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Realized semibetas: Disentangling "good" and "bad" downside risks[☆]

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ABSTRACT

We propose a new decomposition of the traditional market beta into four *semi*betas that depend on the signed covariation between the market and individual asset returns. We show that semibetas stemming from negative market and negative asset return covariation predict significantly higher future returns, while semibetas attributable to negative market and positive asset return covariation predict significantly lower future returns. The two semibetas associated with positive market return variation do not appear to be priced. The results are consistent with the pricing implications from a mean-*semi*variance framework combined with arbitrage risk driving a wedge between the risk premiums for long and short positions. We conclude that rather than betting against the traditional market beta, it is better to bet on and against the "right" semibetas.

1. Introduction

The Capital Asset Pricing Model (CAPM) reigns supreme as the most widely-studied and practically-used model for valuing speculative assets. In its basic form, the model predicts a simple linear relationship between the expected excess return on an asset and the beta of that asset with respect to the aggregate market portfolio. While early empirical evidence largely corroborated this prediction (e.g., Fama et al., 1969; Blume, 1970), an extensive subsequent literature has called into question the ability of the standard market beta to satisfactorily explain the cross-sectional variation in returns, with the estimated risk premiums being too low, often insignificant, and sometimes even negative (e.g., Roll, 1977; Bhandari, 1988; Fama and French, 1992). Numerous explanations have been put forth for these findings, ranging from measurement errors (e.g., Shanken, 1992; Hollstein et al., 2019), to agency problems (Baker et al., 2011), to the need for separate betas associated with cash-flow and discount rate news (Campbell and Vuolteenaho, 2004), to leverage constraints (Frazzini and Pedersen, 2014) and the need for separate liquidity and fundamental betas (Acharya and Pedersen, 2005), to name but a few.

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These "rescue attempts" notwithstanding, another strand of literature, tracing back to the early work by Roy (1952), Markowitz (1959), Hogan and Warren (1972, 1974) and Bawa and Lindenberg (1977), posits that the mean-variance, or quadratic utility, framework underlying the basic CAPM and the resulting security market line and linear beta pricing relationship is too simplistic. If investors are averse to volatility only when it leads to losses, not gains, then the relevant measure of risk is not (total) variance but rather the semivariance of negative returns. This same basic idea also underlies the notion of loss aversion and the prospect theory pioneered by Kahneman and Tversky (1979), as supported by an extensive subsequent experimental literature and other empirical evidence. Intuitively, if investors only care about downside variation, then the covariation associated with a positive aggregate market return should not be priced in equilibrium. These same pricing implications also arise in a setting with disappointment aversion preferences as in Gul (1991), and its generalization in Routledge and Zin (2010), recently explored by Farago and Tedongap (2018). As shown by Anthonisz (2012), they may also be cast in a more traditional stochastic discount factor pricing framework assuming a "kinked" pricing kernel.

Consistent with these ideas, Ang et al. (2006a) find that the downside beta version of the CAPM does a better job than the traditional CAPM in terms of explaining the cross-sectional variation in U.S. equity returns. Post and van Vliet (2004) reach the same conclusion, and Lettau et al. (2014) similarly find that a downside beta version of the CAPM better explains the variation in the returns across other asset classes. By contrast, recent work by Atilgan et al. (2018) has called into question the ability of downside betas to satisfactorily explain the cross-sectional variation in more recent U.S. and international equity returns. Levi and Welch (2020) also conclude that downside betas do not provide superior cross-sectional return predictions compared to the predictability afforded by traditional betas.

Set against this background, we propose a new four-way decomposition of the traditional market beta into four *semi*betas. Our decomposition relies on the newly-developed *semi*covariance concept of Bollerslev et al. (2020a). Letting r and f denote the returns on some risky asset and the aggregate market portfolio, respectively, the four semibetas are then defined as:

$$\beta \equiv \frac{Cov(r,f)}{Var(f)} = \frac{\mathcal{N}+\mathcal{P}+\mathcal{M}^{+}+\mathcal{M}^{-}}{Var(f)} \equiv \beta^{\mathcal{N}}+\beta^{\mathcal{P}}-\beta^{\mathcal{M}^{+}}-\beta^{\mathcal{M}^{-}}. \quad (1)$$

The $\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}^{+}$, and $\mathcal{M}^{-}$ semicovariance components refer to the respective portions of total covariation $Cov(r,f)$ defined by both returns being positive (the "P" state), both returns being negative ("N"), mixed sign with positive market return ("M^{+}"), and mixed sign with negative market return ("M^{-}"). Since the mixed-sign semicovariances are always weakly negative numbers, with lower values indicating stronger covariation, to ease the interpretation of the risk premium estimates in our empirical analyses, we purposely define the mixed-sign semibetas as $\beta^{\mathcal{M}^{+}} \equiv -\mathcal{M}^{+}/Var(f)$ and $\beta^{\mathcal{M}^{-}} \equiv -\mathcal{M}^{-}/Var(f)$.

The traditional CAPM, of course, does not differentiate between any of the four covariation components ($\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}^{+}$, and $\mathcal{M}^{-}$), combining them into a single market β and a single risk premium. The downside version of the CAPM, investigated in the aforementioned studies, effectively combines the pricing of the two negative-market-return covariation components ($\mathcal{N}$ and $\mathcal{M}^{-}$) into a single downside beta and the two positive-market-return covariation components ($\mathcal{P}$ and $\mathcal{M}^{+}$) into a single upside beta, each with their own individual risk premiums. Anticipating our empirical results, we strongly reject these pricing restrictions in the data.

To help further intuit the main idea, Fig. 1 depicts hypothetical bivariate contour plots for the returns on the market and four different illustrative assets, each of which have a traditional CAPM beta equal to one.[1] Since the CAPM betas are the same, the CAPM predicts identical expected returns for all four assets.

Meanwhile, consider the asset in Panel A, which is jointly Normally distributed with the market, and the asset in Panel B, which (contrary to most equity returns) has less correlation during market downturns and greater correlation during market upturns ($\beta^{\mathcal{N}} < \beta^{\mathcal{P}}$). Using the semibeta model results described in Section 4, the annual expected excess return for asset A is 9.45%, while it is only 7.09% for asset B, a finding consistent with investors being particularly averse to downside risk, and thus willing to accept lower expected returns for an asset displaying a desirable dependence structure. On the other hand, the asset depicted in Panel C, which is more strongly correlated with the market during downturns than upturns ($\beta^{\mathcal{N}} > \beta^{\mathcal{P}}$), and so is less desirable from a mean-semivariance perspective, has an expected return of 11.91%, an increase of 2.5% relative to asset A, and 4.8% relative to asset B, two assets with the exact same market beta. Finally, like asset C, the asset in Panel D is more strongly correlated with the market during downturns than upturns ($\beta^{\mathcal{N}} > \beta^{\mathcal{P}}$), however, its mixed semicovariations ($\beta^{\mathcal{M}^{-}} > \beta^{\mathcal{M}^{+}}$) imbue it with superior hedging benefits relative to asset C, and thus a lower expected return of 10.86%.

Counter to the intuition conveyed by Fig. 1 and the implications stemming from the idea that each of the four semibetas may be priced differently, in a frictionless financial market, the risks associated with $\mathcal{N}$ and $\mathcal{M}^{-}$ ($\mathcal{P}$ and $\mathcal{M}^{+}$) should be priced the same, as a short position in an asset simply switches the signs of the corresponding semicovariation components. However, as forcefully argued by Pontiff (1996) and Schleifer and Vishny (1997), legal constraints and charters impede many institutional investors from short-selling, and many individual investors are simply reluctant to sell short, effectively creating limits-to-arbitrage and arbitrage risk (see also the discussion in Hong and Sraer, 2016). This arbitrage risk in turn induces a wedge between the pricing of the $\mathcal{N}$ and $\mathcal{M}^{-}$ ($\mathcal{P}$ and $\mathcal{M}^{+}$) semicovariation components, and the risk premiums associated with the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^{-}}$ ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^{+}}$) semibetas. Intuitively, assets that covary pos-

[1] The distributions are formed using standard Normal marginal distributions and Normal or Clayton copulas to account for different non-Gaussian dependencies, see Supplemental Appendix S1 for details.

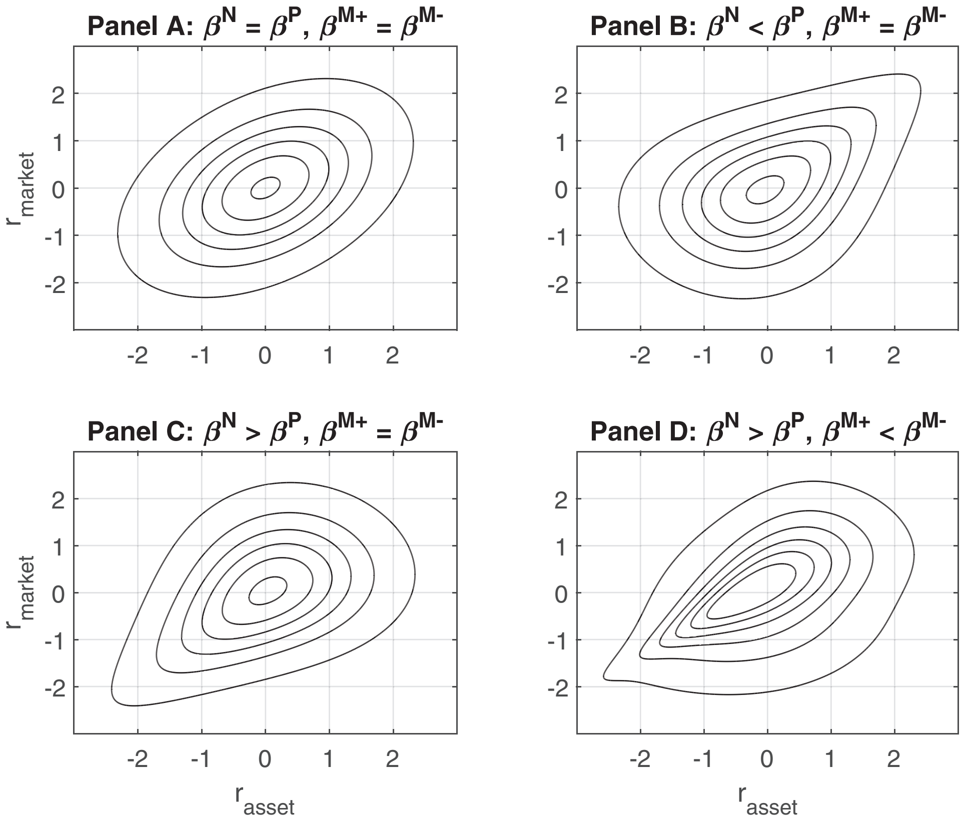

Fig. 1. Hypothetical return distributions. The figure presents isoprobability contours of the bivariate PDFs for four hypothetical return distributions, all of which have standard Normal marginal distributions and all of which imply a CAPM beta of one.

itively with the market when the market is performing poorly will exacerbate downside return variation, while assets that covary negatively with the market when the market is performing poorly help mitigate downside risk. Correspondingly, we find that the former types of assets command higher risk premiums.

True betas and semibetas, of course, are not directly observable. Instead, guided by the burgeoning realized volatility literature, and the in-fill asymptotic arguments championed therein, we rely on so-called realized betas (Barndorff-Nielsen and Shephard, 2004) and semibetas constructed from higher frequency returns over fixed time intervals; for additional discussion of the realized beta concept along with empirical applications, see also Andersen et al. (2006) and Patton and Verardo (2012). Based on these new measures, we offer three main empirical contributions.

Our initial empirical investigations are based on monthly realized semibetas constructed from daily stock returns over the 1963 to 2019 sample period. The estimated semibetas clearly reveal the existence of asymmetric dependencies between the individual stocks and the market beyond those of the linear dependencies captured by the traditional market beta. More importantly, our results strongly support the hypothesis that these non-linear dependencies are priced differently: stocks with higher $\beta^{\mathcal{N}}$ are associated with significantly higher subsequent daily returns; stocks with higher $\beta^{\mathcal{M}^-}$ are associated with significantly lower subsequent daily returns; while neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to carry a significant risk premium. These findings remain robust to the inclusion of a long list of other return predictor variables previously analyzed in the literature. Corroborating the thesis that the difference in the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ may be attributed to market frictions and limits-to-arbitrage, we show that the rejection of the hypothesis that the two risk premiums are identical is stronger for portfolios made up of stocks with higher arbitrage risk, as proxied by the level of idiosyncratic volatility (e.g., Pontiff, 1996; Stambaugh et al., 2015), and stocks that are more difficult to value, as proxied by the rate of turnover (e.g., Harris and Raviv, 1993; Blume et al., 1994; Kumar, 2009). Further underscoring the significance of this difference in the pricing of the semibetas, the two-way decomposition of the traditional market beta into separate up and downside betas previously advocated in the literature (Ang et al., 2006a) is also strongly rejected against the four-way semibeta decomposition proposed here.

Second, hewing more closely to the in-fill asymptotic arguments underlying the statistical consistency of the realized measures, we construct daily realized semibetas based on high-frequency intraday data. Our sample consists of all of the S&P 500 constituent stocks over the 1993–2019 time period. Temporal aggregation generally tends to mute non-linear dependencies in returns, and as such the daily semibetas may better reveal the inherent asymmetric dependencies than the monthly beta measures constrcuted from coarser daily returns. In line with this thesis, using our high-frequency-based beta measures, we arrive at qualitatively very similar, but economically and statistically stronger, conclusions. $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ are priced differently, with estimated annualized risk premiums of 18.10% and 7.82%, respectively, while the estimated risk premiums for $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ are both statistically insignificant at conventional levels. By comparison, the estimated risk premium for the traditional market beta is 4.49%. Further elaborating on the statistical significance of the results, we demonstrate how the daily realized semibetas may also be used in predicting cross-sectional differences in returns over longer weekly and monthly horizons.

Finally, we investigate whether these statistically significant differences in the compensation for the different semibetas also translate into "economically significant" differences in the performance of simple portfolio strategies. We find that a long-short semibeta strategy generates average annual excess returns of 8.17%, and an annualized Sharpe ratio of 0.92. By comparison, similar portfolio strategies based on the standard CAPM betas and the Ang et al. (2006a) downside betas generate excess returns of 5.62% and 7.11%, respectively, with Sharpe ratios of only 0.37 and 0.49. Using the four- and five-factor models of Carhart (1997) and Fama and French (1993, 2015) to assess the risk-adjusted performance, we find annualized alphas of 6.85% and 7.52% respectively, with overwhelmingly significant t-statistics. By comparison, the traditional beta and the downside beta portfolios produce much smaller and at best only borderline significant alphas. Hence, adding to the recent literature and debate about betting on or against beta (see, e.g., Frazzini and Pedersen, 2014; Cederburgh and O'Doherty, 2016; Bali et al., 2017; Novy-Marx and Velikov, 2018; Schneider et al., 2020), we conclude that it is better to bet on and against the "right" semibetas.

In addition to the previous studies on downside risk noted above, our empirical findings are also related to the vast literature on asymmetric dependencies in stock returns, including among others Longin and Solnik (2001), Ang and Chen (2002), Patton (2004), Hong et al. (2006), Elkamhi and Stefanova (2014) and Engle and Mistry (2014). They are also related to the more recent and rapidly growing literature on the pricing of downside tail, or crash, risk, including Bali et al. (2009), Bollerslev and Todorov (2011), Kelly and Jiang (2014), Cremers et al. (2015) Bollerslev et al. (2016), Chabi-Yo et al. (2018), Farago and Tedongap (2018), Barunik and Nevrla (2019), Bondarenko and Bernard (2020), Chabi-Yo et al. (2019), Lu and Murray (2019) and Orlowski et al. (2019). In contrast to all of these studies, however, which rely on the use of options and/or non-linear procedures for assessing the asymmetric joint tail dependencies and the pricing thereof, we maintain a simple linear pricing relationship, together with a simple-to-implement additive decomposition of the traditional market beta into the four semibeta components. Our new semibeta measures are also distinctly different from, and much simpler to implement than, the entropy approach of Jiang et al. (2018) designed to measure asymmetries in up and downside comovements.

The semibetas, and the joint dependencies captured by them, are also related to the notion of coskewness originally proposed by Kraus and Litzenberger (1976), and the corresponding notion of cokurtosis, as investigated empirically by Harvey and Siddique (2000), Dittmar (2002), Conrad et al. (2013), Langlois (2020) and Schneider et al. (2020), among others. We find that the semibetas remain highly significant for explaining the cross-sectional variation controlling for coskewness and cokurtosis, while both of these co-dependency measures are rendered insignificant by the inclusion of the semibeta measures. Our reliance on the new semicovariance concept for decomposing the systematic market risk and defining the semibetas also sets our analysis apart from other recent studies based on the semivariance concept for defining and empirically investigating asset specific "good" and "bad" volatility measures and the separate pricing thereof, as in, for example Feunou et al. (2018), Feunou and Okou (2019) and Bollerslev et al. (2020b).

The remainder of the paper is structured as follows. In Section 2 we discuss our construction of the realized semibetas, along with a brief summary of their empirical distributional features. In Section 3, we present our key empirical findings related to the pricing of the monthly realized semibetas based on firm-level cross-sectional regressions. In Section 4, we discuss our results based on daily semibetas estimated from high-frequency intraday data. In Section 5, we consider the performance of simple semibeta-based portfolio strategies, including comparisons to other similarly constructed beta-based portfolios. Section 6 concludes. Additional empirical results and robustness checks are detailed in an online Supplemental Appendix.

2. Realized semibetas

We begin by formally defining realized semibetas. We then briefly discuss the data that we use in our main empirical investigations, followed by a summary of the salient distributional features of the resulting realized semibeta estimates.

2.1. Definitions

Let $r_{t,k,i}$ denote the "high-frequency" return on asset i over the k^{th} time interval within some fixed time period t, with the concurrent "high-frequency" return for the aggregate market denoted by $f_{t,k}$. To fix ideas, and in accordance with our two separate empirical analyses discussed below, think about k as a day and t as a month, or k as a 15-minute time interval and t as a day. Define the signed intra-period asset returns by $r^{+}_{t,k,i} \equiv \max(r_{t,k,i}, 0)$ and $r^{-}_{t,k,i} \equiv \min(r_{t,k,i}, 0)$, with the signed intra-period market returns defined analogously. The realized semibetas are then defined by:

$$\widehat{\beta}_{t,i}^{\mathcal{N}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}}, \quad \widehat{\beta}_{t,i}^{\mathcal{P}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$
$$\widehat{\beta}_{t,i}^{\mathcal{M}^-} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}}, \quad \widehat{\beta}_{t,i}^{\mathcal{M}^+} \equiv \frac{-\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}} \qquad (2)$$

where m denotes the number of higher-frequency return intervals within each time period. The semibetas provide an exact four-way decomposition of the traditional realized market beta:

$$\widehat{\beta}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f_{t,k}}{\sum_{k=1}^{m} f_{t,k}^{2}} = \widehat{\beta}_{t,i}^{\mathcal{N}} + \widehat{\beta}_{t,i}^{\mathcal{P}} - \widehat{\beta}_{t,i}^{\mathcal{M}^+} - \widehat{\beta}_{t,i}^{\mathcal{M}^-}. \qquad (3)$$

As previously noted, we purposely change the sign on the two mixed semibetas, to make them positive, thereby allowing for an easier interpretation of the correspondingly decomposed risk premium estimates.

Let $\mathcal{RV}_t$ and $\mathcal{COV}_{t,i}$ denote the latent true period t market return variation and covariation between the market return and the return on the individual asset i, with the corresponding true semicovariation measures denoted by $\mathcal{P}_{t,i}$, $\mathcal{N}_{t,i}$, $\mathcal{M}_{t,i}^{+}$ and $\mathcal{M}_{t,i}^{-}$, respectively. Barndorff-Nielsen and Shephard (2004) show that for increasingly finer sampled returns, or $m \to \infty$, realized betas consistently estimate the true latent betas:

$$\widehat{\beta}_{t,i} \xrightarrow{p} \frac{\mathcal{COV}_{t,i}}{\mathcal{RV}_t}. \qquad (4)$$

Similarly, the in-fill asymptotic theory in Bollerslev et al. (2020a) pertaining to realized semicovariances imply that the realized semibetas consistently estimate the true semibetas:

$$\widehat{\beta}_{t,i}^{\mathcal{N}} \xrightarrow{p} \frac{\mathcal{N}_{t,i}}{\mathcal{RV}_t}, \widehat{\beta}_{t,i}^{\mathcal{P}} \xrightarrow{p} \frac{\mathcal{P}_{t,i}}{\mathcal{RV}_t}, \widehat{\beta}_{t,i}^{\mathcal{M}^+} \xrightarrow{p} \frac{-\mathcal{M}_{t,i}^{+}}{\mathcal{RV}_t},$$
$$\widehat{\beta}_{t,i}^{\mathcal{M}^-} \xrightarrow{p} \frac{-\mathcal{M}_{t,i}^{-}}{\mathcal{RV}_t}. \qquad (5)$$

For ease of notation, in the remainder when not necessary we drop the subscripts and hats, and refer to these realized (semi)beta measures simply as β, $\beta^{\mathcal{N}}$, etc.

If the market and individual asset returns were jointly Normally distributed, the four semibetas would convey no new information over and above the conventional market beta. In particular, it follows that under joint Normality:

$$\beta^{\mathcal{N}} = \beta^{\mathcal{P}} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} + \beta \arccos\left(-\frac{\sigma_f}{\sigma_r}\beta\right)\right),$$
$$\beta^{\mathcal{M}^+} = \beta^{\mathcal{M}^-} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} - \beta \arccos\left(\frac{\sigma_f}{\sigma_r}\beta\right)\right).$$

However, if the market and individual asset returns are not Normally distributed, the two concordant semibetas ($\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$) and the two disconcordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$) will generally differ, and each of the four semibetas may convey additional useful information to that of the standard market beta (β). As such, each of the semibetas may also be priced differently.

2.2. Data and summary statistics

Our primary empirical investigations rely on daily data obtained from the Center for Research in Securities Prices (CRSP) database spanning the period from January 1963 to December 2019. We include all stocks with CRSP codes 10 and 11. In line with previous work, we remove all "penny stocks" with prices less than five dollars to help alleviate biases arising from price discreteness (e.g., Harris (1994); Amihud (2002)). All in all, this leaves us with a total of 273,823 firm-month observations.

Panel A of Table 1 reports the time series averages of the cross-sectional means, medians and standard deviations of the resulting monthly (semi)beta estimates averaged across all of the stocks in the sample. Panel B gives the time series averages of the cross-sectional correlations. Consistent with on average positive dependencies between the market and each of the individual stocks, the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) on average far exceed the two discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$). The two concordant semibetas are also correlated more strongly with the traditional market beta (β), and more so than with each other. Nonetheless, the correlations with the traditional beta are still far below unity, suggesting that the semibetas do convey different, and potentially useful, information over and above that of the traditional market beta.

To help further visualize the differences in the betas, Panel A of Fig. 2 depicts their unconditional distributions across all of the days and stocks in the sample. The distribution of conventional betas is centered around one, as expected, and is close to symmetric. Meanwhile, the realized semibetas are all weakly positive by construction, and thus unsurprisingly their distributions are right-skewed. Echoing the summary statistics in Table 1, the semibeta distributions are all centered below unity. Also, the unconditional distributions of the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) are almost indistinguishable, as are the distributions of the two discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$).

The average autocorrelation functions shown in Panel B of Fig. 2 indicate a strong degree of persistence for all of the semibetas, with the autocorrelations remaining in excess of 0.4 at the annual 12*th* lag.[2] Underpinning the cross-sectional return predictability regressions that we rely on in our asset pricing investigations discussed next, the high first-order autocorrelations of around 0.7 for each of the monthly semibetas, also imply that this month's realized semibetas for a given stock provide accurate predictions of next month's semibetas for that same stock.

3. Semibetas and the cross-section of expected returns

We begin our empirical investigations pertaining to the pricing of the non-linear dependencies encoded in the realized semibeta by presenting the results from standard Fama and MacBeth (1973) type cross-sectional

[2] We rely on the instrumental variable approach of Hansen and Lunde (2014), using lags 4 through 10 as instruments, to adjust for measurement errors in the realized betas, thereby allowing for more meaningful comparisons of the autocorrelation functions across the different betas.

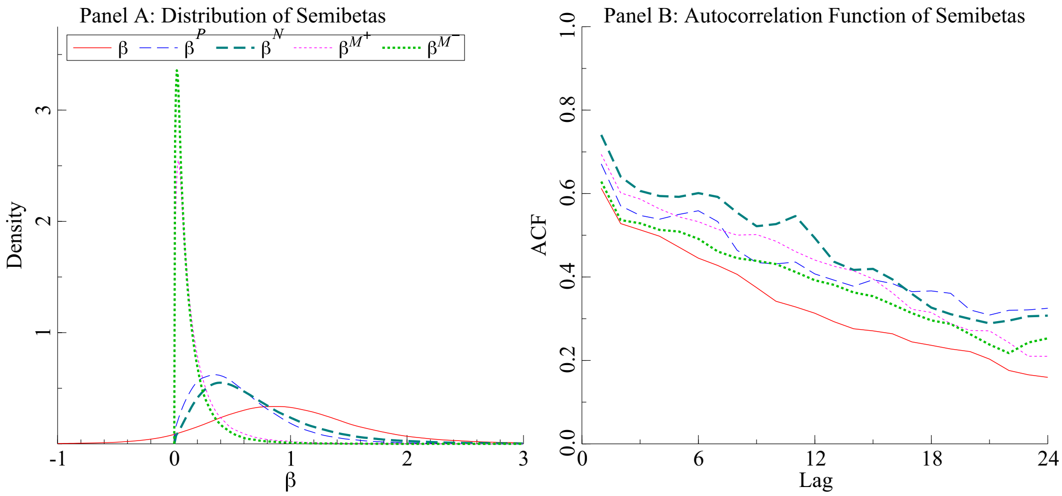

Fig. 2. Unconditional distributions and autocorrelations. Panel A displays kernel density estimates of the unconditional distribution of the monthly realized beta and semibetas averaged across time and stocks. Panel B reports the average autocorrelation functions for the monthly realized beta and semibetas averaged across stocks. The estimates are based on all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2019.

predictive regressions. These regressions conveniently allow for the simultaneous estimation of separate risk premiums for each of the semibetas. In particular, for each month $t = 1, \ldots, T-1$, and all of the stocks $i = 1, \ldots, N_t$, available for month t and $t+1$, we estimate the month $t+1$ risk premiums (λs) for the different semibetas from the cross-sectional regression:

$$r_{t+1,i} = \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}\hat{\beta}^{\mathcal{N}}_{t,i} + \lambda^{\mathcal{P}}_{t+1}\hat{\beta}^{\mathcal{P}}_{t,i} + \lambda^{\mathcal{M}^+}_{t+1}\hat{\beta}^{\mathcal{M}^+}_{t,i} + \lambda^{\mathcal{M}^-}_{t+1}\hat{\beta}^{\mathcal{M}^-}_{t,i} + \epsilon_{t+1,i}. \quad (6)$$

Based on these $T-1$ cross-sectional estimates, we then estimate the average risk premiums associated with each of the semibetas by the time series averages of the lambdas over all of the months in the sample:

$$\hat{\lambda}^j = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\lambda}^j_t \quad j = \mathcal{N}, \mathcal{P}, \mathcal{M}^+, \mathcal{M}^-. \quad (7)$$

The resulting annualized estimates, along with their t-statistics based on Newey-West robust standard errors (using 10 lags), together with the time-series average of the R^2s from the first-stage cross-sectional regressions in Eq. (6), are reported in the second row of Table 2. As a benchmark, the first row of the table reports the estimated risk premium for the traditional CAPM realized beta. Consistent with the basic mean-variance framework, the traditional beta carries a statistically significant risk premium of 4.27% per year. This estimated risk premium is somewhat below the average annual equity risk premium of 6.87% observed over the sample, corroborating the basic premise underlying the "betting against beta" investment strategy (Frazzini and Pedersen, 2014).

Supporting the idea that the semibetas convey additional useful information, the cross-sectional fit, reported in the final column, rises from 2.33% when using the standard CAPM beta to 5.16% when using the semibetas. We can formally test whether this gain in R^2 is statistically significant by noting that the semibeta-based

Table 1
Summary Statistics. Panel A reports the time series averages of the cross-sectional means, medians and standard deviations for the monthly realized semibetas constructed from daily returns. Panel B reports the time series averages of the cross-sectional correlations. The estimates are based on all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2019.

	β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$
Panel A: Summary statistics					
Mean	0.99	0.60	0.76	0.21	0.16
Median	0.92	0.54	0.67	0.15	0.10
St.Dev.	0.76	0.36	0.46	0.21	0.19
Panel B: Correlations					
β	1.00	0.72	0.79	-0.30	-0.29
$\beta^{\mathcal{N}}$		1.00	0.42	0.10	-0.09
$\beta^{\mathcal{P}}$			1.00	-0.07	0.06
$\beta^{\mathcal{M}^+}$				1.00	0.23
$\beta^{\mathcal{M}^-}$					1.00

pricing model reduces to the traditional CAPM model if the semibeta risk premiums satisfy:

$$H^{CAPM}_{0,t}: \ \lambda^{\mathcal{N}}_t = \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t = -\lambda^{\mathcal{M}^-}_t. \quad (8)$$

We reject this restriction at the 5% level for 46.1% of the 684 months in our sample, thus strongly refuting the traditional one-beta model in favor of a model that exploits the additional information contained in the semibetas.

The risk premium estimates reported in the second row of Table 2 highlight the richer pricing implications of the mean-semivariance framework: $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both associated with statistically significant risk premiums, while $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ do not appear to be priced in the cross-section. Underscoring not just the statistical significance of the estimated risk premiums, but also the economic significance, a one standard deviation increase in $\beta^{\mathcal{N}}$ relative to its cross-sectional mean is associated with an increase in the expected annual return of 3.80%, while a one standard deviation increase in $\beta^{\mathcal{M}^-}$ relative

Table 2

Monthly Fama-MacBeth regressions. The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas are calculated from daily data. All of the control variables are measured on the day prior to the monthly returns. The estimates are based on all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2019.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	RV	IVOL	ILLIQ	R^2
4.27												2.33
3.96												
	10.54	1.84	4.59	-6.00								5.16
	4.51	1.17	1.32	-1.97								
	8.74	0.25	5.72	-13.55	-2.56	-0.64	0.05					10.55
	3.61	0.16	1.51	-3.68	-5.05	-0.57	2.05					
	9.78	2.56	8.01	-11.84	-4.82	-1.14	0.06	-0.12	-0.71	0.52	-2.81	13.85
	3.80	1.26	1.89	-3.03	-5.31	-1.00	2.22	-2.07	-1.87	0.39	-4.47	

to its cross-sectional mean lowers the expected return by 1.14%.

3.1. Standard risk factors and controls

A plethora of other risk factors and firm characteristics have, of course, been put forth in the literature as significant drivers of the cross-sectional variation in equity returns; see, e.g., the recent account by Harvey et al. (2016). Focussing on some of the more prominent variables that have received the most attention in the literature, we consider size (ME) (Banz, 1981), book-to-market (BM) (Fama and French, 1993), momentum (MOM) (Jegadeesh and Titman, 1993), return reversals (REV) (Jegadeesh, 1990), idiosyncratic volatility (IVOL) (Ang et al., 2006b), realized volatility (RV) (Andersen et al., 2001), and illiquidity (ILLIQ) (Amihud, 2002). Further details concerning the construction of each of these variables are given in Appendix A.

The third row of Table 2 reports the average risk premium estimates from the cross-sectional regressions that in addition to the semibetas include ME, BM and MOM, mimicking the popular Fama-French-Carhart four factor (FFC4) model. Consistent with the extant literature and results with more recent data, the estimated risk premiums for ME and MOM are both significant, while the premium for BM is insignificant at conventional levels. Correspondingly, the inclusion of the three additional risk factors increases the average cross-sectional R^2 from 5.16% for the regressions based solely on the four semibetas to 10.55% for the semibeta+FFC-based model. Importantly, however, the risk premiums associated with $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain highly statistically significant.

The specification in the bottom row of Table 2 further incorporates REV, RV, IVOL and ILLIQ as additional controls. This further increases the average cross-sectional R^2 to 13.85%. But again, the inclusion of these additional controls does not meaningfully alter the large and highly significant t-statistic associated with $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$. Moreover, the risk premium estimates for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ also remain similar to the estimates in the second row, obtained without the inclusion of any controls, underscoring the robustness of the semibeta pricing.

3.2. Arbitrage risk and semibeta pricing

The semibeta risk premium estimates discussed above are based on the traditional Fama-MacBeth cross-sectional regression approach in (6) involving the returns on long positions in each of the individual stocks. However, it follows readily from the definition of the semibetas in Eq. (2) that the $\hat{\beta}^{\mathcal{N}}_{t,i}$ ($\hat{\beta}^{\mathcal{P}}_{t,i}$) of a long position in stock i equals $-\hat{\beta}^{\mathcal{M}^-}_{t,i}$ ($-\hat{\beta}^{\mathcal{M}^+}_{t,i}$) for a short position in stock i. Hence, in a frictionless market, in which the expected return on a short position is equal to the negative of the expected return on a long position, the risk premium associated with $\hat{\beta}^{\mathcal{N}}_{t,i}$ ($\hat{\beta}^{\mathcal{P}}_{t,i}$) should be equal to the negative of the risk premium associated with $\hat{\beta}^{\mathcal{M}^-}_{t,i}$ ($\hat{\beta}^{\mathcal{M}^+}_{t,i}$). Since most stocks can fairly easily and cheaply be borrowed (see, e.g., D'Avolio, 2002, and the more recent analysis in Henderson et al., 2019), the difference in the absolute values of the significant risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}-}$ may seem puzzling.

However, as argued by Pontiff (1996) and Schleifer and Vishny (1997), with legal restrictions and charters impeding many institutional investors from short-selling, and many individual investors simply reluctant to sell short, this may effectively create limits-to-arbitrage and related arbitrage risks (see also the discussion in Hong and Sraer, 2016). This arbitrage risk in turn may cause systematic risks associated with long and short positions to be priced differently. Flights to liquidity and accompanying downward liquidity spirals, as discussed by Brunnermeier and Pedersen (2009) and Anthonisz and Putnins (2017), might further exacerbate these pricing differences.

To corroborate this conjecture, we follow the literature in using idiosyncratic volatility (IVOL) as a proxy for arbitrage risk (e.g., Pontiff, 1996; Stambaugh et al., 2015). Intuitively, if arbitrageurs are able to neutralize their exposure to benchmark risks, then IVOL, as opposed to total volatility, is naturally interpreted as a measure of arbitrage risk, with higher IVOL implying greater impediment to price-correcting arbitrage. Accordingly, we split the cross-section of stocks into separate groups of stocks with high and low IVOLs, and compare the risk premium estimates for each group. We rely on the same calculation of IVOL as in Ang et al. (2006b) used in the previous section.

To facilitate a direct test of the hypothesis that $\lambda^{\mathcal{N}} = -\lambda^{\mathcal{M}^-}$ for each of the two separate IVOL groups, we reparameterize the cross-sectional regression in (6) as:

$$r_{t+1,i} = \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}(\hat{\beta}^{\mathcal{N}}_{t,i} - \hat{\beta}^{\mathcal{M}^-}_{t,i}) + \lambda^{\mathcal{P}}_{t+1}(\hat{\beta}^{\mathcal{P}}_{t,i} - \hat{\beta}^{\mathcal{M}^+}_{t,i}) + \delta^{\mathcal{M}^+}_{t+1}\hat{\beta}^{\mathcal{M}^+}_{t,i} + \delta^{\mathcal{M}^-}_{t+1}\hat{\beta}^{\mathcal{M}^-}_{t,i} + \epsilon_{t+1,i}. \quad (9)$$

This reparameterization does not change the fit of the regression. Conveniently, however, it allows for the construction of a simple t-test for the hypothesis that the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}-}$ are the same based on the time series average of the $\delta_t^{\mathcal{M}^-}$ estimates:

$$\hat{\delta}^{\mathcal{M}^-} = \frac{1}{T-1}\sum_{t=2}^{T}\hat{\delta}_t^{\mathcal{M}^-}. \quad (10)$$

Implementing these cross-sectional regressions and accompanying simple t-test for the 50% of stocks with the lowest IVOL in each of the months in the sample results in an estimate of $\delta^{\mathcal{M}^-}$ equal to 3.84, with an insignificant t-statistic of 1.02. On the other hand, consistent with the thesis that the different risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}-}$ may be attributed to arbitrage risk, the estimate for $\delta^{\mathcal{M}^-}$ for the 50% of stocks with the highest IVOL equals 10.98, with a significant t-statistic of 2.59.[3]

To further buttress the role played by arbitrage risk and valuation uncertainty, we also consider grouped estimates based on turnover (TO); see Appendix A for details on our calculations of TO. Turnover is generally thought to be higher for stocks that are more difficult to value and subject to greater investor disagreement (e.g., Harris and Raviv, 1993; Blume et al., 1994). As such, stocks with higher turnover arguably also pose greater arbitrage price discrepancies (e.g, Kumar, 2009). Consistent with this view, the estimation results for the TO groupings tell the same general story as the IVOL groupings: the estimate (t-statistic) for $\delta^{\mathcal{M}^-}$ equals 8.38 (2.30) for the 50% of stocks with the highest TO, and hence more difficult to value stocks, compared to 5.60 (1.22) for the 50% of stocks with lower TO, and hence stocks subject to less arbitrage risk.

3.3. Upside and downside betas

In addition to the standard set of predictor variables included in Table 2, other beta decompositions have also been found to improve upon the traditional CAPM. Most closely related to the present analysis are the up and downside betas proposed in the widely-cited study by Ang et al. (2006a). Realized versions of the betas advocated therein are naturally defined as:

$$\hat{\beta}_{t,i}^{+} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f_{t,k}^{+}}{\sum_{k=1}^{m}(f_{t,k}^{+})^2}, \qquad \hat{\beta}_{t,i}^{-} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f_{t,k}^{-}}{\sum_{k=1}^{m}(f_{t,k}^{-})^2}. \quad (11)$$

In contrast to the semibetas proposed here, which account for joint asymmetric dependencies by conditioning the covariation on both the signed market and individual asset returns, the upside and downside betas condition only on the sign of the market return.

For ease of comparison, i the first row in Table 3 we repeat the baseline results for semibetas from Table 2. The second row in Table 3 reports the estimated average risk premiums associated with the upside and downside betas. The results are broadly consistent with the findings of Ang et al. (2006a) in that only β^- carries a significant risk premium. The results are also in line with the estimated risk premiums for the semibetas presented in the top row, which show that only $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$, which account for negative market comovements, are associated with significant risk premiums.

To more directly compare and contrast the pricing of the semibetas with the pricing of the up and downside betas, the third row in Table 3 reports the estimates obtained by including all of the six betas in the same cross-sectional regressions. Despite the relatively high correlation between the semibetas and the up/downside betas,[4] the estimated risk premium for $\beta^{\mathcal{N}}$ clearly stands out as the most significant with a t-statistic of 3.50, followed by the premium for $\beta^{\mathcal{M}^-}$ with a t-statistic of -2.90. Meanwhile, the risk premium for β^- has a t-statistic of only 1.27, suggesting that the information contained in semibetas effectively subsumes the information in the downside beta in terms of explaining the cross-sectional variation in the returns. A joint test that all of the semibeta risk premiums are zero, leaving only the up and downside betas with nonzero risk premiums, also strongly rejects the null with a p-value of less than 0.01. By contrast, a joint test that both of the up and downside beta premiums are zero, leaving only the semibetas with nonzero premiums, fails to reject the null, with a p-value of 0.16.

To facilitate a more direct test of whether the semibetas provide superior cross-sectional pricing predictions compared to the up and downside betas, notice that the latter can be obtained as a weighted sum of the former:

$$\hat{\beta}_{t,i}^{+} = (\widehat{\beta}_{t,i}^{\mathcal{P}} - \widehat{\beta}_{t,i}^{\mathcal{M}^+})\frac{\sum_{k=1}^{m} f_{t,k}^2}{\sum_{k=1}^{m}(f_{t,k}^{+})^2},$$
$$\hat{\beta}_{t,i}^{-} = (\widehat{\beta}_{t,i}^{\mathcal{N}} - \widehat{\beta}_{t,i}^{\mathcal{M}^-})\frac{\sum_{k=1}^{m} f_{t,k}^2}{\sum_{k=1}^{m}(f_{t,k}^{-})^2}.$$

Since the weights on the semibetas only involve functions of market returns, they do not vary in the cross-section. Accordingly, the semibeta pricing model proposed here reduces to the up and downside beta pricing model of Ang et al. (2006a) if the following restrictions hold on a per period basis:

$$H_{0,t}^{UP+DOWN}: \; \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+}. \quad (12)$$

We find that this hypothesis is rejected at the 5% level for 42.0% of the 684 monthly cross-sectional regressions (recall that the stricter CAPM restrictions in (8) were rejected at the 5% level for 46.1% of the months in the sample). Going one step further, we can also test the stronger hypothesis that only downside beta risk is priced:

$$H_{0,t}^{DOWN}: \; \lambda_t^{\mathcal{N}} = -\lambda_t^{\mathcal{M}^-} \cap \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+} = 0. \quad (13)$$

We find that this hypothesis is rejected at the 5% level for 50.5% of the months in the sample.

[3] Additional estimation results for all of the parameters in (9) are available in Supplemental Appendix S2. Naturally, the hypothesis that the (insignificant) risk premiums for $\beta^{\mathcal{P}}$ and $-\beta^{\mathcal{M}+}$ are the same is never rejected for any of the groups by the similarly defined t-statistics for $\delta^{\mathcal{M}^+}$.

[4] Correlations between all of the semibetas and the up/downside betas, along with the other controls, are presented in Appendix B. The time series averages of the cross-sectional correlations between β^+ and $\beta^{\mathcal{P}}$, and β^- and $\beta^{\mathcal{N}}$, in particular, are as high as 0.91, thus hindering a precise estimation of each of the individual risk premiums.

Table 3
Monthly Fama-MacBeth regressions and other measures. The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas, up and downside betas, coskewness and cokurtosis measures are calculated from daily data. The estimates are based on all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2019.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
10.54	1.84	4.59	-6.00					5.16
4.51	1.17	1.32	-1.97					
				1.21	3.23			3.41
				1.85	3.84			
11.53	-2.30	2.82	-11.20	-6.21	1.84			5.48
3.50	-0.55	1.34	-2.90	-0.97	1.27			
						5.44	2.13	1.68
						3.05	2.77	
18.03	-1.59	3.70	-11.32			12.13	-2.64	6.40
5.02	-0.76	1.09	-3.31			4.36	-3.41	

The period-by-period restrictions in (12) and (13) obviously imply that the same restrictions must hold on average. The simple t-statistics for testing the hypothesis that $\lambda^{\mathcal{N}} = -\lambda^{\mathcal{M}^-}$ for the different groups of stocks discussed in Section 3.2 above already pointed to arbitrage risk as the likely culprit behind the rejection of this hypothesis. In other words, the presence of market frictions and limits-to-arbitrage implies that considering only downside betas entails a significant loss of information relative to a model based on downside semibetas.

3.4. Coskewness and cokurtosis

The semibetas succinctly account for non-Normally distributed systematic risks by conditioning on the signed returns. A number of other, more statistically oriented, measures have been explored in the literature as a way to capture non-Normal asymmetric joint return dependencies and the possible pricing thereof. Most notably among these is arguably coskewness, as originally proposed by Kraus and Litzenberger (1976), and analyzed more extensively by Harvey and Siddique (2000) and Christoffersen et al. (2017), among others. Others have similarly argued that cokurtosis appears to be priced in the cross-section (see e.g., Dittmar, 2002 and Ang et al., 2006a). Directly following these studies, we calculate monthly realized coskewness and cokurtosis measures for stock i by:

$$CSK_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^2}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\frac{1}{m}\sum_{j=1}^{m}(f_{t,k}-\bar{f}_t)^2}, \tag{14}$$

$$CKT_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^3}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\left(\frac{1}{m}\sum_{k=1}^{m}(f_{t,k}-\bar{f}_t)^2\right)^{3/2}}, \tag{15}$$

where $\bar{f}_t$ and $\bar{r}_{t,i}$ denote the month t average return on the market and stock i, respectively.

In line with the studies cited above, the estimated risk premiums for the monthly CSK and CKT measures, reported in the fourth row of Table 3, are indeed both statistically significant. Meanwhile, comparing the top row with the penultimate row, shows that the semibeta pricing model fits the data much better than the coskewness/cokurtosis model, with an average cross-sectional R^2 of 5.16% compared to 1.68%. The results in the bottom row show that combining all six measures in a single model further improves the R^2 to 6.40%. Importantly, however, the t-statistics associated with $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ both remain strongly significant in the regression that includes CSK and CKT.

At the same time, joint tests that the semibeta premiums, or the coskewness/cokurtosis premiums, are equal to zero are both rejected at the 5% level. As such, this indicates that while coskewness and cokurtosis have substantially less cross-sectional explanatory power than semibetas, they do contain additional information about non-linear dependencies over and above the semibetas. This is perhaps not surprising, as coskewness and cokurtosis are primarily driven by joint dependencies in the tails, and several recent studies have argued that such systematic tail risks appear to be priced differently from other risks (see, e.g., Kelly and Jiang, 2014; Bollerslev et al., 2016; Chabi-Yo et al., 2019; Orlowski et al., 2019). By contrast, the semibetas advocated here rely on a simple decomposition of the standard covariation with the market and more "normal" systematic risks.

4. High-frequency data and daily semibetas

It is well established that temporally aggregated returns tend to be closer to being Normally distributed than more finely sample returns (see, e.g., Campbell et al., 1997; Engle, 2011). As such, the monthly realized semibetas constructed from daily returns that underlie the results discussed in the previous section may obscure more subtle non-Normal dependencies that would be visible with daily realized semibetas constructed from higher frequency intraday returns. Of course, the theory underlying the realized semibetas consistently estimating the true latent covariation components also formally hinges on the use of ever finer sampled returns over fixed time intervals, which is better mimicked empirically with the use of intraday returns as opposed to daily returns in the construction of the betas.

Hence, we therefore extend our previous analysis by investigating the pricing of daily semibetas constructed from high-frequency intraday data. Our analyses rely on high-frequency data obtained from the Trades and Quotes (TAQ) database. We include all of the S&P 500 constituent stocks during the January 1993 to December 2019 sample period, resulting in a total of 6,799 trading days and 1,182 unique securities. We adopt a 15-minute sampling scheme, or $m = 26$ return observations per day, in our calculations of the realized semibeta measures. This choice strikes a judicious balance between biases induced by market microstructure effects when sampling too finely versus the theoretical continuous-time arguments underlying the consistency of the realized semicovariance measures.[5] We further match the intraday TAQ data and sample of stocks to the CRSP database to obtain the full-day returns for each of the stocks (this also ensures proper handling of stock splits and dividends). All of our subsequent asset pricing investigations are based on these full-day and resulting longer weekly and monthly returns. We also rely on the daily market capitalization for each of the individual stocks from the CRSP database in our construction of the high-frequency value-weighted market index.

A full set of descriptive summary statistics for the resulting daily realized semibetas are provided in Appendix B. The general features fairly closely mirror those of the monthly semibetas discussed in Section 2.2. The average values of the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) exceed the average values of the two discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$), and $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$ are also both more strongly correlated with the traditional market beta (β), than $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$. Underpinning our use of the daily realized semibetas in daily predictive Fama-MacBeth regressions, the daily semibetas are even more strongly autocorrelated than the monthly semibetas, with first-order autocorrelations around 0.9.[6]

Turning to the daily Fama-MacBeth regressions, Table 4 reports the estimated annualized risk premiums, along with their t-statistics based on Newey-West robust standard errors (using 22 lags), together with the time-series average of the R^2s from the first-stage cross-sectional regressions. Consistent with the idea that the use of daily returns in the estimation of monthly semibetas blurs some of the inherent asymmetric dependencies captured by the daily realized semibetas, the daily semibetas reveal even stronger predictive pricing relationships than the corresponding results for the monthly semibetas in Table 2. However, the same key findings remain: the risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both significant, while the risk premiums for $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ are both insignificant. Further mirroring the monthly results, the explanatory power of the semibeta pricing model is again more than double that of the traditional CAPM reported in the top row, with an average cross-sectional R^2 of 5.42% compared to just 2.57%. Tests of the restriction that the risk premiums associated with the four semibetas are indeed the same, corresponding to the $H_{0,t}^{CAPM}$ hypothesis in Eq. (8), are also rejected at the 5% level for 70.0% of the 6,799 days in the sample. Further corroborating the robustness of the daily results, the bottom two rows of Table 4 show that the significance of the risk premiums for the daily $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain intact to the inclusion of the same set of controls considered with the monthly semibetas in Table 2.

Our discussion in Section 3.2 pointed to arbitrage risk as a possible explanation for the difference in the absolute values of the estimated risk premiums for the monthly $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$. To further bolster that conjecture, we repeat the same stratified estimation approach for the daily semibetas, considering separate groups of stocks with high and low arbitrage risks, formally comparing the resulting risk premium estimates with the aid of the reparameterized regression in (9) and t-statistic for testing $\delta^{\mathcal{M}^-}$ equal to zero. In particular, considering the 50% of stocks with the lowest IVOL for each of the days in the sample, the estimate of $\delta^{\mathcal{M}^-}$ equals 2.58, with an insignificant t-statistic of 0.54, while the estimate of $\delta^{\mathcal{M}^-}$ for the 50% of stocks with the highest IVOL equals 23.93, with a highly significant t-statistic of 3.82.[7] Partitioning the sample into the 50% stocks with the highest and lowest daily turnover (TO), the estimates (t-statistics) for $\delta^{\mathcal{M}^-}$ equal -3.89 (-0.79) and 8.04 (2.18), respectively. As such, these results again suggest that the different risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ may at least in part be attributed to arbitrage risk.

In parallel to our analysis of the monthly semibetas in Section 3.3, Table 5 further shows that the inclusion of daily equivalents to the up and downside betas and coskewness and cokurtosis measures do not affect the significance of the daily semibetas. Consistent with Ang et al. (2006a), the estimates in the second row imply that only downside beta risk is priced. However, the inclusion of the semibetas in the cross-sectional regressions, reported in the third row of the table, again renders the estimated risk premiums for both β^+ and β^- insignificant. Hypothesis $H_{0,t}^{UP+DOWN}$ in Eq. (12), corresponding to symmetric pricing of the pairs of semibetas, is also rejected at the 5% level for 63.5% of the 6799 daily cross-sectional regressions, while the stronger $H_{0,t}^{DOWN}$ hypothesis in Eq. (13), corresponding to only downside beta risk being priced, is rejected at the 5% level for an even larger 72.8% of the daily regressions.

In contrast to the monthly results in Table 3, the estimated risk premiums for the daily realized CSK and CKT measures, reported in the fourth row of Table 5, are both insignificant. Meanwhile, as shown in the last row of the table, CSK and CKT both become significant when included together with the semibetas. Importantly,

[5] Although a finer 5-min sampling frequency has often been used in the realized volatility literature for the calculation of univariate realized volatility measures (see, e.g., Liu et al., 2015, and the many references therein), market microstructure effects are further compounded in a multivariate setting by the so-called Epps (1979) effect, which leads to a downward bias in realized covariation measures stemming from asynchronous prices. Correspondingly, we resort to a coarser 15-min sampling frequency, also used by Bollerslev et al. (2020a) in their analysis of realized semicovariances for a similar sample of individual stocks.

[6] The autocorrelations for the daily semibetas are shown in the Supplemental Appendix S3. To further highlight the additional information about asymmetric dependencies conveyed by the daily semibetas, Section S4 in the Supplemental Appendix compares the estimated realized semibetas to the limiting values that would obtain if the daily individual stock and market returns were jointly Normally distributed.

[7] Estimation results pertaining to all of the parameters in (9) are again available in Supplemental Appendix S2.

Table 4

Daily Fama-MacBeth regressions The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas are calculated from 15-minute intraday data. All of the control variables are measured prior to the daily returns, as detailed in Appendix A. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2019 sample period.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	RV	IVOL	ILLIQ	R^2
4.49												2.57
3.42												
	18.10	-0.23	-4.49	-7.82								5.42
	5.40	-0.09	-1.14	-2.19								
	19.02	-4.10	-2.63	-11.41	-1.78	-2.01	0.09					8.62
	5.86	-1.64	-0.73	-3.27	-3.63	-1.94	3.36					
	18.79	-1.57	2.33	-7.18	-2.70	-2.64	0.08	-0.43	0.43	-3.26	-0.83	11.23
	5.87	-0.61	0.68	-2.02	-4.62	-2.55	2.83	-5.20	2.16	-4.45	-2.62	

Table 5

Daily Fama-MacBeth regressions on other measures The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas, up and downside betas, and coskewness and cokurtosis measures are calculated from 15-minute intraday data based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2019 sample period.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
18.10	-0.23	-4.49	-7.82					5.42
5.40	-0.09	-1.14	-2.19					
				-0.37	5.71			3.60
				-0.38	5.32			
15.84	-8.69	7.76	-11.13	-4.27	2.10			6.48
3.89	-1.03	0.78	-2.16	-1.05	0.78			
						-1.86	0.88	1.51
						-0.71	0.96	
26.21	-2.19	-5.06	-14.99			10.93	-3.71	6.31
6.28	-0.73	-1.20	-4.00			3.18	-3.68	

however, the estimated risk premiums for the daily $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain strongly significant in the cross-sectional regressions that include the daily CSK and CKT measures.

4.1. Daily semibetas and longer investment horizons

The strong predictive relationship between the daily realized semibetas and the cross-sectional variation in the subsequent daily returns naturally raises the question of whether this same predictive relationship based on daily semibetas carries over to longer investment horizons? To investigate this, we rely on the identical day t realized semibetas and cross-sectional regression in (6) in which we replace the left-hand-side daily returns with the cumulative returns from day $t+1$ to day $t+h$, setting $h=5$ and $h=20$, corresponding to a "week" and a "month," respectively. This effectively amounts to using daily semibetas to predict multiple daily returns, and the sum thereof, further into the future, and as such one might expect the longer horizon results to be weaker than the results for the one-day-ahead return predictions.

Nonetheless, the results reported in Table 6 are generally in line with the daily return predictions reported in Table 4.[8] They are also consistent with the results for the monthly semibetas and monthly forecast horizons for the wider sample of stocks discussed in Section 3: the estimated risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both highly statistically significant, while neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to be priced. Testing the restrictions implied by the CAPM, as given by $H_{0,t}^{CAPM}$ in Eq. (8), we reject the null at the 5% significance level for 67.2% and 63.3% of the weekly and monthly regressions, respectively. Testing the symmetric pricing restriction, given by hypothesis $H_{0,t}^{UP+DOWN}$ in Eq. (12), we reject the null at the 5% level for 61.5% and 59.0% of the weekly and monthly regressions, respectively. Also, the hypothesis that the risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are the same and that neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ is priced, as stipulated by $H_{0,t}^{DOWN}$ in Eq. (13), is rejected at the 5% level for 69.4% and 66.3% of the weekly and monthly regressions, respectively.

Further corroborating the findings in Table 4 based on a shorter daily investment horizon, the weekly and monthly $\lambda^{\mathcal{N}}$ and $\lambda^{\mathcal{M}^-}$ estimates both remain statistically significant when including the same set of widely used control variables. At the same time, comparing the magnitudes of the estimated semibeta risk premiums, the (annualized) monthly estimates are naturally smaller than the (annualized) weekly estimates, as the strength of the predictability afforded by the daily semibetas diminishes with the return horizon.

5. Betting on, and against, semibetas

To help better assess not just the statistical, but also the economic significance of the semibeta pricing, this section showcases the performance of various semibeta trading strategies. In addition to portfolios that "bet on

[8] To enhance the efficiency of our inference, we purposely rely on overlapping return windows and appropriately adjusted standard errors and t-statistics. However, qualitatively similar findings are obtained with non-overlapping return windows.

Table 6

Daily semibetas and weekly and monthly investment horizons. The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional regressions for predicting the future weekly (5-days) and monthly (20-days) returns. The daily semibetas are calculated from 15-minute intraday data on the last day preceding the return window. All of the control variables are measured prior to the daily returns. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2019 sample period.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	RV	IVOL	ILLIQ	R^2
Panel A: Weekly												
4.74												2.29
4.47												
	12.42	1.28	6.34	-12.30								5.09
	5.98	0.79	2.72	-4.13								
	9.26	0.89	2.38	-12.38	-3.20	-2.77	0.08	-0.30	0.16	-0.99	-0.98	11.44
	5.37	0.60	1.44	-4.33	-5.94	-2.84	2.74	-4.31	1.46	-1.55	-3.99	
Panel B: Monthly												
2.73												1.85
3.07												
	7.20	1.51	3.18	-3.66								4.41
	4.45	1.24	1.70	-1.65								
	4.40	0.56	-0.23	-5.82	-2.82	-2.50	0.07	-0.23	-0.16	0.51	-0.62	11.19
	3.53	0.53	-0.18	-2.80	-6.34	-2.96	2.61	-4.33	-1.93	0.92	-3.76	

$\beta^{\mathcal{N}}$" and "bet against $\beta^{\mathcal{M}^-}$," we also consider a combined semibeta strategy comprised of an equal-weighted combination of "betting on $\beta^{\mathcal{N}}$" and "betting against $\beta^{\mathcal{M}^-}$" portfolios. To most accurately capture the inherent non-linear dependencies underling the differential pricing of the semibetas, we purposely rely on daily realized semibetas. For comparison purposes, we also include a long-short strategy based on the traditional market beta.[9]

To avoid the critiques of Novy-Marx and Velikov (2018), we form the long-short portfolios using well-established methods. Firstly, we estimate betas and semibetas using standard, if modern, methods from high frequency econometrics, as detailed above. We then take a value-weighted long position in the top quintile and a value-weighted short position in the bottom quintile of stocks, rebalanced daily, to obtain zero-cost portfolios. We rely on continuously-compounded, as opposed to arithmetic, returns to facilitate the calculation of the cumulative portfolio returns over longer holding periods. We restrict the sample of stocks to the constituents of the S&P 500 index, thus explicitly excluding small, and potentially difficult to short micro-cap stocks. We use the popular four-factor model of Fama and French (1993) and Carhart (1997) (FFC4), as well as the five-factor model of Fama and French (2015) (FF5) to assess the risk-adjusted performance of the portfolios.

The top panel of Table 7 reports the average returns, standard deviations and annualized Sharpe ratios for the long-short portfolios. The average return on the Semi-β portfolio is nearly double that of the beta portfolio, while the volatility is just over half that of the beta portfolio, combining to yield a Sharpe ratio of 0.92 compared to 0.37 for the traditional market beta portfolio.[10] The latter two columns show that both the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ components of the Semi-β portfolio contribute to its superior performance: the $\beta^{\mathcal{N}}$ portfolio generates much higher returns and comparable volatility to the standard beta portfolio, while the $\beta^{\mathcal{M}^-}$ portfolio generates similar returns with much lower volatility.

The lower panel of Table 7 reports the estimated FFC4 and FF5 alphas and factor loadings for the different portfolios. The traditional beta strategy generates an annualized alpha of 3.94% according to the FF5 factor model, with a t-statistic of 1.98. The beta strategy generates no significant alpha according to the FFC4 factor model. By contrast, the Semi-β strategy, and both of its underlying components, generate large and significant alphas, according to both the FFC4 and FF5 factor models. The annualized alphas range from 5.68% to 8.59%, with the corresponding t-statistics between 3.31 and 6.49.[11] These alphas will, of course, be reduced when accounting for transactions costs, and we analyze this in more detail below.

Looking at the estimated factor loadings, the conventional long-short β portfolio and the $\beta^{\mathcal{N}}$ portfolio exhibit

[9] Supplemental Appendix S8 contains additional results for a portfolio that takes long positions in high $\beta^{\mathcal{N}}$ stocks and short positions in high $\beta^{\mathcal{M}^-}$ stocks, and a portfolio based on long positions in low $\beta^{\mathcal{M}^-}$ stocks and short positions in low $\beta^{\mathcal{N}}$ stocks. The performance of these two additional betting on and against semibeta portfolios are qualitatively similar to that of the semibeta portfolio presented here.

[10] Identical rankings of the four portfolios are obtained when using the sample mean of the squared demeaned negative daily returns in place of the daily sample variance in the calculation of downside Sharpe type ratios. Further details of these additional results are available in the Supplemental Appendix S6.

[11] The inclusion of a betting against beta (BAB) factor in the FF3 model results in an even larger alpha of 9.04% for the Semi β strategy, with a corresponding t-statistic of 8.42. To guard against potential biases in the unconditional alphas arising from temporal variation in conditional betas (see, e.g., Jagannathan and Wang, 1996 and Lewellen and Nagel, 2006), we also calculated conditional alphas following the approach of Cederburgh and O'Doherty (2016, cf. Section II.B). The same general conclusions remain true: the semibeta portfolios result in highly significant positive conditional alphas, while the conditional alphas for the standard beta portfolios are always insignificant. The magnitudes of the average conditional alphas for the Semi-β and $\beta^{\mathcal{N}}$ portfolios are also very similar to the values reported in Table 7, while the average conditional alphas for the $\beta^{\mathcal{M}^-}$ portfolios are marginally lower. Further details of these additional results are available in the Supplemental Appendix S7.

Table 7

Betting on and against semibetas. The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. The $\beta^{\mathcal{N}}$ strategy bets on $\beta^{\mathcal{N}}$, the $\beta^{\mathcal{M}^-}$ strategy bets against $\beta^{\mathcal{M}^-}$, while the Semi-β strategy represents an equally-weighted combination of the former two strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993–2019 sample.

	β		Semi-β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	5.62		8.17		10.02		5.56	
Std dev	15.37		8.86		15.78		7.80	
Sharpe	0.37		0.92		0.63		0.71	
α	2.52	3.94	6.84	7.52	6.89	8.59	6.02	5.68
	1.21	1.98	5.92	6.49	3.31	4.22	3.93	3.65
β_{MKT}	0.57	0.50	0.28	0.25	0.59	0.51	-0.02	-0.01
	75.03	62.98	67.31	53.06	76.91	62.03	-3.22	-2.28
β_{SMB}	0.27	0.15	0.31	0.24	0.39	0.26	0.23	0.22
	18.94	10.67	38.92	28.43	27.12	17.78	21.88	19.12
β_{HML}	-0.01	0.22	-0.01	0.16	-0.06	0.20	0.04	0.12
	-0.42	14.59	-1.10	17.85	-3.98	12.61	3.72	9.97
β_{MOM}	-0.21		-0.16		-0.22		-0.10	
	-20.47		-27.83		-21.16		-13.06	
β_{RMW}		-0.42		-0.25		-0.46		-0.03
		-21.65		-21.46		-22.80		-2.13
β_{CMA}		-0.33		-0.24		-0.40		-0.08
		-14.01		-17.42		-16.37		-4.48
R^2	56.11	58.20	55.83	56.68	58.98	61.96	10.19	8.02

fairly similar FFC4 and FF5 systematic risk exposures. Meanwhile, the estimated factor loadings for the $\beta^{\mathcal{M}^-}$ portfolio are markedly different. In contrast to the other portfolios, the $\beta^{\mathcal{M}^-}$ portfolio is close to market neutral. The FFC4 estimates further suggest that the portfolio contains a higher proportion of value stocks than the other portfolios, while the FF5 estimates point to decidedly lower exposures to the profitability and investment factors than any of the other portfolios. The combined Semi-β strategy naturally reflects these different risk profiles of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ portfolios.[12]

5.1. Betting on the competition

To underscore the superiority of the semibeta portfolio, Table 8 reports the results from analogously constructed up and downside beta, and coskewness and cokurtosis portfolios. Given the pertinent discussion in Ang et al. (2006a) and Harvey and Siddique (2000), we consider value-weighted long-short positions based on the top and bottom quintiles of stocks betting on β^-, against β^+, against CSK, and on CKT. In parallel to the semibeta portfolios discussed above, we also consider equal-weighted combinations of the two respective pairs of portfolios, denoted "$\beta^- - \beta^+$" and "$CKT - CSK$" in the table.[13]

The top panel in Table 8 reveals that only the β^- portfolio has a Sharpe ratio in excess of the conventional beta sorted portfolio. However, the Sharpe ratio of 0.49 for the β^- portfolio is still substantially lower than the Sharpe ratios for all of the semibeta-based strategies in Table 7. The lower panel in Table 8 further shows that the FFC4 and FF5 alphas for the CSK and CKT portfolios are all small and statistically insignificant. Only the β^- portfolio obtains significant alphas of 4.15% and 5.64% for the FFC4 and FF5 factor models, respectively. As one might expect, the estimated risk exposures for the β^- portfolio are fairly similar to the estimates for the semibeta portfolio reported in Table 7. However, despite these similarities in risk profiles, the annualized FFC4 and FF5 alphas for the combined Semi-β portfolio are both larger and much more strongly significant than the alphas for the β^- portfolio, again highlighting the superior performance of the betting on and against semibeta strategy.

5.2. Longer holding periods

The daily rebalancing of the long-short (semi)beta strategies considered in Table 7 may be difficult to implement in practice. To alleviate this concern, we consider the performance of the same portfolio strategies based on less frequent weekly and monthly rebalancing, or equivalently longer weekly and monthly holding periods.

Table 9, in particular, shows that moving to weekly rebalancing adversely affects the traditional beta strategy,

[12] To further explore these differences in risk profiles, we also calculated industry concentrations. The β and $\beta^{\mathcal{N}}$ portfolios again appear fairly similar along that dimension. Most noticeably, the $\beta^{\mathcal{M}^-}$ portfolio on average invest less in HiTech firms and more in Non-durables than the other two portfolios. Moreover, it is generally less concentrated with lower overall industry exposures. Further details are available in the Supplemental Appendix S5.

[13] Ang et al. (2006a) note that β^+ tends to be positively correlated with β, leading to an ambiguous prediction for the sign of the relationship between β^+ and expected returns. To overcome this, they suggest sorting on the "relative" β^+, defined as $\beta - \beta^+$. We also implemented this approach and found that the untabulated resulting portfolio did indeed have a higher Sharpe ratio than the portfolio based solely on β^+. However, the FFC4 and FF5 alphas were small and statistically insignificant.

Table 8
Betting on the competition. The top panel reports annualized descriptive statistics of portfolios formed using up and downside betas, and coskewness and cokurtosis measures, designed to bet on β^-, against β^+, against *CSK*, and on *CKT*. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993–2019 sample period.

	β^- - β^+		β^-		β^+		CKT - CKS		CSK		CKT	
Avg ret	1.83		7.11		-5.12		-0.19		-1.91		1.96	
Std dev	5.34		14.46		13.58		6.11		7.49		9.17	
Sharpe	0.34		0.49		-0.38		0.03		-0.25		0.21	
α	1.46	1.78	4.15	5.64	-2.92	-3.75	-0.56	0.02	-2.12	-1.98	0.67	1.69
	1.33	1.63	1.95	2.70	-1.42	-1.87	-0.52	0.02	-1.42	-1.32	0.43	1.11
β_{MKT}	0.04	0.03	0.52	0.45	-0.45	-0.40	0.13	0.11	0.02	0.01	0.24	0.20
	8.87	5.91	66.70	53.85	-59.32	-49.73	33.19	25.54	2.86	2.11	42.74	33.18
β_{SMB}	0.01	0.01	0.25	0.15	-0.22	-0.13	-0.03	-0.06	0.02	0.03	-0.09	-0.14
	1.66	1.35	17.21	9.92	-15.90	-8.79	-4.50	-7.37	2.37	2.37	-8.43	-12.49
β_{HML}	-0.02	-0.03	-0.03	0.16	-0.02	-0.23	-0.08	-0.05	-0.02	-0.04	-0.13	-0.07
	-2.86	-3.86	-1.81	10.12	-1.21	-14.91	-10.22	-6.62	-2.20	-3.49	-11.91	-5.75
β_{MOM}	0.03		-0.15		0.22		0.00		0.03		-0.03	
	5.62		-14.73		21.42		-0.82		3.49		-4.41	
β_{RMW}		-0.01		-0.37		0.35		-0.10		0.00		-0.20
		-1.20	0.00	-18.07	0.00	17.54	0.00	-9.18	0.00	0.25	0.00	-12.92
β_{CMA}		-0.01		-0.30		0.27		-0.05		0.01		-0.12
		-1.13		-12.13		11.51		-4.38		0.67		-6.67
R^2	2.19	1.71	50.24	52.95	47.83	48.07	16.30	17.70	0.64	0.45	26.09	28.55

Table 9
Betting on and against semibetas with weekly rebalancing. The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. The $\beta^{\mathcal{N}}$ strategy bets on $\beta^{\mathcal{N}}$, the $\beta^{\mathcal{M}^-}$ strategy bets against $\beta^{\mathcal{M}^-}$, while the Semi-β strategy represents an equally-weighted combination of the former two strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced weekly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample period.

	β		Semi-β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	2.33		4.91		6.71		2.41	
Std dev	14.24		8.31		14.57		7.44	
Sharpe	0.16		0.59		0.46		0.32	
α	-0.29	0.95	3.83	4.45	4.21	5.49	2.74	2.71
	-0.14	0.48	3.40	3.84	2.21	2.85	1.95	1.91
β_{MKT}	0.50	0.44	0.24	0.20	0.50	0.44	-0.02	-0.03
	66.99	56.06	57.40	43.90	71.40	56.45	-4.73	-4.81
β_{SMB}	0.26	0.16	0.31	0.25	0.37	0.26	0.25	0.23
	19.05	11.00	40.21	29.34	28.56	18.84	25.74	22.35
β_{HML}	-0.06	0.13	-0.04	0.09	-0.09	0.12	0.01	0.06
	-4.14	8.38	-5.10	10.33	-6.93	8.48	1.15	5.25
β_{MOM}	-0.18		-0.13		-0.20		-0.06	
	-18.29		-23.50		-21.68		-8.09	
β_{RMW}		-0.40		-0.22		-0.40		-0.05
		-20.56		-19.68		-21.02		-3.74
β_{CMA}		-0.24		-0.18		-0.31		-0.05
		-10.46		-12.98		-13.57		-2.79
R^2	50.76	52.63	47.95	48.92	52.68	54.52	10.21	9.65

with the Sharpe ratio falling markedly from 0.37 to 0.16. Moreover, the FF5 alpha that was borderline significant with daily rebalancing becomes small and insignificant. By contrast, the Semi-β strategy reported in the second set of columns continues to outperform. The Sharpe ratio does decrease from 0.92 to 0.59, and the annualized alphas are also somewhat lower than the alphas obtained with more frequent daily rebalancing. However, both the FFC4 and FF5 alphas remain strongly significant, with t-statistics of 3.40 and 3.84, respectively.

Table 10 presents the corresponding results based on even less frequent monthly portfolio rebalancing. The Sharpe ratio for the traditional beta strategy decreases even further to 0.01, and the corresponding FFC4 and FF5 alphas both become negative, albeit not statistically significantly so. The Semi-β portfolio, on the other hand, retains its appeal. The Sharpe ratio of 0.42 is obviously lower than the ratios obtained with daily and weekly rebalancing, and the annualized FFC4 and FF5 alphas are also both less than the corresponding daily and weekly alphas. Still, both of the alphas remain statistically significant, consistent with the analysis in Section 4.1, and the relationship between semibetas and future returns holding true at daily, weekly, and monthly horizons.

Table 10

Betting on and against semibetas with monthly rebalancing. The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. The $\beta^{\mathcal{N}}$ strategy bets on $\beta^{\mathcal{N}}$, the $\beta^{\mathcal{M}^-}$ strategy bets against $\beta^{\mathcal{M}^-}$, while the Semi-β strategy represents an equally weighted combination of the former two strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced monthly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample period.

	β		Semi-β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	0.14		3.44		2.87		3.39	
Std dev	13.52		8.10		13.87		7.16	
Sharpe	0.01		0.42		0.21		0.47	
α	-2.10	-1.17	2.41	3.03	0.69	1.75	3.50	3.70
	-1.09	-0.63	2.13	2.65	0.37	0.94	2.76	2.90
β_{MKT}	0.45	0.41	0.22	0.19	0.45	0.40	0.00	-0.01
	63.99	53.91	53.84	41.86	65.75	53.05	-0.50	-2.26
β_{SMB}	0.23	0.13	0.31	0.25	0.36	0.25	0.27	0.25
	17.68	9.32	40.62	30.25	28.21	18.72	31.07	26.98
β_{HML}	-0.07	0.09	-0.04	0.07	-0.06	0.14	-0.03	0.01
	-5.21	5.97	-5.64	8.39	-4.89	9.85	-3.05	0.53
β_{MOM}	-0.18		-0.12		-0.21		-0.03	
	-19.35		-21.77		-22.74		-5.11	
β_{RMW}		-0.38		-0.23		-0.39		-0.07
		-20.67		-20.13		-21.24		-5.19
β_{CMA}		-0.16		-0.15		-0.26		-0.04
		-7.12		-11.14		-11.88		-2.83
R^2	46.84	47.90	46.72	47.86	49.71	50.86	12.76	12.94

5.3. Transaction costs

The analysis above pertaining to the profitability of the various betting on and against (semi)beta strategies did not take into account the cost of actually implementing the portfolio positions. Such costs are clearly important in practice. Hence, in this section we explicitly consider the impact of transaction costs.

To better replicate the empirical practice of not "trading too much," we focus on the Semi-β portfolio with monthly rebalancing. In parallel with previous studies (e.g., Han, 2006; DeMiguel et al., 2009; Liu, 2009), we assume that the transaction costs are proportional to the turnover of the portfolio, with the portfolio turnover computed simply as the sum of the turnover of the long and short legs of the portfolio. To provide an empirically realistic upper bound (see, e.g., the transaction cost estimates in Novy-Marx and Velikov, 2016), we simply fix the roundtrip trading costs at 0.5% for all of the S&P 500 stocks in the sample.[14]

Rather than trading all the way to the positions that would be "optimal" in the absence of transaction costs, several procedures have been developed in the literature to help mitigate trading costs (e.g., Bertsimas and Lo, 1998; Engle and Ferstenberg, 2007; Obizhaeva and Wang, 2013). These procedures are typically geared towards the specific setting and strategy at hand and can be difficult to realistically implement. Instead, we follow the simple-to-implement idea of Garleanu and Pedersen (2013) of only partially adjusting the portfolio weights each period. Specifically, let ω_t^F denote the vector of (fully-adjusted) semibeta portfolio weights in month t. The partially-adjusted portfolio weights for month t are then obtained as:

$$\omega_t^P = \lambda\omega_{t-1}^P + (1-\lambda)\omega_t^F, \tag{16}$$

where the scalar parameter $0 < \lambda < 1$ governs the speed of adjustment. This same approach has also recently been implemented by Bollerslev et al. (2018). While such a partial-adjustment approach will help mitigate turnover, it will generally also dampen the signal. As such, the benefits will depend in a complicated way on the interaction between the particular strategy and the transaction costs that are incurred, and the best choice of λ must therefore also be determined on a case-by-case basis. We do not attempt to do so here. Instead, in line with similar uses of moving average filters in other situations, volatility estimation included, we simply set $\lambda = 0.95$ and initialize the weights by fixing $\omega_1^P \equiv \omega_1^F$.[15]

Table 11 summarizes the performance of the resulting partially-adjusted Semi-β portfolios. For ease of comparison, the two left-most columns present the results using fully-adjusted portfolio weights with no transaction costs, corresponding to the second set of columns in Table 10. The second set of columns shows the results with transaction costs. As the numbers show, doing so severely affects the performance of the semibeta strategy, resulting in significantly negative alphas. To help combat this detrimental impact of "too much trading," the last

[14] Additional results for other transaction costs assumptions are available in Supplemental Appendix S9.

[15] The Supplemental Appendix S9 contains additional results for alternative choices of λ, which further highlight the trade-off in signal retention and transaction cost reduction. It also contains additional results for alternative, more involved, procedures based on smoothing the semibeta estimates.

Table 11

Betting on and against semibetas with transaction costs. The top panel reports annualized descriptive statistics for the Semi-β portfolios, constructed as an equally-weighted combination of $\beta^{\mathcal{N}}$ portfolios that bet on $\beta^{\mathcal{N}}$, and $\beta^{\mathcal{M}^-}$ portfolios that bet against $\beta^{\mathcal{M}^-}$. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. All of the portfolios are self-financing based on value-weighted long-short positions determined by the combined semibeta strategy rebalanced monthly. The roundtrip transaction cost (T-cost) is set to 0.5%. The two left-most columns are identical to the second set of columns in Table 10 with fully adjusted portfolio weights. The second set of columns incorporate transaction costs. The right two sets of columns report the results based on partially-adjusted portfolio weights as discussed in the main text, without and with transaction costs. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample period.

Adjustment	Full		Full		Partial		Partial	
T-cost	No		Yes		No		Yes	
Avg ret	3.44		-2.75		3.75		3.46	
Std dev	8.10		8.29		7.32		7.32	
Sharpe	0.42		-0.33		0.51		0.47	
α	2.41	3.03	-3.79	-3.15	2.22	3.65	1.93	3.36
	2.13	2.65	-3.34	-2.75	2.45	4.12	2.12	3.79
β_{MKT}	0.22	0.19	0.22	0.19	0.23	0.18	0.23	0.18
	53.84	41.86	54.12	41.89	70.22	50.54	70.19	50.50
β_{SMB}	0.31	0.25	0.32	0.25	0.28	0.23	0.28	0.23
	40.62	30.25	40.84	30.27	46.11	35.67	46.09	35.64
β_{HML}	-0.04	0.07	-0.04	0.08	-0.10	-0.03	-0.10	-0.03
	-5.64	8.39	-5.17	8.71	-15.62	-4.48	-15.60	-4.46
β_{MOM}	-0.12		-0.12		-0.01		-0.01	
	-21.77		-21.59		-2.80		-2.80	
β_{RMW}		-0.23		-0.23		-0.22		-0.22
		-20.13		-20.54		-25.11		-25.11
β_{CMA}		-0.15		-0.15		-0.17		-0.17
		-11.14		-11.03		-16.22		-16.21
R^2	46.72	47.86	44.81	46.08	50.91	56.90	50.89	56.89

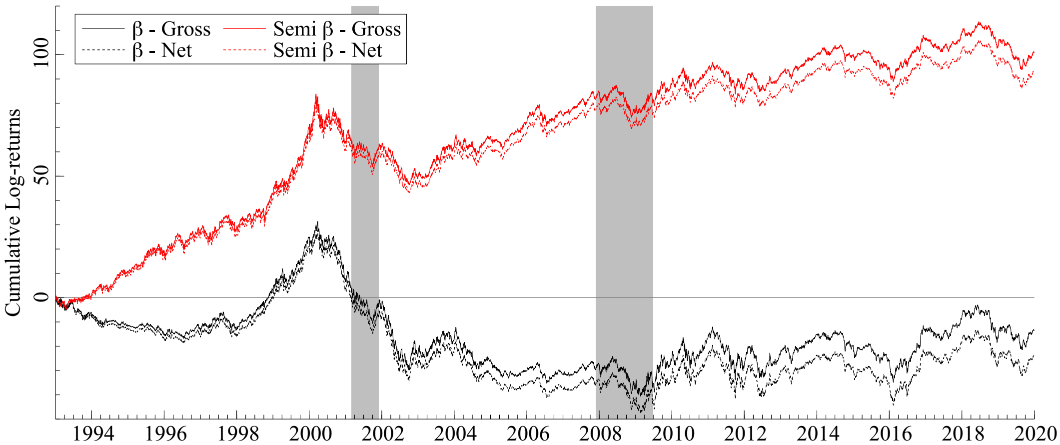

Fig. 3. Cumulative returns for beta and semibeta long-short portfolio strategies. The figure plots the cumulative percentage returns of long-short strategies based on beta and semibeta sorted value-weighted quintile portfolios. The semibeta portfolios are constructed as an equally-weighted combination of $\beta^{\mathcal{N}}$ portfolios that bet on $\beta^{\mathcal{N}}$, and $\beta^{\mathcal{M}^-}$ portfolios that bet against $\beta^{\mathcal{M}^-}$. The shaded region represents NBER recession periods. The beta estimates and portfolio returns are based on all of the S&P 500 constituent stocks and days in the 1993-2019 sample.

two sets of columns present the results for the partially-adjusted portfolios, with and without transaction costs. Interestingly, partially adjusting the portfolio weights slightly improves the performance, even in the absence of transaction costs. Compared to the numbers in the first sets of columns, the average return is slightly higher, while the volatility is slightly lower, resulting in an increase in the Sharpe ratio from 0.42 to 0.51. Likewise, the alphas are also both more strongly significant for the portfolio based on partially-adjusted weights. In other words, trading only part of the way to the target not only reduces turnover, it seemingly also reduces the “noise” in the semibeta estimates, thereby strengthening the signal, and in turn resulting in an overall slightly better performing portfolio. The results in the last set of columns further highlight the advantage of the partial adjustment approach in the presence of transaction costs. Incorporating transaction costs naturally lowers the average returns and Sharpe ratios compared to the partially-adjusted portfolio without transaction costs. However, the performance of the partially-adjusted portfolio clearly beats that of the fully-adjusted portfolio with transaction costs. The FFC4 and FF5 alphas for the partially-adjusted semibeta portfolio also both remain positive and statistically significant, with t-statistics of 2.12 and 3.79, respectively.

To visualize the timing of the returns, and more clearly contrast the performance of the semibeta strategy with the returns based on a traditional long-short beta strategy, Fig. 3 plots the cumulative returns from both. In both cases, we rely on the partially-adjusted portfolio weights. The solid lines depict the cumulative returns ignoring transaction costs. The dashed lines show the returns that incorporate 0.5% roundtrip transaction costs. As the figure shows, the semibeta strategy performs well throughout most of the sample period, resulting in quite high cumulative returns at the end of the sample, even after incorporating transaction costs. By contrast, and consistent with the idea of "betting against beta" advocated by Frazzini and Pedersen (2014), the traditional beta strategy performs poorly over much of the sample period, resulting in negative cumulative returns by the end of the sample, even without incorporating transaction costs.

6. Conclusion

We propose a new additive decomposition of the traditional market beta into four *semi*betas defined by the signed covariation between the market and individual asset returns: $\beta = \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}^+} - \beta^{\mathcal{M}^-}$. Consistent with the implications from a setup in which investors only care about downside risk, we find that only the two semibetas associated with negative market return variation are priced. At the same time, we strongly reject that the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ are the same, as would be implied by a traditional downside beta model. We attribute this difference to arbitrage risk driving a wedge between the compensation for long versus short positions.

The results from a variety of specifications and empirical analyzes, involving different sampling frequencies, prediction horizons and a long list of additional controls, reveal that the risk premium for $\beta^{\mathcal{N}}$ is around double that for $-\beta^{\mathcal{M}^-}$, and close to three times the risk premium for the traditional market β. We further establish that simple trading strategies that bet on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ lead to Sharpe ratios that are more than double that of the market. Accounting for transaction costs, these same long-short semibeta strategies continue to produce economically large and strongly statistically significant risk-adjusted alphas. In conclusion: do not bet on or against beta, bet on *and* against the "right" semibetas.

Appendix A. Additional control variables

- Size (ME). Following Fama and French (1993), a firm's size is measured by its market value of equity: the product of closing price and the number of shares outstanding. We use its natural logarithm to reduce skewness. In the CRSP sample, we use end-of-month values. In the TAQ sample, the value is updated daily.
- Book-to-Market (BM). Following Fama and French (1992), Book-to-Market is computed in June of year t, as the ratio of book value of common equity in fiscal year $t-1$ to the market value of equity in December of year $t-1$. Book value of equity is defined as book value of stockholder' equity (SEQ), plus balance-sheet deferred taxes (TXDB) and investment tax credit (ITCB, if available), minus book value of preferred stock (PSTK). Book-to-Market is updated yearly in both the CRSP and TAQ sample.
- Momentum (MOM). Following Jegadeesh and Titman (1993), momentum is the compound gross return from 12 months to 1 month before the date, i.e. skipping the short-term reversal month. For the CRSP sample we use the compounded $t-12$ to $t-2$ monthly returns. For the TAQ sample, we compound the gross return from day $t-252$ through day $t-21$. The measure is computed only if a minimum of 100 days is available.
- Reversal (REV). Following Jegadeesh (1990) and Lehmann (1990), the short-term reversal is the return for the previous month. For the CRSP sample we use the lagged monthly return, while for the TAQ sample we compound the returns from day $t-20$ to day $t-1$.
- Idiosyncratic Volatility (IVOL). Following Ang et al. (2006b), this is calculated as the standard deviation of the day $t-20$ to $t-1$ residuals from the daily return regression:

$$r_{t,i} - r_t^f = \alpha_i + \beta_i(f_t - r_{t,i}^f) + \gamma_i SMB_{t,i} + \phi_i HML_{t,i} + \epsilon_{t,i},$$

where $r_{t,i}$ and f_t denote the daily stock and market return, r_t^f denotes the risk-free rate, and $SMB_{t,i}$ and $HML_{t,i}$ denote the daily size and value factors for stock i. For the CRSP sample, we compute this value once per month based on the daily returns within the month. For the TAQ sample, we update this value based on the $t-20$ to $t-1$ daily returns.
- Realized Variance (RV). Following Andersen et al. (2001), we calculate realized variances as:

$$RV_{t,i} = \sum_{k=1}^{m} r_{t,k,i}^2.$$

Mirroring our estimation of the realized betas, we set k to a day for the monthly measures used with the CRSP sample, and 15-minutes for the daily measures used with the TAQ sample.
- Illiquidity (ILLIQ). Following Amihud (2002), illiquidity for stock i is defined as:

$$ILLIQ_{t,i} = \frac{1}{N_t}\sum_{j=1}^{N_t}\left(\frac{|r_{t-j,i}|}{volume_{t-j,i} \times price_{t-j,i}}\right).$$

We take the logarithm to reduce the skewness and the impact of outliers. For the CRSP sample we compute the measure based on daily data within the previous month, while for the TAQ sample we use daily data from day $t-20$ to day $t-1$.
- Turnover (TO). Following Kumar (2009), we calculate turnover as volume divided by shares outstanding. For the CRSP sample we use the past month's volume and shares outstanding, while for the TAQ sample we use the previous day's volume and shares outstanding.

Appendix B. Additional summary statistics

Table B.1

Descriptive statistics CRSP sample. Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and downside betas, coskewness and cokurtosis measures are all constructed from 15-minute intraday returns. The sample consists of all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2019.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	RV	IVOL	ILLIQ
Mean	0.99	0.60	0.76	0.21	0.16	1.01	0.96	-0.03	1.28	14.26	0.72	14.99	1.45	5.36	1.51	-11.21
Median	0.92	0.54	0.67	0.15	0.10	0.91	0.90	-0.03	1.35	14.18	0.64	11.04	0.98	3.39	1.32	-11.22
StDev	0.76	0.36	0.46	0.21	0.19	0.99	1.09	0.29	0.82	1.33	0.49	32.17	8.68	8.24	0.83	1.25

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	RV	IVOL	ILLIQ
β	1.00	0.72	0.79	-0.30	-0.29	0.82	0.75	0.03	0.64	0.01	-0.08	0.07	-0.01	0.41	0.26	0.02
$\beta^{\mathcal{P}}$		1.00	0.42	0.10	-0.09	0.34	0.89	-0.28	0.40	-0.11	-0.08	0.06	0.02	0.50	0.43	0.11
$\beta^{\mathcal{N}}$			1.00	-0.07	0.06	0.91	0.33	0.29	0.39	-0.09	-0.08	0.05	-0.04	0.57	0.48	0.10
$\beta^{\mathcal{M}^+}$				1.00	0.23	-0.46	-0.02	-0.13	-0.44	-0.26	-0.01	-0.01	-0.01	0.33	0.47	0.19
$\beta^{\mathcal{M}^-}$					1.00	-0.03	-0.50	0.13	-0.44	-0.22	0.00	-0.01	-0.03	0.35	0.46	0.17
β^+						1.00	0.30	0.31	0.51	0.03	-0.06	0.05	-0.03	0.37	0.24	0.01
β^-							1.00	-0.30	0.53	0.00	-0.07	0.05	0.02	0.27	0.17	0.02
CSK								1.00	0.03	0.01	-0.01	-0.02	-0.03	0.01	-0.01	-0.01
CKT									1.00	0.23	-0.06	0.04	0.01	-0.05	-0.23	-0.15
ME										1.00	-0.22	0.07	0.02	-0.25	-0.36	-0.80
BM											1.00	0.00	0.01	-0.05	-0.04	0.10
MOM												1.00	0.00	0.02	0.00	0.00
REV													1.00	-0.03	-0.03	-0.01
RV														1.00	0.90	0.17
IVOL															1.00	0.27
ILLIQ																1.00

Table B.2

Descriptive statistics TAQ sample. Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and downside betas, coskewness and cokurtosis measures are all constructed from 15-minute intraday returns. The sample consists of all S&P 500 constituent stocks from January 1993 to December 2019.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	RV	IVOL	ILLIQ
Mean	0.95	0.70	0.74	0.28	0.26	0.95	0.93	-0.01	1.38	15.59	0.48	16.46	1.39	8.68	1.57	-14.72
Median	0.86	0.59	0.63	0.17	0.15	0.85	0.84	-0.01	1.47	15.58	0.39	11.47	1.07	3.25	1.31	-14.63
StDev	1.06	0.47	0.50	0.37	0.35	1.34	1.42	0.39	1.25	1.36	0.81	41.01	9.61	60.86	0.99	1.66

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	RV	IVOL	ILLIQ
β	1.00	0.65	0.67	-0.33	-0.33	0.78	0.76	0.00	0.65	0.05	-0.02	0.03	-0.01	0.13	0.12	-0.06
$\beta^{\mathcal{P}}$		1.00	0.44	0.20	0.06	0.27	0.82	-0.31	0.31	-0.14	-0.03	0.00	-0.08	0.47	0.35	0.00
$\beta^{\mathcal{N}}$			1.00	0.07	0.19	0.82	0.28	0.29	0.31	-0.13	-0.03	0.00	0.03	0.47	0.34	0.02
$\beta^{\mathcal{M}^+}$				1.00	0.38	-0.49	-0.04	-0.16	-0.42	-0.29	0.00	-0.05	-0.08	0.54	0.37	0.12
$\beta^{\mathcal{M}^-}$					1.00	-0.05	-0.49	0.17	-0.42	-0.27	0.00	-0.04	0.03	0.52	0.35	0.13
β^+						1.00	0.27	0.33	0.49	0.05	-0.02	0.02	0.07	0.11	0.10	-0.05
β^-							1.00	-0.35	0.49	0.03	-0.02	0.02	-0.09	0.12	0.12	-0.07
CSK								1.00	-0.02	0.00	0.00	-0.01	0.01	0.00	0.00	0.00
CKT									1.00	0.22	-0.03	0.03	0.00	-0.13	-0.13	-0.12
ME										1.00	-0.10	0.09	0.03	-0.33	-0.36	-0.57
BM											1.00	-0.05	-0.01	-0.01	-0.04	0.02
MOM												1.00	0.03	-0.05	-0.03	0.02
REV													1.00	-0.03	0.04	0.03
RV														1.00	0.48	0.14
IVOL															1.00	0.09
ILLIQ																1.00

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