \right) \end{aligned}$$

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- In general we won't know $\mathbf{C}(\boldsymbol{\theta})$ in closed form, but we can nevertheless use it as a model for the true copula $\mathbf{C}^*$.

Illustration of some factor copulas

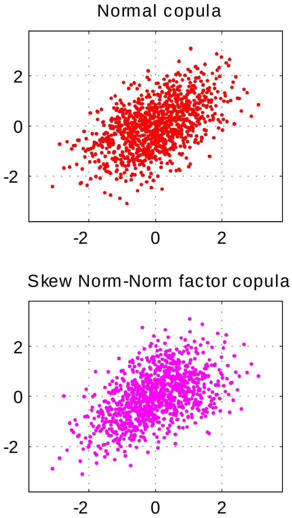
- Consider the following factor structure:

$$\begin{aligned} \text{Let} \quad X_i &= \lambda Z + \varepsilon_i, \quad i = 1, 2, ..., N \\ \varepsilon_i &\sim iid \;\; t(\nu), \quad Z \perp\!\!\!\perp \varepsilon_i \; \forall \; i \\ Z &\sim Skew \;\; t(\nu, \psi) \\ \nu &\in [2, \infty], \quad \psi \in [-0.99, 0] \end{aligned}$$

- We set $\lambda = 1$ so that the factor copula implied by this structure generates linear correlation of 0.5.
- We will first consider some **bivariate** distributions with this structure, and then some **high dimension** distributions.

Scatterplots of joint distributions with factor copulas

Marginal distributions are N(0,1), linear correlation = 0.5.



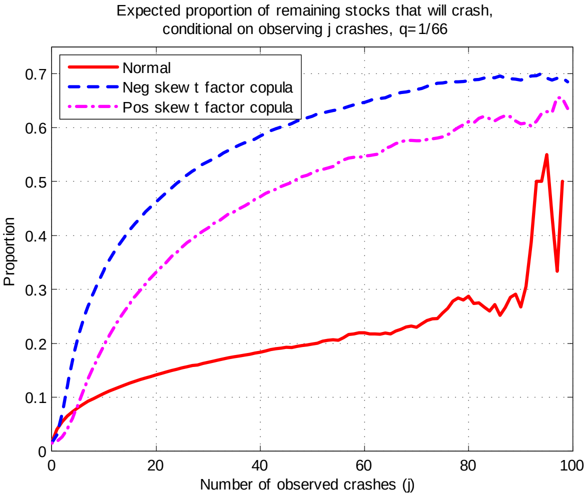
"Crash" dependence

- **Crash dependence** (similar to Embrechts, *et al.*, 2000): Conditional on j variables being in their q tails, what is expected proportion of remaining variables that are in their q tails?

$$\begin{aligned}
\pi_j^q &\equiv \frac{\kappa_j^q}{N-j} \\
\text{where} \quad \kappa_j^q &= E\left[N_q^* \middle| N_q^* \geq j\right] - j \\
N_q^* &\equiv \textstyle\sum_{i=1}^N \mathbf{1}\left\{U_i \leq q\right\}
\end{aligned}$$

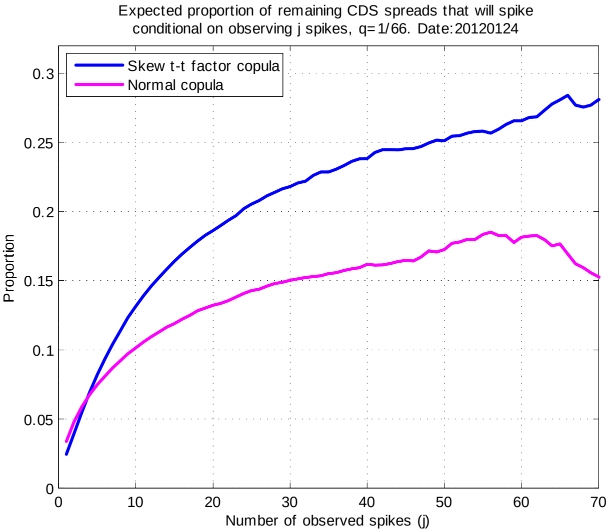
Proportion of remaining stocks that will crash

"Crash" defined as a 1/66 event = once in a quarter for daily asset returns



Proportion of remaining CDS spreads that will spike

"Spike" defined as a 1/66 event = once in a quarter for daily CDS spreads



Outline

Copulas and the probability integral transform

- Two useful results on copulas and transformations of continuous random variables:

1. If $Y_i \sim F_i$, then $U_i \equiv F_i\left(Y_i\right) \sim Unif\left(0,1\right)$
2. If $\left[Y_1, ..., Y_N\right]' \sim \mathbf{F} = \mathbf{C}\left(F_1, ..., F_N\right)$, then $\left[U_1, ..., U_N\right]' \sim \mathbf{C}$

Time-varying copulas with GAS dynamics

- We will model dynamics using the "generalized autoregressive score" or **GAS model** of Creal, Koopman and Lucas (2011, *JAE*).
- This approach models the parameters of the copula as a function of the lagged parameters and the score of the copula likelihood:

$$\text{Let} \quad \mathbf{U}_t|\mathcal{F}_{t-1} \sim \mathbf{C}(\boldsymbol{\kappa}_\mathbf{t})$$

$$\begin{aligned} \text{where} \quad \boldsymbol{\kappa}_{\mathbf{t+1}} &= \omega + \beta \boldsymbol{\kappa}_\mathbf{t} + \alpha \mathbf{s}_\mathbf{t} \\ \mathbf{s}_\mathbf{t} &= S_t \cdot \Delta_t \\ \Delta_t &= \frac{\partial \log \mathbf{c}(\mathbf{u}_t; \boldsymbol{\kappa}_\mathbf{t})}{\partial \boldsymbol{\kappa}_\mathbf{t}} \end{aligned}$$

- A key benefit of this approach is that the "forcing variable" in the model for κ_{t+1} is provided directly by the choice of copula model

Factor copulas with GAS dynamics

- We use the GAS model to capture time-varying dependence by letting the **loadings on the common factor** change through time
- Similar to Engle's DCC model, we impose that α and β are **common** across firms, and allow only the "intercept" parameters to differ
- We also impose that the shape parameters $(\nu_z, \nu_\varepsilon, \psi)$ are **constant**:

$$X_{it} = \lambda_{it} Z_t + \varepsilon_{it}, \quad i = 1, 2, ..., 100$$

$$Z_t \sim Skew\ \ t\left(\nu_z, \psi\right), \ \ \varepsilon_{it} \sim iid\ \ t(\nu_\varepsilon), \quad Z \perp\!\!\!\perp \varepsilon_i \ \forall\ i$$

$$\log \lambda_{it} = \omega_i + \beta \log \lambda_{i,t-1} + \alpha \frac{\partial \log c\left(\mathbf{u}_{t-1} | \boldsymbol{\lambda}_{t-1}, \nu_z, \psi, \nu_\varepsilon\right)}{\partial \lambda_i}$$

Flexible GAS dynamics and "Variance Targeting"

$$X_{it} = \lambda_{it} Z_t + \varepsilon_{it}, \qquad i = 1, 2, ..., 100$$

$$\log \lambda_{it} = \omega_i + \beta \log \lambda_{i,t-1} + \alpha \frac{\partial \log c \left(\mathbf{u}_{t-1} | \boldsymbol{\lambda}_{t-1}, \nu_z, \psi, \nu_\varepsilon \right)}{\partial \lambda_i},$$

- We do not want to numerically estimate ≈ 100 parameters (ω_i)
  - We use a **variance targeting**–type approach
  - We obtain quasi-closed form estimates of the intercept parameters, ω_i, based on sample rank correlations, $\bar{\rho}_{ij}$:

$$\bar{\rho}_{ij} = g\left(\omega_i, \omega_j, \nu_z, \psi, \nu_\varepsilon\right) \Leftrightarrow \omega_i = g^{-1}\left(\bar{\rho}_{ij}, \nu_z, \psi, \nu_\varepsilon\right)$$

  - We then numerically optimize over only $(\alpha, \beta, \nu_z, \psi, \nu_\varepsilon)$

Parsimony / flexibility and factor copulas

- Factor copula with GAS dynamics:

$$X_{it} = \lambda_{g(i),t} Z_t + \varepsilon_{it}, \qquad i = 1, 2, ..., 100$$
$$Z_t \sim Skew\ \ t\left(\nu_z, \psi\right), \ \ \varepsilon_{it} \sim iid\ \ t(\nu_\varepsilon), \ \ Z \perp\!\!\!\perp \varepsilon_i\ \forall\ i$$

$$\log \lambda_{gt} = \omega_g + \beta \log \lambda_{g,t-1} + \alpha \frac{\partial \log c\left(\mathbf{u}_{t-1} | \boldsymbol{\lambda}_{t-1}, \nu_z, \psi, \nu_\varepsilon\right)}{\partial \lambda_g},$$
$$g = 1, 2..., G$$

1. **Equidependence** model: $G = 1$
2. **Block equidependence** model: $G = 5$ (according to industry groups)
3. **Fully flexible**: $G = 100$

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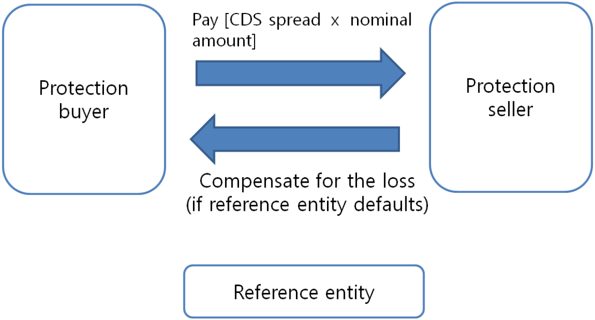
Credit default swaps (CDS)

- A CDS written on firm i (the "reference entity") at date t is a contract in which the buyer agrees to make periodic payments (determined by CDS spread) to the seller until the contract matures (at $t+T$) or a default occurs, whichever happens first.
- If a default occurs before maturity $t+T$, the seller compensates the buyer for the realized credit loss



CDS and implied probabilities of default

- Under some simplifying assumptions (see Carr and Wu, RFS, for eg) it is possible to show that a CDS spread (S_{it}) is given by:

$$S_{it} = P_{it}^{\mathbb{Q}} \times LGD_{it} = P_{it}^{\mathbb{P}} \times \mathcal{M}_{it} \times LGD_{it}$$

  where $P_{it}^{\mathbb{Q}}$ and $P_{it}^{\mathbb{P}}$ are the implied and objective probabilities of default, $\mathcal{M}_{it}$ is the market price of risk, and LGD_{it} is the loss-given-default.

  - This simple expression can also be obtained as a first-order approximation of more complicated formulas when $P_{it}^{\mathbb{Q}} \approx 0$.

- We work with the log-difference of the CDS spread, which yields:

$$\Delta \log S_{it} = \Delta \log P_{it}^{\mathbb{P}} + \Delta \log \mathcal{M}_{it} + \Delta \log LGD_{it}$$

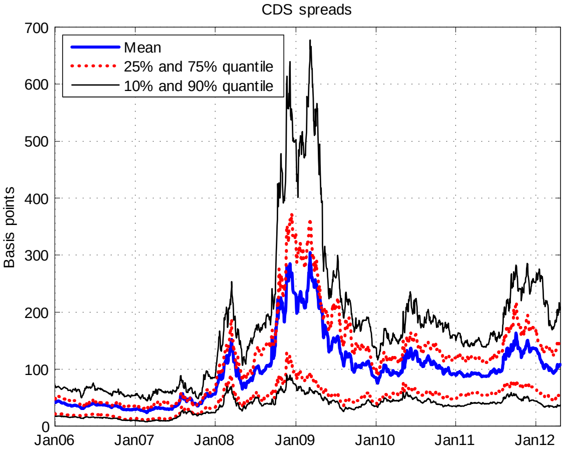
Measures of systemic risk

- Measures of "systemic risk" in financial markets:
  - Adrian and Brunnermeier (2009): **CoVaR**≈quantile of market returns conditional on firm *i* stress
  - Brownlees and Engle (2011): **Marginal Expected Shortfall**≈expected return on firm *i* conditional on market stress
  - Huang, Zhou and Zhu (2009): **price of insurance against system-wide losses**
- Our proposed measure is related to the above:
  - We use CDS spreads to measure individual firm "distress", and then estimate the expected number of firms **simultaneously distressed** given firm *i* in distress
  - It is the "simultaneous" aspect that makes this measure "systemic", and which requires the specification of a model for dependence.

Description of the data

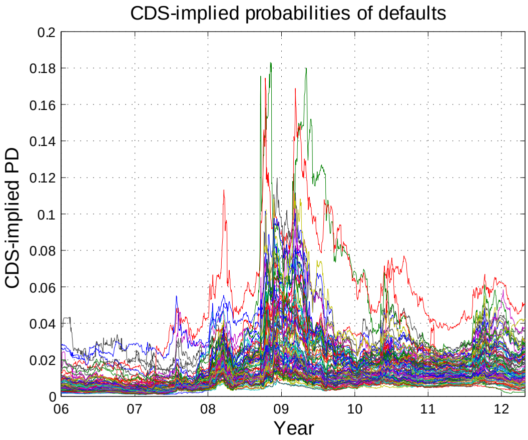
- We use daily CDS spreads for single reference entities from Markit Corporation
- We restrict our attention to 5-yr maturity CDS contracts for U.S. corporations in U.S. dollars for senior subordinated debt
  - This is the most liquid of the CDS contracts
- Use 100 CDS spreads with limited missing data among 125 constituents of a CDS index (North America CDX series 17)
- Our sample period is Jan 2006 – Apr 2012, so T=1644 and N=100

Time series of CDS spreads



CDS-implied probabilities of default

Avg PD = 1.1% = ~A- grade. PD of ~5% is investment/speculative grade cutoff



Summary statistics: CDS spreads

Cross-sectional distribution of individual summary statistics

	Mean	5%	25%	Median	75%	95%
Mean	**97.0**	37.2	53.6	75.0	123.8	200.3
Std dev	70.0	17.3	27.2	47.5	84.3	180.6
Skewness	1.2	0.1	0.7	1.3	1.6	2.5
Kurtosis	5.1	2.2	2.9	4.9	6.5	9.5
5%	23.9	9.0	11.7	18.9	29.9	60.5
25%	42.3	20.4	25.2	35.3	47.5	104.7
Median	85.3	35.1	50.1	**69.4**	113.8	166.2
75%	122.1	46.3	65.9	93.6	154.7	251.1
95%	245.5	72.5	102.6	168.5	313.6	631.9
99%	**338.7**	80.4	122.9	231.3	435.2	827.1

Summary statistics: Log-diff of CDS spreads

Cross-sectional distribution of individual summary statistics

	Mean	5%	25%	Median	75%	95%
Mean	5.6	-1.6	2.6	5.5	8.5	13.8
Std dev	378.9	308.6	347.6	373.5	400.4	476.5
Skewness	1.1	-0.3	0.4	0.8	1.5	3.6
Kurtosis	25.5	7.7	10.3	14.6	25.9	74.8
5%	-514.6	-622.3	-551.3	-509.6	-474.0	-415.7
25%	-144.2	-172.3	-155.6	-145.4	-134.8	-112.0
Median	-2.3	-9.0	-3.6	-0.7	0.0	0.0
75%	132.1	95.2	120.5	131.0	144.4	174.6
95%	570.5	457.8	537.1	568.3	612.8	685.0

CDS data: Industry groups

- Markit Corporation classifies single entities into 5 groups: Consumer, Manufacturing, Finance, Energy, and Telecom
- We use this classification to group firms for one of our specifications:

Group	**Count**
Consumer	34
Manufacturing	21
Finance	16
Energy	12
Telecom	17
Total	100

The CDS "Big Bang"

- With the growth of the CDS market through the 2000s, participants wanted more homogeneous contracts to increase liquidity
- On April 8, 2009, the North American CDS market underwent changes to contract conventions
  - CDS coupons were fixed to be 100 or 500 bp, with upfront payments adjusted accordingly
  - More rigid rules on triggers for "credit events" and auctions that follow such events
  - Move towards central clearing, away from OTC trading
- These changes could potentially change the dynamics of CDS spreads, and we test for these using a simple structural break test:
  - We have 591 pre-break obs, 1053 post-break obs

Did the CDS "Big Bang" cause a structural break?

- **Conditional mean:** we test for changes in all parameters jointly, and find significant changes for 39 firms
- **Conditional variance:** Controlling for changes in the mean, we find breaks in the variance for 66 firms
  - 28 firms have a significant break in both mean and variance
- Thus structural breaks appear to be important for models of CDS spreads for many firms – we allow for these breaks in our analysis below.

Dynamic copulas and the heterogeneous dependence

	Equidep		Block equidep		Flexible	
	Normal	*Factor*	*Normal*	*Factor*	*Normal*	*Factor*
$\omega_{1\to G}$						
α	0.0216	0.0263	0.0293	0.0260	0.1435	0.1714
β	0.8474	0.9072	0.9758	0.9919	0.9753	0.9819
ν	-	12.8236	-	95.6159	-	50.9100
ν_ε	-	5.6297	-	5.2700	-	5.5892
ψ	-	-0.0146	-	0.0932	-	0.1236
$\log L$	38395	40983	38519	41165	39361	**41913**
Rank	6	3	5	2	4	1

Model comparison tests I

- **Hypothesis 1:** *Normal copula as good as Factor copula*

		log L	Diff	Num restrictions	p-value†
Equidep-Static	*Normal*	36185			
	Factor	39508	3322	3	0.000
Equidep-GAS	*Normal*	38395			
	Factor	40983	2588	3	0.000
Block-GAS	*Normal*	38518			
	Factor	41165	2647	3	0.000
Flexible-GAS	*Normal*	39361			
	Factor	41913	2552	3	0.000

★ Factor copula significantly better than Normal copula

Model comparison tests II

- **Hypothesis 2:** *Equidependence as good as Block equidependence*

		log L	Diff	Num restrictions	p-value
Normal-Static	*Equidep*	36185			
	Block	36477	292	4	0.000
Factor-Static	*Equidep*	39508			
	Block	39757	249	4	0.000
Normal-GAS	*Equidep*	38395			
	Block	38518	123	4	0.000
Factor-GAS	*Equidep*	40983			
	Block	41165	182	4	0.000

★ Block equidependence significantly better than Equidependence

Model comparison tests III

- **Hypothesis 3:** *Block equidependence as good as Flexible model*

		log L	Diff	Num restrictions	p-value
Normal-Static	*Block*	36477			
	Flexible	37652	1175	95	0.000
Factor-Static	*Block*	39757			
	Flexible	40628	871	95	0.000
Normal-GAS	*Block*	38518			
	Flexible	39361	842	95	0.000
Factor-GAS	*Block*	41165			
	Flexible	41913	747	95	0.000

★ Flexible model significantly better than Block equidependence

Model comparison tests IV

- **Hypothesis 4:** *Common factor has same tail shape as idio. shocks*

		log L	Diff	Num restrictions	p-value
Block-Static	*Same*	39360			
	Diff	39757	397	1	0.000
Equidep-GAS	*Same*	40868			
	Diff	40983	115	1	0.000
Block-GAS	*Same*	41017			
	Diff	41165	148	1	0.000
Flexible-GAS	*Same*	41740			
	Diff	41913	173	1	0.000

★ Common factor has different (thinner) tails than idio. shocks

Model comparison tests V

- **Hypothesis 5:** *Static copula as good as copula with GAS dynamics*

		log L	Diff	Num restrictions	p-value†
Normal-Block	*Static*	36477			
	GAS	38518	2041	2	0.000
Factor-Block	*Static*	39757			
	GAS	41165	1409	2	0.000
Normal-Flexible	*Static*	37652			
	GAS	39361	1708	2	0.000
Factor-Flexible	*Static*	40628			
	GAS	41913	1285	2	0.000

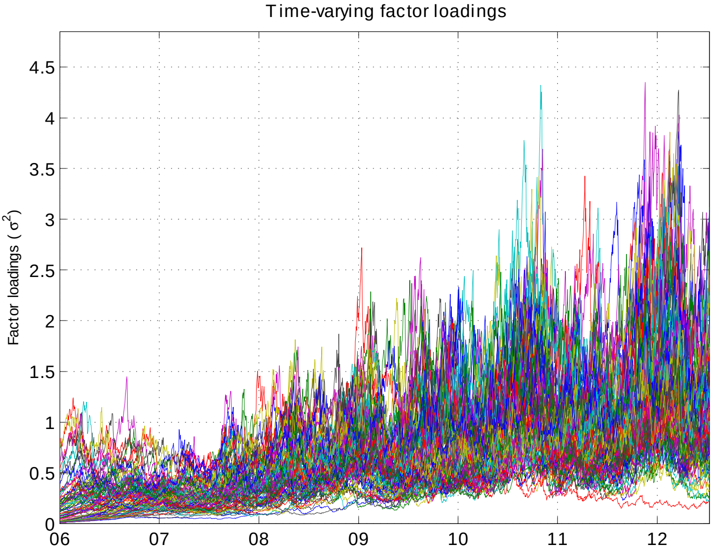
★ GAS dynamics significantly improve fit over no dynamics

Conclusions from model comparisons

- The common factor and idio. shocks are fat-tailed, with idio shocks having fatter tails
  - **Normality is strongly rejected**
- The preferred model allows each firm to have a unique loading on the common factor
  - **Heterogeneous model preferred** over equidependence models
- Time variation in the dependence structure is significant
  - **GAS dynamics** better than no dynamics

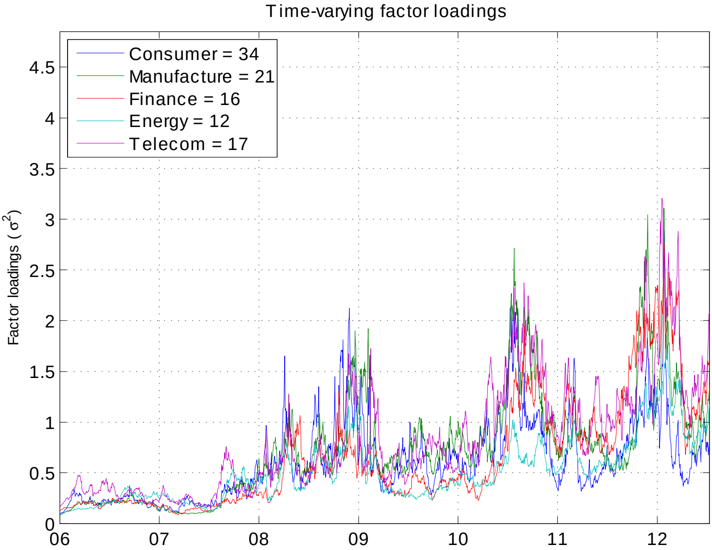
Estimating loadings on the common factor

Loadings on the common factor, for each individual firm



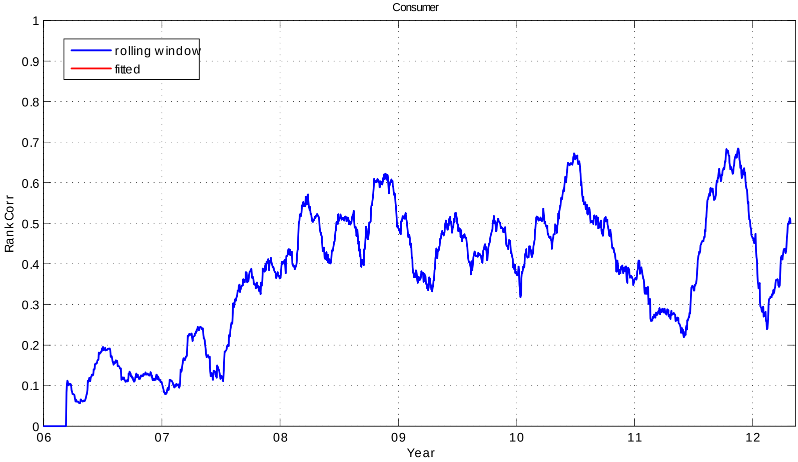
Estimating loadings on the common factor

Loadings on the common factor, for block equidependence model



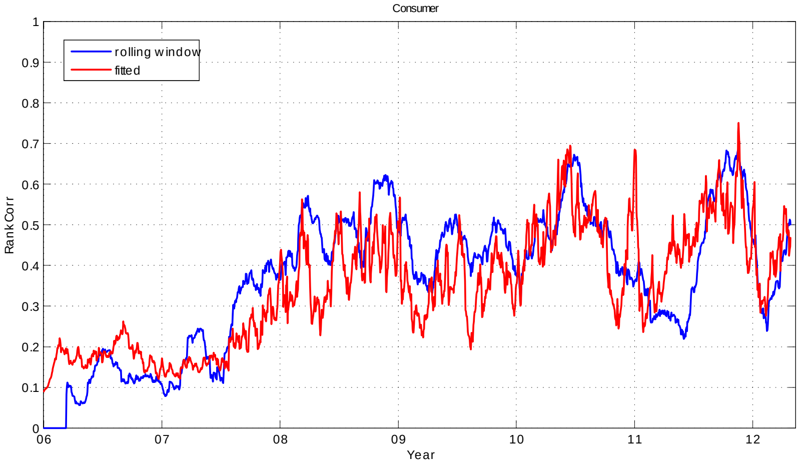
Model-implied and rolling window rank correlations

60-day rolling window rank correlations



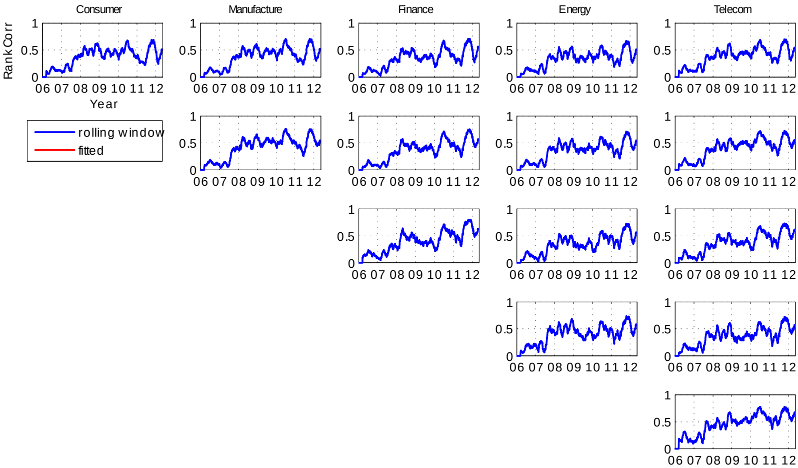
Model-implied and rolling window rank correlations

GAS dynamics match the rolling window correlations reasonably well



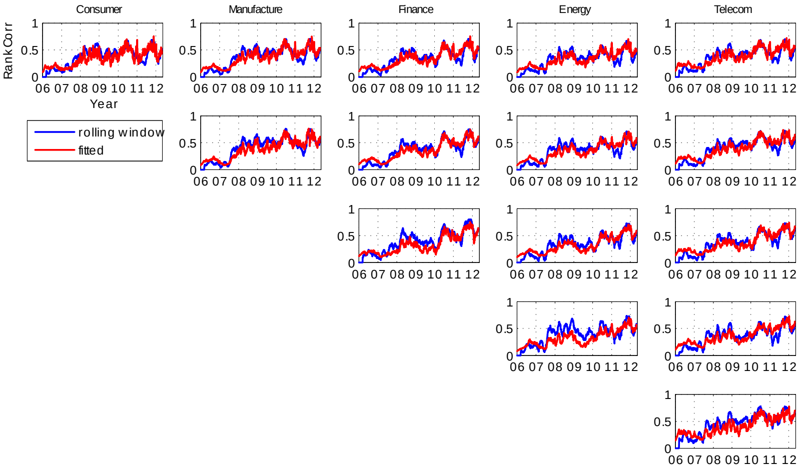
Model-implied and rolling window rank correlations

60-day rolling window rank correlations



Model-implied and rolling window rank correlations

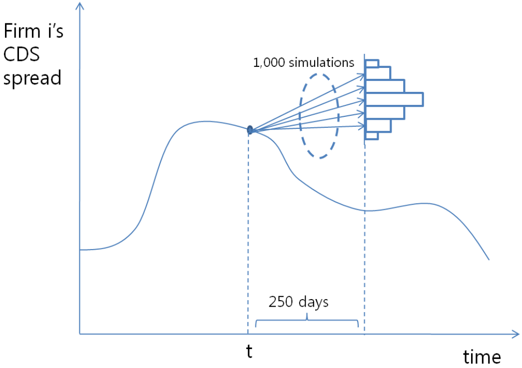
GAS dynamics broadly match the rolling window correlations



Outline

Measuring the probability of systemic distress I

- We now use our model to estimate the prob of systemic distress
- For each day in our sample, we simulate the time-path of all 100 CDS spreads 250 days into the future



Measuring the probability of systemic distress II

- We measure "**distress**" as a firm's one-year-ahead CDS lying above some (high) threshold:

$$D_{it} \equiv \mathbf{1}\left\{S_{it} \geq c_{it}^{*}\right\}$$

- We choose this threshold as the 99% quantile for the CDS spread:

$$\Pr\left[S_{it} \leq c_{it}^{*}\right] = 0.99$$

- In our sample $\bar{c}_{0.99}^{*} = 339$ bps. (Average CDS spread is 97 bps.)

Measuring the probability of systemic distress III

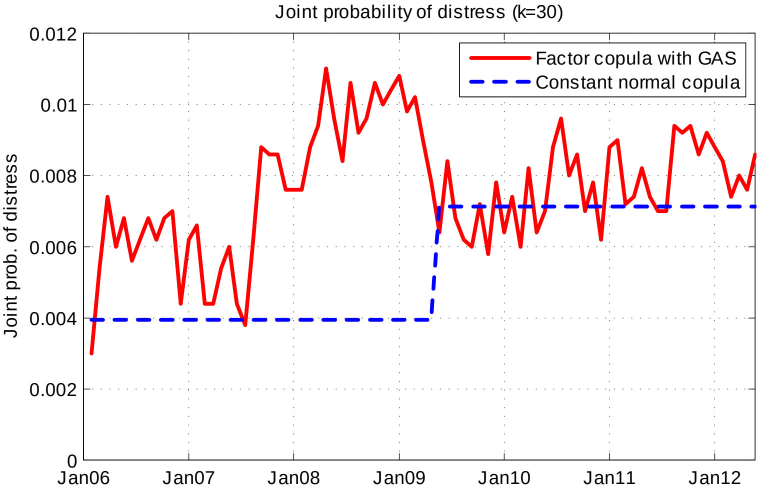
- We measure the "**joint probability of distress**" as the probability that at least k firms are in distress:

$$\color{red}{JPD_{t,k}} = \Pr_t \left[\left(\frac{1}{N} \sum_{i=1}^{100} D_{i,t+250} \right) \geq \frac{k}{N} \right]$$

- We set $k = 30$, but the results are similar for $k = 20$ and $k = 40$.

Joint probability of distress

Prob of systemic distress rose in 2008, and has remained relatively high



Measuring the impact of firm i distress on systemic risk I

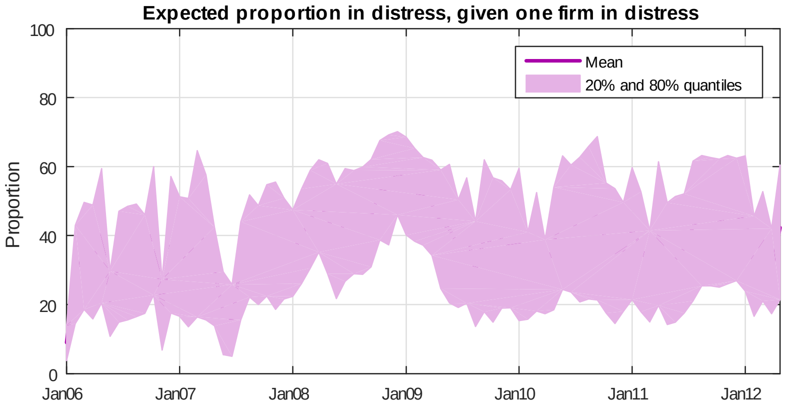
- Our model for all 100 firms allows us to study how distress in **one firm** correlates with **system-wide distress**

- We measure the "**expected proportion in distress**" for firm i as the expected number of firms in distress, given that firm i is in distress:

$$EPD_{i,t} = E_t \left[\frac{1}{N} \sum_{j=1}^{100} D_{j,t+250} \middle| D_{i,t+250} = 1 \right]$$

Expected proportion in distress

Prob of systemic distress rose in 2008, and has remained relatively high



"Most systemic" firms

- Our measure of systemic risk is:

$$EPD_t^i = E_t\left[\frac{1}{N}\sum_{j=1}^{100} D_{j,t+250} \middle| D_{i,t+250} = 1\right]$$

- When this measure is **low** it reveals that firm i being in distress is not a signal of widespread distress (firm i is more **idiosyncratic**)
- When this measure is **high** it reveals that firm i being in distress is a signal of widespread distress (firm i is a **bellwether**)
  - This is different from some other measures (eg, MES): "safer" firms are more likely to be bellwethers than riskier firms.

Expected proportion in distress

	26 January 2009		17 April 2012	
	EPD	**Firm**	**EPD**	**Firm**
Most systemic	78	Lockheed Martin	94	Wal-Mart
2	77	Campbell Soup	88	Baxter Int'l
3	75	Marsh & McLennan	88	Walt Disney
4	75	Baxter Int'l	87	Home Depot
5	74	Goodrich	84	McDonald's
⋮				
96	35	Vornado Realty	12	MetLife
97	34	Gen Elec Capital	11	The GAP
98	34	Johnson Controls	11	Sallie Mae
99	34	Alcoa	11	Comp Sci Corp
Least systemic	33	Sallie Mae	8	Pitney Bowes

Distress spillovers between financial and real sectors

- A particular focus in the systemic risk literature is spillovers of distress from the financial sector to the nonfinancial ("real") sector

- Our sample contains 16 financial firms and 84 nonfinancials, and we next consider the Expected Proportion in Distress across these two classifications

$$EPD_t^{F|F} = E_t\left[\frac{1}{16}\sum_{j=1}^{16} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \mathit{Financial}\right]$$

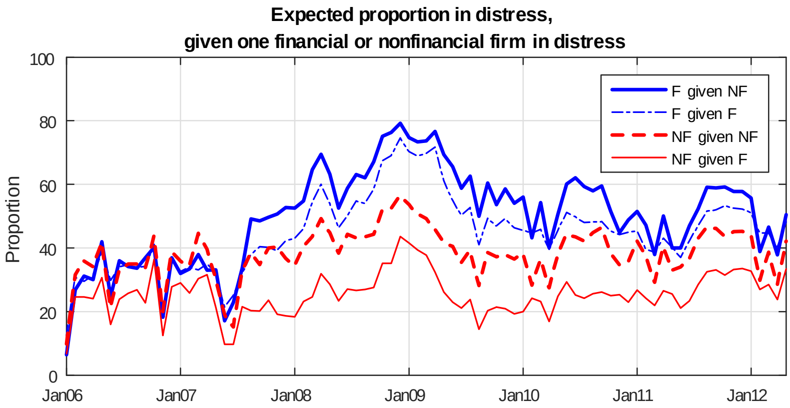
$$EPD_t^{NF|F} = E_t\left[\frac{1}{84}\sum_{j=1}^{84} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \mathit{Financial}\right]$$

$$EPD_t^{F|NF} = E_t\left[\frac{1}{16}\sum_{j=1}^{16} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \mathit{NonFin}\right]$$

$$EPD_t^{NF|NF} = E_t\left[\frac{1}{84}\sum_{j=1}^{84} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \mathit{NonFin}\right]$$

Distress spillovers between financial and real sectors

Spillover seems strongest from real to financial, not the other way around



Estimates of systemic distress from CDS spreads

- Our estimates of the joint conditional distribution of the CDS spreads on 100 US firms over the period Jan 2006–April 2012 reveal:

1. Dependence between CDS spreads rose during the financial crisis of 2008, and has remained high since then
2. The median degree of systemic risk of a firm has nearly doubled since the pre-crisis period
    - Similar to results for European sovereign default probabilities in Zhang, *et al.* (2011)
    - This increase in the probability of *systemic* distress is not reflected in the average probability of default implied from CDS spreads

Summary and conclusion

- We present a simple and flexible class of **dynamic factor copula** models that may be applied in high dimensions.
  - Analytical results on tail dependence available using EVT
  - A "variance targeting" method makes high dim applications feasible
- We applied the new copulas to a collection of 100 daily CDS spreads
  - Among the **highest dimension** copula application to date
  - Evidence of **asymmetric, heterogeneous and time-varying** dependence
  - We find that the **risk of systemic distress** has remained high since the financial crisis