Optimal Monetary and Fiscal Policy for Small Open and Emerging Economies

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A Medium-Scale Model for a Small Open Economy

In this chapter the full model is described, with the characterization of the household and the firms' problem, the policy rules for the government in a competitive equilibrium, the foreign sector and aggregation[1]. Beyond the description of the model, the definitions of a competitive and Ramsey equilibria are presented, as well as the procedures to deal with the problem of a zero lower bound for nominal interest rates and the welfare measure computation. The model is an extension for a small-open economy of the closed-economy model for monetary policy analysis proposed in Christiano, Eichenbaun and Evans (2005)[10] (CEE (2005), henceforth) and Altig, Christiano, Eichenbaun and Linde (2005)[3]. Similar models are used in Adolfson, Laseén, Lindé and Villani (2007)[2] and, more recently, in Christiano, Trabandt and Walentin (2007)[11]. These models for small-open economies combine the basic sticky price framework proposed in Galí and Monacelli (2005)[16] and Monacelli (2005)[28] to add a set of nominal and real frictions based in the formulation of CEE (2005)[10].

In a brief overview, from the household perspective, the model presents external

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[1] A full description of the model and the transformation for stationary form are available in a technical appendix upon request. Also, appendix A lists the final set of equilibrium conditions.

habit persistence in consumption, adjustment costs for investment, portfolio and changing the capacity utilization. Households own capital, demand money to buy consumption goods and set their wages after observing the demand for his specific type of labor. The objective of the household is to maximize the discounted value of expected utility. In order to achieve the objective, households in each period buy both domestically produced and imported goods for consumption, sell their labor to satisfy the demand by the firms after the acceptance of the proposed wage and set the rate of capital utilization. In order to transfer wealth across periods, households trade bonds domestically and in the international financial markets and accumulate capital built from both domestically produced and imported goods. Households are subject to a cash-in-advance constraint, requiring domestic currency to buy a share of total consumption goods.

Firms in the tradable and non-tradable sectors of the domestic economy rent capital and labor from the households to produce goods. They set prices in a Calvo style, with a probability α_i of not adjusting prices in period t. Firms from the tradable sector have to compete with imported goods retailers. These retail firms buy goods produced abroad and sell them domestically, also adjusting prices in a Calvo style in terms of domestic currency. On the other hand, firms from the tradable sector can sell goods for the exported goods retailers. These firms buy domestically produced goods and sell them abroad, setting price in a Calvo style in terms of foreign currency – thus, local currency pricing in both domestic and foreign markets justifies pricing-to-market discrimination and the deviations of the Law of One Price, as commonly seen in the literature[2]. A demand for foreign currency is justified in the model by a working capital constraint for imported goods retailers, with those firms selling bonds to obtain foreign currency to finance the total acquisition of foreign inputs.

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[2] Some of the references in models with at least partial local currency pricing are Kollmann (2002)[22], Ambler, Dib and Rebei (2004)[4], Devereux, Lane and Xu (2006)[15], Christiano, Trabandt and Walentin (2007)[11], Justiniano and Preston (2009)[21],

The government in a competitive equilibrium sets nominal interest rates according to a Taylor rule based on inflation, output gap and changes in the real exchange rate, in order to match an exogenous, time-varying inflation target. In terms of fiscal policy, the government has three instruments available to finance an exogenous stream of consumption: money, bonds sold domestically, and distortionary taxes. The government might tax in different rates consumption and the income from capital, labor and profits. In the competitive equilibrium, taxes on labor are set according to a simple policy rule based on total government liabilities. Taxes on consumption, capital and on profits are exogenous.

The foreign sector is described by a simple VAR including lags of the foreign money supply, output, inflation, interest rates and a measure of the risk premium. The VAR has all shocks identified by a Cholesky decomposition, following the traditional procedure in the literature. The model has a total of 16 shocks, with five of them being from the foreign sector (one for each variable of the VAR), plus the following: one on the price of imported goods in foreign currency; two stationary sectorial productivity shocks; a non-stationary aggregate productivity shock; a non-stationary, investment-specific shock; government spending; three tax shocks; monetary policy shock and a inflation target shock.

The chapter is organized as follows. Sections 1.1 and 1.2 present details of the optimization problem of households and firms, respectively, highlighting the role of the nominal and real frictions added with the literature. A characterization of the government and the foreign economy is provided in sections 1.3 and 1.4. These blocks of the model are very stylized, with shocks driving most of the dynamics of the government and a VAR describing the foreign economy. The aggregation problem and the macroeconomic identities are presented in section 1.5. Section 1.6 presents the steps to obtain the stationary representation of the model, the competitive equilibrium, the Ramsey equilibrium and the computation of welfare measures.

1.1 Households

There is a continuum of infinitely-lived households i ($i \in [0,1]$) populating the domestic economy, each one of them with an endowment of labor type i, $h_t(i)$. There is no population growth and labor can not be sold for firms in the rest of the world. In the intertemporal problem, households maximize discounted utility choosing current period's consumption capacity utilization and investment for each sector, wages, hours worked and the money demand, and next period's foreign and domestic bond holdings and physical capital stock. The general statement of the intertemporal household problem, given the non-Ponzi games constraints, is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma)\log\left(C_t(i)-\zeta C_{t-1}\right)+\gamma \log\left(1-h_t(i)\right)\right]$$

$$\begin{aligned}
s.t.: \quad & P_t\left(1+\tau_t^c\right)C_t(i)+\Upsilon_t^{-1}P_t\left(I_{x,t}^d(i)+I_{n,t}^d(i)\right)+P_tM_t(i)+R_{t-1}B_{h,t}(i) \\
& +S_tR_{t-1}^fIB_t(i)+W_t\tfrac{\phi_w}{2}\left(\tfrac{W_t(i)}{\pi_t^{Xw}W_{t-1}(i)}-\mu^I\right)^2+\tfrac{\psi_1}{2}Y_t\left(\tfrac{B_{t+1}(i)}{Y_t}-\tfrac{B}{Y}\right)^2 \\
& +\tfrac{\psi_2}{2}Y_t\left(\tfrac{S_tIB_{t+1}(i)}{Y_t}-\tfrac{rer\ IB}{Y}\right)^2=P_{t-1}M_{t-1}(i)+\left(1-\tau_t^h\right)W_t(i)h_t(i) \\
& +\left(1-\tau_t^{\phi}\right)P_t\Phi_t(i)+\left(1-\tau_t^k\right)P_t\left[\left(R_{n,t}^k\mu_{n,t}-\Upsilon_t^{-1}a\left(\mu_{n,t}\right)\right)\overline{K}_{n,t}(i)\right. \\
& \left.+\left(R_{x,t}^k\mu_{x,t}-\Upsilon_t^{-1}a\left(\mu_{x,t}\right)\right)\overline{K}_{x,t}(i)\right]+B_{h,t+1}(i)+S_tIB_{t+1}(i)
\end{aligned}$$

$$\overline{K}_{j,t+1}(i)=(1-\delta)\overline{K}_{j,t}(i)+I_{j,t}^d(i)\left(1-\aleph\left(\frac{I_{j,t}^d(i)}{I_{j,t-1}^d(i)}\right)\right)$$

$$a\left(\mu_{j,t}\right)=\theta_1\left(\mu_{j,t}-1\right)+\frac{\theta_2}{2}\left(\mu_{j,t}-1\right)^2$$

$$K_{j,t}=\mu_{j,t}\overline{K}_{j,t}$$

$$\aleph\left(\frac{I^d_{i,t}}{I^d_{i,t-1}}\right)=\frac{\phi_i}{2}\left(\frac{I^d_{i,t}}{I^d_{i,t-1}}-\mu^I\right)^2 \qquad j=\{x,n\}$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t}=\mu^{\Upsilon}_{t+1}=(1-\rho_{\Upsilon})\,\mu^{\Upsilon}+\rho_{\Upsilon}\mu^{\Upsilon}_t+\epsilon^{\Upsilon}_{t+1} \qquad \epsilon^{\Upsilon}_t\sim N\left(0,\sigma_{\Upsilon}\right)$$

$$h_t(i)=\left(\frac{W_t(i)}{W_t}\right)^{-\varpi}h_t$$

$$M_t(i)\geq\nu^m\left(1+\tau^c_t\right)C_t(i)$$

In this problem, β is the intertemporal discount factor of the utility function. The utility function assumes a traditional, log-separable form in terms of consumption and labor, with consumption adjusted by external habit persistence[3]. The degree of habit persistence is defined by the parameter $\zeta\in[0,1)$.

In the model, households accumulate physical capital, $\overline{K}_{j,t}$, for $j=\{x,n\}$ representing the sectors of the economy, buying from the firms investment goods that depreciate at a rate δ. Define Υ^{-1}_t as the non-stationary inverse of the relative price of investment in terms of consumption goods. The relative price of investment goods can also be interpreted as a technology shock affecting the linear production function available to households to transform consumption goods in investment goods[4]. Investment is subject to an adjustment cost $\aleph\left(.\right)$, in the same fashion as in CEE (2005)[10] and Altig, Christiano, Eichenbaun and Linde (2005)[3] such that $\aleph\left(1\right)=0,\aleph'\left(1\right)=0,\aleph''\left(1\right)>0$. The functional form adopted follows Schmitt-Grohé and Uribe (2005b)[33], with μ^I defining the steady state growth of investment. Households rent capital for the firms after setting the rate of capital utilization for each sector ($\mu_{j,t}$), paying a cost given by the function $a\left(\mu_{j,t}\right)$ to change the utilization level in each period and in each sector. The after-tax private return of capital in each sector is defined, thus, as $\left(1-\tau^k_t\right)P_t\left(R^k_{j,t}\mu_{j,t}-\Upsilon^{-1}_t a\left(\mu_{j,t}\right)\right)\overline{K}_{j,t}(i)$.

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[3] In terms of notation, the general variable $x_t(i)$ represents the choice of household i on period t about x. The variable x_t is the aggregate value of $x_t(i)$ for the economy.

[4] See Greenwood, Hercowitz and Krusell (2000)[18] and Schmitt-Grohé and Uribe (2005b)[33].

The supply of labor is decided by each household taking as given the aggregate wage of the economy, the aggregate demand for labor, h_t, and the quadratic adjustment cost function for wages. As a monopolist of a specific type of labor, the household chooses the nominal wage $W(i)$ and supplies all the demanded for labor $h_t(i)$ given the acceptance of $W(i)$. The elasticity of substitution across different types of labor $h_t(i)$ is given by $\varpi > 1$. The nominal wage adjustment cost function allows for partial indexation based on current inflation. The degree of indexation is determined by χ_w ($\chi_w \in [0,1]$). The presence of sticky wages in the model results in an additional distortion, defined by mcw_t, which is equivalent to the markup households impose over real wages since they supply a specific type of labor to the firms. The use of a quadratic adjustment cost in the wage-setting process[5] is consistent with the absence of lump sum instruments to correct for wealth dispersion across households. Wage setting processes based on the Calvo model create dispersion in the wage income across households, and the representative household is recovered through a lump sum subsidy scheme or an asset market structure that is capable to insure all households against this dispersion. Both instruments would be controversial with one of the main objectives of this paper, which is evaluating the optimal policy under the assumption that the government does not have access to any sort of lump sum scheme to support the agents. Another alternative to avoid the dispersion in wages is to assume the presence of a centralized union that coordinates the supply of labor among households, as proposed in Schmitt-Grohé and Uribe (2005b)[33]. The assumption of a labor union with such market power, however, does not seem reasonable for developed small-open economies outside Scandinavian countries[6].

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[5] See, for instance, Chugh (2006)[12] and Garcia-Cicco (2008)[17].

[6] According to data from OECD (2004), only Denmark, Sweden, Finland, Iceland and Belgium presented a steady increase in the percentage of workers associated with an union (trade-union density) from 1960 to 2000. In Latin American economies like Mexico and Chile, recently associated with OECD, the trade-union density is significantly lower compared to Scandinavian countries, and declining since the 1990's (see Visser Martin Tergeist, 2008[19]).

Still in the budget constraint, households are able to allocate wealth over time buying one-period, non-state contingent nominal bonds from the government, $B_{h,t+1}(i)$, or from the rest of the world, $IB_{t+1}(i)$. In the later case, the bonds are priced in foreign currency, and S_t is the nominal exchange rate. In order to adjust its portfolio, and to induce stationarity in the model, the households incurs in adjustment costs, both domestically and in the international financial markets, based on the variance of the stock of bonds as a proportion of the GDP[7]. Households also receive (after-tax) dividends from the firms $\Phi_t(i)$.

Finally, following Schmitt-Grohé and Uribe (2007)[34], households demand money, $M_t(i)$, in a cash-in-advance constraint, in order to pay for a share $\nu^m \geq 0$ of the after-tax consumption. The constraint holds with equality as long as (gross) nominal interest rates, R_t, are larger than unity. The sequence of events in each period for the households is the same as in CEE (2005)[10], with the households first deciding consumption and capital allocation, then deciding, in sequence, the financial portfolio, wages and the labor supply, and the final composition of portfolio between bonds and money. Thus, domestic currency, in this model, is expressed as an end-of-period aggregate.

Define $\widetilde{\lambda}_t/P_t$, $\widetilde{\lambda}_t\widetilde{q}_{j,t}$, $\lambda_t^m\widetilde{\lambda}_t$ and $\left(\widetilde{\lambda}_t\left(1-\tau_t^h\right)W_t\right)/\left(P_t\, mcw_t\right)$ the Lagrange multipliers on the budget constraint, on the capital accumulation equations, on the cash-in-advance constraint and on the labor demand function, respectively. After taking the first order conditions of the Lagrangian of the household's problem, and using the fact that the equilibrium is symmetric (note especially that, in equilibrium, $C_t(i) = C_t$ and $W_t(i) = W_t$), the final set of equilibrium conditions of the intertem-

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[7] See Schmitt-Grohé and Uribe (2003)[31]. The functional form adopted is the same as the model proposed by those authors. However, the use of the ratio to GDP is adopted here to obtain the stationary form of the model.

poral problem of the household is given by:

$$\frac{\left(1-\tau_t^h\right)\widetilde{W}_t}{\left(1+\tau_t^c\right)\left(C_t-\zeta C_{t-1}\right)} = \frac{\gamma}{(1-\gamma)} \frac{mcw_t\left(1+\nu^m\left(\frac{\widetilde{R}_t-1}{\widetilde{R}_t}\right)\right)}{(1-h_t)} \tag{1.1}$$

$$\frac{(1-\gamma)}{C_t-\zeta C_{t-1}} = \left(1+\tau_t^c\right)\widetilde{\lambda}_t\left(1+\nu^m\left(\frac{R_t-1}{R_t}\right)\right) \tag{1.2}$$

$$\widetilde{\lambda}_t\left[1-\psi_1\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)\right] = \beta R_t E_t\left(\frac{\widetilde{\lambda}_{t+1}}{\pi_{t+1}}\right) \tag{1.3}$$

$$\widetilde{\lambda}_t\left[1-\psi_2\left(\frac{S_t IB_{t+1}}{Y_t}-\frac{rer\ IB}{Y}\right)\right] = \beta R_t^f E_t\left(\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}\widetilde{\lambda}_{t+1}\right) \tag{1.4}$$

$$\begin{aligned} &\widetilde{\lambda}_t \widetilde{q}_{x,t} = \\ &\qquad \beta E_t\left\{\widetilde{\lambda}_{t+1}\left[\left(1-\tau_{t+1}^k\right)\left(R_{x,t+1}^k \mu_{x,t+1}-\Upsilon_{t+1}^{-1} a\left(\mu_{x,t+1}\right)\right)+\widetilde{q}_{x,t+1}(1-\delta)\right]\right\} \end{aligned} \tag{1.5}$$

$$\begin{aligned} &\widetilde{\lambda}_t \widetilde{q}_{n,t} = \\ &\qquad \beta E_t\left\{\left[\widetilde{\lambda}_{t+1}\left(1-\tau_{t+1}^k\right)\left(R_{n,t+1}^k \mu_{n,t+1}-\Upsilon_{t+1}^{-1} a\left(\mu_{n,t+1}\right)\right)+\widetilde{q}_{n,t+1}(1-\delta)\right]\right\} \end{aligned} \tag{1.6}$$

$$K_{n,t} = \mu_{n,t}\overline{K}_{n,t} \tag{1.7}$$

$$K_{x,t} = \mu_{x,t}\overline{K}_{x,t} \tag{1.8}$$

$$\theta_1+\theta_2\left(\mu_{n,t}-1\right) = \frac{R_{n,t}^k}{\Upsilon_t^{-1}} \tag{1.9}$$

$$\theta_1+\theta_2\left(\mu_{x,t}-1\right) = \frac{R_{x,t}^k}{\Upsilon_t^{-1}} \tag{1.10}$$

$$R_t = \frac{1}{r_{t,t+1}} \tag{1.11}$$

$$\widetilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)\right)^{-1} \tag{1.12}$$

$$\begin{aligned}\widetilde{\lambda}_t \Upsilon_t^{-1} \quad = \quad & \widetilde{\lambda}_t \widetilde{q}_{x,t} \left[1 - \Psi\left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right) - \left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right) \Psi'\left(\frac{I^d_{x,t}}{I^d_{x,t-1}}\right)\right] \\ & + \beta E_t \left[\widetilde{\lambda}_{t+1} \widetilde{q}_{x,t+1} \left(\frac{I^d_{x,t+1}}{I^d_{x,t}}\right)^2 \Psi'\left(\frac{I^d_{x,t+1}}{I^d_{x,t}}\right)\right] \end{aligned} \tag{1.13}$$

$$\begin{aligned}\widetilde{\lambda}_t \Upsilon_t^{-1} \quad = \quad & \widetilde{\lambda}_t \widetilde{q}_{n,t} \left[1 - \Psi\left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right) - \left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right) \Psi'\left(\frac{I^d_{n,t}}{I^d_{n,t-1}}\right)\right] \\ & + \beta E_t \left[\widetilde{\lambda}_{t+1} \widetilde{q}_{n,t+1} \left(\frac{I^d_{n,t+1}}{I^d_{n,t}}\right)^2 \Psi'\left(\frac{I^d_{n,t+1}}{I^d_{n,t}}\right)\right] \end{aligned} \tag{1.14}$$

$$\overline{K}_{x,t+1}(i) = (1-\delta)\, \overline{K}_{x,t}(i) + I^d_{x,t}(i) \left(1 - \aleph\left(\frac{I^d_{x,t}(i)}{I^d_{x,t-1}(i)}\right)\right) \tag{1.15}$$

$$\overline{K}_{n,t+1}(i) = (1-\delta)\, \overline{K}_{n,t}(i) + I^d_{n,t}(i) \left(1 - \aleph\left(\frac{I^d_{n,t}(i)}{I^d_{n,t-1}(i)}\right)\right) \tag{1.16}$$

$$\begin{aligned}\left(\frac{1-\varpi}{\varpi} + \frac{1}{mcw_t}\right) \varpi h_t \left(1 - \tau_t^h\right) = & -\frac{\phi_w}{\pi_t^{\chi_w - 1}} \left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}}\right) \left(\frac{\widetilde{W}_t}{\pi_t^{\chi_w - 1} \widetilde{W}_{t-1}} - \mu^I\right) \\ & + \beta E_t \left[\frac{\widetilde{\lambda}_{t+1} \phi_w}{\widetilde{\lambda}_t \pi_{t+1}^{\chi_w - 1}} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\right)^2 \left(\frac{\widetilde{W}_{t+1}}{\pi_{t+1}^{\chi_w - 1} \widetilde{W}_t} - \mu^I\right)\right] \end{aligned} \tag{1.17}$$

From the first order equilibrium conditions, notice that the uncovered interest parity (UIP) condition between domestic interest rates and the interest rates in international financial markets is a result of the combination of equations 1.3 and 1.4. The UIP condition holds in its strict sense only in the steady state, since the dynamics is also influenced by the presence of domestic and foreign portfolio adjustment costs. This is a small departure from other studies for small-open economies, like Adolfson, Laseén, Lindé and Villani (2007)[2], where the only source of discrepancy between the domestic and foreign interest rates from the UIP condition is the debt-elastic foreign interest rate, like the one described in Schmitt-Grohé and Uribe (2003)[31]. As the description of the foreign block of the model will make clear, the UIP condition adopted here is a combination of the debt-elastic foreign interest rate and the portfolio adjustment cost proposed in Schmitt-Grohé and Uribe (2003)[31].

In the first stage of the decision in each period, the household also solves a sequence of minimization problems constrained by the CES aggregator function in order to choose the composition of the consumption and investment baskets. Expressing first the consumption problem, households decide the composition between imported and domestically produced goods in the tradable goods basket, and then chooses the optimal composition of tradable and non-tradable goods. For simplicity of exposition, assume also that the portfolio adjustment costs are paid with a share of the consumption goods acquired by the households. As a consequence, the cost minimization problem of the household is given by:

$$\min_{C_{n,t},C_{t,t},C_{m,t},C_{x,t}} P_{n,t}C_{n,t} + P_{t,t}C_{t,t}$$

$$C_t + PAC_{b,t} + PAC_{ib,t} = \left[(1-\omega)^{\frac{1}{\varepsilon}} C_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} C_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.18}$$

$$C_{t,t} = \left[(1-\varkappa)^{\frac{1}{\varrho}} C_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} C_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}} \tag{1.19}$$

$$PAC_{b,t} = \frac{\psi_1}{2} Y_t \left(\frac{B_{h,t+1}}{Y_t} - \frac{B_h}{Y}\right)^2$$

$$PAC_{ib,t} = \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{Y_t} - \frac{rer\ IB}{Y}\right)^2$$

Combine the first order conditions with the expenditure function to obtain the demand for each type of tradable good and the price index of tradable goods:

$$C_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}}\right)^{-\varrho} C_{t,t} \tag{1.20}$$

$$C_{x,t} = (1-\varkappa) \left(\frac{P_{x,t}}{P_{t,t}}\right)^{-\varrho} C_{t,t} \tag{1.21}$$

$$P_{t,t} = \left[(1-\varkappa) P_{x,t}^{1-\varrho} + \varkappa P_{m,t}^{1-\varrho}\right]^{\frac{1}{1-\varrho}}$$

By analogy, the optimal decision between tradable and non-tradable goods and the CPI index is given by:

$$C_{t,t} = \omega \left(\frac{P_{t,t}}{P_t}\right)^{-\varepsilon} (C_t + PAC_{b,t} + PAC_{ib,t}) \tag{1.22}$$

$$C_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t}\right)^{-\varepsilon} (C_t + PAC_{b,t} + PAC_{ib,t}) \tag{1.23}$$

$$P_t = \left[(1-\omega) P_{n,t}^{1-\varepsilon} + \omega P_{t,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Households also solve an equivalent problem when setting the composition of the investment good for each sector. For simplicity, assume that the weights in the basket of goods and the elasticities of substitution among different types of investment goods is the same as the weights and the elasticities for consumption goods. Also, assume that the adjustment costs in capital utilization are paid in terms of aggregate investment. As a consequence, the demands for home produced and imported investment goods are given by:

$$\Upsilon_t^{-1} I_t = \Upsilon_t^{-1} \left(I_{n,t}^d + a(\mu_{n,t}) \overline{K}_{n,t} + I_{x,t}^d + a(\mu_{x,t}) \overline{K}_{x,t}\right) \tag{1.24}$$

$$I_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}}\right)^{-\varrho} I_{t,t} \tag{1.25}$$

$$I_{x,t} = (1-\varkappa) \left(\frac{P_{x,t}}{P_{t,t}}\right)^{-\varrho} I_{t,t} \tag{1.26}$$

$$I_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{1.27}$$

$$I_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{1.28}$$

$$\Upsilon_t^{-1} I_t = \left[(1-\omega)^{\frac{1}{\varepsilon}} I_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} I_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$I_{t,t} = \left[(1-\varkappa)^{\frac{1}{\varrho}} I_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} I_{m,t}^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}$$

1.2 Firms

There are four sectors in the economy, each sector composed by a continuum of firms operating in a monopolistic competitive framework. Firms in the non-tradable (n) and tradable (x) sectors demand labor and capital to produce. Firms in the imported goods (m) sector and in the exported goods (xp) sector buy the final good and sell it to consumers in the domestic economy (for the case of imported goods sector firms) or in the rest of the world (for the case of exported goods sector firms). Firms chooses the amount of production inputs to buy and set new prices according to a probability $\alpha_i, i = \{n, x, m, xp\}$, that is independent across sectors and across firms. If a firm is not allowed to optimize prices in period t, it changes prices according to an indexation rule based on past inflation. Imported goods' firms must finance the total amount of imported goods using only foreign currency. There is no firm entry into or exit out of sector i and also no change of firms across sectors.

The four sectors would result, in a log-linearized model around price stability, in four equations like the New Keynesian Phillips curve describing the dynamics of prices. However, since price stability might not be optimal policy for Ramsey planner, the recursive formulation for the first order condition of firms in terms of prices described in Schmitt-Grohé and Uribe (2005b)[33] is adopted. The recursive formulation is flexible enough to accomodate price stability as one special case, and also allows, for estimation purposes, matching the steady state inflation with the average inflation in the sample.

1.2.1 Domestic non-tradable goods' producers problem:

Firms in the non-tradable sector use capital and labor to produce goods that are used for consumption, investment and spent by the government. The production technology is a simple Cobb-Douglas function with a sectorial stationary productivity shock and a non-stationary, labor-augmenting technological shock. Setting real profits for firm i_n as $\Phi_{n,t}(i_n)$, the problem of the domestic, non-tradable goods producers of type i_n product ($i_n \in [0,1]$) is to maximize the expected discounted stream of profits, subject to the demand for good i_n, the production technology and the aggregate demand for non-tradable goods. In order to solve the problem, firms choose in each period the demand for labor, capital and, if allowed to do so with probability $1-\alpha_n$, they optimize prices. The statement of the problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{n,t} \left(\frac{P_{n,t}(i_n)}{P_{n,t}} D_{n,t}(i_n) - \frac{W_t}{P_{n,t}} h_{n,t}(i_n) - \frac{P_t}{P_{n,t}} R^k_{n,t} K_{n,t}(i_n) \right)$$

$$s.t.: \qquad D_{n,t}(i_n) = \left(\tfrac{P_{n,t}(i_n)}{P_{n,t}} \right)^{-\eta_n} Y_{n,t}$$

$$Y_{n,t} = C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \tfrac{P_t}{P_{n,t}} I_{n,t}$$

$$a_{n,t} K_{n,t}(i_n)^{\theta} \left(z_t h_{n,t}(i_n) \right)^{1-\theta} - z_t^* \chi_n \geq D_{n,t}(i_n)$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \tfrac{z_t^*}{z_t}$$

$$\tfrac{z_{t+1}}{z_t} = \mu^z_{t+1} = (1-\rho_z)\mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}; \quad \rho_z \in [0,1); \quad \epsilon^z_t \sim N(0,\sigma_z)$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon^n_{t+1}; \quad \rho_n \in [0,1); \quad \epsilon^n_t \sim N(0,\sigma_n)$$

In this problem, $a_{n,t}$ is a stationary, sector-specific technology shock, z_t is a labor-augmenting, non-stationary technology shock. The non-stationary shock z_t affects all the firms using labor in production. In order to guarantee zero profits in the steady state, $z_t^* \chi_n$ introduces a fixed cost proportional to the evolution of the non-stationary shocks in production, following CEE (2005)[10], Schmitt-Grohé and Uribe (2005, 2005b)[32][33], among others. Describing the demand for good i_n, parameter η_n is the elasticity of substitution across varieties of non-tradable goods.

From the first order conditions in terms of $h_{n,t}(i_n)$ and $K_{n,t}(i_n)$, it is possible to prove that the capital-labor ratio is the same across firms in the non-tradable sector. As a consequence, the marginal cost across firms is also the same in this sector. Setting $mc_{n,t}$ as the Lagrange multiplier on the firm's demand constraint, the two equilibrium conditions are:

$$\widetilde{W}_t \frac{P_t}{P_{n,t}} = mc_{n,t}(1-\theta)\, a_{n,t} z_t \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta} \tag{1.29}$$

$$R^k_{n,t} \frac{P_t}{P_{n,t}} = mc_{n,t} \theta a_{n,t} \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta - 1} \tag{1.30}$$

Prices are formed in a Calvo style with indexation, where α_n is the probability that firm i_n is not allowed to optimally adjust its price in period t. In the case firms are not allowed to set up prices optimally, they follow the simple rule $P_{n,t}(i_n) = \pi_{n,t-1}^{\kappa_n} P_{n,t-1}(i_n)$, for $0 \leq \kappa_n \leq 1$ and $\pi_{n,t+1} = \frac{P_{n,t+1}}{P_{n,t}}$. Setting the Lagrangean of the problem, considering only the terms relevants for price determination:

$$\begin{aligned} {}_n = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} P_{n,t+s} & \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}} \right)^{1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{1-\eta_n} Y_{n,t+s} \right. \\ & \left. - mc_{n,t+s} \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}} \right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} Y_{n,t+s} \right) \right) \end{aligned}$$

In this problem, $r_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and $\widetilde{P}_{n,t}(i_n)$ is the new price set by firms allowed to adjust prices in period t. The first order condition for firms allowed to adjust prices is:

$$E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}} \right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} \times$$
$$\left(\frac{(\eta_n - 1)}{\eta_n} \frac{\widetilde{P}_{n,t}\left(i_n\right)}{P_{n,t+s}} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right) - mc_{n,t+s} \right) = 0$$

As a consequence of the first order condition, given that mark-up over prices is the same across firms, the symmetric equilibrium is characterized by all firms in the non-tradable sector allowed to adjust prices in period t setting the same price: $\widetilde{P}_{n,t}\left(i_n\right) = \widetilde{P}_{n,t}$. Following Schmitt-Grohé and Uribe (2005b)[33], split the pricing function equation in two parts, X_t^1 and X_t^2, and define $\widetilde{p}_{n,t} = \frac{\widetilde{P}_{n,t}}{P_{n,t}}$ in order to obtain the recursive solution for the problem of the non-tradable goods' producers:

$$X_t^1 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}} \right)^{-1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} mc_{n,t+s}$$

$$X_t^2 = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} \left(\frac{\widetilde{P}_{n,t}}{P_{n,t}} \right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right)^{-\eta_n} \left(\frac{(\eta_n - 1)}{\eta_n} \prod_{k=1}^{s} \left(\frac{\pi_{n,t+k-1}^{\kappa_n}}{\pi_{n,t+k}} \right) \right)$$

The system describing the evolution of non-tradable inflation is given by:

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{-1-\eta_n} mc_{n,t} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}} \right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}} \right)^{-\eta_n} X_{t+1}^1 \tag{1.31}$$

$$X_t^2 = Y_{n,t} \widetilde{p}_{n,t}^{-\eta_n} \frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}} \right)^{-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}} \right)^{1-\eta_n} X_{t+1}^2 \tag{1.32}$$

$$X_t^1 = X_t^2 \tag{1.33}$$

1.2.2 Tradable goods' producers problem:

A tradable goods producer i_x ($i_x \in [0,1]$) solves the same problem as the non-tradable producer, using labor and capital as production factors. The total production of the tradable good is divided between domestic absorption (consumption, investment and government spending) and the demand of a continuum of i_{xp} exporting firms ($D_{xp,t}$). The tradable goods' firm problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{x,t} \left(\frac{P_{x,t}(i_x)}{P_{x,t}} D_{x,t}(i_x) - \frac{\widetilde{W}_t}{P_{x,t}} h_{x,t}(i_x) - \frac{P_t}{P_{x,t}} R^k_{x,t} K_{x,t}(i_x) \right)$$

$$s.t.: \qquad D_{x,t}(i_x) = \left(\tfrac{P_{x,t}(i_x)}{P_{x,t}} \right)^{-\eta_x} Y_{x,t}$$

$$Y_{x,t} = C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \tfrac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t}$$

$$a_{x,t} K_{x,t}(i_x)^{\theta} (z_t h_{x,t}(i_x))^{1-\theta} - z_t^* \chi_x \geq D_{x,t}(i_{tr})$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \tfrac{z_t^*}{z_t}$$

$$\tfrac{z_{t+1}}{z_t} = \mu^z_{t+1} = (1-\rho_z)\mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}; \quad \rho_z \in [0,1); \quad \epsilon^z_t \sim N(0, \sigma_z)$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon^x_{t+1}; \quad \rho_x \in [0,1); \quad \epsilon^x_t \sim N(0, \sigma_x)$$

χ_x is a fixed cost proportional to total output associated with the non-stationary shock in order to guarantee zero profits in steady state. Parameter η_x is the elasticity of substitution across varieties of tradable goods. Setting $mc_{x,t}$ as the Lagrange multiplier on the firm's demand constraint, the solution of the cost minimization problem of the firm in terms of $h_{x,t}(i_x)$ and $K_{x,t}(i_x)$, after using again the fact that the capital-labor ratio is the same across firms, become:

$$\widetilde{W}_t \frac{P_t}{P_{x,t}} = mc_{x,t} (1-\theta) a_{x,t} z_t \left(\frac{K_{x,t}}{z_t h_{x,t}} \right)^{\theta} \tag{1.34}$$

$$R^k_{x,t} \frac{P_t}{P_{x,t}} = mc_{x,t} \theta a_{x,t} \left(\frac{K_{x,t}}{z_t h_{x,t}} \right)^{\theta-1} \tag{1.35}$$

Similar to the firms in the non-tradable sector, price adjustment is based on the Calvo mechanism with indexation to past inflation, with $0 \leq \kappa_x \leq 1$ defining the degree of indexation in the tradable sector. Taking the first order conditions in terms of $\widetilde{P}_{x,t}\left(i_x\right)$, and defining $\pi_{x,t+1}=\frac{P_{x,t+1}}{P_{x,t}}$, the optimal price set by each firm is:

$$E_t\sum_{s=0}^{\infty}\alpha_x^s r_{t,t+s}Y_{x,t+s}P_{x,t+s}\left(\frac{\widetilde{P}_{x,t}\left(i_x\right)}{P_{x,t}}\right)^{-\eta_x}\prod_{k=1}^{s}\left(\frac{\pi_{x,t+k-1}^{\kappa_x}}{\pi_{x,t+k}}\right)^{-\eta_x}\times$$
$$\left(\frac{\left(\eta_x-1\right)}{\eta_x}\frac{\widetilde{P}_{x,t}\left(i_x\right)}{P_{x,t}}\prod_{k=1}^{s}\left(\frac{\pi_{x,t+k-1}^{\kappa_x}}{\pi_{x,t+k}}\right)-mc_{x,t+s}\right)=0$$

As a consequence of the same mark-up over prices across firms, the symmetric equilibrium is characterized by all firms in the tradable sector setting the same price when allowed to optimize, $\widetilde{P}_{x,t}\left(i_x\right)=\widetilde{P}_{x,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Z_t^1, Z_t^2, such that $Z_t^1=Z_t^2$, and $\widetilde{p}_{x,t}=\frac{\widetilde{P}_{x,t}}{P_{x,t}}$:

$$Z_t^1=\widetilde{p}_{x,t}^{-1-\eta_x}Y_{x,t}mc_{x,t}+\alpha_x r_{t,t+1}E_t\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}}\right)^{-\eta_x}Z_{t+1}^1 \tag{1.36}$$

$$Z_t^2=\widetilde{p}_{x,t}^{-\eta_x}Y_{x,t}\frac{\left(\eta_x-1\right)}{\eta_x}+\alpha_x r_{t,t+1}E_t\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}}\right)^{1-\eta_x}Z_{t+1}^2 \tag{1.37}$$

$$Z_t^1=Z_t^2 \tag{1.38}$$

1.2.3 Imported goods' firms problem:

Following Lubik and Schorfheide (2006)[25], deviations from the Law of One price in the model arises as a consequence of price stickiness in imported and exported goods. An imported goods' firm i_m ($i_m\in[0,1]$) buys a bundle of the international homogeneous good[8] and relabel it as an imported good type i_m. In order to buy the goods produced in the rest of the world, the firm needs to make payments using foreign currency. The firm sells intraperiod bonds in foreign markets in order

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[8] Note that, in the model, one country buys a combination of the goods from different countries. As a consequence, there is a gap between the world's CPI (P_t^*) and the price of the bundle imported by a given country ($P_{m,t}^*$).

to get foreign currency, but it does not transfer financial wealth over time. As a consequence, firms do not incur in exposure to risk in the international markets, just an increase in the marginal cost of production. As a timing convention, the bonds traded do not reflect in the end of period balance of payments. The same framework is adopted in Christiano, Trabandt and Walentin (2007)[11] and Mendoza and Yue (2008)[27]. The budget constraint of the exporting firm i_m, expressed in terms of domestic prices, is given by:

$$\frac{S_t P_t^*}{P_t} M_{m,t}^*(i_m) + \frac{S_t}{P_t} B_{m,t+1}^*(i_m) =$$

$$\frac{S_t}{P_t} P_{t-1}^* M_{m,t-1}^*(i_m) + \frac{S_t}{P_t} R_{t-1}^f B_{m,t}^*(i_m) + \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \Phi_{m,t}(i_m)$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Following the assumption that firms do not keep any financial wealth across periods, and that all profits are distributed to the households, obtain the expression for real profits:

$$P_t^* M_{m,t}^*(i_m) + R_t^f B_{m,t+1}^*(i_m) = 0, \qquad \forall t$$

$$\Longrightarrow \frac{S_t P_t^*}{P_t} \left(M_{m,t}^*(i_m) - \frac{M_{m,t}^*(i_m)}{R_t^f} \right) = \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \Phi_{m,t}(i_m)$$

$$\Phi_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_t^*}{P_t} \left(\frac{R_t^f - 1}{R_t^f} \right) M_{m,t}^*(i_m)$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. The imported goods' firm problem is given by:

$$\max_{\widetilde{P}_{m,t}(i_m)} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t} \right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_{m,t}^*}{P_t} \left(\frac{R_t^f - 1}{R_t^f} \right) \frac{P_t^*}{P_{m,t}^*} M_{m,t}^*(i_m) \right]$$

$$s.t. : D_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m)}{P_{m,t}} \right)^{-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right)$$

$$M_{m,t}(i_m) \geq \frac{P^*_{m,t}}{P^*_t} D_{m,t}(i_m)$$

where $P^*_{m,t}$ is the price of the imported good bought by the domestic economy, quoted in foreign prices. Parameter η_m is the elasticity of substitution across varieties of imported goods. Taking the first order conditions in terms of $\widetilde{P}_{m,t}(i_m)$, the price for those firms allowed to optimize prices in period t, and defining $\pi_{m,t+1} = \frac{P_{m,t+1}}{P_{m,t}}$ and $0 \leq \kappa_m \leq 1$ the degree of indexation in the imported goods' sector, the expression for the optimal price set by each firm becomes:

$$E_t \sum_{s=0}^{\infty} \alpha_m^s r_{t,t+s} P_{m,t+s} \left(C_{m,t+s} + \Upsilon^{-1}_{t+s} \frac{P_{t+s}}{P_{m,t+s}} I_{m,t+s} \right) \left(\frac{\widetilde{P}_{m,t}(i_m)}{P_{m,t+s}} \right)^{-\eta_m} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_m}_{m,t+k-1}}{\pi_{m,t+k}} \right)^{-\eta_m} \times$$
$$\left(\frac{(\eta_m - 1)}{\eta_m} \frac{\widetilde{P}_{m,t}(i_m)}{P_{m,t+s}} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_m}_{m,t+k-1}}{\pi_{m,t+k}} \right) - \frac{S_{t+s} P^*_{m,t+s}}{P_{m,t+s}} \left(1 + \frac{R^f_{t+s} - 1}{R^f_{t+s}} \right) \right) = 0$$

In this problem, α_m is the probability that the importing firm i_m is not allowed to adjust its price in period t. As a consequence of the same mark-up over prices across firms (in this case, given by the real exchange rate deflated by the import price level), the symmetric equilibrium is characterized by $\widetilde{P}_{m,t}(i_m) = \widetilde{P}_{m,t}$. The recursive solution for the pricing problem of the importing firms is obtained after properly defining Y^1_t and Y^2_t such that $Y^1_t = Y^2_t$, and $\widetilde{p}_{m,t} = \frac{\widetilde{P}_{m,t}}{P_{m,t}}$:

$$Y^1_t = \widetilde{p}_{m,t}^{-1-\eta_m} \left(C_{m,t} + \Upsilon^{-1}_t \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{S_t P^*_{m,t}}{P_{m,t}} \left(1 + \frac{R^f_t - 1}{R^f_t} \right)$$
$$+ \alpha_m r_{t,t+1} E_t \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-1-\eta_m} \left(\frac{\pi^{\kappa_m}_{m,t}}{\pi^{(1+\eta_m)/\eta_m}_{m,t+1}} \right)^{-\eta_m} Y^1_{t+1} \quad (1.39)$$

$$Y_t^2 = \widetilde{p}_{m,t}^{-\eta_m} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \frac{(\eta_m - 1)}{\eta_m}$$

$$+ \alpha_m r_{t,t+1} E_t \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{\eta_m/(\eta_m-1)}} \right)^{1-\eta_m} Y_{t+1}^2 \quad (1.40)$$

$$Y_t^1 = Y_t^2 \quad (1.41)$$

1.2.4 Exported goods' firms problem:

On the exported goods' side, there is a specific sector consuming tradable goods and, in a Calvo style, setting prices in foreign currency. An exported goods' firm i_{xp} ($i_{xp} \in [0,1]$) buys a share of the final tradable good in the domestic economy and sell it to the rest of the world. Prices are sticky in foreign currency. The exported goods' firm problem is given by:

$$\max_{\widetilde{P}_{x,t}^*(i_{xp})} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{S_t \widetilde{P}_{x,t}^* (i_{xp}) - P_{x,t}}{P_t} \right) D_{xp,t} (i_{xp}) - \left(\frac{R_t - 1}{R_t} \right) M_{xp,t}(i_{xp}) - z_t^* \chi_{xp} \right]$$

$$s.t. : D_{xp,t} (i_{xp}) = \left(\frac{P_{x,t}^* (i_{xp})}{P_{x,t}^*} \right)^{-\eta_{xp}} X_t$$

where χ_{xp} is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Parameter η_{xp} is the foreign elasticity of substitution across varieties of domestic exported goods. Taking the first order conditions in terms of $\widetilde{P}_{x,t}^* (i_{xp})$, and defining $\pi_{x,t+1}^* = \frac{P_{x,t+1}^*}{P_{x,t}^*}$ and $0 \leq \kappa_{xp} \leq 1$ the degree of indexation in the exported goods' sector, the expression for the optimal price set by each firm becomes:

$$E_t \sum_{s=0}^{\infty} \alpha_{xp}^s r_{t,t+s} P_{x,t+s}^* X_s \left(\frac{\widetilde{P}_{x,s}^* (i_{xp})}{P_{x,s}^*} \right)^{-\eta_{xp}-1} \prod_{k=1}^{s} \left(\frac{\left(\pi_{x,t+k-1}^*\right)^{\kappa_{xp}}}{\pi_{x,t+k}^*} \right)^{-\eta_{xp}} \times$$

$$\left(\frac{(\eta_{xp} - 1)}{\eta_{xp}} \frac{\widetilde{P}_{x,t}^* (i_{xp})}{P_{x,t}^*} \prod_{k=1}^{s} \left(\frac{\left(\pi_{x,t+k-1}^*\right)^{\kappa_{xp}}}{\pi_{x,t+k}^*} \right) - \frac{P_{x,s}}{S_s P_{x,s}^*} \right) = 0$$

In this problem, α_{xp} is the probability that an importing firm i_x is not allowed to adjust its price in period t, $P^*_{x,t}$ is the price of the tradable good from the domestic economy quoted in foreign prices. The symmetric equilibrium is again characterized by $\widetilde{P}^*_{x,t}(i_x) = \widetilde{P}^*_{x,t}$. The recursive solution for the pricing problem of the exporting firms is obtained after properly defining U^1_t and U^2_t, such that $U^1_t = U^2_t$, and $\widetilde{p}^*_{x,t} = \frac{\widetilde{P}^*_{x,t}}{P^*_{x,t}}$:

$$U^1_t = \left(\widetilde{p}^*_{x,t}\right)^{-1-\eta_{xp}} X_t \frac{P_{x,t}}{S_t P^*_{x,t}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}}\right)^{-1-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}}\right)^{-\eta_{xp}} U^1_{t+1} \tag{1.42}$$

$$U^2_t = \left(\widetilde{p}^*_{x,t}\right)^{-\eta_{xp}} X_t \frac{(\eta_{xp}-1)}{\eta_{xp}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}}\right)^{-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{\eta_{xp}}{(\eta_{xp}-1)}}}\right)^{1-\eta_{xp}} U^2_{t+1} \tag{1.43}$$

$$U^1_t = U^2_t \tag{1.44}$$

1.3 Government

In the competitive equilibrium of the economy, the government follows basic rules to set monetary and fiscal policy. In terms of monetary policy, a standard Taylor rule includes an autorregressive component, plus the deviations of inflation from an exogenous, autocorrelated inflation target, deviations of output from its steady-state, and changes in the real exchange rate:

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_t}{R}\right) + + (1-\rho_R)\left[\alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi^o_{t+1}}\right) + \alpha_y \log\left(\frac{y_{t+1}}{y}\right) + \alpha_{rer} \log\left(\frac{rer_{t+1}}{rer_t}\right)\right] + \epsilon^R_{t+1} \tag{1.45}$$

$$\pi^o_{t+1} = (1-\rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi^o_t + \epsilon^{\pi^o}_{t+1} \tag{1.46}$$

$$\epsilon^R_t \sim N(0,\sigma_R) \qquad \epsilon^{\pi^o}_t \sim N(0,\sigma_{\pi^o})$$

The government, in order to finance its exogenous expenditures, G_t, collects distortionary taxes on consumption, labor, capital and profits income ($\tau^c_t, \tau^h_t, \tau^k_t$ and τ^ϕ_t), sells bonds domestically, $B_{g,t}$ and controls the money supply, M_t. The government budget constraint is given by:

$$P_t G_t + R_{t-1}B_{g,t} = P_t T_t + P_t M_t + B_{g,t+1} - P_{t-1}M_{t-1}$$

$$G_t = z^*_t g_t$$

$$g_t = (1-\rho_g)\, g + \rho_g g_{t-1} + \epsilon^g_t \qquad \epsilon^g_t \sim N(0,\sigma_g) \tag{1.47}$$

$$\begin{aligned} T_t &= \tau^c_t C_t + \tau^h_t \widetilde{W}_t h_t + \tau^\phi_t \Phi_t \\ &\quad + \tau^k_t \left[\left(R^k_{n,t}\mu_{n,t} - \Upsilon^{-1}_t a(\mu_{n,t})\right)\overline{K}_{n,t} + \left(R^k_{x,t}\mu_{x,t} - \Upsilon^{-1}_t a(\mu_{x,t})\right)\overline{K}_{x,t}\right] \end{aligned} \tag{1.48}$$

Following Schmitt-Grohé and Uribe (2005b)[33], after defining the total real government liabilities (L_t), the evolution of government debt is pinned down by a fiscal policy rule where the government sets income taxation as a function of the gap between the actual liabilities as a proportion of GDP and its steady state value, plus a term related with the output gap, in order to account for the stabilization of the business cycle. Use the definition of net government liabilities to rewrite the budget constraint:

$$L_{t-1} \equiv M_{t-1} + \frac{R_{t-1}}{P_{t-1}}B_{g,t} \tag{1.49}$$

$$\Longrightarrow L_t = \frac{R_t}{\pi_t}L_{t-1} + R_t(G_t - T_t) - (R_t - 1)M_t \tag{1.50}$$

To close the dynamics of the fiscal block, assume that the government follows a fiscal policy rule to determine the labor income taxation, while taxes on capital and profits are exogenous. The assumption of a fiscal policy rule for labor income taxation is an arbitrary choice, since the presence of portfolio adjusment costs in domestic financial markets ensures stationarity in the model. Also, for simplicity, assume that the taxation on profits is constant over time. Notice that taxes on profits are lump sum transfers from the households to the government. In this sense, it does not interfere with the dynamics under the competitive equilibrium, where profits are zero.

$$\tau_t^h - \tau^h = \psi_1 \left(\frac{L_t}{Y_t} - \frac{l}{y} \right) + \psi_2 \left(y_t - y \right) + \epsilon_t^{\tau} \tag{1.51}$$

$$\tau_t^k = \left(1 - \rho_{\tau k} \right) \tau^k + \rho_{\tau k} \tau_{t-1}^k + \epsilon_t^{\tau k} \tag{1.52}$$

$$\tau_t^{\phi} = \tau^{\phi} \tag{1.53}$$

$$\tau_t^c = \left(1 - \rho_c \right) \tau^c + \rho_c \tau_{t-1}^c + \epsilon_t^{\tau c} \tag{1.54}$$

$$\epsilon_t^{\tau h} \sim N\left(0, \sigma_{\tau h}\right) \quad \epsilon_t^{\tau k} \sim N\left(0, \sigma_{\tau k}\right) \quad \epsilon_t^{\tau \phi} \sim N\left(0, \sigma_{\tau \phi}\right) \quad \epsilon_t^{\tau c} \sim N\left(0, \sigma_{c}\right)$$

Additionally, the government solves an equivalent problem as the households to determine their optimal consumption of tradable and non-tradable goods. By assumption, the government does not consume imported goods[9]. The demand for each type of good is given by:

$$G_{n,t} = (1 - \omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} G_t \tag{1.55}$$

$$G_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} G_t \tag{1.56}$$

$$G_t = \left[(1 - \omega)^{\frac{1}{\varepsilon}} G_{n,t}^{\frac{\varepsilon - 1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} G_{t,t}^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}$$

1.4 International Financial Markets and World's Economy

The transmission of shocks from international financial markets assume the existence of an international bond market capable of evaluating country-specific risk on bonds

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[9] The same assumption is used in Lubik and Schorfheide (2006)[25].

issued outside the domestic economy. In this sense, a mechanism to induce stationarity in the style proposed in Schmitt-Grohé and Uribe (2003)[31] can be used to determine (and estimate) the risk premium of the bonds issued in each country as a function of the net foreign position of the economy. The international interest rate is given by:

$$R_t^f = R_t^* \left(1 + \xi_t\right)^{\kappa_1} \left(\frac{S_t IB_{t+1}}{P_t Y_t} / \frac{IB}{Y} \right)^{\kappa_2} \tag{1.57}$$

In this equation, R_t^* is a baseline, risk-free nominal interest rate on bonds traded in international markets; ξ_t is an autonomous shock in the risk premium, associated with the general risk level of the world's economy, with expected value equal to the long run risk premium demanded from the domestic economy, ξ^*; the last term is the gap between total external debt of the domestic economy and its long run level.

The world's economy is modeled by a VAR containing measures of output, y_t^*, inflation, π_t^*, interest rates, R_t^*, growth of money supply, ΔM_t^*, and the risk premium, ξ_t. The objective of the VAR with this specification is to be as close as possible to the empirical studies of identification of shocks in the line of CEE (2005)[10], without imposing a prior theoretical specification for the economy. When compared with Garcia-Cicco (2008)[17], the system of equations here has a different identification assumption for the shocks. Also, the inclusion of the risk premium tries to capture financial shocks that are not only unrelated with country-specific events, but also not associated with changes in foreign monetary policy. International shocks are identified with the Cholesky decomposition of the variance-covariance matrix of residuals. The world's output is added in order to identify supply from demand shocks in changes in the international prices. Thus, the VAR for the rest of the world will provide five shocks for the domestic economy.

$$\begin{bmatrix} \frac{\Delta M^*_t}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R^*_t}{R^*} \\ \frac{\pi^*_t}{\pi^*} \\ \frac{y^*_t}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M^*_{t-1}}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R^*_{t-1}}{R^*} \\ \frac{\pi^*_{t-1}}{\pi^*} \\ \frac{y^*_{t-1}}{y^*} \end{bmatrix} + \begin{bmatrix} \epsilon^{m*}_t \\ \epsilon^{\xi}_t \\ \epsilon^{R*}_t \\ \epsilon^{\pi*}_t \\ \epsilon^{y*}_t \end{bmatrix} \qquad \begin{bmatrix} \epsilon^{m*}_t \\ \epsilon^{\xi}_t \\ \epsilon^{R*}_t \\ \epsilon^{\pi*}_t \\ \epsilon^{y*}_t \end{bmatrix} \overset{iid}{\sim} (0, \sum) \tag{1.58}$$

In the system, A is a 5 by 5 matrix of coefficients, $\sum$ is a 5 by 5 upper triangular matrix of shocks estimated from the unrestricted model using the Cholesky decomposition. Variables are listed from the "more endogenous" to the "more exogenous" variable.

Two assumptions close the relation between prices and quantities of goods between the domestic country and the rest of world. First, assume that households in the rest of the world solve an expenditure minimization problem in order to set the optimal demand for home produced tradable goods. The solution of this problem is given by the demand equation:

$$X_t = \left(\frac{P^*_{x,t}}{P^*_t} \right)^{-\eta^*} z^*_t y^*_t \tag{1.59}$$

Finally, the terms of trade of the domestic economy are defined as the ratio between the exported goods and the imported goods price levels, both quoted in foreign currency. Also, the dynamics of the price of imported goods in foreign currency is given by an error-correction model that ensure the terms of trade becomes stationary, in the line of Garcia-Cicco (2008)[17]. The dynamics of the terms of trade and imported goods' prices are given by:

$$tot_t = \frac{\pi^*_{x,t}}{\pi^*_{m,t}} tot_{t-1} \tag{1.60}$$

$$\frac{\pi^{m*}_t}{\pi^{m*}} = \upsilon_1 \frac{\pi^{m*}_{t-1}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X^*_{t-1} + \epsilon^{\pi m}_t \qquad \epsilon^{\pi m}_t \sim N\left(0, \sigma_{\pi m}\right) \tag{1.61}$$

with $X^*_t = \begin{bmatrix} \frac{\Delta M^*_t}{\Delta M^*} & \frac{\xi_t}{\xi^*} & \frac{R^f_t}{R^*} & \frac{\pi^*_t}{\pi^*} & \frac{y^*_t}{y^*} \end{bmatrix}$.

1.5 Aggregation and Relative Prices

In order to find an expression for the aggregate constraint of the economy, start from the demand faced by a non-tradable producer firm and integrate both sides over all the i_n firms, noting that $h_{n,t} = \int_0^1 h_{n,t}\left(i_n\right) di_n$, and that the capital-labor ratio is constant across all the firms:

$$a_{n,t} K_{n,t}^{\theta} \left(z_t h_{n,t}\right)^{1-\theta} - z_t^* \chi_n = \int_0^1 \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}}\right)^{-\eta_n} di_n \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t}\right)$$

Define $s_{n,t} = \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}}\right)^{-\eta_{n,t}} di_n$ to obtain:

$$a_{n,t} K_{n,t}^{\theta} \left(z_t h_{n,t}\right)^{1-\theta} - z_t^* \chi_n = s_{n,t} \left(C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \frac{P_t}{P_{n,t}} I_{n,t}\right) \tag{1.62}$$

Obtain the recursive form of $s_{n,t}$:

$$s_{n,t} = \int_0^1 \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}}\right)^{-\eta_n} di$$

$$\Longrightarrow s_{n,t} = \left(1 - \alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}}\right)^{\eta_n} s_{n,t-1} \tag{1.63}$$

Also, from the definition of the non-tradable goods price index:

$$P_{n,t} = \left[\int_0^1 P_{n,t}\left(i_n\right)^{1-\eta_n} di\right]^{\frac{1}{1-\eta_n}}$$

$$\Longrightarrow 1 = \left(1 - \alpha_n\right) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}}\right)^{1-\eta_n} \tag{1.64}$$

Equivalent expressions can be written for the resource constraint, price dispersion and the price index of imported and domestically produced tradable goods and the price index of exported goods in foreign currency respectively:

$$D_{m,t} - z_t^* \chi_m = s_{m,t} \left(C_{m,t} + \Upsilon_t^{-1} \frac{P_t}{P_{m,t}} I_{m,t} \right) \tag{1.65}$$

$$s_{m,t} = (1 - \alpha_m) \widetilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1} \tag{1.66}$$

$$1 = (1 - \alpha_m) \widetilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m} \tag{1.67}$$

$$a_{x,t} K_{x,t}^{\theta} \left(z_t h_{x,t} \right)^{1-\theta} - z_t^* \chi_x = s_{x,t} \left(C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \frac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t} \right) \tag{1.68}$$

$$s_{x,t} = (1 - \alpha_x) \widetilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1} \tag{1.69}$$

$$1 = (1 - \alpha_x) \widetilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x} \tag{1.70}$$

$$D_{xp,t} - z_t^* \chi_{xp} = s_{xp,t} X_t \tag{1.71}$$

$$s_{xp,t} = (1 - \alpha_{xp}) \left(\widetilde{p}_{x,t}^{*} \right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^{*}}{\left(\pi_{xp,t-1}^{*} \right)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1} \tag{1.72}$$

$$1 = (1 - \alpha_{xp}) \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{\left(\pi_{x,t-1}^{*} \right)^{\kappa_{xp}}}{\pi_{x,t}^{*}} \right)^{1-\eta_{xp}} \tag{1.73}$$

From the aggregation condition of the labor market, the total amount of work hours supplied by the domestic households is given by:

$$h_{x,t} + h_{n,t} = h_t \tag{1.74}$$

The external equilibrium assumes that the net foreign position of domestic households is proportional to the average trade balance result in steady state. Again, notice that the external equilibrium in the bond markets does not include the bonds issued by imported goods' firms, as they are negotiated and liquidated at the beginning and the end of each period. The description of the net foreign position in terms of domestic currency is given by:

$$P_{x,t}X_t - P_{m,t}D_{m,t}\left[1+\left(\frac{R_t^f-1}{R_t^f}\right)\right] = S_t R_{t-1}^f P_t^* IB_t - S_t P_{t+1}^* IB_{t+1} \tag{1.75}$$

It's also necessary to determine the market clearing conditions for domestic bonds and money market. For simplicity, assume that foreign households and domestic firms do not demand home government bonds. As a consequence:

$$B_{g,t} + B_{h,t} = 0 \tag{1.76}$$

Finally, the gross domestic product is defined as:

$$\begin{aligned} Y_t = C_t + \frac{\psi_1}{2}Y_t\left(\frac{B_{t+1}}{Y_t}-\frac{B}{Y}\right)^2 + \frac{\psi_2}{2}Y_t\left(\frac{S_t IB_{t+1}}{Y_t}-\frac{rer\ IB}{Y}\right)^2 \\ + \Upsilon_t^{-1}I_t + G_t + \frac{P_{x,t}}{P_t}X_t - \frac{P_{m,t}}{P_t}D_{m,t}\left[1+\left(\frac{R_t^f-1}{R_t^f}\right)\right] \end{aligned} \tag{1.77}$$

Aggregate profits are given by:

$$\Phi_t = Y_t - \widetilde{W}_t h_t - R_{n,t}^k \mu_{n,t}\overline{K}_{n,t} - R_{x,t}^k \mu_{x,t}\overline{K}_{x,t} \tag{1.78}$$

1.5.1 Relative prices

The model includes a set of relative prices that are strictly related to some observables of the economy. In terms of dynamics, the set of relative prices in the model is given by:

$$pt_t = \frac{P_{t,t}}{P_t} = \frac{\pi_{t,t}}{\pi_t}\frac{P_{t,t-1}}{P_{t-1}} \tag{1.79}$$

$$pn_t = \frac{P_{n,t}}{P_t} = \frac{\pi_{n,t}}{\pi_t}\frac{P_{n,t-1}}{P_{t-1}} \tag{1.80}$$

$$px_t = \frac{P_{x,t}}{P_{t,t}} = \frac{\pi_{x,t}}{\pi_{t,t}}\frac{P_{x,t-1}}{P_{t,t-1}} \tag{1.81}$$

$$pm_t = \frac{P_{m,t}}{P_{t,t}} = \frac{\pi_{m,t}}{\pi_{t,t}} \frac{P_{m,t-1}}{P_{t,t-1}} \tag{1.82}$$

$$pm_t^* = \frac{P_{m,t}^*}{P_t^*} = \frac{\pi_{m,t}^*}{\pi_t^*} \frac{P_{m,t-1}^*}{P_{t-1}^*} \tag{1.83}$$

$$rer_t = \frac{S_t P_t^*}{P_t} \tag{1.84}$$

1.6 Stationary Form and Equilibrium

The objective of this section is to describe the equilibrium conditions with the necessary adjustments to induce stationarity and characterize the competitive and Ramsey Equilibria. Define the stationary allocations with small letters, such that, for a generic variable X_t and the appropriate trend $\check{Z}_t$, the stationary variable is given by $x_t \equiv X_t/\check{Z}_t$. The model in stationary form is fully described by the stochastic processes for the following sets of variables:

- prices: $\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r_{x,t}^k, r_{n,t}^k, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi_t^*, \pi_{x,t}^*, \pi_t^{m*}, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}_{x,t}^*, pt_t, pn_t, px_t, pm_t, pm_t^*, tot_t$;
- interest rates: $R_t, \widetilde{R}_t, R_t^*, R_t^f$;
- allocations: $c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i_{x,t}^d, i_{n,t}^d, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x_t^1, x_t^2, z_t^1, z_t^2, y_t^1, y_t^2, u_t^1, u_t^2, ib_t, b_{h,t}, \xi_t, \Delta M_t^*, y_t^*, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, g_{x,t}, m_t, \phi_t$;
- government policies: $\tau_t^h, l_t, t_t, b_{g,t}$;
- domestic shocks: $g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o$.

The equations describing the law of motion of the variables are given by a set of equilibrium conditions for the household (equations 1.1-1.28), firms responsible for domestic production (equations 1.29-1.38), exporting and importing firms (equations

1.39-1.44), government (equations 1.45-1.56), foreign sector (equations 1.57-1.61), aggregation and price indexes (equations 1.62-1.78) and relative prices (equations 1.79-1.84). Additionally, there are 4 exogenous processes for sectoral productivity and aggregate productivity growth $(a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon)$. As a consequence, there are 84 equations for endogenous variables[10] and 9 domestic exogenous stochastic processes for a total of 93 variables in the model.

The prices and the shocks are stationary, but the allocations must be normalized in order to ensure stationarity. The set of variables given by $\left\{\overline{K}_{n,t+1}, K_{n,t+1}, \overline{K}_{x,t+1}, K_{x,t+1}, I_t, I_{t,t}, I_{n,t}, I_{m,t}, I_{x,t}, I_{x,t}^d, I_{n,t}^d\right\}$ must be normalized by $z_t^* \Upsilon_t$, while the variables $\{Y_t, C_t, C_{t,t}, C_{n,t}, C_{m,t}, C_{x,t}, W_t, X_t, D_{m,t}, D_{xp,t}, B_{h,t+1}, B_{g,t+1}, IB_{t+1}, M_t, X_t^1, X_t^2, Z_t^1, Z_t^2, Y_t^1, Y_t^2, U_t^1, U_t^2, G_t, G_{t,t}, G_{n,t}, G_{x,t}, L_t, T_t\}$ must be adjusted by z_t^*. Finally, the prices in each sector for renting capital from households $\left\{R_{x,t}^k, R_{n,t}^k\right\}$ and the shadow prices of investment $\{\widetilde{q}_{x,t}, \widetilde{q}_{n,t}\}$ are divided by Υ_t^{-1}, while the Lagrange multiplier of consumption, $\widetilde{\lambda}_t$, is normalized by $(z_t^*)^{-1}$ to obtain λ_t.

1.6.1 Competitive Equilibrium

Given exogenous paths for shocks $\left\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o\right\}$, foreign sector variables $\left\{\Delta M_t^*, \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\right\}$, policy processes for interest rates $\left\{R_t, \widetilde{R}_t, R_t^f\right\}$ and taxes τ_t^h, and initial values for prices $\{\pi_{-1}, \pi_{n,-1}, \pi_{x,-1}, \pi_{t,-1}, \pi_{m,-1}, w_{-1}, pt_{-1}, pn_{-1}, px_{-1}, pm_{-1}, pm_{-1}^*, tot_{-1}\}$ and allocations $\left\{c_{-1}, i_{x,-1}^d, i_{n,-1}^d, \overline{k}_{x,0}, \overline{k}_{n,0}, b_{h,-1}, b_{g,-1}, ib_{-1}, s_{n,-1}, s_{m,-1}, s_{x,-1}, s_{xp,-1}, l_{-1}\right\}$, a stationary competitive equilibrium is a set of processes for prices $\{\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r_{x,t}^k, r_{n,t}^k, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi_{x,t}^*, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}_{x,t}^*, pt_t, pn_t, px_t, pm_t, pm_t^*, tot_t\}$ and allocations $\{c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i_{x,t}^d, i_{n,t}^d, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x_t^1, x_t^2, z_t^1, z_t^2, y_t^1, y_t^2, u_t^1, u_t^2, ib_t, b_{h,t}, b_{g,t}, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, m_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, g_{x,t}, t_t, l_t,\ \phi_t\}$

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[10] Note that equation 1.58 is a 5-variable VAR.

such that, after stationary transformations of the respective equations: a) households maximize utility; b) firms maximize profits; c) government balances its budget; d) markets clear.

1.6.2 Ramsey Equilibrium

The Ramsey equilibrium is evaluated by the "timeless perspective" described in Woodford (2003)[36], where the government is assumed to run the policy committed for a very long time. An alternative interpretation of this approach is that the government can not change its policy from the time when the Ramsey policy is implemented to the next periods. Given that capital is a predetermined variable in the model, the Ramsey planner, without this constraint, could maximize its revenues setting a very high value for τ_t^k at $t = 0$ and run an alternative policy for $t = 1, 2, 3...$ In this sense, this constraint eliminates any dynamics resulting from the initial state of the economy, and the economy fluctuates around its optimal policy steady state.

Given exogenous paths for shocks $\left\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o\right\}$ and foreign sector variables $\left\{\Delta M_t^*, \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\right\}$, previously defined, and a set of initial values for Lagrange multipliers, a Ramsey equilibrium is a set of processes for prices and allocations that maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma)\log\left(C_t(i) - \zeta C_{t-1}\right) + \gamma \log\left(1 - h_t(i)\right)\right]$$

subject to the equilibrium conditions of the competitive equilibrium and $R_t \geq 1$.

A couple of notes regarding the Ramsey equilibrium. First, the Ramsey equilibrium for a small-open economy must explicitly include an extra non-Ponzi game condition for the evolution of government liabilities. As explained in Schmitt-Grohé and Uribe (2003b)[30], the absence of an explicit non-Ponzi game condition for liabilities allows the government to run explosive schemes against the rest of the world as the optimal policy, using its own stock of liabilities to absorb all the shocks. In the model here, even with the government not trading international bonds, the optimal fiscal policy still could result in non-stationary behavior, as the government sets domestic interest rates low enough to induce households to use foreign bonds to allocate resources across time. The presence of portfolio adjustment costs, both in the domestic and in the international financial markets, combined with the risk premium function over foreign interest rate for borrowing in international markets, ensures that the Ramsey problem is stationary. To be more specific, the presence of portfolio adjusment costs in domestic financial markets imposes a discipline for the domestic household when setting its portfolio. The counterpart of the bonds traded in domestic markets is exactly the amount of debt issued by the government. Thus, the same constraint imposed on the household behavior is transfered for the government debt policy.

Second, the restriction that nominal interest rates must be at least larger than zero – the "zero lower bound problem" – presents an issue that must be carefully addressed. The model solution is obtained after a first-order log-linearization of the equilibrium conditions. As a consequence, for very large shocks, the lower bound for interest rates ($R_t \geq 1$) might be violated. In order to handle with this problem, Woodford (2003)[36], Adjemian, Pariès and Moyen (2007)[29] and Batini, Levine and Pearlman (2009)[6], add one extra term to the welfare function of the households, penalizing for high deviations of the interest rates from its steady state level. Here, the penalty function is asymmetric, reducing the welfare only for very low values of the nominal interest rates:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\gamma) \log \left(C_t(i) - \zeta C_{t-1} \right) + \gamma \log \left(1 - h_t(i) \right) + \exp \left[\omega_r \left(R_t / R \right) \right] \right\}$$

The asumetric term allows the Ramsey planner to make a choice between increasing the optimal level of inflation in steady state or reducing the variance of nominal interest rate changes in the dynamics of the optimal policy. Adjemian, Pariès and Moyen (2007)[29] document, in an estimated model for the Euro Area, a probability of 5% to violate the lower bound constraint for interest rates when the estimated model is centered around a steady state of 2% inflation per year under the competitive equilibrium. This probability increases to 13% if the steady state inflation is zero and to 37% under the Ramsey optimal monetary policy.

Parameter ω_r is calibrated to ensure that, in the ergotic distribution of the nominal interest rates, the probability of violating the lower bound of nominal interest rates is arbitrarily small.[11]

1.6.3 Welfare Computation

In order to compute the welfare costs of an alternative policy relative to the time-invariant Ramsey optimal fiscal and monetary policy, denote $\mho_i$ the welfare associated with a given monetary and fiscal policy regime indexed by i, measured in terms of the period utility function of the households following the policy functions c_t^i and h_t^i for consumption and labor supply, respectively[12]. The welfare *conditional on a initial state* at period zero of adopting policy i is defined as:

$$\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right)$$

$$U_t\left(c_t, h_t\right) = (1-\gamma) \log \left(c_t - \frac{\zeta c_{t-1}}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right) + \gamma \log\left(1 - h_t\right)$$

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[11] The solution for the Ramsey problems is computed using the package Dynare for Matlab, combined with Andrew Levin's code to write the problem. For Levin's code, see Levin, Onatski, Williams, and Williams (2006)[23].

[12] We ignore here the term adjusting for the stationary process of consumption in the utility function, $(1-\gamma)\log z_t^*$, since the policies compared here do not change the long run growth rate of the economy, z_t^*. As a consequence, the welfare cost of the alternative policies is not affected by this term.

Note that E_0 defines the expectations operator in terms of period zero. Using equivalent notation, the *unconditional welfare* of adopting policy regime i is defined as:

$$\mho_i^u = E_t \sum_{t=0}^{\infty} \beta^t U_t \left(c_t^i, h_t^i\right)$$

Following Schmitt-Grohé and Uribe (2005 and 2005b)[32][33], the welfare cost λ_c of adopting the alternative policy regime i instead of the Ramsey monetary and fiscal policy r is measured in terms of the share of consumption the households give up in order to be indifferent between the two policy regimes:

$$\begin{aligned}\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right), h_t^i\right) & \\ = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1-\lambda_c)\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right), h_t^r\right) & \quad (1.85)\end{aligned}$$

Using the period utility function of the households, the welfare cost λ_c can be expressed as:

$$\frac{\mho_i^c - \mho_r^c}{(1-\gamma)} = \log\left((1-\lambda_c)\, c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z \left(\mu_0^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) - \log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z \left(\mu_0^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) + \frac{\beta}{1-\beta}\log\left(1-\lambda_c\right)$$

Following again Schmitt-Grohé and Uribe (2005 and 2005b)[32][33], note that, by the expression above, λ_c is a function of the vector of states and shocks of the model, as they determine the welfare and the consumption in each period. In order to make the shocks relevant to welfare, λ_c is computed based on a second order approximation of the equilibrium conditions. Using the authors result, the final expression for the welfare costs of alternative policies, λ_c, for a vector of exogenous shocks with variance σ^2, is given by:

$$\lambda_c = \frac{\mho_{r,\sigma\sigma}^c - \mho_{i,\sigma\sigma}^c}{(1-\gamma)\left(\frac{\beta}{1-\beta} + \frac{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}} - \zeta}\right)} \times \frac{\sigma^2}{2}$$

where $\mho^c_{r,\sigma\sigma}$ and $\mho^c_{i,\sigma\sigma}$ are the second derivatives of the welfare function in terms of the vector of exogenous shocks σ. It is worth noting that this measure of welfare cost will also be used to compare the loss of the Ramsey policy under the constraint of the zero lower bound of interest rates.

2

Optimal Policy: The Ramsey Steady State

The main objective of this chapter is to evaluate the Ramsey planner's choices in terms of steady state policies and allocations with a calibration based on the literature of medium scale macroeconomic models. The simulations performed here do not target matching specific moments. Instead, the main goal is to understand the trade-offs presented in the planner's problem and the optimal responses given restrictions imposed by parametric assumptions and by the number of instruments available for the planner. As a consequence, this section does attempt to highlight differences between SOEs and EMEs, but, instead, tries to clearly state the priorities of the Ramsey planner when defining the optimal policy.

Few authors in the literature provide a comprehensive discussion about the properties of the steady state under the Ramsey policy. Woodford (2003)[36] provides a complete description of the steady state policy of the basic New Keynesian model for closed economies. The author explore the differences in the Ramsey setup when imposing additional restrictions like those included here, as the "timeless perspective" of the Ramsey formulation, discussed in the definition of the Ramsey equilibrium in chapter 1, and one possible solution for the zero lower bound constrain. Still in the closed economy framework, but now dealing with variations of models similar in structure to CEE (2005)[10], Schmitt-Grohé and Uribe (2005, 2005b, 2007)[32] [33] [34] explore the properties of the steady state under Ramsey optimal monetary and fiscal policy. These medium-scale models do not have a closed form solution, like the basic formulations described by Woodford (2003)[36]. As a consequence, the only way to understand and describe optimal policy is using numerical simulations. The results in terms of steady state of prices usually point out for price stability as the main outcome of the Ramsey planner, with small variations depending on the number of nominal and real rigidities included in the model.

The chapter is organized as follows. In the next section, the baseline calibration is described, with details about the big ratios and the range of the parameter simulations. Section 2.2 shows the optimal policy choices of taxes and interest rates for the general model, where the Ramsey planner can make use of all policy instruments available. The next two sections deal with two special cases: first, in section 2.3, the classical problem of choosing the optimal relative taxation between capital and labor is approached in a model where the government has no access to consumption taxes; next, the case where the government can not discriminate between production inputs using taxes is discussed, in a version of the model where the government sets optimal taxation using only income and consumption taxes. Introducing a limiting case, section 2.5 describes the optimal policy when the government has access only to an income tax. Finally, section 2.6 highlights the effects of the correction for the zero lower bound for nominal interest rates in the steady state of inflation and taxes and section 2.7 concludes.

In terms of results, price stability seems to be the main goal of the Ramsey planner around the parameters used in calibration. However, the number of taxes available for the government plays a key role in explaining how the government sets the optimal taxes and interest rates under different assumptions on nominal and real rigidities. To be more specific, the inclusion of consumption taxes as one of the taxes available (but not the single tax instrument) in the model results in price stability as the optimal outcome, eliminating all the trade-offs related to the combination of nominal and real rigidities in the model only with income taxation. This result confirms, for a model with a large number of real and nominal rigidities, designed for small open economies, the propositions in Correia, Nicolini and Teles (2008)[13] regarding the role of a tax over the final good of the economy, vis-a-vis a tax over intermediate inputs. However, it is worth noting that the classical result from Judd (2002)[20] of a high subsidy to capital relative to the returns of labor is robust no matter the set of instruments available to the benevolent government.

2.1 Calibration

The steady state of the model described in the previous chapter is fully characterized by the parameters listed in table 2.1 and the big ratios used to define the structural parameters under the competitive equilibrium. In a brief description of these ratios, assume that, in steady state, the domestic economy and the rest of the world stabilize the price level in all sectors. This assumption, which is common in models where the traditional New Keynesian Phillips curve is adopted[1], implies that there is no persistent loss due to price dispersion across firms in steady state. The growth rate of productivity is set at 2% per year, while, for simplicity, the growth in investment-specific technological shock is set at zero[2]. The growth rate of productivity obviously implies that output per capita and other cointegrated variables grow at the same annual rate. It is assumed that households spend 20% of their time endowment at work ($h = 0.2$), following close the standard calibration proposed in Schmitt-Grohé

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[1] It's worth remember at this point that the New Keynesian Phillips curve is a log-linear approximation of the first order condition of the firms with respect to prices around a steady state of price stability.

[2] The same assumption is made during estimation in chapter 3.

and Uribe (2005b)[33]. When describing the government operations, it's important being able to compare the results in this chapter with other papers in the literature. In this sense, the calibration relies on standard numbers for the United States, with the ratios with respect to GDP of government spending, money supply and net public debt set at 17%, 16.95% and 42%, respectively, following Schmitt-Grohé and Uribe (2005)[33]. The competitive equilibrium value for taxes in the US between 1990 and 2000 are taken from Carey and Rabesona (2003)[8], following an updated methodology derived from Mendoza, Razin and Tesar (1994)[26]. Taxes on capital (τ^k), labor (τ^h) and consumption (τ^c) are set at 39.5%, 23.4% and 6.4%. In the foreign sector, assume that the risk premium is set to zero and the trade balance is in equilibrium.

The choice of parameter values, presented in table 2.1, is based on the literature of medium scale models for economies with a similar set of rigidities as those presented here. Some values are standard, like the depreciation rate of capital at 10% per year ($\delta = 0.025$), the capital share representing 30% of output and the discount factor β targeting an annualized real interest rate of 4% in the balanced growth path. The final value of β, higher than the usual calibrations in RBC models, is comparable to other studies where there is a non-stationary component in productivity[3]. Due to the absence of empirical estimates for the share of tradable goods in the GDP, assume that these goods represent around 55% of the consumption basket. The share of imported goods in the aggregate consumption is set at 20%.

For didactic purposes, assume that the price elasticity of demand for each sector in the economy, $\eta_i, i = \{n, x, m, xp\}$, is the same. There is empirical evidence supporting the hypothesis that firms trading in foreign markets, especially exporting firms, present higher markups over prices when compared to firms trading only in the

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[3] It's worth noting that the value of β will change during the estimation procedure, since the growth rate of productivity will be calibrated to match the average growth of the economy in the dataset utilized.

Parameter	Description	Value	Source
δ	Depreciation rate*	0.025	
θ	Capital share*	0.3	
β	Discount factor	0.9952	
ω	Share of tradable goods*	0.55	
$\varkappa$	Share of imports in tradable*	0.36364	
$\eta_n = \eta_x$	Price elasticity demand domestic goods	5	SGU (2007)[34]
η_m	Price elasticity demand imported goods	5	
η_{xp}	Price elasticity demand exported goods	5	
$\alpha_n = \alpha_x$	Calvo parameter domestic goods	0.6	CEE (2005)[10]
α_m	Calvo parameter imported goods	0.6	
α_{xp}	Calvo parameter exported goods	0.6	
ζ	Habit persistence	0.55	SW (2003)[35]
ϖ	Elast. subst. across labor types	21	CEE (2005)[10]
κ_1	Elast. R_t^f to exogenous risk premium	1	
κ_2	Elast. R_t^f to net foreign position	1	
η^*	Elast. subst. domestic exports to ROW	1	
θ_2/θ_1	Adjustment of capacity utilization	2.02	SGU (2005b)[33]

Table 2.1: Calibration for Steady State. (*) Same calibration used in estimation.

domestic goods market (see De Loecker and Warzynski, 2009[24]). When estimating the model, these differences will be noticed. Parameters η_i are calibrated such that the firm markup is set at 25% (see Basu and Fernald, 1997[5], and Schmitt-Grohé and Uribe, 2007[34]). The same reasoning is used to set, in the standard calibration, the Calvo parameter assigning the probability of a firm not adjusting its prices. Following the estimation of CEE (2005)[10], $\alpha_i, i = \{n, x, m, xp\}$, is set to 0.6.

Estimates of the habit persistence parameter in consumption are very unstable, with severe implication for the other parameters of the model. Justiniano and Preston (2009)[21] report, for different dataset and assumptions on the structural model, values in the range of $[0.05, 0.82]$. Garcia-Cicco (2008)[17] estimates a structural model with Mexican data and finds, in the baseline estimation, a value of 0.83, decreasing to 0.13 depending on the assumptions of the model structure. Given the wide dispersion of estimates, for this chapter assume that, in the baseline scenario, the habit persistence parameter ζ is set at 0.55, following the estimate of Smets and Wouters (2003)[35] in a model for the Euro Area.

The elasticity of substitution across different labor types, ϖ, is usually calibrated in the literature. For emerging economies, Garcia-Cicco (2008)[17], based on the literature for emerging economies, sets a value where the markup of wages over the marginal rate of substitution between labor and leisure equals 100% ($\varpi = 2$). This is the same value calibrated in Smets and Wouters (2003)[35]. The estimation of CEE (2005)[10] is the baseline for the calibration used in Altig, Christiano, Eichenbaun and Lindé (2005)[3], Schmitt-Grohé and Uribe (2005, 2005b)[32][33] and Adolfson, Laseén, Lindé and Villani (2007)[2]. The estimation implies a markup of only 5% ($\varpi = 21$). For this chapter, the baseline scenario assumes the small markup value, leaving the analysis of different degrees of wage stickiness in the estimation.[4]

Given the functional form adopted for the cost of adjusting the capacity utilization, and the hypothesis that, in the steady state of the competitive equilibrium, the economy operates at full capacity ($\mu_n = \mu_n = 1$), the computation of the steady state demands a value for the ratio of parameters θ_2/θ_1. The value used for the simulations follows Schmitt-Grohé and Uribe (2005b)[33], based on the estimation of Altig, Christiano, Eichenbaun and Lindé (2005)[3]. Despite the fact that this ratio is irrelevant for the steady state under the competitive equilibrium, the possibility of different levels of capacity utilization under the Ramsey policy forces an assumption regarding this value.

For the remaining parameters, the price elasticity of demand from the rest of the world for the domestic good, η^*, is set at unity, just like the elasticity of the domestic bonds traded in foreign markets to the world's risk premium and the domestic economy's net foreign asset position (κ_1 and κ_2). These parameters are used only

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[4] Notice, however, that there is an equivalence in terms of results in the steady state depending on the combination of values between habit persistence and wage stickness. This result better explained in section 2.3.

to determine the level of foreign variables, without any repercussion to the domestic economy's steady state.

2.2 An overview: The case with all taxes available

Consider the case where the government has access to all the fiscal policy instruments described in the model in chapter 1: taxes on consumption (τ^c), labor income (τ^h), capital income (τ^k), profits (τ^ϕ), the control of money supply (m) and debt (b_g). Despite providing large degrees of freedom for the Ramsey planner to optimize the objective function, this exercise provides a benchmark for the results in the following sections. The combination of income and consumption taxes has been explored in simple monetary models usually without capital. Examples can be found, in a flexible price environment for closed economies, in Chari, Christiano and Kehoe (1996)[9] and De Fiore and Teles (2003)[14] and for models with sticky prices in Correia, Nicolini and Teles (2008)[13]. The main focus of these papers is establishing the conditions under which the Friedman rule can be sustained as the optimal monetary policy framework. In the open economy framework, Adao, Correia and Teles (2009)[1] show, in a model without capital and with restrictions on labor mobility, that if each country in a single currency area can tax domestic consumption and labor income, the real exchange rate is completely irrelevant to characterize the optimal allocations and, as a consequence, welfare.

Table 2.2 describes the optimal choices of nominal interest rates and taxes on capital, labor and consumption under different assumptions regarding nominal rigidities, indexation, parameters characterizing the open economy and taxation on profits. The results in line 1 are based on the standard calibration described in the previous section. The baseline scenario assumes that the tax on profits is set at the same rate as the tax on capital ($\tau^\phi = \tau^k$) and that there is no indexation in prices.

There are two striking results in the first panel of table 2.2. First, the degree of

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.35	100	-100	Baseline scenario
2	0	0	-3.85	0.00	-15.35	100	-100	
3	0.6	0	0.00	4.00	-15.35	100	-100	
4	0	0.6	0.00	4.00	-15.35	100	-100	
				Indexation:				
5	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
				Open Economy:				
10	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.13	100	-100	$\varkappa = 0.01$
14	0.6	0.6	0.00	4.00	-13.11	100	-100	$\omega = 0.01$
15	0.6	0.6	0.10	4.10	-15.27	100	-100	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.35	100	-100	$\pi = 1.0074$ (3%p.y.)
				Profit Taxation:				
17	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 0$
18	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 1$

Table 2.2: Optimal inflation and taxes. Baseline scenario: $\pi^* = \pi^o = 0$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $tb/y = 0$; $\tau^\phi = \tau^k$.

nominal rigidity does not affect the optimal policy in terms nominal interest rates, as there are only two possible outcomes regarding monetary policy, irrespective of the values for the Calvo parameters: price stability or the Friedman rule. The Friedman rule is the optimal policy under price flexibility or under conditions where the output loss due to sticky prices are removed, like some cases of indexation described below. For every other combination of parameters setting price rigidities, price stability is the optimal policy outcome. Second, also irrespective of the main parameters of the model, taxation on labor is set at 100%, while the tax on consumption is, in fact, a subsidy of 100%. As a matter of fact, the two results might be connected. De Fiore and Teles (2003)[14] and Correia, Nicolini and Teles (2008)[13] show that, if the conditions for uniform taxation on consumption goods are satisfied[5] and the Friedman rule is the optimal policy, than consumption must be fully subsidized and labor income fully taxed. One of the reasons for this result is that the number of policy instruments is at least enough to eliminate the distortions generated from different frictions affecting the steady state of households and firms allocations. If this is the case, than money becomes nonessential, in the sense that any level of money holdings satisfy the households' equilibrium conditions, and the Ramsey planner eliminates the cost of shopping fully subsidizing household consumption.

There are three motives to believe that the result described in De Fiore and Teles (2003)[14] and Correia, Nicolini and Teles (2008)[13] also applies in this framework for small-open economies. First, the log-separable utility function in consumption and labor satisfies the implementation conditions for uniform taxation in consumption. Second, the robustness of results in terms of the taxes in consumption and labor even under different parameterization of the model. Third, as described in lines 17 and 18, is the fact that the Ramsey policy remained exactly the same as in the baseline calibration under different assumptions for taxation on profits. As discussed in Schmitt-Grohé and Uribe (2005b)[33], profits are a lump sum transfer from firms to the households. If allowed to set it optimally, the Ramsey planner chooses to confiscate all income from profits to finance its spending with minimum distortion of the households and firms allocations, setting $\tau^{\phi} = 1$. In the model proposed here, with the current number of policy instruments, the Ramsey planner is indifferent to the inclusion of a lump sum instrument.

The second panel of table 2.2 shows alternative scenarios regarding indexation. Three possible combinations of scenarios can generate the Friedman rule as an outcome for monetary policy. In line 5, with the economy under full indexation, the

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[5] These conditions are the separability between labor and consumption and homotheticity in consumption in the utility function.

output loss due to price dispersion across firms is eliminated, generating, in steady state, a similar framework to complete price flexibility. However, as line 6 shows, it is necessary full indexation for both production and retail firms, since the policy with indexation present only on production is very similar to the baseline scenario in line 1. Lines 7, 8 and 9 show that the Friedman rule might return as a policy outcome with indexation in domestic production firms if prices for imported goods firms are flexible. Thus, the Friedman rule under the current tax system may still emerge as a solution under a restrict set of conditions: price flexibility; full indexation in prices of domestic firms and retailers; full indexation in prices of domestic firms with price flexibility of imported goods' retailers.

When structural parameters are changed, the tax instrument affected is the capital taxation, which is, under the baseline calibration, a subsidy. The intuition for the subsidy in capital was developed in Judd (2002)[20], where the presence of imperfect competition in product markets creates a distortion proportional to the price markup resulting from imperfect competition on the household's intertemporal substitution of consumption. This distortion is increasing over time. Schmitt-Grohé and Uribe (2005b)[33] explore the properties of the subsidy for case with depreciation and time-varying capital utilization in a closed economy. Lines 13 and 14 in the table, however, show that parameters related to the open economy framework affect the size of the subsidy, as it declines with a reduction for the demand of imported goods. In order to understand the result, note that the steady state of capital taxes and the return on capital are given by:

$$\tau^k = 1 - \left(r_i^k \mu_i - a\left(\mu_i\right)\right)^{-1} \left(\frac{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}{\beta} - 1 + \delta\right)$$

$$r_i^k = mc_i \theta \left(\frac{k_i}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} h_i}\right)^{\theta-1}$$

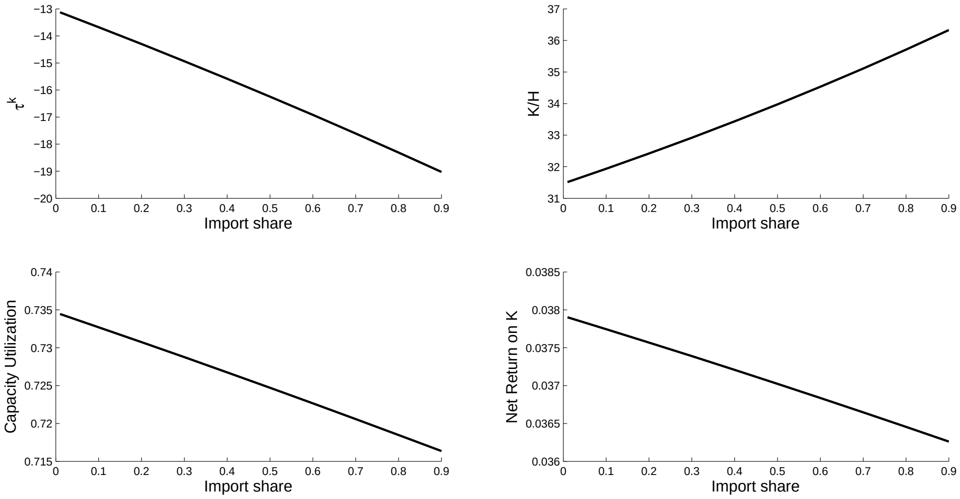

FIGURE 2.1: Optimal taxation on capital and openness

As $\varkappa$ or ω approach zero, the share of total investment based on domestic production increases, as the total demand of imported goods (c_m and i_m) decreases[6]. Without the imported good, the demand for domestic investment goods increases, also increasing the marginal return on capital (r_i^k) and reducing the subsidy necessary to reduce the distortions from the price markup. Figure 2.1 shows the size of the subsidy on capital, the capital-labor ratio, the rate of capital utilization and the marginal return of capital net of adjustment costs ($r_i^k \mu_i - a(\mu_i)$) as a function of the share of imported goods in the tradable goods basket $\varkappa$.

Finally, the only difference in allocations and policies in this setup of taxes was found when the inflation in steady state for the foreign economy was larger than zero. However, as the result in line 15 shows, the increase in domestic inflation is smaller than the change in foreign inflation: an inflation of 3% per year in the rest of the world results in an increase of less than 0.1% in the inflation under the optimal

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[6] Remember, from the optimal choice of the households based on the CES aggregator that $\varkappa$ is the ratio of imported goods in the basket of tradable goods and ω is the share of tradable goods in the basket of total demand.

Ramsey policy. Also notice that this small deviation is a consequence only of foreign inflation, as a positive value in the underlying competitive equilibrium does not alter results from the baseline scenario.

Given the results on taxes and inflation, the next three sections gain in importance, as the constraint in the number of policy instruments will describe the policy choices of the Ramsey planner. From results in this section, it is already known, so far, that price rigidities restrict the policy choices between price stability and the Friedman rule. The presence of markups over prices also implies, just like as in Judd (2002)[20], that the optimal tax on capital is actually a subsidy. The constraint in the number of instruments makes explicit a ranking of preferences in terms of which distortions in the model must be addressed with higher priority. It also provides a robustness test for the hypothesis of price stability as the major goal for monetary policy, as inflation, seen as a tax on money holdings, gains in importance as the alternative instruments become unavailable.

2.3 The case without consumption taxes

In this section, consider the case where the government can not directly tax consumption ($\tau^c = 0, \forall t$). Despite not being able to tax consumption, the Ramsey planner is still capable of perfectly discriminating the household's sources of income, as it is allowed to tax labor, capital and profits income at different rates. In order to understand the importance of the tax discrimination between labor and capital income, consider first the stationary transformation of the intertemporal Euler equation of the households when consumption can not be taxed:

$$\left(1-\tau^h\right) w\,(1-h) = \frac{\gamma}{(1-\gamma)} mcw \left(1 - \frac{\zeta}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) c \left[1 + \nu^m \left(\frac{R-1}{R}\right)\right]$$

The expression in the left-hand side is the (after-tax) value of leisure. On the right-hand side, the second term, $mcw = \frac{\varpi}{\varpi - 1}$ in steady state, is the wedge between wages and the marginal disutility of labor, generated by wage stickiness, while the third term, $\left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right) c$, is the consumption adjusted for habit persistence. The last term, $\left[1 + \nu^m \left(\frac{R-1}{R}\right)\right]$, is a consequence of the money demand by the households. Thus, there are two policy instruments in this equation (nominal interest rates and labor income taxes) set by the Ramsey planner and three distortions: habit persistence, wage gap and the financial friction due to the cash-in-advance constraint. Note that, by construction, the distortions generated by habit persistence and wage gap operate with opposite signs: on the one hand, high elasticity of substitution across labor types, given by ϖ in the equation describing the demand for labor of household i, $h_t(i)$, implies low distortions from wage stickiness, with mcw having a lower bound at one; on the other hand, high values of ζ imply high degree of habit persistence on consumption, with the after-tax value of leisure approaching zero[7]. As a consequence, for an appropriate combination of ζ and ϖ, it is possible that the wage gap and the habit persistence offset each other, making the cash-in-advance constraint the only relevant distortion in the labor-leisure allocation.

The perfect discrimination between capital and labor income through taxes still allows the Ramsey planner to formulate fiscal and monetary policies resulting in prices and allocations close to the Friedman rule. In order to understand why, note that with the taxation on capital designed to eliminate the wedge from the markup over prices, labor taxation can be used to minimize the gap between the efficient and the distorted intratemporal labor allocation. As seen above, for special cases of the parameters characterizing the rigidities from wage stickiness and habit persistence,

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[7] It's obvious that, for the model where the productivity growth is equal to zero, the expression $\left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)$ is exactly bounded between zero and one.

the cost of holding money might emerge as the most relevant distortion affecting the labor allocation. Thus, in the limiting case of this special combination of parameters, the Friedman rule surges as the optimal policy outcome.

The role of real rigidities and the possibility of a monetary policy outcome close to the Friedman rule can be better visualized in figure 2.2. In the figure, the vertical axe shows the Ramsey inflation as a function of the Calvo probability of a firm in the domestic production sector (α_n) and the imported goods sector (α_m) changing prices. The figure has two surfaces, with the surface computed for the baseline scenario overlapping the surface of the scenario assuming a high degree of habit persistence on consumption ($\zeta = 0.90$). Note, in the figure, that inflation is decreasing as prices in both sectors become more flexible, with a policy close to the Friedman rule achieved when prices in both sectors are close to full flexibility. Also note that the economy with high habit persistence presents policies closer to the Friedman rule even in the presence of a larger degree of price rigidity. In the intratemporal Euler equation described above, the increase in ζ reduces the distortion generated by wage stickiness, allowing the Ramsey planner to reduce the cost of holding money even in the presence of price rigidity.

Table 2.3 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities. Line number 1 presents the baseline scenario, with no indexation, taxation of profits in the same level as the taxation on capital and price stability in the underlying competitive equilibrium both domestically and in the rest of the world. The baseline scenario is characterized by a small deflation, and, as expected, a combination of subsidy on tax capital and high taxes on labor income. The second line of the table confirms the result that under flexible prices the Friedman rule is the optimal outcome of the Ramsey problem, as monetary policy tries to eliminate the cost of carrying money. However, as lines 3 and 4 show, the mild deflation returns as a result if there is heterogeneity in price rigidity across sectors.

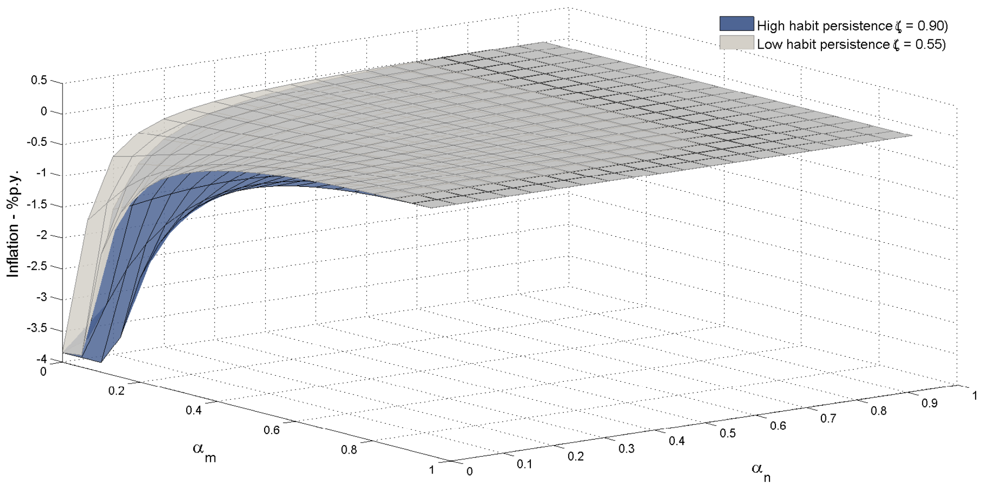

Figure 2.2: Real rigidities and optimal inflation: no consumption taxes

The second panel of table 2.3 confirms the scenarios presented in the previous section, regarding indexation, where the Friedman rule is optimal.

The third panel of table 2.3 details the optimal policy changing some parameters that are specific to the small-open economy described by the model. First, in lines 10-12, note that price flexibility for firms in the exported goods' sector do not alter the results in terms of indexation and price flexibility described in the first two panels. This is also the same result obtained in the previous section. In lines 13 and 14, reducing the share of tradable goods, ω, to 1% of the total domestic absorption, and the share of imported goods, $\varkappa$ to 1% of the domestic absorption of tradable goods, respectively, the ”closed” economy features negative inflation in steady state, in a result similar to Schmitt-Grohé and Uribe (2005b)[33][8]. Note that the reduction of the capital tax subsidy as ω and $\varkappa$ converges to zero also shows up in this framework. Lines 15 and 16 also confirm the results of a small effect of foreign inflation in the

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[8] The baseline results in Schmitt-Grohé and Uribe (2005b)[33] of a small positive inflation in steady state are a consequence of the assumption of a lump sum transfer from the government to the households, with inflation acting as a tax on consumption. When this transfers are set to zero, the small deflation emerges as the optimal policy outcome.

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	Obs.:
1	0.6	0.6	-0.11	3.88	-16.12	30.40	Baseline scenario
2	0	0	-3.85	0.00	-15.05	39.07	
3	0.6	0	-0.15	3.84	-16.12	30.41	
4	0	0.6	-0.44	3.54	-16.12	30.48	
Indexation:							
5	0.6	0.6	-3.85	0.00	-15.03	39.18	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
9	0	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
Open Economy:							
10	0.6	0.6	-0.11	3.88	-16.12	30.40	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.05	39.07	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	-0.44	3.54	-16.12	30.48	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	-0.19	3.80	-14.47	29.25	$\varkappa = 0.01$
14	0.6	0.6	-0.19	3.80	-14.46	29.24	$\omega = 0.01$
15	0.6	0.6	-0.01	3.99	-16.05	30.28	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	-0.11	3.88	-16.12	30.40	$\pi = 1.0074$ (3%p.y.)
Profit Taxation:							
17	0.6	0.6	-0.02	3.98	-15.35	36.49	$\tau^\phi = 0$
18	0.6	0.6	-0.02	3.98	-15.34	36.49	$\tau^\phi = 1$

Table 2.3: Optimal inflation and taxes – no consumption taxes. Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; tb/y=0.

determination of domestic optimal level of inflation.

Finally, changes in the profit taxation indeed affect the allocations and the optimal policy in this framework for taxes. The setting of a fixed rate for taxes on profits, instead of a choice based also on the taxation of capital, as in the baseline scenario, allows the Ramsey planner to approximate the optimal policy to price stability. Most of the adjustment to the change in the tax structure is reflected in the optimal level of debt supported by the households.

In this exercise, one of the policy instruments was removed, when compared to the setup in the previous section. Two results were robust to this formulation: the

Friedman rule as the optimal outcome under flexible prices or indexation structure simulating the flexible price economy; and the subsidy to capital compared to the high taxes on labor income, in order to eliminate the distortions discussed in Judd (2002)[20]. Despite low, inflation under different levels of price rigidity was not zero, like in the previous section. The next section will show that the consumption tax play a critical role in this regard.

2.4 The case of income and consumption taxes

Assume, for this section, that the government is constrained on taxing all income at the same tax rate – thus, $\tau^k = \tau^h = \tau^\phi = \tau^y, \forall t$. However, differently from the previous section, the government can optimally set the taxation on consumption. Despite both taxes affect the intratemporal decision between labor and consumption of the household, the distinction and optimal mix between taxes on income and on consumption is relevant because of the additional dimensions of each tax in the model. Taxes on consumption will affect the transaction technology of the economy, since, for every unit of domestic of foreign good consumed, households must pay the tax. On the other hand, the taxation on income will distort the intratemporal decisions of capital accumulation, based on the net expected return of capital in the next period, and the use of the current stock of capital, since there is an allowance for changing the capacity utilization of the economy.

Table 2.4 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities, with the first line again describing the baseline scenario. The first block of results confirms the observations made in the previous section. The Ramsey planner still tries to tax labor more than capital, relying, in this case, in very high consumption taxes to pay for a subsidy on capital – in this case, the subsidy is implemented through negative income taxation. Regarding the optimal choice of interest rates, the model with income and consumption taxes replicates the results presented in section 2.2, where inflation departs from zero only in conditions of full price flexibility or in the case of indexation where the distortions from price stickiness are eliminated. In this cases, the optimal policy approaches the Friedman rule. In Correia, Nicolini and Teles (2008)[13], the authors prove that allocations in a closed economy without capital are the same under flexible and sticky prices as the optimal outcome of the Ramsey problem, irrespective of the degree of price stickiness. The important constraint necessary to prove the result is the minimum number of instruments (taxes) to operate fiscal policy, where consumption taxes is one of those taxes. Here, not only the model is designed to describe an open economy, but also the presence of capital add one more source of revenues from income taxation, increasing the complexity of the model. In this sense, the results of Correia, Nicolini and Teles (2008)[13] seem to be robust to such expansion of the model.

The intuition for the result in Correia, Nicolini and Teles (2008)[13] is that, under the appropriate set of instruments, consumption tax operates as a state-contingent price used by the government to replicate the Pareto efficient allocations even under sticky prices. It is worth noting that it's only the consumption tax that is capable to operate like this: the same results can not be implemented if the tax system discriminates the income of capital and labor, as seen in the previous section. Thus, not only the number of instruments is relevant, but also that one of this instruments is the consumption tax. The optimal policy for prices, on the other hand, depends on the degree of price rigidity. Under flexible prices, taxes do not need to account for the distorting markups in production, and the optimal policy is the Friedman rule. Under sticky prices, price stability over all periods and states is the optimal outcome, as the elimination of production markups dominate the objective function of the planner, just like in section 2.2. In this sense, consumption taxes assume the role of debt in models with complete markets, with very high volatility in order to insure households against all possible states of world.

	α_n	α_m	π	R	$\tau^y(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.16	79.69	Baseline scenario
2	0.0	0.0	-3.71	0.15	-15.16	82.30	
3	0.6	0.0	0.00	4.00	-15.16	79.69	
4	0.0	0.6	0.00	4.00	-15.16	79.69	
Indexation:							
5	0.6	0.6	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
7	0	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
Open Economy:							
10	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0$
11	0.6	0.6	-3.79	0.06	-15.16	82.36	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.12	83.84	$\varkappa = 0$
14	0.6	0.6	0.00	4.00	-13.12	83.84	$\omega = 0$
15	0.6	0.6	0.10	4.10	-15.10	79.85	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.14	79.60	$\pi = 1.0074$ (3%p.y.)

Table 2.4: Optimal inflation and taxes – consumption and income taxes. Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; tb/y=0.

In order to check the robustness of the Ramsey steady state with this combination of taxes, it becomes critical to evaluate prices and taxes under different parametrization of the nominal and real rigidities affecting the intratemporal Euler equation describing the consumption-leisure trade-offs. The stationary transformation of the intratemporal Euler equation when the government has access to consumption taxes is given by:

$$\frac{\left(1-\tau^h\right)}{\left(1+\tau^c\right)} w\left(1-h\right) = \frac{\gamma}{\left(1-\gamma\right)} mcw \left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) c\left[1+\nu^m\left(\frac{R-1}{R}\right)\right]$$

Again, as described in the previous section, there are three main rigidities affecting the intratemporal choice: habit persistence in consumption (given by parameter ζ), wage stickiness (setting a wedge given by mcw) and the cash-in-advance constraint

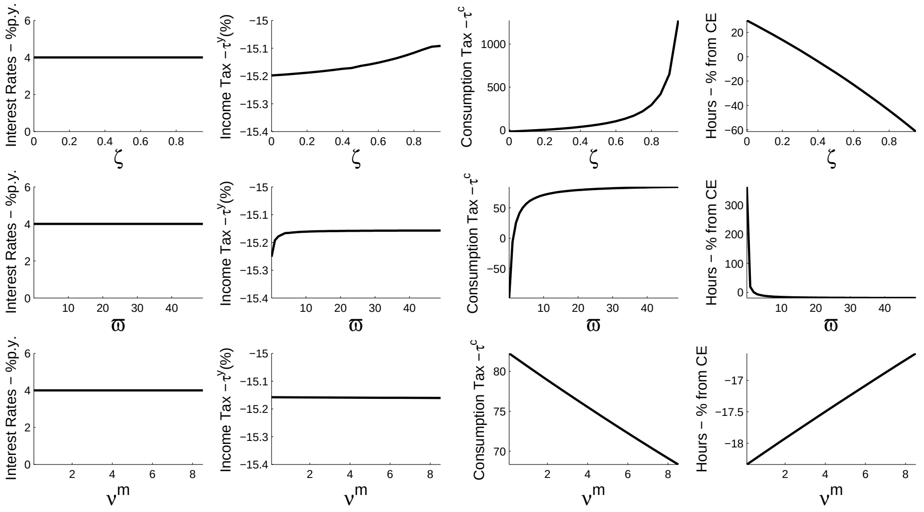

FIGURE 2.3: Optimal taxes and inflation: consumption and income taxes

(restricted by parameter ν^m). Figure 2.3 describe the taxes and nominal interest rate choices under all possible parameter combinations for these wedges. Each line of the figure shows how interest rate, taxes and the optimal labor supply responds when one of the parameters change along the interval of possible values. Notice that the Friedman rule is not a possible outcome, even eliminating most of the domestic rigidities present in the model. The optimal nominal interest rate does not diverge from 4% per year, implying that price stability is the optimal outcome. Income taxation also does not change significantly, always very close to the values found in the previous section as the subsidy for capital. The main adjustment, as expected, is on the consumption tax and on the labor supply, measured as the percentage deviation of the hours supplied under the Ramsey policy relative to the assumption of hours in the steady state of the competitive equilibrium.

This section characterized an extension of the results in Correia, Nicolini and Teles (2008)[13], showing the role of consumption taxes, even when the set of fiscal policy instruments is not as complete as in section 2.2. However, not only the number of instruments, but also the composition of the instrument set play a critical role in obtaining the result, as seen by the comparison of results with the previous section. In both cases, the Ramsey planner could set two taxes. However, in the previous section, a mild deflation was the optimal outcome in the Ramsey policy.

2.5 The case of an income tax

In this section, consider the situation where the government is restricted to operate fiscal policy setting only a distortionary tax on total income ($\tau^h = \tau^k = \tau^\phi = \tau^y$, $\tau^c = 0$). The case of a single tax on income is extensively explored in the literature when considering a single distortionary fiscal policy instrument. For closed economies, Schmitt-Grohé and Uribe (2005b)[33] details the steady state and the dynamics in a medium scale model with several rigidities. For an open economy, Benigno and De Paoli (2009)[7] explore the dynamics of a very simple model for small-open economies distorting the household intratemporal condition with a tax on total income. Ambler, Dib and Rebei (2004)[4], despite the focus on optimal monetary policy, also evaluates the dynamics of a small open economy with income taxes.

When the government has access to only one tax, it becomes impossible to set any sort of discrimination between production factors or between the demand of intermediate and final goods. With a single tax instrument and the money demand driven by the cash-in-advance constraint, the income tax is set in such a way that the government budget constraint is balanced. As a consequence of the lack of instruments, inflation is considered as an additional tax from the Ramsey planner's perspective. This is the main result from the first panel of table 2.5, since, for low levels of price rigidity, inflation is significantly larger than zero. Note that this is the opposite relation presented, for instance, in section 2.3, where inflation was decreasing as a function of price rigidity. For a simple comparison with the previous case, figure 2.4 replicates the same simulation presented in figure 2.2, showing two

	α_n	α_m	π	R	$\tau^y(\%)$	Obs.:
1	0.6	0.6	1.89	5.97	8.63	Baseline scenario
2	0.01	0.01	174.41	185.38	-0.29	
3	0.6	0.01	2.45	6.55	8.60	
4	0.01	0.6	6.11	10.35	8.11	
Open Economy:						
5	0.6	0.6	1.89	5.97	8.63	$\alpha_{xp} = 0.001$
6	0.6	0.6	6.14	10.38	8.11	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
7	0.6	0.6	2.44	6.54	11.37	$\varkappa = 0.05$
8	0.6	0.6	2.47	6.57	11.49	$\omega = 0.05$
9	0.6	0.6	1.97	6.05	8.69	$\pi = \pi^* = 1.0074$ (3%p.y.)
10	0.6	0.6	1.89	5.97	8.65	$\pi = 1.0074$ (3%p.y.)

Table 2.5: Optimal inflation and taxes – income taxes. Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $tb/y = 0$.

surfaces relating the Calvo parameter for domestic and imported goods' firms and the optimal of inflation for two different degrees of habit persistence in consumption.

There are two determinants of this result. First, notice, from the intratemporal Euler condition of the households setting the consumption-labor choice, that the money demand by the households sets up the nominal interest rate as an equivalent instrument as a tax in labor income. Thus, from the intratemporal allocation, a tax on labor income is observationally equivalent to an increase in nominal interest rates. Second, there is the objective of the Ramsey planner, present in all the simulations so far, in discriminating the returns from capital to the returns from labor, subsidizing the first at the expense of the later. Given that the nominal interest rate does not directly affect the capital allocation, raising inflation and the long run nominal interest rate is equivalent to imposing a large tax on labor. In order to confirm this result, simulations of the model without the cash-in-advance constraint result to price stability as the main outcome of the Ramsey policy with this configuration of taxes.[9]

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[9] Results available upon request.

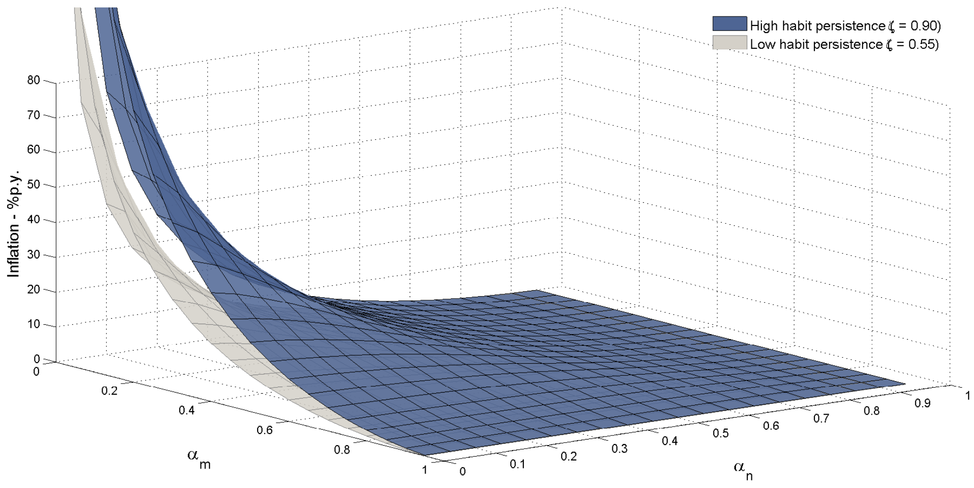

FIGURE 2.4: Real rigidities and optimal inflation: single income tax

Results in this section confirm, even under this extreme assumption regarding the number of taxes, the main priorities of the Ramsey planner, as discussed in the previous sections. The major difference is the instruments used in order to implement the optimal policy. Notice that, under this restricted set of instruments, the government still implements a relative subsidy on capital, even at the expense of an output loss due to the increase in inflation.

2.6 The zero lower bound and the steady state

The steady state of the Ramsey problem with the proposed correction for the zero lower bound problem for nominal interest rates highlights some of the trade-offs previously discussed in this chapter. The asymmetric term added in the objective function of the Ramsey planner shifts inflation to a higher level, reducing the probability that the zero lower bound for nominal interest rates is violated. Table 2.6 shows the optimal policy under different assumptions for domestic price rigidity and values for parameter ω_r, comparing the results with the baseline scenario presented

	α_n	ω_r	π	R	$\tau^k(\%)$	$\tau^h(\%)$	$\tau^c(\%)$	Obs.:
Full Set of Taxes								
1	0.6	0	0.00	4.00	-15.35	100.00	-100.00	Baseline
2	0.6	0.01	0.76	4.79	-15.36	100.00	-100.00	
3	0.6	0.1	6.09	10.33	-15.45	100.00	-100.00	
4	0	0.01	2.88	7.00	-15.36	100.00	-100.00	
5	0	0.1	15.67	20.30	-15.28	100.00	-100.00	
No Consumption Tax – $\tau^c = 0$								
6	0.6	0	-0.11	3.88	-16.12	30.40	0.00	Baseline
7	0.6	0.01	0.66	4.68	-16.16	29.90	0.00	
8	0.6	0.1	6.12	10.36	-14.97	37.62	0.00	
9	0	0.01	2.40	6.49	-16.16	29.50	0.00	
10	0	0.1	15.98	20.62	-14.79	36.23	0.00	
Consumption and Income Taxes – τ^c and $\tau^h = \tau^k = \tau^\phi = \tau^y$								
11	0.6	0	0.00	4.00	-15.16	-15.16	79.69	Baseline
12	0.6	0.01	0.77	4.80	-15.19	-15.19	79.68	
13	0.6	0.1	6.11	10.35	-15.47	-15.47	80.65	
14	0	0.01	2.88	6.99	-15.20	-15.20	78.37	
15	0	0.1	15.79	20.42	-15.68	-15.68	76.71	

Table 2.6: Ramsey policy and the zero lower bound. Baseline scenario: $\pi^* = \pi^o = 0\%$; $\varkappa = 0.363$; $\omega = 0.55$; $\alpha_{xp} = 0.6$; $\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 0$; $\tau^\phi = \tau^k$; tb/y=0.

in the previous sections. The first result detailed in table 2.6 is that the size of the adjustment is a function of the nominal rigidities in the model. The more flexible prices are, the smaller parameter ω_r must be in order to generate a significant deviation of the Ramsey policy for inflation from the baseline scenario. This is seen, for instance, in the first panel, comparing the results in lines 2 and 4, where the only difference between the two economies is the value of the Calvo parameter for domestic producers (α_n).

An important topic to be considered when implementing the adjustment to the zero lower bound problem is the new steady state values of the fiscal policy instruments. As the results in table 2.6 show, the number of instruments available plays a role again when a higher inflation is imposed as the Ramsey solution. In the first panel, when the government has all taxes available, the adjustment is made on the subsidy on capital, as the main result of the section 2.2 regarding taxes on consumption and on labor income remains the same. Without at least one of the taxes of the model, the final adjustment is distributed among the instruments remaining.

Notice, first, that the adjustment of the taxes related to capital is not linear as ω_r increases. In fact, there is an u-shaped behavior of capital taxes, mostly due to the cost of adjusting the capacity utilization in the model. Consider the effects of an increase in ω_r when the government has access to all taxes. This is an interesting case to evaluate, as the capital tax is the only instrument adjusted with the increase in inflation. For low levels of ω_r, inflation raises the markup over prices, changing the returns on capital and on labor. In order to compensate for the distortion from inflation, the Ramsey planner initially increases the subsidy on capital and reducing the steady state of the capacity utilization. However, deviations from the full utilization of capital increases the quadratic cost of adjustment of capacity utilization, reducing the net return of capital. There is obviously a limit in the reduction of the net return of capital. This is exactly the point where the Ramsey planner stops increasing the capital subsidy together with ω_r, starting the movement in the opposite direction. Figure 2.5 shows the capital tax, inflation, marginal costs and the rate of capital utilization as a function of ω_r for different assumptions regarding nominal rigidity. Note that the u-shaped path for capital taxes follows a close path to the rate of capital utilization, just as described.

2.7 Conclusions

In this chapter, the main results confirm the propositions in Correia, Nicolini and Teles (2008)[13] about the relevance of consumption tax in the determination of the optimal policy in terms of inflation. For every scenario not associated with the conditions of flexible prices, price stability is the optimal outcome of the Ramsey

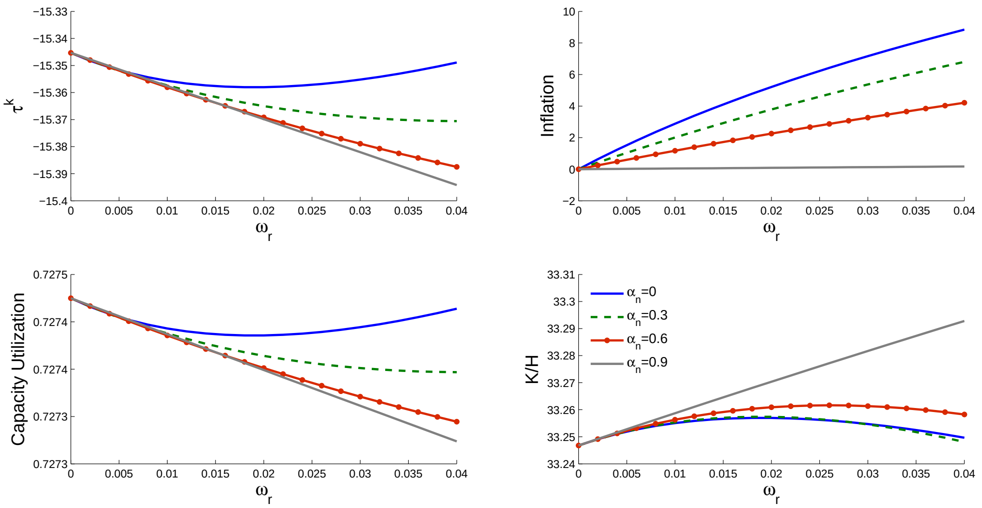

Figure 2.5: Optimal taxes and the zero lower bound for interest rates

problem. Also, the subsidy for capital is robust to every formulation in the model, confirming that the price markups distortions are the main target of the benevolent government when setting its policy. The small-open economy framework does not have a large influence in the determination of the steady state, mainly because the relevant distortions are still associated with the intertemporal and the intratemporal Euler equations of the household – structures that are irrelevant to the setup of an open economy.

The next chapter presents the details of the estimation and the first comparison between emerging and developed, small-open economies. However, the characterization of the steady state of this economies will appear only in chapter 4, as the dynamics of Ramsey problem is described by a first order approximation of the non-linear problem.

Appendix A

Stationary Equilibrium Conditions

In order to transform the model for the stationary form, first note that:

$$z_t^* = z_t \Upsilon_t^{\frac{\theta}{1-\theta}} \Longrightarrow \frac{z_t^*}{z_{t-1}^*} = \frac{z_t}{z_{t-1}} \left(\frac{\Upsilon_t}{\Upsilon_{t-1}} \right)^{\frac{\theta}{1-\theta}} = \mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{\theta}{1-\theta}}$$

Also:

$$\frac{z_t^* \Upsilon_t}{z_{t-1}^* \Upsilon_{t-1}} = \left(\frac{z_t^*}{z_{t-1}^*} \right) \mu_t^\Upsilon = \left(\frac{z_t \Upsilon_t^{\frac{\theta}{1-\theta}}}{z_{t-1} \Upsilon_{t-1}^{\frac{\theta}{1-\theta}}} \right) \mu_t^\Upsilon = \mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{1}{1-\theta}}$$

The stationary equilibrium conditions of the model are:

$$\frac{(1-\tau_t^h)}{(1+\tau_t^c)} w_t = \frac{\gamma}{(1-\gamma)} \left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \right) \frac{mcw_t \left(1 + \nu^m \left(\frac{\widetilde{R}_t - 1}{\widetilde{R}_t} \right) \right)}{(1-h_t)}$$

$$\left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \right)^{-1} (1-\gamma) = (1+\tau_t^c) \lambda_t \left(1 + \nu^m \frac{R_t - 1}{R_t} \right)$$

$$\lambda_t \left[1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right) \right] = \beta R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1} \left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon \right)^{\frac{\theta}{1-\theta}} \right)} \right)$$

$$\lambda_t \left[1 - \psi_2 \left(\frac{rer_t \; ib_{t+1}}{y_t} - \frac{rer \; ib}{y}\right)\right] = \beta R_t^f E_t \left(\frac{rer_{t+1}}{rer_t} \frac{\lambda_{t+1}}{\pi^*_{t+1} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)}\right)$$

$$k_{x,t} = \mu_{x,t} \overline{k}_{x,t}$$

$$k_{n,t} = \mu_{n,t} \overline{k}_{n,t}$$

$$\theta_1 + \theta_2 \left(\mu_{x,t} - 1\right) = r^k_{x,t}$$

$$\theta_1 + \theta_2 \left(\mu_{n,t} - 1\right) = r^k_{n,t}$$

$$R_t = \frac{1}{r_{t,t+1}}$$

$$\widetilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y}\right)\right)^{-1}$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon^n_{t+1}$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon^x_{t+1}$$

$$\lambda_t q_{x,t} = \beta E_t \left\{ \frac{\left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}}{\mu^\Upsilon_{t+1}} \lambda_{t+1} \times \right.$$

$$\left. \left[\quad \left(1 - \tau^k_{t+1}\right) \left(r^k_{x,t+1} \mu_{x,t+1} - a\left(\mu_{x,t+1}\right)\right) + q_{x,t+1}\left(1 - \delta\right) \quad \right] \right\}$$

$$\lambda_t q_{n,t} = \beta E_t \left\{ \frac{\left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}}\right)^{-1}}{\mu^\Upsilon_{t+1}} \lambda_{t+1} \times \right.$$

$$\left. \left[\quad \left(1 - \tau^k_{t+1}\right) \left(r^k_{n,t+1} \mu_{n,t+1} - a\left(\mu_{n,t+1}\right)\right) + q_{n,t+1}\left(1 - \delta\right) \quad \right] \right\}$$

$$\lambda_t = \lambda_t q_{x,t} \left[1 - \frac{\phi_i}{2} \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right.$$

$$\left. - \phi_i \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} \right) \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right) \right]$$

$$+ \beta E_t \left[\lambda_{t+1} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{x,t+1}}{\mu^\Upsilon_{t+1}} \times \right.$$

$$\left. \phi_i \left(\frac{i^d_{x,t+1}}{i^d_{x,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i^d_{x,t+1}}{i^d_{x,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{1}{1-\theta}} - \mu^V \right) \right]$$

$$\lambda_t = \lambda_t q_{n,t} \left[1 - \frac{\phi_i}{2} \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right.$$

$$\left. - \phi_i \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} \right) \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right) \right]$$

$$+ \beta E_t \left[\lambda_{t+1} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{n,t+1}}{\mu^\Upsilon_{t+1}} \times \right.$$

$$\left. \phi_i \left(\frac{i^d_{n,t+1}}{i^d_{n,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i^d_{n,t+1}}{i^d_{n,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1} \right)^{\frac{1}{1-\theta}} - \mu^V \right) \right]$$

$$\begin{aligned}
\overline{k}_{x,t+1} &= (1-\delta) \frac{\overline{k}_{x,t}}{\mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}}} + i^d_{x,t} \left(1 - \frac{\phi_i}{2} \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right) \\
\overline{k}_{n,t+1} &= (1-\delta) \frac{\overline{k}_{n,t}}{\mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}}} + i^d_{n,t} \left(1 - \frac{\phi_i}{2} \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t \right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right)
\end{aligned}$$

$$\left(\frac{\varpi-1}{\varpi}+\frac{1}{mcw_t}\right)\varpi h_t\left(1-\tau_t^h\right)=$$

$$-\frac{\phi_w}{\pi_t^{\chi_w-1}}\left(\frac{w_t}{w_{t-1}}\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)\left(\frac{w_t}{\pi_t^{\chi_w-1}w_{t-1}}\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}-\mu^I\right)$$

$$+\beta E_t\left[\frac{\lambda_{t+1}\phi_w}{\lambda_t\pi_{t+1}^{\chi_w-1}}\left(\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)^{-1}\times\right.$$

$$\left.\left(\frac{w_{t+1}}{w_t}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right)^2\left(\frac{w_{t+1}}{\pi_{t+1}^{\chi_w-1}w_t}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}}-\mu^I\right)\right]$$

$$c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{b_h}{y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2=\left[(1-\omega)^{\frac{1}{\varepsilon}}c_{n,t}^{\frac{\varepsilon-1}{\varepsilon}}+\omega^{\frac{1}{\varepsilon}}c_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$c_{t,t}=\left[(1-\varkappa)^{\frac{1}{\varrho}}c_{x,t}^{\frac{\varrho-1}{\varrho}}+\varkappa^{\frac{1}{\varrho}}c_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}}$$

$$c_{m,t}=\varkappa\left(pm_t\ pt_t\right)^{-\varrho}c_{t,t}$$

$$c_{x,t}=(1-\varkappa)\left(px_t\ pt_t\right)^{-\varrho}c_{t,t}$$

$$c_{t,t}=\omega\left(pt_t\right)^{-\varepsilon}\left(c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{B}{Y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2\right)$$

$$c_{n,t}=(1-\omega)\left(pn_t\right)^{-\varepsilon}\left(c_t+\frac{\psi_1}{2}y_t\left(\frac{b_{h,t+1}}{y_t}-\frac{B}{Y}\right)^2+\frac{\psi_2}{2}y_t\left(\frac{rer_t\ ib_t}{y_t}-\frac{rer\ ib}{y}\right)^2\right)$$

$$i_t=i_{n,t}^d+a\left(\mu_{n,t}\right)\frac{\overline{k}_{n,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}}+i_{x,t}^d+a\left(\mu_{x,t}\right)\frac{\overline{k}_{x,t}}{\mu_t^z\left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}}$$

$$i_{m,t}=\varkappa\left(pm_t\ pt_t\right)^{-\varrho}i_{t,t}$$

$$i_{x,t}=(1-\varkappa)\left(px_t\ pt_t\right)^{-\varrho}i_{t,t}$$

$$i_{t,t}=\omega\left(pt_t\right)^{-\varepsilon}i_t$$

$$i_{n,t}=(1-\omega)\left(pn_t\right)^{-\varepsilon}i_t$$

$$w_t = pn_t mc_{n,t}\left(1-\theta\right)a_{n,t}\left(\frac{k_{n,t}}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}h_{n,t}}\right)^{\theta}$$

$$r_{n,t}^k = pn_t mc_{n,t}\theta a_{n,t}\left(\frac{k_{n,t}}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}h_{n,t}}\right)^{\theta-1}$$

$$w_t = px_t mc_{x,t}\left(1-\theta\right)a_{x,t}\left(\frac{k_{x,t}}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}h_{x,t}}\right)^{\theta}$$

$$r_{x,t}^k = px_t mc_{x,t}\theta a_{x,t}\left(\frac{k_{x,t}}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}h_{x,t}}\right)^{\theta-1}$$

$$x_t^1 = \widetilde{p}_{n,t}^{-1-\eta_n}\left(c_{n,t}+g_{n,t}+\frac{i_{n,t}}{pn_t}\right)mc_{n,t}$$

$$+E_t\alpha_n r_{t,t+1}\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n}\mu_{t+1}^z\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}x_{t+1}^1$$

$$x_t^2 = \widetilde{p}_{n,t}^{-\eta_n}\left(c_{n,t}+g_{n,t}+\frac{i_{n,t}}{pn_t}\right)\frac{(\eta_n-1)}{\eta_n}$$

$$+E_t\alpha_n r_{t,t+1}\left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n}\left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n}\mu_{t+1}^z\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}x_{t+1}^2$$

$$z_t^1 = \widetilde{p}_{x,t}^{-1-\eta_x}\left(c_{x,t}+g_{t,t}+\frac{pt_t}{px_t}i_{x,t}+d_{xp,t}\right)mc_{x,t}$$

$$+E_t\alpha_x r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}}\right)^{-\eta_x}\mu_{t+1}^z\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}z_{t+1}^1$$

$$z_t^2 = \widetilde{p}_{x,t}^{-\eta_x}\left(c_{x,t}+g_{t,t}+\frac{pt_t}{px_t}i_{x,t}+d_{xp,t}\right)\frac{(\eta_x-1)}{\eta_x}$$

$$+E_t\alpha_x r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x}\left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}}\right)^{1-\eta_x}\mu_{t+1}^z\left(\mu_{t+1}^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}z_{t+1}^2$$

$$\begin{aligned} y_t^1 = {} & \widetilde{p}_{m,t}^{-1-\eta_m}\left(c_{m,t} + i_{m,t}\frac{pt_t}{pm_t}\right) rer_t \frac{pt_t\; pm_t^*}{pm_t}\left(1+\frac{R_t^f-1}{R_t^f}\right) \\ & + E_t\alpha_m r_{t,t+1}\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-1-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}}\right)^{-\eta_m}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} y_{t+1}^1 \end{aligned}$$

$$\begin{aligned} y_t^2 = {} & \widetilde{p}_{m,t}^{-\eta_m}\left(c_{m,t} + i_{m,t}\frac{pt_t}{pm_t}\right)\frac{(\eta_m-1)}{\eta_m} \\ & + E_t\alpha_m r_{t,t+1}\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}}\right)^{1-\eta_m}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} y_{t+1}^2 \end{aligned}$$

$$\begin{aligned} u_t^1 = {} & \left(\widetilde{p}_{x,t}^*\right)^{-1-\eta_{xp}} x_t\frac{px_t\; pt_t}{rer_t\; pm_t^*\; tot_t} \\ & + E_t\alpha_{xp} r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*}\right)^{-1-\eta_{xp}}\left(\frac{\left(\pi_{x,t}^*\right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^*\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}}\right)^{-\eta_{xp}}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} u_{t+1}^1 \end{aligned}$$

$$\begin{aligned} u_t^2 = {} & \left(\widetilde{p}_{x,t}^*\right)^{-\eta_{xp}} x_t\frac{(\eta_{xp}-1)}{\eta_{xp}} \\ & + E_t\alpha_{xp} r_{t,t+1}\left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*}\right)^{-\eta_{xp}}\left(\frac{\left(\pi_{x,t}^*\right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^*\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}}\right)^{1-\eta_{xp}}\mu_{t+1}^z\left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} u_{t+1}^2 \end{aligned}$$

$$\begin{aligned} x_t^1 &= x_t^2 \\ z_t^1 &= z_t^2 \\ y_t^1 &= y_t^2 \\ u_t^1 &= u_t^2 \end{aligned}$$

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_{t+1}}{R}\right) +$$

$$+ (1-\rho_R)\left[\alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi^o_{t+1}}\right) + \alpha_y \log\left(\frac{y_{t+1}}{y}\right) + \alpha_{rer}\log\left(\frac{rer_{t+1}}{rer_t}\right)\right] + \epsilon^R_{t+1}$$

$$\pi^o_{t+1} = (1-\rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi^o_t + \epsilon^{\pi^o}_{t+1}$$

$$t_t = \tau^c_t c_t + \tau^h_t w_t h_t + \tau^k_t\left[\left(r^k_{n,t}\mu_{n,t} - a\left(\mu_{n,t}\right)\right)\overline{k}_{n,t} + \left(r^k_{x,t}\mu_{x,t} - a\left(\mu_{x,t}\right)\right)\overline{k}_{x,t}\right] + \tau^\phi_t \phi_t$$

$$g_t = \left(1-\rho_g\right) g + \rho_g g_{t-1} + \epsilon^g_t$$

$$l_t = m_t + R_t b_{g,t+1}$$

$$l_t = \frac{R_t}{\pi_t}\frac{l_{t-1}}{\mu^z_t\left(\mu^\Upsilon_t\right)^{\frac{\theta}{1-\theta}}} + R_t\left(g_t - t_t\right) - \left(R_t - 1\right) m_t$$

$$\tau^h_t - \tau^h = \psi_1\left(\frac{l_t}{y_t} - \frac{l}{y}\right) + \psi_2\left(y_t - y\right) + \epsilon^\tau_t$$

$$\tau^k_t = \left(1-\rho_{\tau k}\right)\tau^k + \rho_{\tau k}\tau^k_{t-1} + \epsilon^{\tau k}_t$$

$$\tau^\phi_t = \tau^\phi$$

$$\tau^c_t = \left(1-\rho_{\tau c}\right)\tau^c + \rho_{\tau c}\tau^c_{t-1} + \epsilon^{\tau c}_t$$

$$g_{n,t} = \left(1-\omega\right)\left(pn_t\right)^{-\varepsilon} g_t$$

$$g_{t,t} = \omega\left(pt_t\right)^{-\varepsilon} g_t$$

$$R^f_t = R^*_t\left(1+\xi_t\right)^{\kappa_1}\left(\frac{rer_t ib_t}{y_t} / \frac{ib}{y}\right)^{\kappa_2}$$

$$\begin{bmatrix} \frac{\Delta M^*_t}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R^*_t}{R^*} \\ \frac{\pi^*_t}{\pi^*} \\ \frac{y^*_t}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M^*_{t-1}}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R^*_{t-1}}{R^*} \\ \frac{\pi^*_{t-1}}{\pi^*} \\ \frac{y^*_{t-1}}{y^*} \end{bmatrix} + \begin{bmatrix} \varepsilon^{m*}_t \\ \varepsilon^{\xi}_t \\ \varepsilon^{R*}_t \\ \varepsilon^{\pi*}_t \\ \varepsilon^{y*}_t \end{bmatrix}$$

$$x_t = \left(pm^*_t\ tot_t\right)^{-\eta*} y^*_t$$

$$tot_t = \frac{\pi_{x,t}^*}{\pi_t^*} tot_{t-1}$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = \upsilon_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X_{t-1} + \varepsilon_t^{\pi m}$$

$$a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_{n,t}^{1-\theta} - \chi_n = s_{n,t} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right)$$

$$s_{n,t} = (1-\alpha_n) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}} \right)^{\eta_n} s_{n,t-1}$$

$$1 = (1-\alpha_n) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}} \right)^{1-\eta_n}$$

$$d_{m,t} - \chi_m = s_{m,t} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right)$$

$$s_{m,t} = (1-\alpha_m) \widetilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}} \right)^{\eta_m} s_{m,t-1}$$

$$1 = (1-\alpha_m) \widetilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}} \right)^{1-\eta_m}$$

$$a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_{x,t}^{1-\theta} - \chi_x = s_{x,t} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right)$$

$$s_{x,t} = (1-\alpha_x) \widetilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}} \right)^{\eta_x} s_{x,t-1}$$

$$1 = (1-\alpha_x) \widetilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}} \right)^{1-\eta_x}$$

$$d_{xp,t} - \chi_{xp} = s_{xp,t} x_t$$

$$s_{xp,t} = (1-\alpha_{xp}) \left(\widetilde{p}_{x,t}^* \right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{\left(\pi_{xp,t-1}^*\right)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1}$$

$$1 = (1-\alpha_{xp}) \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{\left(\pi_{x,t-1}^*\right)^{\kappa_{xp}}}{\pi_{x,t}^*} \right)^{1-\eta_{xp}}$$

$$h_{x,t} + h_{n,t} = h_t$$

$$y_t = c_t + g_t + i_t + px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t}\left[1 + \left(\frac{R_t^f - 1}{R_t^f}\right)\right]$$

$$+ \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y}\right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t\ ib_t}{y_t} - \frac{rer\ ib}{y}\right)^2$$

$$b_{g,t} + b_{h,t} = 0$$

$$px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t}\left[1 + \left(\frac{R_t^f - 1}{R_t^f}\right)\right] = rer_t R_{t-1}^f \frac{ib_t}{\mu_t^z \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} - rer_t \pi_{t+1}^* ib_{t+1}$$

$$\phi_t = y_t - w_t h_t - r_{n,t}^k \mu_{n,t} \overline{k}_{n,t} - r_{x,t}^k \mu_{x,t} \overline{k}_{x,t}$$

$$m_t = \nu^m \left(1 + \tau_t^c\right) c_t$$

$$pt_t = \frac{\pi_{t,t}}{\pi_t} pt_{t-1}$$

$$pn_t = \frac{\pi_{n,t}}{\pi_t} pn_{t-1}$$

$$px_t = \frac{\pi_{x,t}}{\pi_{t,t}} px_{t-1}$$

$$pm_t = \frac{\pi_{m,t}}{\pi_{t,t}} pm_{t-1}$$

$$pm_t^* = \frac{\pi_{m,t}^*}{\pi_t^*} pm_{t-1}^*$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu_{t+1}^{\Upsilon} = \left(1 - \rho_{\Upsilon}\right) \mu^{\Upsilon} + \rho_{\Upsilon} \mu_t^{\Upsilon} + \epsilon_{t+1}^{\Upsilon}$$

$$\frac{z_{t+1}}{z_t} = \mu_{t+1}^z = \left(1 - \rho_z\right) \mu^z + \rho_z \mu_t^z + \epsilon_{t+1}^z$$

Appendix B

Steady State Conditions: Competitive Equilibrium

This section describes the sequence of equations necessary to compute the steady state of the competitive equilibrium of the assuming that the values related to income taxation are known. The taxation on consumption is obtained using the government budget constraint, assuming that the steady state level of debt-output ratio is known. Given steady state values for taxes $\tau^h, \tau^k, \tau^\phi$, parameter values for $\beta, \theta, \delta, \omega, \varkappa, \mu^z, \mu^\Upsilon, \eta_x, \eta_n, \eta_m, \eta_{xp}, \varpi, \kappa_1, \alpha_x, \alpha_m, \alpha_{xp}, \alpha_n,$ and steady state values for $h, R^*/R^f, tb/y, \tau/y, \pi^o, \pi^*, g/y, b/y, m/y, imp/y$ and the share of non-tradable goods in the output, there are 86 variables and 9 parameters to be computed in the steady state of the competitive equilibrium. The set of variables is given by: $\Big\{\pi, \pi_n, \pi_m, \pi_t, \pi_x, \pi^*_m, \pi^*_x, a_n, a_x, q_n, q_x, \mu_x, \mu_n, mcw_t, \Delta M^*, pt, pn, px, pm, pm^*, R, r, \widetilde{R}, R^f, \xi, R^*, \widetilde{p}_x, s_x, \widetilde{p}_m, s_m, \widetilde{p}^*_x, s_{xp}, \widetilde{p}_n, s_n, mc_n, mc_x, rer, r^k_n, r^k_x, h_x, h_n, \overline{k}_x, \overline{k}_n, k_x, k_n, i^d_x, i^d_n, i, g, g_t, g_n, c, c_t, c_n, c_x, c_m, i_n, i_t, i_x, i_m, ib, x, b_g, m, l, d_m, w, d_{xp}, \tau^c, tot, y^*, x^1, x^2, y^1, y^2, z^1, z^2, u^1, u^2, \lambda, y, \phi\Big\}$. The set of parameters is given by: $\{\theta_1, \theta_2, \nu^m, \chi_n, \chi_x, \chi_m, \chi_{xp}, \gamma\}$.

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \pi^o$$

$$\pi_m^* = \pi_x^* = \Delta M^* = \pi^*$$

$$a_n = a_x = 1$$

$$q_n = q_x = 1$$

$$\mu_x = \mu_n = 1$$

$$mcw_t = \frac{\varpi}{\varpi - 1}$$

$$pt = 1 \qquad pn = 1 \qquad px = 1 \qquad pm = 1 \qquad pm^* = 1$$

$$R = \tfrac{\pi}{\beta}\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}} \qquad r = \tfrac{1}{R} \qquad \widetilde{R} = R^f = R \qquad \xi = \left(\tfrac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad R^* = \left(\tfrac{R^*}{R^f}\right) R^f$$

$$\widetilde{p}_x = \left(\tfrac{1-\alpha_x \pi_x^{(\kappa_x - 1)(1-\eta_x)}}{1-\alpha_x}\right)^{\frac{1}{1-\eta_x}} \qquad s_x = \tfrac{(1-\alpha_x)\widetilde{p_x}^{-\eta_x}}{1-\alpha_x \pi_x^{\eta_x(1-\kappa_x)}}$$

$$\widetilde{p}_m = \left(\tfrac{1-\alpha_m \pi_m^{(\kappa_m - 1)(1-\eta_m)}}{1-\alpha_m}\right)^{\frac{1}{1-\eta_m}} \qquad s_m = \tfrac{(1-\alpha_m)\widetilde{p_m}^{-\eta_m}}{1-\alpha_m \pi_m^{\eta_m(1-\kappa_m)}}$$

$$\widetilde{p}_x^* = \left(\tfrac{1-\alpha_{xp}(\pi_x^*)^{(\kappa_{xp} - 1)(1-\eta_{xp})}}{1-\alpha_{xp}}\right)^{\frac{1}{1-\eta_{xp}}} \qquad s_{xp} = \tfrac{(1-\alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1-\alpha_{xp}(\pi_x^*)^{\eta_{xp}(1-\kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\tfrac{1-\alpha_n \pi_n^{(\kappa_n - 1)(1-\eta_n)}}{1-\alpha_n} \right)^{\frac{1}{1-\eta_n}} \qquad s_n = \tfrac{(1-\alpha_n)\widetilde{p}_n^{-\eta_n}}{1-\alpha_n \pi_n^{\eta_n (1-\kappa_n)}}$$

$$mc_n = \widetilde{p}_n \frac{1 - \alpha_n \ r \ \pi_n^{-\eta_n\left(\kappa_n - \frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_n \ r \ \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$mc_x = \widetilde{p}_x \frac{1 - \alpha_x \ r \ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_x \ r \ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$rer = \widetilde{p}_m \left(1 + \frac{R^f - 1}{R^f}\right)^{-1} \frac{pm}{pt \ pm^*} \frac{1 - \alpha_m \ r \ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_m \ r \ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$r_n^k = \left(1 - \tau^k\right)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$r_x^k = \left(1 - \tau^k\right)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$\frac{k_x}{h_x} = \mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} \left(\frac{r_x^k}{mc_x \theta}\right)^{\frac{1}{\theta - 1}}$$

$$\tfrac{h_x}{h_n} = \tfrac{mc_x Y_x}{mc_n Y_n} \qquad h = 0.2 \Longrightarrow h_n = h \left(1 + \tfrac{mc_x Y_x}{mc_n Y_n}\right)^{-1} \qquad h_x = \tfrac{Y_x}{Y_n} \tfrac{mc_x}{mc_n} h_n$$

$$k_x = \overline{k}_x = \tfrac{k_x}{h_x} h_x \qquad k_n = \overline{k}_n = h_n \tfrac{k_x}{h_x} \left(\tfrac{mc_x}{mc_n}\right)^{\frac{1}{\theta}}$$

$$i_x^d = \left(1 - \tfrac{(1-\delta)}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \overline{k}_x \qquad i_n^d = \left(1 - \tfrac{(1-\delta)}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \overline{k}_n$$

$$i = i_x^d + i_n^d \qquad \theta_1 = r_x^k \qquad \theta_2 = \theta_1 \tfrac{\theta_2}{\theta_1}$$

$$w = mc_n \left(1-\theta\right) \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{\theta-1}} \frac{k_n \mu_n}{h_n}\right)^\theta$$

$$g = \frac{g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$g_n = \left(1-\omega\right) g \qquad g_t = \omega g$$

$$c = \left(1 - \frac{tb}{y}\right) \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) - g - i$$

$$c_n = \left(1-\omega\right) c \qquad c_t = \omega c \qquad c_x = \left(1-\varkappa\right) c_t \qquad c_m = \varkappa c_t$$

$$i_n = \left(1-\omega\right) i \qquad i_t = \omega i \qquad i_x = \left(1-\varkappa\right) i_t \qquad i_m = \varkappa i_t$$

$$\frac{ib}{y} = \frac{tb}{y} \left[rer \left(\frac{R^f}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - \pi^*\right)\right]^{-1}$$

$$ib = \frac{ib}{y}\left(wh + r_x^k\frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$x = \frac{tb}{y}\left(wh + r_x^k\frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) + d_m\left(1 + \frac{R^f - 1}{R^f}\right)$$

$$m = \frac{m}{y}\left(wh + r_x^k\frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$b_g = \frac{b_g}{y}\left(wh + r_x^k\frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$\frac{l}{y} = \frac{m}{y} + R\frac{b_g}{y}$$

$$l = \frac{l}{y}\left(wh + r_x^k\frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$d_m = s_m\left(c_m + i_m\tfrac{pt}{pm}\right) \Longrightarrow \chi_m = 0 \qquad d_{xp} = s_{xp}x \Longrightarrow \chi_{xp} = 0$$

$$d_{xp} = s_{xp}x$$

$$\tau^c = \left\{R\left[g - \tau^h wh - \tau^k\left(r_n^k k_n + r_x^k k_x\right)\right] - \left(R-1\right)m - l\left(1 - \frac{R}{\pi\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)\right\}\left(cR\right)^{-1}$$

$$\nu^m = \frac{m}{\left(1+\tau^c\right) c}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)\left(\eta_{xp}-1\right)} \frac{1-\alpha_{xp}\; r\; \left(\pi_x^*\right)^{\left(1-\eta_{xp}\right)\left(\kappa_{xp}-\frac{\left(1+\eta_{xp}\right)}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_{xp}\; r\; \left(\pi_x^*\right)^{\left(-\eta_{xp}\right)\left(\kappa_{xp}-\frac{\left(1+\eta_{xp}\right)}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\; pt}{rer\; pm^*}\right)$$

$$y^* = x\, tot^{\eta *}$$

$$\chi_n = \left(\frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_n^{1-\theta} - s_n \left(c_n + g_n + \frac{P}{P_n} i_n\right)$$

$$\chi_x = \left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_x^{1-\theta} - s_x \left(c_x + g_t + \frac{P_t}{P_{x,t}} i_x + d_{xp}\right)$$

$$x^1 = \frac{\widetilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right) mc_n}{1-\alpha_n\; r\; \pi_n^{-\eta_n\left(\kappa_n-\frac{\left(1+\eta_n\right)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p}_n^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right)}{1-\alpha_n\; r\; \pi_n^{\left(1-\eta_n\right)\left(\kappa_n-\frac{\eta_n}{\left(\eta_n-1\right)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{\left(\eta_n-1\right)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p}_m^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right) rer \frac{pt\; pm^*}{pm} \left(1 + \frac{R^f-1}{R^f}\right)}{1-\alpha_m\; r\; \pi_m^{\left(-\eta_m\right)\left(\kappa_m-\frac{\left(1+\eta_m\right)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y^2 = \frac{\widetilde{p}_m^{-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right)}{1-\alpha_m\; r\; \pi_m^{\left(1-\eta_m\right)\left(\kappa_m-\frac{\left(1+\eta_m\right)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{\left(\eta_m-1\right)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p}_x^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right) mc_x}{1 - \alpha_x \; r \; \pi_x^{-\eta_x \left(\kappa_x - \frac{(1+\eta_x)}{\eta_x} \right)} \mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p}_x^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right)}{1 - \alpha_x \; r \; \pi_x^{(1-\eta_x) \left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)} \right)} \mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x \; \left(\widetilde{p}_x^* \right)^{-1-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^* \right)^{(-\eta_{xp}) \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \left(\frac{px \; pt}{rer \; pm^* \; tot} \right)$$

$$u^2 = \frac{x \; \left(\widetilde{p}_x^* \right)^{-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^* \right)^{(1-\eta_{xp}) \left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$\frac{\gamma}{(1-\gamma)} = \frac{\left(1 - \tau^h \right) w \left(1 - h \right)}{mcw \left(1 + \tau^c \right) \left(1 + \nu^m \left(\frac{R-1}{R} \right) \right) c \left(1 - \frac{\zeta}{\mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \right)}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^\Upsilon \right)^{\frac{\theta}{1-\theta}}} \right)^{-1} \frac{(1-\gamma)}{\left(1 + \tau^c \right) \left(1 + \nu^m \frac{R-1}{R} \right)}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon \right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon \right)^{\frac{1}{1-\theta}}}$$

Appendix C

Ramsey Steady State

The Ramsey solution assumes the same parameters from the competitive equilibrium to compute allocations and prices, including those derived implicitly in the steady state computation. The Ramsey equilibrium is characterized by no inflation dispersion across sectors (thus, relative prices remain set at unity) and the Ramsey planner has the domestic nominal interest rates (R) and taxes (τ^h, τ^k, τ^c) as instruments to maximize the objective function, taking as given the values for domestic government expenditure, g, the taxation over profits, τ^ϕ, and the steady state values for the rest of the world.

$$\tau^h = \tau^h \qquad \tau^k = \tau^k \qquad \tau^c = \tau^c \qquad \tau^\phi = \tau^\phi \qquad R = R$$

$$R^* = R^* \qquad g = g$$

$$\pi^* = \pi_x^* = \pi_m^* = \Delta M^*$$

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \frac{\beta R}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$a_n = a_x = pm = px = pt = pn = pm^* = 1$$

$$mcw = \tfrac{\varpi}{\varpi - 1} \qquad r = \tfrac{1}{R} \qquad R^f = \tfrac{\pi^*}{\pi} R \qquad \xi = \left(\tfrac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad \widetilde{R} = R$$

$$\widetilde{p}_x^* = \left(\tfrac{1-\alpha_x (\pi_x^*)^{(\kappa_{xp}-1)(1-\eta_{xp})}}{1-\alpha_{xp}}\right)^{\frac{1}{1-\eta_{xp}}} \qquad s_{xp} = \tfrac{(1-\alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1-\alpha_{xp}(\pi_x^*)^{\eta_{xp}(1-\kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\tfrac{1-\alpha_n \pi_n^{(\kappa_n-1)(1-\eta_n)}}{1-\alpha_n}\right)^{\frac{1}{1-\eta_n}} \qquad s_n = \tfrac{(1-\alpha_n)\widetilde{p}_n^{-\eta_n}}{1-\alpha_n \pi_n^{\eta_n(1-\kappa_n)}}$$

$$\widetilde{p}_x = \left(\tfrac{1-\alpha_x \pi_x^{(\kappa_x-1)(1-\eta_x)}}{1-\alpha_x}\right)^{\frac{1}{1-\eta_x}} \qquad s_x = \tfrac{(1-\alpha_x)\widetilde{p}_x^{-\eta_x}}{1-\alpha_x \pi_x^{\eta_x(1-\kappa_x)}}$$

$$\widetilde{p}_m = \left(\tfrac{1-\alpha_m \pi_m^{(\kappa_m-1)(1-\eta_m)}}{1-\alpha_m}\right)^{\frac{1}{1-\eta_m}} \qquad s_m = \tfrac{(1-\alpha_m)\widetilde{p}_m^{-\eta_m}}{1-\alpha_m \pi_m^{\eta_m(1-\kappa_m)}}$$

$$rer = \widetilde{p}_m \left(1 + \frac{R^f - 1}{R^f}\right)^{-1} \frac{pm}{pt\ pm^*} \frac{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$mc_x = \widetilde{p}_x \frac{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$mc_n = \widetilde{p}_n \frac{1-\alpha_n\ r\ \pi_n^{-\eta_n\left(\kappa_n-\frac{(1+\eta_n)}{\eta_n}\right)}\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n-\frac{\eta_n}{(\eta_n-1)}\right)}\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\frac{(\eta_n-1)}{\eta_n}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)(\eta_{xp}-1)}\frac{1-\alpha_{xp}\ r\ \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp}-\frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_{xp}\ r\ \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp}-\frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\left(\frac{px\ pt}{rer\ pm^*}\right)$$

$$q_x = 1 \qquad q_n = 1$$

$$g_n = (1-\omega)\,g \qquad g_t = \omega g$$

$$\mu_n = \sqrt{\frac{2}{\theta_2}\left[\left(1-\tau^k\right)^{-1}\left(\frac{\mu^{\Upsilon}\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}\right)}{\beta}-1+\delta\right)-\theta_1+\frac{\theta_2}{2}\right]}$$

$$\mu_x = \sqrt{\frac{2}{\theta_2}\left[\left(1-\tau^k\right)^{-1}\left(\frac{\mu^{\Upsilon}\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}\right)}{\beta}-1+\delta\right)-\theta_1+\frac{\theta_2}{2}\right]}$$

$$r_x^k = \theta_2\left(\mu_x-1\right)+\theta_1 \qquad r_n^k = \theta_2\left(\mu_n-1\right)+\theta_1$$

$$\tfrac{k_x}{h_x} = \mu_x\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}\left(\tfrac{r_x^k}{mc_x\theta}\right)^{\frac{1}{\theta-1}} \qquad \tfrac{k_n}{h_n} = \mu_n\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}\left(\tfrac{r_n^k}{mc_n\theta}\right)^{\frac{1}{\theta-1}}$$

$$w = mc_n\left(1-\theta\right)\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{\theta-1}}\ \tfrac{k_n\mu_n}{h_n}\right)^{\theta}\left(1+\nu_f^m\tfrac{(R-1)}{R}\right)^{-1} \qquad \tfrac{h_x}{h_n} = \tfrac{mc_xY_x}{mc_nY_n}$$

In order to calculate the amount of labor used in domestic production, use the non-tradable sector equilibrium condition:

$$s_n\left(c_n+g_n+i_n\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n$$

$$s_n\left(1-\omega\right)\left(c+g+i_x^d+i_n^d+a\left(\mu_n\right)k_n+a\left(\mu_x\right)k_x\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(c+g+\left(1-\frac{1-\delta}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}h_n+\frac{k_x}{h_x}h_x\right)\right)+\chi_n=\\ \left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n\end{aligned}$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(c+g\right)+\chi_n=\left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n\\ -s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\end{aligned}$$

$$\begin{aligned}s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R\left(1-h\right)}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n=\\ \left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta}h_n-s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\end{aligned}$$

$$s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n=$$

$$\left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n+$$

$$s_n\left(1-\omega\right)\left[\left(\frac{w\left(1-\tau^h\right)R\left(1+\frac{h_x}{h_n}\right)h_n}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}\right)\right.$$

$$\left.-\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\right]$$

Set:

$$\begin{aligned}
HN_1 &= s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n \\
HN_2 &= \left(\frac{1}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta \\
HN_3 &= s_n\left(1-\omega\right)\left(\frac{w\left(1-\tau^h\right)R\left(1+\frac{h_x}{h_n}\right)}{mcw\left(1+\tau^c\right)\left(R+\nu^m\left(R-1\right)\right)\gamma\left(1-\frac{\zeta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)}\right) \\
HN_4 &= s_n\left(1-\omega\right)\left(1-\frac{1-\delta}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)
\end{aligned}$$

Then:

$$h_n = \frac{HN_1}{HN_2 + HN_3 - HN_4}$$

$$h = \left(1 + \frac{h_x}{h_n}\right) h_n$$

$$h_x = h - h_n$$

Continuing with the steady state calculation:

$$k_x = \tfrac{k_x}{h_x} h_x \qquad k_n = \tfrac{k_n}{h_n} h_n \qquad \overline{k}_x = k_x/\mu_x \qquad \overline{k}_n = k_n/\mu_n$$

$$i_x^d = \left(1 - \tfrac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \tfrac{k_x}{h_x} h_x \qquad i_n^d = \left(1 - \tfrac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \tfrac{k_n}{h_n} h_n$$

$$i = i_x^d + i_n^d + +a\left(\mu_n\right) \overline{k}_n + a\left(\mu_x\right) \overline{k}_x$$

$$i_n = (1-\omega)\, i \qquad i_r = \omega i \qquad i_x = (1-\varkappa)\, i_r \qquad i_m = \varkappa i_r$$

$$c_n = \left(\left(\frac{1}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n}\right)^\theta h_n - \chi_n\right) \frac{1}{s_n} - g_n - i_n$$

$$c = \tfrac{c_n}{(1-\omega)} \qquad c_t = \omega c \qquad c_x = (1-\varkappa)\, c_t \qquad c_m = \varkappa c_t$$

$$d_{xp} = \left(\left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_x^{1-\theta} - \chi_x\right) \frac{1}{s_x} - c_x - g_t - i_x$$

$$x = \left(\chi_{xp} - d_{xp}\right) / s_{xp}$$

$$y^* = x \, tot^{\eta *} \qquad d_m = \chi_m - s_m \left(c_m + \tfrac{pt}{pm} i_m\right)$$

$$ib = \frac{x - d_m \left(1 + \frac{\left(R^f - 1\right)}{R^f}\right)}{rer} \left(\frac{R^f}{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} - \pi^*\right)^{-1}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f}\right)$$

$$m = \nu^m \left(1 + \tau^c\right) c$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}$$

$$b_g = \left(l - m\right) R^{-1}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right)^{-1} \frac{\left(1 - \gamma\right)}{\left(1 + \tau^c\right) \left(1 + \nu^m \frac{R-1}{R}\right)}$$

$$x^1 = \frac{\widetilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right) mc_n}{1 - \alpha_n \; r \; \pi_n^{-\eta_n \left(\kappa_n - \frac{\left(1+\eta_n\right)}{\eta_n}\right)} \mu^z \left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p}_n^{-\eta_n}\left(c_n + g_n + \frac{i_n}{pn}\right)}{1 - \alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p}_m^{-1-\eta_m}\left(c_m + i_m\frac{pt}{pm}\right) rer\frac{pt\ pm^*}{pm}\left(1 + \frac{R^f - 1}{R^f}\right)}{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y_t^2 = \frac{\widetilde{p}_m^{-\eta_m}\left(c_m + i_m\frac{pt}{pm}\right)}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p}_x^{-1-\eta_x}\left(c_x + g_t + \frac{pt}{px}i_x + d_{xp}\right) mc_x}{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p}_x^{-\eta_x}\left(c_x + g_t + \frac{pt}{px}i_x + d_{xp}\right)}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-1-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\left(\frac{px\ pt}{rer\ pm^*\ tot}\right)$$

$$u^2 = \frac{x\ \left(\widetilde{p}_x^*\right)^{-\eta_{xp}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)}\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$
\begin{aligned}
l = \Big\{ R \left[g - \tau^h w h - \tau^k \left(\left(r_n^k - a\left(\mu_n\right)\right) k_n + \left(r_x^k - a\left(\mu_x\right)\right) k_x \right) - \tau^c c - \tau^\phi \phi \right] \\
- \left(R - 1\right) m \Big\} \left(1 - \frac{R}{\pi \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)^{-1}
\end{aligned}
$$

Appendix D

Welfare Cost Measurement

Following Schmitt-Grohé and Uribe (2006 and 2007)[**?**][34], the welfare cost λ_c of adopting the alternative policy regime i instead of the Ramsey monetary and fiscal policy r is measured in terms of the share of consumption the households give up in order to be indifferent between the two policy regimes:

$$\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left(\left(c_t^i - \frac{\zeta c_{t-1}^i}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right), h_t^i \right) =$$

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Using the period utility function of the households, the welfare cost λ_c is given by:

$$\mho_i^c = E_0 \sum_{t=0}^{\infty} \beta^t U_t \left((1-\lambda_c) \left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right), h_t^r \right)$$

Plug the period utility function for period zero and decompose the infinite sum:

$$\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \gamma\log\left(1-h_0^r\right) + E_0\sum_{t=1}^{\infty}\beta^t U_t\left((1-\lambda_c)\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right), h_t^r\right)$$

Sum and subtract $(1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right)$ in the right-side of the equation:

$$\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) - (1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + (1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \gamma\log\left(1-h_0^r\right) + E_0\sum_{t=1}^{\infty}\beta^t U_t\left((1-\lambda_c)\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right), h_t^r\right)$$

Decompose, from the infinite sum, the term of the welfare cost of the alternative policy, using the fact that the utility function is log-linear in consumption:

$$\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) - (1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + (1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \gamma\log\left(1-h_0^r\right) + E_0\sum_{t=1}^{\infty}\beta^t(1-\gamma)\log\left(1-\lambda_c\right) + E_0\sum_{t=1}^{\infty}\beta^t U_t\left(\left(c_t^r - \frac{\zeta c_{t-1}^r}{\mu_t^z\left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right), h_t^r\right)$$

Note that the third, fourth and last term of the right-hand side equal the welfare of the Ramsey policy, $\mho_r^c$:

$$\mho_i^c = (1-\gamma)\log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right)$$

$$-(1-\gamma)\log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \mho_r^c + E_0\sum_{t=1}^{\infty}\beta^t(1-\gamma)\log(1-\lambda_c)$$

Organizing the terms:

$$\frac{\mho_i^c - \mho_r^c}{(1-\gamma)} = \log\left((1-\lambda_c)\,c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) - \log\left(c_0^r - \frac{\zeta c_{-1}^r}{\mu_0^z\left(\mu_0^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}\right) + \frac{\beta}{1-\beta}\log(1-\lambda_c)$$

Now, approximate the welfare cost λ_c by a second-order Taylor expansion around the vector of disturbances σ to obtain:

$$\lambda_c \approx \overline{\lambda}_c + \lambda_{c,\sigma}\sigma + \lambda_{c,\sigma\sigma}\frac{\sigma^2}{2}$$

Following the results in Schmitt-Grohé and Uribe (2006 and 2007)[?][34], note that $\overline{\lambda}_c$ vanishes, because all the policies considered here do not alter the steady state of the economy, and $\lambda_{c,\sigma} = 0$. The second total derivative of the equation provides the welfare measure:

$$\frac{\mho_{i,\sigma\sigma}^c - \mho_{r,\sigma\sigma}^c}{(1-\gamma)} = -\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)\lambda_{c,\sigma\sigma}$$

$$\Longrightarrow \lambda_{c,\sigma\sigma} = \frac{\mho_{r,\sigma\sigma}^c - \mho_{i,\sigma\sigma}^c}{(1-\gamma)\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)}$$

$$\Longrightarrow \lambda_c \approx \left(\frac{\mho_{r,\sigma\sigma}^c - \mho_{i,\sigma\sigma}^c}{(1-\gamma)\left(\frac{\mu_0}{\mu_0-\zeta} + \frac{\beta}{1-\beta}\right)}\right)\frac{\sigma^2}{2}$$

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