

Time-Varying Systemic Risk: Evidence from a Dynamic Copula Model of CDS Spreads

Dong Hwan Oh and Andrew J. Patton

23 May 2013

S.A.1 Additional tables

Table S1: Simulation results for the “heterogeneous dependence” model

	True	Bias	Std	Median	90%	10%	Diff (90%-10%)
ω_1	-0.030	0.004	0.017	-0.022	-0.005	-0.052	0.047
ω_2	-0.029	0.004	0.018	-0.022	-0.005	-0.048	0.043
ω_3	-0.029	0.004	0.016	-0.021	-0.005	-0.043	0.038
ω_4	-0.028	0.003	0.017	-0.023	-0.005	-0.047	0.042
ω_5	-0.028	0.004	0.016	-0.020	-0.005	-0.046	0.041
ω_6	-0.027	0.004	0.016	-0.020	-0.003	-0.044	0.040
ω_7	-0.026	0.003	0.016	-0.022	-0.004	-0.042	0.038
ω_8	-0.026	0.003	0.016	-0.020	-0.005	-0.043	0.038
ω_9	-0.025	0.003	0.015	-0.019	-0.005	-0.041	0.036
ω_{10}	-0.025	0.002	0.016	-0.019	-0.005	-0.041	0.036
ω_{11}	-0.024	0.002	0.015	-0.018	-0.004	-0.038	0.033
ω_{12}	-0.023	0.003	0.013	-0.018	-0.004	-0.037	0.032
ω_{13}	-0.023	0.003	0.014	-0.018	-0.004	-0.038	0.033
ω_{14}	-0.022	0.003	0.012	-0.018	-0.004	-0.035	0.031
ω_{15}	-0.022	0.002	0.013	-0.019	-0.004	-0.043	0.039
ω_{16}	-0.021	0.002	0.013	-0.016	-0.003	-0.034	0.031
ω_{17}	-0.020	0.003	0.011	-0.015	-0.003	-0.032	0.029
ω_{18}	-0.020	0.002	0.013	-0.015	-0.003	-0.033	0.030
ω_{19}	-0.019	0.002	0.012	-0.016	-0.003	-0.031	0.028
ω_{20}	-0.019	0.002	0.011	-0.015	-0.003	-0.033	0.030
ω_{21}	-0.018	0.003	0.010	-0.013	-0.003	-0.028	0.025
ω_{22}	-0.017	0.002	0.010	-0.013	-0.003	-0.028	0.025
ω_{23}	-0.017	0.003	0.009	-0.013	-0.003	-0.025	0.022
ω_{24}	-0.016	0.002	0.010	-0.013	-0.003	-0.024	0.021
ω_{25}	-0.016	0.000	0.010	-0.014	-0.003	-0.030	0.027

**Table S1: Simulation results for the “heterogeneous dependence” model
(continued)**

	True	Bias	Std	Median	90%	10%	Diff (90%-10%)
ω_{26}	-0.015	0.001	0.010	-0.012	-0.003	-0.028	0.025
ω_{27}	-0.014	0.000	0.011	-0.011	-0.003	-0.028	0.025
ω_{28}	-0.014	0.001	0.009	-0.011	-0.002	-0.023	0.021
ω_{29}	-0.013	0.000	0.009	-0.011	-0.002	-0.025	0.023
ω_{30}	-0.012	0.001	0.008	-0.010	-0.002	-0.022	0.020
ω_{31}	-0.012	0.000	0.008	-0.010	-0.002	-0.022	0.020
ω_{32}	-0.011	0.001	0.008	-0.008	-0.002	-0.019	0.017
ω_{33}	-0.011	0.001	0.007	-0.009	-0.002	-0.017	0.015
ω_{34}	-0.010	-0.001	0.008	-0.009	-0.002	-0.021	0.019
ω_{35}	-0.009	0.000	0.008	-0.008	-0.002	-0.020	0.018
ω_{36}	-0.009	0.000	0.007	-0.008	-0.002	-0.018	0.016
ω_{37}	-0.008	0.001	0.005	-0.006	-0.001	-0.014	0.013
ω_{38}	-0.008	0.001	0.006	-0.005	-0.001	-0.016	0.015
ω_{39}	-0.007	0.001	0.005	-0.005	-0.001	-0.013	0.012
ω_{40}	-0.006	-0.001	0.005	-0.006	-0.002	-0.015	0.014
ω_{41}	-0.006	-0.003	0.007	-0.007	-0.002	-0.019	0.017
ω_{42}	-0.005	0.000	0.004	-0.005	-0.001	-0.010	0.009
ω_{43}	-0.005	0.001	0.004	-0.003	0.000	-0.009	0.008
ω_{44}	-0.004	0.000	0.004	-0.003	0.000	-0.010	0.010
ω_{45}	-0.003	-0.001	0.005	-0.003	0.000	-0.010	0.010
ω_{46}	-0.003	0.001	0.003	-0.002	0.002	-0.007	0.008
ω_{47}	-0.002	-0.001	0.003	-0.002	0.000	-0.006	0.006
ω_{48}	-0.002	-0.001	0.003	-0.001	0.001	-0.006	0.008
ω_{49}	-0.001	-0.001	0.004	-0.001	0.002	-0.006	0.008
ω_{50}	0.000	-0.002	0.004	-0.002	0.001	-0.007	0.008
ω_{51}	0.000	-0.002	0.003	-0.001	0.001	-0.006	0.007
ω_{52}	0.001	-0.001	0.004	0.000	0.004	-0.003	0.007
ω_{53}	0.002	0.000	0.004	0.000	0.006	-0.003	0.009
ω_{54}	0.002	-0.003	0.004	-0.001	0.002	-0.005	0.007
ω_{55}	0.003	-0.001	0.004	0.001	0.007	-0.003	0.010
ω_{56}	0.003	-0.002	0.003	0.001	0.007	-0.002	0.009
ω_{57}	0.004	0.000	0.004	0.003	0.010	0.000	0.010
ω_{58}	0.005	-0.002	0.004	0.001	0.008	-0.001	0.009
ω_{59}	0.005	-0.002	0.003	0.003	0.008	0.000	0.008
ω_{60}	0.006	-0.002	0.005	0.003	0.010	0.000	0.010

**Table S1: Simulation results for the “heterogeneous dependence” model
(continued)**

	True	Bias	Std	Median	90%	10%	Diff (90%-10%)
ω_{61}	0.006	-0.002	0.005	0.003	0.010	0.000	0.010
ω_{62}	0.007	-0.001	0.005	0.004	0.013	0.001	0.012
ω_{63}	0.008	-0.003	0.005	0.003	0.012	0.000	0.011
ω_{64}	0.008	-0.002	0.005	0.004	0.013	0.001	0.012
ω_{65}	0.009	-0.002	0.006	0.005	0.013	0.000	0.013
ω_{66}	0.009	-0.003	0.005	0.006	0.014	0.001	0.013
ω_{67}	0.010	-0.004	0.006	0.005	0.013	0.000	0.013
ω_{68}	0.011	-0.004	0.006	0.005	0.013	0.001	0.013
ω_{69}	0.011	-0.002	0.007	0.008	0.022	0.002	0.020
ω_{70}	0.012	-0.004	0.007	0.007	0.017	0.001	0.016
ω_{71}	0.012	-0.003	0.007	0.009	0.019	0.001	0.017
ω_{72}	0.013	-0.005	0.007	0.007	0.016	0.001	0.015
ω_{73}	0.014	-0.004	0.008	0.008	0.020	0.001	0.019
ω_{74}	0.014	-0.004	0.009	0.008	0.023	0.002	0.021
ω_{75}	0.015	-0.004	0.008	0.009	0.019	0.002	0.017
ω_{76}	0.016	-0.005	0.009	0.008	0.025	0.002	0.023
ω_{77}	0.016	-0.003	0.009	0.011	0.026	0.002	0.024
ω_{78}	0.017	-0.004	0.009	0.010	0.024	0.002	0.022
ω_{79}	0.017	-0.004	0.010	0.011	0.032	0.002	0.030
ω_{80}	0.018	-0.004	0.009	0.012	0.026	0.002	0.024
ω_{81}	0.019	-0.006	0.009	0.011	0.024	0.002	0.022
ω_{82}	0.019	-0.005	0.010	0.012	0.026	0.003	0.024
ω_{83}	0.020	-0.005	0.010	0.012	0.029	0.002	0.027
ω_{84}	0.020	-0.004	0.012	0.013	0.033	0.004	0.030
ω_{85}	0.021	-0.006	0.011	0.014	0.032	0.002	0.030
ω_{86}	0.022	-0.006	0.011	0.014	0.029	0.003	0.026
ω_{87}	0.022	-0.006	0.013	0.015	0.032	0.003	0.029
ω_{88}	0.023	-0.006	0.011	0.014	0.032	0.004	0.028
ω_{89}	0.023	-0.006	0.012	0.016	0.033	0.003	0.030
ω_{90}	0.024	-0.006	0.012	0.016	0.036	0.003	0.033
ω_{91}	0.025	-0.005	0.014	0.017	0.036	0.004	0.033
ω_{92}	0.025	-0.005	0.014	0.018	0.039	0.003	0.036
ω_{93}	0.026	-0.007	0.012	0.018	0.038	0.003	0.035
ω_{94}	0.026	-0.006	0.015	0.018	0.040	0.004	0.036
ω_{95}	0.027	-0.006	0.014	0.018	0.040	0.004	0.036

**Table S1: Simulation results for the “heterogeneous dependence” model
(continued)**

	True	Bias	Std	Median	90%	10%	Diff (90%-10%)
ω_{96}	0.028	-0.007	0.015	0.019	0.042	0.004	0.038
ω_{97}	0.028	-0.006	0.015	0.019	0.042	0.004	0.038
ω_{98}	0.029	-0.006	0.015	0.020	0.045	0.004	0.041
ω_{99}	0.029	-0.008	0.013	0.020	0.038	0.004	0.034
ω_{100}	0.030	-0.007	0.016	0.021	0.040	0.004	0.036
α	0.050	-0.006	0.015	0.045	0.062	0.023	0.039
β	0.980	0.002	0.012	0.983	0.997	0.966	0.031
ν^{-1}	0.200	-0.002	0.009	0.199	0.209	0.186	0.023
ψ_z	0.100	0.008	0.032	0.111	0.152	0.064	0.088

Notes: This table presents results from the simulation study described in Section 3 of the paper. Panel C contains results for the “heterogeneous dependence” model.

Table S2: Static copula model estimation results

	Equidep			Block equidep			Heterog		
	Normal		Factor	Normal		Factor	Normal		Factor
	Pre	Post		Pre	Post		Pre	Post	
$\lambda_{1 \rightarrow G}$	0.326 (0.126)	0.817 (0.099)	0.336 (0.118)	0.802 (0.098)	—	—	—	—	—
ν_z^{-1}	—	—	0.108 (0.027)	0.009 (0.025)	—	0.097 (0.028)	0.011 (0.019)	—	0.111 (0.034)
ν_ϵ^{-1}	—	—	0.214 (0.015)	0.205 (0.010)	—	0.198 (0.014)	0.213 (0.009)	—	0.191 (0.013)
ψ_z	—	—	0.055 (0.062)	0.080 (0.056)	—	0.085 (0.066)	0.092 (0.060)	—	0.105 (0.063)
$\log \mathcal{L}$	36,185		39,508	36,477		39,757	37,652		40,628
AIC	-72,366		-79,000	-72,934		-79,482	-74,904		-80,844
BIC	-72,364		-78,990	-72,922		-79,463	-74,661		-80,594

Notes: This table presents parameter estimates for two versions of the factor copula (Normal and Skew t - t), each with one of three degrees of heterogeneity of dependence (equidependence, block equidependence, and heterogeneous dependence). All models are imposed to be constant through time, and so the GAS parameters are fixed at zero. Standard errors based on the stationary bootstrap of Politis and Romano (1994) are presented below the estimated parameters. All models are allowed to have a structural break on April 8, 2009 (see Section 4.4 of the paper), and we denote parameters from the first and second sub-samples as “Pre” and “Post.” The log-likelihood at the estimated parameters and the Akaike and Bayesian Information criteria are presented in the bottom three rows. The factor loadings (λ_i) for the block equidependence and heterogeneous dependence models are not reported to conserve space.

Table S3: Simulation results for MLE with different numbers of quadrature nodes

	10 nodes			50 nodes			150 nodes		
	λ	ν^{-1}	ψ	λ	ν^{-1}	ψ	λ	ν^{-1}	ψ
True	1.000	0.250	-0.500	1.000	0.250	-0.500	1.000	0.250	-0.500
Bias	0.039	-0.062	-0.072	0.023	-0.018	-0.039	0.026	-0.002	-0.026
Std	0.126	0.038	0.203	0.144	0.049	0.168	0.135	0.045	0.144
Median	1.021	0.193	-0.532	1.014	0.236	-0.510	1.027	0.251	-0.500
90% quant	1.204	0.239	-0.341	1.208	0.293	-0.339	1.187	0.305	-0.349
10% quant	0.878	0.136	-0.990	0.846	0.163	-0.751	0.856	0.188	-0.704
diff (90%-10%)	0.326	0.103	0.649	0.362	0.130	0.412	0.331	0.118	0.355

Notes: This table presents the results from the simulation study described in Appendix A of the paper.

S.A.2 “Variance targeting” assumptions

In Figure 1 we present simulation evidence supporting the applicability of the assumptions underlying Proposition 1 of the paper. In both panels we use a simulation with 50,000 observations to estimate the true functions. The left panel shows the mapping from rank correlation to linear correlation. This mapping changes slightly with the shape parameters $(\theta_z, \theta_\varepsilon)$, but we see that for all choices presented the function is indeed strictly increasing, supporting assumption (d). We further see that for all three shape parameter choices the function φ is close to being the identity function, and we invoke this approximation in our estimation to increase computational speed. The right panel plots the mapping from rank correlation to log factor loadings, and we see that the true mapping is reasonably approximated by a straight line, particularly for values of rank correlation near the sample average rank correlation in our application, which is around 0.4, supporting assumption (c).

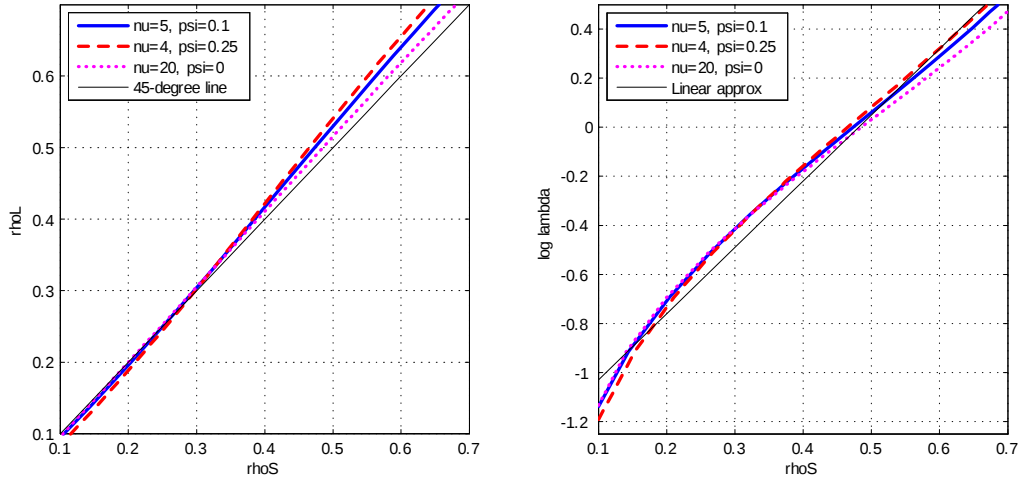


Figure 1: *The left panel plots the mapping from rank correlation to linear correlation for various choices of shape parameters in the factor copula. The right panel compares the true mappings from rank correlation to log-lambda with a linear approximation.*