

Supplemental Appendix for

**Dynamic Factor Copula Models
with Estimated Cluster Assignments**

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Table S1: Simulation results for time-varying copulas, G=20

Panel A: Parameter estimation accuracy

	True	Gaussian		<i>t</i>		skew <i>t</i>	
		Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
ω_1^M	0.04	0.043	0.007	0.042	0.007	0.044	0.008
ω_2^M	0.04	0.043	0.006	0.041	0.008	0.043	0.008
ω_3^M	0.04	0.043	0.006	0.042	0.007	0.043	0.008
ω_4^M	0.04	0.043	0.007	0.042	0.007	0.044	0.008
ω_5^M	0.04	0.044	0.007	0.042	0.007	0.043	0.007
ω_6^M	0.04	0.043	0.007	0.042	0.006	0.043	0.007
ω_7^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_8^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_9^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_{10}^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_{11}^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_{12}^M	0.04	0.043	0.006	0.042	0.007	0.043	0.008
ω_{13}^M	0.04	0.043	0.007	0.042	0.006	0.044	0.007
ω_{14}^M	0.04	0.044	0.007	0.042	0.007	0.043	0.008
ω_{15}^M	0.04	0.043	0.007	0.043	0.007	0.044	0.008
ω_{16}^M	0.04	0.043	0.007	0.042	0.007	0.043	0.007
ω_{17}^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_{18}^M	0.04	0.043	0.006	0.041	0.007	0.043	0.008
ω_{19}^M	0.04	0.043	0.008	0.042	0.006	0.044	0.007
ω_{20}^M	0.04	0.043	0.007	0.042	0.007	0.043	0.008
ω_1^C	0.04	0.044	0.009	0.045	0.009	0.049	0.042
ω_2^C	0.04	0.045	0.009	0.045	0.010	0.049	0.032
ω_3^C	0.04	0.045	0.008	0.045	0.010	0.049	0.036
ω_4^C	0.04	0.045	0.009	0.045	0.009	0.049	0.031
ω_5^C	0.04	0.044	0.008	0.045	0.009	0.049	0.036
ω_6^C	0.04	0.045	0.008	0.045	0.010	0.049	0.043
ω_7^C	0.04	0.044	0.007	0.045	0.010	0.048	0.034
ω_8^C	0.04	0.044	0.008	0.045	0.010	0.049	0.038
ω_9^C	0.04	0.044	0.007	0.045	0.010	0.049	0.039
ω_{10}^C	0.04	0.044	0.008	0.045	0.010	0.049	0.039
ω_{11}^C	0.04	0.044	0.007	0.045	0.010	0.049	0.035

Notes: This table continues on the next page.

Table S1: Simulation results for time-varying copulas, G=20 (continued)

Panel A: Parameter estimation accuracy (continued)							
	True	Gaussian		t		skew t	
		Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
ω_{12}^C	0.04	0.045	0.009	0.045	0.010	0.049	0.039
ω_{13}^C	0.04	0.045	0.008	0.045	0.009	0.049	0.037
ω_{14}^C	0.04	0.044	0.008	0.045	0.009	0.048	0.035
ω_{15}^C	0.04	0.045	0.008	0.045	0.009	0.049	0.037
ω_{16}^C	0.04	0.045	0.009	0.045	0.010	0.049	0.043
ω_{17}^C	0.04	0.044	0.009	0.045	0.010	0.049	0.036
ω_{18}^C	0.04	0.045	0.009	0.045	0.010	0.049	0.037
ω_{19}^C	0.04	0.045	0.008	0.045	0.010	0.049	0.040
ω_{20}^C	0.04	0.045	0.009	0.045	0.010	0.048	0.036
α^M	0.02	0.020	0.002	0.020	0.002	0.020	0.002
β^M	0.90	0.891	0.014	0.894	0.015	0.895	0.017
α^C	0.02	0.020	0.002	0.020	0.003	0.020	0.003
β^C	0.90	0.890	0.018	0.889	0.022	0.879	0.091
ν	5.00			5.001	0.068	5.017	0.105
ζ	-0.10					-0.100	0.007

Panel B: Group assignment estimation accuracy			
Number incorrect			
0	100	100	99
1	0	0	1
≥ 2	0	0	0

Panel C: Estimation details						
	Clustering	Copula	Clustering	Copula	Clustering	Copula
Time (min)	21.64	42.6	21.59	56.4	22.85	81.6
EM (iter)	85.5	-	85.3	-	85.6	-

Notes: This table presents results from 100 simulations from Gaussian, t , and skew t factor copulas with 20 groups and GAS dynamics. Panel A presents results on estimation accuracy of the copula parameters, Panel B presents results on estimation accuracy of the group assignments, and Panel C presents average estimation time (for the two stages of estimation) and EM iterations based using a machine with an Intel Xeon processor, with ten cores and clock speed of 2.60GHz.

Table S2: List of firms used in the empirical analysis

Ticker	Name	SIC	Ticker	Name	SIC	Ticker	Name	SIC
AAPL	Apple	35	DVN	Devon	13	NKE	Nike	30
ABT	Abbott Lab.	28	EBAY	Ebay	73	NOV	Nat. Oilwell	35
ACN	Accenture	67	EMR	Emerson Ele	35	NSC	Norfolk South	40
ADBE	Adobe	73	ETR	Entergy Corp	49	NVDA	Nvidia	36
AEP	Ame Elec Pow	49	EXC	Exelon	49	ORCL	Oracle	73
AGN	Actavis	28	F	Ford	37	OXY	Occidental	13
AIG	Ame Inter Group	63	FCX	Freeport Memo	10	PCLN	Priceline	73
ALL	Allstate	63	FDX	Fedex	45	PEP	Pepsico	20
AMGN	Amgen	28	GD	Gen Dynamics	37	PFE	Pfizer	28
AMT	American Tower	48	GE	Gen Electric	35	PG	Procter Gamble	28
AMZN	Amazon	73	GILD	Gilead	28	PM	Philip Morris	21
APA	Apache	13	GOOGL	Google	73	QCOM	Qualcomm	36
AVP	Avon Products	28	GS	Goldman Sachs	62	RF	Regions Fin	60
AXP	Amex	60	HAL	Halliburton	13	RTN	Raytheon	38
BA	Boeing	37	HD	Home Depot	52	SBUX	Starbucks	58
BAC	Bank Of Am	60	HON	Honeywell Int	37	SLB	Schlumberger	13
BAX	Baxter	38	HPQ	Hewlett Pac	35	SNS	Steak N Shake	58
BHI	Baker Hughes	35	IBM	IBM	35	SO	Southern Co	49
BIIB	Biogen	28	INTC	Intel	36	SPG	Simon Property	67
BK	Bank Of NY	60	JNJ	Johnson & J	28	T	A T & T	48
BLK	Blackrock	62	JPM	Jpmorgan	60	TGT	Target	53
BMJ	Bristol-Myers	28	KO	Coca Cola	20	TMO	Thermo Fisher	38
C	Citigroup Inc	60	LLY	Lilly Eli	28	TXN	Texas Instru	36
CAT	Caterpillar	35	LMT	Lockheed Mar	37	UNH	Unitedhealth	63
CL	Colgate Palmo	28	LOW	Lowe's	52	UNP	Union Pacific	40
CMCSA	Comcast	48	MA	Mastercard	73	UPS	United Parcel	42
COF	Capital One	60	MCD	Mcdonalds	58	USB	U S Bancorp	60
COP	Conocophillips	13	MDLZ	Mondelez	20	V	Visa	61
COST	Costco	53	MDT	Medtronic	38	VZ	Verizon	48
CPB	Campbell Soup	20	MET	Metlife	63	WBA	Walgreens	59
CRM	Salesforce	73	MMM	3M	38	WFC	Wells Fargo	60
CSCO	Cisco Sys	36	MO	Altria	21	WMB	Williams Co	49
CVS	C V S Health	59	MRK	Merck	28	WMT	Walmart	53
CVX	Chevron	13	MS	Morgan Stanley	60	WY	Weyerhaeuser	08
DHR	Danaher	38	MSFT	Microsoft	73	XOM	Exxon Mobil	13
DIS	Disney Walt	48	NEE	Nextera Energy	49	XRX	Xerox	35
DUK	Duke Energy	49	NFLX	Netflix	78			

	Description	Num		Description	Num		Description	Num
SIC 0	Forestry, Agri.	1	SIC 3	Manuf: elec, mach	26	SIC 6	Finance, Ins	19
SIC 1	Mining, construct.	9	SIC 4	Transprt, comm's	16	SIC 7	Services	10
SIC 2	Manuf: food, furn.	19	SIC 5	Trade	10	Total		110

Table S3: Estimation results for the 1-digit SIC model

Panel A: Parameter estimation accuracy						
	Gaussian		<i>t</i>		skew <i>t</i>	
	Est.	Std Dev	Est.	Std Dev	Est.	Std Dev
ω_1^M	0.105	0.015	0.020	0.007	0.020	0.007
ω_2^M	0.074	0.012	0.014	0.005	0.014	0.005
ω_3^M	0.095	0.015	0.018	0.007	0.018	0.007
ω_4^M	0.073	0.013	0.014	0.005	0.014	0.005
ω_5^M	0.074	0.012	0.014	0.005	0.014	0.005
ω_6^M	0.107	0.017	0.021	0.008	0.020	0.008
ω_7^M	0.086	0.013	0.017	0.006	0.016	0.006
ω_1^C	0.006	0.002	0.006	0.002	0.006	0.002
ω_2^C	0.003	0.001	0.003	0.001	0.003	0.001
ω_3^C	0.001	0.000	0.001	0.000	0.001	0.000
ω_4^C	0.003	0.001	0.003	0.001	0.003	0.001
ω_5^C	0.002	0.001	0.003	0.001	0.003	0.001
ω_6^C	0.003	0.001	0.004	0.001	0.004	0.001
ω_7^C	0.003	0.001	0.003	0.001	0.004	0.001
α^M	0.019	0.001	0.008	0.002	0.008	0.002
β^M	0.890	0.017	0.979	0.008	0.979	0.008
α^C	0.003	0.001	0.003	0.001	0.003	0.001
β^C	0.993	0.002	0.993	0.002	0.993	0.002
ν			32.170	1.390	30.913	2.464
ζ					-0.227	0.004

Panel B: Estimation details			
$\log \mathcal{L}$	71237.13	74912.38	74997.21
AIC	-142438	-149787	-149954
BIC	-142333	-149676	-149838
Time (hours)	0.82	0.96	1.42

Notes: This table presents the estimated parameters and standard errors for a factor copula when the group assignments are based on one-digit SIC codes, leading to seven groups.

Table S4: Estimation results for the optimal 21 group model

Panel A: Parameter estimation accuracy

	Gaussian		t		skew t	
	Est.	Std Dev	Est.	Std Dev	Est.	Std Dev
ω_1^M	0.086	0.007	0.010	0.004	0.010	0.003
ω_2^M	0.143	0.007	0.017	0.005	0.017	0.005
ω_3^M	0.105	0.007	0.012	0.004	0.012	0.004
ω_4^M	0.129	0.008	0.015	0.005	0.015	0.005
ω_5^M	0.086	0.006	0.010	0.003	0.010	0.003
ω_6^M	0.087	0.006	0.010	0.004	0.010	0.004
ω_7^M	0.100	0.005	0.012	0.004	0.012	0.004
ω_8^M	0.101	0.006	0.012	0.004	0.012	0.004
ω_9^M	0.081	0.006	0.010	0.004	0.010	0.003
ω_{10}^M	0.082	0.006	0.010	0.003	0.010	0.003
ω_{11}^M	0.129	0.007	0.015	0.005	0.015	0.005
ω_{12}^M	0.086	0.006	0.010	0.004	0.010	0.004
ω_{13}^M	0.087	0.005	0.010	0.004	0.010	0.003
ω_{14}^M	0.117	0.007	0.014	0.005	0.014	0.005
ω_{15}^M	0.130	0.006	0.015	0.005	0.015	0.005
ω_{16}^M	0.135	0.009	0.016	0.005	0.016	0.005
ω_{17}^M	0.108	0.007	0.013	0.004	0.013	0.004
ω_{18}^M	0.059	0.005	0.007	0.003	0.007	0.002
ω_{19}^M	0.165	0.009	0.019	0.006	0.020	0.006
ω_{20}^M	0.090	0.006	0.011	0.004	0.011	0.004
ω_{21}^M	0.141	0.009	0.017	0.006	0.017	0.006
ω_1^C	0.003	0.002	0.003	0.001	0.003	0.001
ω_2^C	0.006	0.003	0.005	0.001	0.005	0.001
ω_3^C	0.006	0.003	0.005	0.001	0.005	0.001
ω_4^C	0.003	0.002	0.003	0.000	0.003	0.000
ω_5^C	0.003	0.002	0.003	0.001	0.003	0.001
ω_6^C	0.005	0.002	0.004	0.001	0.004	0.001
ω_7^C	0.003	0.002	0.003	0.000	0.003	0.000
ω_8^C	0.002	0.001	0.001	0.000	0.001	0.000
ω_9^C	0.009	0.004	0.007	0.001	0.007	0.001
ω_{10}^C	0.003	0.002	0.003	0.000	0.003	0.000

Notes: This table continues on the next page.

Table S4: Estimation results for the optimal 21 group model, continued

Panel A: Parameter estimation accuracy (continued)						
	Gaussian		t		skew t	
	Est.	Std Dev	Est.	Std Dev	Est.	Std Dev
ω_{11}^C	0.006	0.003	0.005	0.001	0.005	0.001
ω_{12}^C	0.003	0.002	0.003	0.000	0.003	0.000
ω_{13}^C	0.000	0.000	0.000	0.000	0.000	0.000
ω_{14}^C	0.003	0.002	0.002	0.000	0.002	0.000
ω_{15}^C	0.002	0.001	0.002	0.000	0.002	0.000
ω_{16}^C	0.004	0.002	0.003	0.000	0.003	0.000
ω_{17}^C	0.007	0.004	0.006	0.001	0.006	0.001
ω_{18}^C	0.001	0.001	0.001	0.001	0.001	0.001
ω_{19}^C	0.008	0.004	0.007	0.001	0.007	0.001
ω_{20}^C	0.004	0.002	0.003	0.000	0.003	0.000
ω_{21}^C	0.008	0.004	0.006	0.001	0.006	0.001
α^M	0.038	0.002	0.013	0.002	0.012	0.002
β^M	0.885	0.007	0.986	0.005	0.986	0.005
α^C	0.007	0.001	0.008	0.001	0.008	0.001
β^C	0.993	0.003	0.995	0.001	0.995	0.001
ν			33.965	1.321	33.788	1.573
ζ					-0.394	0.041

Panel B: Estimation details			
$\log \mathcal{L}$	86056.86	89506.49	89624.77
AIC	-172022	-178919	-179154
BIC	-171754	-178645	-178874
Time (clustering) (hrs)	8.63	8.63	8.63
Time (copula) (hrs)	1.37	1.38	1.43
EM iterations	98.7	98.7	98.7

Notes: This table presents the estimated parameters and standard errors for a factor copula when the group assignments are estimated from the data. The number of groups is set to 21 and 100 randomly-chosen starting values for group assignment estimation are used. The average estimation time and EM iterations presented in Panel B are based on a machine with 28 cores.

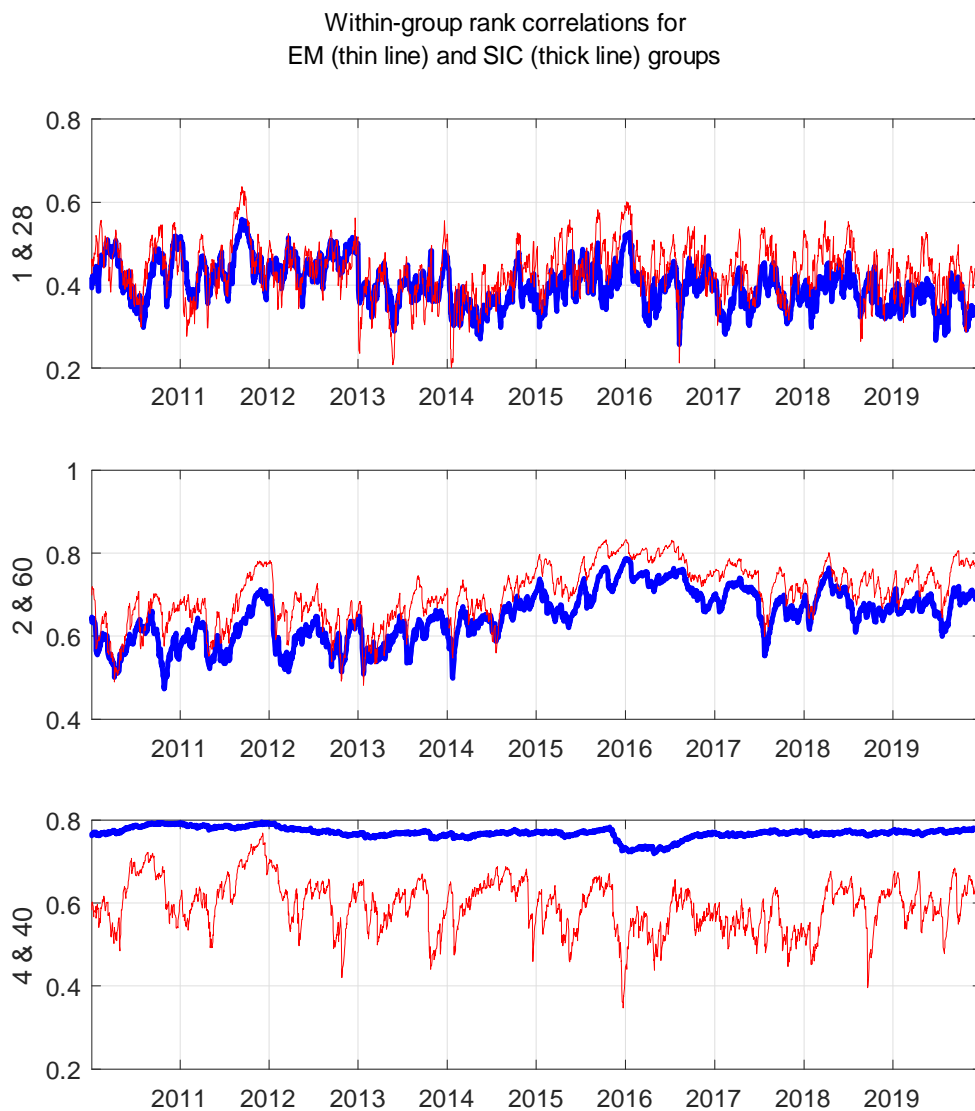


Figure S1: *Time series plots of model-implied within-group rank correlations. The upper panel presents estimated group 1 and SIC group 28; the middle panel presents estimated group 2 and SIC group 60; the lower panel presents estimated group 4 and SIC group 40.*

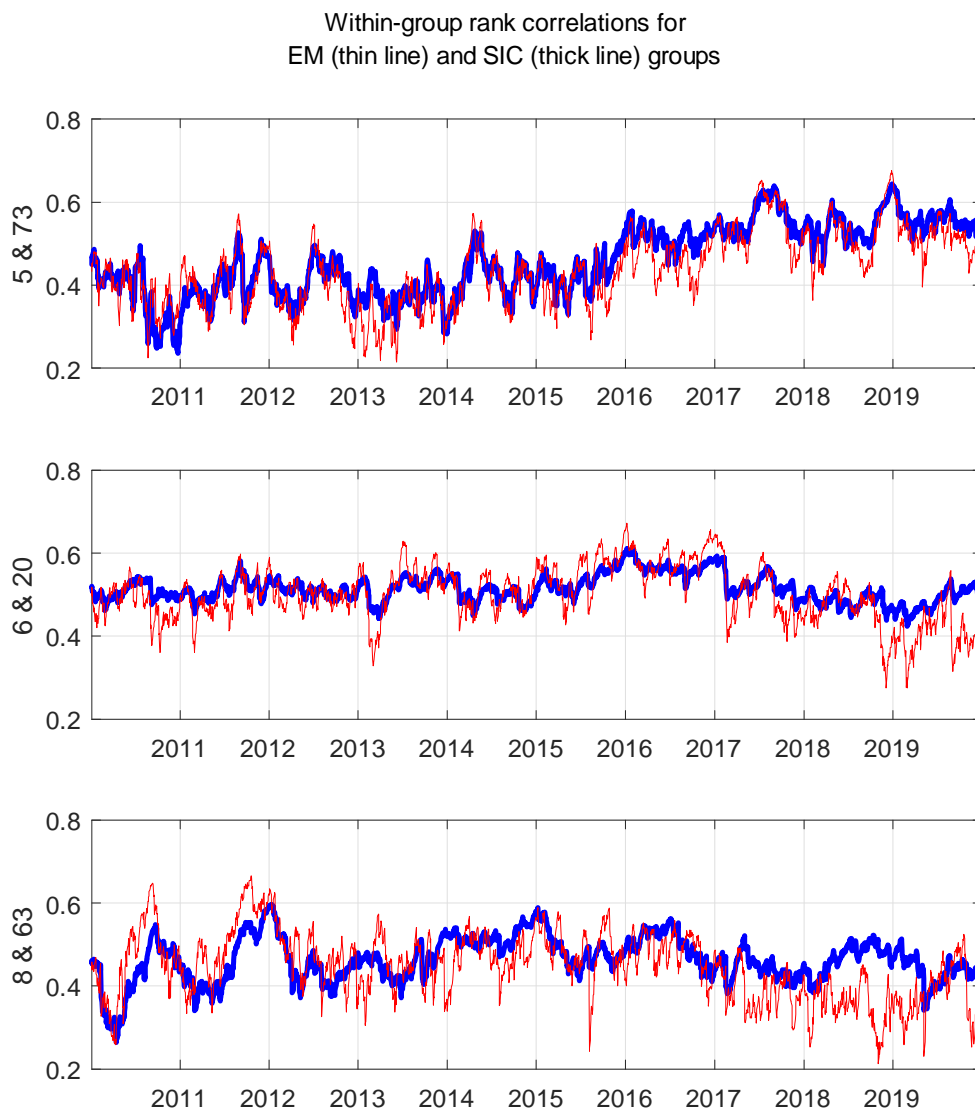


Figure S2: *Time series plots of model-implied within-group rank correlations. The upper panel presents estimated group 5 and SIC group 73; the middle panel presents estimated group 6 and SIC group 20; the lower panel presents estimated group 8 and SIC group 63.*

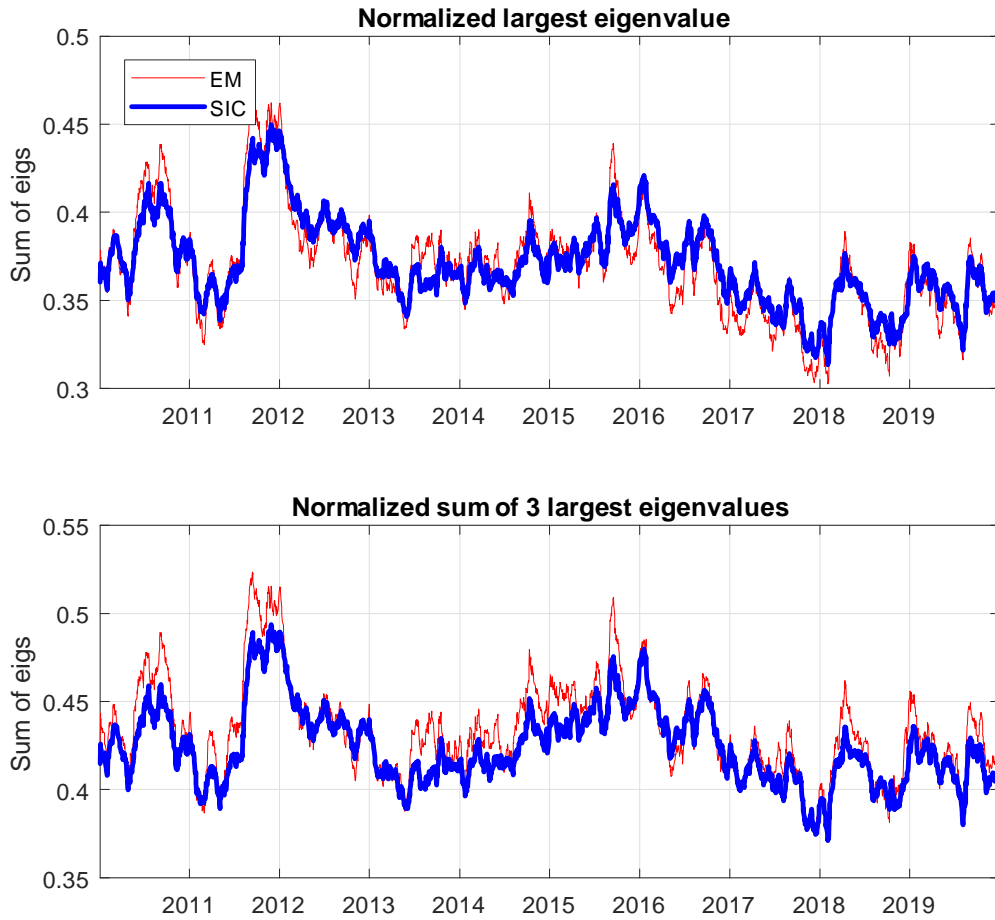


Figure S3: *The upper panel presents the largest eigenvalues of the conditional rank correlation matrices, divided by 110, the number of assets for the 2-digit SIC-based model and the optimal EM-based factor copula model, both of which have a total of 22 factors. The lower panel presents the sum of the 3 largest eigenvalues of the conditional rank correlation matrices, also divided by 110.*