Optimal Monetary and Fiscal Policy in a Medium Scale Model for Small-Open and Emerging Economies – Technical Appendix

Angelo M. Fasolo*

Duke University — Central Bank of Brazil

February 22, 2010

Abstract

This appendix describes the model, equilibrium conditions, steady-state and welfare computations of the competitive and the Ramsey equilibria of the model in "Optimal Monetary and Fiscal Policy in a Medium Scale Model for Small-Open and Emerging Economies".

---

*Acknowledgments: I would like to thank CAPES/Brazilian Ministry of Education and the Central Bank of Brazil for financial support during the PhD. However, none of these institutions or people are responsible for eventual errors and omissions in the paper, which are entirely my fault. Also, all the views expressed in this work represent only my opinions, not necessarily reflecting those of the Central Bank of Brazil and CAPES or its staff.

1 The Model

In this section, the full model is described, with the characterization of the household and the firms' problem, the policy rules for the government in a competitive equilibrium, the foreign sector and aggregation. From the household perspective, the model presents external habit persistence in consumption, adjustment costs for investment, portfolio and changing the capacity utilization. Households own capital, demand money to buy consumption goods and set their wages after observing the demand for his specific type of labor.

Firms in the tradable and non-tradable sectors of the domestic economy rent capital and labor from the households to produce goods. They set prices in a Calvo style, with a probability α_i of not adjusting prices in period t. Firms from the tradable sector have to compete with imported goods retailers. These retail firms buy goods produced abroad and sell domestically, also adjusting prices in a Calvo style in terms of domestic currency. On the other hand, firms from the tradable sector can sell goods for the exported goods retailers. These firms buy domestically produced goods and sell in abroad, setting price in a Calvo style in terms of foreign currency. A demand for foreign currency is justified in the model by a working capital constraint for imported goods retailers, with those firms selling bonds to obtain foreign currency to finance the total aquisition of foreign inputs.

The government in a competitive equilibrium sets nominal interest rates according to a Taylor rule based on inflation, output gap and changes in the real exchange rate, in order to match an exogenous, time-varying inflation target. In terms of fiscal policy, the government has three instruments available to finance an exogenous stream of consumption: money, bonds sold domestically, and distortionary taxes. The government might tax in different rates consumption and the income from capital, labor and profits. In the competitive equilibrium, taxes on labor are set according to a simple policy rule based on total government liabilities. Taxes on consumption, capital and on profits are exogenous.

The foreign sector is described by a simple VAR including lags of the foreign money supply, output, inflation, interest rates and a measure of the risk premium. The VAR has all shocks identified by a Cholesky decomposition, following the traditional procedure in the literature. The model has a total of 16 shocks, with five of them being from the foreign sector (one for each variable of the VAR), plus the following: one on the price of imported goods in foreign currency; two stationary sectorial productivity shocks; a non-stationary aggregate productivity shock; a non-stationary, investment-specific shock; government spending; three tax shocks; monetary policy shock and a inflation target shock.

1.1 Households

There is a continuum of infinitely-lived households i ($i \in [0,1]$) populating the domestic economy, each one of them offering for domestic firms a labor type i, $h_t(i)$. There is no population growth and labor can not be sold for firms in the rest of the world. In the intertemporal problem, households maximize discounted utility choosing current period's consumption capacity utilization and investment for each sector, wages, hours worked and the money demand, and next period's foreign and domestic bond holdings and physical capital stock. The general statement of the intertemporal household problem, given the non-Ponzi games constraints, is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma)\log\left(C_t(i) - \zeta C_{t-1}\right) + \gamma \log\left(1 - h_t(i)\right)\right]$$

$$s.t.: \qquad P_t\left(1+\tau_t^c\right)C_t(i) + \Upsilon_t^{-1} P_t \left(I_{x,t}^d(i) + I_{n,t}^d(i)\right) + P_t M_t(i) + R_{t-1} B_{h,t}(i) + S_t R_{t-1}^f IB_t(i)$$

$$+W_t \tfrac{\phi_w}{2}\left(\tfrac{W_t(i)}{\pi_t^{\chi_w} W_{t-1}(i)} - \mu^I\right)^2 + \tfrac{\psi_1}{2} Y_t \left(\tfrac{B_{t+1}(i)}{Y_t} - \tfrac{B}{Y}\right)^2 + \tfrac{\psi_2}{2} Y_t \left(\tfrac{S_t IB_{t+1}(i)}{Y_t} - \tfrac{rer\ IB}{Y}\right)^2 =$$

$$\left(1-\tau_t^h\right) W_t(i) h_t(i) + \left(1-\tau_t^{\phi}\right) P_t \Phi_t(i) + \left(1-\tau_t^k\right) P_t \left[\left(R_{n,t}^k \mu_{n,t} - \Upsilon_t^{-1} a\left(\mu_{n,t}\right)\right) \overline{K}_{n,t}(i)\right.$$

$$\left. + \left(R_{x,t}^k \mu_{x,t} - \Upsilon_t^{-1} a\left(\mu_{x,t}\right)\right) \overline{K}_{x,t}(i)\right] + P_{t-1} M_{t-1}(i) + B_{h,t+1}(i) + S_t IB_{t+1}(i)$$

$$\overline{K}_{j,t+1}(i) = (1-\delta)\overline{K}_{j,t}(i) + I_{j,t}^d(i)\left(1 - \aleph\left(\frac{I_{j,t}^d(i)}{I_{j,t-1}^d(i)}\right)\right)$$

$$K_{j,t} = \mu_{j,t}\overline{K}_{j,t} \qquad a\left(\mu_{j,t}\right) = \theta_1\left(\mu_{j,t} - 1\right) + \tfrac{\theta_2}{2}\left(\mu_{j,t} - 1\right)^2$$

$$\aleph\left(\tfrac{I_{i,t}^d}{I_{i,t-1}^d}\right) = \tfrac{\phi_i}{2}\left(\tfrac{I_{i,t}^d}{I_{i,t-1}^d} - \mu^I\right)^2 \qquad j = \{x, n\}$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu_{t+1}^{\Upsilon} = \left(1-\rho_{\Upsilon}\right)\mu^{\Upsilon} + \rho_{\Upsilon}\mu_t^{\Upsilon} + \epsilon_{t+1}^{\Upsilon} \qquad \epsilon_t^{\Upsilon} \sim N\left(0, \sigma_{\Upsilon}\right)$$

$$h_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\varpi} h_t \qquad M_t(i) \geq \nu^m \left(1+\tau_t^c\right) C_t(i)$$

In this problem, β is the intertemporal discount factor of the utility function. The utility function assumes a traditional, log-separable form in terms of consumption and labor, with consumption adjusted by external habit persistence[1]. In the budget constraint, define Υ_t^{-1} as the non-stationary inverse of the relative price of investment in terms of consumption goods. The households also set the rate of capital utilization for each sector ($\mu_{j,t}$), paying a cost given by the function $a\left(\mu_{j,t}\right)$

---

[1] In terms of notation, the general variable $x_t(i)$ represents the choice of household i on period t about x. The variable x_t is the aggregate value of $x_t(i)$ for the economy.

to change the utilization level in each period and in each sector j, $j = \{x, n\}$. Thus, $\overline{K}_{j,t}$ is the stock of physical capital accumulated in sector j, while $K_{j,t}$ is the capital actually used by firms in production. $\aleph(.)$ defines a function for the investment adjustment cost, in the same fashion as in Christiano, Eichenbaun and Evans (2005) [10] and Altig, Christiano, Eichenbaun and Linde (2005) such that $\aleph(1) = 0, \aleph'(1) = 0, \aleph''(1) > 0$. The functional form adopted follows Schmitt-Grohé and Uribe (2007) [38], with μ^I defining the steady state growth of investment. Still in the budget constraint, households are able to allocate wealth over time buying one-period, non-state contingent nominal bonds from the government, $B_{h,t+1}(i)$, or from the rest of the world, $IB_{t+1}(i)$. In the later case, the bonds are priced in foreign currency, and S_t is the nominal exchange rate. In order to adjust its portfolio, and to induce stationarity in the model, the households incurs in a small porfolio adjustment cost based on the variance of the stock of bonds as a proportion of the economy's GDP[2]. Households also receive (after-tax) dividends from the firms $\Phi_t(i)$.

The supply of labor is decided by each household taking as given the aggregate wage of the economy, the aggregate demand for labor, h_t, and the quadratic adjustment cost function for wages. As a monopolist of a specific type of labor, the household chooses the nominal wage $W(i)$ and supplies all the demanded for labor $h_t(i)$ given the acceptance of $W(i)$. The elasticity of substitution across different types of labor $h_t(i)$ is given by ϖ. The nominal wage adjustment cost function allows for partial indexation based on current inflation. The degree of indexation is determined by χ_w ($\chi_w \in [0, 1]$). The presence of sticky wages in the model results in an additional distortion, defined by mcw_t, which is equivalent to the markup households impose over real wages since they supply a specific type of labor to the firms.

Finally, following Schmitt-Grohé and Uribe (2007b) [39], households demand money, $M_t(i)$, in a cash-in-advance constraint, in order to pay for a share $\nu^m \geq 0$ of the after-tax consumption. The constraint holds with equality as long as (gross) nominal interest rates are larger than unity.

Define $\widetilde{\lambda}_t/P_t, \widetilde{\lambda}_t\widetilde{q}_{j,t}, \lambda_t^m\widetilde{\lambda}_t$ and $\left(\widetilde{\lambda}_t\left(1-\tau_t^h\right)W_t\right)/\left(P_t mcw_t\right)$ the Lagrange multipliers on the budget constraint on the capital accumulation equations, on the cash-in-advance constraint and on the labor demand function, and define $\widetilde{W_t} = W_t/P_t$ the (aggregate) real wages. The Lagrangean of the

---

[2] See Schmitt-Grohé and Uribe (2003(b)) [36]. The functional form adopted is the same as the model proposed by those authors. However, the use of the ratio to GDP is adopted here to obtain the stationary form of the model.

household's problem is given by:

$$
\begin{aligned}
\mathcal{L}_h = E_0 \sum_{t=0}^{\infty} \beta^t \bigg\{ & (1-\gamma)\log\left(C_t(i) - \zeta C_{t-1}\right) + \gamma \log\left(1 - h_t(i)\right) - \frac{\widetilde{\lambda}_t}{P_t} \big[P_t \left(1+\tau_t^c\right) C_t(i) + P_t M_t(i) \\
& + R_{t-1} B_{h,t}(i) + S_t R^f_{t-1} IB_t(i) + \frac{\psi_1}{2} Y_t \left(\frac{B_{h,t+1}(i)}{Y_t} - \frac{B_h}{Y} \right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}(i)}{Y_t} - \frac{rer\ IB}{Y} \right)^2 \\
& + W_t \frac{\phi_w}{2} \left(\frac{W_t(i)}{\pi_t^{\chi_w} W_{t-1}(i)} - \mu^I \right)^2 + \Upsilon_t^{-1} P_t \left[I^d_{x,t}(i) + I^d_{n,t}(i) \right] - P_{t-1} M_{t-1}(i) - B_{h,t+1}(i) \\
& - S_t IB_{t+1}(i) - \left(1 - \tau_t^{\phi}\right) P_t \Phi_t(i) di - \left(1 - \tau_t^h\right) W_t(i) \left(\frac{W_t(i)}{W_t} \right)^{-\varpi} h_t \\
& - \left(1 - \tau_t^k\right) P_t \left(\left(R^k_{x,t} \mu_{x,t} - \Upsilon_t^{-1} a\left(\mu_{x,t}\right) \right) \overline{K}_{x,t}(i) + \left(R^k_{n,t} \mu_{n,t} - \Upsilon_t^{-1} a\left(\mu_{n,t}\right) \right) \overline{K}_{n,t}(i) \right) \big] \\
& - \widetilde{\lambda}_t \widetilde{q}_{x,t} \left[\overline{K}_{x,t+1}(i) - (1-\delta) \overline{K}_{x,t}(i) - I^d_{x,t}(i) \left(1 - \aleph \left(\frac{I^d_{x,t}(i)}{I^d_{x,t-1}(i)} \right) \right) \right] \\
& - \widetilde{\lambda}_t \widetilde{q}_{n,t} \left[\overline{K}_{n,t+1}(i) - (1-\delta) \overline{K}_{n,t}(i) - I^d_{n,t}(i) \left(1 - \aleph \left(\frac{I^d_{n,t}(i)}{I^d_{n,t-1}(i)} \right) \right) \right] \\
& - \lambda_t^m \widetilde{\lambda}_t \left[\nu^m \left(1+\tau_t^c\right) C_t(i) - M_t(i) \right] + \frac{\widetilde{\lambda}_t \left(1 - \tau_t^h\right) W_t}{P_t mcw_t} \left[\left(\frac{W_t(i)}{W_t} \right)^{-\varpi} h_t - h_t(i) \right] \bigg\}
\end{aligned}
$$

The first order conditions in terms of $C_t(i), h_t(i), B_{h,t+1}(i), IB_{t+1}(i), M_t(i), \overline{K}_{x,t+1}(i), \overline{K}_{n,t+1}(i)$, $\mu_{x,t}, \mu_{n,t}, I^d_{x,t}(i), I^d_{n,t}(i)$ and $W_t(i)$ of the problem, using the fact that equilibrium is symmetric (especially note that $C_t(i) = C_t$ and $W_t(i) = W_t$), are given by:

$$
\frac{(1-\gamma)}{C_t - \zeta C_{t-1}} - \left(1+\tau_t^c\right) \widetilde{\lambda}_t - \nu^m \left(1+\tau_t^c\right) \lambda_t^m \widetilde{\lambda}_t = 0
$$

$$
-\frac{\gamma}{1-h_t} + \frac{\widetilde{\lambda}_t \left(1-\tau_t^h\right)}{mcw_t} \frac{W_t}{P_t} = 0
$$

$$
\frac{\widetilde{\lambda}_t}{P_t} \psi_1 \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y} \right) - \frac{\widetilde{\lambda}_t}{P_t} + \beta R_t E_t \left(\frac{\widetilde{\lambda}_{t+1}}{P_{t+1}} \right) = 0
$$

$$
\frac{S_t}{P_t} \widetilde{\lambda}_t \psi_2 \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y} \right) - \frac{S_t}{P_t} \widetilde{\lambda}_t + \beta R^f_t E_t \left(S_{t+1} \frac{\widetilde{\lambda}_{t+1}}{P_{t+1}} \right) = 0
$$

$$
-\widetilde{\lambda}_t + \lambda_t^m \widetilde{\lambda}_t + \beta E_t \left(\frac{\widetilde{\lambda}_{t+1} P_t}{P_{t+1}} \right) = 0
$$

$$
\Longrightarrow \widetilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y} \right) \right)^{-1}
$$

$$
-\widetilde{\lambda}_t \widetilde{q}_{x,t} + \beta E_t \widetilde{\lambda}_{t+1} \left(1 - \tau^k_{t+1}\right) \left(R^k_{x,t+1} \mu_{x,t+1} - \Upsilon_{t+1}^{-1} a\left(\mu_{x,t+1}\right) \right) + \beta E_t \widetilde{\lambda}_{t+1} \widetilde{q}_{x,t+1} (1-\delta) = 0
$$

$$
-\widetilde{\lambda}_t \widetilde{q}_{n,t} + \beta E_t \widetilde{\lambda}_{t+1} \left(1 - \tau^k_{t+1}\right) \left(R^k_{n,t+1} \mu_{n,t+1} - \Upsilon_{t+1}^{-1} a\left(\mu_{n,t+1}\right) \right) + \beta E_t \widetilde{\lambda}_{t+1} \widetilde{q}_{n,t+1} (1-\delta) = 0
$$

$$
\widetilde{\lambda}_t \left[\Upsilon_t^{-1} a'\left(\mu_{x,t}\right) - R^k_{x,t} \right] = 0
$$

$$
\widetilde{\lambda}_t \left[\Upsilon_t^{-1} a'\left(\mu_{n,t}\right) - R^k_{n,t} \right] = 0
$$

$$\widetilde{\lambda}_t \Upsilon_t^{-1} - \widetilde{\lambda}_t \widetilde{q}_{x,t} + \widetilde{\lambda}_t \widetilde{q}_{x,t} \aleph\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right) + \widetilde{\lambda}_t \widetilde{q}_{x,t}\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right) \aleph'\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right) \\ - \beta E_t \widetilde{\lambda}_{t+1} \widetilde{q}_{x,t+1}\left(\frac{I_{x,t+1}^d}{I_{x,t}^d}\right)^2 \aleph'\left(\frac{I_{x,t+1}^d}{I_{x,t}^d}\right) = 0$$

$$\widetilde{\lambda}_t \Upsilon_t^{-1} - \widetilde{\lambda}_t \widetilde{q}_{n,t} + \widetilde{\lambda}_t \widetilde{q}_{n,t} \aleph\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right) + \widetilde{\lambda}_t \widetilde{q}_{n,t}\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right) \aleph'\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right) \\ - \beta E_t \widetilde{\lambda}_{t+1} \widetilde{q}_{n,t+1}\left(\frac{I_{n,t+1}^d}{I_{n,t}^d}\right)^2 \aleph'\left(\frac{I_{n,t+1}^d}{I_{n,t}^d}\right) = 0$$

$$(\varpi - 1)\frac{\widetilde{\lambda}_t\left(1-\tau_t^h\right)}{P_t} h_t + \frac{\widetilde{\lambda}_t}{P_t}\frac{\phi_w W_t}{\pi_t^{\chi_w} W_{t-1}}\left(\frac{W_t}{\pi_t^{\chi_w} W_{t-1}} - \mu^I\right) \\ - \frac{\widetilde{\lambda}_t\left(1-\tau_t^h\right)}{P_t mcw_t}\varpi h_t - \beta E_t \frac{\widetilde{\lambda}_{t+1}}{P_{t+1}}\frac{\phi_w}{\pi_{t+1}^{\chi_w}}\left(\frac{W_{t+1}}{W_t}\right)^2\left(\frac{W_{t+1}}{\pi_{t+1}^{\chi_w} W_t} - \mu^I\right) = 0$$

The final set of equilibrium conditions of the household is given by:

$$\frac{\left(1-\tau_t^h\right)\widetilde{W}_t}{\left(1+\tau_t^c\right)\left(C_t - \zeta C_{t-1}\right)} = \frac{\gamma}{(1-\gamma)}\frac{mcw_t\left(1+\nu^m\left(\frac{\widetilde{R}_t - 1}{\widetilde{R}_t}\right)\right)}{\left(1-h_t\right)} \tag{1}$$

$$\frac{(1-\gamma)}{C_t - \zeta C_{t-1}} = \left(1+\tau_t^c\right)\widetilde{\lambda}_t\left(1+\nu^m\left(\frac{R_t - 1}{R_t}\right)\right) \tag{2}$$

$$\widetilde{\lambda}_t\left[1-\psi_1\left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)\right] = \beta R_t E_t\left(\frac{\widetilde{\lambda}_{t+1}}{\pi_{t+1}}\right) \tag{3}$$

$$\widetilde{\lambda}_t\left[1-\psi_2\left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y}\right)\right] = \beta R_t^f E_t\left(\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}\widetilde{\lambda}_{t+1}\right) \tag{4}$$

$$\widetilde{\lambda}_t \widetilde{q}_{x,t} = \beta E_t\left\{\widetilde{\lambda}_{t+1}\left[\left(1-\tau_{t+1}^k\right)\left(R_{x,t+1}^k \mu_{x,t+1} - \Upsilon_{t+1}^{-1} a\left(\mu_{x,t+1}\right)\right) + \widetilde{q}_{x,t+1}(1-\delta)\right]\right\} \tag{5}$$

$$\widetilde{\lambda}_t \widetilde{q}_{n,t} = \beta E_t\left\{\left[\widetilde{\lambda}_{t+1}\left(1-\tau_{t+1}^k\right)\left(R_{n,t+1}^k \mu_{n,t+1} - \Upsilon_{t+1}^{-1} a\left(\mu_{n,t+1}\right)\right) + \widetilde{q}_{n,t+1}(1-\delta)\right]\right\} \tag{6}$$

$$K_{n,t} = \mu_{n,t}\overline{K}_{n,t} \tag{7}$$

$$K_{x,t} = \mu_{x,t}\overline{K}_{x,t} \tag{8}$$

$$\theta_1 + \theta_2\left(\mu_{n,t} - 1\right) = \frac{R_{n,t}^k}{\Upsilon_t^{-1}} \tag{9}$$

$$\theta_1 + \theta_2\left(\mu_{x,t} - 1\right) = \frac{R_{x,t}^k}{\Upsilon_t^{-1}} \tag{10}$$

$$R_t = \frac{1}{r_{t,t+1}} \tag{11}$$

$$\widetilde{R}_t = R_t \left(1-\psi_1\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)\right)^{-1} \tag{12}$$

$$\begin{aligned}
\widetilde{\lambda}_t \Upsilon_t^{-1} &= \widetilde{\lambda}_t \widetilde{q}_{x,t}\left[1-\Psi\left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right)-\left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right)\Psi'\left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right)\right] \\
&\quad +\beta E_t\left[\widetilde{\lambda}_{t+1}\widetilde{q}_{x,t+1}\left(\frac{I^d_{x,t+1}}{I^d_{x,t}}\right)^2\Psi'\left(\frac{I^d_{x,t+1}}{I^d_{x,t}}\right)\right]
\end{aligned} \tag{13}$$

$$\begin{aligned}
\widetilde{\lambda}_t \Upsilon_t^{-1} &= \widetilde{\lambda}_t \widetilde{q}_{n,t}\left[1-\Psi\left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right)-\left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right)\Psi'\left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right)\right] \\
&\quad +\beta E_t\left[\widetilde{\lambda}_{t+1}\widetilde{q}_{n,t+1}\left(\frac{I^d_{n,t+1}}{I^d_{n,t}}\right)^2\Psi'\left(\frac{I^d_{n,t+1}}{I^d_{n,t}}\right)\right]
\end{aligned} \tag{14}$$

$$\overline{K}_{x,t+1}(i) = (1-\delta)\,\overline{K}_{x,t}(i) + I^d_{x,t}(i)\left(1-\aleph\left(\frac{I^d_{x,t}(i)}{I^d_{x,t-1}(i)}\right)\right) \tag{15}$$

$$\overline{K}_{n,t+1}(i) = (1-\delta)\,\overline{K}_{n,t}(i) + I^d_{n,t}(i)\left(1-\aleph\left(\frac{I^d_{n,t}(i)}{I^d_{n,t-1}(i)}\right)\right) \tag{16}$$

$$\begin{aligned}
\left(\frac{1-\varpi}{\varpi}+\frac{1}{mcw_t}\right)\varpi h_t\left(1-\tau^h_t\right) &= -\frac{\phi_w}{\pi_t^{\chi_w-1}}\left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}}\right)\left(\frac{\widetilde{W}_t}{\pi_t^{\chi_w-1}\widetilde{W}_{t-1}}-\mu^I\right) \\
&\quad +\beta E_t\left[\frac{\widetilde{\lambda}_{t+1}\phi_w}{\widetilde{\lambda}_t\pi_{t+1}^{\chi_w-1}}\left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\right)^2\left(\frac{\widetilde{W}_{t+1}}{\pi_{t+1}^{\chi_w-1}\widetilde{W}_t}-\mu^I\right)\right]
\end{aligned} \tag{17}$$

The household also solves a sequence of minimization problems constrained by the CES aggregator function in order to choose first between imported and domestically produced goods in the tradable goods basket, and then chooses the optimal composition of tradable and non-tradable goods:

$$\begin{gathered}
\min_{C_{n,t},C_{t,t},C_{m,t},C_{x,t}} P_{n,t}C_{n,t}+P_{t,t}C_{t,t} \\
C_t+\frac{\psi_1}{2}Y_t\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)^2+\frac{\psi_2}{2}Y_t\left(\frac{S_t IB_{t+1}}{Y_t}-\frac{rer\ IB}{Y}\right)^2=\left[(1-\omega)^{\frac{1}{\varepsilon}}C_{n,t}^{\frac{\varepsilon-1}{\varepsilon}}+\omega^{\frac{1}{\varepsilon}}C_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}
\end{gathered} \tag{18}$$

$$C_{t,t}=\left[(1-\varkappa)^{\frac{1}{\varrho}}C_{x,t}^{\frac{\varrho-1}{\varrho}}+\varkappa^{\frac{1}{\varrho}}C_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}} \tag{19}$$

Combine the first order conditions with the expenditure function to obtain the demand for each type of tradable good and the price index of tradable goods:

$$C_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}}\right)^{-\varrho} C_{t,t} \tag{20}$$

$$C_{x,t} = (1-\varkappa) \left(\frac{P_{x,t}}{P_{t,t}}\right)^{-\varrho} C_{t,t} \tag{21}$$

$$P_{t,t} = \left[(1-\varkappa) P_{x,t}^{1-\varrho} + \varkappa P_{m,t}^{1-\varrho}\right]^{\frac{1}{1-\varrho}}$$

By analogy, the optimal decision between tradable and non-tradable goods and the CPI index is given by:

$$C_{t,t} = \omega \left(\frac{P_{t,t}}{P_t}\right)^{-\varepsilon} \left(C_t + \frac{\psi_1}{2} Y_t \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y}\right)^2\right) \tag{22}$$

$$C_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t}\right)^{-\varepsilon} \left(C_t + \frac{\psi_1}{2} Y_t \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y}\right)^2\right) \tag{23}$$

$$P_t = \left[(1-\omega) P_{n,t}^{1-\varepsilon} + \omega P_{t,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Households also solve an equivalent problem when setting the composition of the investment good for each sector. For simplicity, assume that the weights in the basket of goods and the elasticities of substitution among different types of investment goods is the same as the weights and the elasticities for consumption goods. Also, assume that the adjustment costs in the portfolios of domestic and foreign bonds and in capital utilization are allocated in the aggregate investment. As a consequence, the demands for home produced and imported investment goods are given by:

$$\Upsilon_t^{-1} I_t = \Upsilon_t^{-1} \left(I_{n,t}^d + a\left(\mu_{n,t}\right) \overline{K}_{n,t} + I_{x,t}^d + a\left(\mu_{x,t}\right) \overline{K}_{x,t}\right) \tag{24}$$

$$I_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}}\right)^{-\varrho} I_{t,t} \tag{25}$$

$$I_{x,t} = (1-\varkappa) \left(\frac{P_{x,t}}{P_{t,t}}\right)^{-\varrho} I_{t,t} \tag{26}$$

$$I_{t,t} = \omega \left(\frac{P_{t,t}}{P_t}\right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{27}$$

$$I_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t}\right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{28}$$

$$\Upsilon_t^{-1} I_t = \left[(1-\omega)^{\frac{1}{\varepsilon}} I_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} I_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$I_{t,t} = \left[(1-\varkappa)^{\frac{1}{\varrho}} I_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} I_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}}$$

1.2 Firms' Problem

There are four sectors in the economy, each sector composed by a continuum of firms operating in a monopolistic competitive framework. Firms in the non-tradable (n) and tradable (x) sectors demand labor and capital to produce. Firms in the imported goods (m) sector and in the exported goods (xp) sector buy the final good and sell it to consumers in the domestic economy (for the case of imported goods sector firms) or in the rest of the world (for the case of exported goods sector firms). Firms chooses the amount of production inputs to buy and set new prices according to a probability $\alpha_i, i = \{n, x, m, xp\}$, that is independent across sectors and across firms. Imported good's firms must finance the total amount of imported goods using only foreign currency.

1.2.1 Domestic non-tradable goods' producers problem:

Firms in the non-tradable sector use capital and labor to produce goods that are used for consumption, investment and spent by the government. The production technology is a simple Cobb-Douglas function with a sectorial stationary productivity shock and a non-stationary, labor-augmenting technological shock. Setting real profits for firm i_n as $\Phi_{n,t}(i_n)$, the general problem of the domestic, non-tradable goods producers of type i_n product ($i_n \in [0,1]$) is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{n,t} \left(\frac{P_{n,t}(i_n)}{P_{n,t}} D_{n,t}(i_n) - \frac{W_t}{P_{n,t}} h_{n,t}(i_n) - \frac{P_t}{P_{n,t}} R^k_{n,t} K_{n,t}(i_n) \right)$$

$$\begin{array}{lc}
s.t.: & D_{n,t}(i_n) = \left(\frac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_n} Y_{n,t} \\
 & Y_{n,t} = C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t} \\
 & a_{n,t} K_{n,t}(i_n)^{\theta} \left(z_t h_{n,t}(i_n) \right)^{1-\theta} - z_t^* \chi_n \geq D_{n,t}(i_n) \\
 & \Upsilon_t^{\frac{\theta}{1-\theta}} = \frac{z_t^*}{z_t} \\
 & \frac{z_{t+1}}{z_t} = \mu^z_{t+1} = (1-\rho_z)\mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}; \quad \rho_z \in [0,1); \quad \epsilon^z_t \sim N(0, \sigma_z) \\
 & \log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon^n_{t+1}; \quad \rho_n \in [0,1); \quad \epsilon^n_t \sim N(0, \sigma_n)
\end{array}$$

The firms maximize profits subject to the demand for its good, the fixed cost of production and subject to two productivity shocks. In each period, firms choose the number of hours to buy from households and the amount of capital to rent. Also, if allowed, firms set their prices optimally. In this problem, $a_{n,t}$ is a stationary, sector-specific technology shock, z_t is a labor-augmenting, non-stationary technology shock. The non-stationary shock z_t affects all the firms using labor in production. In order to guarantee zero profits in the long run, $z_t^* \chi_n$ introduces a fixed cost proportional to the evolution of the non-stationary shocks in production, following Schmitt-Grohé and Uribe (2007) [38].

From the first order conditions in terms of $h_{n,t}(i_n)$ and $K_{n,t}(i_n)$, it is possible to prove that the capital-labor ratio is the same across firms in the non-tradable sector. As a consequence, the marginal cost across firms is also the same in this sector. Setting $mc_{n,t}$ as the Lagrange multiplier on the firm's demand constraint, the two equilibrium conditions are:

$$\widetilde{W}_t \frac{P_t}{P_{n,t}} = mc_{n,t} \left(1-\theta\right) a_{n,t} z_t \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta} \tag{29}$$

$$R^k_{n,t} \frac{P_t}{P_{n,t}} = mc_{n,t} \theta a_{n,t} \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta-1} \tag{30}$$

Prices are formed in a Calvo style with indexation, where α_n is the probability that firm i_n is not allowed to optimally adjust its price in period t. In the case firms are not allowed to set up prices optimally, they follow the simple rule $P_{n,t}\left(i_n\right) = \pi^{\kappa_n}_{n,t-1} P_{n,t-1}\left(i_n\right)$, for $0 \leq \kappa_n \leq 1$ and $\pi_{n,t+1} = \frac{P_{n,t+1}}{P_{n,t}}$. Setting the Lagrangean of the problem, considering only the terms relevants for price determination:

$$\begin{aligned}\mathcal{L}_n = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} P_{n,t+s} &\left(\left(\frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}}\right)^{1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{1-\eta_n} Y_{n,t+s} \right. \\ &\left. - mc_{n,t+s} \left(\left(\frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}}\right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{-\eta_n} Y_{n,t+s} \right) \right)\end{aligned}$$

In this problem, $r_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and $\widetilde{P}_{n,t}\left(i_n\right)$ is the new price set by firms allowed to adjust prices in period t. The first order condition for firms allowed to adjust prices is:

$$\begin{aligned}E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}}\right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{-\eta_n} \times \\ \left(\frac{\left(\eta_n - 1\right)}{\eta_n} \frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right) - mc_{n,t+s} \right) = 0\end{aligned}$$

As a consequence of the first order condition, given that mark-up over prices is the same across firms, the symmetric equilibrium is characterized by all firms in the non-tradable sector allowed to adjust prices in period t will set the same price: $\widetilde{P}_{n,t}\left(i_n\right) = \widetilde{P}_{n,t}$.

Following Schmitt-Grohé and Uribe (2006) [37], split the pricing function equation in two parts, X^1_t and X^2_t, and define $\widetilde{p}_{n,t} = \frac{\widetilde{P}_{n,t}}{P_{n,t}}$ in order to obtain the recursive solution for the problem of the non-tradable goods' producers:

$$X^1_t = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} \left(\frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)\right)$$

The recursive form for X_t^1 is given by:

$$X_t^1 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^1 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \frac{P_{n,t+s-1}}{P_{n,t+s-1}} \cdots \frac{P_{n,t+1}}{P_{n,t+1}} \frac{P_{n,t}}{P_{n,t}} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^1 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \pi_{n,t+k} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + E_t \sum_{s=1}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \pi_{n,t+k} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + E_t \sum_{s=1}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\pi_{n,t+k}^{-\frac{1}{\eta_n}} \frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + E_t \sum_{s=1}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}^{\frac{1}{\eta_n}} \pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s}$$

$$\begin{aligned} X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n} \times \\ \sum_{s=1}^{\infty} \alpha_n^{s-1} r_{t+1,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t+1}}{P_{n,t+1}}\right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{-\eta_n} mc_{n,t+s} \end{aligned}$$

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n} X_{t+1}^1$$

The recursive form for X_t^2 is given by:

$$X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \frac{P_{n,t+s-1}}{P_{n,t+s-1}} \cdots \frac{P_{n,t+1}}{P_{n,t+1}} \frac{P_{n,t}}{P_{n,t}} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^{s} \pi_{n,t+k} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = Y_{n,t} \widetilde{p}_{n,t}^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} + E_t \sum_{s=1}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n} \prod_{k=1}^{s} \pi_{n,t+k} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = Y_{n,t}\widetilde{p}_{n,t}^{-\eta_n}\frac{(\eta_n - 1)}{\eta_n} + E_t\sum_{s=1}^{\infty}\alpha_n^s r_{t,t+s}Y_{n,t+s}\left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n}\prod_{k=1}^{s}\left(\pi_{n,t+k}^{\frac{1}{1-\eta_n}}\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = Y_{n,t}\widetilde{p}_{n,t}^{-\eta_n}\frac{(\eta_n - 1)}{\eta_n} + E_t\sum_{s=1}^{\infty}\alpha_n^s r_{t,t+s}Y_{n,t+s}\left(\frac{\widetilde{P}_{n,t}}{P_{n,t}}\right)^{-\eta_n}\prod_{k=1}^{s}\left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}^{\frac{1}{\eta_n-1}}\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = Y_{n,t}\widetilde{p}_{n,t}^{-\eta_n}\frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1}E_t\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n}\times$$
$$\sum_{s=1}^{\infty}\alpha_n^{s-1}r_{t+1,t+s}Y_{n,t+s}\left(\frac{\widetilde{P}_{n,t+1}}{P_{n,t+1}}\right)^{-\eta_n}\prod_{k=1}^{s}\left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}}\right)^{1-\eta_n}$$

$$X_t^2 = Y_{n,t}\widetilde{p}_{n,t}^{-\eta_n}\frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1}E_t\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n}X_{t+1}^2$$

The system describing the evolution of non-tradable inflation is given by:

$$X_t^1 = Y_{n,t}\widetilde{p}_{n,t}^{-1-\eta_n}mc_{n,t} + \alpha_n r_{t,t+1}E_t\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n}X_{t+1}^1 \quad (31)$$

$$X_t^2 = Y_{n,t}\widetilde{p}_{n,t}^{-\eta_n}\frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1}E_t\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n}X_{t+1}^2 \quad (32)$$

$$X_t^1 = X_t^2 \quad (33)$$

1.2.2 Tradable goods' producers problem:

A tradable goods producer i_x ($i_x \in [0,1]$) solves the same problem as the non-tradable producer, using labor and capital as a production factors. The total production of the tradable good is divided between domestic absorption (consumption, investment and government spending) and the demand of a continuum of i_{xp} exporting firms ($D_{xp,t}$). The tradable goods' firm problem is given by:

$$\max E_0\sum_{t=0}^{\infty}r_{0,t}P_{x,t}\left(\frac{P_{x,t}(i_x)}{P_{x,t}}D_{x,t}(i_x) - \frac{\widetilde{W}_t}{P_{x,t}}h_{x,t}(i_x) - \frac{P_t}{P_{x,t}}R_{x,t}^k K_{x,t}(i_x)\right)$$

$$s.t.: \quad D_{x,t}(i_x) = \left(\tfrac{P_{x,t}(i_x)}{P_{x,t}}\right)^{-\eta_x}Y_{x,t}$$

$$Y_{x,t} = C_{x,t} + G_{t,t} + \Upsilon_t^{-1}\tfrac{P_t}{P_{x,t}}I_{x,t} + D_{xp,t}$$

$$a_{x,t}K_{x,t}(i_x)^{\theta}(z_t h_{x,t}(i_x))^{1-\theta} - z_t^*\chi_x \geq D_{x,t}(i_{tr})$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \tfrac{z_t^*}{z_t}$$

$$\tfrac{z_{t+1}}{z_t} = \mu_{t+1}^z = (1-\rho_z)\mu^z + \rho_z\mu_t^z + \epsilon_{t+1}^z; \quad \rho_z \in [0,1); \quad \epsilon_t^z \sim N(0,\sigma_z)$$

$$\log a_{x,t+1} = \rho_x\log a_{x,t} + \epsilon_{t+1}^x; \quad \rho_x \in [0,1); \quad \epsilon_t^x \sim N(0,\sigma_x)$$

χ_x is a fixed cost proportional to total output associated with the non-stationary shock in order to guarantee zero profits in steady state. Setting $mc_{x,t}$ as the Lagrange multiplier on the firm's demand constraint, the solution of the cost minimization problem of the firm in terms of $h_{x,t}\left(i_x\right)$ and $K_{x,t}\left(i_x\right)$, after using again the fact that the capital-labor ratio is the same across firms, become:

$$\widetilde{W}_t \frac{P_t}{P_{x,t}} = mc_{x,t}\left(1-\theta\right) a_{x,t} z_t \left(\frac{K_{x,t}}{z_t h_{x,t}}\right)^{\theta} \tag{34}$$

$$R^k_{x,t} \frac{P_t}{P_{x,t}} = mc_{x,t} \theta a_{x,t} \left(\frac{K_{x,t}}{z_t h_{x,t}}\right)^{\theta-1} \tag{35}$$

Similar to the firms in the non-tradable sector, price adjustment is based on the Calvo mechanism with indexation to past inflation, with $0 \leq \kappa_x \leq 1$ defining the degree of indexation in the tradable sector. Taking the first order conditions in terms of $\widetilde{P}_{x,t}\left(i_x\right)$, and defining $\pi_{x,t+1} = \frac{P_{x,t+1}}{P_{x,t}}$, the optimal price set by each firm is:

$$E_t \sum_{s=0}^{\infty} \alpha_x^s r_{t,t+s} Y_{x,t+s} P_{x,t+s} \left(\frac{\widetilde{P}_{x,t}\left(i_x\right)}{P_{x,t}}\right)^{-\eta_x} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_x}_{x,t+k-1}}{\pi_{x,t+k}}\right)^{-\eta_x} \times \left(\frac{\left(\eta_x - 1\right)}{\eta_x} \frac{\widetilde{P}_{x,t}\left(i_x\right)}{P_{x,t}} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_x}_{x,t+k-1}}{\pi_{x,t+k}}\right) - mc_{x,t+s}\right) = 0$$

As a consequence of the same mark-up over prices across firms, the symmetric equilibrium is characterized by all firms in the tradable sector setting the same price when allowed to optimize, $\widetilde{P}_{x,t}\left(i_x\right) = \widetilde{P}_{x,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Z^1_t, Z^2_t, such that $Z^1_t = Z^2_t$, and $\widetilde{p}_{x,t} = \frac{\widetilde{P}_{x,t}}{P_{x,t}}$:

$$Z^1_t = \widetilde{p}_{x,t}^{-1-\eta_x} Y_{x,t} mc_{x,t} + \alpha_x r_{t,t+1} E_t \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x} \left(\frac{\pi^{\kappa_x}_{x,t}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}}\right)^{-\eta_x} Z^1_{t+1} \tag{36}$$

$$Z^2_t = \widetilde{p}_{x,t}^{-\eta_x} Y_{x,t} \frac{\left(\eta_x - 1\right)}{\eta_x} + \alpha_x r_{t,t+1} E_t \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-\eta_x} \left(\frac{\pi^{\kappa_x}_{x,t}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}}\right)^{1-\eta_x} Z^2_{t+1} \tag{37}$$

$$Z^1_t = Z^2_t \tag{38}$$

1.2.3 Imported goods' firms problem:

Following Lubik and Schorfheide (2006) [27], deviations from the Law of One price in the model arrises as a consequence of price stickiness in imported and exported goods. An imported goods' firm i_m ($i_m \in [0,1]$) buys a bundle of the international homogeneous good[3] and relabel it as an imported

---

[3] Note that, in the model, one country buys a combination of the goods from different countries. As a consequence, there is a gap between the world's CPI (P^*_t) and the imported good bundle imported by a given country ($P^*_{m,t}$).

good type i_m. In order to buy the goods produced in the rest of the world, the firm needs to finance a share of the total using foreign currency. The firm sells intraperiod bonds in foreign markets in order to get foreign currency, but it does not transfer financial wealth over time. As a consequence, firms do not incur in exposure to risk in the international markets, just an increase in the marginal cost of production. As a timing convention, the bonds traded do not reflect in the end of period balance of payments. The same framework is adopted in Christiano, Trabandt and Walentin (2007) [11] and Mendoza and Yue (2008) [29]. The budget constraint of the exporting firm i_m, expressed in terms of domestic prices, is given by:

$$\begin{aligned}&\frac{S_t P_t^*}{P_t} M_{m,t}^*(i_m) + \frac{S_t}{P_t} B_{m,t+1}^*(i_m) = \\ &\quad \frac{S_t}{P_t} P_{t-1}^* M_{m,t-1}^*(i_m) + \frac{S_t}{P_t} R_{t-1}^f B_{m,t}^*(i_m) + \left(\frac{P_{m,t}\left(i_m\right) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}\left(i_m\right) - z_t^* \chi_m - \Phi_{m,t}(i_m)\end{aligned}$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Following the assumption that firms do not keep any financial wealth across periods, and that all profits are distributed to the households, obtain the expression for real profits:

$$P_t^* M_{m,t}^*(i_m) + R_t^f B_{m,t+1}^*(i_m) = 0, \qquad \forall t$$

$$\Longrightarrow \frac{S_t P_t^*}{P_t}\left(M_{m,t}^*(i_m) - \frac{M_{m,t}^*(i_m)}{R_t^f}\right) = \left(\frac{P_{m,t}\left(i_m\right) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}\left(i_m\right) - z_t^* \chi_m - \Phi_{m,t}(i_m)$$

$$\Phi_{m,t}(i_m) = \left(\frac{P_{m,t}\left(i_m\right) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}\left(i_m\right) - z_t^* \chi_m - \frac{S_t P_t^*}{P_t}\left(\frac{R_t^f - 1}{R_t^f}\right) M_{m,t}^*(i_m)$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. The imported goods' firm problem is given by:

$$\max_{\widetilde{P}_{m,t}(i_m)} E_0 \sum_{t=0}^{\infty} r_{0,t}\left[\left(\frac{P_{m,t}\left(i_m\right) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}\left(i_m\right) - z_t^* \chi_m - \frac{S_t P_{m,t}^*}{P_t}\left(\frac{R_t^f - 1}{R_t^f}\right) \frac{P_t^*}{P_{m,t}^*} M_{m,t}^*(i_m)\right]$$

$$s.t.: D_{m,t}\left(i_m\right) = \left(\frac{P_{m,t}\left(i_m\right)}{P_{m,t}}\right)^{-\eta_m}\left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t}\right)$$

$$M_{m,t}\left(i_m\right) \geq \frac{P_{m,t}^*}{P_t^*} D_{m,t}\left(i_m\right)$$

where $P_{m,t}^*$ is the price of the imported good bought by home economy, quoted in foreign prices. Taking the first order conditions in terms of $\widetilde{P}_{m,t}\left(i_m\right)$, the price for those firms allowed to optimize prices in period t, and defining $\pi_{m,t+1} = \frac{P_{m,t+1}}{P_{m,t}}$ and $0 \leq \kappa_m \leq 1$ the degree of indexation in the imported goods' sector, the expression for the optimal price set by each firm becomes:

$$E_t \sum_{s=0}^{\infty} \alpha_m^s r_{t,t+s} P_{m,t+s} \left(C_{m,t+s} + \Upsilon_{t+s}^{-1} \frac{P_{t+s}}{P_{m,t+s}} I_{m,t+s} \right) \left(\frac{\widetilde{P}_{m,t}\left(i_m\right)}{P_{m,t+s}} \right)^{-\eta_m} \prod_{k=1}^{s} \left(\frac{\pi_{m,t+k-1}^{\kappa_m}}{\pi_{m,t+k}} \right)^{-\eta_m} \times$$
$$\left(\frac{(\eta_m - 1)}{\eta_m} \frac{\widetilde{P}_{m,t}\left(i_m\right)}{P_{m,t+s}} \prod_{k=1}^{s} \left(\frac{\pi_{m,t+k-1}^{\kappa_m}}{\pi_{m,t+k}} \right) - \frac{S_{t+s} P_{m,t+s}^{*}}{P_{m,t+s}} \left(1 + \frac{R_{t+s}^{f} - 1}{R_{t+s}^{f}} \right) \right) = 0$$

In this problem, α_m is the probability that importing firm i_m is not allowed to adjust its price in period t. As a consequence of the same mark-up over prices across firms (in this case, given by the real exchange rate deflated by the import price level), the symmetric equilibrium is characterized by $\widetilde{P}_{m,t}\left(i_m\right) = \widetilde{P}_{m,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Y_t^1 and Y_t^2 such that $Y_t^1 = Y_t^2$, and $\widetilde{p}_{m,t} = \frac{\widetilde{P}_{m,t}}{P_{m,t}}$:

$$Y_t^1 = \widetilde{p}_{m,t}^{-1-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{S_t P_{m,t}^{*}}{P_{m,t}} \left(1 + \frac{R_t^f - 1}{R_t^f} \right)$$
$$+ \alpha_m r_{t,t+1} E_t \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-1-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{-\eta_m} Y_{t+1}^1 \quad (39)$$

$$Y_t^2 = \widetilde{p}_{m,t}^{-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{(\eta_m - 1)}{\eta_m}$$
$$+ \alpha_m r_{t,t+1} E_t \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{\eta_m/(\eta_m - 1)}} \right)^{1-\eta_m} Y_{t+1}^2 \quad (40)$$

$$Y_t^1 = Y_t^2 \quad (41)$$

1.2.4 Exported goods' firms problem:

On exported goods' side, there is a specific sector consuming tradable goods and, in a Calvo style, setting prices in foreign currency. An exported goods' firm i_{xp} ($i_{xp} \in [0,1]$) buys a share of the final tradable good in the domestic economy and sell it to the rest of the world. Price is sticky in foreign currency. The exported goods' firm problem is given by:

$$\max_{\widetilde{P}_{x,t}^{*}(i_{xp})} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{S_t \widetilde{P}_{x,t}^{*}\left(i_{xp}\right) - P_{x,t}}{P_t} \right) D_{xp,t}\left(i_{xp}\right) - \left(\frac{R_t - 1}{R_t} \right) M_{xp,t}(i_{xp}) - z_t^{*} \chi_{xp} \right]$$
$$s.t. : D_{xp,t}\left(i_{xp}\right) = \left(\frac{P_{x,t}^{*}\left(i_{xp}\right)}{P_{x,t}^{*}} \right)^{-\eta_{xp}} X_t$$

where χ_{xp} is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Taking the first order conditions in terms of $\widetilde{P}^*_{x,t}\left(i_{xp}\right)$, and defining $\pi^*_{x,t+1} = \frac{P^*_{x,t+1}}{P^*_{x,t}}$ and $0 \leq \kappa_{xp} \leq 1$ the degree of indexation in the exported goods' sector, the expression for the optimal price set by each firm becomes:

$$E_t \sum_{s=0}^{\infty} \alpha_{xp}^s r_{t,t+s} P^*_{x,t+s} X_s \left(\frac{\widetilde{P}^*_{x,s}\left(i_{xp}\right)}{P^*_{x,s}} \right)^{-\eta_{xp}-1} \prod_{k=1}^{s} \left(\frac{\left(\pi^*_{x,t+k-1}\right)^{\kappa_{xp}}}{\pi^*_{x,t+k}} \right)^{-\eta_{xp}} \times$$
$$\left(\frac{\left(\eta_{xp}-1\right)}{\eta_{xp}} \frac{\widetilde{P}^*_{x,t}\left(i_{xp}\right)}{P^*_{x,t}} \prod_{k=1}^{s} \left(\frac{\left(\pi^*_{x,t+k-1}\right)^{\kappa_{xp}}}{\pi^*_{x,t+k}} \right) - \frac{P_{x,s}}{S_s P^*_{x,s}} \right) = 0$$

In this problem, α_{xp} is the probability that importing firm i_x is not allowed to adjust its price in period t, $P^*_{x,t}$ is the price of the tradable good from the domestic economy quoted in foreign prices. The symmetric equilibrium is again characterized by $\widetilde{P}^*_{x,t}\left(i_x\right) = \widetilde{P}^*_{x,t}$. The recursive solution for the pricing problem of the exporting firms is obtained after properly defining U_t^1 and U_t^2, such that $U_t^1 = U_t^2$, and $\widetilde{p}^*_{x,t} = \frac{\widetilde{P}^*_{x,t}}{P^*_{x,t}}$:

$$U_t^1 = \left(\widetilde{p}^*_{x,t}\right)^{-1-\eta_{xp}} X_t \frac{P_{x,t}}{S_t P^*_{x,t}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}} \right)^{-1-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{-\eta_{xp}} U_{t+1}^1 \tag{42}$$

$$U_t^2 = \left(\widetilde{p}^*_{x,t}\right)^{-\eta_{xp}} X_t \frac{\left(\eta_{xp}-1\right)}{\eta_{xp}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}} \right)^{-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{\eta_{xp}}{(\eta_{xp}-1)}}} \right)^{1-\eta_{xp}} U_{t+1}^2 \tag{43}$$

$$U_t^1 = U_t^2 \tag{44}$$

1.3 Government

In the competitive equilibrium of the economy, the government follows basic rules to set monetary and fiscal policy. In terms of monetary policy, a standard Taylor rule includes an autorregressive component, plus the deviations of inflation from an exogenous, autocorrelated inflation target, deviations of output from its steady-state, and changes in the real exchange rate:

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_t}{R}\right) + \\ + (1-\rho_R)\left[\alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi^o_{t+1}}\right) + \alpha_y \log\left(\frac{y_{t+1}}{y}\right) + \alpha_{rer}\log\left(\frac{rer_{t+1}}{rer_t}\right)\right] + \epsilon^R_{t+1} \quad (45)$$

$$\pi^o_{t+1} = (1-\rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi^o_t + \epsilon^{\pi^o}_{t+1} \quad (46)$$

$$\epsilon^R_t \sim N(0,\sigma_R) \qquad \epsilon^{\pi^o}_t \sim N(0,\sigma_{\pi^o})$$

The government, in order to finance its exogenous expenditures, G_t, collects distortionary taxes on consumption, labor, capital and profits income ($\tau^c_t, \tau^h_t, \tau^k_t$ and τ^ϕ_t), sells bonds domestically, $B_{g,t}$ and controls the money supply, M_t. The government budget constraint is given by:

$$P_tG_t + R_{t-1}B_{g,t} = P_tT_t + P_tM_t + B_{g,t+1} - P_{t-1}M_{t-1}$$

$$G_t = z^*_t g_t$$

$$g_t = \left(1-\rho_g\right)g + \rho_g g_{t-1} + \epsilon^g_t \qquad \epsilon^g_t \sim N(0,\sigma_g) \quad (47)$$

$$T_t = \tau^c_t C_t + \tau^h_t \widetilde{W}_t h_t + \tau^\phi_t \Phi_t \\ + \tau^k_t\left[\left(R^k_{n,t}\mu_{n,t} - \Upsilon^{-1}_t a\left(\mu_{n,t}\right)\right)\overline{K}_{n,t} + \left(R^k_{x,t}\mu_{x,t} - \Upsilon^{-1}_t a\left(\mu_{x,t}\right)\right)\overline{K}_{x,t}\right] \quad (48)$$

Following Schmitt-Grohé and Uribe (2006) [37], after defining the total real government liabilities (L_t), the evolution of government debt is pinned down by a fiscal policy rule where the government sets income taxation as a function of the gap between the actual liabilities as a proportion of GDP and its steady state value, plus a term related with the output gap, in order to account for the stabilization of the business cycle. Use the definition of net government liabilities to rewrite the budget constraint:

$$L_{t-1} \equiv M_{t-1} + \frac{R_{t-1}}{P_{t-1}}B_{g,t} \quad (49)$$

$$\Longrightarrow L_t = \frac{R_t}{\pi_t}L_{t-1} + R_t\left(G_t - T_t\right) - \left(R_t - 1\right)M_t \quad (50)$$

To close the dynamics of the fiscal block, assume that the government follows a fiscal policy rule to determine the labor income taxation, while taxes on capital and profits are exogenous. The assumption of a fiscal policy rule for labor income taxation is an arbitrary choice, since the presence of portfolio adjusment costs in domestic financial markets ensures stationarity in the model. Also, for simplicity, assume that the taxation on profits is constant over time. Notice that taxes on profits are lump sum transfers from the households to the government. In this sense, it does not interfere with the dynamics under the competitive equilibrium, where profits are zero.

$$\tau_t^h - \tau^h = \psi_1 \left(\frac{L_t}{Y_t} - \frac{l}{y} \right) + \psi_2 \left(y_t - y \right) + \epsilon_t^{\tau} \tag{51}$$

$$\tau_t^k = \left(1 - \rho_{\tau k} \right) \tau^k + \rho_{\tau k} \tau_{t-1}^k + \epsilon_t^{\tau k} \tag{52}$$

$$\tau_t^{\phi} = \tau^{\phi} \tag{53}$$

$$\tau_t^c = \left(1 - \rho_c \right) \tau^c + \rho_c \tau_{t-1}^c + \epsilon_t^{\tau c} \tag{54}$$

$$\epsilon_t^{\tau h} \sim N\left(0, \sigma_{\tau h}\right) \quad \epsilon_t^{\tau k} \sim N\left(0, \sigma_{\tau k}\right) \quad \epsilon_t^{\tau \phi} \sim N\left(0, \sigma_{\tau \phi}\right) \quad \epsilon_t^{\tau c} \sim N\left(0, \sigma_{c}\right)$$

Additionally, the government solves an equivalent problem as the households to determine their optimal consumption of tradable and non-tradable goods. By assumption, the government does not consume imported goods[4]. The demand for each type of good is given by:

$$G_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} G_t \tag{55}$$

$$G_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} G_t \tag{56}$$

$$G_t = \left[(1-\omega)^{\frac{1}{\varepsilon}} G_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} G_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

1.4 International Financial Markets and World's Economy

The transmission of shocks from international financial markets assume the existence of an international bond market capable of evaluating country-specific risk on bonds issued outside the domestic economy. In this sense, a mechanism to induce stationarity in the style proposed in Schmitt-Grohé and Uribe (2003(b)) [36] can be used to determine (and estimate) the risk premium of the bonds issued in each country as a function of the net foreign position of the economy. The international interest rate is given by:

$$R_t^f = R_t^* \left(1 + \xi_t \right)^{\kappa_1} \left(\frac{S_t IB_{t+1}}{P_t Y_t} / \frac{IB}{Y} \right)^{\kappa_2} \tag{57}$$

In this equation, R_t^* is a baseline, risk-free nominal interest rate on bonds traded in international markets; ξ_t is an autonomous shock in the risk premium, associated with the general risk level of the world's economy, with expected value equal to the long run risk premium demanded from the

---

[4] The same assumption is used in Lubik and Schorfheide (2006).

domestic economy, ξ^*; the last term is the gap between total external debt of the domestic economy and the long run level of external debt.

The world's economy is modeled by a VAR containing measures of output, y_t^*, inflation, π_t^*, interest rates, R_t^*, growth of real money supply, ΔM_t^*, and the risk premium, ξ_t. The inclusion of the risk premium tries to capture financial shocks that are not only unrelated with country-specific events, but also not associated with changes in foreign monetary policy. International shocks are identified with the Cholesky decomposition of the variance-covariance matrix of residuals. The world's output is added in order to identify supply from demand shocks in changes in the international prices. Thus, the VAR for the rest of the world will provide five shocks for the domestic economy.

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{m*} \\ \epsilon_t^{\xi} \\ \epsilon_t^{R*} \\ \epsilon_t^{\pi*} \\ \epsilon_t^{y*} \end{bmatrix} \qquad \begin{bmatrix} \epsilon_t^{m*} \\ \epsilon_t^{\xi} \\ \epsilon_t^{R*} \\ \epsilon_t^{\pi*} \\ \epsilon_t^{y*} \end{bmatrix} \overset{iid}{\sim} (0, \sum) \tag{58}$$

In the system, A is a 5 by 5 matrix of coefficients, $\sum$ is a 5 by 5 upper triangular matrix of shocks. Variables are listed from the "more endogenous" to the "more exogenous" variable.

Two assumptions closes the relation between prices and quantities of goods between the domestic country and the rest of world. First, assume that households in the rest of the world solve an expenditure minimization problem in order to set the optimal demand for home produced tradable goods. The solution of this problem is given by the demand equation:

$$X_t = \left(\frac{P_{x,t}^*}{P_t^*} \right)^{-\eta^*} z_t^* y_t^* \tag{59}$$

Finally, the terms of trade of the domestic economy are defined as the ratio between the exported goods price and the imported goods price level, both quoted in foreign currency. The dynamics of the terms of trade are given by:

$$tot_t = \frac{\pi_{x,t}^*}{\pi_{m,t}^*} tot_{t-1} \tag{60}$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = \upsilon_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X_{t-1}^* + \epsilon_t^{\pi m} \qquad \epsilon_t^{\pi m} \sim N\left(0, \sigma_{\pi m}\right) \tag{61}$$

with $X_t^* = \begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} & \frac{\xi_t}{\xi^*} & \frac{R_t^f}{R^*} & \frac{\pi_t^*}{\pi^*} & \frac{y_t^*}{y^*} \end{bmatrix}$.

1.5 Aggregation and Relative Prices

In order to find an expression for the aggregate constraint of the economy, start from the demand faced by a non-tradable producer firm and integrate both sides over all the i_n firms, noting that $h_{n,t} = \int_0^1 h_{n,t}\left(i_n\right) di_n$ and that the capital-labor ratio is constant across all the firms:

$$a_{n,t} K_{n,t}^{\theta} \left(z_t h_{n,t}\right)^{1-\theta} - z_t^* \chi_n = \int_0^1 \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}}\right)^{-\eta_n} di_{\not\eta} \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t}\right)$$

Define $s_{n,t} = \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}}\right)^{-\eta_{n,t}} di_n$ to obtain:

$$a_{n,t} K_{n,t}^{\theta} \left(z_t h_{n,t}\right)^{1-\theta} - z_t^* \chi_n = s_{n,t} \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t}\right) \tag{62}$$

Obtain the recursive form of $s_{n,t}$:

$$\begin{aligned}
s_{n,t} &= \int_0^1 \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}}\right)^{-\eta_n} di \\
s_{n,t} &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \int_0^1 \left(\frac{\widetilde{P}_{n,t-1}\left(i_n\right) \pi_{n,t-1}^{\kappa_n}}{P_{n,t}}\right)^{-\eta_n} di \\
s_{n,t} &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \int_0^1 \left(\frac{\widetilde{P}_{n,t-1}\left(i_n\right) \pi_{n,t-1}^{\kappa_n}}{P_{n,t}}\right)^{-\eta_n} di \\
s_{n,t} &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(1-\alpha_n\right) \left(\frac{\widetilde{P}_{n,t-1} \pi_{n,t-1}^{\kappa_n}}{P_{n,t}}\right)^{-\eta_n} + \alpha_n^2 \left(1-\alpha_n\right) \left(\frac{\widetilde{P}_{n,t-2} \pi_{n,t-1}^{\kappa_n} \pi_{n,t-2}^{\kappa_n}}{P_{n,t}}\right)^{-\eta_n} + \ldots \\
s_{n,t} &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \sum_{j=1}^{\infty} \alpha_n^j \left(\frac{\widetilde{P}_{n,t-j} \prod_{s=1}^{j} \pi_{n,t-j-1+s}^{\kappa_n}}{P_{n,t}}\right)^{-\eta_n} \\
\Longrightarrow s_{n,t} &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}}\right)^{\eta_n} s_{n,t-1} \qquad (63)
\end{aligned}$$

Also, from the definition of the non-tradable goods price index:

$$\begin{aligned}
P_{n,t} &= \left[\int_0^1 P_{n,t}\left(i_n\right)^{1-\eta_n} di\right]^{\frac{1}{1-\eta_n}} \\
P_{n,t}^{1-\eta_n} &= \left(1-\alpha_n\right) \widetilde{P}_{n,t}^{1-\eta_n} + \alpha_n \left(P_{n,t-1} \pi_{n,t-1}^{\kappa_n}\right)^{1-\eta_n} \\
1 &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{P_{n,t-1} \pi_{n,t-1}^{\kappa_n}}{P_{n,t}}\right)^{1-\eta_n} \\
\Longrightarrow 1 &= \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}}\right)^{1-\eta_n} \qquad (64)
\end{aligned}$$

Equivalent expressions can be written for the resource constraint, price dispersion and the price index of imported and domestically produced tradable goods and the price index of exported goods in foreign currency:

$$D_{m,t} - z_t^* \chi_m = s_{m,t} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \tag{65}$$

$$s_{m,t} = (1-\alpha_m)\, \widetilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1} \tag{66}$$

$$1 = (1-\alpha_m)\, \widetilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m} \tag{67}$$

$$a_{x,t} K_{x,t}^{\theta} \left(z_t h_{x,t} \right)^{1-\theta} - z_t^* \chi_x = s_{x,t} \left(C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \frac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t} \right) \tag{68}$$

$$s_{x,t} = (1-\alpha_x)\, \widetilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1} \tag{69}$$

$$1 = (1-\alpha_x)\, \widetilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x} \tag{70}$$

$$D_{xp,t} - z_t^* \chi_{xp} = s_{xp,t} X_t \tag{71}$$

$$s_{xp,t} = (1-\alpha_{xp}) \left(\widetilde{p}_{x,t}^* \right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{\left(\pi_{xp,t-1}^* \right)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1} \tag{72}$$

$$1 = (1-\alpha_{xp})\, \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{\left(\pi_{x,t-1}^* \right)^{\kappa_{xp}}}{\pi_{x,t}^*} \right)^{1-\eta_{xp}} \tag{73}$$

From the aggregation condition of the labor market, the total amount of work hours supplied by the domestic households is given by:

$$h_{x,t} + h_{n,t} = h_t \tag{74}$$

The external equilibrium assumes that the net foreign position of domestic households is proportional to the average trade balance result in steady state. Again, notice that the external equilibrium in the bond markets do not include the bonds issued by imported goods's firms, as they are negotiated and liquidated at the beginning and the end of each period. The description of the net foreign position in terms of domestic currency is given by:

$$P_{x,t} X_t - P_{m,t} D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] = S_t R_{t-1}^f P_t^* IB_t - S_t P_{t+1}^* IB_{t+1} \tag{75}$$

It's also necessary to determine the market clearing conditions for domestic bonds and money market. For simplicity, assume that foreign households and domestic firms do not demand home government bonds. As a consequence:

$$B_{g,t} + B_{h,t} = 0 \tag{76}$$

Finally, the gross domestic product is defined as:

$$\begin{aligned} Y_t = C_t + \frac{\psi_1}{2} Y_t \left(\frac{B_{t+1}}{Y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y} \right)^2 \\ + \Upsilon_t^{-1} I_t + G_t + \frac{P_{x,t}}{P_t} X_t - \frac{P_{m,t}}{P_t} D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] \end{aligned} \tag{77}$$

Aggregate profits are given by:

$$\Phi_t = Y_t - \widetilde{W}_t h_t - R^k_{n,t} \mu_{n,t} \overline{K}_{n,t} - R^k_{x,t} \mu_{x,t} \overline{K}_{x,t} \tag{78}$$

1.5.1 Relative prices

The model includes a set of relative prices that are strictly related to some observables of the economy. In terms of dynamics, the set of relative prices in the model are given by:

$$pt_t = \frac{P_{t,t}}{P_t} = \frac{\pi_{t,t}}{\pi_t} \frac{P_{t,t-1}}{P_{t-1}} \tag{79}$$

$$pn_t = \frac{P_{n,t}}{P_t} = \frac{\pi_{n,t}}{\pi_t} \frac{P_{n,t-1}}{P_{t-1}} \tag{80}$$

$$px_t = \frac{P_{x,t}}{P_{t,t}} = \frac{\pi_{x,t}}{\pi_{t,t}} \frac{P_{x,t-1}}{P_{t,t-1}} \tag{81}$$

$$pm_t = \frac{P_{m,t}}{P_{t,t}} = \frac{\pi_{m,t}}{\pi_{t,t}} \frac{P_{m,t-1}}{P_{t,t-1}} \tag{82}$$

$$pm^*_t = \frac{P^*_{m,t}}{P^*_t} = \frac{\pi^*_{m,t}}{\pi^*_t} \frac{P^*_{m,t-1}}{P^*_{t-1}} \tag{83}$$

$$rer_t = \frac{S_t P^*_t}{P_t} \tag{84}$$

2 Stationary Form and Equilibrium

The objective of this section is to describe the equilibrium conditions with the necessary adjustments to induce stationarity and characterize the competitive and Ramsey Equilibria. Define the stationary allocations with small letters, such that, for a generic variable X_t and the appropriate trend $\check{Z}_t$, the stationary variable is given by $x_t \equiv X_t/\check{Z}_t$. The model in stationary form is fully described by the stochastic processes for the following sets of variables:

- prices: $\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r^k_{x,t}, r^k_{n,t}, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi^*_t, \pi^*_{x,t}, \pi^{m*}_t, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}^*_{x,t}, pt_t, pn_t, px_t, pm_t, pm^*_t, tot_t$;
- interest rates: $R_t, \widetilde{R}_t, R^*_t, R^f_t$;

- allocations: $c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i^d_{x,t}, i^d_{n,t}, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x^1_t, x^2_t, z^1_t, z^2_t, y^1_t, y^2_t, u^1_t, u^2_t, ib_t, b_{h,t}, \xi_t, \Delta M^*_t, y^*_t, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, g_{x,t}, m_t, \phi_t$;
- government policies: $\tau^h_t, l_t, t_t, b_{g,t}$;
- domestic shocks: $g_t, \tau^k_t, \tau^\phi_t, \tau^c_t, a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t, \pi^o_t$.

The equations describing the law of motion of the variables are given by a set of equilibrium conditions for the household (equations 1-28), firms responsible for domestic production (equations 29-38), exporting and importing firms (equations 39-44), government (equations 45-56), foreign sector (equations 57-61), aggregation and price indexes (equations 62-78) and relative prices (equations 79-84). Additionally, there are 4 exogenous processes for sectoral productivity and aggregate productivity growth $(a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t)$. As a consequence, there are 84 equations for endogenous variables[5] and 9 domestic exogenous stochastic processes for a total of 93 variables in the model.

The prices and the shocks are stationary, but the allocations must be normalized in order to ensure stationarity. The set of variables given by $\{\overline{K}_{n,t+1}, K_{n,t+1}, \overline{K}_{x,t+1}, K_{x,t+1}, I_t, I_{t,t}, I_{n,t}, I_{m,t}, I_{x,t}, I^d_{x,t}, I^d_{n,t}\}$ must be normalized by $z^*_t \Upsilon_t$, while the variables $\{Y_t, C_t, C_{t,t}, C_{n,t}, C_{m,t}, C_{x,t}, W_t, X_t, D_{m,t}, D_{xp,t}, B_{h,t+1}, B_{g,t+1}, IB_{t+1}, M_t, X^1_t, X^2_t, Z^1_t, Z^2_t, Y^1_t, Y^2_t, U^1_t, U^2_t, G_t, G_{t,t}, G_{n,t}, G_{x,t}, L_t, T_t\}$ must be adjusted by z^*_t. Finally, the prices in each sector for renting capital from households $\{R^k_{x,t}, R^k_{n,t}\}$ and the shadow prices of investment $\{\widetilde{q}_{x,t}, \widetilde{q}_{n,t}\}$ are divided by Υ^{-1}_t, while the Lagrange multiplier of consumption, $\widetilde{\lambda}_t$, is normalized by $(z^*_t)^{-1}$ to obtain λ_t.

2.1 Competitive Equilibrium

Definition 1 *Given exogenous paths for shocks* $\left\{g_t, \tau^k_t, \tau^\phi_t, \tau^c_t, a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t, \pi^o_t\right\}$*, foreign sector variables* $\left\{\Delta M^*_t, \xi_t, R^*_t, \pi^*_t, y^*_t, \pi^*_{m,t}\right\}$*, policy processes for interest rates* $\left\{R_t, \widetilde{R}_t, R^f_t\right\}$ *and taxes* τ^h_t*, and initial values for prices* $\left\{\pi_{-1}, \pi_{n,-1}, \pi_{x,-1}, \pi_{t,-1}, \pi_{m,-1}, w_{-1}, pt_{-1}, pn_{-1}, px_{-1}, pm_{-1}, pm^*_{-1}, tot_{-1}\right\}$ *and allocations* $\left\{c_{-1}, i^d_{x,-1}, i^d_{n,-1}, \overline{k}_{x,0}, \overline{k}_{n,0}, b_{h,-1}, b_{g,-1}, ib_{-1}, s_{n,-1}, s_{m,-1}, s_{x,-1}, s_{xp,-1}, l_{-1}\right\}$*, a stationary competitive equilibrium is a set of processes for prices* $\{\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r^k_{x,t}, r^k_{n,t}, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi^*_{x,t}, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}^*_{x,t}, pt_t, pn_t, px_t, pm_t, pm^*_t, tot_t\}$ *and allocations* $\{c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i^d_{x,t}, i^d_{n,t}, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x^1_t, x^2_t, z^1_t, z^2_t, y^1_t, y^2_t, u^1_t, u^2_t, ib_t, b_{h,t}, b_{g,t}, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, m_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, g_{x,t}, t_t, l_t, \ \phi_t\}$ *such that, after stationary transformations of the respective equations: a) Households maximize utility; b) Firms maximize profits; c) Government balances its budget; d) Markets clear.*

---

[5] Note that equation 58 is a 5-variable VAR.

2.2 Ramsey Equilibrium

The Ramsey equilibrium is evaluated by the "timeless perspective" described in Woodford (2003) [45], where the government is assumed to run the policy committed for a very long time. In this sense, any dynamics resulting from the initial state of the economy is eliminated, and the economy fluctuates around its steady-state.

Definition 2 *Given exogenous paths for shocks* $\left\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o\right\}$ *and foreign sector variables* $\left\{\Delta M_t^*, \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\right\}$, *previously defined, and a set of initial values for Lagrange multipliers, a Ramsey equilibrium is a set of processes for prices and allocations that maximize*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma) \log \left(C_t(i) - \zeta C_{t-1}\right) + \gamma \log \left(1 - h_t(i)\right)\right]$$

subject to the equilibrium conditions of the competitive equilibrium and $R_t \geq 1$.

2.3 Stationary Equilibrium Conditions

In order to transform the model for the stationary form, first note that:

$$z_t^* = z_t \Upsilon_t^{\frac{\theta}{1-\theta}} \Longrightarrow \frac{z_t^*}{z_{t-1}^*} = \frac{z_t}{z_{t-1}} \left(\frac{\Upsilon_t}{\Upsilon_{t-1}}\right)^{\frac{\theta}{1-\theta}} = \mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}$$

Also:

$$\frac{z_t^* \Upsilon_t}{z_{t-1}^* \Upsilon_{t-1}} = \left(\frac{z_t^*}{z_{t-1}^*}\right) \mu_t^\Upsilon = \left(\frac{z_t \Upsilon_t^{\frac{\theta}{1-\theta}}}{z_{t-1} \Upsilon_{t-1}^{\frac{\theta}{1-\theta}}}\right) \mu_t^\Upsilon = \mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}$$

The stationary equilibrium conditions of the model are:

$$\frac{\left(1-\tau_t^h\right)}{\left(1+\tau_t^c\right)} w_t = \frac{\gamma}{(1-\gamma)} \left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) \frac{mcw_t \left(1 + \nu^m \left(\frac{\widetilde{R}_t - 1}{\widetilde{R}_t}\right)\right)}{\left(1 - h_t\right)}$$

$$\left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)^{-1} (1-\gamma) = \left(1+\tau_t^c\right) \lambda_t \left(1 + \nu^m \frac{R_t - 1}{R_t}\right)$$

$$\lambda_t \left[1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y}\right)\right] = \beta R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1} \left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)}\right)$$

$$\lambda_t \left[1 - \psi_2 \left(\frac{rer_t\ ib_{t+1}}{y_t} - \frac{rer\ ib}{y}\right)\right] = \beta R_t^f E_t \left(\frac{rer_{t+1}}{rer_t} \frac{\lambda_{t+1}}{\pi_{t+1}^* \left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)}\right)$$

$$\lambda_t q_{x,t} = \beta E_t \left\{ \frac{\left(\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}}{\mu^\Upsilon_{t+1}} \lambda_{t+1}\left[\left(1-\tau^k_{t+1}\right)\left(r^k_{x,t+1}\mu_{x,t+1} - a\left(\mu_{x,t+1}\right)\right) + q_{x,t+1}\left(1-\delta\right)\right]\right\}$$

$$\lambda_t q_{n,t} = \beta E_t \left\{ \frac{\left(\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}}{\mu^\Upsilon_{t+1}} \lambda_{t+1}\left[\left(1-\tau^k_{t+1}\right)\left(r^k_{n,t+1}\mu_{n,t+1} - a\left(\mu_{n,t+1}\right)\right) + q_{n,t+1}\left(1-\delta\right)\right]\right\}$$

$$k_{x,t} = \mu_{x,t}\overline{k}_{x,t}$$

$$k_{n,t} = \mu_{n,t}\overline{k}_{n,t}$$

$$\theta_1 + \theta_2\left(\mu_{x,t}-1\right) = r^k_{x,t}$$

$$\theta_1 + \theta_2\left(\mu_{n,t}-1\right) = r^k_{n,t}$$

$$R_t = \frac{1}{r_{t,t+1}}$$

$$\widetilde{R}_t = R_t\left(1-\psi_1\left(\frac{b_{h,t+1}}{y_t}-\frac{b_h}{y}\right)\right)^{-1}$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon^n_{t+1}$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon^x_{t+1}$$

$$\begin{aligned}\lambda_t = {} & \lambda_t q_{x,t}\left[1-\frac{\phi_i}{2}\left(\frac{i^d_{x,t}}{i^d_{x,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)^2 - \phi_i\left(\frac{i^d_{x,t}}{i^d_{x,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}\right)\left(\frac{i^d_{x,t}}{i^d_{x,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)\right] \\ & +\beta E_t\left[\lambda_{t+1}\left(\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}\frac{q_{x,t+1}}{\mu^\Upsilon_{t+1}}\phi_i\left(\frac{i^d_{x,t+1}}{i^d_{x,t}}\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}}\right)^2\left(\frac{i^d_{x,t+1}}{i^d_{x,t}}\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}}-\mu^V\right)\right]\end{aligned}$$

$$\begin{aligned}\lambda_t = {} & \lambda_t q_{n,t}\left[1-\frac{\phi_i}{2}\left(\frac{i^d_{n,t}}{i^d_{n,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)^2 - \phi_i\left(\frac{i^d_{n,t}}{i^d_{n,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}\right)\left(\frac{i^d_{n,t}}{i^d_{n,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)\right] \\ & +\beta E_t\left[\lambda_{t+1}\left(\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}\frac{q_{n,t+1}}{\mu^\Upsilon_{t+1}}\phi_i\left(\frac{i^d_{n,t+1}}{i^d_{n,t}}\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}}\right)^2\left(\frac{i^d_{n,t+1}}{i^d_{n,t}}\mu^z_{t+1}\left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}}-\mu^V\right)\right]\end{aligned}$$

$$\begin{aligned}\overline{k}_{x,t+1} &= \left(1-\delta\right)\frac{\overline{k}_{x,t}}{\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}} + i^d_{x,t}\left(1-\frac{\phi_i}{2}\left(\frac{i^d_{x,t}}{i^d_{x,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)^2\right) \\ \overline{k}_{n,t+1} &= \left(1-\delta\right)\frac{\overline{k}_{n,t}}{\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}} + i^d_{n,t}\left(1-\frac{\phi_i}{2}\left(\frac{i^d_{n,t}}{i^d_{n,t-1}}\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}}-\mu^V\right)^2\right)\end{aligned}$$

$$\left(\frac{\varpi-1}{\varpi}+\frac{1}{mcw_t}\right)\varpi h_t\left(1-\tau_t^h\right)=-\frac{\phi_w}{\pi_t^{\chi_w-1}}\left(\frac{w_t}{w_{t-1}}\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)\left(\frac{w_t}{\pi_t^{\chi_w-1}w_{t-1}}\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}-\mu^I\right)$$
$$+\beta E_t\left[\frac{\lambda_{t+1}\phi_w}{\lambda_t\pi_{t+1}^{\chi_w-1}}\left(\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)^{-1}\left(\frac{w_{t+1}}{w_t}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)^2\left(\frac{w_{t+1}}{\pi_{t+1}^{\chi_w-1}w_t}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}-\mu^I\right)\right]$$

$$c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{b_h}{y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2=\left[(1-\omega)^{\frac{1}{\varepsilon}}c_{n,t}^{\frac{\varepsilon-1}{\varepsilon}}+\omega^{\frac{1}{\varepsilon}}c_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$
$$c_{t,t}=\left[(1-\varkappa)^{\frac{1}{\varrho}}c_{x,t}^{\frac{\varrho-1}{\varrho}}+\varkappa^{\frac{1}{\varrho}}c_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}}$$
$$c_{m,t}=\varkappa\left(pm_t\ pt_t\right)^{-\varrho}c_{t,t}$$
$$c_{x,t}=(1-\varkappa)\left(px_t\ pt_t\right)^{-\varrho}c_{t,t}$$
$$c_{t,t}=\omega\left(pt_t\right)^{-\varepsilon}\left(c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{B}{Y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2\right)$$
$$c_{n,t}=(1-\omega)\left(pn_t\right)^{-\varepsilon}\left(c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{B}{Y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2\right)$$
$$i_t=i_{n,t}^d+a\left(\mu_{n,t}\right)\frac{\overline{k}_{n,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}}+i_{x,t}^d+a\left(\mu_{x,t}\right)\frac{\overline{k}_{x,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}}$$
$$i_{m,t}=\varkappa\left(pm_t\ pt_t\right)^{-\varrho}i_{t,t}$$
$$i_{x,t}=(1-\varkappa)\left(px_t\ pt_t\right)^{-\varrho}i_{t,t}$$
$$i_{t,t}=\omega\left(pt_t\right)^{-\varepsilon}i_t$$
$$i_{n,t}=(1-\omega)\left(pn_t\right)^{-\varepsilon}i_t$$
$$w_t=pn_tmc_{n,t}(1-\theta)a_{n,t}\left(\frac{k_{n,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}h_{n,t}}\right)^\theta$$
$$r_{n,t}^k=pn_tmc_{n,t}\theta a_{n,t}\left(\frac{k_{n,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}h_{n,t}}\right)^{\theta-1}$$
$$w_t=px_tmc_{x,t}(1-\theta)a_{x,t}\left(\frac{k_{x,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}h_{x,t}}\right)^\theta$$
$$r_{x,t}^k=px_tmc_{x,t}\theta a_{x,t}\left(\frac{k_{x,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}h_{x,t}}\right)^{\theta-1}$$

$$\begin{aligned}x_t^1 = \widetilde{p}_{n,t}^{-1-\eta_n}\left(c_{n,t}+g_{n,t}+\frac{i_{n,t}}{pn_t}\right)mc_{n,t}\\ + E_t\alpha_n r_{t,t+1}\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}x_{t+1}^1\end{aligned}$$

$$\begin{aligned}x_t^2 = \widetilde{p}_{n,t}^{-\eta_n}\left(c_{n,t}+g_{n,t}+\frac{i_{n,t}}{pn_t}\right)\frac{(\eta_n-1)}{\eta_n}\\ + E_t\alpha_n r_{t,t+1}\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}x_{t+1}^2\end{aligned}$$

$$\begin{aligned}z_t^1 = \widetilde{p}_{x,t}^{-1-\eta_x}\left(c_{x,t}+g_{t,t}+\frac{pt_t}{px_t}i_{x,t}+d_{xp,t}\right)mc_{x,t}\\ + E_t\alpha_x r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}}\right)^{-\eta_x}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}z_{t+1}^1\end{aligned}$$

$$\begin{aligned}z_t^2 = \widetilde{p}_{x,t}^{-\eta_x}\left(c_{x,t}+g_{t,t}+\frac{pt_t}{px_t}i_{x,t}+d_{xp,t}\right)\frac{(\eta_x-1)}{\eta_x}\\ + E_t\alpha_x r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}}\right)^{1-\eta_x}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}z_{t+1}^2\end{aligned}$$

$$\begin{aligned}y_t^1 = \widetilde{p}_{m,t}^{-1-\eta_m}\left(c_{m,t}+i_{m,t}\frac{pt_t}{pm_t}\right)rer_t\frac{pt_t\;pm_t^*}{pm_t}\left(1+\frac{R_t^f-1}{R_t^f}\right)\\ + E_t\alpha_m r_{t,t+1}\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-1-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}}\right)^{-\eta_m}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}y_{t+1}^1\end{aligned}$$

$$\begin{aligned}y_t^2 = \widetilde{p}_{m,t}^{-\eta_m}\left(c_{m,t}+i_{m,t}\frac{pt_t}{pm_t}\right)\frac{(\eta_m-1)}{\eta_m}\\ + E_t\alpha_m r_{t,t+1}\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}}\right)^{1-\eta_m}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}y_{t+1}^2\end{aligned}$$

$$\begin{aligned}u_t^1 = \left(\widetilde{p}_{x,t}^*\right)^{-1-\eta_{xp}}x_t\frac{px_t\;pt_t}{rer_t\;pm_t^*\;tot_t}\\ + E_t\alpha_{xp} r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*}\right)^{-1-\eta_{xp}}\left(\frac{\left(\pi_{x,t}^*\right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^*\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}}\right)^{-\eta_{xp}}\mu_{t+1}^{z}\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}u_{t+1}^1\end{aligned}$$

$$u_t^2 = \left(\widetilde{p}_{x,t}^*\right)^{-\eta_{xp}} x_t \frac{\left(\eta_{xp}-1\right)}{\eta_{xp}} + E_t \alpha_{xp} r_{t,t+1} \left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*}\right)^{-\eta_{xp}} \left(\frac{\left(\pi_{x,t}^*\right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^*\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}}\right)^{1-\eta_{xp}} \mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} u_{t+1}^2$$

$$x_t^1 = x_t^2$$

$$z_t^1 = z_t^2$$

$$y_t^1 = y_t^2$$

$$u_t^1 = u_t^2$$

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_{t+1}}{R}\right) + \left(1-\rho_R\right)\alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi_{t+1}^o}\right) + \alpha_y \log\left(\frac{y_{t+1}}{y}\right) + \alpha_{rer} \log\left(\frac{rer_{t+1}}{rer_t}\right) + \epsilon_{t+1}^R$$

$$\pi_{t+1}^o = (1-\rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi_t^o + \epsilon_{t+1}^{\pi^o}$$

$$t_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k \left[\left(r_{n,t}^k \mu_{n,t} - a\left(\mu_{n,t}\right)\right)\overline{k}_{n,t} + \left(r_{x,t}^k \mu_{x,t} - a\left(\mu_{x,t}\right)\right)\overline{k}_{x,t}\right] + \tau_t^\phi \phi_t$$

$$g_t = \left(1-\rho_g\right) g + \rho_g g_{t-1} + \epsilon_t^g$$

$$l_t = m_t + R_t b_{g,t+1}$$

$$l_t = \frac{R_t}{\pi_t} \frac{l_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} + R_t\left(g_t - t_t\right) - \left(R_t - 1\right) m_t$$

$$\tau_t^h - \tau^h = \psi_1 \left(\frac{l_t}{y_t} - \frac{l}{y}\right) + \psi_2 \left(y_t - y\right) + \epsilon_t^\tau$$

$$\tau_t^k = \left(1-\rho_{\tau k}\right)\tau^k + \rho_{\tau k}\tau_{t-1}^k + \epsilon_t^{\tau k}$$

$$\tau_t^\phi = \tau^\phi$$

$$\tau_t^c = \left(1-\rho_{\tau c}\right)\tau^c + \rho_{\tau c}\tau_{t-1}^c + \epsilon_t^{\tau c}$$

$$g_{n,t} = \left(1-\omega\right)\left(pn_t\right)^{-\varepsilon} g_t$$

$$g_{t,t} = \omega \left(pt_t\right)^{-\varepsilon} g_t$$

$$R_t^f = R_t^* \left(1+\xi_t\right)^{\kappa_1} \left(\frac{rer_t ib_t}{y_t} / \frac{ib}{y}\right)^{\kappa_2}$$

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m*} \\ \varepsilon_t^\xi \\ \varepsilon_t^{R*} \\ \varepsilon_t^{\pi*} \\ \varepsilon_t^{y*} \end{bmatrix}$$

$$x_t = \left(pm_t^* \, tot_t\right)^{-\eta*} y_t^*$$

$$tot_t = \frac{\pi_{x,t}^*}{\pi_t^*} tot_{t-1}$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = \upsilon_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X_{t-1} + \varepsilon_t^{\pi m}$$

$$a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_{n,t}^{1-\theta} - \chi_n = s_{n,t} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right)$$

$$s_{n,t} = (1-\alpha_n) \widetilde{p_{n,t}^{-\eta_n}} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}} \right)^{\eta_n} s_{n,t-1}$$

$$1 = (1-\alpha_n) \widetilde{p_{n,t}^{1-\eta_n}} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}} \right)^{1-\eta_n}$$

$$d_{m,t} - \chi_m = s_{m,t} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right)$$

$$s_{m,t} = (1-\alpha_m) \widetilde{p_{m,t}^{-\eta_m}} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1}$$

$$1 = (1-\alpha_m) \widetilde{p_{m,t}^{1-\eta_m}} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m}$$

$$a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_{x,t}^{1-\theta} - \chi_x = s_{x,t} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right)$$

$$s_{x,t} = (1-\alpha_x) \widetilde{p_{x,t}^{-\eta_x}} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1}$$

$$1 = (1-\alpha_x) \widetilde{p_{x,t}^{1-\eta_x}} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x}$$

$$d_{xp,t} - \chi_{xp} = s_{xp,t} x_t$$

$$s_{xp,t} = (1-\alpha_{xp}) \left(\widetilde{p}_{x,t}^*\right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{\left(\pi_{xp,t-1}^*\right)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1}$$

$$1 = (1-\alpha_{xp}) \widetilde{p_{xp,t}^{1-\eta_{xp}}} + \alpha_{xp} \left(\frac{\left(\pi_{x,t-1}^*\right)^{\kappa_{xp}}}{\pi_{x,t}^*} \right)^{1-\eta_{xp}}$$

$$h_{x,t} + h_{n,t} = h_t$$

$$b_{g,t} + b_{h,t} = 0$$

$$px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] = rer_t R_{t-1}^f \frac{ib_t}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - rer_t \pi_{t+1}^* ib_{t+1}$$

$$\begin{aligned} y_t = c_t + g_t + i_t + px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] \\ + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t\ ib_t}{y_t} - \frac{rer\ ib}{y} \right)^2 \end{aligned}$$

$$\phi_t = y_t - w_t h_t - r^k_{n,t} \mu_{n,t} \overline{k}_{n,t} - r^k_{x,t} \mu_{x,t} \overline{k}_{x,t}$$

$$m_t = \nu^m \left(1 + \tau^c_t\right) c_t$$

$$pt_t = \frac{\pi_{t,t}}{\pi_t} pt_{t-1}$$

$$pn_t = \frac{\pi_{n,t}}{\pi_t} pn_{t-1}$$

$$px_t = \frac{\pi_{x,t}}{\pi_{t,t}} px_{t-1}$$

$$pm_t = \frac{\pi_{m,t}}{\pi_{t,t}} pm_{t-1}$$

$$pm^*_t = \frac{\pi^*_{m,t}}{\pi^*_t} pm^*_{t-1}$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu^{\Upsilon}_{t+1} = \left(1 - \rho_{\Upsilon}\right) \mu^{\Upsilon} + \rho_{\Upsilon} \mu^{\Upsilon}_t + \epsilon^{\Upsilon}_{t+1}$$

$$\frac{z_{t+1}}{z_t} = \mu^z_{t+1} = \left(1 - \rho_z\right) \mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}$$

3 Steady State Conditions: Competitive Equilibrium

This section describes the sequence of equations necessary to compute the steady state of the competitive equilibrium of the assuming that the values related to income taxation are known. The taxation on consumption is obtained using the government budget constraint, assuming that the steady state level of debt-output ratio is known. Given steady state values for taxes $\tau^h, \tau^k, \tau^\phi$, parameter values for $\beta, \theta, \delta, \omega, \varkappa, \mu^z, \mu^\Upsilon, \eta_x, \eta_n, \eta_m, \eta_{xp}, \varpi, \kappa_1, \alpha_x, \alpha_m, \alpha_{xp}, \alpha_n$, and steady state values for $h, R^*/R^f, tb/y, \tau/y, \pi^o, \pi^*, g/y, b/y, m/y, imp/y$ and the share of non-tradable goods in the output, there are 86 variables and 9 parameters to be computed in the steady state of the competitive equilibrium. The set of variables is given by: $\{\pi, \pi_n, \pi_m, \pi_t, \pi_x, \pi_m^*, \pi_x^*, a_n, a_x, q_n, q_x, \mu_x, \mu_n, mcw_t, \Delta M^*, pt, pn, px, pm, pm^*, R, r, \widetilde{R}, R^f, \xi, R^*, \widetilde{p}_x, s_x, \widetilde{p}_m, s_m, \widetilde{p}_x^*, s_{xp}, \widetilde{p}_n, s_n, mc_n, mc_x, rer, r_n^k, r_x^k, h_x, h_n, \overline{k}_x, \overline{k}_n, k_x, k_n, i_x^d, i_n^d, i, g, g_t, g_n, c, c_t, c_n, c_x, c_m, i_n, i_t, i_x, i_m, ib, x, b_g, m, l, d_m, w, d_{xp}, \tau^c, tot, y^*, x^1, x^2, y^1, y^2, z^1, z^2, u^1, u^2, \lambda, y, \phi\}$. The set of parameters is given by: $\left\{\theta_1, \theta_2, \nu^m, \chi_n, \chi_x, \chi_m, \chi_{xp}, \gamma\right\}$.

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \pi^o$$

$$\pi_m^* = \pi_x^* = \Delta M^* = \pi^*$$

$$a_n = a_x = 1$$

$$q_n = q_x = 1$$

$$\mu_x = \mu_n = 1$$

$$mcw_t = \frac{\varpi}{\varpi - 1}$$

$$pt = 1 \qquad pn = 1 \qquad px = 1 \qquad pm = 1 \qquad pm^* = 1$$

$$R = \tfrac{\pi}{\beta}\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}} \qquad r = \tfrac{1}{R} \qquad \widetilde{R} = R^f = R \qquad \xi = \left(\tfrac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad R^* = \left(\tfrac{R^*}{R^f}\right) R^f$$

$$\widetilde{p}_x = \left(\tfrac{1-\alpha_x \pi_x^{(\kappa_x - 1)(1-\eta_x)}}{1-\alpha_x}\right)^{\frac{1}{1-\eta_x}} \qquad s_x = \tfrac{(1-\alpha_x)\widetilde{p}_x^{-\eta_x}}{1-\alpha_x \pi_x^{\eta_x(1-\kappa_x)}}$$

$$\widetilde{p}_m = \left(\tfrac{1-\alpha_m \pi_m^{(\kappa_m-1)(1-\eta_m)}}{1-\alpha_m}\right)^{\frac{1}{1-\eta_m}} \qquad s_m = \tfrac{(1-\alpha_m)\widetilde{p}_m^{-\eta_m}}{1-\alpha_m \pi_m^{\eta_m(1-\kappa_m)}}$$

$$\widetilde{p}_x^* = \left(\tfrac{1-\alpha_{xp}(\pi_x^*)^{(\kappa_{xp}-1)(1-\eta_{xp})}}{1-\alpha_{xp}}\right)^{\frac{1}{1-\eta_{xp}}} \qquad s_{xp} = \tfrac{(1-\alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1-\alpha_{xp}(\pi_x^*)^{\eta_{xp}(1-\kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\tfrac{1-\alpha_n \pi_n^{(\kappa_n-1)(1-\eta_n)}}{1-\alpha_n}\right)^{\frac{1}{1-\eta_n}} \qquad s_n = \tfrac{(1-\alpha_n)\widetilde{p}_n^{-\eta_n}}{1-\alpha_n \pi_n^{\eta_n(1-\kappa_n)}}$$

$$mc_n = \widetilde{p}_n \frac{1-\alpha_n\ r\ \pi_n^{-\eta_n\left(\kappa_n-\frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n-\frac{\eta_n}{(\eta_n-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n-1)}{\eta_n}$$

$$mc_x = \widetilde{p}_x \frac{1-\alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x-\frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x-\frac{\eta_x}{(\eta_x-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x-1)}{\eta_x}$$

$$rer = \widetilde{p}_m \left(1+\frac{R^f-1}{R^f}\right)^{-1} \frac{pm}{pt\ pm^*} \frac{1-\alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m-\frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m-\frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m-1)}{\eta_m}$$

$$r_n^k = \left(1-\tau^k\right)^{-1} \left[\beta^{-1}\mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$r_x^k = \left(1-\tau^k\right)^{-1} \left[\beta^{-1}\mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$\frac{k_x}{h_x} = \mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} \left(\frac{r_x^k}{mc_x\theta}\right)^{\frac{1}{\theta-1}}$$

$$\tfrac{h_x}{h_n} = \tfrac{mc_x Y_x}{mc_n Y_n} \qquad h = 0.2 \Longrightarrow h_n = h\left(1+\tfrac{mc_x Y_x}{mc_n Y_n}\right)^{-1} \qquad h_x = \tfrac{Y_x}{Y_n}\tfrac{mc_x}{mc_n}h_n$$

$$k_x = \overline{k}_x = \tfrac{k_x}{h_x}h_x \qquad k_n = \overline{k}_n = h_n \tfrac{k_x}{h_x}\left(\tfrac{mc_x}{mc_n}\right)^{\frac{1}{\theta}}$$

$$i_x^d = \left(1-\tfrac{(1-\delta)}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)\overline{k}_x \qquad i_n^d = \left(1-\tfrac{(1-\delta)}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)\overline{k}_n$$

$$i = i_x^d + i_n^d \qquad \theta_1 = r_x^k \qquad \theta_2 = \theta_1 \tfrac{\theta_2}{\theta_1}$$

$$w = mc_n \left(1-\theta\right) \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{\theta-1}} \frac{k_n \mu_n}{h_n}\right)^\theta$$

$$g = \frac{g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$g_n = \left(1-\omega\right) g \qquad g_t = \omega g$$

$$c = \left(1 - \frac{tb}{y}\right) \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) - g - i$$

$$c_n = \left(1-\omega\right) c \qquad c_t = \omega c \qquad c_x = \left(1-\varkappa\right) c_t \qquad c_m = \varkappa c_t$$

$$i_n = \left(1-\omega\right) i \qquad i_t = \omega i \qquad i_x = \left(1-\varkappa\right) i_t \qquad i_m = \varkappa i_t$$

$$\frac{ib}{y} = \frac{tb}{y} \left[rer \left(\frac{R^f}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - \pi^*\right)\right]^{-1}$$

$$ib = \frac{ib}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$x = \frac{tb}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) + d_m \left(1 + \frac{R^f - 1}{R^f}\right)$$

$$m = \frac{m}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$b_g = \frac{b_g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$\frac{l}{y} = \frac{m}{y} + R\frac{b_g}{y}$$

$$l = \frac{l}{y}\left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$d_m = s_m \left(c_m + i_m \tfrac{pt}{pm}\right) \Longrightarrow \chi_m = 0 \qquad d_{xp} = s_{xp} x \Longrightarrow \chi_{xp} = 0$$

$$d_{xp} = s_{xp} x$$

$$\tau^c = \left\{R\left[g - \tau^h wh - \tau^k \left(r_n^k k_n + r_x^k k_x\right)\right] - \left(R-1\right)m - l\left(1 - \frac{R}{\pi \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)\right\}\left(cR\right)^{-1}$$

$$\nu^m = \frac{m}{\left(1+\tau^c\right)c}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)\left(\eta_{xp}-1\right)} \frac{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(1-\eta_{xp}\right)\left(\kappa_{xp} - \frac{\left(1+\eta_{xp}\right)}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(-\eta_{xp}\right)\left(\kappa_{xp} - \frac{\left(1+\eta_{xp}\right)}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\ pt}{rer\ pm^*}\right)$$

$$y^* = x\, tot^{\eta *}$$

$$\chi_n = \left(\frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_n^{1-\theta} - s_n\left(c_n + g_n + \frac{i_n}{pn}\right)$$

$$\chi_x = \left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_x^{1-\theta} - s_x\left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right)$$

$$x^1 = \frac{\widetilde{p_n}^{-1-\eta_n}\left(c_n + g_n + \frac{i_n}{pn}\right) mc_n}{1 - \alpha_n\ r\ \pi_n^{-\eta_n\left(\kappa_n - \frac{\left(1+\eta_n\right)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p_n}^{-\eta_n}\left(c_n + g_n + \frac{i_n}{pn}\right)}{1 - \alpha_n\ r\ \pi_n^{\left(1-\eta_n\right)\left(\kappa_n - \frac{\eta_n}{\left(\eta_n - 1\right)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{\left(\eta_n - 1\right)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p_m}^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right) rer \frac{pt\ pm^*}{pm} \left(1 + \frac{R^f-1}{R^f}\right)}{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y_t^2 = \frac{\widetilde{p_m}^{-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right)}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p_x}^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right) mc_x}{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p_x}^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right)}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-1-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\ \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\ pt}{rer\ pm^*\ tot}\right)$$

$$u^2 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(1-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\ \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{\left(\eta_{xp} - 1\right)}{\eta_{xp}}$$

$$\frac{\gamma}{(1-\gamma)} = \frac{\left(1 - \tau^h\right) w \left(1 - h\right)}{mcw \left(1 + \tau^c\right) \left(1 + \nu^m \left(\frac{R-1}{R}\right)\right) c \left(1 - \frac{\zeta}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)^{-1} \frac{(1-\gamma)}{\left(1 + \tau^c\right) \left(1 + \nu^m \frac{R-1}{R}\right)}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f}\right)$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}$$

4 Ramsey Steady State

The Ramsey solution assumes the same parameters from the competitive equilibrium to compute allocations and prices, including those derived implicitly in the steady state computation. The Ramsey equilibrium is characterized by no inflation dispersion across sectors (thus, relative prices remain set at unity) and the Ramsey planner has the domestic nominal interest rates (R) and taxes (τ^h, τ^k, τ^c) as instruments to maximize the objective function, taking as given the values for domestic government expenditure, g, the taxation over profits, τ^ϕ, and the steady state values for the rest of the world.

$$\tau^h = \tau^h \qquad \tau^k = \tau^k \qquad \tau^c = \tau^c \qquad \tau^\phi = \tau^\phi \qquad R = R$$

$$R^* = R^* \qquad g = g$$

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \frac{\beta R}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$\pi^* = \pi_x^* = \pi_m^* = \Delta M^*$$

$$a_n = a_x = pm = px = pt = pn = pm^* = 1$$

$$mcw = \frac{\varpi}{\varpi - 1} \qquad r = \frac{1}{R} \qquad R^f = \frac{\pi^*}{\pi} R \qquad \xi = \left(\frac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad \widetilde{R} = R$$

$$\widetilde{p}_x^* = \left(\frac{1 - \alpha_x (\pi_x^*)^{(\kappa_{xp} - 1)(1 - \eta_{xp})}}{1 - \alpha_{xp}}\right)^{\frac{1}{1 - \eta_{xp}}} \qquad s_{xp} = \frac{(1 - \alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1 - \alpha_{xp} (\pi_x^*)^{\eta_{xp}(1 - \kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\frac{1 - \alpha_n \pi_n^{(\kappa_n - 1)(1 - \eta_n)}}{1 - \alpha_n}\right)^{\frac{1}{1 - \eta_n}} \qquad s_n = \frac{(1 - \alpha_n) \widetilde{p}_n^{-\eta_n}}{1 - \alpha_n \pi_n^{\eta_n (1 - \kappa_n)}}$$

$$\widetilde{p}_x = \left(\frac{1 - \alpha_x \pi_x^{(\kappa_x - 1)(1 - \eta_x)}}{1 - \alpha_x}\right)^{\frac{1}{1 - \eta_x}} \qquad s_x = \frac{(1 - \alpha_x) \widetilde{p}_x^{-\eta_x}}{1 - \alpha_x \pi_x^{\eta_x (1 - \kappa_x)}}$$

$$\widetilde{p}_m = \left(\frac{1 - \alpha_m \pi_m^{(\kappa_m - 1)(1 - \eta_m)}}{1 - \alpha_m}\right)^{\frac{1}{1 - \eta_m}} \qquad s_m = \frac{(1 - \alpha_m) \widetilde{p}_m^{-\eta_m}}{1 - \alpha_m \pi_m^{\eta_m (1 - \kappa_m)}}$$

$$rer = \widetilde{p}_m \left(1 + \frac{R^f - 1}{R^f}\right)^{-1} \frac{pm}{pt\ pm^*} \frac{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$mc_x = \widetilde{p}_x \frac{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$mc_n = \widetilde{p}_n \frac{1 - \alpha_n\ r\ \pi_n^{-\eta_n\left(\kappa_n - \frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)\left(\eta_{xp} - 1\right)} \frac{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(1-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\ pt}{rer\ pm^*}\right)$$

$$q_x = 1 \qquad q_n = 1$$

$$g_n = (1 - \omega)\, g \qquad g_t = \omega g$$

$$\mu_n = \sqrt{\frac{2}{\theta_2}\left[\left(1 - \tau^k\right)^{-1}\left(\frac{\mu^\Upsilon\left(\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)}{\beta} - 1 + \delta\right) - \theta_1 + \frac{\theta_2}{2}\right]}$$

$$\mu_x = \sqrt{\frac{2}{\theta_2}\left[\left(1 - \tau^k\right)^{-1}\left(\frac{\mu^\Upsilon\left(\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)}{\beta} - 1 + \delta\right) - \theta_1 + \frac{\theta_2}{2}\right]}$$

$$r_x^k = \theta_2\left(\mu_x - 1\right) + \theta_1 \qquad r_n^k = \theta_2\left(\mu_n - 1\right) + \theta_1$$

$$\tfrac{k_x}{h_x} = \mu_x \mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} \left(\tfrac{r_x^k}{mc_x \theta}\right)^{\frac{1}{\theta-1}} \qquad \tfrac{k_n}{h_n} = \mu_n \mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} \left(\tfrac{r_n^k}{mc_n \theta}\right)^{\frac{1}{\theta-1}}$$

$$w = mc_n\,(1 - \theta)\left(\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{\theta-1}}\ \tfrac{k_n \mu_n}{h_n}\right)^\theta \qquad \tfrac{h_x}{h_n} = \tfrac{mc_x Y_x}{mc_n Y_n}$$

In order to calculate the amount of labor used in domestic production, use the non-tradable sector equilibrium condition:

$$s_n\left(c_n+g_n+i_n\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n$$

$$s_n\left(1-\omega\right)\left(c+g+i_x^d+i_n^d+a\left(\mu_n\right)k_n+a\left(\mu_x\right)k_x\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n$$

$$s_n\left(1-\omega\right)\left(c+g+\left(1-\frac{1-\delta}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}h_n+\frac{k_x}{h_x}h_x\right)\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(c+g\right)+\chi_n=&\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n\\&-s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\end{aligned}$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R\left(1-h\right)}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n=&\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n\\&-s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\end{aligned}$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n=&\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n+\\s_n\left(1-\omega\right)\Bigg[\left(\frac{w\left(1-\tau^h\right)R\left(1+\frac{h_x}{h_n}\right)h_n}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}\right)&\\-\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\Bigg]&\end{aligned}$$

Set:

$$\begin{aligned}
HN_1 &= s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n \\
HN_2 &= \left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta} \\
HN_3 &= s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R\left(1+\frac{h_x}{h_n}\right)}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}\right) \\
HN_4 &= s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)
\end{aligned}$$

Then:

$$h_n = \frac{HN_1}{HN_2+HN_3-HN_4}$$

$$h = \left(1+\frac{h_x}{h_n}\right)h_n$$

$$h_x = h-h_n$$

Continuing with the steady state calculation:

$$k_x = \tfrac{k_x}{h_x}h_x \qquad k_n = \tfrac{k_n}{h_n}h_n \qquad \overline{k}_x = k_x/\mu_x \qquad \overline{k}_n = k_n/\mu_n$$

$$i_x^d = \left(1-\tfrac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)\tfrac{k_x}{h_x}h_x \qquad i_n^d = \left(1-\tfrac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)\tfrac{k_n}{h_n}h_n$$

$$i = i_x^d + i_n^d + +a\left(\mu_n\right)\overline{k}_n + a\left(\mu_x\right)\overline{k}_x$$

$$i_n = \left(1-\omega\right)i \qquad i_r = \omega i \qquad i_x = \left(1-\varkappa\right)i_r \qquad i_m = \varkappa i_r$$

$$c_n = \left(\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n-\chi_n\right)\frac{1}{s_n}-g_n-i_n$$

$$c = \tfrac{c_n}{\left(1-\omega\right)} \qquad c_t = \omega c \qquad c_x = \left(1-\varkappa\right)c_t \qquad c_m = \varkappa c_t$$

$$d_{xp} = \left(\left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_x^{1-\theta} - \chi_x \right) \frac{1}{s_x} - c_x - g_t - i_x$$

$$x = \left(\chi_{xp} - d_{xp}\right) / s_{xp}$$

$$y^* = x\, tot^{\eta *} \qquad d_m = \chi_m - s_m \left(c_m + \tfrac{pt}{pm} i_m \right)$$

$$ib = \frac{x - d_m \left(1 + \frac{\left(R^f - 1\right)}{R^f}\right)}{rer} \left(\frac{R^f}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - \pi^* \right)^{-1}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$m = \nu^m \left(1 + \tau^c\right) c$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}$$

$$l = \left[R \left[g - \tau^h wh - \tau^k \left(\left(r_n^k - a\left(\mu_n\right)\right) k_n + \left(r_x^k - a\left(\mu_x\right)\right) k_x \right) - \tau^c c - \tau^\phi \phi \right] - \left(R - 1\right) m \right] \left(1 - \frac{R}{\pi \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)^{-1}$$

$$b_g = \left(l - m\right) R^{-1}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)^{-1} \frac{\left(1 - \gamma\right)}{\left(1 + \tau^c\right) \left(1 + \nu^m \frac{R-1}{R}\right)}$$

$$x^1 = \frac{\widetilde{p_n}^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right) mc_n}{1 - \alpha_n \; r \; \pi_n^{-\eta_n \left(\kappa_n - \frac{\left(1+\eta_n\right)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p_n}^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right)}{1 - \alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p_m}^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right) rer \frac{pt\ pm^*}{pm} \left(1 + \frac{R^f - 1}{R^f}\right)}{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y^2 = \frac{\widetilde{p_m}^{-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right)}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p_x}^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right) mc_x}{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p_x}^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right)}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-1-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\ pt}{rer\ pm^*\ tot}\right)$$

$$u^2 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{\left(1-\eta_{xp}\right)\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{\left(\eta_{xp} - 1\right)}{\eta_{xp}}$$

5 Welfare Cost Measurement

Following Schmitt-Grohé and Uribe (2006 and 2007) [37] [38], the welfare cost λ_c of adopting the alternative policy regime i instead of the Ramsey monetary and fiscal policy r is measured in terms of the share of consumption the households give up in order to be indifferent between the two policy regimes:

$$\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^{\tilde{z}} \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right) = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^{\tilde{z}} \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Using the period utility function of the households, the welfare cost λ_c is given by:

$$\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^{\tilde{z}} \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Plug the period utility function for period zero and decompose the infinite sum:

$$\begin{aligned} \mho_i^c = (1-\gamma) \log & \left((1-\lambda_c)\, c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}} \left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right) \\ & + \gamma \log \left(1 - h_0^r\right) + E_0 \sum_{t=1}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^{\tilde{z}} \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \end{aligned}$$

Sum and subtract $(1-\gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}} \left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right)$ in the right-side of the equation:

$$\begin{aligned} \mho_i^c = (1-\gamma) \log & \left((1-\lambda_c)\, c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}} \left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right) \\ & - (1-\gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}} \left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right) + (1-\gamma) \log \left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}} \left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right) \\ & + \gamma \log \left(1 - h_0^r\right) + E_0 \sum_{t=1}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^{\tilde{z}} \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right) \end{aligned}$$

Decompose, from the infinite sum, the term of the welfare cost of the alternative policy, using the fact that the utility function is log-linear in consumption:

$$\begin{aligned}\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) \\ -(1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + (1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) \\ +\gamma\log\left(1-h_0^r\right) + E_0\sum_{t=1}^{\infty}\beta^t(1-\gamma)\log(1-\lambda_c) + E_0\sum_{t=1}^{\infty}\beta^t U_t\left(\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^{\tilde{z}}\left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right), h_t^r\right)\end{aligned}$$

Note that the third, fourth and last term of the right-hand side equal the welfare of the Ramsey policy, $\mho_r^c$:

$$\begin{aligned}\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) \\ -(1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \mho_r^c + E_0\sum_{t=1}^{\infty}\beta^t(1-\gamma)\log(1-\lambda_c)\end{aligned}$$

Organizing the terms:

$$\frac{\mho_i^c - \mho_r^c}{(1-\gamma)} = \log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) - \log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^{\tilde{z}}\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \frac{\beta}{1-\beta}\log(1-\lambda_c)$$

Now, approximate the welfare cost λ_c by a second-order Taylor expansion around the vector of disturbances σ to obtain:

$$\lambda_c \approx \overline{\lambda}_c + \lambda_{c,\sigma}\sigma + \lambda_{c,\sigma\sigma}\frac{\sigma^2}{2}$$

Following the results in Schmitt-Grohé and Uribe (2006 and 2007) [37] [38], note that $\overline{\lambda}_c$ vanishes, because all the policies considered here do not alter the steady state of the economy, and $\lambda_{c,\sigma} = 0$. The second total derivative of the equation provides the welfare measure:

$$\begin{aligned}\frac{\mho_{i,\sigma\sigma}^c - \mho_{r,\sigma\sigma}^c}{(1-\gamma)} &= -\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)\lambda_{c,\sigma\sigma} \\ \Longrightarrow \lambda_{c,\sigma\sigma} &= \frac{\mho_{r,\sigma\sigma}^c - \mho_{i,\sigma\sigma}^c}{(1-\gamma)\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)} \\ \Longrightarrow \lambda_c &\approx \left(\frac{\mho_{r,\sigma\sigma}^c - \mho_{i,\sigma\sigma}^c}{(1-\gamma)\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)}\right)\frac{\sigma^2}{2}\end{aligned}$$

References

[1] ADOLFSON, Malin, LASEÉN, Stefan, LINDÉ, Jesper and VILLANI, Mattias, 2007. *"Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through",* Journal of International Economics, 72(2), pages 481-511.

[2] ADJEMIAN, Stéphane, PARIÈS, Matthieu D., and MOYEN, Stéphane, 2007. *"Optimal Monetary Policy in an Estimated DSGE for the Euro Area"*, European Central Bank Working Paper number 803.

[3] AGUIAR, Mark, and GOPINATH, Gita, 2007. *"Emerging Market Business Cycles: The Cycle is the Trend"*, Journal of Political Economy, vol.115(1), pages 69-102.

[4] AMBLER, Steve, DIB, Ali, and REBEI, Nooman, 2004. *"Optimal Taylor Rules in an Estimated Model of a Small Open Economy"*, Bank of Canada Working Paper Series number 04-36.

[5] AN, Sungbae, and SCHORFHEIDE, Frank, 2007. *"Bayesian Analysis of DSGE Models"*, Econometric Reviews, volume 26(2), pages 113-172.

[6] BATINI, Nicoletta, LEVINE, Paul and PEARLMAN, Joseph, 2009. *"Monetary and Fiscal Rules in an Emerging Small Open Economy"*, International Monetary Fund, IMF Working Paper, WP/09/22.

[7] BENIGNO, Gianluca, and DE PAOLI, Bianca, 2009. *"On the International Dimension of Fiscal Policy",* Centre for Economic Performance Discussion Paper number 905.

[8] CANOVA, Fabio, 2007. *"Methods for Applied Macroeconomic Research"*, Princeton University Press.

[9] CASTILLO, Paul, MONTORO, Carlos and TUESTA, Vicente, 2006. *"An Estimated Stochastic General Equilibrium Model with Partial Dollarization: A Bayesian Approach"*. Central Bank of Chile, Working Paper number 381.

[10] CHRISTIANO, Lawrence J., EICHENBAUM, Martin, and EVANS, Charles C., 2005. *"Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy"*, Journal of Political Economy, vol.113(1), pages 1-45.

[11] CHRISTIANO, Lawrence J., TRABANDT, Mathias, and WALENTIN, Karl, 2007. *"Introducing Financial Frictions and Unemployment Into a Small Open Economy Model",* Sveriges Riksbank Working Paper Series number 214.

[12] CORSETTI, Giancarlo, DEDOLA, Luca, and LEDUC, Sylvain, 2007. *"Optimal Monetary Policy and the Sources of Local-Currency Price Stability",* NBER Working Papers number 13544.

[13] DE PAOLI, Bianca, 2009. *"Monetary Policy and Welfare in a Small Open Economy",* Journal of International Economics, Elsevier, vol. 77(1), pages 11-22.

[14] DEVEREUX, Michael B., LANE, Philip R., and XU, Juanyi, 2006. *"Exchange Rates and Monetary Policy in Emerging Market Economies",* Economic Journal, Royal Economic Society, vol. 116, issue 511, pages 478-506.

[15] DUFOUR, Jean-Marie, KHALAF, Lynda, and KICHIAN, Maral, 2009. *"Assessing Indexation-Based Calvo Inflation Models"*, Bank of Canada Working Paper 2009-7.

[16] ELEKDAG, Selim, JUSTINIANO, Alejandro, and TCHAKAROV, Ivan, 2005. *"An Estimated Small Open Economy Model of the Financial Accelerator"*, International Monetary Fund, IMF Working Paper, WP/05/44.

[17] ELEKDAG, Selim, and TCHAKAROV, Ivan, 2007. *"Balance Sheets, Exchange Rate Policy, and Welfare"*, Journal of Economic Dynamics and Control, volume 31, pages 3986-4015.

[18] FELICES, Guillermo, and TUESTA, Vicente, 2007. "Monetary Policy in a Dual Currency Environment", Banco Central de Reserva del Perú, Working Paper Series number 2007-006.

[19] GALÍ, Jordi, and MONACELLI, Tommaso, 2005. *"Monetary Policy and Exchange Rate Volatility in a Small Open Economy",* Review of Economic Studies, Blackwell Publishing, vol. 72(3), pages 707-734.

[20] GARCIA-CICCO, Javier, 2008. *"Estimating Models for Monetary Policy Analysis in Emerging Countries"*, manuscript, Duke University.

[21] HAMILTON, James D., 1994. *"Time Series Analysis"*, Princeton University Press.

[22] JUSTINIANO, Alejandro, and PRESTON, Bruce, 2008. *"Monetary Policy and Uncertainty in an Empirical Small Open Economy Model",* Journal of Applied Econometrics, forthcoming.

[23] KOLLMANN, Robert, 2002. *"Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles",* Journal of Monetary Economics, Elsevier, vol. 49(5), pages 989-1015.

[24] LAXTON, Douglas, and PESENTI, Paolo, 2003. *"Monetary Rules for Small, Open, Emerging Economies"*, Journal of Monetary Economics, Elsevier, vol. 50(5), pages 1109-1146.

[25] LEVIN, Andrew T., ONATSKI, Alexei, WILLIAMS, John C., WILLIAMS, Noah, 2005. *"Monetary Policy under Uncertainty in Microfounded Macroeconometric Models."* In: NBER Macroeconomics Annual 2005, GERTLER, Mark, ROGOFF, Kenneth, MIT Press.

[26] LEVIN, Andrew T., and WILLIAMS, John C., 2003. *"Robust Monetary Policy with Competing Reference Models",* Journal of Monetary Economics, Elsevier, vol. 50(5), pages 945-975.

[27] LUBIK, Thomas, and SCHORFHEIDE, Frank, 2006. *"A Bayesian Look at New Open Economy Macroeconomics"*, NBER Macroeconomics Annual 2005, MIT Press, pages 297-346.

[28] LUCAS, Robert E. Jr., and STOCKEY, Nancy L., 1983. *"Optimal Fiscal and Monetary Policy in an Economy without Capital",* Journal of Monetary Economics, Elsevier, vol. 12(1), pages 55-93.

[29] MENDOZA, Enrique G., and YUE, Vivian Z., 2008. *"A Solution to the Default Risk-Business Cycle Disconnect",* NBER Working Papers number 13861.

[30] MONACELLI, Tommaso, 2005. *“Monetary Policy in a Low Pass-through Environment”,* Journal of Money, Credit and Banking, vol. 37, pages 1047–1066.

[31] RABANAL, Pau, and RUBIO-RAMIREZ, Juan, 2005. *"Comparing New-Keynesian Models of the Business Cycle: A Bayesian Approach",* Journal of Monetary Economics, Elsevier, vol. 52(6), pages 1151-1166.

[32] RAMSEY, Frank P., 1927. *"A Contribution of the Theory of Taxation"*, Economic Journal 37, pages 47-61.

[33] RAVENNA, Federico, and NATALUCCI, Fabio M., 2008. *"Monetary Policy Choices in Emerging Market Economies: The Case of High Productivity Growth"*, UC Santa Cruz Center for International Economics Working Paper, number 08-01.

[34] ROTEMBERG, Julio J. and WOODFORD, Michael, 1997. *"An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy"*, NBER Macroeconomics Annual, MIT Press, pages 297-346.

[35] SCHMITT-GROHÉ, Stephanie, and URIBE, Martin, 2003. *"Anticipated Ramsey Reforms and the Uniform Taxation Principle: The Role of International Financial Markets"*, European Central Bank Working Paper number 210.

[36] SCHMITT-GROHÉ, Stephanie, and URIBE, Martin, 2003(b). *"Closing Small Open Economy Models"*, Journal of International Economics, Elsevier, vol.61(1), pages 163-185.

[37] SCHMITT-GROHÉ, Stephanie, and URIBE, Martin, 2006. *"Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model",* NBER Macroeconomics Annual 2005, MIT Press, pages 383-425.

[38] SCHMITT-GROHÉ, Stephanie, and URIBE, Martin, 2007. *"Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model"*, in Monetary Policy Under Inflation Targeting, edited by Klaus Schmidt-Hebbel and Rick Mishkin, Central Bank of Chile, Santiago, Chile, p. 125-186.

[39] SCHMITT-GROHÉ, Stephanie, and URIBE, Martin, 2007(b). *"Optimal Simple and Implementable Monetary and Fiscal Rules"*, Journal of Monetary Economics, Elsevier, vol. 54(6), pages 1702-1725.

[40] SILVEIRA, Marcos Antonio C. da, 2006. *"A Small Open Economy as a Limit Case of a Two-Country New Keynesian DSGE Model: a Bayesian Estimation with Brazilian Data"*. Instituto de Pesquisa Econômica Aplicada – IPEA, Texto para Discussão 1252a.

[41] SMETS, Frank R., and WOUTERS, Rafael, 2002. *"Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy"*, Journal of Monetary Economics, Elsevier, vol. 49(5), pages 947-981.

[42] SMETS, Frank R., and WOUTERS, Rafael, 2003. *"An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area"*, Journal of European Economic Association, volume 1 (5), pages 1123-1175.

[43] SMETS, Frank R., and WOUTERS, Rafael, 2005. *"Comparing Shocks and Frictions in US and Euro Area Business Cycles: a Bayesian DSGE Approach"*, Journal of Applied Econometrics, vol. 20(2), pages 161-183.

[44] SVENSSON, Lars E.O., 2000. *"Open-Economy Inflation Targeting",* Journal of International Economics, 50, pages 155-183.

[45] WOODFORD, Michael, 2003. *"Interest and Prices: Foundations of a Theory of Monetary Policy"*, Princeton University Press.