Time-Varying Liquidity in Hedge Fund Returns

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Introduction

- Hedge funds are a large and fast-growing sector of the economy
  - Around $US 1.4 trillion under management and growing at 20% per year
- Unlike traditional fund managers, little is known in detail about the strategies employed by hedge funds
  - Strategies are known to be dynamic, with fast turnover, involving long and short positions, and often using relatively illiquid assets
- The (il)liquidity of hedge fund investments is currently attracting much attention, by investors and by regulators
  - The SEC, the FSA and the Chairman of the Federal Reserve have all mentioned the issue of hedge fund liquidity in the last 12 months.

Hedge fund liquidity

- "Liquidity" is a hard concept to define and measure
  - Most definitions suggest that a liquid asset is one that is possible to trade in large quantities, quickly, and at "low" cost.
- Standard proxies for liquidity (bid-ask spreads, volume of trade, depth at the best bid and ask quotes) are not available/relevant for hedge funds
- To overcome this difficulty, we use a proxy for liquidity from time series analysis: *autocorrelation* (aka serial correlation), motivated by a recent paper by Getmansky, Lo and Makarov (2004, *JFE*)

Autocorrelation in hedge fund returns

- Unlike most other financial assets, hedge funds often generate returns that are highly autocorrelated.

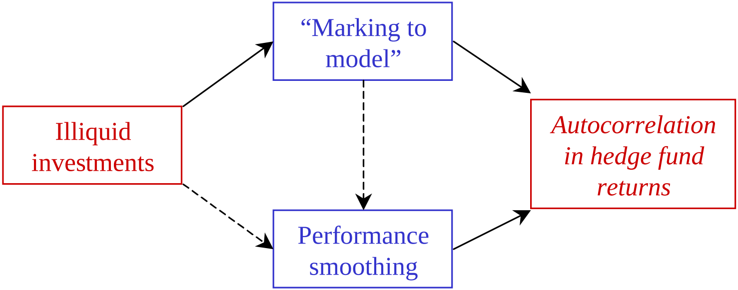
Style	$Corr\left[r_t, r_{t-1}\right]$
Convertible arbitrage	30.93
Merger arbitrage	20.67
Fixed income	19.59
Distressed securities	18.63
Equity hedge	13.99
Market neutral	13.85
Global macro	9.51
Equity nonhedge	6.97
S&P 500	1.61
FTSE	1.74

Autocorrelation and liquidity

- Getmansky et al. systematically analyse various sources of the observed autocorrelation in hedge fund returns:

    1. Time-varying expected returns - *not large enough under realistic parameter values*
    2. Time-varying leverage - *not large enough under realistic parameter values*
    3. Fee structures of hedge funds - *induces correlation in net returns of the wrong sign*
    4. Market inefficiencies - *if true, would soon be eliminated (by hedge fund managers themselves)*
    5. **Illiquidity and smoothed returns**

Illiquidity and autocorrelation



Our modelling strategy

- If we can condition on the work of Getmansky et al. and believe that the autocorrelation in reported hedge fund returns is driven primarily by the exposure of the fund to illiquid assets, then:
- By modelling *time-varying* autocorrelation in reported hedge fund returns we may gain some insight into the *time-varying liquidity* of hedge fund investments
- Further, by examining which variables best explain movements in autocorrelations we can determine the variables that have the greatest impact on hedge fund liquidity.

Previous research on hedge funds

- **Hedge fund performance:**
  - Ackermann, McEnally and Ravenscraft (1999, *JFE*)
  - Agarwal and Naik (2000, *JFQA*)
  - Fung and Hsieh (1997 *RFS*, 2002 *FAJ*)
  - Liang (1999, 2001, *FAJ*)
  - Kosowski, Naik and Teo (2006, *JFE*)
  - Fung, Hsieh, Naik and Ramadorai (2006, *mimeo*), amongst many others.
- **Risk/return characteristics:**
  - Mitchell and Pulvino (2001, *JF*)
  - Fung and Hsieh (2001, *RFS*)
  - Agarwal and Naik (2004, *RFS*), amongst others.
- **Hedge fund liquidity:**
  - Getmansky, Lo and Makarov (2004, *JFE*)
  - Aragon (2006, *JFE*)
  - Bollen and Krepley-Pool (2006, *JFQA*)

Contributions of our paper

Using returns on over 600 individual hedge funds in eight different styles, from the CISDM hedge fund database, over 1994 - 2004:

1. We propose a flexible model to capture the time variation in the autocorrelation of hedge fund returns, nesting the GLM model as a special case, thus providing a test for time-varying liquidity.
   - We find statistically significant evidence of time-varying liquidity for all 8 styles considered.
2. We consider a variety of candidate variables to capture time-varying liquidity, and control for aggregate market liquidity before testing the significance of other variables.
   - The most important factors appear to be the returns on stock and bond indices

The model of Getmansky, Lo and Makarov (2004)

- GLM suggest considering reported hedge fund returns as a linear combination of current and lagged *true* returns on the fund:

$$\begin{aligned} r_{it}^{o} &= \theta_{0i} r_{it} + \theta_{1i} r_{it-1} + \ldots + \theta_{qi} r_{it-q} \\ \text{s.t.} \quad 1 &= \theta_{0i} + \theta_{1i} + \ldots \theta_{qi} \end{aligned}$$

- If the true returns can be taken as serially uncorrelated, then this is a MA(q) model for observed hedge fund returns.

- In this framework, the parameter θ_{0i} is a natural summary measure for the degree of liquidity.

The Getmansky, Lo and Makarov model

$$r_{it}^{o} = \theta_0 r_{it} + \theta_1 r_{it-1} + \theta_2 r_{it-2}$$

Style	$\hat{\theta}_0$	$\hat{\rho}_1\%$	$\hat{\rho}_2\%$
Convertible arbitrage	0.73	30.93	12.81
Merger arbitrage	0.82	20.67	11.63
Fixed income	0.82	19.59	11.13
Distressed securities	0.84	18.63	7.81
Equity hedge	0.92	13.99	6.88
Market neutral	0.88	13.85	7.54
Global macro	0.97	9.51	1.98
Equity nonhedge	0.97	6.97	-0.01

A model for time-varying hedge fund liquidity

- We extend the GLM model by allowing the parameters of that model to vary through time:

$$\begin{aligned} r^o_{it} &= \theta_{0it} r_{it} + \theta_{1it} r_{it-1} + \ldots + \theta_{qit} r_{it-q} \\ \text{s.t. } 1 &= \theta_{0it} + \theta_{1it} + \ldots \theta_{qit} \ \forall \ t \end{aligned}$$

- We maintain the structure of the GLM model, but we consider letting the amount of "smoothing" in observed returns vary through time.

- It is clear that without some further structure this model will have too many parameters to be reliably estimated.

A model for time-varying hedge fund liquidity, cont'd

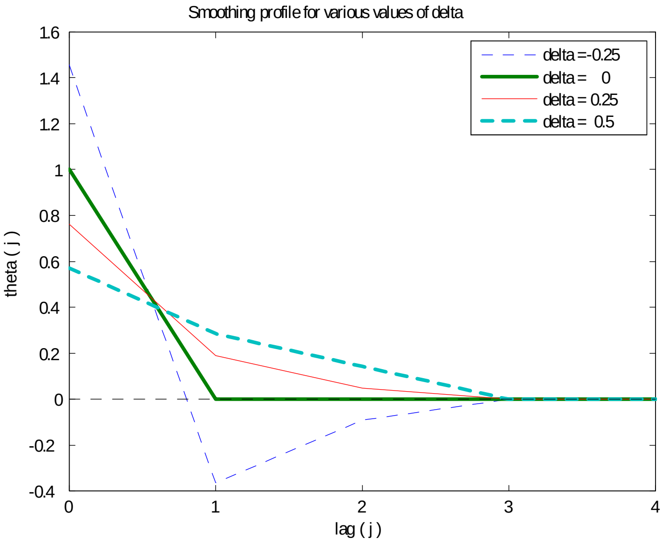
- We constrain each parameter θ_{ijt} to be a function of a single liquidity "index" , δ_{it}, which determines the liquidity of fund i at time t.

- We also constrain the parameters to decline geometrically to zero, similar to an AR process:

$$
\begin{aligned}
\theta_{ijt} &= \theta_{i0t} \cdot sgn(\delta_{it}) \left|\delta_{it}\right|^{j}, \quad j = 1, 2, \ldots, q \\
\theta_{i0t} &= 1/\bar{\theta}_{it} \\
\text{where } \bar{\theta}_{it} &= 1 + sgn(\delta_{it}) \sum_{j=1}^{q} \left|\delta_{it}\right|^{j}
\end{aligned}
$$

- We tested the assumption of geometrically declining weights for the constant (GLM) model for each fund, and it was rejected only for 5% of funds (the size of the test). Thus that restriction is reasonable.

“Smoothing profile” for various values of delta



A model for time-varying hedge fund liquidity, cont'd

- By constraining each parameter θ_{ijt} to be a function of a single liquidity "index" , δ_{it}, we reduce the number of time-varying parameters from q to just 1.

- We model δ_{it} as a function of observable liquidity factors, both common and fund-specific:

$$\begin{aligned}
\delta_{it} &= \Lambda(X_t'\lambda_i + Z_{it}'\phi_i) \\
X_t'\lambda_i &= \beta_i + \gamma_{i1}X_{1t} + \ldots\gamma_{iM}X_{Mt} \\
Z_{it}'\phi_i &= \phi_{i1}Z_{1t} + \ldots + \phi_{ip}Z_{pt} \\
\text{where}\quad \Lambda(z) &= \left(1 - e^{-z}\right) / \left(1 + e^{-z}\right)
\end{aligned}$$

- The GLM model is obtained as a special case when $\gamma_{i1} = \ldots = \gamma_{iM} = \phi_{i1} = \ldots = \phi_{ip} = 0$

A model for time-varying hedge fund liquidity, cont'd

- The above model for time-varying liquidity has a total of $3 + M + p$ parameters per hedge fund. We consider 7 common factors and 2 fund-specific factors, leading to 12 parameters per fund.

- To increase the power of the test for time-varying liquidity we pool the funds by investment style, and assume that the factor coefficients are common across all funds in the same style.
  - i.e. we assume $\gamma_{ik} = \bar{\gamma}_k$ and $\phi_{il} = \bar{\phi}_l$ for all funds in the same style.

- We do *not* initially assume that the "intercept" terms, β_i, are constant across funds in the same style.
  - However, we tested this restriction and found it could not be rejected for any of the 8 styles we consider, and so we further impose that $\beta_i = \bar{\beta}$ for all funds in the same style.

Estimating the parameters of the model

- We assume that the "true returns" r_{it} are serially uncorrelated.
- We do not make any assumptions about cross-sectional correlation between the true returns, and we do not exploit possible correlation in estimation.
- Similarly, we do not assume normality or homoscedasticity (we interpret our estimator as a QMLE) and use robust standard errors.
- We obtain our parameter estimates from

$$\hat{\boldsymbol{\theta}}_T \equiv \arg\max_{\boldsymbol{\theta}} \sum_{i=1}^{k} \frac{1}{T_i - S_i + 1} \sum_{t=S_i}^{T_i} \log f\left(r_{it}; \boldsymbol{\theta}\right)$$

where the first and last observation on fund i are denoted S_i and T_i, and f is the Normal density with mean given by the MA(2) model above and constant variance σ_i^2.

Hedge fund liquidity factors

- We consider a variety of variables that might naturally be thought to affect the liquidity of hedge funds' investments.
  - Our initial model includes the contemporaneous values of these variables, so they might be thought of as simple explanatory variables.
  - Our second model uses the first lags of these variables, which leads to a model for predicting hedge fund liquidity.
- The variables we consider generally all have both an "innocent" and a "less-than-innocent" rationale for affecting the degree of smoothing in hedge fund returns: marking-to-market vs. performance smoothing.
  - We do not attempt to disentangle these effects.
  - Our results might be used to evaluate the empirical validity of a theoretical model to distinguish between these explanations.

Hedge fund liquidity factors, cont'd

- **Market returns:** one-month returns on the S&P500 and the Lehman Brothers aggregate bond index to proxy for equity and bond returns.
- **Market volatility:** realised volatility from daily returns on the S&P500 index and the Lehman Brothers bond index.
- **Equity market liquidity:** the Pastor-Stambaugh (2003) liquidity index to proxy for aggregate equity market liquidity.
- **Bond market liquidity:** bid-ask spreads from the U.S. Treasury bill market to proxy for bond market liquidity, as suggested by Fleming (2003) and Goldreich, Hanke and Nath (2005)

Hedge fund liquidity factors, cont'd

- **Calendar effects:** a dummy for the April - September months. Most funds audits are in December (Liang 2003) but there is also evidence of seasonality in liquidity (Hong and Yu 2005)

- **Net fund flows:** we compute net fund flows as

$$NFF_{it} = \frac{NAV_{it} - NAV_{it-1}}{NAV_{it-1}} - r_{it}^{o}$$

  We use a forward-looking 3-month average of this variable in the "contemporaneous" model, to account for the redemption notice period.

- **Sign of the "true" return:** motivated by the work of Bollen and Krepley-Pool (2006): $S_{it} \equiv sgn\left(r_{it}\right)$. The coefficient on this variable is estimated via an iterative procedure.

A small simulation study

- To determine whether our model and tests work well in finite samples we conducted a small simulation study:

$$r_{it}^{o} = (1 - \theta_{i1} - \theta_{i2})\, r_{it} + \theta_{i1} r_{i,t-1} + \theta_{i2} r_{i,t-2},$$

$$\text{where} \quad \begin{bmatrix} r_{it} \\ \mathbf{f}_t \end{bmatrix} \sim iid\ N\left(\begin{bmatrix} \mu_i \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \sigma_i^2 & \mathbf{0}_N' \\ \mathbf{0}_N & I_N \end{bmatrix} \right)$$

- We set $T \in \{75, 150, 500\}$, $K = \{1, 10, 50\}$, and $N = \{1, 4, 8\}$.
- We calibrated the values for $(\mu_i, \sigma_i^2, \theta_{i1}, \theta_{i2})$ from a randomly selected subset of our funds.

Missing observations

- We randomly selected fifty (the largest value of K we considered in the simulation) funds from our sample and recorded the dates of each fund's first and last observation, t_i^{first} and t_i^{last}.
- From these, we computed

$$\tau_i^{first} = \frac{t_i^{first}}{T}, \quad \tau_i^{last} = \frac{t_i^{last}}{T}$$

  which reflect the proportions of each sample that were missing from the start and end of the sample for fund i.
- To replicate the missing data in our simulation we used these values of $\left(\tau_i^{first}, \tau_i^{last}\right)$, $i = 1, 2, ..., 50$ to determine which observations we should "throw away".

Finite-sample properties of individual t-tests

Proportion of rejections at the nominal 5% level

		No missing obs, no 'true return' factor		With missing obs and 'true return' factor	
Number of funds:		$K = 1$	$K = 50$	$K = 1$	$K = 50$
Sample size	Num. of factors				
$T = 75$	$N = 1$	0.09	0.06	0.10	0.03
$T = 150$	$N = 1$	0.05	0.06	0.07	0.04
$T = 500$	$N = 1$	0.02	0.04	0.03	0.04
$T = 75$	$N = 8$	0.23	0.06	0.11	0.04
$T = 150$	$N = 8$	0.08	0.06	0.10	0.04
$T = 500$	$N = 8$	0.03	0.06	0.03	0.05

Finite-sample properties of joint chi2-tests

Proportion of rejections at the nominal 5% level

		No missing obs, no ‘true return’ factor		With missing obs and ‘true return’ factor	
Number of funds:		$K = 1$	$K = 50$	$K = 1$	$K = 50$
Sample size	Num. of factors				
$T = 75$	$N = 1$	0.09	0.06	0.17	0.03
$T = 150$	$N = 1$	0.05	0.06	0.06	0.01
$T = 500$	$N = 1$	0.02	0.04	0.02	0.02
$T = 75$	$N = 8$	0.78	0.21	0.98	0.18
$T = 150$	$N = 8$	0.45	0.12	0.79	0.06
$T = 500$	$N = 8$	0.05	0.06	0.17	0.06

Summary of results from simulation study

- Overall the results suggest that our models and tests have reasonable properties in finite samples.
- For our parameter values ($T = 75 \sim 150$, $K = 20 \sim 120$, $N = 7 \sim 8$) we found that the t-tests had good properties, and the χ^2 tests were slightly over-sized.
- In all cases we found that pooling the data across funds improved the finite-sample size of the tests, supporting our modelling strategy
    - Our motivation for doing this was that it would improve the *power*, but checking this is beyond the scope of this paper.

Description of the data

- We use monthly returns and accompanying information on both live and "dead" funds from the CISDM database, over the period January 1993 to August 2004 (140 observations).
- We consider 8 fund styles: Merger Arbitrage, Distressed Securities, Equity Hedge, Equity Nonhedge, Market Neutral, Fixed Income Arbitrage, Convertible Arbitrage, Global Macro.
- We only study funds with at least 48 months of observations, which leaves us with a total of 609 individual hedge funds.

Description of the data, cont'd

Average of the moments across funds in a given style

Category	K	Mean	SD	Skew	Kurt	$\hat{\theta}_0$
Mkt neutral	121	0.96	3.88	0.44	6.57	0.88
Eq hedge	58	0.99	5.18	0.01	6.40	0.92
Eq nonhedge	20	1.24	8.25	0.17	4.86	0.97
Global macro	90	0.96	5.30	0.28	5.60	0.97
Distressed	72	1.08	3.82	-0.14	7.83	0.84
Merger arb	106	0.89	3.04	-0.17	6.70	0.82
Conv. arb	106	1.02	2.11	-0.14	7.18	0.73
Fixed income	36	0.58	2.37	-2.27	17.10	0.82

t-statistics on liquidity factors across fund styles

Equity-based styles

A negative coefficient implies that as the factor rises, liquidity also rises

Factor	Market neutral	Equity hedge	Equity nonhedge	Merger arb.	Dist-ressed
Stock ret	**−2.11**	**−3.16**	−1.37	−1.22	−0.70
Stock vol	0.38	−0.18	−0.28	1.23	1.94
Stock liq	0.95	1.46	1.21	−1.39	−1.21
Bond ret	**2.41**	**2.90**	0.71	0.90	0.98
Bond vol	0.58	1.05	0.10	0.03	0.03
Bond liq	0.02	0.00	0.00	0.01	−0.01
Winter?	0.19	−0.53	0.49	0.88	1.24
Net flow	0.29	0.27	−0.77	**−3.18**	0.80
"True ret"	0.60	0.37	0.00	−0.81	−0.50
p-value	**0.02**	**0.00**	**0.00**	**0.00**	**0.00**

t-statistics on liquidity factors across fund styles

Non-equity based styles

A negative coefficient implies that as the factor rises, liquidity also rises

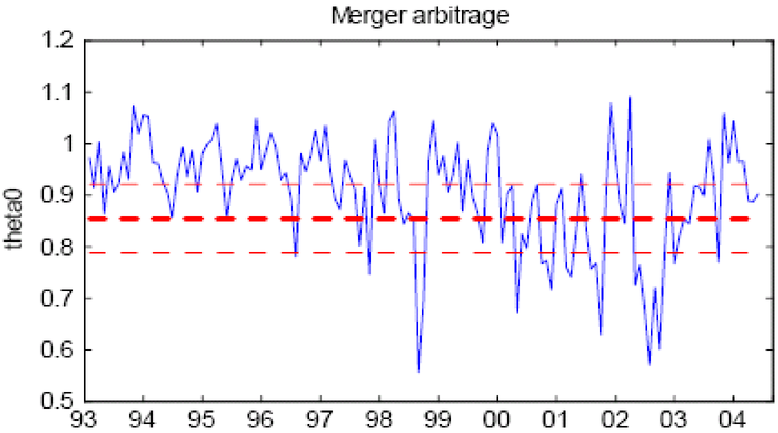
Factor	Conv. arb.	Fixed income	Global macro
Stock ret	−0.11	0.66	−1.16
Stock vol	**4.30**	**2.21**	−0.47
Stock liq	0.35	−0.03	0.88
Bond ret	0.30	−0.83	**2.54**
Bond vol	−0.40	0.80	1.20
Bond liq	−1.72	0.00	0.01
Winter?	−0.27	0.06	−0.18
Net flow	−1.85	**2.16**	**1.98**
"True ret"	−1.25	−**3.05**	0.97
p-value	**0.00**	**0.00**	0.15

Summary of results

- Our most prominent result is the strong evidence in favour of time-varying liquidity: the assumption of constant liquidity is rejected at the 5% significance level for 7 out of 8 styles.
- The coefficient on the return on the S&P500 is negative for 7 out of 8 styles (though only significant for 2).
- The coefficient on the return on the Bond index is positive for 7 out of 8 styles (and significant for 3).
- Net fund flows are significant for 3 out of 8 styles, negative for 1 and positive for 2.

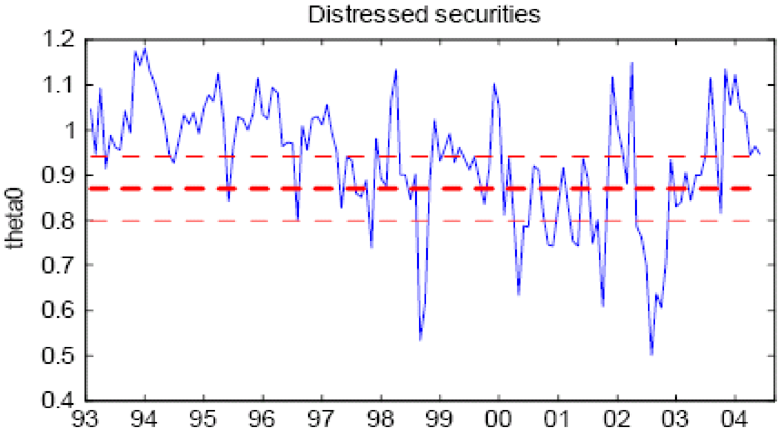
Plot of theta0 for Merger Arbitrage funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



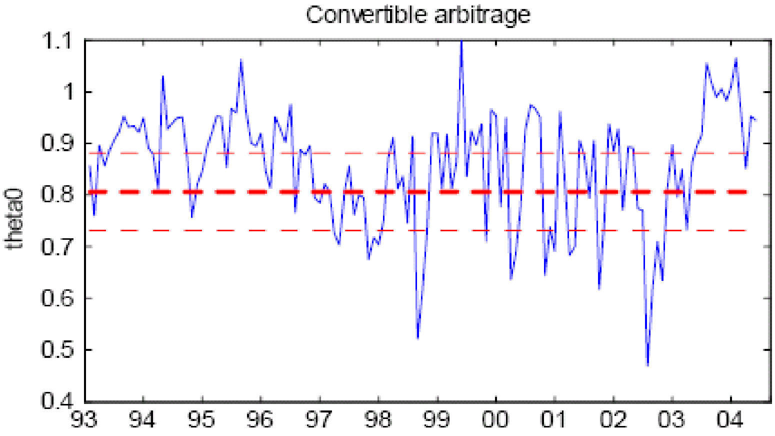
Plot of theta0 for Distressed Securities funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



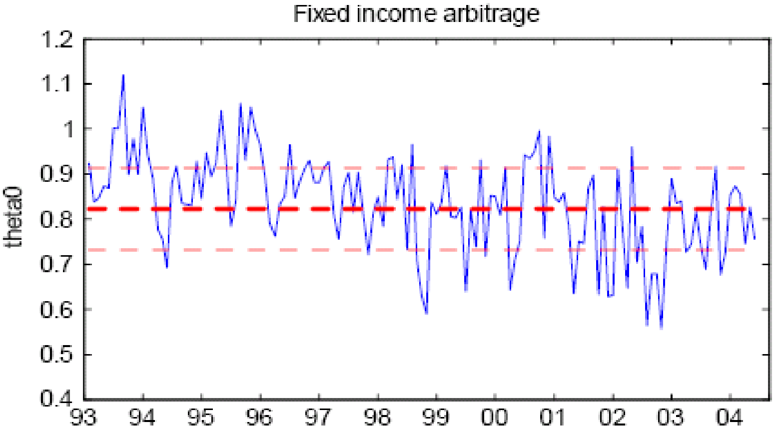
Plot of theta0 for Convertible Arbitrage funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



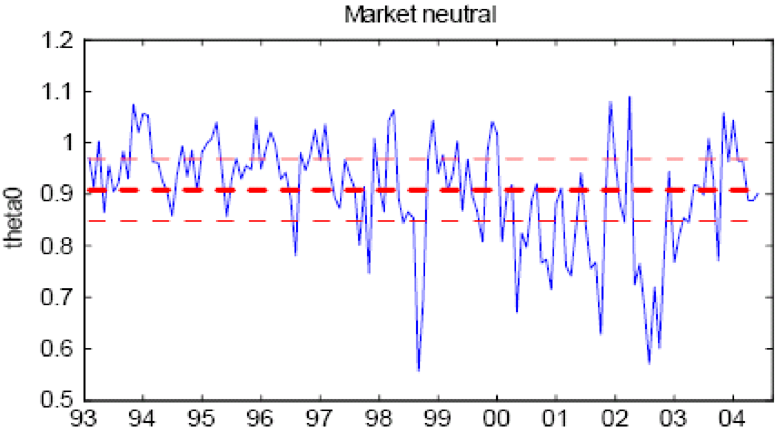
Plot of theta0 for Fixed Income funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



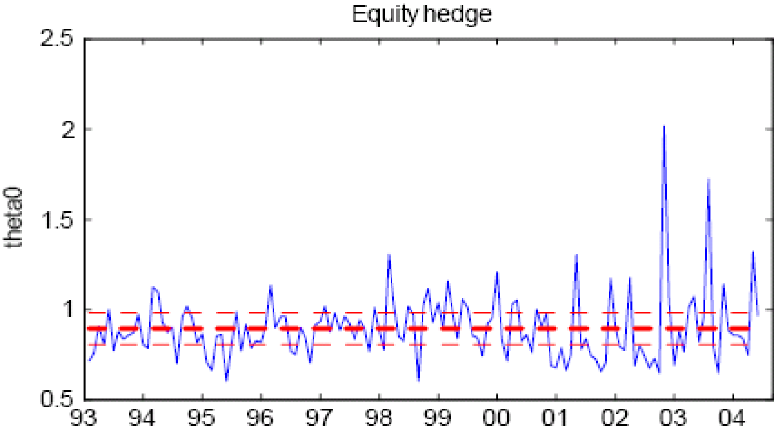
Plot of theta0 for Market Neutral funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



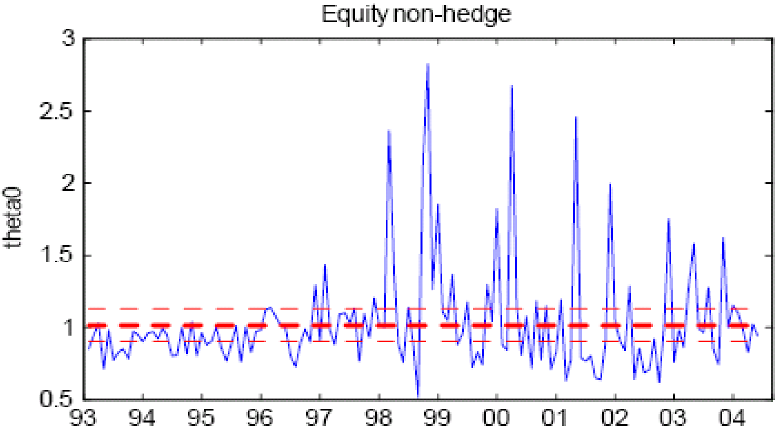
Plot of theta0 for Equity Hedge funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



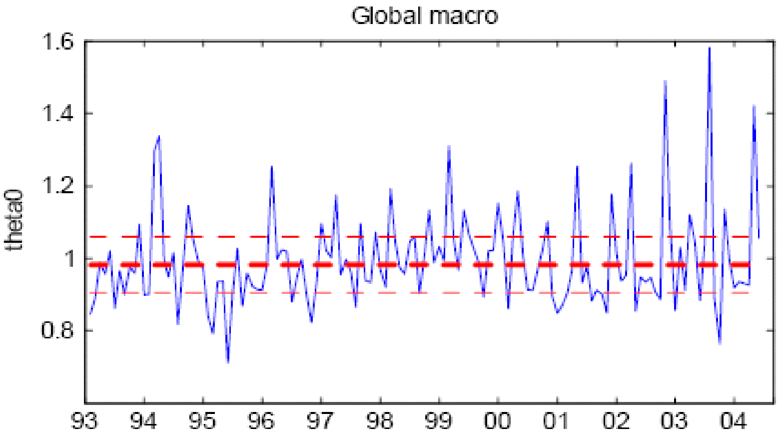
Plot of theta0 for Equity Non-hedge funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



Plot of theta0 for Global Macro funds

GLM theta0 with 95% confidence interval, and our estimated time-varying theta0



Conclusions

- We proposed an model for time-varying hedge fund liquidity, building on the connection between liquidity and autocorrelation established by Getmansky, Lo and Makarov (2004).
- Our model allows us to test for the importance of several factors jointly or separately, controlling for, e.g., aggregate market liquidity.
- In our empirical study of over 600 individual hedge funds, we found strong evidence of time-varying liquidity for all hedge fund styles.
    - We found that hedge fund liquidity falls following a decline in the equity market, and rises with a decline in the bond market
- We did not find evidence that liquidity varies through the year, nor with the "true" return, when controls for other factors are included.