Supplemental Appendix to:
Realized SemiCovariances: Looking for Signs of Direction Inside the Covariance Matrix

This version: September 22, 2017

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This supplemental appendix contains additional, more detailed, simulation and empirical results for Bollerslev, Patton, and Quaedvlieg (2017).

- Section S1 expands on the simulation results in Section 2.3, by providing rejection rates for additional parameter choices, as well as simulations which incorporate a diurnal pattern in the spot-volatility. In addition to the more detailed results for the test of $\mathcal{P} = \mathcal{N}$, we also provide additional results pertaining to the test of $\mathcal{M}^+ = \mathcal{M}^-$.

- Section S2 elaborates on the test results in Section 3.2, by providing an expanded list of events associated with high rejection rates. It also further compares the statistical tests for equality between semicovariances to other existing high-frequency measures and tests.

- Section S3 expands on the results in Section 3.3, by providing additional summary statistics and test results related to the time-series dynamics of the various semicovariance components.

- Section S4 expands on the results in Section 3.4, by providing more detailed univariate portfolio forecasting results, as well as results pertaining to multivariate covariance matrix forecasting.

S1. Additional Simulation Results

This section contains a number of additional simulation results. Table S.1 expands on Table 1 in the main text, by including additional values of m and ρ. Table S.2 provides the same results for the test of $\mathcal{M}^+ = \mathcal{M}^-$. Table S.3 presents simulations under the null in the presence of time-varying spot volatility as captured by a diurnal pattern, defined as

$$\varsigma_u = C + Ae^{-au} + Be^{-b(1-u)}, \tag{S1.1}$$

where, following Andersen, Dobrev, and Schaumburg (2012), we set the periodicity parameters to $A = 0.75$, $B = 0.25$, $C = 0.88929198$, and $a = b = 10$, respectively.

The results in Tables S.1 and S.3 are in line with those in the main text. The additional sampling frequency, $m = 390$, exhibits more accurate size and more powerful tests. Time-varying spot-volatility has no real impact on the finite-sample properties of the test, as indeed, it does not violate any assumptions of the theory.

Table S.2 presents additional results for $\mathcal{M}^+ = \mathcal{M}^-$, where the estimator of the asymptotic variance is again based on (23), and denoted $\tilde{\pi}$. The results are qualitatively similar to those of the tests for $\mathcal{P} = \mathcal{N}$, with overall accurate size and good power. The main difference is evident for small sample sizes: whereas the properties of the test for $\mathcal{P} = \mathcal{N}$ improve when the correlation increases, the properties of the test for $\mathcal{M}^+ = \mathcal{M}^-$ slightly worsen, which is most evident for $m = 26$ at the 1% level. The intuition is straightforward; when the correlation is high more observations fall within the concordant quadrants, and $\mathcal{P}$ and $\mathcal{N}$ can be estimated more precisely, while the opposite holds for the mixed-sign semicovariances which are increasingly imprecise for high correlations.

Table S.1: Additional Simulation Results for Testing $\mathcal{P} = \mathcal{N}$

m	ρ	$\pi^{(m)}$			$\pi^{(m)}_{Tr}$		
		10%	5%	1%	10%	5%	1%
				$\mathcal{H}_0$			
390	0	0.100	0.047	0.010	0.118	0.061	0.012
390	0.5	0.104	0.054	0.010	0.138	0.076	0.022
390	0.7	0.100	0.052	0.010	0.137	0.080	0.023
78	0	0.108	0.053	0.009	0.130	0.070	0.018
78	0.5	0.099	0.048	0.009	0.153	0.092	0.028
78	0.7	0.104	0.052	0.009	0.159	0.097	0.028
26	0	0.116	0.052	0.006	0.144	0.083	0.027
26	0.5	0.116	0.056	0.010	0.180	0.113	0.046
26	0.7	0.114	0.058	0.010	0.181	0.115	0.047
				$\mathcal{H}_A$: Cojumps			
390	0	0.000	0.000	0.000	0.999	0.999	0.998
390	0.5	0.000	0.000	0.000	0.999	0.998	0.997
390	0.7	0.000	0.000	0.000	0.998	0.997	0.995
78	0	0.001	0.001	0.000	0.992	0.990	0.984
78	0.5	0.006	0.001	0.000	0.973	0.962	0.928
78	0.7	0.009	0.002	0.000	0.953	0.939	0.893
26	0	0.010	0.003	0.000	0.936	0.920	0.885
26	0.5	0.028	0.008	0.001	0.848	0.807	0.722
26	0.7	0.034	0.010	0.002	0.802	0.753	0.652
				$\mathcal{H}_A$: Asymmetric Correlations			
390	0	0.964	0.933	0.810	0.970	0.939	0.829
390	0.5	0.931	0.874	0.697	0.942	0.897	0.757
390	0.7	0.909	0.850	0.656	0.933	0.890	0.743
78	0	0.960	0.920	0.755	0.960	0.925	0.782
78	0.5	0.915	0.855	0.644	0.934	0.887	0.725
78	0.7	0.900	0.826	0.602	0.922	0.870	0.702
26	0	0.944	0.870	0.553	0.944	0.877	0.584
26	0.5	0.898	0.804	0.493	0.914	0.838	0.557
26	0.7	0.880	0.781	0.478	0.903	0.820	0.549

Note: The table reports the rejection frequencies for testing the "daily" $\mathcal{P} = \mathcal{N}$ based on 10,000 replications. The $\pi^{(m)}$ and $\pi^{(m)}_{Tr}$ versions of the tests rely on the standard and truncated variance for normalizing the difference. The data are generated from a continuous price process $d\mathbf{P}_u = \boldsymbol{\sigma}_u d\mathbf{W}_u$ aggregated to the one, five, and fifteen "minute" frequency, corresponding to $m = 390$, $m = 78$, and $m = 26$ observation per "day." The $\mathcal{H}_0$ hypothesis postulates a constant spot covariance matrix, $\boldsymbol{\sigma}_u\boldsymbol{\sigma}'_u \equiv [(1-\rho)\mathbf{I}_2 + \rho\mathbf{J}'_2]$ for all u. The first $\mathcal{H}_A$ alternative add cojumps of random size $N(8/\sqrt{78}, 2/\sqrt{78})$ at a single random time each "day." The second $\mathcal{H}_A$ alternative have dynamically varying spot correlations, determined by $[(1-(\rho+0.05))\mathbf{I}_2 + (\rho+0.05)\mathbf{J}_2]$ if $W_{u,1} < 0$, and $[(1-(\rho-0.05))\mathbf{I}_2 + (\rho-0.05)\mathbf{J}_2]$ if $W_{u,1} > 0$.

Table S.2: Additional Simulation Results for Testing $\mathcal{M}^+ = \mathcal{M}^-$

m	ρ	$\tilde{\pi}^{(m)}$			$\tilde{\pi}^{(m)}_{Tr}$		
		10%	5%	1%	10%	5%	1%
				$\mathcal{H}_0$			
390	0	0.099	0.049	0.009	0.113	0.059	0.014
390	0.5	0.099	0.048	0.010	0.100	0.049	0.009
390	0.7	0.102	0.051	0.010	0.100	0.051	0.010
78	0	0.108	0.049	0.008	0.125	0.070	0.017
78	0.5	0.104	0.047	0.005	0.108	0.051	0.008
78	0.7	0.105	0.045	0.005	0.109	0.047	0.005
26	0	0.116	0.055	0.007	0.142	0.076	0.026
26	0.5	0.116	0.042	0.003	0.118	0.047	0.005
26	0.7	0.096	0.030	0.001	0.099	0.031	0.001
				$\mathcal{H}_A$: Cojumps			
390	0	0.101	0.050	0.009	0.111	0.057	0.012
390	0.5	0.100	0.052	0.009	0.103	0.052	0.009
390	0.7	0.104	0.050	0.009	0.106	0.053	0.010
78	0	0.107	0.051	0.009	0.116	0.059	0.012
78	0.5	0.106	0.050	0.006	0.108	0.050	0.007
78	0.7	0.102	0.045	0.005	0.110	0.050	0.007
26	0	0.123	0.054	0.008	0.128	0.064	0.014
26	0.5	0.107	0.040	0.003	0.110	0.040	0.004
26	0.7	0.089	0.028	0.002	0.102	0.033	0.002
				$\mathcal{H}_A$: Asymmetric Correlations			
390	0	0.965	0.935	0.806	0.971	0.941	0.837
390	0.5	0.995	0.987	0.939	0.995	0.988	0.945
390	0.7	0.999	0.997	0.985	0.999	0.997	0.984
78	0	0.959	0.921	0.751	0.964	0.928	0.786
78	0.5	0.992	0.976	0.884	0.991	0.978	0.894
78	0.7	0.998	0.994	0.944	0.999	0.995	0.945
26	0	0.943	0.866	0.549	0.950	0.882	0.586
26	0.5	0.979	0.922	0.602	0.978	0.927	0.621
26	0.7	0.986	0.939	0.655	0.989	0.945	0.656

Note: The table reports the rejection frequencies for testing the "daily" $\mathcal{M}^+ = \mathcal{M}^-$ based on 10,000 replications. See Table S.1 for details.

Table S.3: Additional Simulation Results Including Diurnal Patterns

		10%	5%	1%	10%	5%	1%
m	ρ	$\mathcal{P} = \mathcal{N}$			$\mathcal{M}^+ = \mathcal{M}^-$		
390	0	0.102	0.053	0.011	0.100	0.056	0.011
390	0.5	0.099	0.050	0.008	0.104	0.052	0.012
390	0.7	0.096	0.046	0.010	0.102	0.048	0.013
78	0	0.104	0.050	0.008	0.109	0.053	0.008
78	0.5	0.105	0.053	0.008	0.109	0.049	0.006
78	0.7	0.106	0.053	0.011	0.106	0.047	0.005
26	0	0.114	0.051	0.006	0.112	0.050	0.006
26	0.5	0.119	0.055	0.009	0.104	0.040	0.002
26	0.7	0.119	0.057	0.011	0.092	0.028	0.002

Note: See Tables S.1 and S.2. The table reports the rejection frequencies for testing the "daily" $\mathcal{P} = \mathcal{N}$ and $\mathcal{M}^+ = \mathcal{M}^-$, using $\pi^{(m)}$. The results are based on 10,000 replications under the $\mathcal{H}_0$ hypothesis that $\boldsymbol{\sigma}_u \boldsymbol{\sigma}'_u \equiv \varsigma_u[(1-\rho)\mathbf{I}_2 + \rho \mathbf{J}_2]$, with the ς_u diurnal pattern as in Andersen, Dobrev, and Schaumburg (2012).

S2. Economic Content Semicovariance Inequality

S2.1. Additional Event Study Results

Table S.4 provides more detailed information on the events coinciding with high rejection frequencies of semicovariance equality, which are presented in Table 2 in the main text. Here we provide the top 5 rejection days in each direction for the three version of the tests. The top panel uses "raw" returns only, and rejection may be due to either jumps or asymmetric correlations. The middle panel improves this test's properties by using the jump-robust estimator of the asymptotic variance, based on truncated returns. The bottom panel uses jump-robust estimators of the asymptotic variance as well as the semicovariances, such that rejection is due to asymmetric correlations.

First, as discussed in the main text, the rejection dates based on $\mathbf{\Pi}_{Tr}^{(m)}$ are typically associated with clear marcoeconomic news announcements, whereas the results based on $\mathbf{\Pi}^{(m)}$ tend to coincide with 'softer' news. This remains mostly true for the expanded set of events associated with a large fraction of rejections presented in this table. However, the double truncated test introduced here offers even stronger evidence of that tendency.

In particular, the double truncated test in the bottom panel helps distinct which events are likely associated with price jumps, and which events are likely associated with asymmetry in correlations. Indeed, events which lead to rejections in the double truncated tests are unlikely due surely due to up- and downside correlation asymmetry, such that the remaining events in the two top panels are most likely associated with price jumps. The double truncated version tends to reject on days associated with these 'softer' events, while the remaining event days are almost all associated with Federal Reserve announcements.

Table S.4: Additional Top Rejection Days

Date	%	Event
		Results based on $\mathcal{P}^{(m)}$, $\mathcal{N}^{(m)}$ and $\boldsymbol{\pi}^{(m)}$
$\mathcal{P} > \mathcal{N}$		
26-11-2008	62	Bank of America acquisition of Merill Lynch approval
07-07-2010	62	Greek parliament passes pension reform
13-06-2013	55	US retail sales higher than expected
04-10-2011	51	Dexia Bank 'teetered on the brink of collapse', talk of haircut Greek debt
20-07-2010	49	Federal Reserve announces reduction of Asset Buying Program
$\mathcal{N} > \mathcal{P}$		
25-02-2013	69	Italian elections
21-06-2012	68	Rumors of Moody's downgrade for global banks
01-06-2011	65	Moody's cut Greece's bond rating by three notches
03-02-2014	64	Janet Yellen sworn in as new Fed chair
21-09-2011	60	IMF Global Financial Stability Reports European countries must be ready to recapitalize their banks. Moody's cut Bank of America's rating by two notches.
		Results based on $\mathcal{P}^{(m)}$, $\mathcal{N}^{(m)}$ and $\boldsymbol{\pi}^{(m)}_{Tr}$
$\mathcal{P} > \mathcal{N}$		
18-12-2013	94	Federal Reserve announces reduction of Asset Buying Program
18-09-2013	85	Federal Reserve announces sustaining Asset Buying Program
18-09-2007	84	Federal Reserve lowers rate in response to housing 'correction'
13-09-2012	80	Federal Reserve continues buying Mortgage Backed Securities
04-10-2011	78	Dexia Bank 'teetered on the brink of collapse', talk of haircut Greek debt
$\mathcal{N} > \mathcal{P}$		
11-12-2007	93	Federal Reserve drops rate by 25 basis points.
19-06-2013	89	Federal Reserve continues buying Mortgage Backed Securities
27-10-1997	85	Mini-crash and NYSE circuit breaker trading halt
25-02-2013	81	Italian elections
21-06-2012	81	Rumors of Moody's downgrade for global banks
		Results based on $\mathcal{P}^{(m)}_{Tr}$, $\mathcal{N}^{(m)}_{Tr}$ and $\boldsymbol{\pi}^{(m)}_{Tr}$
$\mathcal{P} > \mathcal{N}$		
07-07-2010	59	Greek parliament passes pension reform
26-11-2008	56	Bank of America acquisition of Merill Lynch approval
23-03-2009	54	Treasury announces plan to buy up billions in bad bank assets
13-06-2013	52	US retail sales higher than expected
23-08-2011	47	Gaddafi overthrown. 'Things are so bad people expect Bernanke to step in later this week'
$\mathcal{N} > \mathcal{P}$		
25-02-2013	63	Italian elections
03-02-2014	52	Janet Yellen sworn in as new Fed chair
01-06-2011	58	Moody's cut Greece's bond rating by three notches
04-08-2011	54	Credit downgrade of US debt from AAA to AA+
21-06-2012	51	Rumors of Moody's downgrade for global banks

Note: The table reports the five days of most one-sided rejections for the test of $\mathcal{P}_{ij} = \mathcal{N}_{ij}$ in either direction based on three different combinations of estimators. $\mathcal{P}^{(m)}$,$\mathcal{N}^{(m)}$ and $\boldsymbol{\pi}^{(m)}$ are the standard estimators while, $\mathcal{P}^{(m)}_{Tr}$,$\mathcal{N}^{(m)}_{Tr}$ and $\boldsymbol{\pi}^{(m)}_{Tr}$ are based on truncated returns, leading to different degrees of jump-robustness. The first column provides the date, the second column provides the fraction of pairs where equality is rejected in that direction. The final column provides a description of the event.

S2.2. Tests for $\mathcal{P} = \mathcal{N}$ and Other High-Frequency Measures

In order to highlight the uniqueness of the events that semicovariances uncover, we analyze rejection frequencies conditional on some other high-frequency measures. In particular we consider the events where one, or both of the stocks jump, as well as days where the SPY index jumps. We conduct daily jump tests using the MaxLog version of the Barndorff-Nielsen and Shephard (2006) test using MedRV and MedRQ to estimate continuous variation and integrated quarticity, and set the level of the test at 1%. Next, we check whether rejections are more likely in high volatility periods, governed by the distribution of RV. Finally, we take the related measure of Signed Jump Variation (SJV) introduced by Patton and Sheppard (2015), which is the difference between the positive and negative semivariance. We consider various quantiles of its distribution, as well as tests of the SJV being equal to zero, using the result in Equation (17).

We perform a simple non-parametric analysis in Table S.5 where we report the rejection frequencies of $\mathcal{P} = \mathcal{N}$ conditional on the various measures. Note that due to the large amount of observations all differences are statistically significant, but the differences are not necessarily large in terms of increasing the probability of rejection. As stated before, unconditionally we reject equality in about 9% of the pair-days. Rejections are actually less frequent on days where either of the stocks or the market jumps. We do see a slightly larger amount of rejections on high volatility pair-days, but the result is not particularly striking as the rejection frequencies increases to a mere 11% for the top percentile of pair-days, clearly not explaining rejection of equality. Finally, consider the rejection frequencies conditional on Signed Jump Variation, which is a significant sorting variable. In particular, on days where both SJVs are either significantly positive or negative, we also reject equality of the semicovariances in around 80% of the cases. Obviously, SJV is closely related to $\mathcal{P} = \mathcal{N}$ as the semicovariances are a function of the semivariances. On pair-days where both positive semivariances are significantly higher than the negative semivariances, we would expect the semicovariances to also differ, unless the semicorrelation drops proportionally.

Table S.5: Conditional Rejection Frequencies

	$\mathcal{P} > \mathcal{N}$	$\mathcal{N} > \mathcal{P}$	Either
Unconditional	0.045	0.045	0.090
Jump tests			
One Jump	0.039	0.038	0.077
Both Jump	0.030	0.028	0.058
Market Jump	0.051	0.031	0.082
RV distribution			
RV $<$ 1% Quantile	0.051	0.030	0.080
RV $<$ 10% Quantile	0.052	0.036	0.088
RV $<$ 25% Quantile	0.051	0.040	0.091
RV $<$ 50% Quantile	0.048	0.043	0.091
RV $>$ 50% Quantile	0.042	0.047	0.089
RV $>$ 75% Quantile	0.042	0.048	0.090
RV $>$ 90% Quantile	0.044	0.049	0.094
RV $>$ 99% Quantile	0.050	0.059	0.108
SJV tests			
Both SJV < 0	0.000	0.827	0.827
One SJV < 0	0.000	0.284	0.284
Both SJV $= 0$	0.025	0.027	0.053
Opposing SJV	0.017	0.021	0.038
One SJV > 0	0.261	0.000	0.262
Both SJV > 0	0.787	0.000	0.787
SJV distribution			
SJV $<$ 1% Quantile	0.003	0.202	0.204
SJV $<$ 10% Quantile	0.005	0.133	0.138
SJV $<$ 25% Quantile	0.006	0.116	0.121
SJV $<$ 50% Quantile	0.008	0.082	0.090
SJV $>$ 50% Quantile	0.082	0.008	0.091
SJV $>$ 75% Quantile	0.108	0.007	0.115
SJV $>$ 90% Quantile	0.122	0.006	0.128
SJV $>$ 99% Quantile	0.184	0.003	0.187

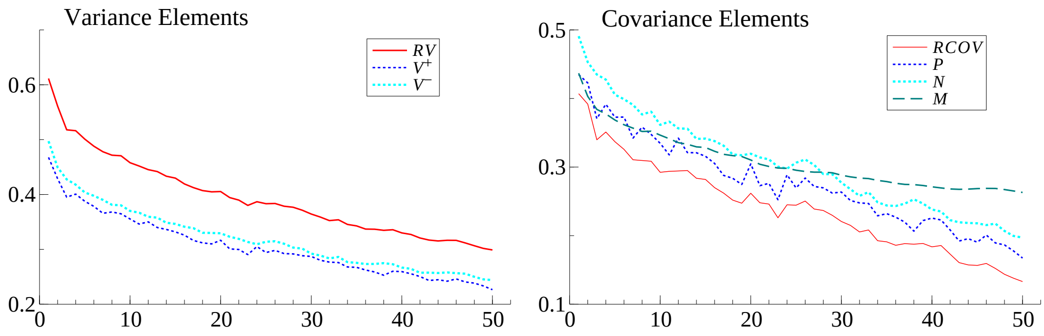
Note: This table provides rejection frequencies of the null that $\mathcal{P} = \mathcal{N}$, conditional on a number of events. Jump tests are based on the MaxLog version of the jump test proposed in Barndorff-Nielsen and Shephard (2006). The quantiles of RV are quantiles of the average realized variance of the pair-day. The SJV tests panel is based on rejection of the semivariances being equal at the 5% level. The quantiles of SJV are quantiles of the average SJV of the pair-day.

S3. More results on semicovariance dynamics

S3.1. Unadjusted Autocorrelation Functions

Figure S.1 provides the empirical autocorrelation function of the semicovariances. This figure is the analogue to Figure 4 in the main text, but ignoring the fact that the semicovariances are measured with error. Interestingly, the realized variance appears more persistent than the semivariances. Obviously, the variance uses twice the number of observations the semivariances use, and the results in the paper suggest that this difference in persistence is due to differences in measurement error, and therefore spurious. The results for the semicovariances on the other hand, are mostly in line with those in the paper.

Figure S.1: Unadjusted Autocorrelations



Note: The graph plots the empirical autocorrelation functions for each of the individual elements of the semicovariance matrix. The autocorrelations are averaged over 1,000 random pairs of stocks.

S3.2. Formal tests of semicovariance block structure.

Here, we present results for formal tests of the possible 'block structures' in the vector HAR model for $[\mathcal{P}, \mathcal{N}, \mathcal{M}]$ uncovered in Table 3 in the main text. First, we test in-sample significance of the parameters using standard F-tests (at 99% level) with Newey and West (1987) standard errors. For each hypothesis, we test whether the parameters on the daily, weekly, and monthly lags of the relevant semicovariances are jointly significant. Then, we consider a small out-of-sample analysis, where we compare the out-of-sample performance of the restricted and unrestricted models using Diebold and Mariano (1995) tests of forecasts based on rolling windows containing 1,000 observations. Analogous to the in-sample tests, we perform one-sided tests, where the alternative is that the unrestricted model's out-of-sample mean square error is lower than that of the restricted model, again at the 99% level.

We consider a total of thirteen restrictions, described in Table S.6. The last two columns present the in- and out-of-sample rejection frequencies. The in-sample tests typically reject the null hypotheses, indicating that the apparent block structures in the parameter estimates are not formally supported by the data from a statistical perspective. However, the tests are based on large samples, and as such the power is likely very high, meaning that small deviations from the block structure may be sufficient to reject the null. While most of the rejection frequencies are above 0.9, the hypotheses related to the significance of $\mathcal{P}$ are rejected less frequently than all others, suggesting that $\mathcal{P}$ plays a less important role for describing the dynamic dependencies in the covariances. In contrast to the in-sample tests, the out-of-sample tests almost never reject the parameter restrictions leading to the block structure. This is consistent with the idea that more parsimonious models tend to perform well out-of-sample.

Table S.6: Testing model restrictions

	LHS	Restriction on RHS	In-Sample Rej.	Out-of-Sample Rej.
H_1:	$\mathcal{P}_{ij,t}$	$\mathcal{P}_{ij} = 0$	0.786	0.002
H_2:	$\mathcal{P}_{ij,t}$	$\mathcal{M}_{ij} = 0$	0.852	0.004
H_3:	$\mathcal{P}_{ij,t}$	$\mathcal{P}_{ij} = \mathcal{M}_{ij} =0$	0.976	0.004
H_4:	$\mathcal{N}_{ij,t}$	$\mathcal{M}_{ij} = 0$	0.764	0.002
H_5:	$\mathcal{N}_{ij,t}$	$\mathcal{P}_{ij} = 0$	0.832	0.002
H_6:	$\mathcal{N}_{ij,t}$	$\mathcal{P}_{ij} = \mathcal{M}_{ij} = 0$	0.960	0.002
H_7:	$\mathcal{M}_{ij,t}$	$\mathcal{P}_{ij} = 0$	0.596	0.000
H_8:	$\mathcal{M}_{ij,t}$	$\mathcal{N}_{ij} = 0$	0.924	0.192
H_9:	$\mathcal{M}_{ij,t}$	$\mathcal{P}_{ij} = \mathcal{N}_{ij} = 0$	0.994	0.198
H_{10}:	$\mathcal{P}_{ij,t}$, $\mathcal{N}_{ij,t}$, $\mathcal{M}_{ij,t}$	The block structure holds	1.000	0.054
H_{11}:	$RCOV_{ij,t}$	$\mathcal{P}_{ij} = 0$	0.724	0.002
H_{12}:	$RCOV_{ij,t}$	$\mathcal{M}_{ij} = 0$	0.904	0.000
H_{13}:	$RCOV_{ij,t}$	$\mathcal{P}_{ij} = \mathcal{M}_{ij} = 0$	0.954	0.002

Note: This table tests hypothesis on restrictions in the model (25) of the main text. Each hypothesis states in which equation the restriction is tested, along with the restriction. Each restriction implies a zero coefficient on the daily, weekly and monthly lag. The in-sample 1%-level rejection frequencies are based on an F-test. The out-of-sample rejections are based on a Diebold-Mariano test at 1% where rejection means the unrestricted models' forecasts are better.

S4. Semicovariances and Volatility Forecasting

S4.1. More Detailed Univariate Forecasting Results

In this section we provide more details on the univariate portfolio forecasting results of Section 2.1 in the main paper. We expand the set of models under consideration, provide detailed results on more portfolio sizes and use the Model Confidence Set of Hansen et al. (2011) to assess statistical significance of the improvements in forecasting precision.

For notational simplicity, let $RV_d \equiv RV_{t-1}$, $RV_w \equiv RV_{t-2|t-5}$ and $RV_m \equiv RV_{t-6|t-22}$, respectively, with similar definitions applied for the realized semicovariance measures.

We consider the following set of models for forecasting RV^p, succinctly expressed as functions of the specific realized measures used in the construction of the forecasts:

1a) $RV^p_{t+1|t} = f(RV^p_d, RV^p_w, RV^p_m)$
1b) $RV^p_{t+1|t} = f(\mathcal{V}^{+,p}_d, \mathcal{V}^{-,p}_d, RV^p_w, RV^p_m)$
2) $RV^p_{t+1|t} = f(\mathcal{P}^p_d, \mathcal{N}^p_d, \mathcal{M}^p_d, RV^p_w, RV^p_m)$
3) $RV^p_{t+1|t} = f(\mathcal{P}^p_d, \mathcal{P}^p_w, \mathcal{P}^p_m, \mathcal{N}^p_d, \mathcal{N}^p_w, \mathcal{N}^p_m, \mathcal{M}^p_d, \mathcal{M}^p_w, \mathcal{M}^p_m)$
4) $RV^p_{t+1|t} = f(\mathcal{N}^p_d, \mathcal{N}^p_w, \mathcal{N}^p_m, \mathcal{M}^p_d, \mathcal{M}^p_w, \mathcal{M}^p_m)$
5) $RV^p_{t+1|t} = f(\mathcal{N}^p_d, \mathcal{N}^p_w, \mathcal{N}^p_m, \mathcal{M}^p_m)$
6) $RV^p_{t+1|t} = f(\mathcal{N}^p_d, \mathcal{N}^p_w, \mathcal{N}^p_m)$

In addition to the models for directly forecasting RV^p, we also consider a model in which we construct separate forecasts for $\mathcal{P}^p$, $\mathcal{N}^p$, and $\mathcal{M}^p$, and then add up the forecasts to arrive at a forecast for RV.

7) $RV^p_{t+1|t} = f_{\mathcal{P}}(\mathcal{P}^p_d, \mathcal{P}^p_w, \mathcal{P}^p_m) + f_{\mathcal{N}}(\mathcal{N}^p_d, \mathcal{N}^p_w, \mathcal{N}^p_m) + f_{\mathcal{M}}(\mathcal{M}^p_d, \mathcal{M}^p_w, \mathcal{M}^p_m)$

where $f_{\mathcal{P}}$, $f_{\mathcal{N}}$, and $f_{\mathcal{M}}$ are models analogous to those in models (1a) to (6) above, but with the dependent variable set to $\mathcal{P}^p$, $\mathcal{N}^p$, and $\mathcal{M}^p$ respectively.

Models 1a, 1b, 3, and 5 are referred to as HAR, SHAR, SCHAR and SCHAR-r respectively in the main paper.

We follow the forecasting set-up discussed in the main paper. The detailed results for portfolios of size $N = 1, 2, 5, 10, 50, 100$ are reported in Table S.7. The first column pertaining to each of the different loss functions reports the average loss across time and across sets of stocks. For each of the 500 samples, we also compute the ratio of each model's average loss to that of the standard HAR; the baseline Model 1. The second column pertaining to each of the loss functions presents the average of these ratios over all of the random samples. Finally, the third column presents the fraction of samples for which each of the models were included in the 80% Model Confidence Set (MCS) of Hansen, Lunde, and Nason (2011). For each of the columns and each matrix size N we report the best model in boldface. For the two loss columns this is the lowest number, for the MCS this is the highest number. For the univariate case, $\mathcal{M}$ is always zero and $\mathcal{P}$ and $\mathcal{N}$ are simply the semivariances, so we only report the loss based on Models 1a, 1b, 6 and 7.

The table clearly shows that for portfolios comprised of a large number of stocks, utilizing the information in the realized semicovariances can result in significantly better performance than the benchmark models that only use realized variances (or realized semivariances) estimated from the univariate portfolio returns. For $N \geq 50$, the best-performing model under both MSE and QLIKE loss is Model 5 (labelled "SCHAR-r" in the main paper) which uses all three lags (daily, weekly and monthly) of $\mathcal{N}^p$ and $\mathcal{M}^p$, but excludes lags of $\mathcal{P}_t^p$ altogether. For N between 2 and 10 the best-performing model under MSE loss is a more parsimonious version of that model, which includes all three lags of $\mathcal{N}^p$, but only the monthly lag of $\mathcal{M}^p$, while under QLIKE the preferred model is Model 2, which only decomposes the daily lag of $RCOV$. Consistent with the results in Patton and Sheppard (2015), the SHAR model (Model 1b) performs the best among the models that do not use any cross-sectional information ($N = 1$).

Second, the Model Confidence Set approach generally exhibits quite low power, failing to distinguish models in a large fraction of the cases. However, the fact that for almost all randomly selected collections of assets model 5 ends up in the MCS suggests this model provides the best forecasts out of those considered. It is natural that the parsimonious specifications 5 and 6 should do so well, but the results indicate that the monthly lag of the mixed-sign semicovariance $\mathcal{M}$ truly helps in predicting future volatility, consistent with its strong persistence documented in Figure 4.

Table S.7: Portfolio RV^p forecasts

Model	MSE Average	MSE Ratio	MSE MCS	QLIKE Average	QLIKE Ratio	QLIKE MCS
			N = 1			
1a	35.112	1.000	0.886	0.239	1.000	0.728
1b	**34.981**	**0.997**	**0.914**	**0.238**	**0.998**	**0.864**
6	35.209	1.001	0.882	0.243	1.022	0.250
7	35.326	1.020	0.740	0.247	1.036	0.034
			N = 2			
1a	8.094	1.000	0.760	0.184	1.000	0.652
1b	7.951	0.988	0.868	0.184	0.998	0.656
2	7.946	0.986	0.914	**0.183**	**0.995**	**0.880**
3	8.009	0.998	0.854	0.446	1.165	0.372
4	7.899	0.982	0.892	0.191	1.056	0.376
5	7.833	0.972	0.910	0.188	1.043	0.442
6	**7.800**	**0.966**	**0.962**	0.185	1.005	0.650
7	8.064	1.012	0.714	0.187	1.020	0.138
			N = 5			
1a	3.014	1.000	0.466	0.151	1.000	0.544
1b	2.815	0.966	0.698	0.150	0.994	0.582
2	2.804	0.958	0.828	**0.150**	**0.994**	**0.736**
3	2.810	0.969	0.770	0.206	1.322	0.360
4	2.721	0.934	0.806	0.153	1.014	0.516
5	2.679	0.922	0.848	0.156	1.034	0.654
6	**2.662**	**0.916**	**0.962**	0.151	1.003	0.704
7	2.935	1.010	0.408	0.154	1.021	0.194
			N = 10			
1a	1.849	1.000	0.306	0.141	1.000	0.442
1b	1.671	0.966	0.554	0.139	0.986	0.612
2	1.662	0.949	0.700	0.142	0.994	0.686
3	1.643	0.955	0.690	0.210	1.318	0.402
4	1.602	0.922	0.760	0.141	0.996	0.604
5	1.567	**0.908**	0.862	**0.139**	**0.979**	**0.816**
6	**1.560**	0.911	**0.892**	0.141	1.021	0.714
7	1.778	1.025	0.228	0.145	1.033	0.172
			N = 50			
1a	0.335	1.000	0.254	0.130	1.000	0.462
1b	0.284	0.951	0.428	0.125	0.963	0.788
2	0.288	0.954	0.566	0.133	1.021	0.638
3	0.284	0.944	0.674	0.140	1.087	0.682
4	0.284	0.933	0.600	0.135	1.135	0.750
5	0.273	**0.878**	**0.986**	**0.121**	**0.931**	**0.990**
6	**0.272**	0.885	0.896	0.126	0.963	0.822
7	0.312	1.050	0.134	0.136	1.049	0.240
			N = 100			
1a	0.048	1.000	0.282	0.119	1.000	0.492
1b	0.045	0.935	0.550	0.115	0.957	0.806
2	0.047	0.964	0.538	0.125	1.041	0.552
3	0.045	0.976	0.538	0.236	1.495	0.766
4	0.045	0.960	0.458	0.155	1.677	0.798
5	**0.041**	**0.862**	**0.988**	**0.111**	**0.925**	**0.994**
6	0.042	0.866	0.934	0.112	0.933	0.862
7	0.050	1.056	0.130	0.126	1.055	0.286

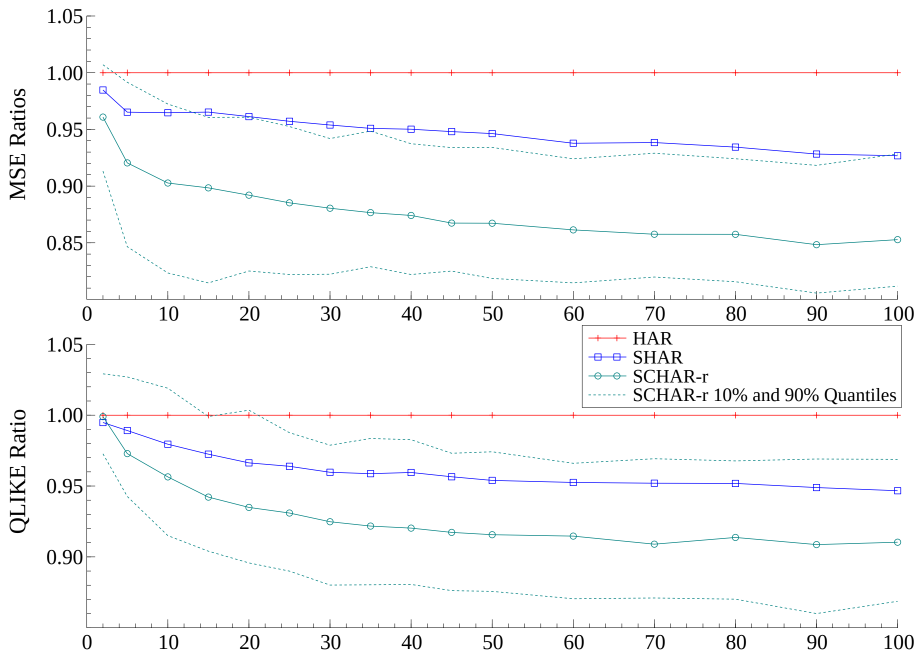
Note: This table provides aggregate forecasting results for portfolio variance, resulting from portfolios sized $N = 1, ..., 100$. Each number is based on 500 randomly selected sets out of the 749 S&P500 stocks. The Average column provides the average loss over time and sets of series. The Ratio column provides the average over all sets of stocks of the ratio of time-averaged loss of model k relative to model 1. Finally, the MCS column provides the fraction of sets each respective model was part of the 80% Model Confidence Set. Boldface denotes the best model per dimension N and column.

S4.1.1. Univariate Forecasting: Including the overnight return

All the results in the paper are based on open-to-close return variation. In this section we present the results of portfolio RV^p prediction when the realized measures are based on close-to-close returns. Figure S.2 presents the analogue to Figure 5 in the main paper, while Table S.8 is the analogue to S.7 of this Appendix.

The results for both the figure and the table are qualitatively similar to those excluding the overnight return. As Table S.8 demonstrates, the absolute loss is typically higher. This is in line with earlier work that demonstrates that the overnight variation is difficult to predict (see e.g. Andersen, Bollerslev, and Huang, 2011, who treat the overnight returns as jumps, requiring a separate specification). The ranking, as well as the relative improvements, of the various models are almost identical to the results which exclude the overnight.

Figure S.2: Median Loss Ratios: RV^p prediction, overnight included



Note: The graph plots the median loss ratios as a function of the number of stocks in the portfolio, N. The ratio is calculated as the average loss of the models divided by the average loss of of the standard HAR, based on 500 random samples of N stocks.

Table S.8: Portfolio RV^p forecasts, overnight included

Model	MSE Average	MSE Ratio	MSE MCS	QLIKE Average	QLIKE Ratio	QLIKE MCS
			N = 1			
1a	36.422	1.000	**0.912**	0.239	1.000	0.750
1b	**36.265**	**0.996**	0.906	**0.238**	**0.997**	**0.932**
6	36.449	1.004	0.894	0.243	1.024	0.320
7	36.686	1.032	0.722	0.246	1.040	0.024
			N = 2			
1a	8.896	1.000	0.804	0.183	1.000	0.612
1b	8.713	0.980	0.862	0.183	0.996	0.658
2	8.716	0.979	0.904	**0.182**	**0.993**	**0.886**
3	8.748	0.982	0.912	0.199	1.081	0.444
4	8.626	0.968	0.908	0.187	1.018	0.514
5	8.540	0.960	0.924	0.184	1.003	0.568
6	**8.499**	**0.956**	**0.984**	0.182	0.995	0.800
7	8.865	0.998	0.768	0.186	1.016	0.182
			N = 5			
1a	3.258	1.000	0.462	0.146	1.000	0.448
1b	3.027	0.955	0.704	0.144	0.989	0.586
2	3.012	0.948	0.840	0.144	0.984	0.760
3	3.008	0.951	0.790	0.186	1.232	0.444
4	2.930	0.932	0.852	0.146	0.995	0.624
5	2.883	0.921	0.884	0.144	0.985	0.778
6	**2.869**	**0.919**	**0.968**	**0.143**	**0.980**	**0.808**
7	3.177	0.996	0.398	0.148	1.017	0.262
			N = 10			
1a	1.847	1.000	0.324	0.137	1.000	0.428
1b	1.673	0.948	0.608	0.136	0.990	0.590
2	1.670	0.936	0.750	0.134	0.981	0.644
3	1.643	0.931	0.730	0.172	1.216	0.422
4	1.608	0.915	0.818	0.135	0.981	0.644
5	**1.575**	**0.900**	**0.906**	**0.132**	**0.963**	**0.876**
6	1.569	0.905	0.884	0.135	0.983	0.702
7	1.786	1.008	0.252	0.140	1.028	0.248
			N = 50			
1a	0.441	1.000	0.230	0.126	1.000	0.374
1b	0.365	0.940	0.446	0.121	0.954	0.808
2	0.369	0.952	0.582	0.126	0.999	0.670
3	0.352	0.939	0.698	0.151	1.195	0.752
4	0.358	0.929	0.642	0.130	1.047	0.772
5	0.353	**0.871**	**0.984**	**0.117**	**0.922**	**0.984**
6	**0.351**	0.879	0.922	0.120	0.944	0.874
7	0.412	1.047	0.152	0.133	1.052	0.208
			N = 100			
1a	0.061	1.000	0.246	0.115	1.000	0.430
1b	0.057	0.927	0.622	0.109	0.947	0.840
2	0.058	0.963	0.612	0.117	1.020	0.652
3	0.056	0.984	0.542	0.155	1.396	0.794
4	0.056	0.963	0.492	0.148	1.331	0.838
5	**0.052**	**0.860**	**0.986**	**0.105**	**0.916**	**0.986**
6	0.052	0.865	0.966	0.106	0.921	0.900
7	0.063	1.061	0.140	0.122	1.060	0.258

Note: This table provides aggregate forecasting results for portfolio variance (including overnight variation), resulting from portfolios sized $N = 1, ..., 100$. Each number is based on 500 randomly selected sets out of the 749 S&P500 stocks. The Average column provides the average loss over time and sets of series. The Ratio column provides the average over all sets of stocks of the ratio of time-averaged loss of model k relative to model 1. Finally, the MCS column provides the fraction of sets each respective model was part of the 80% Model Confidence Set. Boldface denotes the best model per dimension N and column.

S4.2. Multivariate Forecasting

In this section our aim is to forecast the full $(N \times N)$ realized covariance matrix, rather than the portfolio variance considered in the main paper.

We consider a set of seven different models. All of the models rely on standard HAR-type specifications, along the lines of the vech-HAR model of Chiriac and Voev (2011), in which we estimate scalar autoregressive coefficients, but individual constant terms, based on standard OLS procedures. That is, each individual element has identical dynamics, but different unconditional means.

1) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{RCOV}_d, \mathbf{RCOV}_w, \mathbf{RCOV}_m)$
2) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{P}_d, \mathbf{N}_d, \mathbf{M}_d, \mathbf{RCOV}_w, \mathbf{RCOV}_m)$
3) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{P}_d, \mathbf{P}_w, \mathbf{P}_m, \mathbf{N}_d, \mathbf{N}_w, \mathbf{N}_m, \mathbf{M}_d, \mathbf{M}_w, \mathbf{M}_m)$
4) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{N}_d, \mathbf{N}_w, \mathbf{N}_m, \mathbf{M}_d, \mathbf{M}_w, \mathbf{M}_m)$
5) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{N}_d, \mathbf{N}_w, \mathbf{N}_m, \mathbf{M}_m)$
6) $\mathbf{RCOV}_{t+1|t} = f(\mathbf{N}_d, \mathbf{N}_w, \mathbf{N}_m)$

In parallel to the univariate case, as a final model we also consider the sum of the forecasts from three separate vech-HAR models for $\mathbf{P}$, $\mathbf{N}$, and $\mathbf{M}$.

7) $\mathbf{RCOV}_{t+1|t} = f_{\mathbf{P}}(\mathbf{P}_d, \mathbf{P}_w, \mathbf{P}_m) + f_{\mathbf{N}}(\mathbf{N}_d, \mathbf{N}_w, \mathbf{N}_m) + f_{\mathbf{M}}(\mathbf{M}_d, \mathbf{M}_w, \mathbf{M}_m)$

Similar to the univariate case, for each N, ranging from 2 to 100, we take 500 random sets of stocks, and compute the semicovariances on their overlapping sample. We evaluate the performance of the forecasts using the multivariate analogues to MSE and QLIKE:

$$Frobenius(\mathbf{H}_t, \mathbf{\Sigma}_t) = \mathrm{Tr}((\mathbf{\Sigma}_t - \mathbf{H}_t)'(\mathbf{\Sigma}_t - \mathbf{H}_t))/N^2 \quad \text{(S4.1)}$$

$$QLIKE(\mathbf{H}_t, \mathbf{\Sigma}_t, \tilde{\mathbf{\Sigma}}_t) = \mathrm{Tr}(\mathbf{H}_t^{-1}\mathbf{\Sigma}_t) - \log|\mathbf{H}_t^{-1}\tilde{\mathbf{\Sigma}}_t| - N \quad \text{(S4.2)}$$

where $\mathbf{H}_t$ refers to the forecast, and $\mathbf{\Sigma}_t$ denotes the ex-post estimate of the covariance matrix. In the results reported on below, we use $\mathbf{RCOV}_t$ as our proxy for $\mathbf{\Sigma}_t$.

The $QLIKE$ loss function in equation (S4.2) is a modification of the original QLIKE loss function, also known as James and Stein (1961) loss, which is obtained if $\tilde{\mathbf{\Sigma}}_t = \mathbf{\Sigma}_t$. The original loss function is not well defined if $\mathbf{\Sigma}_t$ is only positive semidefinite. We define $\tilde{\mathbf{\Sigma}}_t$ as:

$$\tilde{\mathbf{\Sigma}}_t \equiv (1-\lambda_t)\,\mathbf{\Sigma}_t^* + \lambda_t Diag\{\mathbf{\Sigma}_t^*\} \tag{S4.3}$$

$$\text{where} \quad \lambda_t = \arg\min(\lambda \in \{0, 0.1, ..., 1\} : |\tilde{\mathbf{\Sigma}}_t| > 0) \tag{S4.4}$$

$$\text{and} \quad \mathbf{\Sigma}_t^* \equiv \mathbf{I_N} \odot \max\{\mathbf{\Sigma}_t, 0.001 \cdot \mathbf{J_N}\} + (\mathbf{J_N} - \mathbf{I_N}) \odot \mathbf{\Sigma}_t \tag{S4.5}$$

In words, equations (S4.3)-(S4.4) shrink the off-diagonal elements of the matrix towards zero, and equation (S4.5) ensures that the diagonals are at least 0.001. This modification affects only the normalization of this loss function, not the ranking of competing forecasts.

The results for matrices of size $N = 1, 2, 5, 10, 50, 100$ are reported in Table S.9, in the same format as Table S.7. In the univariate case ($N = 1$) the models based on semivariances fail to significantly improve on the standard HAR model. For $N \geq 2$, however, the standard HAR model is *never* preferred under Frobenius loss, while under QLIKE loss it is never preferred for $N \geq 10$. In vast dimensions ($N = 100$), Model 2 (where the daily lag of **RCOV** is decomposed into semicovariances) and Model 3 (where all lags of **RCOV** are decomposed into semicovariances) almost always outperform the other models. Whereas in the univariate setting parsimonious specifications appear to be preferred, in the multivariate setting the large cross-section allows us to pin down the parameters precisely, and the large models utilizing the full decomposition lead to the best forecasts.

While the reductions in loss reported in Table S.9 may appear numerically small, especially compared to those in the univariate case, they are statistically significant.[28] In particular, for $N = 100$ the best models are in the MCS 93% and 85% of the samples for Frobenius and QLIKE loss, respectively. The next best model is part of the MCS for only 38% and 21% of the samples, respectively.

To help visualize the performance of the models across different matrix dimensions, Figure S.3 provides the multivariate analogue to Figure 5 in the main paper. It plots the median loss ratios across the 500 samples for N ranging from 1 to 100. To avoid cluttering the figure we only include the results for Models 1, 2 and 3. In addition, to illustrate the cross-sectional variation in out-performance, we also include the cross-sectional 10% and 90% quantiles of loss ratios for Model 3. The figure reveals that the gains in forecast performance obtained from decomposing

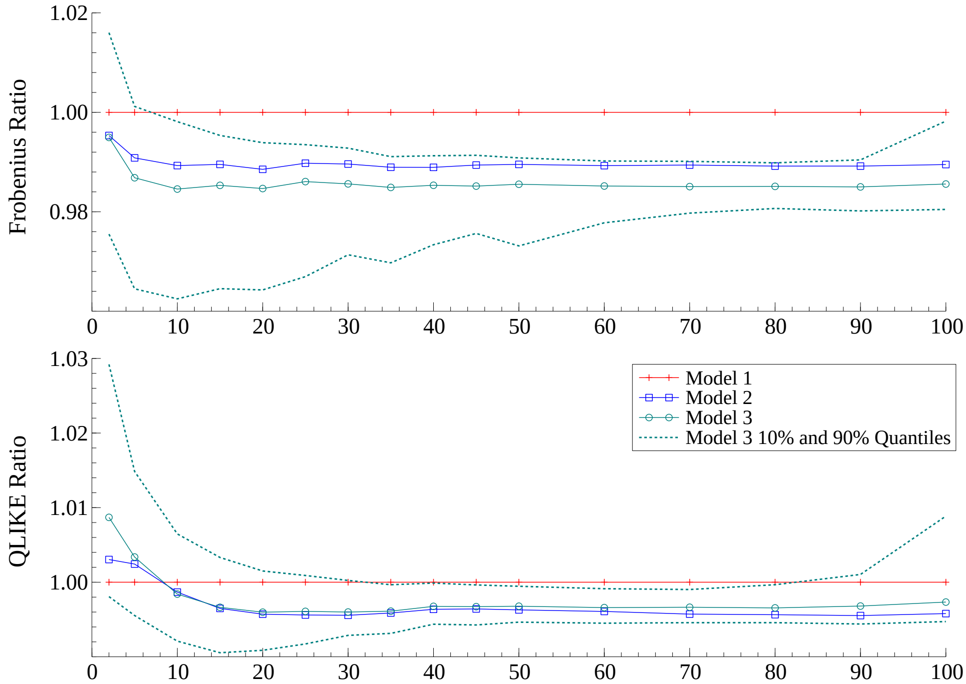
[28] The estimation error in the realized covariances also means that the reductions are underestimated numerically. Andersen, Bollerslev, and Meddahi (2005) show how to correct this underestimation in univariate applications. And a similar correction could in principle be employed here, although it would not affect the relative ordering of the models.

Table S.9: **RCOV** forecasts

	Frobenius			QLIKE		
Model	Average	Ratio	MCS	Average	Ratio	MCS
			N = 1			
1	**35.112**	**1.000**	0.870	**0.239**	**1.000**	**0.970**
6	35.209	1.001	**0.874**	0.243	1.022	0.342
7	35.326	1.021	0.686	0.247	1.037	0.022
			N = 2			
1	18.087	1.000	0.882	**0.581**	**1.000**	**0.928**
2	**17.971**	**0.995**	**0.924**	0.584	1.005	0.576
3	17.983	0.996	0.918	0.590	1.017	0.416
4	18.111	1.001	0.794	0.599	1.032	0.032
5	18.104	1.001	0.800	0.599	1.031	0.038
6	18.310	1.013	0.166	0.627	1.080	0.006
7	18.206	1.017	0.696	0.595	1.025	0.038
			N = 5			
1	9.546	1.000	0.586	**2.154**	**1.000**	**0.738**
2	9.397	0.988	0.852	2.161	1.004	0.420
3	**9.367**	**0.985**	**0.948**	2.166	1.005	0.482
4	9.417	0.992	0.656	2.202	1.023	0.022
5	9.433	0.993	0.620	2.210	1.026	0.020
6	9.818	1.047	0.062	2.777	1.283	0.014
7	9.563	1.010	0.572	2.169	1.008	0.314
			N = 10			
1	5.943	1.000	0.280	6.333	1.000	0.386
2	5.815	0.986	0.678	6.329	1.000	0.384
3	**5.786**	**0.982**	**0.952**	**6.327**	**0.999**	**0.572**
4	5.806	0.992	0.454	6.424	1.015	0.006
5	5.815	0.994	0.406	6.443	1.018	0.002
6	6.222	1.151	0.020	8.382	1.508	0.006
7	5.926	1.005	0.380	6.328	0.999	0.456
			N = 50			
1	1.372	1.000	0.026	155.347	1.000	0.096
2	1.347	0.988	0.424	**154.762**	**0.996**	**0.820**
3	1.351	**0.984**	**0.986**	154.842	0.997	0.460
4	**1.337**	0.994	0.138	155.535	1.002	0.002
5	1.341	0.996	0.124	155.831	1.002	0.002
6	1.575	1.307	0.006	169.305	1.106	0.014
7	1.363	1.000	0.038	155.506	1.001	0.010
			N = 100			
1	0.677	1.000	0.116	431.189	1.000	0.260
2	0.674	0.994	0.398	**430.952**	**0.999**	**0.804**
3	**0.671**	**0.990**	**0.928**	433.053	1.002	0.224
4	0.704	1.041	0.086	444.891	1.082	0.008
5	0.744	1.099	0.044	449.751	1.055	0.016
6	0.948	1.419	0.004	440.220	1.023	0.014
7	0.679	1.002	0.066	433.445	1.006	0.000

Note: This table provides aggregate forecasting results for covariance matrices of size $N = 1, ..., 100$. Each number is based on 500 randomly selected sets out of the 749 S&P500 stocks. We report the results for the models (1) to (7) defined above. The Average column provides the average loss over time and sets of series. The Ratio column provides the average over all sets of stocks of the ratio of time-averaged loss of model k relative to model 1. Finally, the MCS column provides the fraction of sets each respective model was part of the 80% Model Confidence Set. Boldface denotes the best model per dimension N and column.

Figure S.3: Median loss ratios: **RCOV** prediction



Note: The graph plots the median loss ratios for two of the best performing model as a function of the size of the covariance matrix, N. The ratio is calculated as the average loss of the models divided by the average loss of the standard HAR (Model 1). The medians are based on 500 random samples of stocks.

the realized covariance matrix into its semicovariance components appear even for relatively small values of N. Interestingly, the gains remain roughly constant for N ranging between 20 and 100. The 10% and 90% quantiles further show that these gains are present for almost all samples: under Frobenius loss, the 90% quantile is below one for all $N \geq 5$, while for QLIKE it is below one for N between 30 and 80.

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