Supplemantary Appendix to Loss in the Time of Cholera: Long-run Impact of a Disease Epidemic on the Urban Landscape (Not for Publication)

Additional Theoretical Results to Subsection 1.3

Investments/maintenance

Consider the situation, when the owner has already got all n rich tenants and deviates to low investment for now (and then he is expected to invest low forever). Then he gains k in the current period, but next period with probability q he signs a contract with a rent $W^r - c_0^r - d$, which is by d lower than in the case of high investment strategy, therefore his expected loss from signing a worse contract is $\frac{d}{1-\delta(1-\frac{q}{n})}$. Hence a condition for high investment with two rich tenants is:

$$k - \delta q \frac{d}{1-\delta(1-\frac{q}{n})} \leq 0$$

Now assume this inequality holds and, moreover, the owner with $m-1$ poor tenants always invests high. Now consider the situation, when the owner still has m poor tenants and deviates to low investment until he has $m-1$ poor tenants. Then the landlord gains k in current period, next period with probability $\frac{m}{n}q$ he signs a contract with a new rich tenant with a rent $W^r - c_0^r - (1-\delta(1-\frac{q}{n}))d$ (a new rich tenant knows that her losses are just for one period, because after hiring her the landlord switches to high investment); with probability $(1-\frac{m}{n})q$ the owner renews a contract with an existing rich tenant. The tenant knows that she will face additional disutility d, while having m poor neighbours, that is equivalent to increasing c_m^r by d. Also if she moves into the state with $m-1$ poor she still faces d, but only for 1 period, which is equivalent to increasing c_{m-1}^r by $(1-\delta(1-q\frac{m}{n}))d$. Hence, from (6) we get that r_m decreases by $\frac{(1+\delta\frac{m}{n}q)(1-\delta+\delta\frac{q}{n})}{1-\delta+\delta\frac{m+1}{n}q}d$.

Hence a condition for high investment with m poor tenants is:

$$k - \delta\left[\frac{m}{n}qd + (1-\frac{m}{n})q\frac{1+\delta\frac{m}{n}q}{1-\delta+\delta\frac{m+1}{n}q}d\right] \leq 0$$

or, equivalently,

$$\frac{k}{d} \leq \frac{\delta\frac{q}{n}[n - m\delta + \frac{m(n+1)}{n}\delta q]}{1-\delta+\frac{m+1}{n}\delta q}$$

Poor types also willing to pay premium for rich neighbors

Here the payoff under the always rich strategy does not change. Consider the always poor strategy. Reinterpret costs c_k^p that poor tenants face from having

k poor neighbours as costs d_{n-1-k} from having $n-1-k$ rich neighbours, so $d_i = c^p_{n-1-i}$. Then all formulas for the always rich strategy can be rewritten, if we redefine state as a number of rich tenants, use W^p instead of W^r and costs d_i instead of c_i.

$$U_{poor}(S_r, x) = U_{current}(S_r) + g_{n-1-x}$$

$$g_k = \frac{(1-\delta+\delta q)}{(1-\delta)(1-\delta(1-\frac{q}{n}))} W^p - \sum_{i=0}^{k} b_{ik} d_i =$$
$$= \frac{(1-\delta+\delta q)}{(1-\delta)(1-\delta(1-\frac{q}{n}))} W^p - \sum_{j=n-1-k}^{n-1} b_{n-1-j,k} c^p_j,$$

where b_{ik} are taken from (7).

We can conclude that a landlord, having x poor tenants prefers the always rich strategy to the always poor one if $f_x > g_x$ or, equivalently,

$$W^r - W^p > (1-\delta)\left(1-\delta(1-\frac{q}{n})\right) \sum_{i=0}^{x} \frac{\frac{x!}{i!}(x+1-i)(\delta\frac{q}{n})^{x-i}}{\prod_{m=i}^{x+1}(1-\delta(1-q\frac{m}{n}))} c^r_i -$$
$$-(1-\delta)\left(1-\delta(1-\frac{q}{n})\right) \sum_{j=x}^{n-1} \frac{\frac{(n-1-x)!}{(n-1-j)!}(j+1-x)(\delta\frac{q}{n})^{j-x}}{\prod_{m=n-1-j}^{n-x}(1-\delta(1-q\frac{m}{n}))} c^p_j$$

Multiple owners

First note that since $W^r - c^r_1 < W^p$, it is always better for an owner to acquire a poor tenant if the other apartment has a poor tenant and the belief is that the other apartment owner plays an always poor strategy. Assume now that the owner of apartment 1 has a vacancy when the other apartment currently has a rich tenant, but he believes that the other apartment owner from now on will follow the always poor strategy. We call the owner's strategy initially rich if he acquires a rich tenant in such situations, and only.

If the owner chooses the always poor strategy, then he receives W^p in every period and $U_{poor} = \frac{W^p}{1-\delta}$.

Now we concentrate on the initially rich strategy. Consider a rich tenant, who pays rent r per period. If her contract gets renegotiated, she gets $V(out) = -\frac{W^r}{1-\delta}$. Her expected utility from renting depends on her current neighbour's type.

$$V(poor) = -(r + c^r_1) + \delta[(1-q)V(poor) + \frac{q}{2}V(poor) + \frac{q}{2}V(out)]$$
$$V(poor) = \frac{-(r+c^r_1) + \delta\frac{q}{2}V(out)}{1-\delta(1-\frac{q}{2})} = \frac{-(r+c^r_1) - \frac{q}{2}\frac{\delta}{1-\delta}W^r}{1-\delta(1-\frac{q}{2})}$$

$$V(rich) = -(r + c_0^r) + \delta[(1-q)V(rich) + \frac{q}{2}V(poor) + \frac{q}{2}V(out)]$$

$$V(rich) = \frac{-r + \delta\frac{q}{2}V(out)}{1-\delta(1-\frac{q}{2})} - \frac{1}{1-\delta(1-q)}c_0^r - \frac{\delta\frac{q}{2}}{(1-\delta(1-q))(1-\delta(1-\frac{q}{2}))}c_1^r$$

The apartment owner chooses rent r^* by making the tenant indifferent between renting and outside option: $V(rich) = V(out)$. Hence:

$$\frac{-r^* + \delta\frac{q}{2}V(out)}{1-\delta(1-\frac{q}{2})} - \frac{1}{1-\delta(1-q)}c_0^r - \frac{\delta\frac{q}{2}}{(1-\delta(1-q))(1-\delta(1-\frac{q}{2}))}c_1^r = V(out)$$

$$r^* = W^r - \frac{1-\delta+\frac{1}{2}\delta q}{1-\delta+\delta q}c_0^r - \frac{\frac{1}{2}\delta q}{1-\delta+\delta q}c_1^r \tag{1}$$

The apartment owner prefers the initially rich strategy to the always poor strategy if $r^* > W^p$, which is equivalent to:

$$W^r - W^p > \frac{1-\delta+\frac{1}{2}\delta q}{1-\delta+\delta q}c_0^r + \frac{\frac{1}{2}\delta q}{1-\delta+\delta q}c_1^r$$

This condition guarantees that (rich,rich) becomes an absorbing state.

No price disrimination

First consider the always poor strategy. If there is a vacancy and a poor tenant, the apartment owner sets a price W^p. As $r^* < W^p$, only poor type tenants apply. Then as we know from the proof of Proposition 1:

$$U_{poor} = \frac{r_0}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta)(1-\delta+\delta\frac{q}{2})}W^p$$

Now consider the always rich strategy of the apartment owner. If there is a vacancy and a rich tenant, the owner sets a price $W^r - c_0^r$ and only rich types apply. If there is a vacancy and a poor tenant in the other apartment, then the owner sets a price r^*, which attracts both types of tenants. Under the always rich strategy the apartment owner's expected continuation payoff depends on the current state, which is determined by the current tenants' rents (we also use an index to distinguish between rich and poor types, who both can pay r^*).

$$U(W^r - c_0^r, W^r - c_0^r) = \frac{2(W^r - c_0^r)}{1-\delta}$$

$$U(W^r - c_0^r, r^*_{rich}) = \frac{r^*}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta)(1-\delta+\delta\frac{q}{2})}(W^r - c_0^r)$$

First consider states with r^*_{rich}:

$$U(r^*_{poor}, r^*_{rich}) = 2r^* + \delta \left[\left(1 - \frac{q}{2}\right) U(r^*_{poor}, r^*_{rich}) + \frac{q}{2} U(W^r - c^r_0, r^*_{rich}) \right]$$

$$U(r^*_{poor}, r^*_{rich}) = \frac{2r^* + \delta \frac{q}{2} U(W^r - c^r_0, r^*_{rich})}{1 - \delta + \delta \frac{q}{2}}$$

$$U(r_0, r^*_{rich}) = (r_0 + r^*) + \delta \left[\left(1 - \frac{q}{2}\right) U(r_0, r^*_{rich}) + \frac{q}{2} U(W^r - c^r_0, r^*_{rich}) \right]$$

$$U(r_0, r^*_{rich}) = \frac{r_0 + r^* + \delta \frac{q}{2} U(W^r - c^r_0, r^*_{rich})}{1 - \delta + \delta \frac{q}{2}}$$

Further consider (r^*_{poor}, r^*_{poor}):

$$\begin{aligned} U(r^*_{poor}, r^*_{poor}) &= 2r^* + \delta(1-q) U(r^*_{poor}, r^*_{poor}) + \\ &+ \delta q [\pi U(r^*_{poor}, r^*_{rich}) + (1-\pi) U(r^*_{poor}, r^*_{poor})] \\ U(r^*_{poor}, r^*_{poor}) &= \frac{2(1-\delta) + \delta q (1 + 2\pi)}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} r^* + \\ &+ \frac{\delta^2 \frac{q^2}{2} \pi}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} U(W^r - c^r_0, r^*_{rich}) \end{aligned}$$

Next, find the owner's utility at (r_0, r^*_{poor}):

$$\begin{aligned} U(r_0, r^*_p) &= (r_0 + r^*) + \delta \,(1-q)\, U(r_0, r^*_p) + \\ + \delta \frac{q}{2} [\pi U(r^*_r, r^*_p) + (1-\pi) U(r^*_p, r^*_p)] &+ \delta \frac{q}{2} [\pi U(r_0, r^*_r) + (1-\pi) U(r_0, r^*_p)] \\ U(r_0, r^*_p) &= \frac{r_0}{1 - \delta + \delta \frac{q}{2}} + \frac{1 - \delta + \delta q (1 + \pi)}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} r^* + \\ &+ \frac{\delta^2 \frac{q^2}{2} \pi}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} U(W^r - c^r_0, r^*_{rich}) \end{aligned}$$

The apartment owner's expected utility under the always rich strategy $U_{rich} = \pi U(r_0, r^*_{rich}) + (1-\pi) U(r_0, r^*_{poor})$, hence:

$$\begin{aligned} U_{rich} &= \frac{r_0}{1 - \delta + \delta \frac{q}{2}} + \frac{1 - \delta + \delta q}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} r^* + \\ &+ \frac{\delta \frac{q}{2} \pi (1 - \delta + \delta q)}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} U(W^r - c^r_0, r^*_{rich}) = \\ &= \frac{r_0}{1 - \delta + \delta \frac{q}{2}} + \frac{1 - \delta + \delta q}{(1 - \delta)(1 - \delta + \delta \frac{q}{2})} W^r - \\ - \frac{\delta \frac{q}{2} [(1-\delta)(1+\pi) + 2\delta q \pi]}{(1-\delta)(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} c^r_0 &- \frac{1 - \delta + \delta \frac{q}{2}(1 + \pi)}{(1 - \delta + \delta \frac{q}{2})(1 - \delta + \delta q \pi)} c^r_1 \end{aligned}$$

The always rich strategy gives higher expected payoff than the always poor ($U_{rich} > U_{poor}$), if:

$$W^r - W^p > \frac{\delta\frac{q}{2}[(1-\delta)(1+\pi)+2\delta q\pi]}{(1-\delta+\delta q)(1-\delta+\delta q\pi)}c_0^r + \frac{(1-\delta)(1-\delta+\delta\frac{q}{2}(1+\pi))}{(1-\delta+\delta q)(1-\delta+\delta q\pi)}c_1^r$$

Future changes in attractiveness of the block

1) Consider the situation that when having all current poor tenants, the apartment owner prefers hiring poor tenants now and in the future when there is a vacancy to always hiring middle types, always hiring rich and always hiring very rich types.

From Proposition 1 we know that this happens if correspondingly:

$$W^m - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))}c_i^m$$

$$W^r - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))}c_{i,0}^r$$

$$W^v - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))}c_{i,0,0}^v$$

Assume also that, when having all current rich tenants, the apartment owner prefers hiring rich tenants now and in the future to always hiring very rich tenants. That happens if:

$$W^v - W^r < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))}c_{0,0,i}^v - c_{0,0}^r$$

2) Now suppose that the block becomes more attractive relative to other blocks, in that all types are willing to pay a higher rent than before. But differentially so: poor types by d_p, middle types by d_m, rich types by d_r and very rich by d_v, with $d_v > d_r > d_m > d_p$.

Then, having all current rich tenants, the apartment owner changes his behaviour and starts hiring very rich tenants now and in the future if:

$$W^v - W^r < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))}c_{0,0,i}^v - c_{0,0}^r <$$
$$< W^v - W^r + d_v - d_r$$

Also after the change in block attractiveness, having all current poor tenants, the apartment owner prefers hiring middle tenants now and in the future when there is a vacancy, but doesn't prefer always hiring rich or very rich ones if:

$$W^m - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_i^m <$$

$$< W^m - W^p + d_m - d_p$$

$$W^r - W^p + d_r - d_p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_{i,0}^r$$

$$W^v - W^p + d_v - d_p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_{i,0,0}^v$$

Therefore, there exists a parameter range for which originally both an all poor and an all rich block are stable, and after the increase in the attractiveness of the district, the poor block transitions to a middle-class one, while the rich block transforms to a very rich one.