Risk Price Variation: The Missing Half of the Cross-Section of Expected Returns

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Motivation

- **Where do differences in expected returns come from?**
- Typical models of expected returns are of the form,

$$E\left[r_i^e\right] = \sum_{k=1}^{K} \underbrace{\beta_{ik}}_{\text{amount of risk}} \times \underbrace{\lambda_k}_{\text{price of risk}},$$

  where λ_k is the expected compensation to factor f_k

- The conversation typically focuses on the set of **factors** f
  - Our embarrassment of riches: hundreds of candidates for f!
- This focus is correct only in a Law-of-One-Price world with
  - Costless portfolio formation
  - Frictionless borrowing
  - Integrated markets, etc...
- **This paper is about cross-sectional variation in risk prices (λ)**

Finding Variation in Risk Prices

How can we identify differences in λ? Two approaches:

1. Use economic intuition to conjecture groups and test for equal λs
   - **Problem**: only in certain cases do we know how to group assets **ex ante**
   - How do we guard against data snooping?
   - What if conjectured segments are incorrect or unimportant?
2. Group together assets based on estimated λs ("let the data speak")
   - **Problem**: λs are slopes across assets, that is, the clustering characteristic $\lambda_k^{(i)}$ depends on the other assets in its group
   - Typical off-the-shelf clustering technologies like k-means cannot accommodate this dependence

- We contribute an approach to **estimate** and **test** for variation in λ across assets based on methods in machine learning

Outline

1. Motivation
2. Estimation and testing
   - Fama-MacBeth with unobserved clusters: EM algorithm
   - Testing the null of a single cluster: Subsamples and permutations
   - Simulation results
3. Empirical evidence of risk price variation
   - Statistical evidence for segmentation
   - Economic magnitudes: $E[r]$ variation and Sharpe ratios
   - Detailed examples: US stocks, int'l stocks, multiple asset classes
   - Comparison of multiple-cluster and omitted-factor explanations
4. Summary and conclusion

Related Literature

- **Segmentation / Differences in Cross-Sectional Risk Premia**
  - Investors: Merton (1987); Kadlec & McConnell (1994); Foerster & Karolyi (1999)
  - Market Cap: Hong, Lim, and Stein (2000); Grinblatt & Moskowitz (2004); Israel & Moskowitz (2013)
  - Asset Class: Fama & French (1993); He, Kelly, & Manela (2016)
  - Region: Errunza & Losq (1985), Bekaert & Harvey (1995), Hou, Karolyi, & Kho (2011); Fama & French (2012), inter alia
- **Financial Frictions and Asset Prices**
  - Limited Arbitrage: Gârleanu & Pedersen (2011); Gromb & Vayanos (2002, 2018); Shleifer & Vishny (1997)
  - Limited Participation: Greenwald, Lettau, & Ludvigson (2016); Lettau, Ludvigson, & Ma (2018); Mankiw & Zeldes (1991)
- **Parameter Estimation with Unobserved Heterogeneity**
  - Clustering: MacQueen (1967) (k-means); Dempster, Laird, & Rubin (1977) (expectation maximization)
  - Panel Heterogeneity: Hahn & Moon (2010); Lin & Ng (2012); Saradis & Weber (2015); Bonhomme & Manresa (2015)

Group Assignment and Parameter Estimation

- Consider a candidate set of G groups of assets. We want to solve

$$\left(\hat{\Gamma}, \hat{\Lambda}\right) = \arg\min_{\Gamma,\Lambda} \sum_{i,t} \left(r_{it} - \alpha^{(\gamma_i)} - \sum_k \beta_{ik} \lambda_{kt}^{(\gamma_i)} \right)^2$$

  where

  - Γ is a $N \times 1$ vector of group assignments, $\gamma_i \in 1, \ldots, G$
  - Λ is a $T \times K \times G$ tensor of factor realizations, $\lambda_{kt}^{(g)}$ for each of T dates, K factors, and G groups

- That is a lot of parameters to estimate

- And we don't have differentiability for γ_i, which complicates most standard solution methods

Expectation Maximization to the Rescue

1. Fixing group assignments delivers cross-sectional slopes (λ) via OLS:

$$r_{it} = \alpha_t^{(g)} + \sum_k \beta_{ik} \lambda_{kt}^{(g)} + \epsilon_{it}, \ \forall \gamma_i = g, \, g = 1, ..., G, \, t = 1, \ldots, T$$

2. Fixing cross-sectional slopes delivers group assignments by minimizing fitting errors across groups for each stock

$$\gamma_i = \arg \min_{g \in \{1,...,G\}} \left\{ \left(r_{it} - \alpha_t^{(g)} - \sum_k \beta_{ik} \lambda_{kt}^{(g)} \right)^2 \right\}$$

- Iterating between holding fixed group assignments and lambdas is **expectation maximization**
  - Importantly this cycle converges to a (local) optimum
  - See the paper for discussion of multi-start and genetic algorithm methods used to achieve global optima (▸ Local and Global Optima)

Testing for Multiple Clusters

- Our EM approach finds group assignments and risk prices that maximize the explanatory power of a factor model given a fixed number of groups, G
- To address formally whether there is evidence of heterogeneous risk prices, we need to **test for multiple clusters**
  - Of course adding clusters improves model fit, but is the improvement in fit "big enough" to justify adding so many parameters?
- Note that **standard approaches to testing for segmentation fail:**
  1. A standard test comparing estimated risk prices leads to severe size distortions because groups are estimated
  2. Existing work that accounts for this estimation step, e.g. Bonhomme and Manresa (2015), requires clusters to be "well-separated," which is not true under the null of unified prices

Testing for Multiple Clusters

- We **split our data into subsamples**, $\mathcal{R}$ and $\mathcal{P}$, to overcome size distortion issues:
  1. Estimate cluster assignments on subsample $\mathcal{R}$ (impose "no small groups" assumption)
  2. Estimate cross-sectional slopes on $\mathcal{P}$, given $\hat{\Gamma}_R$, via G simple FMB regressions.
- If dependence between $\mathcal{R}$ and $\mathcal{P}$ samples is limited, this split eliminates the overfitting problem arising from estimated clusters

Test statistics and inference

- Our two test statistics are:

$$F^{Avg} = \frac{1}{(G-1)K} \sum_{g=1}^{G-1} \Delta\bar{\lambda}^{(g,g+1)\prime} \left(\hat{\Sigma}_{\bar{\lambda}^{(g)}} + \hat{\Sigma}_{\bar{\lambda}^{(g+1)}} \right)^{-1} \Delta\bar{\lambda}^{(g,g+1)}$$

$$F^{Dyn} = \frac{1}{(G-1)KP} \sum_{g=1}^{G-1} \sum_{t\in\mathcal{P}} \Delta\lambda_t^{(g,g+1)\prime} \left(\hat{\Sigma}_{\lambda_t^{(g)}} + \hat{\Sigma}_{\lambda_t^{(g+1)}} \right)^{-1} \Delta\lambda_t^{(g,g+1)}$$

- We obtain critical values for these using a **permutations** approach (Lehmann and Romano, 2005):
  - Compute the above test statistics for M randomly assigned group assignments (i.e., permutations)
  - p-value is proportion of permutation stats larger than the test stat
  - Not necessary if FMB model is correctly specified; much better finite-sample properties than standard F tests when **misspecified**

Simulation study

- We use US domestic equity portfolios (P3 in next section) to calibrate parameters of (null) DGP

$$r_{it} = \beta_i f_t + \epsilon_{it}$$

- 16 different specifications:
  1. "Daily" data or "Monthly" data ($T = 10000$ or $T = 300$)
  2. Small or large cross-section ($N = 75$ or $N = 225$)
  3. CAPM or Carhart factor model ($K = 1$ or $K = 4$)
  4. iid or GARCH in volatility [will only show GARCH results below]
- $M = 500$ permutations, $S = 500$ replications of each design.
- Tables below show rejection frequencies of 5% level tests.
- Computing time for this simulation study is $\approx 80,000$ CPU hours

Simulation results: T=300 months

Rejection frequencies are all reasonably close to 0.05, except when N,K are large

N	*K*	*Test*	**G=2**	**=3**	**=4**	**=5**	**∈2–5**
75	**1**	**Avg**	0.04	0.05	0.07	0.07	0.07
75	**4**	**Avg**	0.06	0.06	0.06	0.04	0.07
225	**1**	**Avg**	0.04	0.05	0.05	0.03	0.05
225	**4**	**Avg**	0.07	0.09	0.09	0.08	0.11
75	**1**	**Dyn**	0.05	0.05	0.05	0.13	0.08
75	**4**	**Dyn**	0.06	0.03	0.02	0.05	0.04
225	**1**	**Dyn**	0.06	0.06	0.05	0.06	0.06
225	**4**	**Dyn**	0.13	0.13	0.12	0.07	0.17

Simulation results: T=10,000 days

Rejection frequencies are all reasonably close to 0.05

N	*K*	*Test*	**G=2**	**=3**	**=4**	**=5**	$\in$**2–5**
75	**1**	**Avg**	0.06	0.08	0.07	0.05	0.07
75	**4**	**Avg**	0.05	0.05	0.04	0.05	0.07
225	**1**	**Avg**	0.04	0.06	0.07	0.06	0.05
225	**4**	**Avg**	0.06	0.06	0.06	0.06	0.08
75	**1**	**Dyn**	0.09	0.04	0.01	0.08	0.07
75	**4**	**Dyn**	0.04	0.01	0.03	0.07	0.04
225	**1**	**Dyn**	0.07	0.08	0.09	0.08	0.08
225	**4**	**Dyn**	0.07	0.08	0.07	0.04	0.06

Data

- Throughout, portfolios are our base unit of analysis (▸ Why Portfolios?)
- **Domestic Equities: 1963–2016 (Fama-French Daily)**
  - **P1** (N=75): 25 size-value, 25 size-market beta, 25 size-momentum
  - **P2** (N=115): P1 + 10 size, 10 B/M, 10 market beta, 10 momentum
  - **P3** (N=234): P2 + 49 industry, 10 investment, 10 profitability, 25 size-investment, 25 size-profitability
  - **P4** (N=100): 100 placebo portfolios with PERMNOs selected at random

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- **International Equities: 1991–2016 (Fama-French Monthly)**
  - **P5** (N=100): 25 size-value for North America, Europe, Japan, and Asia-Pacific regions
  - **P6** (N=200): P4 + 25 size-momentum for each region

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- **Multi-Asset Class: 1970–2012 (He-Kelly-Manela Monthly)**
  - **P7** (N=98): 25 size-value, 23 commodities, 10 maturity Treasury bonds, 10 yield corporate bonds, 18 moneyness-maturity-C/P options, 12 FX
  - **P8** (N=148): P6 + 25 size-market beta, 25 size-momentum

Segmentation Everywhere: Domestic Equity Portfolios

Dynamic test rejects everywhere, Avg test rejects less for P1

		Equal Avg Risk Prices		Equal Dyn Risk Prices	
	Model	1963–2016	1999–2016	1963–2016	1999–2016
P1	CAPM	0.312	0.561	0.026	0.006
	FF3F	0.057	0.011	0.000	0.000
	Carhart	0.050	0.102	0.000	0.000
	FF5F	0.078	0.375	0.000	0.000
	HKM	0.236	0.057	0.037	0.001
	HXZQ	0.000	0.671	0.000	0.000
	Carhart+3	0.005	1.000	0.000	0.000
P3	CAPM	0.052	0.006	0.000	0.000
	FF3F	0.000	0.000	0.000	0.000
	Carhart	0.000	0.000	0.000	0.000
	FF5F	0.000	0.000	0.000	0.000
	HKM	0.543	0.007	0.000	0.000
	HXZQ	0.000	0.000	0.000	0.000
	Carhart+3	0.000	0.004	0.000	0.000

Segmentation Everywhere: Placebo Portfolios

Neither test rejects more than expected by chance for placebo portfolios

		Equal Avg Risk Prices		**Equal Dyn Risk Prices**	
	Model	1963–2016	1999–2016	1963–2016	1999–2016
P4	CAPM	0.082	0.618	0.086	1.000
	FF3F	0.579	0.101	1.000	1.000
	Carhart	0.594	0.822	0.602	0.262
	FF5F	0.153	0.883	1.000	1.000
	HKM	1.000	0.711	0.015	0.466
	HXZQ	0.246	0.197	1.000	0.198
	Carhart+3	0.599	1.000	0.118	0.041

We find no segmentation when risk prices are the same!

(formalized in our simulation study)

Segmentation Everywhere: Int'l Equity Portfolios

Average and Dynamic tests reject everywhere

		Equal Avg Risk Prices		Equal Dyn Risk Prices	
	Model	1991–2016	2004–2016	1991–2016	2004–2016
P5	CAPM	0.000	0.000	0.000	0.000
	FF3F	0.000	0.000	0.000	0.000
	Carhart	0.000	0.000	0.000	0.000
	FF5F	0.000	0.000	0.000	0.000
	HKM	0.000	0.000	0.000	0.000
	HXZQ	–	–	–	–
	Carhart+3	0.002	0.001	0.000	0.000
P6	CAPM	0.000	0.000	0.000	0.000
	FF3F	0.000	0.000	0.000	0.000
	Carhart	0.000	0.000	0.000	0.000
	FF5F	0.000	0.000	0.000	0.000
	HKM	0.000	0.000	0.000	0.000
	HXZQ	–	–	–	–
	Carhart+3	0.000	0.000	0.000	0.000

Segmentation Everywhere: Multi-Asset Class Portfolios

Average and Dynamic tests reject everywhere

		Equal Avg Risk Prices		Equal Dyn Risk Prices	
	Model	1986–2010	1998–2010	1986–2010	1998–2010
P7	CAPM	0.000	0.000	0.000	0.000
	FF3F	0.000	0.000	0.000	0.000
	Carhart	0.000	0.000	0.000	0.000
	FF5F	0.000	0.000	0.000	0.000
	HKM	0.000	0.000	0.000	0.000
	HXZQ	0.000	0.000	0.000	0.000
	Carhart+3	0.000	0.000	0.000	0.000
P8	CAPM	0.000	0.000	0.000	0.000
	FF3F	0.000	0.000	0.000	0.000
	Carhart	0.000	0.000	0.000	0.000
	FF5F	0.000	0.000	0.000	0.000
	HKM	0.000	0.000	0.000	0.000
	HXZQ	0.000	0.000	0.000	0.000
	Carhart+3	0.000	0.000	0.000	0.000

Detailed Example: International Equity Portfolios

International Equity Portfolios (P6): Global Carhart, 1991–2016

Table: Determining the number of clusters

	# Clusters (G)					
	1	**2**	**3**	**4**	**5**	**2–5**
Avg test *p*-val	–	0.000	0.000	**0.000**	0.000	0.000
Dyn test *p*-val	–	0.000	0.000	**0.000**	0.000	0.000
LL ($\times 10^{-4}$)	5.498	6.003	6.195	**6.318**	6.328	
AIC ($\times 10^{-4}$)	-10.852	-11.718	-11.958	**-12.060**	-11.935	

Detailed Example: International Equity Portfolios

International Equity Portfolios (P6): Global Carhart, 1991–2016

Table: Parameter estimates of 1- and G^*- cluster models

	G=1	G=4				
	All	**Grp 1**	**Grp 2**	**Grp 3**	**Grp 4**	$p_F\left(\bar{\lambda}=\right)$
$\bar{\lambda}_{MKT}$	4.24	-1.57	-12.06	-0.55	2.03	0.12
t-stat	(0.73)	(-0.33)	(-2.38)	(-0.14)	(0.26)	
$\bar{\lambda}_{HML}$	1.07	2.86	9.09	4.38	6.48	0.11
t-stat	(0.42)	(1.22)	(2.36)	(1.35)	(2.22)	
$\bar{\lambda}_{SMB}$	0.05	2.82	-0.55	0.73	2.93	0.61
t-stat	(0.02)	(1.64)	(-0.16)	(0.37)	(1.02)	
$\bar{\lambda}_{UMD}$	8.07	5.79	18.69	12.16	4.00	0.00
t-stat	(2.79)	(1.96)	(3.59)	(3.70)	(0.93)	
R^2_G	0.76	0.94	0.89	0.95	0.94	
$R^2_{Combined}$	0.76		0.93			

Detailed Example: International Equity Portfolios

International Equity Portfolios (P6): Global Carhart, 1991–2016

Table: Estimated group memberships

	G=1	G=4			
	All	**Grp 1**	**Grp 2**	**Grp 3**	**Grp 4**
NA	50	50	0	0	0
AP	50	0	50	0	0
EU	50	0	0	50	0
JP	50	0	0	0	50
N_G	200	50	50	50	50
T	6783	312	312	312	312
Conjectured labels:		NA	AP	EU	JP

Interpretation: Regional stock markets are internally integrated and (perfectly) **externally segmented**

Detailed Example: Domestic Equity Portfolios

Domestic Equity Portfolios (P3): Domestic Carhart, 1963–2016

Table: Determining the number of clusters

	# Clusters (G)					
	1	**2**	**3**	**4**	**5**	**2–5**
Avg test *p*-val	–	**0.000**	0.487	0.490	0.000	0.000
Dyn test *p*-val	–	**0.000**	0.000	0.000	0.000	0.000
LL ($\times 10^{-6}$)	6.44	**6.51**	6.53	6.54	6.55	
AIC ($\times 10^{-6}$)	-12.81	**-12.89**	-12.86	-12.82	-12.79	

Detailed Example: Domestic Equity Portfolios

Domestic Equity Portfolios (P3): Domestic Carhart, 1963–2016

Table: Parameter estimates of 1- and G^*- cluster models

	G=1	G=2		
	All	**Grp 1**	**Grp 2**	$p_F\left(\bar{\lambda} =\right)$
$\bar{\lambda}_{MKT}$	-1.13	-0.52	2.33	0.20
t-stat	(-0.44)	(-0.16)	(1.02)	
$\bar{\lambda}_{HML}$	3.79	2.12	8.12	0.00
t-stat	(2.26)	(1.35)	(3.66)	
$\bar{\lambda}_{SMB}$	1.60	2.21	-2.54	0.03
t-stat	(0.98)	(1.21)	(-0.86)	
$\bar{\lambda}_{UMD}$	7.11	5.57	10.43	0.00
t-stat	(3.46)	(2.91)	(4.01)	
R^2_G	0.91	0.90	0.94	
$R^2_{Combined}$	0.91	0.92		

Detailed Example: Domestic Equity Portfolios

Domestic Equity Portfolios (P3): Domestic Carhart, 1963–2016

Table: Estimated group memberships

	G=1	G=2	
	All	Grp 1	Grp 2
ME 1-3	81	0	81
ME 4-5	54	54	0
Industry	49	44	5
Other	50	50	0
N_G	234	148	86
T	13469	13469	13469
	Conjectured labels:	Large cap.	Small cap.

Interpretation: Market capitalization is the **single most important** determinant of risk-price heterogeneity in domestic equity portfolios

Detailed Example: Multi-Asset Class Portfolios

Cross-Asset Class Portfolios (P8): He, Kelly, and Manela (2017) Factors, 1986–2010

Table: Determining the number of clusters

	# Clusters (G)					
	1	**2**	**3**	**4**	**5**	**2–5**
Avg test *p*-val	–	0.000	0.000	0.000	**0.000**	0.000
Dyn test *p*-val	–	0.000	0.000	0.000	**0.000**	0.000
LL ($\times 10^{-6}$)	5.19	5.46	5.55	5.61	**5.66**	
AIC ($\times 10^{-6}$)	-10.30	-10.75	-10.83	-10.88	**-10.89**	

Detailed Example: Multi-Asset Class Portfolios

Cross-Asset Class Portfolios (P8): He, Kelly, and Manela (2017) Factors, 1986–2010

Table: Parameter estimates of 1- and G^*- cluster models

	G=1	G=5					
	All	**G1**	**G2**	**G3**	**G4**	**G5**	$p_F\left(\bar{\lambda}=\right)$
$\bar{\alpha}$	0.62	-31.05	2.67	-0.13	11.42	0.54	0.00
t-stat	(6.41)	(-4.53)	(4.02)	(-0.05)	(2.76)	(6.81)	
$\bar{\lambda}_{MKT}$	7.14	45.85	10.18	10.57	-2.39	7.33	0.00
t-stat	(2.22)	(4.77)	(1.30)	(2.53)	(-0.48)	(2.10)	
$\bar{\lambda}_{HKM}$	9.30	-48.38	22.84	14.43	-8.47	9.91	0.06
t-stat	(1.18)	(-1.34)	(1.75)	(1.15)	(-0.87)	(1.14)	
R_G^2	0.74	0.98	0.58	0.85	0.91	0.84	
$R_{Combined}^2$	0.74			0.88			

Detailed Example: Multi-Asset Class Portfolios

Cross-Asset Class Portfolios (P8): He, Kelly, and Manela (2017) Factors, 1986–2010

Table: Estimated group memberships

	G=1	G=5				
	All	**G1**	**G2**	**G3**	**G4**	**G5**
Options	18	18	0	0	0	0
Commod.	23	0	14	5	0	4
US Bonds	20	0	16	0	0	4
FX	12	0	0	11	0	1
Stocks	75	2	0	16	46	11
N_G	148	20	30	32	46	20
T	300	300	300	300	300	300
Conjectured labels:		Options	Commod. / Bonds	FX+	Stocks	Other

Interpretation: Options, commodities and bonds, FX and some stock portfolios, and other domestic stock portfolios have **very different risk prices**, even when confronted by a unifying intermediary-asset pricing model

Man vs Machine: Ex ante vs estimated clusters

- We now consider portfolios hand-constructed to be related to market segmentation. We use four measures from the literature:
  1. **Institutional ownership ratio** = prop of equity held by institutional investors (Gompers and Metrick, 2001, *QJE*)
  2. **Institutional ownership concentration** = Herfindahl-Hirschman index of institutional ownership (Lewellen, 2011, *JFE*)
  3. **Average effective quoted spread** (Holden and Jacobsen, 2014, *JF*)
  4. **Price impact** (Holden and Jacobsen, 2014, *JF*)

  - We reverse the order of IOR so that all go from high to low liquidity
- We form one-way decile portfolios (+ pfs for IOC=1 and PI<0).
- We also form two-way quintile portfolios interacting liquidity with size.
- In total we have 42 one-way + 110 two-way sort portfolios.

Man vs Machine: Ex ante vs estimated clusters

Liquidity sort portfolios: Domestic Carhart, 1993–2016

Table: Parameter estimates of 1- and 2-cluster models

	G=1	**G=2**		
	All	**High liq**	**Low liq**	$p_F\ (\bar{\lambda} =)$
$\bar{\lambda}_{MKT}$	13.53	11.16	22.88	0.00
t-stat	(4.12)	(2.76)	(2.93)	
$\bar{\lambda}_{HML}$	2.33	0.46	4.31	0.00
t-stat	(1.19)	(0.21)	(1.44)	
$\bar{\lambda}_{SMB}$	1.24	2.40	-1.18	0.20
t-stat	(0.55)	(1.16)	(-0.41)	
$\bar{\lambda}_{UMD}$	12.02	10.99	26.59	0.00
t-stat	(3.45)	(1.85)	(3.60)	

Man vs Machine: Ex ante vs estimated clusters

Liquidity sort portfolios: Domestic Carhart, 1993–2016

Table: Determining the number of clusters

	# Clusters (G)					
	1	**2**	**3**	**4**	**5**	**2–5**
Avg test *p*-val	–	**0.000**	0.487	0.033	0.060	0.000
Dyn test *p*-val	–	**0.000**	0.000	0.000	0.001	0.000
LL ($\times 10^{-6}$)	6.44	**6.51**	6.53	6.54	6.55	
AIC ($\times 10^{-6}$)	-12.81	**-12.89**	-12.86	-12.82	-12.79	

Man vs Machine: Ex ante vs estimated clusters

Liquidity sort portfolios: Domestic Carhart, 1993–2016

Table: Estimated group memberships

	G=1	G=2	
	All	**Grp 1**	**Grp 2**
Liq only	42	41	1
Liq x ME 5	22	21	1
Liq x ME 1-4	88	0	88
N_G	152	62	90
	Conjectured labels:	Large cap.	Small cap.

Man vs Machine: Ex ante vs estimated clusters

- Using existing knowledge to form sort portfolios from measures of liquidity, we find strong evidence of differences in risk prices across U.S. equities.
  - This confirms many existing studies in this area.
- When we **estimate** the optimal groups for the portfolios, we also find strong evidence of differences in risk prices.
  - However we find that the leading source of heterogeneity among these portfolios is *market capitalization*, not liquidity.
- We also considered the same analysis using only the one-way liquidity sort portfolios, effectively shutting down heterogeneity in the size direction.
  - We again find significant evidence of market segmentation, and now the estimated groups are based on liquidity.

Conclusion

- We develop a new methodology for **detecting and estimating unobserved variation in risk prices**
- Frictions matter: **risk prices vary across assets**
  - Every portfolio set/factor model considered shows significant variation in λ
  - Gains from allowing for multiple clusters are comparable to gains from moving from a simpler factor model to a more sophisticated model (e.g., CAPM to FF3, or FF3 to FF5)
  - **New frontier: understand, explain, and exploit variation in λs**
- This feature poses a challenge to much of empirical asset pricing
  - Implications for portfolio choice, security pricing, performance evaluation
  - The zoo of **"expected return factors"** may be a side effect of **heterogeneous risk prices**

Segmentation Everywhere: Summary

- **Statistical evidence** of segmented markets is **ubiquitous**. For the tests of equal factor dynamics:
  1. Domestic equities: 80/81 tests reject the null of a single cluster
  2. International equities: all 36 tests reject with p-val=0.000
  3. Multi-asset class portfolios: all 42 tests reject with p-val=0.000
- Differences in **average** risk prices are also strongly significant, reject null for 57/81, 35/36 and 41/42 cases
- But are violations of unified risk pricing also **economically meaningful?**

Economic vs. Statistical Significance of Segmentation

- We measure **economic significance** in two ways:

  1. **Increased explanatory power** for cross-section of expected returns:

$$\frac{\sigma^2_{G*}\left(\bar{r}\right)}{\sigma^2_1\left(\bar{r}\right)} \equiv \frac{var_i\left(\frac{1}{T}\sum_{t=1}^{T}\left(\hat{\alpha}_t^{(\hat{\gamma}_i)} + \hat{\beta}_i\hat{\lambda}_t^{(\hat{\gamma}_i)}\right)\right)}{var_i\left(\frac{1}{T}\sum_{t=1}^{T}\left(\tilde{\alpha}_t + \hat{\beta}_i\tilde{\lambda}_t\right)\right)}$$

  2. Improvements of maximal, in-sample **Sharpe ratio**:

$$\Delta SR_{G*} \equiv \sqrt{\mu'_\Lambda \Sigma_\Lambda^{-1}\mu_\Lambda} - \sqrt{\mu'_\lambda \Sigma_\lambda^{-1}\mu_\lambda}$$

Economic Importance: Domestic Portfolios

Gains in explanatory power of around 15–70%, increases in SR of around 0.15–0.80

	Model	$\sigma^2(\bar{r}_{G*})$ / $\sigma^2(\bar{r}_1)$		$SR_{G*} - SR_1$	
		63–16	**99–16**	**63–16**	**99–16**
P1	CAPM	3.77	161.88	0.26	0.15
	FF3F	1.81	1.80	0.74	-0.09
	Carhart	1.03	1.38	0.10	0.16
	FF5F	*	1.10	*	0.15
	HKM	8.25	5.30	0.33	0.38
	HXZQ	1.16	2.67	0.67	0.29
	Carhart+3	*	*	*	*
P3	CAPM	2.75	6.16	0.17	0.48
	FF3F	1.67	1.75	0.82	0.47
	Carhart	1.41	1.55	0.86	0.85
	FF5F	1.49	1.22	0.54	0.14
	HKM	6.25	10.42	0.08	0.72
	HXZQ	1.51	2.39	0.69	0.69
	Carhart+3	1.20	1.23	0.51	0.32

Economic Importance: International Portfolios

Gains in explanatory power of 100-300%, increases in SR of around 0.4-0.8

		$\sigma^2(\bar{r}_{G*})$ / $\sigma^2(\bar{r}_1)$		$SR_{G*} - SR_1$	
	Model	**91–16**	**04–16**	**91–16**	**04–16**
P5	CAPM	7.20	1.34	0.55	0.06
	FF3F	5.00	1.25	0.51	0.13
	Carhart	5.61	1.07	1.31	0.25
	FF5F	1.33	1.11	0.54	0.92
	HKM	4.15	1.30	0.40	0.48
	HXZQ	–	–	–	–
	Carhart+3	2.22	1.12	0.64	0.77
P6	CAPM	3.98	1.21	0.89	0.71
	FF3F	3.06	1.46	1.13	0.93
	Carhart	4.07	1.37	1.62	0.75
	FF5F	2.10	1.07	1.27	0.70
	HKM	3.62	1.23	1.16	0.79
	HXZQ	–	–	–	–
	Carhart+3	2.34	1.08	1.61	0.99

Economic Importance: Multi-Asset Class Portfolios

Gains in explanatory power of 5-30%, increases in SR of around 0.4-0.9

		$\sigma^2(\bar{r}_{G*})$ / $\sigma^2(\bar{r}_1)$		$SR_{G*} - SR_1$	
	Model	**86-10**	**98-10**	**86-10**	**98-10**
P7	CAPM	11.97	69.22	1.03	1.70
	FF3F	1.33	1.84	0.55	0.87
	Carhart	0.81	1.13	0.61	0.68
	FF5F	1.15	2.15	0.42	0.93
	HKM	7.48	19.69	0.79	1.40
	HXZQ	1.05	2.69	0.44	1.06
	Carhart+3	0.90	0.97	0.56	1.12
P8	CAPM	4.93	5.95	1.11	0.68
	FF3F	1.27	1.34	0.80	1.26
	Carhart	1.08	2.48	0.87	1.80
	FF5F	1.23	2.80	0.95	1.93
	HKM	4.47	2.40	0.87	1.02
	HXZQ	1.21	1.56	0.90	1.41
	Carhart+3	1.18	1.08	1.47	1.37

Omitted Factors or Differences in Risk Prices?

- **Whence all the segmentation?**
- One possibility: omitted factors masquerade as clusters
- To see why, consider two simple models:

$$\text{Model 1:} \quad r_{it} = \alpha_t^{(1)} \mathbf{1}\left(i \in G_1\right) + \alpha_t^{(2)} \mathbf{1}\left(i \in G_2\right) + \epsilon_{it},$$
$$\text{Model 2:} \quad r_{it} = \tilde{\alpha}_t + \beta_i \eta_t + \tilde{\epsilon}_{it}.$$

- If the factor model (Model 2) is true, estimating the two-cluster model (Model 1) gives

$$\widehat{\Delta\alpha_t} = \eta_t \left(E\left[\beta_i | i \in G_1\right] - E\left[\beta_i | i \in G_2\right]\right).$$

$\Longrightarrow$ We obtain separation in cross-sectional slopes so long as the average loadings β_i differ between “clusters” and the factor is priced

- The reverse also occurs: clusters can manifest as new “factors”

Distinguishing Between Clusters and Factors

- Idea: Test whether **comparably parsimonious factor models** explain the cross-section of returns as well as the cluster model. Approach:

1. Choose the "best" cluster and factor models on the $\mathcal{R}$ sample
   - AIC-optimal number of clusters, G^*, and group assignments.
   - Extract K^* extra factors (PCAs), and store the PCA weights.
2. Compare the MSEs in the $\mathcal{P}$ sample using Rivers & Vuong (2002)
   - Although this is on the $\mathcal{P}$ sample, this is an "in-sample" comparison

- We use three choices for K^*:
  1. $K_1^* = 3$: an ad hoc, uniform choice for number of extra factors
  2. $K_2^* = G^* - 1$: the same number of additional partitions of the data
  3. $K_3^* =$ AIC-optimal, up to a maximum of $(G^* - 1)(K + 1) - 1$

Extra Clusters or Factors: Domestic Equity Portfolios

Omitted factors are comparably important for P1 and large factor models

		1963–2016			**1999–2016**		
	Model	K_1^*	K_2^*	K_3^*	K_1^*	K_2^*	K_3^*
	CAPM	−−	0	+++	−−	0	−−
	FF3F	−−−	0	0	−−	0	−−
	Carhart	+++	+++	+++	++	+++	+++
P1	FF5F	*	*	*	−	+++	0
	HKM	−−−	0	0	++	+++	+++
	HXZQ	+++	+++	+++	*	*	*
	Carhart+3	*	*	*	*	*	*
	CAPM	+++	+++	+++	0	−−	0
	FF3F	+++	+++	+++	0	+++	−−−
	Carhart	+++	+++	+++	+++	+++	−−
P3	FF5F	+++	+++	0	0	+++	−−
	HKM	0	+++	+++	+++	+++	−−−
	HXZQ	+++	+++	+++	+++	+++	0
	Carhart+3	+++	+++	+++	+++	+++	+++

Extra Clusters or Factors: International Portfolios

Multiple risk prices are generally favored, and universally so for more variegated portfolio sets

	Model	1991–2016			2004–2016		
		K_1^*	K_2^*	K_3^*	K_1^*	K_2^*	K_3^*
	CAPM	+++	+++	+++	0	+++	0
	FF3F	+++	+++	+++	−−−	+++	−−−
	Carhart	−−−	+++	−−−	−−−	+++	−−−
P5	FF5F	+++	+++	+++	+++	+++	+++
	HKM	+++	+++	+++	+++	+++	+++
	HXZQ						
	Carhart+3	+++	+++	+++	+++	+++	+++
	CAPM	+++	+++	+++	+++	+++	+++
	FF3F	+++	+++	0	+++	+++	+++
	Carhart	+++	+++	+++	+++	+++	+++
P6	FF5F	0	+++	0	+++	+++	+++
	HKM	+++	+++	+++	+++	+++	+++
	HXZQ						
	Carhart+3	+++	+++	+++	+++	+++	0

Extra Clusters or Factors: Multi-Asset Class Portfolios

Multiple risk prices are almost always strongly preferred

		1986–2010			1998–2010		
	Model	K_1^*	K_2^*	K_3^*	K_1^*	K_2^*	K_3^*
P7	CAPM	+++	+++	+++	+++	+++	+++
	FF3F	+++	+++	+++	+++	+++	+++
	Carhart	+++	+++	+++	+++	+++	+++
	FF5F	+++	+++	+++	+++	+++	++
	HKM	+++	+++	+++	+++	+++	+++
	HXZQ	+++	+++	+++	+++	+++	+++
	Carhart+3	+++	+++	+++	+++	+++	+++
P8	CAPM	+++	+++	++	+++	+++	0
	FF3F	+++	+++	0	+++	+++	+++
	Carhart	+++	+++	+++	+++	+++	+++
	FF5F	+++	+++	+++	+++	+++	0
	HKM	+++	+++	+++	+++	+++	+++
	HXZQ	+++	+++	+++	+++	+++	+++
	Carhart+3	+++	+++	+++	+++	+++	+++

Null and Alternative Hypotheses

- We consider two tests. The null in both is of **no segmentation / equal risk prices / no "very good deals"**:

$$H_0: \ \bar{\lambda}_k^{(1)} = \bar{\lambda}_k^{(2)} = \ldots = \bar{\lambda}_k^{(G)} \ \forall k$$
$$vs. \ \ H_1: \ \bar{\lambda}_k^{(g)} \neq \bar{\lambda}_k^{(g')} \text{ for some } k, g, g'.$$

  and

$$H_0: \ \lambda_{kt}^{(1)} = \lambda_{kt}^{(2)} = \ldots = \lambda_{kt}^{(G)} \ \forall k, t$$
$$vs. \ \ H_1: \ \lambda_{kt}^{(g)} \neq \lambda_{kt}^{(g')} \text{ for some } k, g, g', t.$$

- The first test generalizes FMB-style t tests to look at differences in **expected returns** across clusters
- The second tests enriches the first by adding information from the **dynamics** of cross-sectional slopes
- Note: these tests do not consider $\bar{\alpha}^{(g)}$ or $\alpha_t^{(g)}$ because our focus is on risk price heterogeneity, not on zero-beta rates

Why Portfolios? Back

- We use portfolios rather than individual stocks for several reasons:
  1. To increase the stability of security risk characteristics over time;
  2. To decrease the measurement error in betas through diversification of idiosyncratic risk;
  3. To reduce the sparsity of the matrix of realized returns; and
  4. To lessen computational cost (by 1–2 orders of magnitude)
- Merton (1973)'s intertemporal CAPM implies that all multifactor-minimum variance efficient investments are spanned by $K+1$ factor-mimicking portfolios $\Longrightarrow$ **"portfolios are enough"**
- Caveat: **cluster assignments obtained using portfolio returns do not generally apply to portfolio constituents**
  - Comovements among securities influence portfolio dynamics and cluster assignments

Local and Global Optima

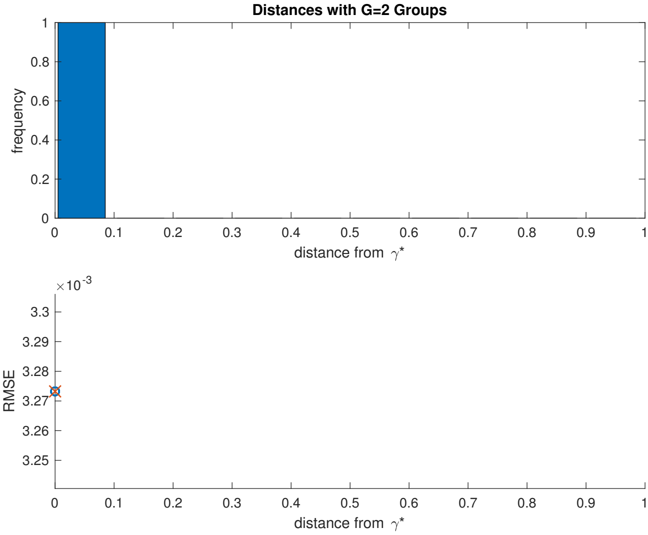
◂ Back

- We use two procedures to find global optima:
  1. Multi-start with $2N$ starting group assignments selected using a generalized version of k-means++
     - We use k-means++ initialization because our EM procedure can be recast as an extension of k-means
  2. Genetic algorithm solutions to optimal group assignment as a mixed-integer programming problem (MATLAB's implementation)
- Appendix A.1 of the paper provides further details
- Appendix A.2 demonstrates that global optimization is sometimes—but not always—important for obtaining the global-best group assignments

Example 1: Local and Global Optima Coincide

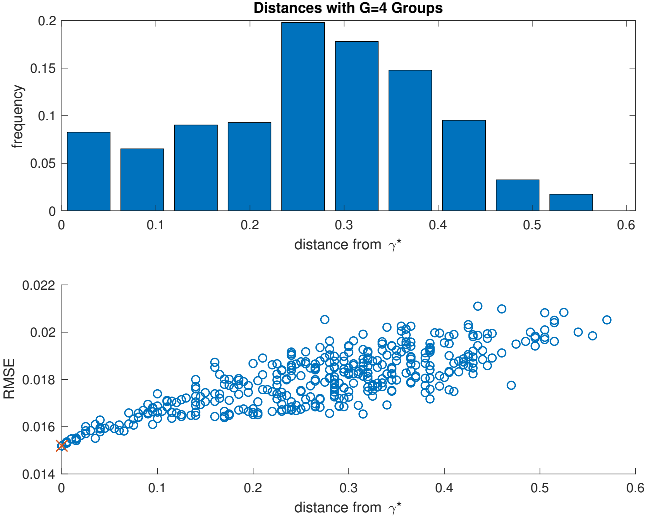
Back

Domestic Equity Portfolios: Domestic Carhart, 1963-2016



Example 2: Local and Global Optima are "Close" ◂ Back

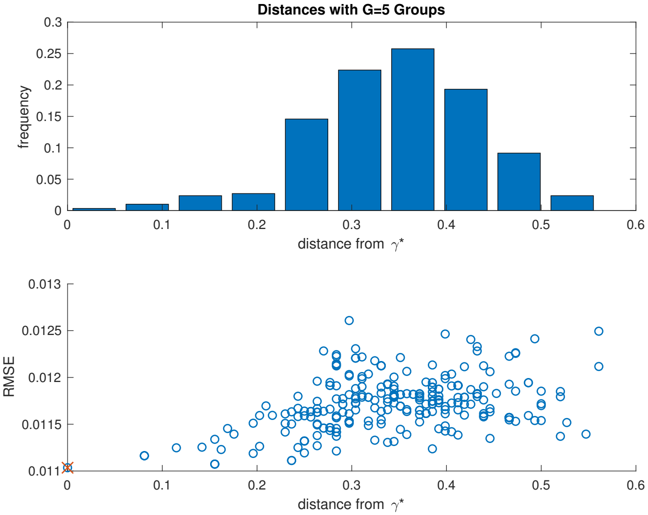
International Equity Portfolios: Global Carhart, 1991-2016



Example 3: Global Optimum is Isolated

Back

Cross-Asset Class Portfolios: He, Kelly, and Manela (2016) Factors, 1986-2010



Group Stability: Domestic Equity Portfolios

Back

Domestic equity portfolio assignments are stable over time

	Model	**P1**	**P2**	**P3**
Period	CAPM	0.89	0.64	0.61
2	FF3F	0.96	0.51	0.63
1981–1998	Carhart	0.92	0.82	**0.68**
Period	FF5F	0.85	0.96	0.90
3	HKM	0.81	0.77	0.69
1999–2016	HXZQ	0.85	0.60	0.51
	Carhart+3	1.00	0.82	0.58

Table reports maximal proportion of group labels in common over all permutations of group labels

Group Stability: International Equity Portfolios ◂ Back

International equity portfolio assignments are highly stable over time

	Model	P5	P6
Period	CAPM	0.87	0.68
1	FF3F	0.61	0.84
1991–2003	Carhart	0.74	**0.78**
Period	FF5F	0.99	0.70
2	HKM	0.55	0.91
2004–2016	HXZQ	–	–
	Carhart+3	0.59	0.65

Table reports maximal proportion of group labels in common over all permutations of group labels

Group Stability: Multi-Asset Class Portfolios

Back

Dimensions of heterogeneity among asset classes change over time

	Model	**P7**	**P8**
Period 1	CAPM	0.70	0.70
1986–1997	FF3F	0.56	0.57
	Carhart	0.57	0.55
Period 2	FF5F	0.73	0.76
1998–2010	HKM	0.62	**0.55**
	HXZQ	0.56	0.57
	Carhart+3	0.88	0.53

Table reports maximal proportion of group labels in common over all permutations of group labels