

# Daily House Price Indexes: Construction, Modeling, and Longer-Run Predictions\*

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## Abstract

We construct *daily* house price indexes for ten major U.S. metropolitan areas. Our calculations are based on a comprehensive database of several million residential property transactions and a standard repeat-sales method that closely mimics the procedure used in the construction of the popular monthly Case-Shiller house price indexes. Our new daily house price indexes exhibit dynamic features similar to those of other daily asset prices, with mild autocorrelation and strong conditional heteroskedasticity. The correlations across house price index returns are low at the daily frequency, but rise monotonically with the return horizon, and are commensurate with existing empirical evidence for existing monthly and quarterly house price series.

Timely and accurate measures of house prices are important in a variety of applications, and are particularly valuable during times of turbulence, such as the recent housing crisis. To quantify the informational advantage of our daily index, we show that a relatively simple multivariate time series model for the daily house price index returns, explicitly allowing for commonalities across cities and GARCH effects, produces forecasts of monthly house price changes that are superior to various alternative forecast procedures based on lower frequency data.

**Keywords:** Data aggregation; Real estate prices; Forecasting; Time-varying volatility

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# 1 Introduction

For many U.S. households their primary residence represents their single largest financial asset holding: the Federal Reserve estimated the total value of the U.S. residential real estate market at \$16 trillion at the end of 2011, compared with \$18 trillion for the U.S. stock market (as estimated by the Center for Research in Security Prices). Consequently, changes in housing valuations importantly affect households' saving and spending decisions, and in turn the overall growth of the economy. A number of studies (e.g., Case et al., 2011) have also argued that the wealth effect of the housing market for aggregate consumption is significantly larger than that of the stock market. The recent economic crisis, which arguably originated with the precipitous drop in housing prices beginning in 2006, directly underscores this point. Despite all of this, and in sharp contrast to most other financial asset classes, aggregate price indexes for residential real estate valuations are only available at relatively low monthly or quarterly frequencies.

Set against this background, we provide a new set of *daily* house price indexes for ten major U.S. metropolitan areas. To the best of our knowledge, this represents the first set of house price indexes at the daily frequency. Our construction is based on a comprehensive database of publicly recorded residential property transactions. We show that the dynamic dependencies in the new daily housing price series closely mimic those of other asset prices (see, e.g., Tsay, 2005, for a discussion of financial time series), and that these dynamic dependencies along with the cross-city correlations are well described by a standard multivariate GARCH type model. This relatively simple daily model in turn allows for the construction of improved longer-run monthly and quarterly housing price forecasts compared with forecasts based on existing monthly and/or quarterly indexes.

Our new daily house price indexes are based on the same “repeat-sales” methodology as the popular S&P/Case-Shiller monthly indexes (see Shiller, 1991), and the Office of Federal Housing Enterprise Oversight’s quarterly indexes. However, the coarser monthly and quarterly frequency of reporting employed in the existing indexes ignores potentially im-

portant information in the daily records of housing transactions, and is likely to result in “aggregation biases” if the true index changes at a higher frequency than the measurement period. Aggregating the indexes to lower frequencies also reduces their volatility, thereby underestimating the true risk of the housing market.

More timely house prices are of direct interest to policy makers, central banks, developers and lenders alike. Also, even though actual housing decisions are made relatively infrequently, potential buyers and sellers could still benefit from more timely price indicators. The need for higher frequency daily indexing is perhaps most acute in periods when prices change rapidly, with high volatility, as observed during the recent financial crisis and its aftermath. To illustrate, Figure 1 shows our new daily house price index along with the oft-cited monthly S&P/Case-Shiller index for Los Angeles from September 2008 through September 2010. The precipitous drop in the daily index over the first six months clearly leads the monthly index. Importantly, the daily index also shows the uptick in housing valuations that occurred around April 2009 some time in advance of the monthly index. Similarly, the more modest rebound that occurred in early 2010 is also first clearly manifest in the daily index.

Systematically analyzing the features of the dynamics of the new daily house price indexes for all of the ten metropolitan areas in our sample, we find that, in parallel to the daily returns on most other broadly defined asset classes, they exhibit only mild predictability in the mean, but strong evidence of volatility clustering. We show that the volatility clustering within and across the different house price indexes can be satisfactorily described by a multivariate GARCH model. The correlation between the daily returns on the city indexes is much lower than the correlation observed for the existing monthly return indexes. However, as we temporally aggregate the daily returns to monthly and quarterly frequencies, we find that the correlations increase to levels consistent with the ones observed for existing lower frequency indexes. Furthermore, we document that the new daily indexes do indeed result in improved forecasts, not solely in that they more

quickly identifying turning points as suggested by Figure 1 for Los Angeles, but also more generally for longer forecast horizons and other sample periods. This holds true for the city-specific housing returns and a composite index, thus directly underscoring the informational advantages of the new daily index developed here vis-a-vis the existing monthly published indexes.

The rest of the paper is organized as follows. The next section provides a review of house price index construction and formally describes the S&P/Case-Shiller methodology. Section 3 describes the data and the construction of our new daily prices series. Section 4 briefly summarizes the dynamic and cross-sectional dependencies in the daily series, and presents our simple multivariate GARCH model designed to account for these dependencies. Section 5 demonstrates how the new daily series and our modeling thereof may be used in more accurately forecasting the corresponding longer-run returns. Section 6 concludes. A Supplemental Appendix contains additional details and empirical results.

## 2 House price index methodologies

The construction of house price indexes is plagued by two major difficulties. Firstly, houses are heterogeneous assets; each house is a unique asset, in terms of its location, characteristics, maintenance status, etc., all of which affect its price. House price indexes aim to measure the price movements of a hypothetical house of average quality, with the assumption that average quality remains the same across time. In reality, average quality has been increasing over time, because newly-built houses tend to be of higher quality and more in line with current households' requirements than older houses. Detailed house qualities are not always available or not directly observable, so when measuring house prices at an aggregate level, it is difficult to take the changing average qualities of houses into consideration. The second major difficulty is sale infrequency. For example, the average time interval between two successive transactions of the same property is about six years in Los

Angeles, based on our data set described in Section 3 below. Related to that, the houses sold at each point in time may not be a representative sample of the overall housing stock.

Three main methodologies have been used to overcome the above-mentioned difficulties in the construction of reliable house price indexes (see, e.g., the surveys by Cho, 1996; Rappaport, 2007; Ghysels et al., 2013). The simplest approach relies on the median value of all transaction prices in a given period. The National Association of Realtors employ this methodology and publishes median prices of existing home sales monthly for both the national and four Census regions. The median price index has the obvious advantage of calculation simplicity, but it does not control for heterogeneity of the houses actually sold.

A second, more complicated, approach uses a hedonic technique, to price the “average quality” house by explicitly pricing its specific attributes. The U.S. Census Bureau constructs its Constant Quality (Laspeyres) Price Index of New One-Family Houses Sold using a hedonic method. Although this method does control for the heterogeneity of houses sold, it also requires much richer data than are typically available.

A third approach relies on repeat sales. This is the method used by both Standard & Poor’s and the Office of Federal Housing Enterprise Oversight (OFHEC). The repeat sales model was originally introduced by Bailey, Muth, and Nourse (1963), and subsequently modified by Case and Shiller (1989). The specific model currently used to construct the S&P/Case-Shiller indexes was proposed by Shiller (1991) (see Clapp and Giaccotto, 1992; Meese and Wallace, 1997, for a comparison of the repeat-sales method with other approaches).

As the name suggests, the repeat sales method estimates price changes by looking at repeated transactions of the same house. This provides some control for the heterogeneity in the characteristics of houses, while only requiring data on transaction prices and dates. The basic models, however, are subject to some strong assumptions (see, e.g., the discussion in Cho, 1996; Rappaport, 2007). Firstly, it is assumed that the quality of a given house remains unchanged over time. In practice, of course, the quality of most houses

changes through aging, maintenance or reconstruction. This in turn causes a so-called “renovation bias.” Secondly, repeat sales indexes exploit information only from houses that have been sold at least twice during the sampling period. This subset of all houses may not be representative of the entire housing stock, possibly resulting in a “sample-selection bias.” Finally, as noted above, all of the index construction methods are susceptible to “aggregation bias” if the true average house price fluctuates within the estimation window.

Our new daily home price indexes are designed to mimic the popular S&P/Case-Shiller house price indexes for the “typical” prices of single-family residential real estate. They are based on a repeat sales method and the transaction dates and prices for all houses that sold at least twice during the sample period. If a given house sold more than twice, then only the non-overlapping sale pairs are used. For example, a house that sold three times generates included sale pairs from the first and second transaction, and the second and third transaction; the pair formed by the first and third transaction is not included.

Specifically, for a house  $j$  that sold at times  $s$  and  $t$  at prices  $H_{j,s}$  and  $H_{j,t}$ , the repeat sales model postulates that,

$$\beta_t H_{j,t} = \beta_s H_{j,s} + \sqrt{2}\sigma_w w_{j,t} + \sqrt{(t-s)}\sigma_v v_{j,t}, \quad 0 \leq s < t \leq T, \quad (1)$$

with the value of the house price index at time  $\tau$  is defined by the inverse of  $\beta_\tau$ . The last two terms on the right-hand side account for “errors” in the sale pairs, with  $\sqrt{2}\sigma_w w_{j,t}$  representing the “mispricing error,” and  $\sqrt{(t-s)}\sigma_v v_{j,t}$  representing the “interval error.” Mispricing errors are included to allow for imperfect information between buyers and sellers, potentially causing the actual sale price of a house to differ from its “true” value. The interval error represents a possible drift over time in the value of a given house away from the overall market trend, and is therefore scaled by the (square root of the) length of the time interval between the two transactions. The error terms  $w_{j,t}$  and  $v_{j,t}$  are assumed independent and identically standard normal distributed.

The model in (1) and the corresponding error structure naturally lend itself to estimation by a multi-stage generalized least square type procedure (for additional details, see Case and Shiller, 1987). The base period of the S&P/ Case-Shiller indexes is January 2000. All index values prior to the base period are estimated simultaneously. After the base period, the index values are estimated using a chain-weighting procedure that conditions on all previous values. This chain-weighting procedure is used to prevent revisions of previously published index values. Finally, the S&P/Case-Shiller indexes are smoothed by repeating a given transaction in three successive months, so that the index for a given month is based on sale pairs for that month and the preceding two months (see the Index Construction Section of S&P/Case-Shiller Home Price Index Methodology).

### 3 Daily house price indexes

We focus our analysis on the ten largest Metropolitan Statistical Areas (MSAs), as measured in the year 2000 (further details pertaining to the counties included in each of the ten MSAs are provided in Table A.1 of the Supplementary Appendix).

#### 3.1 Data and data cleaning

The transaction data used in our daily index estimation is obtained from DataQuick, a property information company. This database contains detailed transactions of more than one hundred million properties in the United States. For most of the areas, the historical transaction records extends from the late 1990s to 2012, with some large metropolitan areas, such as Boston and New York, having transactions recorded as far back as 1987. Properties are uniquely identified by property IDs, which enable us to identify sale pairs. We rely U.S. Standard Use Codes contained in the DataQuick database to identify transactions of single-family residential homes.

Our data cleaning rules are based on the same filters used by S&P/Case-Shiller in the

construction of their monthly indexes. In brief, we remove any transaction that are not “arms length,” using a flag for such transactions available in the database. We also remove transactions with “unreasonably” low or high sale prices (below \$5000 or above \$100 million, and those generating an average annual return of below -50% or above 100%), as well as any sales pair with an interval of less than six months. Sale pairs are also excluded if there are indications that major improvements have been made between the two transactions, although such indications are not always present in the database. For the Los Angeles MSA, for example, this yields a total of 877,885 “clean” sale pairs, representing an average of 180 *daily* sale pairs over the estimation period. Additional details for all ten MSAs are provided in Table A.2 of the Supplementary Appendix.

### 3.2 Estimation of the daily index

The repeat-sales index estimation based on equation (1) is not computationally feasible at the daily frequency, as it involves the simultaneous estimation of several thousand parameters: the daily time spans for the ten MSAs range from 2837 for Washington D.C. to 4470 days for New York. To overcome this difficulty, we use an expanding-window estimation procedure: we begin by estimating daily index values for the final month in an initial start-up period, imposing the constraint that all of the earlier months in the period have only a single *monthly* index value. Restricting the daily values to be the same within each month for all but the last month drastically reduces the dimensionality of the estimation problem. We then expand the estimation period by one month, and obtain daily index values for the new “last” month. We continue this expanding estimation procedure through to the end of our sample period. (This estimation method results in an index that is “revision proof,” in that earlier values of the index do not change when later data becomes available.) Finally, similar to the S&P/Case-Shiller methodology, we normalize all of the individual indexes to 100 based on their average values in the year 2000.

One benefit of the estimation procedure we adopt is that it is possible to formally test



whether the “raw” daily price series actually exhibit significant intra-monthly variation. In particular, following the approach used by Calhoun et al. (1995) to test for “aggregation biases,” we test the null hypothesis that the estimates of  $\beta_{i,\tau}$  for MSA  $i$  are the same for all days  $\tau$  within a given calendar month against the alternative that these estimates differ within the month. These tests strongly reject the null for all months and all ten metropolitan areas; further details concerning the actual F-tests are available upon request. We show below that this statistically significant intra-monthly variation also translates into economically meaningful variation and corresponding gains in forecast accuracy compared to the forecasts based on coarser monthly index values only.

### 3.3 Noise filtering

The raw daily house price indexes are subject to measurement errors, due to the relatively few transactions that are available on a given day. (The average number ranges from 49 for Las Vegas to 180 for Los Angeles.) To help alleviate this problem, it is useful to further clean the data in an effort to extract more accurate estimates of the the true latent daily price series. We rely on a standard Kalman filter-based approach to do so. Specifically, let  $P_{i,t}$  denote the true latent index for MSA  $i$  at time  $t$ . We assume that the “raw” price indexes constructed in the previous section,  $P_{i,t}^* = 1/\beta_{i,t}$ , are related to the true latent price indexes by,

$$\log P_{i,t}^* = \log P_{i,t} + \eta_{i,t}, \quad (2)$$

where the  $\eta_{i,t}$  measurement errors are assumed to be serially uncorrelated. For simplicity of the filter, we further assume that the true index follows a random walk with drift,

$$r_{i,t} \equiv \Delta \log P_{i,t} = \mu_i + u_{i,t}, \quad (3)$$

where  $\eta_{i,t}$  and  $u_{i,t}$  are mutually uncorrelated. It follows readily by substitution that,

$$r_{i,t}^* \equiv \Delta \log P_{i,t}^* = r_{i,t} + \eta_{i,t} - \eta_{i,t-1}. \quad (4)$$

Combining (3) and (4), this in turn implies an MA(1) error structure for the “raw” returns, with the value of the MA coefficient determined by the variances of  $\eta_{i,t}$  and  $u_{i,t}$ ,  $\sigma_\eta^2$  and  $\sigma_u^2$ . This simple MA(1) structure is consistent with the sample autocorrelations for the raw return series reported in Figure A.1 in the Supplementary Appendix.

Interpreting equations (3) and (4) as a simple state-space system,  $\mu$ ,  $\sigma_\eta^2$  and  $\sigma_u^2$  may easily be estimated by standard (quasi-)maximum likelihood methods. This also allows for the easy filtration of the “true” daily returns,  $r_{i,t}$ , by a standard Kalman filter; see, e.g., Hamilton (1994). The Kalman filter implicitly assumes that  $\eta_{i,t}$  and  $u_{i,t}$  are *iid* normal. If the assumption of normality is violated, the filtered estimates are interpretable as best linear approximations. The Kalman filter parameter estimates reported in the Supplementary Appendix imply that the noise-to-signal ( $\sigma_\eta/\sigma_u$ ) ratios for the daily index returns range from a low of 6.48 (Los Angeles) to a high of 15.18 (Boston), underscoring the importance of filtering out the noise.

The filtered estimates of the latent “true” daily price series for Los Angeles are depicted in Figure 2 (similar plots for all ten cities are available in Figure A.2 in the Supplementary Appendix). For comparison, we also include the raw daily prices and the monthly S&P/Case-Shiller index. Looking first at the top panel for the year 2000, the figure clearly illustrates how the filtered daily index mitigates the noise in the raw price series. At the same time, the filtered prices also point to discernable within month variation compared to the step-wise constant monthly S&P/Case-Shiller index.

The bottom panel of Figure 2 reveals a similar story for the full 1995-2012 sample period. The visual differences between the daily series and the monthly S&P/Case-Shiller index are obviously less glaring on this scale. Nonetheless, the considerable (excessive) vari-

ation in the raw daily prices coming from the noise is still evident. We will consequently refer to and treat the filtered series as *the* daily house price indexes in the sequel.

Before turning to our empirical analysis and modeling of the dynamic dependencies in the daily series, it is instructive to more formally contrast the information inherent in the daily indexes with the traditional monthly S&P/Case-Shiller index.

### 3.4 Comparisons with the monthly S&P/Case-Shiller index

Like the monthly S&P/Case-Shiller indexes, our daily house price indexes are based on all publicly available property transactions. However, the complicated non-linear transformations of the data used in the construction of the indexes prevent us from expressing the monthly indexes as explicit functions of the corresponding daily indexes. Instead, as a simple way to help gauge the relationship between the indexes, and the potential loss of information in going from the daily to the monthly frequency, we consider the linear projection of the monthly S&P/Case-Shiller returns for MSA  $i$ , denoted  $r_{i,t}^{S\&P}$ , on 60 lagged values of the corresponding daily index returns,

$$r_{i,t}^{S\&P} = \delta(L)r_{i,t} + \varepsilon_{i,t} \equiv \sum_{j=0}^{59} \delta_j L^j r_{i,t} + \varepsilon_{i,t}, \quad (5)$$

where  $L^j r_{i,t}$  refers to the daily return on the  $j^{th}$  day before the last day of month  $t$ . (As discussed further below, all of the price series appear to be non-stationary. We consequently formulate the projection in terms of returns as opposed to the price levels.) The inclusion of 60 daily lags match the three-month smoothing window used in the construction of the monthly S&P/Case-Shiller indexes, discussed in Section 2. The true population coefficients in the linear  $\delta(L)$  filter are, of course, unknown, however they are readily estimated by ordinary least squares (OLS).

The OLS estimates for  $\delta_{j=0,\dots,59}$  obtained from the single regression that pools the returns for all ten MSAs are reported in the top panel of Figure 3. Each of the individual

coefficients are obviously subject to a fair amount of estimation error. At the same time, there is a clear pattern in the estimates for  $\delta_j$  across lags, naturally suggesting the use of a polynomial approximation in  $j$  to help smooth out the estimation error. The solid line in the figure shows the resulting nonlinear least squares (NLS) estimates obtained from a simple quadratic approximation. The corresponding  $R^2$ s for the unrestricted OLS and the NLS fit ( $\hat{\delta}_j = 0.1807 + 0.0101j - 0.0002j^2$ ) are 0.860 and 0.851, indicating only a slight deterioration in the accuracy of the fit by imposing a quadratic approximation to the lag coefficients. Moreover, even though the monthly S&P/Case-Shiller returns are not an exact linear function of the daily returns, the simple relationship dictated by  $\delta(L)$  accounts for the majority of the monthly variation.

To further illuminate the features of the approximate linear filter linking the monthly returns to the daily returns, consider the gain and the phase of  $\delta(L)$ ,

$$G(\omega) = \left[ \sum_{j=0}^{59} \sum_{k=0}^{59} \delta_j \delta_k \cos(|j-k|\omega) \right]^{1/2}, \quad \omega \in (0, \pi), \quad (6a)$$

$$\theta(\omega) = \tan^{-1} \left( \frac{\sum_{j=0}^{59} \delta_j \sin(j\omega)}{\sum_{j=0}^{59} \delta_j \cos(j\omega)} \right), \quad \omega \in (0, \pi). \quad (6b)$$

Looking first at the gains in Figures 3b and 3c, the unrestricted OLS estimates and the polynomial NLS estimates give rise to similar conclusions. The filter effectively down-weights all of the high-frequency variation (corresponding to periods less than around 70 days), while keeping all of the low-frequency information (corresponding to periods in excess of 100 days). As such, potentially valuable information for forecasting changes in house prices is obviously lost in the monthly aggregate. Further along these lines, Figures 3d and 3e show the estimates of  $\frac{\theta(\omega)}{\omega}$ , or the number of days that the filter shifts the daily returns back in time across frequencies. Although the OLS and NLS estimates differ somewhat for the very highest frequencies, for the lower frequencies (periods in excess of 60 days) the filter systematically shifts the daily returns back in time by about 30 days. This corresponds

roughly to one-half of the three month (60 business days) smoothing window used in the construction of the monthly S&P/Case-Shiller index.

In sum, the monthly S&P/Case-Shiller indexes essentially “kill” all of the within quarter variation inherent in the new daily indexes, while delaying all of the longer-run information by more than a month. We turn next to a more detailed analysis of the time series properties of the new daily indexes.

## 4 Time series modeling of daily housing returns

To facilitate the formulation of a multivariate model for all of the ten city indexes, we restrict our attention to the common sample period from June 2001 to September 2012. Excluding weekends and federal holidays, this yields 2,843 daily observations.

### 4.1 Summary statistics

Summary statistics for each of the ten daily series are reported in Table 1. Panel A gives the sample means and standard deviations for each of the index levels. Standard unit root tests clearly suggest that the price series are non-stationary, and as such the sample moments in Panel A need to be interpreted with care; further details concerning the unit root tests are available upon request. In the sequel, we therefore focus on the easier-to-interpret daily return series.

The daily sample mean returns reported in Panel B are generally positive, ranging a low of -0.006 (Las Vegas) to a high of 0.015 (Los Angeles and Washington D.C.). The standard deviation of the most volatile daily returns 0.599 (Chicago) is double that of the least volatile returns 0.291 (New York). The first-order autocorrelations are fairly close to zero for all of the cities, but the Ljung-Box  $\chi^2_{10}$  tests for up to tenth order serial correlation indicate significant longer-run dynamic dependencies in many of the series.

The corresponding results for the squared daily returns reported in Panel C indicate

very strong dynamic dependencies. This is also immediately evident from the plot of the ten daily return series in Figure 4, which show a clear tendency for large returns in an absolute sense to be followed by other large absolute returns. This directly mirrors the ubiquitous volatility clustering widely documented in the literature for other daily speculative returns (e.g., Tsay, 2005). Further, consistent with the evidence for other financial asset classes, there is also a clear commonality in the volatility patterns across the ten series.

## 4.2 Modeling conditional mean dependencies

The summary statistics discussed above point to the existence of some, albeit relatively mild, dynamic dependencies in the daily conditional means for most of the cities. Some of these dependencies may naturally arise from a common underlying dynamic factor that influences housing valuations nationally. In order to accommodate both city specific and national effects within a relatively simple linear structure, we postulate the following model for the conditional means of the daily returns,

$$E_{t-1}(r_{i,t}) = c_i + \rho_{i1}r_{i,t-1} + \rho_{i5}r_{i,t-5} + \rho_{im}r_{i,t-1}^m + b_{ic}r_{c,t-1}^m, \quad (7)$$

where  $r_{i,t}^m$  refers to the (overlapping) “monthly” returns defined by the summation of the corresponding daily returns,

$$r_{i,t}^m = \sum_{j=0}^{19} r_{i,t-j}, \quad (8)$$

and the composite (national) return  $r_{c,t}$  is defined as a weighted average of the individual city returns,

$$r_{c,t} = \sum_{i=1}^{10} w_i r_{i,t}, \quad (9)$$

with the weights identical to the ones used in the construction of the composite ten city monthly S&P/Case Shiller index, which are 0.212, 0.074, 0.089, 0.037, 0.050, 0.015, 0.055, 0.118, 0.272, and 0.078. The own fifth lag of the returns is included to account for any weekly calendar effects. The inclusion of the own monthly returns and the composite monthly

returns provides a parsimonious way of accounting for longer-run city-specific and common national dynamic dependencies. This particular formulation is partly motivated by the Heterogeneous Autoregressive (HAR) model proposed by Corsi (2009) for modeling so-called realized volatilities, and we will refer to it as an HAR-X model for short. We estimate this model for the conditional mean simultaneously with the model for the conditional variance described in the next section via quasi-maximum likelihood.

The estimation results in Table 2 reveal that the  $\rho_1$  and  $\rho_5$  coefficients associated with the own lagged returns are mostly, though not uniformly, insignificant when judged by the robust standard errors reported in parentheses. Meanwhile, the  $b_c$  coefficients associated with the composite monthly return are significant for nine out of the ten cities. Still, the one-day return predictability implied by the model is fairly modest, with the average daily  $R^2$  across the ten cities equal to 0.024, ranging from a low of 0.007 (Denver) to a high of 0.049 (San Francisco). This mirrors the low  $R^2$ 's generally obtained from time series modeling of other daily financial returns.

The adequacy of the common specification for the conditional mean in equation (7) is broadly supported by the tests for up to tenth-order serial correlation in the residuals  $\varepsilon_{i,t} \equiv r_{i,t} - E_{t-1}(r_{i,t})$  from the model reported in Panel C of Table 2. Only two of the tests are significant at the 5% level (San Francisco and Washington, D.C.) when judged by the standard  $\chi^2_{10}$  distribution. At the same time, the tests for serial correlation in the squared residuals  $\varepsilon_{i,t}^2$  from the model, given in the bottom two rows of Panel C, clearly indicate strong non-linear dependencies in the form of volatility clustering.

### 4.3 Modeling conditional variance and covariance dependencies

Numerous parametric specifications have been proposed in the literature to describe volatility clustering in asset returns. Again, in an effort to keep our modeling procedures simple and easy to implement, we rely on the popular GARCH(1,1) model (Bollerslev, 1986) for

describing the dynamic dependencies in the conditional variances for all of the ten cities,

$$Var_{t-1}(r_{i,t}) \equiv h_{i,t} = \omega_i + \kappa_i \varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}. \quad (10)$$

The results from estimating this model jointly with the the conditional mean model described in the previous section are reported in Panel B of Table 2 together with robust standard errors following Bollerslev and Wooldridge (1992) in parentheses.

The estimated GARCH parameters are all highly statistically significant and fairly similar across cities. Consistent with the results obtained for other daily financial return series, the estimates for the sum  $\kappa + \lambda$  are all very close to unity (and just above for Chicago, at 1.002) indicative of a highly persistent, but eventually mean-reverting, time-varying volatility process.

Wald tests for up to tenth-order serial correlation in the resulting standardized residuals,  $\varepsilon_{i,t}/h_{i,t}^{1/2}$ , reported in Panel C, suggest that little predictability remains, with only one city (San Francisco) rejecting the null of no autocorrelation. The tests for serial correlation in the squared standardized residuals,  $\varepsilon_{i,t}^2/h_{i,t}$ , reject the null for four cities, perhaps indicative of some remaining predictability in volatility not captured by this relatively simple model. However for the majority of cities the specification in equation (10) appears to provide a satisfactory fit. The dramatic reduction in the values of the test statistics for the squared residuals compared to the values reported in the second row of Panel C is particularly noteworthy.

The univariate HAR-X-GARCH models defined by equations (7) and (10) indirectly incorporate commonalities in the cross-city returns through the composite monthly returns  $r_{c,t}$  included in the conditional means. The univariate models do not, however, explain the aforementioned commonalities in the volatilities observed across cities and the corresponding dynamic dependencies in the conditional covariances of the returns.

The Constant Conditional Correlation (CCC) model proposed by Bollerslev (1990)



provides a particularly convenient framework for jointly modeling the ten daily return series by postulating that the temporal variation in the conditional covariances are proportional to the products of the conditional standard deviations. Specifically, let  $\mathbf{r}_t \equiv [r_{1,t}, \dots, r_{10,t}]'$  and  $D_t \equiv \text{diag}\{h_{1t}^{1/2}, \dots, h_{10,t}^{1/2}\}$  denote the  $10 \times 1$  vector of daily returns and  $10 \times 10$  diagonal matrix with the GARCH conditional standard deviations along the diagonal, respectively. The GARCH-CCC model for the conditional covariance matrix of the returns may then be succinctly expressed as,

$$\text{Var}_{t-1}(\mathbf{r}_t) = D_t R D_t, \quad (11)$$

where  $R$  is a  $10 \times 10$  matrix with ones along the diagonal and the conditional correlations in the off-diagonal elements. Importantly, the  $R$  matrix may be efficiently estimated by the sample correlations for the  $10 \times 1$  vector of standardized HAR-X-GARCH residuals; i.e., the estimates of  $D_t^{-1} [\mathbf{r}_t - E_{t-1}(\mathbf{r}_t)]$ . The resulting estimates are reported in Table A.5 in the Supplementary Appendix.

We also experimented with the estimation of the Dynamic Conditional Correlation (DCC) model of Engle (2002), resulting in only a very slight increase in the maximized value of the (quasi-) log-likelihood function. Hence, we conclude that the relatively simple multivariate HAR-X-GARCH-CCC model defined by equations (7), (10), and (11) provides a satisfactory fit to the joint dynamic dependencies in the conditional first and second order moments of the ten daily housing return series.

#### 4.4 Temporal aggregation and housing return correlations

The estimated conditional correlations from the HAR-X-GARCH-CCC model for the daily index returns reported in the Supplementary Appendix average only 0.022. By contrast the unconditional correlations for the monthly S&P/Case Shiller indexes calculated over the same time period average 0.708, and range from 0.382 (Denver–Las Vegas) to 0.926

(Los Angeles–San Diego). The discrepancy between the two sets of numbers may appear to call into question the integrity of our new daily indexes and/or the time-series models for describing the dynamic dependencies therein, however conditional daily correlations and the unconditional monthly correlations are not directly comparable. In an effort to more directly compare the longer-run dependencies inherent in our new daily indexes with the traditional monthly S&P/Case Shiller indexes, we aggregate our daily return indexes to a monthly level by summing the daily returns within a month (20 days). The unconditional sample correlations for these new monthly returns are reported in the lower triangle of Panel B in Table 3. These numbers are obviously much closer, but generally still below the 0.708 average unconditional correlation for the published monthly S&P/Case Shiller indexes.

However, as previously noted, the monthly S&P/Case Shiller indexes are artificially “smoothed,” by repeating each sale pair in the two months following the actual sale. As such, a more meaningful comparison of the longer-run correlations inherent in our new daily indexes with the correlations in the S&P/Case Shiller indexes is afforded by the unconditional quarterly (60 days) correlations reported in the upper triangle of Panel B in Table 3. There, we find an average correlation of 0.668, and a range of 0.317 (Denver–Las Vegas) to 0.906 (Los Angeles–San Diego), which are quite close to the corresponding numbers for the published S&P/Case Shiller indexes.

These comparisons, of course, say nothing about the validity of the HAR-X-GARCH-CCC model for the daily returns, and the low daily *conditional* correlations estimated by that model. As a further model specification check, we therefore also consider the model-implied longer-run correlations, and study how these compare with the sample correlations for the actual longer-run aggregate returns.

The top number in each element of Panels A and B of Table 3 gives the median model-implied unconditional correlations for the daily, weekly, monthly, and quarterly return horizons, based on 500 simulated sample paths. The bottom number in each element is the

corresponding sample correlations for the actual longer-run aggregated returns. Although the daily unconditional correlations in Panel A are all close to zero, the unconditional correlations implied by the model gradually increase with the return horizon, and almost all of the quarterly correlations are in excess of one-half. Importantly, the longer-run model-implied correlations are all in line with their unconditional sample analogues.

To further illuminate this feature, Figure 5 presents the median model-implied and sample correlations for return horizons ranging from one-day to a quarter, along with the corresponding simulated 95% confidence intervals implied by the model for the Los Angeles–New York city pair. The model provides a very good fit across all horizons, with the actual correlations well within the confidence bands. The corresponding plots for all of the 45 city pairs, presented in Figure A.3 in the Supplementary Appendix, tell a similar story.

Taken as whole these results clearly support the idea that the longer-run cross-city dependencies inherent in our new finer sample daily house price series are consistent with those in the published coarser monthly S&P/Case Shiller indexes. The results also confirm that the joint dynamic dependencies in the daily returns are well described by the relatively simple HAR-X-GARCH-CCC model, in turn suggesting that this model could possibly be used in the construction of improved house price forecasts over longer horizons.

## 5 Forecasting housing returns

One of the major potential benefits from higher frequency data is the possibility of constructing more accurate forecasts by using models that more quickly incorporate new information. The plot for Los Angeles discussed in the introduction alludes to this idea. In order to more rigorously ascertain the potential improvements afforded by the daily house price series and our modeling thereof, we consider a comparison of the forecasts from the daily HAR-X-GARCH-CCC model with different benchmark alternatives.

Specifically, consider the problem of forecasting the 20-day (“monthly”) return on the

house price index for MSA  $i$ ,

$$r_{i,t}^{(m)} \equiv \sum_{j=0}^{19} r_{i,t-j} \quad (12)$$

for forecast horizons ranging from  $h = 20$  days ahead to  $h = 1$  day ahead. When  $h = 20$  this corresponds to a simple one-step ahead forecast for one-month returns, but for  $h < 20$  an optimal forecast will contain a mixture of observed data and a forecast for the return over the remaining part of the month. We will use the period June 2001 to June 2009 as our in-sample period, and the period July 2009 to September 2012 as our out-of-sample period, with all of the model parameters estimated once over the fixed in-sample period.

Our simplest benchmark forecast is based purely on end-of-month data, and is therefore *not* updated as the horizon shrinks. We will consider a simple AR(1) for these monthly returns,

$$r_{i,t}^{(m)} = \phi_0 + \phi_1 r_{i,t-20}^{(m)} + e_{i,t}. \quad (13)$$

As the forecast is not updated through the month, the forecast made at time  $t-h$  is simply the AR(1) forecast made at time  $t-20$ ,

$$\hat{r}_{i,t-h}^{Mthly} = \hat{\phi}_0 + \hat{\phi}_1 r_{i,t-20}^{(m)}. \quad (14)$$

Our second benchmark forecast is again purely based on monthly data, but now we allow the forecaster to update the forecast at time  $t-h$ , which may be in the middle of a month. We model the incorporation of observed data by allowing the forecaster to take a linear combination of the monthly return observed on day  $t-h$  and the one-month-ahead forecast made on that day,

$$\hat{r}_{i,t-h}^{Interp} = \left(1 - \frac{h}{20}\right) r_{i,t-h}^{(m)} + \frac{h}{20} \left(\hat{\phi}_0 + \hat{\phi}_1 r_{i,t-h}^{(m)}\right). \quad (15)$$

Our third forecast fully exploits the daily return information, by using the actual returns from time  $t-19$  to  $t-h$  as the first component of the forecast, as these are part of

the information set at time  $t - h$ , and then using a “direct projection” method to obtain a forecast for the remaining  $h$ -day return based on the one-month return available at time  $t - h$ . Specifically,

$$\hat{r}_{i,t-h}^{Direct} = \sum_{j=h}^{19} r_{i,t-j} + \hat{\beta}_0^{(h)} + \hat{\beta}_1^{(h)} r_{i,t-h}^{(m)}, \quad (16)$$

where  $\beta_0^{(h)}$  and  $\beta_1^{(h)}$  are estimated from the projection:

$$\sum_{j=0}^{h-1} r_{i,t-j} = \beta_0^{(h)} + \beta_1^{(h)} r_{i,t-h}^{(m)} + u_{i,t}. \quad (17)$$

Finally, we consider a forecast based on the HAR-X-GARCH-CCC model presented in the previous section. Like the third forecast, this forecast uses the actual returns from time  $t - 19$  to  $t - h$  as the first component, and then iterates the expression for the conditional daily mean in equation (7) forward to get forecasts for the remaining  $h$  days,

$$\hat{r}_{i,t-h}^{HAR} = \sum_{j=h}^{19} r_{i,t-j} + \sum_{j=0}^{h-1} \hat{E}_{t-h} [r_{i,t-j}]. \quad (18)$$

Given the construction of the target variable, we expect the latter three forecasts (“Interp”, “Direct”, “HAR”) to all beat the “Mthly” forecast for all horizons less than 20 days. If intra-monthly returns have dynamics that differ from those of monthly returns, then we expect the latter two forecasts to beat the “Interp” forecast. Finally, if the HAR-X-GARCH-CCC model presented in the previous section provides a better description of the true dynamics than a simple direct projection, then we would expect the fourth forecast to beat the third.

Figure 6 shows the resulting Root Mean Squared Errors (RMSEs) for the four forecasts as a function of the forecast horizon, when evaluated over the July 2009 to September 2012 out-of-sample period. The first striking, though not surprising, feature is that exploiting higher frequency (intra-monthly) data leads to smaller forecast errors than a forecast based purely on monthly data. All three of the forecasts that use intra-monthly infor-

mation out-perform the model based solely on end-of-month data. The only exception to this is for Las Vegas at the  $h = 20$  horizon, where the HAR model slightly under-performs the monthly model.

Another striking feature of Figure 6 is that the more accurate modeling of the daily dynamic dependencies afforded by the HAR-X-GARCH-CCC model results in lower RMSEs across *all* forecasts horizons for eight of the ten cities. For San Francisco and Las Vegas the direct projection forecasts perform essentially as well as the HAR forecasts, and for Denver and Los Angeles the improvement of the HAR forecast is small (but positive for all horizons). For some of the cities (Boston, Miami and Washington D.C., in particular) the improvements are especially dramatic over longer horizons.

The visual impression from Figure 6 is formally underscored by Diebold-Mariano tests, reported in Table 4. Not surprisingly, the HAR forecasts significantly outperform the monthly forecasts for horizons of 1, 5 and 10 days, for all ten cities and the composite index. At the one-month horizon, a tougher comparison for the model, the HAR forecasts are significantly better than the monthly model forecasts for four out of ten cities, as well as the composite index, and are never significantly beaten by the monthly model forecasts. Almost identical conclusions are drawn when comparing the HAR forecasts to the “interpolation” forecasts, supporting the conclusion that the availability of daily data clearly holds the promise of more accurate forecasts, particularly over shorter horizons, but also even at the monthly level.

The bottom row of each panel in Table 4 compares the HAR forecasts with those from a simple direct projection model. Such forecasts have often been found to perform well in comparison with “iterated” forecasts from more complicated dynamic models. By contrast, the Diebold-Mariano tests reported here suggest that the more complicated HAR forecasts generally perform better than the direct projection forecasts. For no city-horizon pair does the direct projection forecast lead to significantly lower out-of-sample forecast RMSE than the HAR forecasts, while for many city-horizon pairs the reverse is true. In particular, for

Boston, Miami and Washington D.C., the HAR forecasts significantly beat the direct projection forecasts across all four horizons, and for the composite index this is true for all but the shortest horizon.

## 6 Conclusion

We present a set of new *daily* house price indexes for ten major U.S. Metropolitan Statistical Areas spanning the period from June 2001 to September 2012. The indexes are based on the repeat sales method of Shiller (1991), and use a comprehensive database of several million publicly recorded residential property transactions. We demonstrate that the dynamic dependencies in the new daily housing price series closely mimic those of other financial asset prices, and that the dynamics, along with the cross-city correlations, are well described by a standard multivariate GARCH-type model. We find that this simple daily model allows for the construction of improved daily, weekly, and monthly housing price forecasts compared to the forecasts based solely on monthly price indexes.

The new “high frequency” house price indexes developed here open the possibility for many other applications. Most directly, by providing more timely estimates of movements in the housing market, the daily series should be of immediate interest to policy makers and central banks. In a related context, the series may also prove useful in further studying the microstructure of the housing market. At a broader level, combining the daily house price series with other daily estimates of economic activity should afford better and more up-to-date insights into changes in the macro economy. Along these lines, the series also hold the promise for the construction of more accurate forecasts for other macro economic and financial time series. We leave all of these issues for future research.

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**Table 1:** Daily summary statistics

	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
	<u>Panel A: Level</u>									
Mean	177.764	145.561	128.901	118.049	162.896	136.511	164.473	137.702	159.450	170.039
Std. dev.	41.121	13.381	21.631	4.605	48.351	48.568	34.058	27.169	25.877	34.830
	<u>Panel B: Returns</u>									
Mean	0.015	0.008	-0.002	0.003	0.006	-0.006	0.010	0.005	0.011	0.015
Std.dev.	0.347	0.351	0.599	0.303	0.428	0.370	0.387	0.509	0.291	0.502
AR(1)	-0.059	0.047	0.008	-0.018	-0.034	0.061	-0.005	-0.113	0.049	-0.018
LB(10)	67.877	21.935	24.362	16.838	17.742	59.549	15.065	269.509	13.335	24.977
	<u>Panel C: Squared returns</u>									
Mean	0.121	0.123	0.358	0.092	0.183	0.137	0.150	0.259	0.085	0.252
Std. dev.	0.200	0.260	1.269	0.242	0.336	0.369	0.270	0.616	0.170	0.607
AR(1)	0.113	0.102	0.075	0.021	0.107	0.071	0.037	0.042	0.042	0.132
LB(10)	182.307	109.914	102.316	33.414	445.189	85.348	50.715	179.632	53.109	106.434

Note: The table reports summary statistics for each of the ten MSAs for the June 2001 to September 2012 sample period, a total of 2,843 daily observations. AR(1) denotes the first order autocorrelation coefficient. LB(10) refers to the Ljung-Box portmanteau test for up to tenth order serial correlation. The 95% critical value for this test is 18.31.

**Table 2:** Daily HAR-X-GARCH models

$$r_{i,t} = c_i + \rho_{i,1}r_{i,t-1} + \rho_{i,5}r_{i,t-5} + \rho_{i,m}r_{i,t-1}^m + b_{i,c}r_{c,t-1}^m + \varepsilon_{i,t}$$

$$\varepsilon_{i,t}|\Omega_{t-1} \sim N(0, h_{i,t})$$

$$h_{i,t} = \omega_i + \kappa_i\varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}$$

	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
<u>Panel A: Mean</u>										
$c (\times 10^{-2})$	1.710 (0.678)	-0.302 (0.769)	0.094 (0.163)	-0.074 (5.338)	1.152 (0.942)	-0.111 (0.368)	0.240 (3.221)	-0.222 (0.223)	0.908 (0.538)	1.245 (0.884)
$\rho_1$	-0.080 (0.020)	0.030 (0.022)	0.005 (0.011)	-0.015 (0.052)	-0.034 (0.020)	0.004 (0.016)	-0.037 (0.020)	-0.094 (0.018)	0.040 (0.020)	0.012 (0.024)
$\rho_5$	0.054 (0.020)	0.009 (0.017)	-0.006 (0.010)	0.010 (0.101)	-0.006 (0.032)	0.006 (0.039)	-0.036 (0.022)	0.160 (0.022)	0.004 (0.017)	0.032 (0.020)
$\rho_m$	-0.014 (0.007)	-0.014 (0.005)	-0.023 (0.007)	-0.011 (0.008)	-0.008 (0.006)	0.017 (0.004)	-0.013 (0.006)	-0.014 (0.006)	-0.029 (0.006)	-0.035 (0.007)
$b_c$	0.059 (0.009)	0.039 (0.007)	0.049 (0.008)	0.020 (0.018)	0.060 (0.008)	0.035 (0.007)	0.060 (0.010)	0.056 (0.009)	0.054 (0.006)	0.084 (0.010)
$R^2$	0.039	0.018	0.009	0.007	0.027	0.044	0.030	0.049	0.033	0.027

**Table 2:** Continued

	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
<u>Panel B: Variance</u>										
$\omega (\times 10^{-2})$	0.013	0.230	0.075	0.215	0.016	0.014	0.024	0.023	0.041	0.067
	(0.015)	(0.074)	(0.058)	(0.103)	(0.014)	(0.013)	(0.028)	(0.026)	(0.023)	(0.043)
$\kappa$	0.020	0.056	0.056	0.034	0.013	0.017	0.014	0.016	0.026	0.032
	(0.008)	(0.010)	(0.009)	(0.012)	(0.003)	(0.006)	(0.007)	(0.006)	(0.005)	(0.006)
$\lambda$	0.979	0.926	0.946	0.943	0.986	0.982	0.985	0.983	0.969	0.965
	(0.009)	(0.012)	(0.009)	(0.017)	(0.002)	(0.006)	(0.008)	(0.007)	(0.006)	(0.007)
$\kappa + \lambda$	0.999	0.982	1.002	0.977	0.999	0.999	0.999	0.999	0.995	0.998
<u>Panel C: Serial correlation tests</u>										
$\varepsilon_{i,t}$	16.325	10.934	15.178	11.144	8.952	18.086	8.953	25.641	7.133	18.906
	(0.091)	(0.363)	(0.126)	(0.346)	(0.537)	(0.054)	(0.537)	(0.004)	(0.713)	(0.042)
$\varepsilon_{i,t}^2$	92.430	62.011	56.910	22.875	150.471	46.849	41.513	72.156	36.577	36.247
	(0.000)	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\varepsilon_{i,t} h_{i,t}^{-1/2}$	11.003	11.878	15.071	14.344	6.576	20.148	7.677	18.762	6.386	12.855
	(0.357)	(0.293)	(0.130)	(0.158)	(0.765)	(0.028)	(0.660)	(0.043)	(0.782)	(0.232)
$\varepsilon_{i,t}^2 h_{i,t}^{-1}$	12.511	24.289	24.616	25.424	9.426	4.946	16.156	40.312	8.650	11.998
	(0.252)	(0.007)	(0.006)	(0.005)	(0.492)	(0.895)	(0.095)	(0.000)	(0.566)	(0.285)

Note: Panel A and B report Quasi Maximum Likelihood Estimates (QMLE) of HAR-X-GARCH models with robust standard errors in parentheses. Panel C reports Wald test statistics for up to tenth order serial correlation in the (squared) residuals and standardized residuals, with corresponding p-values in parentheses.

**Table 3:** Unconditional return correlations for different return horizons

	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
	Panel A: Daily (lower triangle) and Weekly (upper triangle)									
Los Angeles	–	0.117 0.065	0.061 0.124	0.066 0.073	0.197 0.219	0.172 0.250	0.198 0.240	0.280 0.309	0.164 0.145	0.156 0.204
Boston	0.017 0.026	–	0.033 0.068	0.068 0.128	0.139 0.130	0.133 0.121	0.143 0.063	0.118 0.054	0.105 0.128	0.120 0.129
Chicago	0.002 0.019	−0.007 −0.001	–	0.025 0.108	0.077 0.149	0.058 0.064	0.049 0.042	0.084 0.148	0.102 0.115	0.068 0.089
Denver	0.001 −0.003	0.023 0.031	−0.002 −0.003	–	0.105 0.100	0.092 0.110	0.100 0.090	0.060 0.106	0.053 0.006	0.084 0.090
Miami	0.072 0.069	0.047 0.043	0.024 0.046	0.044 0.047	–	0.173 0.239	0.178 0.214	0.165 0.176	0.187 0.169	0.150 0.183
Las Vegas	0.060 0.077	0.051 0.049	0.015 0.032	0.038 0.027	0.053 0.054	–	0.165 0.209	0.147 0.162	0.123 0.060	0.142 0.173
San Diego	0.077 0.072	0.059 0.053	−0.006 0.022	0.045 0.042	0.056 0.060	0.058 0.065	–	0.171 0.263	0.148 0.169	0.137 0.127
San Francisco	0.183 0.235	0.037 0.038	0.037 0.065	0.006 −0.003	0.057 0.060	0.052 0.068	0.069 0.066	–	0.138 0.137	0.136 0.151
New York	0.032 0.041	0.011 0.000	0.047 0.061	−0.009 −0.002	0.065 0.063	0.010 −0.002	0.027 0.029	0.024 0.031	–	0.149 0.088
Washington, D.C.	0.047 0.045	0.038 0.034	0.017 0.024	0.032 0.041	0.041 0.038	0.049 0.034	0.033 0.027	0.038 0.038	0.044 0.043	–

**Table 3:** Continued

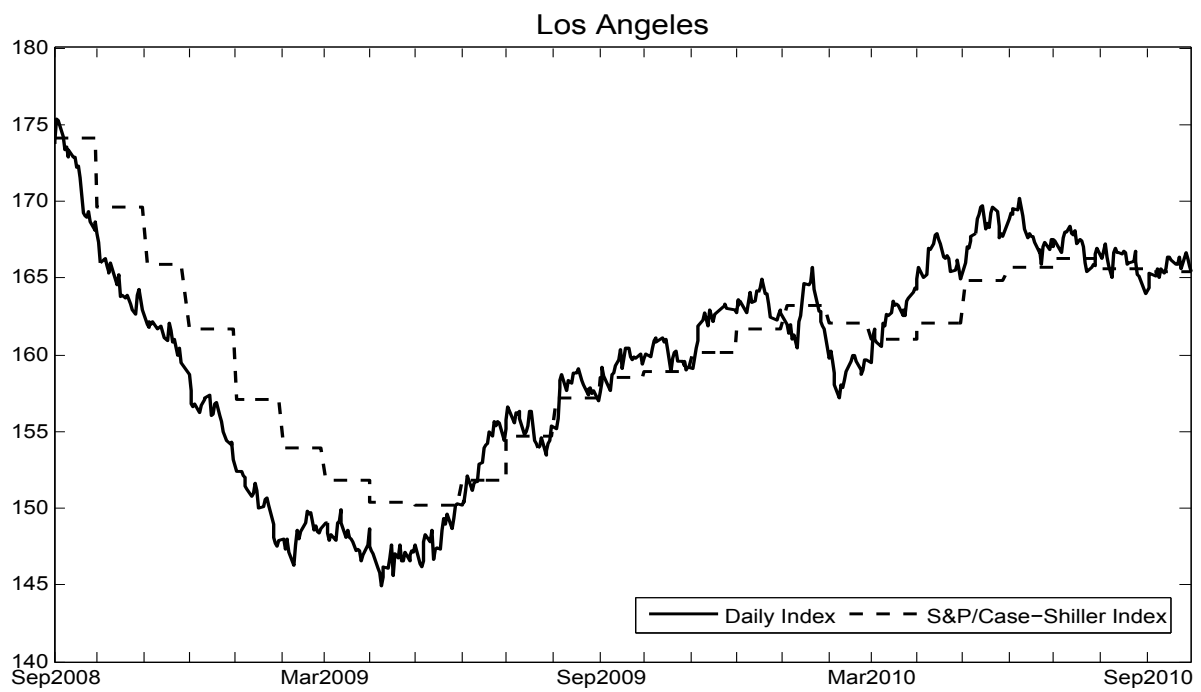
	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
	Panel B: Monthly (lower triangle) and Quarterly (upper triangle)									
Los Angeles	–	0.634 0.621	0.530 0.602	0.463 0.506	0.730 0.852	0.600 0.837	0.731 0.906	0.724 0.834	0.759 0.747	0.733 0.856
Boston	0.382 0.348	–	0.451 0.655	0.400 0.559	0.616 0.507	0.533 0.522	0.624 0.673	0.594 0.623	0.643 0.735	0.627 0.688
Chicago	0.266 0.344	0.207 0.320	–	0.323 0.502	0.519 0.612	0.417 0.510	0.513 0.567	0.500 0.667	0.572 0.767	0.532 0.675
Denver	0.251 0.355	0.210 0.254	0.138 0.293	–	0.457 0.370	0.391 0.317	0.454 0.557	0.416 0.625	0.458 0.411	0.456 0.513
Miami	0.493 0.619	0.384 0.277	0.274 0.355	0.271 0.239	–	0.591 0.797	0.696 0.769	0.669 0.754	0.734 0.761	0.697 0.801
Las Vegas	0.395 0.633	0.328 0.322	0.210 0.233	0.229 0.201	0.404 0.547	–	0.589 0.782	0.558 0.657	0.599 0.659	0.582 0.708
San Diego	0.497 0.626	0.388 0.307	0.260 0.276	0.266 0.351	0.468 0.570	0.400 0.497	–	0.678 0.822	0.731 0.711	0.694 0.824
San Francisco	0.511 0.623	0.334 0.288	0.253 0.404	0.216 0.427	0.424 0.527	0.343 0.417	0.435 0.600	–	0.700 0.663	0.677 0.791
New York	0.505 0.478	0.384 0.415	0.318 0.427	0.247 0.149	0.499 0.496	0.383 0.354	0.480 0.430	0.431 0.394	–	0.738 0.761
Washington, D.C.	0.469 0.603	0.366 0.375	0.277 0.385	0.253 0.309	0.444 0.515	0.368 0.444	0.433 0.551	0.414 0.486	0.478 0.437	–

Note: Model-implied correlations are upper numbers and data-based correlations are in smaller font just below. Daily, weekly, monthly and quarterly horizons correspond to 1, 5, 20, 60 days respectively.

**Table 4:** Diebold-Mariano forecast comparison tests

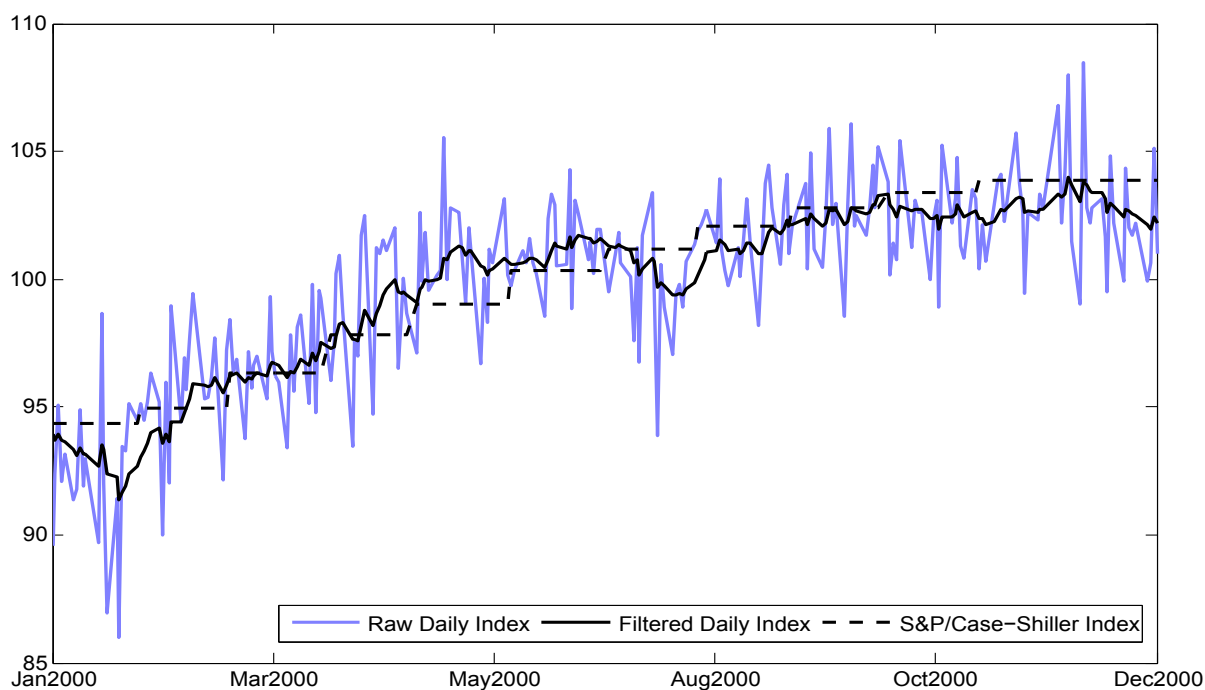
	Composite	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, D.C.
	Panel A: One-day-ahead ( $h = 1$ )										
Mthly v.s. HAR	9.240	8.337	7.378	10.060	9.845	8.680	9.981	9.929	8.067	8.981	9.142
Interp v.s. HAR	8.707	10.171	7.623	6.242	9.249	11.415	8.569	10.786	7.865	8.609	10.293
Direct v.s. HAR	1.599	1.381	2.943	-0.176	1.224	2.785	0.126	-0.276	3.139	-0.012	2.173
	Panel B: One-week-ahead ( $h = 5$ )										
Mthly v.s. HAR	4.956	4.458	3.876	5.412	5.126	5.087	6.682	6.581	4.258	5.268	4.981
Interp v.s. HAR	4.071	2.964	4.856	5.466	6.724	5.882	4.501	4.761	5.349	4.588	5.304
Direct v.s. HAR	4.495	1.200	3.580	1.514	1.141	2.669	-0.298	0.768	-0.373	0.562	3.212
	Panel C: Two-weeks-ahead ( $h = 10$ )										
Mthly v.s. HAR	4.544	2.751	3.799	6.647	4.343	4.078	5.204	5.847	3.453	5.261	4.392
Interp v.s. HAR	4.372	1.478	3.617	4.586	4.042	3.333	2.489	3.598	2.954	2.973	3.798
Direct v.s. HAR	5.668	0.828	3.567	2.640	0.763	2.585	-0.214	1.342	-0.381	0.964	3.563
	Panel D: One-month-ahead ( $h = 20$ )										
Mthly v.s. HAR	—	—	—	—	—	—	—	—	—	—	—
Interp v.s. HAR	6.762	0.623	3.553	4.117	0.830	2.211	-0.511	1.777	0.941	1.909	4.268
Direct v.s. HAR	—	—	—	—	—	—	—	—	—	—	—

Note: The table reports the Diebold-Mariano test statistics for equal predictive accuracy against the alternative that the HAR forecast outperforms the other three forecasts, Mthly, Interp and Direct. The test statistics are asymptotically standard Normal under the null of equal predictive accuracy. The tests are based on the out-of-sample period from July 2009 to September 2012. The Mthly, Interp and Direct models are all identical when  $h = 20$ , so only one set of test statistics are reported in Panel D.

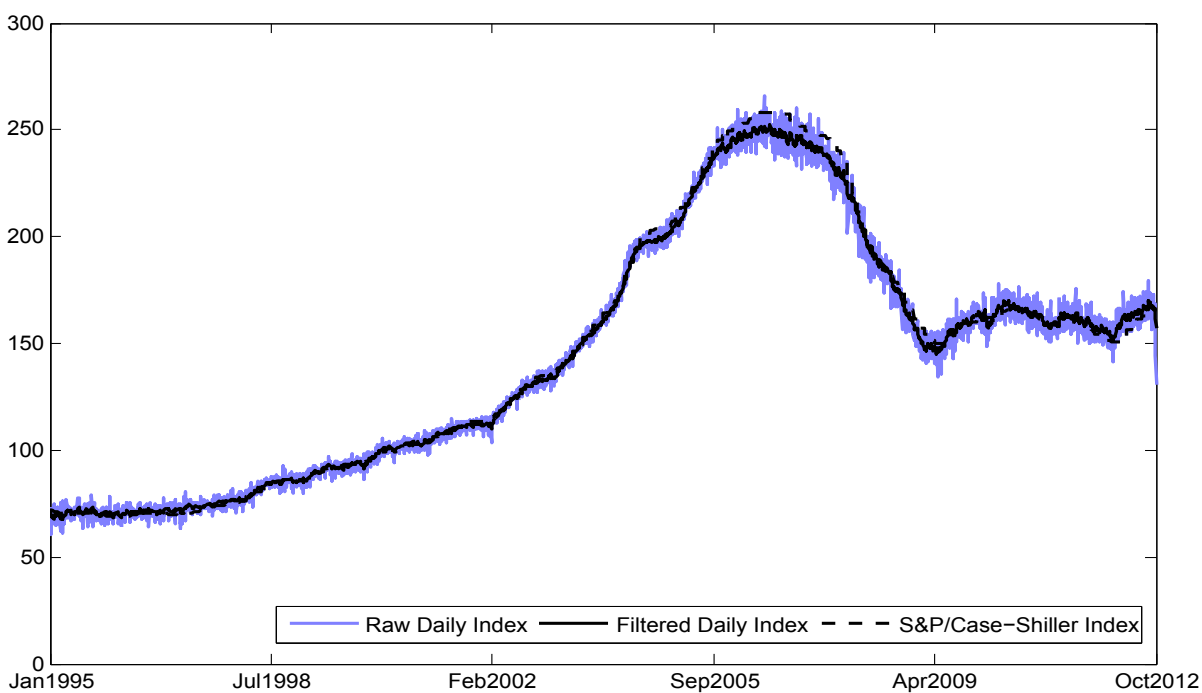


**Figure 1:** Daily and monthly house price indexes for Los Angeles



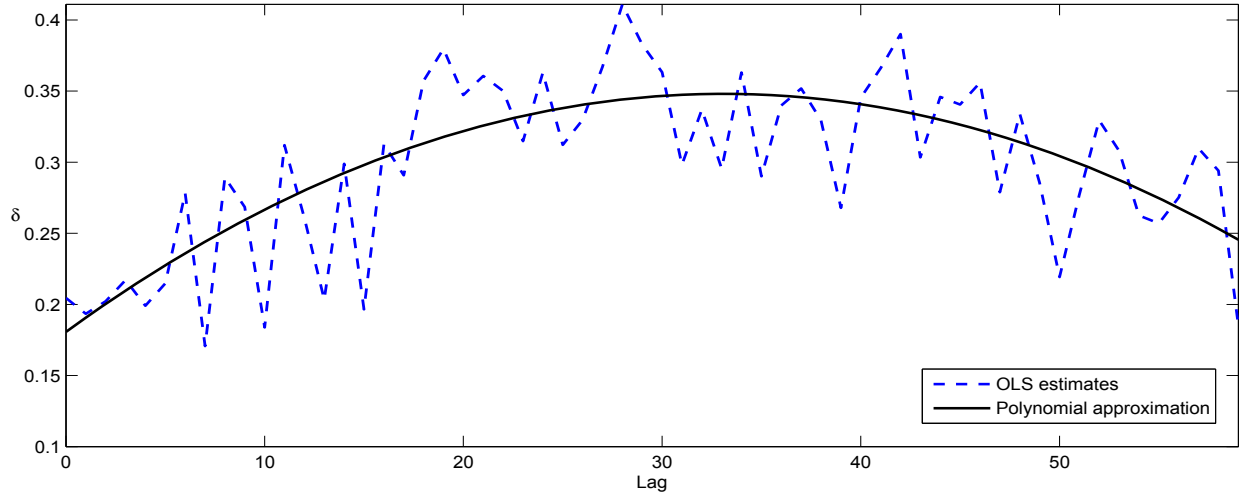


(a) January 3, 2000 to December 29, 2000

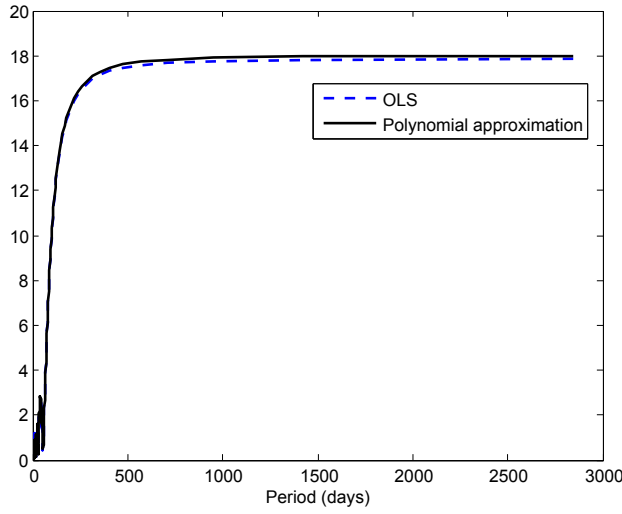


(b) January 3, 1995 to October 23, 2012

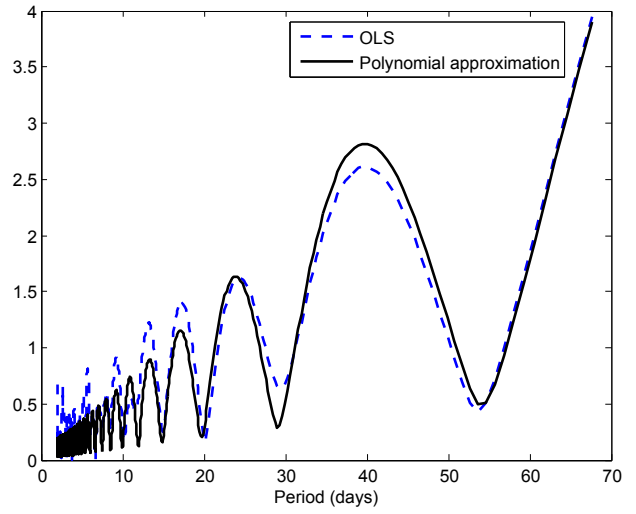
**Figure 2:** Raw and filtered daily house price indexes for Los Angeles



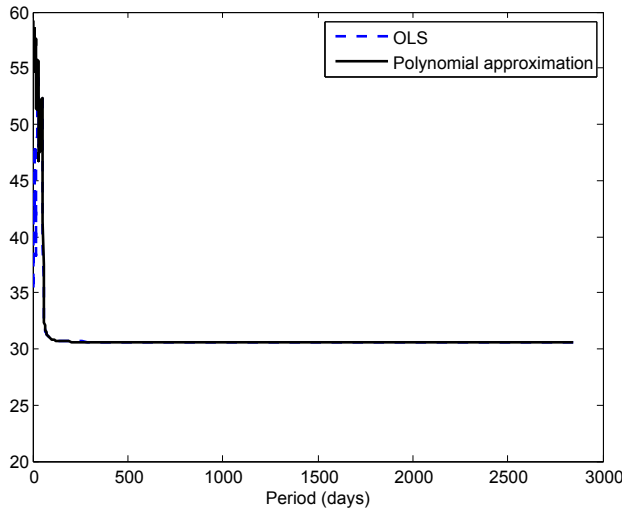
(a) Estimated  $\delta(L)$  filter coefficients



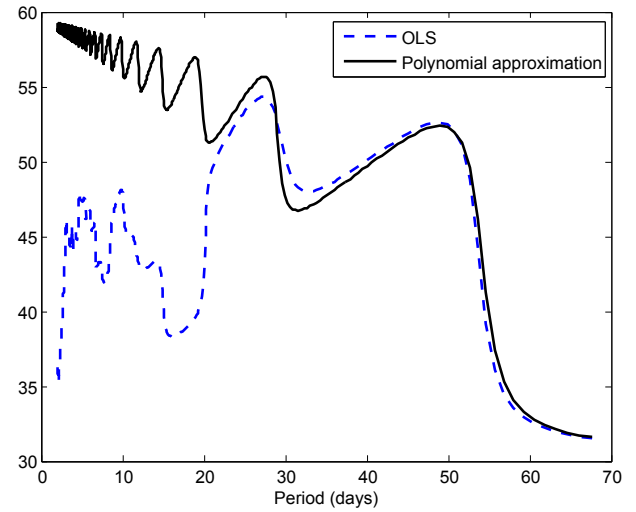
(b) Gain (all periods)



(c) Gain (shorter-run periodicities)

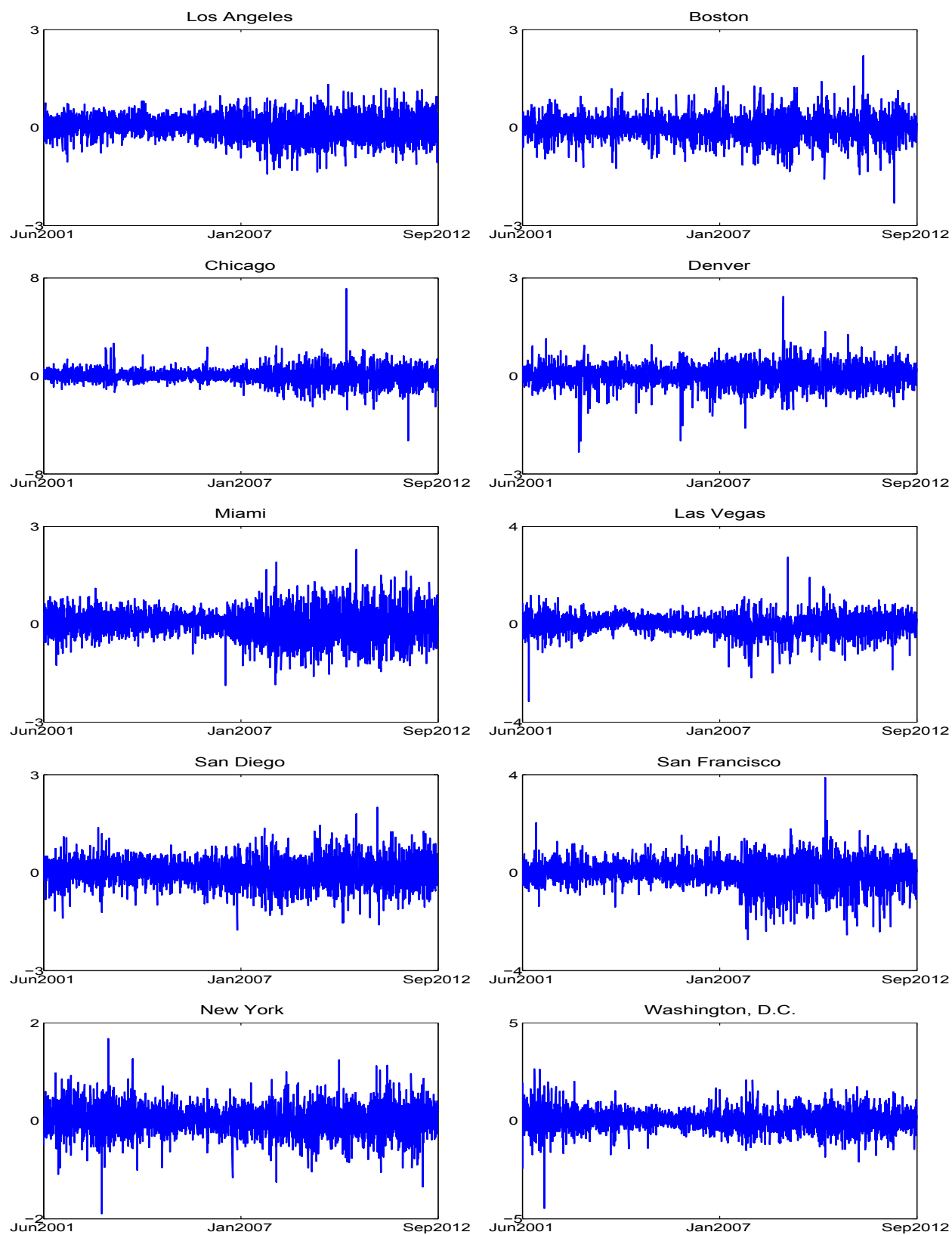


(d) Shift (all periods)

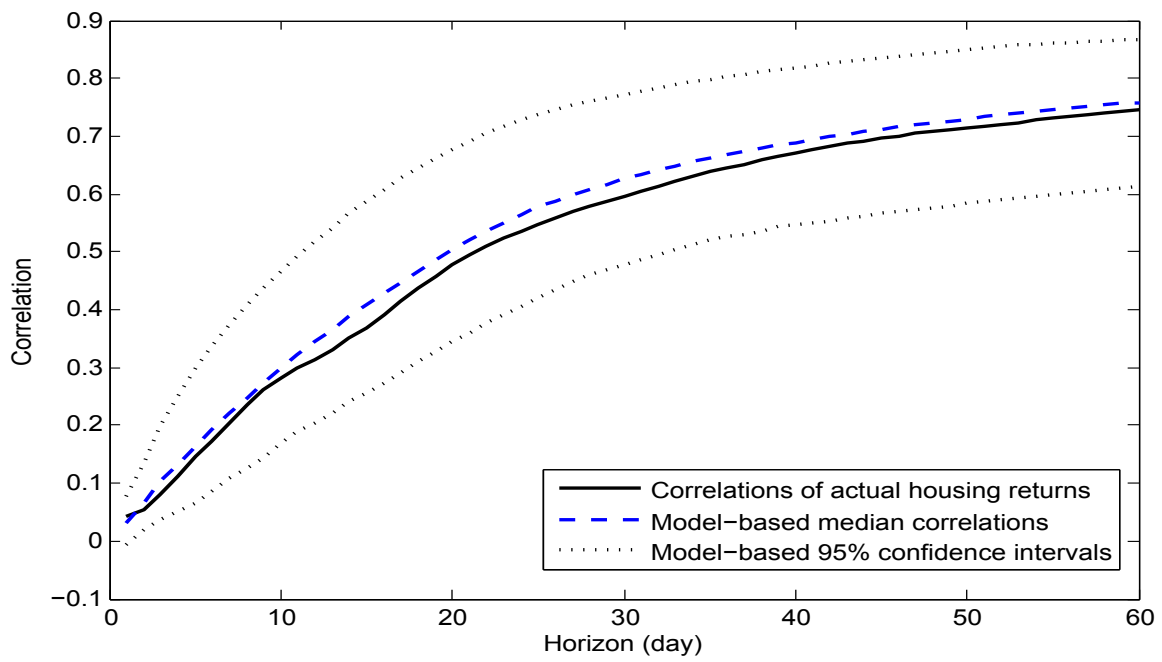


(e) Shift (shorter-run periodicities)

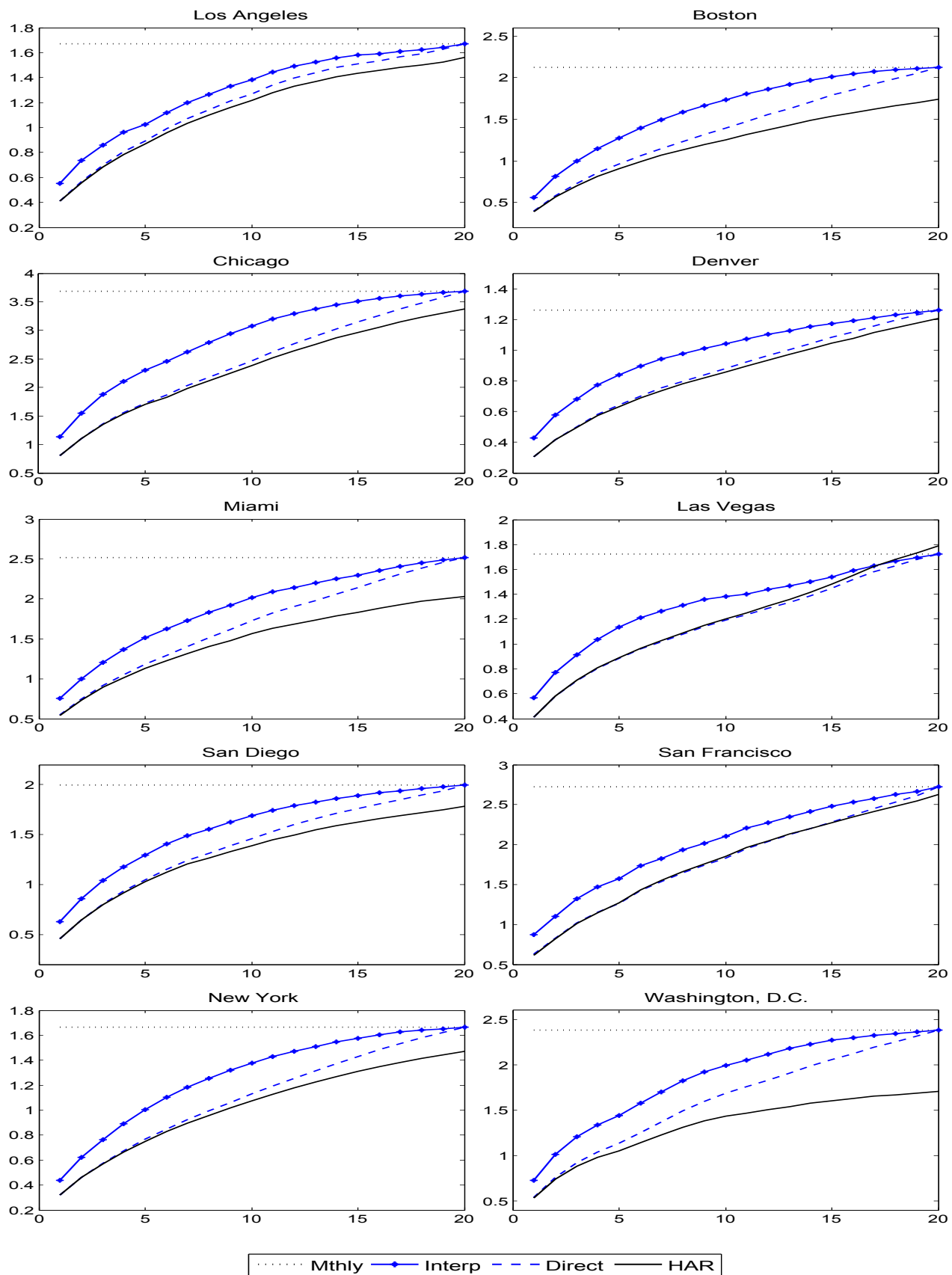
**Figure 3:** Characteristics of the  $\delta(L)$  filter



**Figure 4:** Daily housing returns



**Figure 5:** Unconditional return correlations for Los Angeles and New York



**Figure 6:** Forecast RMSEs as a function of forecast horizon (1 to 20 days)