Time-Varying Systemic Risk: Evidence from a Dynamic Copula Model of CDS Spreads

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“Systemic risk” is broadly defined as the risk of a crash in a large number of firms. It is an “extreme event” in two directions:

1. A **large loss** (ie, a left-tail realization for stock returns)
2. Across a **large proportion** of firms under analysis

There are a variety of methods for studying risk and dependence for small collections of assets, but a relative paucity of models of dependence for large collections of assets.

- There is a growing literature on models for **large covariance matrices** (eg, Engle and Kelly, 2008, Engle, Shephard and Sheppard, 2008, Hautsch, Kyj and Oomen, 2010)
- We propose a new **high dimension copula-based model** that builds on this literature
Main contributions of this paper

1. A flexible, simple, class of **dynamic factor copula** models that can be applied in high dimensional problems.
   - Closed-form expression for these models not generally available, but analytical results on tail dependence available using EVT
   - A “variance targeting” method makes high dimension application feasible

2. An application to **CDS spreads on 100 US firms** with focus on systemic risk:
   - We find significant evidence of tail dependence, asymmetric dependence, and heterogeneous dependence.
   - We find that the risk of systemic distress has increased since the financial crisis

Oh & Patton (2016)
1 Introduction

2 Dynamic high dimension copula models
   ■ Factor copulas
   ■ “GAS” dynamics
   ■ Simulation results

3 Measuring time-varying systemic risk using CDS data
   ■ Description of CDS spread data
   ■ Models for the joint distribution of CDS spreads
   ■ Estimates of systemic risk

4 Conclusion
Copula-based models for economic dependence I

- **Time-varying, low dimension, copulas:**
  - *Stochastic Vol-type*: Hafner and Manner (2012, JAE)

- **High dimension, constant, copulas:**
  - Normal, Student’s *t*, etc.
  - *Factor copulas*: Oh and Patton (2014, JBES)
Time-varying and high $(N \geq 10)$ dimension copulas:

<table>
<thead>
<tr>
<th>Authors</th>
<th>$N$</th>
<th>Copula</th>
<th>Dynamics</th>
<th>Estim</th>
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<tr>
<td>Zhang, et al. (2011, wp)</td>
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<td>Skew $t$</td>
<td>GAS</td>
<td>ML</td>
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<td>Vine</td>
<td>SV</td>
<td>SML</td>
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<td>This paper</td>
<td>100</td>
<td>Factor</td>
<td>GAS</td>
<td>ML</td>
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</table>
Consider a vector of \( n \) variables, \( \mathbf{Y} \), with some joint distribution \( F^* \), marginal distributions \( F_i^* \), and copula \( C^* \)

\[
[Y_1, \ldots, Y_N]' \equiv \mathbf{Y} \sim F^* = C^* (F_1^*, \ldots, F_N^*)
\]
Consider a vector of $n$ variables, $\mathbf{Y}$, with some joint distribution $F^*$, marginal distributions $F_i^*$, and copula $C^*$

$$[Y_1, ..., Y_N]' \equiv \mathbf{Y} \sim F^* = C^* (F_1^*, ..., F_N^*)$$

Oh and Patton (2015, JBES) propose a model for $C^*$ as the copula $C(\theta)$ implied by the following model:

Let $X_i = \lambda_i Z + \varepsilon_i$, $i = 1, 2, ..., N$

$Z \sim F_Z(\theta)$, $\varepsilon_i \sim iid F_\varepsilon(\theta)$, $Z \perp \varepsilon_i \ \forall \ i$

So $[X_1, ..., X_N]' \equiv \mathbf{X} \sim F_X(\theta) = C(G_1(\theta), ..., G_N(\theta); \theta)$
A simple factor copula model

- Consider a vector of \( n \) variables, \( \mathbf{Y} \), with some joint distribution \( F^* \), marginal distributions \( F_i^* \), and copula \( C^* \):

\[
[Y_1, \ldots, Y_N]' \equiv \mathbf{Y} \sim F^* = C^* (F_1^*, \ldots, F_N^*)
\]

- Oh and Patton (2015, *JBES*) propose a model for \( C^* \) as the copula \( C(\theta) \) implied by the following model:

Let \( X_i = \lambda_i Z + \varepsilon_i, \ i = 1, 2, \ldots, N \)

\[
Z \sim F_Z(\theta), \ \varepsilon_i \sim iid \ F_\varepsilon(\theta), \ \mathbf{Z} \perp \varepsilon_i \ \forall \ i
\]

So \( [X_1, \ldots, X_N]' \equiv \mathbf{X} \sim F_X(\theta) = C(G_1(\theta), \ldots, G_N(\theta); \theta) \)

- In general we won’t know \( C(\theta) \) in closed form, but we can nevertheless use it as a model for the true copula \( C^* \).
Consider the following factor structure:

Let \( X_i = \lambda Z + \varepsilon_i \), \( i = 1, 2, \ldots, N \)
\( \varepsilon_i \sim iid \ t(\nu) \), \( Z \perp \varepsilon_i \ \forall \ i \)
\( Z \sim Skew \ t(\nu, \psi) \)
\( \nu \in [2, \infty) \), \( \psi \in [-0.99, 0] \)

We set \( \lambda = 1 \) so that the factor copula implied by this structure generates linear correlation of 0.5.

We will first consider some **bivariate** distributions with this structure, and then some **high dimension** distributions.
Scatterplots of joint distributions with factor copulas
Marginal distributions are $N(0, 1)$, linear correlation $= 0.5$. 

- Normal copula
- $t(4)$-$t(4)$ factor copula
- Skew Norm-Norm factor copula
- Skew $t(4)$-$t(4)$ factor copula

Figure: Oh & Patton (2016) Systemic Risk and Copulas
“Crash” dependence

- **Crash dependence** (similar to Embrechts, *et al.*, 2000): Conditional on \( j \) variables being in their \( q \) tails, what is expected proportion of remaining variables that are in their \( q \) tails?

\[
\pi_j^q \equiv \frac{\kappa_j^q}{N - j}
\]

where

\[
\kappa_j^q = E \left[ N_q^* | N_q^* \geq j \right] - j
\]

\[
N_q^* \equiv \sum_{i=1}^{N} 1 \{ U_i \leq q \} 
\]
Proportion of remaining stocks that will crash

“Crash” defined as a 1/66 event = once in a quarter for daily asset returns

Expected proportion of remaining stocks that will crash, conditional on observing j crashes, q=1/66

Oh & Patton (2016)
Proportion of remaining CDS spreads that will spike

“Spike” defined as a 1/66 event = once in a quarter for daily CDS spreads

Expected proportion of remaining CDS spreads that will spike conditional on observing $j$ spikes, $q=1/66$. Date: 20120124

Num of observed spikes ($j$)

Proportion

Skew t-t factor copula

Normal copula

Oh & Patton (2016)
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Two useful results on copulas and transformations of continuous random variables:

1. If $Y_i \sim F_i$, then $U_i \equiv F_i(Y_i) \sim \text{Unif}(0,1)$

2. If $[Y_1, \ldots, Y_N]' \sim \mathbf{F} = \mathbf{C}(F_1, \ldots, F_N)$, then $[U_1, \ldots, U_N]' \sim \mathbf{C}$
We will model dynamics using the “generalized autoregressive score” or **GAS model** of Creal, Koopman and Lucas (2011, *JAE*).

This approach models the parameters of the copula as a function of the lagged parameters and the score of the copula likelihood:

Let \( U_t \mid \mathcal{F}_{t-1} \sim C(\kappa_t) \)

where

\[
\kappa_{t+1} = \omega + \beta \kappa_t + \alpha s_t \\
\Delta_t = S_t \cdot \Delta_t \\
s_t = \frac{\partial \log c(u_t; \kappa_t)}{\partial \kappa_t}
\]

A key benefit of this approach is that the “forcing variable” in the model for \( \kappa_{t+1} \) is provided directly by the choice of copula model.
We use the GAS model to capture time-varying dependence by letting the loadings on the common factor change through time.

Similar to Engle’s DCC model, we impose that $\alpha$ and $\beta$ are common across firms, and allow only the “intercept” parameters to differ.

We also impose that the shape parameters $(\nu_z, \nu_\varepsilon, \psi)$ are constant:

$$X_{it} = \lambda_{it} Z_t + \varepsilon_{it}, \quad i = 1, 2, \ldots, 100$$

$$Z_t \sim \text{Skew } t(\nu_z, \psi), \quad \varepsilon_{it} \sim \text{iid } t(\nu_\varepsilon), \quad Z \perp \varepsilon_i \quad \forall \ i$$

$$\log \lambda_{it} = \omega_i + \beta \log \lambda_{i,t-1} + \alpha \frac{\partial \log c(u_{t-1} | \lambda_{t-1}, \nu_z, \psi, \nu_\varepsilon)}{\partial \lambda_i}$$
Flexible GAS dynamics and “Variance Targeting”

\[ X_{it} = \lambda_{it} Z_t + \varepsilon_{it}, \quad i = 1, 2, ..., 100 \]
\[ \log \lambda_{it} = \omega_i + \beta \log \lambda_{i,t-1} + \alpha \frac{\partial \log c(u_{t-1}|\lambda_{t-1}, \nu_z, \psi, \nu_\varepsilon)}{\partial \lambda_i}, \]

- We do not want to numerically estimate \( \approx 100 \) parameters \( \omega_i \)
- We use a **variance targeting**–type approach
- We obtain quasi-closed form estimates of the intercept parameters, \( \omega_i \), based on sample rank correlations, \( \bar{\rho}_{ij} \):

\[ \bar{\rho}_{ij} = g(\omega_i, \omega_j, \nu_z, \psi, \nu_\varepsilon) \Leftrightarrow \omega_j = g^{-1}(\bar{\rho}_{ij}, \nu_z, \psi, \nu_\varepsilon) \]
- We then numerically optimize over only \((\alpha, \beta, \nu_z, \psi, \nu_\varepsilon)\)
Parsimony / flexibility and factor copulas

- Factor copula with GAS dynamics:

\[ X_{it} = \lambda_{g(i),t} Z_t + \varepsilon_{it}, \quad i = 1, 2, \ldots, 100 \]

\[ Z_t \sim Skew \ t(\nu_z, \psi), \quad \varepsilon_{it} \sim iid \ t(\nu_{\varepsilon}), \quad Z \perp \varepsilon_i \ \forall \ i \]

\[ \log \lambda_{gt} = \omega_g + \beta \log \lambda_{g,t-1} + \alpha \frac{\partial \log c(u_{t-1} | \lambda_{t-1}, \nu_z, \psi, \nu_{\varepsilon})}{\partial \lambda_g}, \]

\[ g = 1, 2, \ldots, G \]

1. **Equidependence** model: \( G = 1 \)

2. **Block equidependence** model: \( G = 5 \) (according to industry groups)

3. **Fully flexible**: \( G = 100 \)
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Credit default swaps (CDS)

- A CDS written on firm $i$ (the “reference entity”) at date $t$ is a contract in which the buyer agrees to make periodic payments (determined by CDS spread) to the seller until the contract matures (at $t + T$) or a default occurs, whichever happens first.

- If a default occurs before maturity $t + T$, the seller compensates the buyer for the realized credit loss.
Under some simplifying assumptions (see Carr and Wu, RFS, for eg) it is possible to show that a CDS spread \( S_{it} \) is given by:

\[
S_{it} = P^Q_{it} \times LGD_{it} = P^P_{it} \times M_{it} \times LGD_{it}
\]

where \( P^Q_{it} \) and \( P^P_{it} \) are the implied and objective probabilities of default, \( M_{it} \) is the market price of risk, and \( LGD_{it} \) is the loss-given-default.

This simple expression can also be obtained as a first-order approximation of more complicated formulas when \( P^Q_{it} \approx 0 \).

We work with the log-difference of the CDS spread, which yields:

\[
\Delta \log S_{it} = \Delta \log P^P_{it} + \Delta \log M_{it} + \Delta \log LGD_{it}
\]
Measures of systemic risk

- Measures of “systemic risk” in financial markets:
  - Adrian and Brunnermeier (2009): **CoVaR**\(\approx\) quantile of market returns conditional on firm \(i\) stress
  - Brownlees and Engle (2011): **Marginal Expected Shortfall**\(\approx\) expected return on firm \(i\) conditional on market stress
  - Huang, Zhou and Zhu (2009): **price of insurance against system-wide losses**

- Our proposed measure is related to the above:
  - We use CDS spreads to measure individual firm “distress”, and then estimate the expected number of firms **simultaneously distressed** given firm \(i\) in distress
  - It is the “simultaneous” aspect that makes this measure “systemic”, and which requires the specification of a model for dependence.
Description of the data

- We use daily CDS spreads for single reference entities from Markit Corporation

- We restrict our attention to 5-yr maturity CDS contracts for U.S. corporations in U.S. dollars for senior subordinated debt
  - This is the most liquid of the CDS contracts

- Use 100 CDS spreads with limited missing data among 125 constituents of a CDS index (North America CDX series 17)

- Our sample period is Jan 2006 – Apr 2012, so $T=1644$ and $N=100$
Time series of CDS spreads

- Mean
- 25% and 75% quantile
- 10% and 90% quantile

Oh & Patton (2016) Systemic Risk and Copulas
CDS-implied probabilities of default

Avg PD = 1.1% = ~A- grade. PD of ~5% is investment/speculative grade cutoff

Oh & Patton (2016)
## Summary statistics: CDS spreads

Cross-sectional distribution of individual summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>5%</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>95%</th>
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<tr>
<td>Mean</td>
<td>97.0</td>
<td>37.2</td>
<td>53.6</td>
<td>75.0</td>
<td>123.8</td>
<td>200.3</td>
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<tr>
<td>Std dev</td>
<td>70.0</td>
<td>17.3</td>
<td>27.2</td>
<td>47.5</td>
<td>84.3</td>
<td>180.6</td>
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<td>Skewness</td>
<td>1.2</td>
<td>0.1</td>
<td>0.7</td>
<td>1.3</td>
<td>1.6</td>
<td>2.5</td>
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<tr>
<td>Kurtosis</td>
<td>5.1</td>
<td>2.2</td>
<td>2.9</td>
<td>4.9</td>
<td>6.5</td>
<td>9.5</td>
</tr>
<tr>
<td>5%</td>
<td>23.9</td>
<td>9.0</td>
<td>11.7</td>
<td>18.9</td>
<td>29.9</td>
<td>60.5</td>
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<td>25%</td>
<td>42.3</td>
<td>20.4</td>
<td>25.2</td>
<td>35.3</td>
<td>47.5</td>
<td>104.7</td>
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<tr>
<td>Median</td>
<td>85.3</td>
<td>35.1</td>
<td>50.1</td>
<td>69.4</td>
<td>113.8</td>
<td>166.2</td>
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<tr>
<td>75%</td>
<td>122.1</td>
<td>46.3</td>
<td>65.9</td>
<td>93.6</td>
<td>154.7</td>
<td>251.1</td>
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<td>95%</td>
<td>245.5</td>
<td>72.5</td>
<td>102.6</td>
<td>168.5</td>
<td>313.6</td>
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<tr>
<td>99%</td>
<td>338.7</td>
<td>80.4</td>
<td>122.9</td>
<td>231.3</td>
<td>435.2</td>
<td>827.1</td>
</tr>
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</table>

Oh & Patton (2016) Systemic Risk and Copulas
### Summary statistics: Log-diff of CDS spreads

Cross-sectional distribution of individual summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>5%</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.6</td>
<td>-1.6</td>
<td>2.6</td>
<td>5.5</td>
<td>8.5</td>
<td>13.8</td>
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<td>Std dev</td>
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<td>347.6</td>
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<td>Skewness</td>
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<td>-0.3</td>
<td>0.4</td>
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<td>3.6</td>
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<tr>
<td>Kurtosis</td>
<td>25.5</td>
<td>7.7</td>
<td>10.3</td>
<td>14.6</td>
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<td>74.8</td>
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<tr>
<td>5%</td>
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<td>25%</td>
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<td>Median</td>
<td>-2.3</td>
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<td>-3.6</td>
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<td>0.0</td>
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<tr>
<td>75%</td>
<td>132.1</td>
<td>95.2</td>
<td>120.5</td>
<td>131.0</td>
<td>144.4</td>
<td>174.6</td>
</tr>
<tr>
<td>95%</td>
<td>570.5</td>
<td>457.8</td>
<td>537.1</td>
<td>568.3</td>
<td>612.8</td>
<td>685.0</td>
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</table>
Markit Corporation classifies single entities into 5 groups: Consumer, Manufacturing, Finance, Energy, and Telecom

We use this classification to group firms for one of our specifications:

<table>
<thead>
<tr>
<th>Group</th>
<th>Count</th>
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<td>Consumer</td>
<td>34</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>21</td>
</tr>
<tr>
<td>Finance</td>
<td>16</td>
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<tr>
<td>Energy</td>
<td>12</td>
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<tr>
<td>Telecom</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>
The CDS “Big Bang”

- With the growth of the CDS market through the 2000s, participants wanted more homogeneous contracts to increase liquidity.

- On April 8, 2009, the North American CDS market underwent changes to contract conventions:
  - CDS coupons were fixed to be 100 or 500 bp, with upfront payments adjusted accordingly.
  - More rigid rules on triggers for “credit events” and auctions that follow such events.
  - Move towards central clearing, away from OTC trading.

- These changes could potentially change the dynamics of CDS spreads, and we test for these using a simple structural break test:
  - We have 591 pre-break obs, 1053 post-break obs.
Did the CDS “Big Bang” cause a structural break?

- **Conditional mean:** we test for changes in all parameters jointly, and find significant changes for 39 firms

- **Conditional variance:** Controlling for changes in the mean, we find breaks in the variance for 66 firms
  
  - 28 firms have a significant break in both mean and variance

- Thus structural breaks appear to be important for models of CDS spreads for many firms – we allow for these breaks in our analysis below.
Dynamic copulas and the heterogeneous dependence

<table>
<thead>
<tr>
<th></th>
<th>Equidep</th>
<th></th>
<th>Block equidep</th>
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<td>Factor</td>
<td>Normal</td>
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<tr>
<td>( \omega_1 \rightarrow G )</td>
<td></td>
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<tr>
<td>( \alpha )</td>
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<td>( \beta )</td>
<td>0.8474</td>
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<td>( \nu )</td>
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<td>( \nu_\varepsilon )</td>
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<td>( \psi )</td>
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<td>Rank</td>
<td>6</td>
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</table>

Oh & Patton (2016)
Hypothesis 1: *Normal copula as good as Factor copula*

<table>
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<tr>
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<th>Distribution</th>
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<th>p-value$^+$</th>
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Factor copula significantly better than Normal copula
Hypothesis 2: *Equidependence as good as Block equidependence*

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Block equidependence significantly better than Equidependence
- **Hypothesis 3:** Block equidependence as good as Flexible model

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★ Flexible model significantly better than Block equidependence
Hypothesis 4: *Common factor has same tail shape as idio. shocks*

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★ Common factor has different (thinner) tails than idio. shocks
Hypothesis 5: *Static copula as good as copula with GAS dynamics*

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GAS dynamics significantly improve fit over no dynamics
Conclusions from model comparisons

- The common factor and idio. shocks are fat-tailed, with idio shocks having fatter tails
  - **Normality is strongly rejected**

- The preferred model allows each firm to have a unique loading on the common factor
  - **Heterogeneous model preferred** over equidependence models

- Time variation in the dependence structure is significant
  - **GAS dynamics** better than no dynamics
Estimating loadings on the common factor

Loadings on the common factor, for each individual firm
Estimating loadings on the common factor
Loadings on the common factor, for block equidependence model

Time-varying factor loadings

Factor loadings ($\sigma^2$)

- Consumer = 34
- Manufacture = 21
- Finance = 16
- Energy = 12
- Telecom = 17
Model-implied and rolling window rank correlations

60-day rolling window rank correlations

Oh & Patton (2016)
Model-implied and rolling window rank correlations

GAS dynamics match the rolling window correlations reasonably well

Oh & Patton (2016)
Model-implied and rolling window rank correlations

60-day rolling window rank correlations

Oh & Patton (2016)
Model-implied and rolling window rank correlations

GAS dynamics broadly match the rolling window correlations
1 Introduction

2 Dynamic high dimension copula models
   - Factor copulas
   - “GAS” dynamics
   - Simulation results

3 Measuring time-varying systemic risk using CDS data
   - Description of CDS spread data
   - Models for the joint distribution of CDS spreads
   - Estimates of systemic risk

4 Conclusion
We now use our model to estimate the probability of systemic distress.

For each day in our sample, we simulate the time-path of all 100 CDS spreads 250 days into the future.
We measure "distress" as a firm’s one-year-ahead CDS lying above some (high) threshold:

\[ D_{it} \equiv 1 \{ S_{it} \geq c^*_it \} \]

We choose this threshold as the 99% quantile for the CDS spread:

\[ \Pr [ S_{it} \leq c^*_it ] = 0.99 \]

In our sample \( \bar{c}^*_{0.99} = 339 \) bps. (Average CDS spread is 97 bps.)
We measure the “joint probability of distress” as the probability that at least $k$ firms are in distress:

$$JPD_{t,k} = \Pr_t \left[ \left( \frac{1}{N} \sum_{i=1}^{100} D_{i,t+250} \right) \geq \frac{k}{N} \right]$$

We set $k = 30$, but the results are similar for $k = 20$ and $k = 40$. 
Joint probability of distress

Prob of systemic distress rose in 2008, and has remained relatively high

Oh & Patton (2016)
Our model for all 100 firms allows us to study how distress in one firm correlates with system-wide distress.

We measure the “expected proportion in distress” for firm $i$ as the expected number of firms in distress, given that firm $i$ is in distress:

$$EPD_{i,t} = E_t \left[ \frac{1}{N} \sum_{j=1}^{100} D_{j,t+250} \mid D_{i,t+250} = 1 \right]$$
Expected proportion in distress
Prob of systemic distress rose in 2008, and has remained relatively high

Oh & Patton (2016) Systemic Risk and Copulas
“Most systemic” firms

- Our measure of systemic risk is:

\[
EPD^i_t = E_t \left[ \frac{1}{N} \sum_{j=1}^{100} D_{j,t+250} \mid D_{i,t+250} = 1 \right]
\]

- When this measure is low it reveals that firm \( i \) being in distress is not a signal of widespread distress (firm \( i \) is more idiosyncratic).

- When this measure is high it reveals that firm \( i \) being in distress is a signal of widespread distress (firm \( i \) is a bellwether).

- This is different from some other measures (eg, MES): “safer” firms are more likely to be bellwethers than riskier firms.
Expected proportion in distress

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Oh & Patton (2016) Systemic Risk and Copulas
A particular focus in the systemic risk literature is spillovers of distress from the financial sector to the nonfinancial ("real") sector.

Our sample contains 16 financial firms and 84 nonfinancials, and we next consider the Expected Proportion in Distress across these two classifications.

\[
\begin{align*}
EPD_t^{F|F} &= E_t \left[ \frac{1}{16} \sum_{j=1}^{16} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \text{Financial} \right] \\
EPD_t^{NF|F} &= E_t \left[ \frac{1}{84} \sum_{j=1}^{84} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \text{Financial} \right] \\
EPD_t^{F|NF} &= E_t \left[ \frac{1}{16} \sum_{j=1}^{16} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \text{NonFin} \right] \\
EPD_t^{NF|NF} &= E_t \left[ \frac{1}{84} \sum_{j=1}^{84} D_{j,t+250} \middle| D_{i,t+250} = 1, i \in \text{NonFin} \right]
\end{align*}
\]
Distress spillovers between financial and real sectors

Spillover seems strongest from real to financial, not the other way around

Oh & Patton (2016)
Our estimates of the joint conditional distribution of the CDS spreads on 100 US firms over the period Jan 2006–April 2012 reveal:

1. Dependence between CDS spreads rose during the financial crisis of 2008, and has remained high since then.

2. The median degree of systemic risk of a firm has nearly doubled since the pre-crisis period.

- Similar to results for European sovereign default probabilities in Zhang, et al. (2011)
- This increase in the probability of systemic distress is not reflected in the average probability of default implied from CDS spreads.
Summary and conclusion

- We present a simple and flexible class of **dynamic factor copula** models that may be applied in high dimensions.
  - Analytical results on tail dependence available using EVT
  - A “variance targeting” method makes high dim applications feasible

- We applied the new copulas to a collection of 100 daily CDS spreads
  - Among the **highest dimension** copula application to date
  - Evidence of **asymmetric, heterogeneous and time-varying** dependence
  - We find that the **risk of systemic distress** has remained high since the financial crisis