INVESTMENTS IN SOCIAL TIES, RISK SHARING AND INEQUALITY

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Abstract. This paper investigates stable and efficient networks in the context of risk-sharing, when it is costly to establish and maintain relationships that facilitate risk-sharing. We find a novel trade-off between efficiency and equality. The most stable efficient networks also generate the most inequality. The result extends to correlated income structures with individuals split into groups, such that incomes across groups are less correlated but these relationships are more costly. We find that more central agents have better incentives to form across-group links, reaffirming the efficiency benefits of having highly central agents and thus the efficiency inequality trade-off. Our results are robust to many extensions. In general, endogenously formed networks in the risk sharing context tend to exhibit highly asymmetric structures, and stark inequalities in consumption levels.

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1. Introduction

In the context of missing formal insurance markets and limited access to lending and borrowing, incomes may be smoothed through informal risk-sharing agreements that utilize social connections. A large theoretical and empirical literature studies how well informal arrangements replace the missing markets.[1] However, the existing literature does not investigate a potential downside to these agreements: if people's network position affects the share of surplus generated by risk sharing they appropriate, social investments may be distorted and inequality may endogenously arise.[2]

Our starting premise is that social networks are endogenous and that their structure affects how the surplus from risk sharing is split. There is growing empirical evidence that risk-sharing networks respond to financial incentives, and that in general risk-sharing networks form endogenously, in a way that depends on the economic environment: see for example recent work by Binzel et al. (2017) and Banerjee et al. (2014b,c), which in different contexts look at how social networks respond to the introduction of financial instruments such as savings vehicles or microfinance. Our main goal is to develop a theoretical framework that can be used to think about the endogeneity of risk sharing networks, and to aid understanding about how these networks change after certain economic interventions, or more generally after changes in the economic environment.

We provide an examination of these issues, by considering a simple two stage model. In the first stage villagers invest in costly bilateral relationships (as in Myerson (1991) and Jackson and Wolinsky (1996)), knowing that in the second stage they will reach informal risk-sharing agreements. These agreements determine how the surplus generated by risk sharing is distributed, and they depend on the endogenous structure of the social network from the first stage. In this way we elucidate new costs associated with informal risk-sharing. Once incomes have been realized, risk sharing typically reduces inequality by smoothing incomes. Nevertheless, asymmetric equilibrium networks can still generate inequality in expected utilities terms. Agents occupying more advantageous positions in the social network may appropriate considerably more of the benefits generated by risk sharing.

For analytical tractability, in our benchmark model we impose several specific assumptions: agents have CARA utilities, their income realizations are jointly normal, and that surplus is negotiated according to a particular bargaining process, split-the-difference negotiations (Stole and Zwiebel (1996)). In Section 6 we extend our main results to more general settings, dropping all of the specific assumptions above, and showing that the results are robust

[1]An incomplete list of papers includes Rosenzweig (1988), Fafchamps (1992), Coate and Ravallion (1993), Townsend (1994), Udry (1994), Ligon, Thomas and Worrall (2002), Fafchamps and Gubert (2007), Bloch, Genicot, and Ray (2008), Angelucci and di Giorgi (2009), Jackson, Rodriguez-Barraquer and Tan (2012), Ambrus, Mobius and Szeidl (2014).

[2]Previous works that do consider endogenously formed networks include Bramoullé and Kranton (2007a,b) in the theoretical literature and Attanasio et al. (2012) in the experimental literature. For a related paper outside the networks framework, see Glaeser et al. (2002).

to introducing features missing from the benchmark model such as imperfect risk-sharing, enforcement constraints or the possibility of some coalitional deviations.

In the second stage of our model, pairs of agents who have formed a connection *commit* to a bilateral risk-sharing agreement (transfers contingent on income realizations). We investigate agreements satisfying two simple properties. First we require agreements to be pairwise efficient, in that no pair of directly connected agents leave gains from trade on the table.[3] Second, following Stole and Zwiebel (1996), we require the agreements to be robust to "split-the-difference" renegotiations.[4] We show that this leads to the surplus being divided by the Myerson value,[5] a network-specific version of the Shapley value.[6] The transfers required to implement the agreements we identify are particularly simple. Each agent receives an equal share of aggregate realized income (as in Bramoullé and Kranton, 2007a) and on top of that state independent transfers are made.[7]

A key implication of the Myerson value determining the division of surplus is that agents who are more centrally located, in a certain sense, receive a higher share of the surplus. Moreover, in our risk-sharing context it implies that agents receive larger payoffs from providing "bridging links" to otherwise socially distant agents than from providing local connections.[8] Empirical evidence supports this feature of our model—see Goyal and Vega-Redondo (2007), and references therein from the organizational literature: Burt (1992), Podolny and Baron (1997), Ahuja (2000), and Mehra et al. (2001).

Our analysis considers a community that comprises different groups where all agents within each group are ex-ante identical, and establishing links within groups is cheaper than across groups. We also assume that the income realizations of agents within groups are more positively correlated than across groups. Groups can represent different ethnic groups or castes in a given village, or different villages.

[3]Although we consider a model in which there is perfect bilateral risk sharing, we could easily extend the model so that some income is perfectly observed, some income is private, and there is perfect risk sharing of observable income and no risk sharing of unobservable income. This would be consistent with the theoretical predictions of Cole and Kocherlakota (2001) and the empirical findings of Kinnan (2011). In the CARA utilities setting, such unobserved income outside the scope of the risk-sharing arrangement does not affect our results.

[4]Stole and Zwiebel (1996) model bargaining between many employees and an employer. This scenario can be represented by a star network with the employer at the center. We extend their approach to general network structures.

[5]For related noncooperative foundations for the Myerson value, see Fontenay and Gans (2014) and Navarro and Perea (2013). Slikker (2007) also provides noncooperative foundations, although the game analyzed is not decentralized: offers are made at the coalitional level.

[6]The Myerson value is also often assumed in social networks contexts on normative grounds, as a fair allocation: see a related discussion on pp. 422–425 of Jackson (2010).

[7]For investigations of the division of surplus in social networks in other contexts, see Calvo-Armengol (2001, 2003), Corominas-Bosch (2004), Manea (2011), Kets et al. (2011) and Elliott and Nava (2016).

[8]More precisely, in Section 4 we introduce the concept of Myerson distance to capture the social distance between agents in the network, and show that a pair of agents' payoffs from forming a relationship are increasing in this measure.

We first consider the case of homogeneous agents, that is, when there is only one group. Using the inclusion–exclusion principle from combinatorics,[9] we develop a new metric to describe how far apart two agents located in a network are, which we call the Myerson distance. Using this distance we provide a complete characterization of stable networks.

We find that even when agents are ex-ante identical, efficient networks might only be stable if they are extremely asymmetric, thereby identifying a novel trade-off between efficiency and inequality. Among all possible efficient network structures, we find that the most stable (in the sense of being stable for the largest set of parameter values) results in the most unequal division of surplus (for any inequality measure in the Atkinson class). Conversely, the least stable efficient network entails the most equal division of surplus among all efficient networks. Although agents are ex-ante identical, efficiency considerations push the structure of social connections towards asymmetric outcomes that elevate certain individuals. Socially central individuals emerge endogenously from risk-sharing considerations alone. The intuition for this result is that the star network minimizes distance between periphery agents and hence provides the least incentives for them to establish nonessential and therefore socially inefficient extra links.

Turning attention to the case of multiple groups, we find that across-group underinvestment (no connection between two groups even though it would be socially efficient) becomes an issue when the cost of maintaining links across groups is sufficiently high.[10] The reason is that the agents who establish the first connection across groups receive less than the social surplus generated by the link, providing positive externalities for peers in their groups. To consider which agents are best incentivised to provide across-group links we introduce a new measure of network centrality which we term Myerson centrality. Agents more central in this sense have better incentives to provide across-group links. This provides a second force pushing some agents within a group to be more central than others. For example, with two groups, we show that the most stable efficient network structure involves stars within groups, connected by their centers. This reinforces the trade-off between efficiency and equality in the many-groups context.

Beyond this central takeaway, our results also suggest that within homogeneous groups the likely source of inefficiency is overinvestment, as agents might spend too much time building social capital, in order to occupy more central positions in the network. On the other hand, accross groups (communities) it is more likely to expect underinvestment inefficiency, as the agents who establish the first connections do not receive the full social benefits of the link, and they exert a positive externality to other agents in their groups. Empirical work suggests that both these types of inefficiencies in investmenys into social capital arising in our model are possible, in different contexts. Austen-Smith and Fryer (2005) cites numerous references from sociology and anthropology, suggesting that members of poor communities allocate

[9]See Chapter 10 in van Lint and Wilson (2001).

[10]While across-group overinvestment remains possible, the main concern when across-group link costs are relatively high is underinvestment.

inefficiently large amounts of time to activities maintaining social ties, instead of productive activities. In contrast, Feigenberg et al. (2013) find evidence in a microfinance setting that it is relatively easy to experimentally intervene and create social ties among people that yield substantial benefits, suggesting underinvestment in social relationships.

We provide several generalizations of the model that show that the main insights from the benchmark model are robust, and identify additional channels strengthening the results. When bilateral transactions are costly and hence risk-sharing is imperfect, efficiency requires shorter path-lengths between agents, adding an additional force for the emergence of central agents with direct connections to many others in the society. When relationship may fail such that transfers between the affected agents are not possible, redundancy needs to built into the network and the most robust and cost-effective way to do that is to have some very central agents. When risk-sharing agreements between two agents can only be enforced if they have a common friend (in the spirit of Jackson et al. (2012) and Renou and Tomala (2012)), the efficient network structure most robust to a simple form of coalitional deviations is a star structure of triangles, with one highly connected agent in the center. We also show that if there are some exogenously given links, like family relationships, partitioning society to various components then the most stable efficient network will again require a star structure of these components, with one central component connecting to all others. Finally, using the Moore bound from graph theory, we show that there is a basic tension between efficiency and equality/stability that does not depend on the fine details of the modeling of the economic interactions.

Among the theoretical studies on social networks and informal risk sharing that are most related to ours are Bramoullé and Kranton (2007a,b), Bloch et al. (2008), Jackson et al. (2012), Billand et al. (2012), Ali and Miller (2013, 2016), Ambrus et al. (2014) and Ambrus Gao and Milan (2016). Many of these papers focus on the enforcement issues we mainly abstract from, and investigate how social capital can be used to sustain cooperation for lower discount factors than would otherwise be possible. We take a complementary approach and instead focus on the distribution of surplus and the incentives this creates for social investments. One way of viewing our approach is an assumption on the discount factor in a dynamic version of our model. As long as the discount factor is high enough, our equilibrium agreements satisfy the necessary incentive compatibility constraints to be able to be enforced in equilibrium of the dynamic game. And for a range of discount factors below this threshold, our results on enforcement in our extensions section apply.

Among the aforementioned papers, Bramoullé and Kranton (2007a,b) and Billand et al. (2012) investigate endogenously and costly formed networks. Bramoullé and Kranton's (2007a,b) model assumes that the surplus on a connected income component is equally distributed, independently of the network structure. This rules out the possibility of overinvestment or inequality, and leads to different types of stable networks than in our model. Instead of assuming optimal risk-sharing arrangements, Billand et al. (2012) assume an exogenously given social norm, which prescribes that high-income agents transfer a fixed amount of resources to all low-income neighbors. This again leads to very different predictions regarding the types of networks that form in equilibrium.

More generally, understanding the structure of endogenously formed networks is important. Establishing and maintaining social connections (relationships) is costly, in terms of time and other resources. However, on top of direct consumption utility, such links can yield many economic benefits. Papers studying the structure of formed networks in different contexts include Jackson and Wolinsky (1996), Bala and Goyal (2000), Kranton and Minehart (2001), Hojman and Szeidl (2008), and Elliott (2015). Although we study a specific problem tailored to risk sharing in villages, the general structure of our problem is relevant to other applications.[11]

The remainder of the paper is organized as follows. Section 2 describes risk sharing on a fixed network. In Section 3 we introduce a game of network formation with costly link formation. We focus on the structure of networks formed within a single group in Section 4 and then turn to networks spanning multiple groups in Section 5. Section 6 provides generalizations and extensions of our benchmark model, while Section 7 concludes.

2. Preliminaries and Risk Sharing on a Fixed Network

To study social investments and the structure of formed networks, first we need to specify what risk-sharing arrangements take place once the network is formed. Below we introduce an economy in which agents face random income realizations, introduce some basic network terminology, and discuss risk-sharing arrangements for a given network.

2.1. **The socio-economic environment.** We denote the set of agents in our model by $\mathbf{N}$, and assume that they are partitioned into a set of groups $\mathbf{M}$. We let $G : \mathbf{N} \to \mathbf{M}$ be a function that assigns each agent to a group; i.e., if $G(i) = g$ then agent i is in group g. One interpretation of the group partitioning is that $\mathbf{N}$ represents individuals in a region (such as a district or subdistrict), and groups correspond to different villages in the region. Another possible interpretation is that $\mathbf{N}$ represents individuals in a village, and the groups correspond to different castes.

Agents in $\mathbf{N}$ face uncertain income realizations, with expected value μ and variance σ^2 for each agent. We assume that the correlation coefficient between the incomes of any two agents within the same group is ρ_w, while between the incomes of any two agents not in the same group it is $\rho_a < \rho_w$.[12] That is, we assume that incomes are more positively correlated

[11]For a different and more specific application, suppose researchers can collaborate on a project. Each researcher brings something heterogenous and positive to the value of the collaboration, so that the value of the collaboration is increasing in the set of agents involved. Collaboration is possible only when it takes place among agents who are directly connected to another collaborator and surplus is split according to the Myerson value (as in our work, motivated by robustness to renegotiations). Such a setting fits into our framework.

[12]It is well-known that for a vector of random variables, not all combinations of correlations are possible. We implicitly assume that our parameters are such that the resulting correlation matrix is positive semidefinite.

within groups than across groups, so that all else equal, social connections across groups have a higher potential for risk sharing.

Although we introduce the possibility of correlated incomes in a fairly stylized way, our paper is one of the first to permit differently correlated incomes between different pairs of agents. Such correlations are central to the effectiveness of risk-sharing arrangements, as shown below.

We refer to possible realizations of the vector of incomes as *states*, and denote a generic state by ω. We let $y_i(\omega)$ denote the income realization of agent i in state ω. Agents can redistribute realized incomes; hence their consumption levels can differ from their realized incomes.

In our benchmark model we make a number of simplifying assumptions. First, we assume that all agents have constant absolute risk aversion (CARA) utility functions:

$$v(c_i) = -\frac{1}{\lambda}e^{-\lambda c_i}, \tag{1}$$

where c_i is agent i's consumption and $\lambda > 0$ is the coefficient of absolute risk aversion. Second, we assume that incomes are jointly normally distributed.[13] The assumption of CARA utilities, together with jointly normally distributed incomes, greatly enhances the tractability of our model: as we show below it leads to a transferable utility environment in which the implemented risk-sharing arrangements are relatively simple. This utility formulation can be considered a theoretical benchmark case with no income effects. We generalize the theory relaxing these and other assumptions in Section 6.

2.2. **Basic network terminology.** Before proceeding, we introduce some standard terminology from network theory. A social network L is an undirected network, with nodes $\mathbf{N}$ corresponding to the different agents, and links representing social connections. Abusing notation we also let L denote the set of links in the network. We will refer to the agents linked to agent i, $\mathbf{N}(i; L) := \{j : l_{ij} \in L\} \subset \mathbf{N}$, as i's *neighbors*. Where there should be no confusion we abuse notation by writing $\mathbf{N}(i)$ instead of $\mathbf{N}(i; L)$. The *degree centrality* of an agent is simply the number of neighbors she has (i.e., the cardinality of $\mathbf{N}(i; L)$). An agent's neighbors can be partitioned according to the groups they belong to. Let $\mathbf{N}_g(i; L)$ be i's neighbors on network L from group g. A *walk* is a sequence of different agents $\{i, k, k', \ldots, k'', j\}$ such that every pair of adjacent agents in the sequence is linked. A *path* is a walk in which all agents are different. The *path length* of a path is the number of agents in the path.

We will sometimes refer to subsets of agents $\mathbf{S} \subseteq \mathbf{N}$ and denote the subnetworks they generate by $L(\mathbf{S}) := \{l_{ij} \in L : i, j \in \mathbf{S}\}$. A subset of agents $\mathbf{S} \subseteq \mathbf{N}$ is *path-connected* on L if, for each $i \in \mathbf{S}$ and each $j \in \mathbf{S}$, there exists a path connecting i and j. For any network there is a unique partition of $\mathbf{N}$ such that there are no links between agents in different

[13]This specification implies that we cannot impose a lower bound on the set of feasible consumption levels. As we show below, our framework readily generalizes to arbitrary income distributions, but the assumption of normally distributed shocks simplifies the analysis considerably.

partition elements but all agents within a partition element are path-connected. We refer to these partition elements as *network components*. A *shortest path* between two path-connected agents i and j is a path connecting i and j with a lower path length than any other. The *diameter* of a network component $C \subset L$ is $d(C)$, the maximum value—taken over all pairs of agents in C—of the length of a shortest path. A network component is a *tree* when there is a unique path between any two agents in the component. A *line network* is the unique (tree) network, up to a relabeling of agents, in which there is a path from one (end) agent to the other (end) agent that passes through all other agents. A *star network* is the unique tree network, up to a relabeling of agents, in which one (center) agent is connected to all other agents.

2.3. **Risk-Sharing Agreements.** In our benchmark model we assume that income can be directly shared between agents $i, j \in \mathbf{N}$ if and only if they are connected, i.e., $l_{ij} \in L$.[14] We let agents' income realizations be publicly observed within their network component, so agents can make transfer arrangements contingent on it. We consider this environment with perfectly observable incomes within a component as a benchmark which is a relatively good description of village societies in which people closely monitor each other. It is also straightforward to extend the model so that some income is publicly observed (and shared) while the remaining income is privately observed (and never shared). Results are very similar for this more general setting.[15]

Formally, a risk-sharing agreement on a network L specifies transfer $t_{ij}(\omega, L) = -t_{ji}(\omega, L)$ between neighboring agents i and j for every possible state ω. Abusing notation where there should be no confusion we sometimes drop the second argument and write $t_{ij}(\omega)$ instead of $t_{ij}(\omega, L)$. The interpretation is that in state ω agent i is supposed to transfer $t_{ij}(\omega)$ units of consumption to agent j if $t_{ij}(\omega) > 0$, and receives this amount from agent j if $t_{ij}(\omega) < 0$. Given a transfer arrangement between neighboring agents, agent i's consumption in state ω is $c_i(\omega) = y_i(\omega) - \sum_{j \in \mathbf{N}(i)} t_{ij}(\omega)$. It is straightforward to show that state-contingent consumption plans $(c_i(\cdot))_{i \in \mathbf{N}}$ are feasible, that is they can be achieved by bilateral transfers between neighboring agents, if and only if for each component C, containing agents S, $\sum_{i \in S} c_i(\omega) = \sum_{i \in S} y_i(\omega)$ for every state ω.

A basic assumption we make in our model is that given all other risk-sharing arrangements, an agreement reached by linked agents i and j must leave no gains from trade on the table.

[14] In Section 6, motivated by the literature on self-enforcing risk-sharing agreements, we relax this assumption and instead consider the possibility that a link can be used for risk-sharing if and only if it is supported (i.e., the two individuals have a friend in common).

[15] Kinnan (2011) finds evidence that hidden income can explain imperfect risk sharing in Thai villages relative to the enforceability and moral hazard problems we are abstracting from. Cole and Kocherlakota (2001) show that when individuals can privately store income, state-contingent transfers are not possible and risk sharing is limited to borrowing and lending.

There must be no other agreement that can make both i and j strictly better off holding fixed the agreements of other players. We call such transfers *pairwise efficient*.[16]

By the well-known Borch rule (see Borch (1962), Wilson (1968)) a necessary and sufficient condition for this property is that for all neighboring agents i and j,

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)}\right)\Big/\left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)}\right) = \left(\frac{\partial v_i(c_i(\omega'))}{\partial c_i(\omega')}\right)\Big/\left(\frac{\partial v_j(c_j(\omega'))}{\partial c_j(\omega')}\right) \tag{2}$$

for every pair of states ω and ω'. But if this holds for all neighboring agents i and j then the same condition must hold for all pairs of agents on a component of L, independently of whether they are directly or indirectly connected. Hence, pairwise-efficient risk-sharing arrangements are equivalent to Pareto-efficient agreements at the component level. For this reason, below we establish some important properties of Pareto-efficient risk-sharing arrangements on components.

Proposition 1 shows that the CARA utilities framework has the convenient property that expected utilities are transferable, in the sense defined by Bergstrom and Varian (1985). This can be used to show that ex-ante Pareto efficiency is equivalent to minimizing the sum of the variances, and it is achieved by agreements that in every state split the sum of the incomes on each network component equally among the members and then adjust these shares by state-independent transfers. The latter determine the division of the surplus created by the risk sharing agreement. We emphasize that this result does not require any assumption on the distribution of incomes, only that agents have CARA utilities.

Proposition 1. *For CARA utility functions certainty-equivalent units of consumption are transferable across agents, and if $L(\mathbf{S})$ is a network component, the Pareto frontier of ex-ante risk-sharing agreements among agents in $\mathbf{S}$ is represented by a simplex in the space of certainty-equivalent consumption. The ex-ante Pareto-efficient risk-sharing agreements for agents in $\mathbf{S}$ are those that satisfy*

$$\min \sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) \quad \textit{subject to} \quad \sum_{i\in\mathbf{S}} c_i(\omega) = \sum_{i\in\mathbf{S}} y_i(\omega) \quad \textit{for every state } \omega,$$

and they consist of agreements of the form

$$c_i(\omega) = \frac{1}{|\mathbf{S}|}\sum_{k\in\mathbf{S}} y_k(\omega) + \tau_i \quad \textit{for every } i \in \mathbf{S} \textit{ and state } \omega,$$

where $\tau_i \in \mathbb{R}$ is a state-independent transfer made to i and $\sum_{k\in\mathbf{S}} \tau_k = 0$.

The proof of Proposition 1 is in Appendix I. Proposition 1 implies that the total surplus generated by efficient risk-sharing arrangements is an increasing function of the reduction in aggregate consumption variance (the sum of consumption variances). For a general distribution of shocks, this function can be complicated. However, if shocks are

[16]More formally, transfers $\{t_{ij}(\omega, L)\}_{\omega\in\Omega,\, ij:l_{ij}\in L}$ are pairwise efficient for a network L if there is no pair of agents $ij : l_{ij} \in L$ and no alternative transfers $\{t'_{ij}(\omega, L)\}_{\omega\in\Omega,\, ij:l_{ij}\in L}$ such that $t'_{kl}(\omega, L) = t_{kl}(\omega, L)$ for all $kl \neq ij$ and all $\omega \in \Omega$, that gives both i and j strictly higher expected utility.

jointly normally distributed then $c_i = \frac{1}{|\mathbf{S}|}\sum_{k\in\mathbf{S}} y_k + \tau_i$ is also normally distributed, and $\mathrm{E}(v(c_i)) = \mathrm{E}(c_i) - \frac{\lambda}{2}\operatorname{Var}(c_i)$.[17] Hence in this case the total social surplus generated by efficient risk-sharing agreements is proportional to the aggregate consumption variance reduction. This greatly simplifies the computation of surpluses in the analysis below.

We use $TS(L)$ to denote the expected total surplus generated by an ex-ante Pareto-efficient risk-sharing agreement on network L, relative to agents consuming in autarky:

$$TS(L) := CE\Big(\Delta\operatorname{Var}(L,\emptyset)\Big), \tag{3}$$

where, for $L' \subset L$, $\Delta\operatorname{Var}(L, L')$ is the additional variance reduction obtained by efficient risk-sharing on network L instead of L', and $CE(\cdot)$ denotes the certainty-equivalent value of a variance reduction.

For a network L, consisting of a single component, if all agents are from the same group then as there are CARA utility functions and normally distributed incomes

$$TS(L) = CE\Big(\Delta\operatorname{Var}(L,\emptyset)\Big) = \frac{\lambda}{2}\Big(\Delta\operatorname{Var}(L,\emptyset)\Big) = \frac{\lambda}{2}(n-1)\sigma^2(1-\rho_w) = (n-1)V, \tag{4}$$

where $V := \frac{\lambda}{2}\sigma^2(1-\rho_w)$.

2.4. **Division of Surplus.** The assumption that neighboring agents make pairwise-efficient risk-sharing agreements pins down agreements up to state-independent transfers between neighboring agents, but does not constrain the latter transfers (hence the division of surplus) in any way. In our benchmark model, to determine these transfers, we follow the approach in Stole and Zwiebel (1996) and require that agreements are robust to split-the-difference renegotiations. This implies that the transfer is set in a way such that the incremental benefit that the link provides to the two agents is split equally between them. This can be interpreted as a social norm. For a detailed motivation of this assumption, and for noncooperative microfoundations, see Stole and Zwiebel (1996) and Brügemann et al. (2018a). In Section 6 we consider a much larger set of risk-sharing agreements and show that our main results still hold.

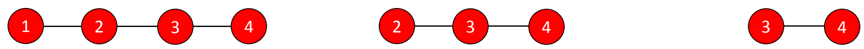
Splitting the incremental benefits of a risk sharing link equally between two agents requires calculating the expected payoffs i and j would receive if they did not have an agreement. We therefore have to consider what agreements would prevail on the network without l_{ij} to find the risk sharing agreements i and j can reach on L, and so on. This results in a recursive system of conditions.

More formally, for a network L a contingent transfer scheme

$$\mathcal{T}(L) := \{t_{ij}(\omega, L')\}_{\omega\in\Omega,\, L'\subseteq L,\, ij:l_{ij}\in L}, \tag{5}$$

specifies all transfers made in all subnetworks of L in all states of the world. The expected utility of agent i on a network $L' \subseteq L$ given a contingent transfer scheme $\mathcal{T}(L)$ is denoted

[17]See, for example, Arrow (1965).



$NTS = 3V$ $NTS = 2V$ $NTS = V$

(A) Line network (B) Three nodes (C) Two nodes

FIGURE 1. To find (gross) expected utilities that are robust to split-the-difference renegotiations on the (formed) line network shown we need to consider the expected utilities that would be obtained on all subnetworks.

$u_i(L', \mathcal{T}(L))$. Where there should be no confusion, we will abuse notation and drop the second argument.

For any network L, the expected utility vector $(u_1, ..., u_{|\mathbf{N}|})$ is *robust to split-the-difference renegotiation* if there is a contingent transfer scheme $\mathcal{T}(L)$ such that $u_i = u_i(L, \mathcal{T}(L))$ for every $i \in \mathbf{N}$ and the following conditions hold:

(i) $u_i(L') - u_i(L' \setminus \{l_{ij}\}) = u_j(L') - u_j(L' \setminus \{l_{ij}\})$ for every $l_{ij} \in L'$ and $L' \subseteq L$;
(ii) transfers $\{t_{ij}(\omega, L')\}_{\omega\in\Omega,\, ij:l_{ij}\in L'}$ are pairwise efficient for all $L' \subseteq L$.

Suppose all agents are from the same group, we have CARA utilities, incomes are normally distributed and we want to find payoffs robust to split-the-difference renegotiation for the line network shown in Figure 1a. A first necessary condition is that agents 1 and 2 benefit equally from their link so that $u_1(L) - u_1(L \setminus \{l_{12}\}) = u_2(L) - u_2(L \setminus \{l_{12}\})$. But in order to ensure this condition is satisfied, we need to know $u_1(L \setminus \{l_{12}\})$ and $u_2(L \setminus \{l_{12}\})$. Normalizing the autarky utility of all agents to 0, without the link l_{12} agent 1 is isolated so $u_1(L \setminus \{l_{12}\}) = 0$. However, to find $u_2(L \setminus \{l_{12}\})$ we need to find payoffs for the three node network in Figure 1b. For this network robustness to split-the-difference renegotiation requires that $u_2(L\setminus\{l_{12}\}) - u_2(L\setminus\{l_{12}, l_{23}\}) = u_3(L\setminus\{l_{12}\}) - u_3(L\setminus\{l_{12}, l_{23}\})$. While $u_2(L\setminus\{l_{23}, l_{23}, \}) = 0$, we need to consider the two node network shown in Figure 1c to find $u_3(L \setminus \{l_{12}, l_{23}\})$. For this network, payoffs must satisfy $u_3(L\setminus\{l_{12}, l_{23}\}) - u_3(L\setminus\{l_{12}, l_{23}, l_{34}\}) = u_4(L\setminus\{l_{12}, l_{23}\}) - u_4(L \setminus \{l_{12}, l_{23}, l_{34}\})$. As $u_3(L \setminus \{l_{12}, l_{23}, l_{34}\}) = u_4(L \setminus \{l_{12}, l_{23}, l_{34}\}) = 0$, the above condition simplifies to $u_3(L \setminus \{l_{12}, l_{23}\}) = u_4(L \setminus \{l_{12}, l_{23}\}) = V/2$, where the last equality follows from pairwise efficiency. Considering the three node network again, we now have the condition $u_2(L \setminus \{l_{12}\}) = u_3(L \setminus \{l_{12}\}) - V/2$. As the link l_{23} generates an incremental surplus of V to be split between agents 2 and 3, pairwise efficiency implies that $u_2(L \setminus \{l_{12}\}) = V/2$ and $u_3(L \setminus \{l_{12}\}) = V$. Finally, returning to the line network, we now have $u_1(L) = u_2(L) - V/2$. As the link l_{12} generates incremental surplus of V, $u_1(L) = V/2$ and $u_2(L) = V$.[18]

[18]This argument only outlines why the payoffs $u_1(L) = V/2$ and $u_2(L) = V$ are necessary for robustness to split-the-difference renegotiations. By considering all other subnetworks, it can be shown that the payoffs $u_1(L) = u_4(L) = V/2$ and $u_2(L) = u_3(L) = V$ are the unique payoffs that are robust to split-the-difference renegotiations.

Below we show that the requirement of robustness to split-the-difference renegotiation implies that the total surplus created by the risk-sharing agreement is divided among agents according to the Myerson value (Myerson 1977, 1980). The Myerson value is a cooperative solution concept defined in transferable utility environments that is a network-specific version of the Shapley value. The basic idea behind it is the same as for the Shapley value.[19] For any order of arrivals of the players, the incremental contribution of an agent i to the total surplus can be derived as the difference between the total surpluses generated by subnetwork $L(\mathbf{S})$ and subnetwork $L(\mathbf{S} \setminus \{i\})$ if agents $\mathbf{S} \setminus \{i\}$ arrive before i. It is easy to see that, for any arrival order, the total surplus generated by L gets exactly allocated to the set of all agents. The Myerson value then allocates the average incremental contribution of a player to the total surplus, taken over all possible orders of arrivals (permutations) of the players, as the player's share of the total surplus. Thus, agent i's Myerson value is[20]

$$MV_i(L) := \sum_{\mathbf{S} \subseteq \mathbf{N}} \frac{(|\mathbf{S}|-1)!(|\mathbf{N}|-|\mathbf{S}|)!}{|\mathbf{N}|!} \Big(TS(L(\mathbf{S})) - TS(L(\mathbf{S} \setminus \{i\})) \Big). \tag{6}$$

Proposition 2. *For any network L, any risk-sharing agreement that is robust to split-the-difference renegotiation yields expected payoffs to agents equal to their Myerson values: $u_i(L) = MV_i(L)$.*

Proof. Theorem 1 of Myerson (1980) states that there is a unique rule for allocating surplus for all subnetworks of L that satisfies the requirements of efficiency at the component level (note that this is an implicit requirement in Myerson's definition of an allocation rule) and, what Myerson (1980) defines as the equal-gains principle. Moreover, the expected payoff the above rule allocates to any player i is MV_i. Requirement (i) in our definition of robustness to split-the-difference renegotiation is equivalent to the equal-gains principle as defined in Myerson (1980). Theorem 1 of Wilson (1968) implies that efficiency at the component level is equivalent to pairwise efficiency between neighboring agents, which is requirement (ii) in our definition of robustness to split-the-difference renegotiation. The result then follows immediately from Theorem 1 of Myerson (1980). □

Proposition 2 is a direct implication of Myerson's axiomatization of the value. A special case of Proposition 2 is Theorem 1 of Stole and Zwiebel (1996), which in effect restricts

[19]We therefore follow Hart and Moore (1990), among others, in using the Shapley value to study investment decisions.

[20]Our assumption that there is perfect risk sharing among path-connected agents ensures that a coalition of path-connected agents generates the same surplus regardless of the exact network structure connecting them. This means that we are in the communication game world originally envisaged by Myerson. We do not require the generalization of the Myerson value to network games proposed in Jackson and Wolinsky (1996), which somewhat confusingly is also commonly referred to as the Myerson value. See Ambrus et al. (2016) for a model of informal risk-sharing in which the exact shape of the network matters in terms of the surplus that agents can attain.

attention to a star network.[21] Our contribution is to point out that their connection between robustness to split-the-difference renegotiations and the Shapley value can be extended to apply to all networks.[22]

The above result shows that any decentralized negotiation procedure between neighboring agents that satisfies two natural properties (not leaving surplus on the table, and robustness to split-the-difference negotiations) leads to the total surplus created by risk-sharing divided according to the Myerson value, and to state-independent transfers between neighboring agents that implement this surplus division. Hence, from now on we assume that all agents expect the surplus to be divided according to the Myerson value implied by the network that eventually forms.

Although we followed a decentralized approach to get to the implication that surplus is divided by the Myerson value, we note that on normative grounds such a division is also cogent in contexts in which there is a centralized community level negotiation over the division of surplus. This is because the Myerson value is a formal way of defining the fair share of an individual from the social surplus, as his average incremental contribution to the total social surplus (where the average is taken across all possible orders of arrival of different players, in the spirit of the Shapley value).

3. Investing in Social Relationships—Benchmark Case

Having defined how formed networks map into risk-sharing arrangements, we can now consider agents' incentives to make investments into social capital, which we think of as the set of relationships that enable risk sharing. We begin by providing the overall framework for the analysis. Then we look at a special case of our model, in which there is a single group. Building on these results we then consider the multiple group case.

In this section we formalize a game of network formation in which establishing links is costly, define efficient networks and identify different types of investment inefficiency.

We consider a two-period model in which in period 1 all agents simultaneously choose which other agents they would like to form links with, and in period 2 agents agree upon the ex-ante Pareto-efficient risk-sharing agreement specified in the previous section (i.e., the total surplus from risk sharing is distributed according to the Myerson value), for the network formed in the first period.[23]

Implicit in our formulation of the timing of the game is the view that relationships are formed over a longer time horizon than that in which agreements are reached about risk

[21] Relative to Myerson's axiomatization, Stole and Zwiebel (1996) generate the key system of equations through considering robustness to renegotiations as we describe above, while Myerson wrote down the system of equations based only on fairness considerations. Stole and Zwiebel (1996) also provide non-cooperative bargaining foundations that underpin this system.

[22] Brügemann et al. (2018b) undertake a related exercise regarding the non-cooperative result in Stole and Zwiebel (1996)

[23] For a complementary treatment of endogenously formed networks when surplus is split according to the Myerson value, see Pin (2011).

sharing. By the time such agreements are being negotiated, the network structure is fixed, and investments into forming social relationships are sunk. In addition, as mentioned in the introduction, the second stage agreements can be viewed as a reduced form treatment of a dynamic game with many state realizations—as long as the discount factor is high enough, our agreements will satisfy the required incentive compatibility constraints for an equilibrium. We also relax our assumptions on enforcement in Section 6.

In period 1 the solution concept we apply to identify which networks form is pairwise stability. The collection of links formed is social network L, and agent i pays a cost $\kappa_w > 0$ for each link i has to someone in the same group, and $\kappa_a > \kappa_w$ for each links i has to someone from a different group. Normalizing the utility from autarky to 0, we abuse notation[24] and let agent i's *net* expected utility if network L forms be

$$u_i(L) = MV_i(L) - |\mathbf{N}_{G(i)}(i;L)|\kappa_w - \Big(|\mathbf{N}(i;L)| - |\mathbf{N}_{G(i)}(i;L)|\Big)\kappa_a. \tag{7}$$

A network L is *pairwise stable* with respect to expected utilities $\{u_i(L)\}_{i\in\mathbf{N}}$ if and only if for all $i, j \in \mathbf{N}$, (i) if $l_{ij} \in L$ then $u_i(L) - u_i(L \setminus \{l_{ij}\}) \geq 0$ and $u_j(L) - u_j(L \setminus \{l_{ij}\}) \geq 0$; and (ii) if $l_{ij} \notin L$ then $u_i(L \cup l_{ij}) - u_i(L) > 0$ implies $u_j(L \cup l_{ij}) - u_j(L) < 0$. In words, pairwise stability requires that no two players can both strictly benefit by establishing an extra link with each other, and no player can benefit by unilaterally deleting one of his links. From now on we will use the terms pairwise-stable and stable interchangeably.

Existence of a pairwise-stable network in our model follows from a result in Jackson (2003), stating that whenever payoffs in a simultaneous-move network formation game are determined based on the Myerson value, there exists a pairwise-stable network.

Our specification assumes that two agents forming a link have to pay the same cost for establishing the link. However, the set of stable networks would remain unchanged if we allowed the agents to share the total costs of establishing a link arbitrarily.[25] This is because for any link, the Myerson value rewards the two agents establishing the link symmetrically. Hence the agents can find a split of the link-formation cost such that establishing the link is profitable for both of them if and only if it is profitable for both of them to form the link with an equal split of the cost. Given this we stick with the simpler model with exogenously given costs.

A network L is *efficient* when there is no other network L'—and no risk sharing agreement on L'—that can make everyone at least as well off as they were on L and someone strictly better off. Let $|L_w|$ be the number of within-group links, and let $|L_a|$ be the number of

[24]In the previous section when investments had already been sunk we used $u_i(L)$ to denote i's expected payoff before link formation costs.

[25]More precisely, we could allow agents to propose a division of the costs of establishing each link as well as indicating who they would like to link to, and a link would then form only if both agents indicate each other and they propose the same split of the cost. A network would then be stable if it is a Nash equilibrium of this expanded network formation game and if there is no new link $l_{ij} \notin L$, and some split of the cost of forming this link, that would make both i and j strictly better off if formed.

across-group links. As expected utility is transferable in certainty-equivalent units, efficient networks must maximize the net total surplus $NTS(L)$:

$$NTS(L) := TS(L) - 2|L_w|\kappa_w - 2|L_a|\kappa_a, \tag{8}$$

Clearly, two necessary conditions for a network to be efficient are that the removal of a set of links does not increase $NTS(L)$ and the addition of a set of links does not increase $NTS(L)$. If there exists a set of links the removal of which increases $NTS(L)$, we will say there is *overinvestment* inefficiency. If there exists a set of links the addition of which increases $NTS(L)$, we will say there is *underinvestment* inefficiency.[26] A network is *robust to underinvestment* if there is no underinvestment inefficiency and no agent can strictly benefit from deleting a link that would result in underinvestment inefficiency. A network is *robust to overinvestment* if there is no overinvestment inefficiency and no pair of agent i, j can both strictly benefit from creating the link l_{ij}.

We will say that a link l_{ij} is *essential* if after its removal i and j are no longer path-connected while it is *superfluous* if after its removal i and j are still path-connected.

Remark 3. *Preventing overinvestment requires that all links be essential. Superfluous links create no social surplus and are costly. In all efficient networks, therefore, every component must be a tree.*

Real world networks among villagers are a long way from being trees. If our model perfectly captured network formation Remark 3 would imply that there is substantial overinvestment. However, our model is stylized, and this result needs to be applied with caution. For example, while there may be overinvestment, our assumption that all links are costly to form is unlikely to hold. Family ties or the time villagers spend working together might permit relationships to be formed without any additional investment. We discuss in Section 6.4 how, what we view as the main insights of our results, extend to a setting in which some links are free to form.

In most of the analysis below, we focus on investigating the relationship between stable networks and efficient networks. Additionally, we investigate the amount of inequality prevailing in equilibria in our model. For this, we will use the Atkinson class of inequality measures (Atkinson, 1970). Specifically we consider a welfare function $W : \mathbb{R}^{|\mathbf{N}|} \to \mathbb{R}$ that maps a profile of expected utilities into the real line such that

$$W(u) = \sum_{i \in \mathbf{N}} f(u_i), \tag{9}$$

where $f(\cdot)$ is assumed to be an increasing, strictly concave and differentiable function. The concavity of $f(\cdot)$ captures the social planner's preference for more equal income distributions.

[26]Note that these definitions are not mutually exclusive (there can be both underinvestment and overinvestment inefficiency) or collectively exhaustive (inefficient networks can have neither underinvestment nor overinvestment inefficiency if an increase in the net total surplus is only possible by the simultaneous addition and removal of links).

Supposing all agents instead received the same expected utility u', we can pose the question what aggregate expected utility is required to keep the level of the welfare function constant.[27] In other words we find the scalar $u' : |\mathbf{N}|f(u') = \sum_{i\in\mathbf{N}} f(u_i)$. Letting $\overline{u} = (1/|\mathbf{N}|)\sum_{i\in\mathbf{N}} u_i$ be the mean expected utility, Atkinson's inequality measure (or index) is given by

$$I(f) = 1 - \frac{u'}{\overline{u}} \in [0,1]. \tag{10}$$

We let $\mathcal{I}$ be the set (class) of Atkinson inequality measures and note that any $I(f) \in \mathcal{I}$ equals zero if and only if all agents receive the same expected utility.[28] There is an infinite set of inequality measures in the Atkinson class, and two different inequality measures in the class can rank the inequality of two distributions differently. However, there are certain pairs of distributions are ranked the same way by all members of the class, such as when one distribution is a mean-preserving spread of the other one.

4. Within-Group Networks

In this section we assume that $|\mathbf{M}| = 1$, that is, that agents are ex-ante symmetric, and any differences in their outcomes stem from their stable positions on the social network. This will lay the foundations for the more general case considered in the next section.

We begin our investigation by proving a general characterization of the set of stable networks. Recall that a *path* between i and j is a walk in which no agent is visited more than once. If there are K paths between i and j on the network L, we let $\mathbf{P}(i,j,L) = \{P_1(i,j,L), \ldots, P_K(i,j,L)\}$ be the set of these paths. For every $k \in \{1,\ldots,K\}$, let $|P_k(i,j,L)|$ be the cardinality of the set of agents on the path $P_k(i,j,L)$.[29] We can now use these definitions to define a quantity that captures how far away two agents are on a network in terms of the probability that for a random arrival order they will be connected without a direct link when the second of the two agents arrives. We will refer to this distance as the agents' Myerson distance:

$$md(i,j,L) := \frac{1}{2} - \sum_{k=1}^{|\mathbf{P}(i,j,L)|} (-1)^{k+1} \left(\sum_{1\le i_1<\cdots<i_k\le|\mathbf{P}(i,j,L)|} \left(\frac{1}{|P_{i_1} \cup \cdots \cup P_{i_k}|} \right) \right). \tag{11}$$

This expression calculates the probability that for a random arrival order the link l_{ij} will be essential immediately after i arrives,[30] using the classic *inclusion–exclusion principle* from combinatorics. This probability is important because it affects i's incentives to link to j.

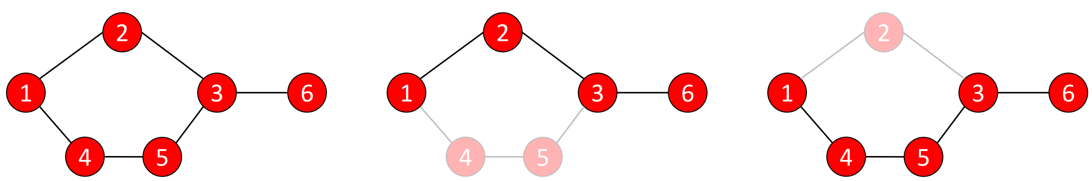
[27] This exercise is analogous to the certainty equivalent exercise that can be undertaken for an agent facing stochastic consumption.

[28] As $f(\cdot)$ approaches the linear function the social planner cares less about inequality and $I(f) \to 0$. Nevertheless, strict concavity prevents $I(f)$ equaling 0 unless all agents receive the same expected utility.

[29] For example, for a path $P_k(i,j,L) = \{i,i',i'',j\}$, $|P_k(i,j,L)| = 4$ and for a path $P_{k'}(i,j,L) = \{i,i',i''',i'''',j\}$, $|P_{k'}(i,j,L)| = 5$. Finally, we will let $|P_k(i,j,L) \cup P_{k'}(i,j,L)| = 5$ denote number of different agents on path $P_k(i,j,L)$ or path $P_{k'}(i,j,L)$.

[30] If for a given arrival order, agents $\mathbf{S} \subseteq \mathbf{N}$ arrive before i, then l_{ij} is essential immediately after i arrives if it is essential on the network $L(\mathbf{S} \cup \{i\})$.

As an illustration, consider the network shown in Panel (A) of Figure 2. The Myerson value allocates each agent their average marginal contribution to total surplus, where the average is taken over all possible arrival orders. For example, for the network shown in Figure 2 consider the arrival order $1, 2, 5, 6, 3, 4$. When agent 1 is added there are no other agents and so no links are formed. Thus 1's marginal contribution to total surplus is 0. Then agent 2 is added and the link l_{12} is formed. This link is essential on this network permitting risk sharing between agents 1 and 2 that wasn't previously possible. As a result, by equation 4, the total surplus generated by risk sharing increases from 0 to V. Thus 2's marginal contribution to total surplus, for this arrival order, is V. When 5 and 6 are added no new links are formed and no additional risk sharing is possible—their marginal contributions are 0. However, the arrival of 3 results in the formation of the links l_{23}, l_{35} and l_{36}. All of these links are essential and risk sharing among agents $1, 2, 5, 6$ and 3 becomes possible. This increases total surplus to $4V$ by equation 4, so 3's marginal contribution to total surplus is $3V$. Finally, adding 4 the links l_{14} and l_{45} are formed, and this permits risk sharing to also include 4 increasing total surplus to $5V$. So 4's marginal contribution to total surplus is V.



(A) Network (B) Path 1 (C) Path 2

FIGURE 2. Paths connecting nodes 1 and 6.

Whenever a link is formed that is essential for a given arrival order, it contributes V to total surplus, while whenever a link is superfluous for a given arrival order, it makes a marginal contribution of 0 to total surplus.[31] Consider now the incentives agent 1 has to form a superfluous link to agent 6. To calculate this we need to know the probability with which such a link would be essential for a random arrival order. There are three ways in which the link l_{16} might not be essential upon i's arrival. First, with probability 1/2 agent 6 arrives after agent 1 and the link l_{16} will be formed on 6's arrival instead of 1's. Second, Path 1 shown in Panel (B) of Figure 2 might be present. This will be the case if and only if agents 2, 3 and 6 arrive before agent 1. The probability that agent 1 is last to arrive of these 4 agents is 1/4. Finally, Path 2 shown in Panel (C) of Figure 2 might be present. This occurs if and only if agents 3, 4 ,5 and 6 arrive before 1. The probability that 1 is last to arrive of these 5 agents is 1/5.

If these three possibilities were mutually exclusive, then the probability the link l_{16} would be formed and essential upon 1's arrival would be: $1 - 1/2 - 1/4 - 1/5$. The probability that

[31]Note that in the arrival order considered in the preceding paragraph, 4's marginal contribution to total surplus would still have been V without the link l_{14} (l_{45}) as long as the link l_{45} (l_{14}) was still formed.

agent 6 arrives after agent 1 is mutually exclusive from the probability that either Path 1 or Path 2 is present, because both these paths need agent 6 to arrive before agent 1. However, it is possible for both Path 1 and Path 2 to be formed upon 1's arrival. Indeed, this occurs if and only if agent 1 is the last agent to arrive, which happens with probability 1/6. So the probability that at least one of the two paths to agent 6 is present upon 1's arrival is $1/4 + 1/5 - 1/6$. We need to subtract the probability 1/6 to avoid double counting the event that both paths are present. Thus, the probability that the link l_{16} will be essential upon 1's arrival, is $1 - 1/2 - 1/4 - 1/5 + 1/6 = md(1, 6, L)$.

Lemma 4. *If all agents are from the same group network L is pairwise stable if and only if*

$$\begin{array}{rrcll} (i) & md(i,j,L\setminus\{l_{ij}\}) & \geq & \kappa_w/V & \text{for all } l_{ij}\in L, \text{ and} \\ (ii) & md(i,j,L) & \leq & \kappa_w/V & \text{for all } l_{ij}\notin L. \end{array}$$

The proof is relegated to Appendix I. Recall from equation 3 that the social benefits of a link is proportional to the variance reduction it generates. For a single group, if a link l_{ij} is essential in the network $L \cup \{l_{ij}\}$, then this variance reduction is $\Delta\,\mathrm{Var}(L \cup \{l_{ij}\}, L) = (1 - \rho_w)\sigma^2$.

The crucial feature of this expression is that it does not depend on size of the network components the link l_{ij} connects on L. Although in general the size of these components does affect the consumption variance, two effects exactly offset each other.[32] On the one hand, in larger components there are more people to benefit from the essential link. On the other hand, people are already able to smooth there consumption more effectively.

As the social value of a non-essential, or superfluous link, is always zero the total surplus generated by a network L takes a very simple form. Let $\Upsilon(L)$ be the number of network components on L. Then

$$TS(L) = CE\Big(\Delta\,\mathrm{Var}(L,\emptyset)\Big) = \Big(|\mathbf{N}| - \Upsilon(L)\Big)\frac{\lambda}{2}\Big(1-\rho_w\Big)\sigma^2 = \Big(|\mathbf{N}| - \Upsilon(L)\Big)V. \tag{12}$$

Since the surplus created by any essential link is V, the total gross surplus is equal to this constant times the number of network component reductions obtained relative to the empty network.

To consider individual incentives to form links we can use the definition of the Myerson value and consider the average marginal contribution an agent makes to total surplus over all possible arrival orders. Specifically, we want to consider the increase in i's Myerson value due to a link l_{ij}. The link l_{ij} will reduce the number of components in the network by one when i arrives, relative to the counterfactual component reduction without l_{ij}, if and only if j has already arrived and there is no other path between i and j. In other words, the

[32] Let $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ be the network components of agent i and agent j on network $L \setminus \{l_{ij}\}$, and let $|\mathbf{S}_1| = s_1$ and $|\mathbf{S}_2| = s_2$. Then the sum of consumption variances on $L(\mathbf{S}_1)$ and $L(\mathbf{S}_2)$ (with Pareto efficient risk sharing) are $\frac{s_1+s_1(s_1-1)\rho_w}{s_1}\sigma^2$ and $\frac{s_2+s_2(s_2-1)\rho_w}{s_2}\sigma^2$, respectively. Once S_1 and S_2 are connected through l_{ij}, the sum of consumption variances on $L(\mathbf{S}_1 \cup \mathbf{S}_2)$ becomes $\frac{s_1+s_2+(s_1+s_2)(s_1+s_2-1)\rho_w}{s_1+s_2}\sigma^2$. This implies that the consumption variance reduction induced by the link l_{ij} is $\Delta\,\mathrm{Var}(L \cup \{l_{ij}\}, L) = (1 - \rho_w)\sigma^2$.

link increases i's marginal contribution to total surplus if and only if it is essential when i is added. Moreover, for the permutations in which l_{ij} is essential it contributes V to i's marginal contribution to total surplus. Averaging over arrival order, the value to i of the link $l_{ij} \in L$ is $md(i, j, L \setminus \{l_{ij}\})V$, while the value to establishing a new link $l_{ij} \notin L$ is $md(i, j, L)V$.

If a link l_{ij} is essential on L then for any arrival order, there will always be a component reduction of 1 when the later of i or j is added. Therefore, $md(i, j, L) = 1/2$, and l_{ij} will be formed as long as $V > 2\kappa_w$. As V is the social value of forming the link and $2\kappa_w$ is the total cost of forming it, when all agents are from the same group there is never underinvestment in a stable network or overinvestment in an essential link.

Proposition 5. *If all agents are from the same group then there is never underinvestment in a stable network. Furthermore, there is never overinvestment in an essential link.*

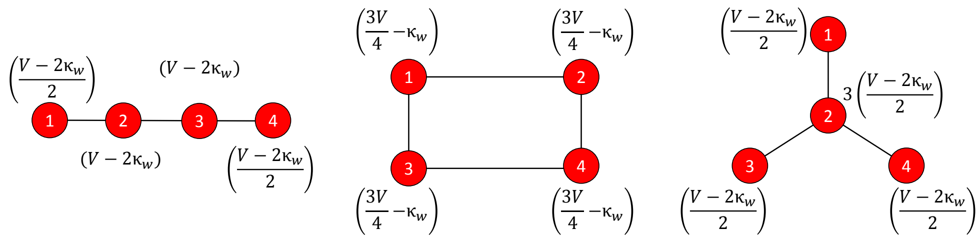
The proof is relegated to Appendix I. When all agents are from the same group Proposition 5 establishes that there is never overinvestment in an essential link, but overinvestment in superfluous links is possible. If the costs of link formation are low enough then agents will receive sufficient benefits from establishing superfluous links to be incentivized to do so. Even if a link l_{ij} is superfluous on L, for some arrival orders it will be essential on the induced subnetwork at the moment when i is added and make a positive marginal contribution to total surplus.[33] An example of such overinvestment is shown in Section E of the Supplementary Appendix.

An immediate implication of Proposition 5 is that if all agents are from the same group and $2\kappa_w > V$ then the only stable network is the empty one and this network is efficient, while if $2\kappa_w < V$ then all stable networks have only one network component (all agents are path-connected). For the remainder of the paper we focus on the parameter range for which the empty network is inefficient for a single group and assume $2\kappa_w < V$. We refer to this as our *regularity condition* and omit it from the statement of subsequent results.

Under this regularity condition the set of efficient networks are the set of tree networks in which all agents are path-connected. In other words, all agents must be in the same component and all links must be essential. We will now focus on which, if any, of these efficient networks are stable. As agents are well incentivized to form essential links, the only reason an efficient network will not be stable is if two agents have a profitable deviation by forming an additional (superfluous) link.

Figure 3 illustrates three networks: A line (Figure 3a); a circle (Figure 3b) and a star (Figure 3c). While the line and star networks are efficient, the circle network is not as it includes a superfluous link. Among the two efficient networks, the star is more stable than the line. Applying Lemma 4, whenever the line is stable so is the star but there are parameter values for which the star is stable and the line is not. While the star is more stable than the line, it also results in more inequality. The expected utility distribution obtained on the

[33] Consider, for examples, arrival orders in which i arrives first and j arrives second.



(A) Line (B) Circle (C) Star

FIGURE 3. Three possible network structures for connecting 4 agents and the resulting net payoffs.

line network can be generated from that obtained on the star network by the best off agent (agent 2) transferring $(V - 2\kappa_w)/2 > 0$ units of expected utility to one of the worst off agents (agent 3). This is enough to ensure that the expected utility distribution on the star is more unequal than the expected utility distribution on the line for any inequality measure in the Atkinson class. We generalize these insights in Proposition 6.

Proposition 6. *Suppose all agents are from the same group.*

(i) *If there exists an efficient stable network then star networks are stable, and for a non-empty range of parameter specifications only star networks are stable. If a line network is stable then all efficient networks are stable.*

(ii) *For all inequality measures in the Atkinson class, among the set of efficient network, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

The proof is in Appendix I but we provide some intuition after we discuss the result. Proposition 6 states that, in a certain sense, among the set of efficient networks the star is the most stable but maximizes inequality, while the line minimizes inequality but is least stable. This indicates a novel tension between stability/efficiency and inequality. For example, in contrast, Pycia (2012) studies when stable coalitional structures exist and finds that stable coalitions are more likely to exist when the bargaining functions of agents are more equal.

To gain intuition for Proposition 6, recall that an efficient network will be stable if and only if no pair of players have a profitable deviation in which they form a superfluous link. By Lemma 4 the incentives for two agents to form such a link are strictly increasing in their Myerson distance. Thus, a network is stable if and only if the pair of agents furthest apart from each other, in terms of their Myerson distance, cannot benefit from forming a link. As efficient networks are tree networks, the Myerson distance between any two agents depends only the length of the unique path between them.[34] The longest path between any pair of agents is, by definition, the diameter of the network $d(L)$. So, an efficient network is stable if

[34]Suppose d is the number of agents on the unique path connecting i and j. The probability that this path exists when agent i arrives is $1/d$. In addition, if agent j has not yet arrived, which occurs with probability $1/2$, i would not benefit from the link l_{ij}, so i's expected payoff from forming a superfluous link to j is

and only if its diameter is sufficiently small. More precisely, an efficient network L is stable if and only if its diameter is weakly less than $\overline{d}(\kappa_w, V)$, where $\overline{d}(\kappa_w, V)$ is increasing in κ_w, decreasing in V and integer valued.[35]

Let $\mathcal{L}^e(\mathbf{N})$ be the set of efficient networks. Star networks have the smallest diameter among networks within this set, while line networks have the largest diameter among networks within this set. This establishes part (i) of Proposition 6.

To gain intuition for part (ii) a first step is noting that on any efficient network agents' net payoffs are proportional to their degrees (i.e., the number of neighbours they have):[36] $u_i(L) = |\mathbf{N}(i; L)|(V/2 - \kappa_w)$. The key insight is then showing that for any network in the set of efficient networks $\mathcal{L}^e(\mathbf{N})$, the star network can be obtained by rewiring the network (deleting a link $l_{ij} \in L$ and adding a link $l_{ik} \notin L$) in such a way that at each step we increase the degree of the agent who already has the highest degree, reduce the degree of some other agent and obtain a new network in $\mathcal{L}^e(\mathbf{N})$. This process transfers expected utility to the agent with the highest expected payoff from some other agent, thereby increasing inequality for any inequality measure in the Atkinson class. Likewise, we can obtain the line network from any network in the set $\mathcal{L}^e(\mathbf{N})$ by rewiring the network to decrease the degree of the agent with the highest degree at every step. This transfers expected utility from the agent with the highest expected payoff to some other agent, thereby decreasing inequality for any inequality measure in the Atkinson class.

To summarize, this section identifies a novel downside to informal risk sharing agreements. Even when investments into social capital are efficient, the networks that can be supported in equilibrium generate social inequality, and this translates into (potentially severe) financial inequality. The setting we have used to identify this tension between efficiency/stability and inequality is very stylized in a number of dimensions. In the rest of the paper, we extend our baseline model in a variety of directions to demonstrate that this basic tension is extremely robust. First we partition agents into multiple groups and generalize the joint income distribution (Section 5). Then, in Section 6 we show robustness to (i) simultaneously generalizing the key assumptions in our baseline model while retaining its basic structure; and (ii) incorporating enforcement. We also show in this section that the same tension appears in an alternative model with imperfect risk-sharing and demonstrate a fundamental trade-off between equality and efficiency/stability that is present extremely generally by building on some well known graph-theory results.

5. Connections Across Groups

We now generalize our model by permitting multiple groups. These different groups might correspond to people from different villages, different occupations, or different social status

$(1 - 1/2 - 1/d)V$. We also note that as d gets large, this converges to $V/2$ which is the value i receives from forming an essential link.

[35]We show in the proof of Proposition 6 that $\overline{d}(\kappa_w, V) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.

[36]This is also known as an agent's degree centrality.

groups, such as castes. We will first show that (under our regularity condition) there is still never any underinvestment within a group. However, this does not apply to links that bridge groups. As, by assumption, incomes are more correlated within a group than across a group, there can be significant benefits from establishing such links and not all these benefits accrue to the agents forming the link. Intuitively, an agent establishing a bridging link to another group provides other members of his group with access to a less correlated income stream, which benefits them. As agents providing such bridging links are unable to appropriate all the benefits these links generate, and these links are relatively costly to establish, there can be underinvestment.

To analyze the incentives to form links within a group, we first need to consider the variance reduction obtained by a within-group link. Such a link may now connect two otherwise separate components consisting of arbitrary distributions of agents from different groups. Suppose the agents in $\mathbf{S}_0 \cup \cdots \cup \mathbf{S}_k$ and the agents in $\widehat{\mathbf{S}}_0 \cup \cdots \cup \widehat{\mathbf{S}}_k$ form two distinct network components, where for every $i \in \{0, ..., k\}$, the agents in $\mathbf{S}_i$ and those in $\widehat{\mathbf{S}}_i$ are all from group i. Consider now a potential link l_{ij} connecting the two otherwise disconnected components. Letting s_0 be the number of agents in group 0, the variance reduction obtained is:[37]

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[(1-\rho_w) + \frac{\sum_{i=0}^k \left(\hat{s}_i \sum_{j=0}^k s_j - s_i \sum_{j=0}^k \hat{s}_j \right)^2}{\left(\sum_{i=0}^k s_i\right)\left(\sum_{i=0}^k \hat{s}_i\right)\left(\sum_{i=0}^k s_i + \hat{s}_i\right)} (\rho_w - \rho_a) \right] \sigma^2. \tag{13}$$

The key feature of this variance reduction is that it is always weakly greater than $(1-\rho_w)\sigma^2$, which is the variance reduction we found in the previous section when all agents were from the same group. Thus, the presence of across-group links only increases the incentives for within-group links to be formed. A within-group link can now give (indirect) access to less correlated incomes from other groups and so is weakly more valuable. This implies that there will still be no underinvestment under our regularity condition that $2\kappa_w < V$.[38] The above reasoning is formalized by Proposition 7.

Proposition 7. *There is no underinvestment between any two agents from the same group in any stable network.*

The proof of Proposition 7 is in Appendix I. While underinvestment is not possible within group, it is possible across groups. An example of this is shown in Section E of the Supplementary Appendix. Although when all agents are from the same group the value of an

[37]By definition

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \operatorname{Var}(L(\mathbf{S}_0, ..., \mathbf{S}_k)) + \operatorname{Var}(L(\widehat{\mathbf{S}_0}, ..., \widehat{\mathbf{S}_k})) - \operatorname{Var}(L(\mathbf{S}_0 \cup \widehat{\mathbf{S}_0}, ..., \mathbf{S} \cup \widehat{\mathbf{S}_k})).$$

Recalling that

$$\operatorname{Var}(L(\mathbf{S}_0, \mathbf{S}_1, ..., \mathbf{S}_k)) = \left(\sum_{i=0}^k (s_i + s_i(s_i - 1)\rho_w) + 2\rho_a \sum_{i=0}^{k-1} (s_i \sum_{j=i+1}^k s_j) \right) \sigma^2 \Big/ \sum_{i=0}^k s_i \ ,$$

some algebra yields the result.

[38]Recall that this regularity condition just requires that it is efficient for two agents in the same group, both without any other connections, to form a link.

essential link does not depend on the sizes of the components it connects, the value of an essential link connecting two different groups of agents increases in the sizes of the components. To demonstrate this formally, consider an isolated group that has no across-group connections and consider the incentives for a first such connection to be formed. Thus the first component consist of agents from a single group, say group 0. We let the second component consist of agents from one or more of the other groups (1 to k). The variance reduction obtained by connecting these two components is

$$\Delta \operatorname{Var}(L \cup l_{ij}, L) = \left[(1-\rho_w) + \frac{\hat{s}_0 \left(\left(\sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2 \right)}{\left(\sum_{i=1}^k s_i \right) \left(\hat{s}_0 + \sum_{i=1}^k s_i \right)} (\rho_w - \rho_a) \right] \sigma^2, \tag{14}$$

which is increasing in $\hat{s}_0$:

$$\frac{\partial \Delta \operatorname{Var}(L \cup l_{ij}, L)}{\partial \hat{s}_0} = \frac{\left(\sum_{i=1}^k s_i \right)^2 + \sum_{i=1}^k s_i^2}{\left(\hat{s}_0 + \sum_{i=1}^k s_i \right)^2} (\rho_w - \rho_a) \sigma^2 > 0. \tag{15}$$

The inequality follows since $\rho_w > \rho_a$. Thus if agents i and j who connect two otherwise unconnected groups they receive a strictly smaller combined private benefit than the social value of the link. To see why, suppose that on the network L the link l_{ij} is essential, and without l_{ij} there would be two components, the first connecting agents from group $G(i)$ and the second connecting agents from group $G(j) \neq G(i)$. Consider the Myerson value calculation. For arrival orders in which i or j is last to arrive, the value of the additional variance reduction due to l_{ij} obtained upon the arrival of the later of i or j, is the same as its marginal social value, i.e., the value of variance reduction obtained by l_{ij} on L. For any other arrival order the value of variance reduction due to l_{ij} when the later of i or j arrives is strictly less. Averaging over these arrival orders, the link l_{ij} contributes less to i and j's combined Myerson values than its social value, leading to the possibility of underinvestment.

Besides underinvestment, overinvestment is also possible across groups. Forming superfluous links will increase an agent's share of surplus without improving overall risk sharing and can therefore create incentives to overinvest. Nevertheless, when κ_a is relatively high, underinvestment rather than overinvestment in across-group links will be the main efficiency concern. In many settings, within-group links are relatively cheap to establish in comparison to across-group links. For example, when the different groups correspond to different castes, it can be quite costly to be seen interacting with members of the other caste (e.g., Srinivas (1962), Banerjee et al. (2013b)). Motivated by this, and because across-group links are typically far sparser than within-group links, we focus our attention on this parameter region. More concretely, below we investigate what within-group network structures create the best incentives to form across-group links and what network structures minimize the incentives for overinvestment within group. Remarkably, we find that these two forces push within-group network structures in the same direction, and in both cases towards inequality in the society.

We begin by considering within-group overinvestment, which corresponds to the formation of superfluous links within a group. We found in the previous section that when all agents are from the same group the star is the efficient network that minimized the incentives for overinvestment. However, once we include links to other groups, the analysis is more complicated. The variance reduction a within-group link generates is still 0 if the link is superfluous, but when the link is essential it depends on the distribution of agents across the different groups the link grants access to. Moreover, the variance reduction may be decreasing or increasing in the numbers of people in those groups.[39] This makes the Myerson value calculation substantially more complicated. When all agents were from the same group all that mattered was whether the link was essential when added. Now, for each arrival order in which the link is essential, we also need to keep track of the distribution of agents across the different groups that are being connected. Nevertheless, our earlier result generalizes to this setting, although the argument establishing the result is more subtle.

To state the result, it is helpful to define a new network structure. A *center-connected star* network is a network in which all within-group network structures are stars and all across-group links are held by the center agents in these stars. We denote the set of center-connected star networks by $\mathcal{L}^{CCS}$.

Proposition 8. *If any efficient network L is robust to overinvestment within group, then any center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to overinvestment within group. Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to overinvestment within group but L is not.*

The proof of Proposition 8 is in Appendix I. In Proposition 6 we found that when all agents are from the same group, incentives for overinvestment (within group) are minimized by forming a (within-group) star. However, the incentives to form superfluous within-group links are weakly greater when someone within the group holds an across-group link (see equation 13). We can therefore think of the incentives for over-investment we found in Proposition 6 as a lower bound on the minimal incentives we can hope to obtain once there are across-group links. A key step in the proof of Proposition 8 shows that this lower bound is obtained by all center-connected star networks.

Consider a center-connected star network L'. As the agent at the center of a within-group star, agent k, has a link to all agents within the same group, we can focus on the incentives of two non-center agents from the same group, i and j, to form a superfluous link. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. On the induced subnetwork $L'(\mathbf{S})$ either l_{ij} is superfluous or else $k \notin \mathbf{S}$. This implies that no across-group links are present whenever the

[39]In the case of an essential across-group link that connects agents from just one group to agents from other groups, the comparative statics are unambiguous. In this case, the variance reduction is increasing in the sizes of the groups connected (see inequality (15)).

additional link l_{ij} makes a positive marginal contribution. Hence considering different arrival orders, the average marginal contribution of such a link when it is added is the same on the star network with no across-group links as for a center-connected star network: The lower bound on within-group overinvestment incentives is obtained.

We now consider the within-group network structures that maximize the incentives for an across-group link to be formed. We have already established that the marginal contribution of a first bridging link to the total surplus is increasing in the sizes of the groups it connects. By the Myerson calculation, the agents with the strongest incentives to form such links are then those who will be linked to the greatest number of other agents within their group when they arrive. The result below formalizes this intuition.

Let $\mathcal{A}(\mathbf{S}_k)$ be the set of possible arrival orders for the agents in $\mathbf{S}_k$. For any arrival order $A \in \mathcal{A}(\mathbf{S})$, let $\mathbf{T}_i(A)$ be the set of agents to whom i is path-connected on $L(\mathbf{S}')$, where $\mathbf{S}'$ is the set of agents (including i) that arrive weakly before i. Let $T_i^{(m)}$ be a random variable, taking values equal to the cardinality of $\mathbf{T}_i(A)$, where A is selected uniformly at random from those arrival orders in which i is the m-th agent to arrive.

We will say that agent $i \in \mathbf{S}_k$ is more *Myerson central* (from now on, simply more central, for brevity) within his group than agent $j \in \mathbf{S}_k$ if $T_i^{(m)}$ first-order stochastically dominates $T_j^{(m)}$ for all $m \in \{1, 2, ..., |\mathbf{S}_k|\}$.[40] In other words, considering all the arrival orders in which i is the m-th agent to arrive, and all the arrival orders in which j is the m-th agent to arrive, the size of i's component at i's arrival is larger than that of j's at j's arrival in the sense of first-order stochastic dominance.[41] This measure of centrality provides a partial ordering of agents.

Lemma 9. *Suppose agents in $\mathbf{S}_0$ form a network component, and all other agents in $\mathbf{N}$ form another network component. Let $i, i' \in \mathbf{S}_0$ and let $j \notin \mathbf{S}_0$. If i is more central within group than i', then i receives a higher payoff from forming l_{ij} than i' receives from forming $l_{i'j}$:*

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L).$$

The proof is relegated to Appendix I. The key step in the proof pairs the arrival orders of a more central agents with a less central agent, so that in each case the more central agent is connected to weakly more people in the same group upon his arrival, and to the same set of people from other groups. Such a pairing of arrival orders is possible from the definition of centrality, and in particular the first-order stochastic dominance it requires.

Lemma 9 shows that more central agents have better incentives to form intergroup links. We can then consider the problem of maximizing the incentives to form intergroup links by

[40] We also use this notion of centrality to compare the within-group centrality of the same agent on two different network structures. To avoid repetition we do not state the slightly different definition that would apply this situation.

[41] An alternative and equivalent definition is that i is more central than j if there exists a bijection $B : \mathcal{A}(\mathbf{S}_k) \to \mathcal{A}(\mathbf{S}_k)$ such that $|\mathbf{T}_i(A)| \geq |\mathbf{T}_j(B(A))|$ and $A(i) = A'(j)$, where $A(i)$ is i's position in the arrival order A and $A' = B(P)$.

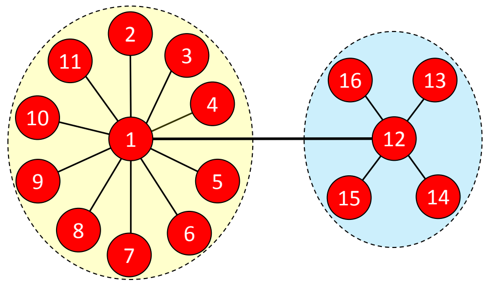


FIGURE 4. Center-connected within-group stars, in a context with two groups.

choosing the within-group network structures (networks containing only within-group links). We will say that the within-group network structures that achieve these maximum possible incentives are most robust to underinvestment inefficiency across groups.

Proposition 10. *If any efficient network L is robust to underinvestment across group, then some center-connected star network $L' \in \mathcal{L}^{CCS}$ is also robust to underinvestment across group. Moreover, if $L \notin \mathcal{L}^{CCS}$, then for a range of parameter specifications the center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but L is not.*

The proof of Proposition 10 is in Appendix I. Intuition can be gained from Lemma 9. This Lemma shows that agents have better incentives to provide a bridging link across group when they are more central within their own group. Thus to maximize the incentives of an agent to provide an across-group link, we need to maximize the centrality of this agent within group. This is achieved by any network that directly connects this agent to all others in the same group. However, only one of these within-group network structures can be part of an efficient network, and this is the star network, with the agent providing the across-group link at the center.

Figure 4 shows a center-connected star network when there are two groups. As long as it is efficient for these groups to be connected, center-connected star networks and only the center-connected star networks minimize the incentives for within-group overinvestment (by Proposition 9) and minimize the incentives for across-group underinvestment (by Proposition 10).

The above results further reinforce the tension between efficiency and equality. However, one subtlety relative to the one group case is that while the center-connected star network maximizes social inequality, in terms of agents' degrees, among all efficient networks, additional assumptions are required on the parameters of the model to ensure that such networks also maximize income inequality for all inequality measures in the Atkinson class. Taking the link formation costs as sunk and considering only second period income, income inequality is maximized by the center-connected-stars network (among efficient networks). However, while the formation of the across-group link benefits the center agents, and increases their expected net payoffs (including link formation costs), unlike the formation of other links, it also increases the expected net payoffs of other individuals in the group.

6. Extensions

The model we presented in the previous sections abstracts away from many potentially relevant aspects of informal risk-sharing, such as enforcement issues or coalitional (group) deviations. Additionally, it imposes specific functional forms on the income distribution and utility functions, and it assumes a micro-founded, but specific, rule of sharing the surplus generated by risk-sharing. These assumptions and modeling choices were made for analytical tractability. In this section we demonstrate that our model can be generalized and extended in many directions, and the main qualitative finding, that there is a tension between equality and efficiency when incentive compatibility is required, remains intact. Our other results, on under- and overinvestment inefficiencies, can also be extended to more general environments than the baseline model, and we provide some results in this direction in the Supplementary Appendix. We also limit attention in this section to homogeneous groups. Extensions of results involving multiple groups are in the Supplementary Appendix.

6.1. **More general environments and surplus sharing rules.** First we examine how the main insights from the baseline model extend when we allow for more general income distributions, utility functions and risk sharing arrangements.

Outside the CARA-normal specification of the model expected utilities are in general nontransferable, so we need to take a more general approach to modeling the risk-sharing arrangements. Let $v_i(c_i)$ be the utility function for agent i, mapping second period consumption into utility. We assume that $v_i = v_j = v$ for all i and j, and that v is strictly increasing and strictly concave. Let $\mathcal{P}$ be the distribution incomes are drawn from.

Let $\mathcal{L}$ be the set of all possible networks for agents in $\mathbf{N}$. We assume there is a unique risk-sharing arrangement that will be implemented for any possible network $L \in \mathcal{L}$, and that agents correctly anticipate the risk-sharing agreement that will obtain. These risk-sharing arrangements, which depend on the social network, might be dictated by social conventions, or they can be outcomes of negotiation processes for transfer arrangements once the network is formed. Let $\tau(L)$ be the transfer arrangement and $u_i^\tau(L)$ be the expected second period consumption utility of agent i implied by $\tau(L)$.[42]

We continue to assume that for every $L \in \mathcal{L}$, $\tau(L)$ specifies a pairwise-efficient risk-sharing arrangement $\tau_{ij}(L)$ for every pair of agents i, j linked in L. As shown earlier, this is equivalent to $\tau(L)$ being Pareto efficient at a component level. Agent i maximizes the difference between expected utility from the second period risk sharing (given by u_i^τ) and her costs of establishing links. Let $C_i(L)$ be the set of agents on the same component as i given L.

[42]More precisely, utility function v_i, the distribution of income realizations and transfer arrangement $\tau(L)$ jointly determine $u_i^\tau(L)$.

Next we impose a series of assumptions on $\tau(\cdot)$. We do not claim that the above assumptions hold universally when informal risk-sharing takes place, but they are relatively weak requirements that are natural in many settings. Our main objective is to demonstrate that our qualitative results hold for a much broader class of models than the CARA-normal setting with surplus division governed by the Myerson value.

Assumption 11. (a) *$u_i^\tau(L \cup \{l_{ij}\}) > u_i^\tau(L)$ for every $L \in \mathcal{L}$, $i, j \in \mathbf{N}$ and $l_{ij} \notin L$.*

(b) *$u_k^\tau(L \cup \{l_{ij}\}) \geq u_k^\tau(L)$ for every $L \in \mathcal{L}$, $i, j, k \in \mathbf{N}$ and $C_i(L) \neq C_j(L)$.*

(c) *If $l_{ij} \notin L$, then*

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|),$$

when the function $g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|)$ is increasing in the distance measure $d(i,j,L)$ and $d : \overline{\mathbf{N}}^2 \times \mathcal{L} \to \mathbb{R}_{++}$ satisfies the following properties:

(i) *If i and j are in different components on L, then $d(i,j,L) = \overline{d}$, with $\overline{d}$ strictly greater than the maximum possible distance between any two path-connected agents.*

(ii) *d depends only on paths (thus ignoring walks with cycles).*

(iii) *Let S_{ij} be the set of paths between i and j and S_{kl} be the set of paths between k and l. We assume $d(i,j;L) > d(k,l;L)$ if there exists a matching function[43] $\mu \in \mathcal{M}(S, S')$ such that each path between i and j is matched to a shorter path between k and l, and all such paths between k and l are independent (do not pass through any of the same nodes as each other).*

(d) *For all networks L,*

$$c_w/2 < \min_{L,i,j \ st. \ C_i(L) \neq C_j(L)} u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L).$$

We maintain Assumption 11 for the rest of this subsection. Part (a) requires that establishing a link always strictly increases the connecting agents' expected consumption utilities. Part (b) requires that the formation of an essential link imposes no negative pecuniary externalities on other agents. Part (c) extends the idea that the private benefits two agents receive from establishing a link should be increasing in the distance between them, while permitting these private benefits to also depend on the sizes of the components being connected. The notion of distance used in this assumption is relatively broad. The class of distance measures permitted includes the Myerson distance which was found to matter in our baseline model, as well as many others. Note that the requirement (iii) on the distance measure only provides a weak partial ordering for the distances between agents. Part (d) requires the cost of forming a link to be small relative to the private benefits of establishing an essential link. For general utility functions and transfer arrangements, there is in general no guarantee that there is

[43]For two sets S and S' we define $\mathcal{M}(S, S')$ as the set of matching functions $\mu : S \to S' \cup \{\emptyset\}$, such that for $s \in S$ if $\mu(s) \neq \emptyset$ then $\mu(s) \neq \mu(t)$ for all $t \in S \setminus \{s\}$. Thus every $\mu \in \mathcal{M}(S, S')$ maps each element of S into a different element of S', or else the empty set.

not underinvestment. Part (d) restricts attention to parts of the parameter space in which underinvestment is ruled out. While this is a nontrivial assumption, it is realistic in many settings.

A network is Pareto efficient if there is a feasible transfer agreement that could be reached on that network such that there is no other network, feasible transfer agreement pair in which all agents are weakly better off and some agents are strictly better off.

Proposition 12. *A network is Pareto efficient if and only if it is a tree connecting all agents.*

Note that for any non-essential link $|C_i(L)| = |C_i(L) \cup \{l_{ij}\})|$. Thus the marginal benefits from i and j forming a superfluous links depend only on the distance between i and j on L and the number of agents in their component. The latter is n for any efficient network, by Proposition 12. Thus, for an efficient network L by Assumption 11 part (c), the marginal benefit i and j receive from forming a superfluous link depends only on the unique path length between i and j, and is strictly increasing in this path length. Thus an efficient network will be stable if and only the maximum distance between any two agents is sufficiently low. The next Corollary formally states this result.

Corollary 13. *An efficient network is stable if and only if its diameter is sufficiently small.*

A network is *least* stable within a class of networks, when its stability implies the stability of any other network in that class. A network is *most* stable within a class of networks, when its instability implies the instability of any other network in that class.

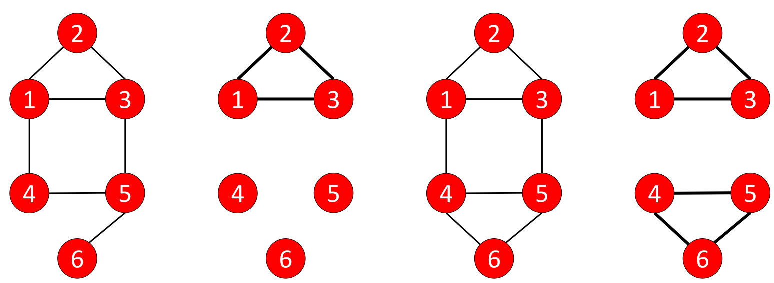
Proposition 14. (i) *The most stable efficient network is the star.*
(ii) *The least stable efficient network is the line.*

In this generalized framework further assumptions are needed to guarantee that the star is the least equitable and that the line is the most equitable tree network for *all* inequality measures within the Atkinson class. The next proposition provides one sufficient condition, requiring that if one efficient network can be obtained from another one by rewiring exactly one link then only the utilities of those agents who gain or lose a link are affected.

Proposition 15. *Suppose that for all pairs of efficient networks L and L' such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then for all inequality measures in the Atkinson class, among the set of efficient networks, star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.*

6.2. **Enforcement through supported risk sharing and coalitional deviations.** So far we have abstracted from enforcement problems. In this section we extend the model to capture the idea that having friends in common can reduce an agent's incentives to renege on an agreement. This might be because the friend in common is able to monitor actions and identify the guilty party in a dispute, or because reneging on the agreement will lead to a damaging reputation loss with the friend in common. While it is beyond the scope of this paper to fully explore these issues, and there is a vibrant literature that focuses on network based enforcement of agreements (see, for example, Jackson et al. (2012), Wolitzky (2012), Ali and Miller (2013, 16), Ambrus et al. (2014), Nava and Piccione (2014) and Ambrus et al. (2016)), in this Section, motivated by this literature, we model the value of friends in common for enforcement by assuming that risk sharing between two agents is possible if, and only if, those two agents have a friend in common. This is known as closure (Coleman, 1988) and has long been thought important for cooperation because it enables collective sanctions to imposed on a deviating agent—if an agent cheats on one of their neighbors, there are friends in common that can also punish the deviating agent.

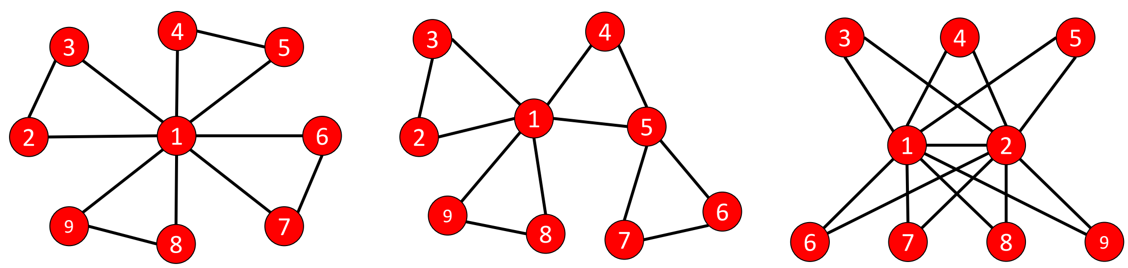
A link in L is supported and can be used for risk-sharing if, and only if, it is part of a triangle (i.e., the complete network among three agents). Let L' be the spanning subgraph of L which contains only supported links. An illustrations of this is provided in Figure 5. Risk-sharing arrangements, and rent distribution, are is as in Section 2. The only difference is that now risk-sharing takes place on the network $L'(L)$ instead of L (but agents continue to pay to form links in L). As in this setting it takes more than two agents to facilitate risk-sharing, we also require robustness of the network to the minimal coalitional deviations necessary to prevent the empty network from being stable.



(A) L_1 (B) L'_1 (C) L_2 (D) L'_2

FIGURE 5. Panel (A) provides an example of a network among six villagers, while panel (B) shows which of these links support risk sharing. Panel (C) provides a different example of network among six villagers, while panel (D) shows which of these links support risk sharing.

Before we can state our main result for this section we need some new terminology. A network L is a *tree-union of triangles* if it can be expressed as the union of m (non-node-disjoint) subnetworks ordered as $\{L(N_1), \dots, L(N_m)\}$, such that $\cup_{i=1}^{k} N_i \cap N_{k+1} = 1$ and each subnetwork $L(N_i)$ is a triangle. Thus each subnetwork in the sequence is a triangle that has exactly one node in common with the union of all the nodes in the subnetworks preceding it in the sequence. Panels (A) and (B) of Figure 6 illustrate two tree unions of triangles. The tree union of triangles illustrated in Panel (A) is known the *Friendship graph*, or Windmill network. These networks are tree-unions of triangles in which all triangles have the same node in common.



(A) (B) (C)

FIGURE 6. Panel (A) The shows the friendship graph connecting 9 villagers. Panel(B) shows a different tree union of triangles connecting nine villagers. Panel (C) shows an alternative network connecting 9 villagers in which all villagers are also able to risk share, but all triangles share a common link.

The cost of forming a link is κ, and also as before, we continue to focus on the parameter range for which risk sharing among all agents is always efficient. As before, the surplus obtained from enabling risk-sharing among two groups of agents is V. Proposition 16 shows it is efficient for all agents to risk-share if and only if $V \geq 3\kappa$, and that the efficient networks are then tree-unions of triangles. Thus, in comparison to Section 2 where agreements didn't need to be supported to be enforceable, tree unions of triangles play the role of tree networks.

Proposition 16. *Suppose the number of villagers $n \geq 3$ is odd.*

(i) *If risk-sharing among all n agents is efficient, then the efficient risk-sharing networks are tree-unions of triangles.*
(ii) *Risk sharing among all n agents is efficient for all n if and only if $V \geq 3\kappa$.*

All proofs for this section are relegated to the Supplementary Appendix. To gain intuition for this result, first observe that any link that is not supported is costly to form but cannot be used for risk-sharing. While in principle such a link might still be valuable as a means for supporting an agreement on another link, this requires a triangle to be formed with the link which would make it supported. Thus in an efficient network all links must be supported, and part of a triangle. Given this, the most efficient way to organize links (among an odd number of agents) is to form a tree union of triangles. This creates distinct triangles in which no link is shared by two triangles. As a comparison, note that there are 15 links formed in the network depicted in panel (c) of Figure 6, where all risk-sharing triangles share a common link, while there are only 12 links in the tree union of triangles illustrated in panels (a) and (b).

Jackson et al. (2012) find a class of networks they call social quilts to be those that can supporting risk-sharing agreements based on renegotiation proofness. Interestingly, tree unions of triangles are social quilts. The networks we identify through efficiency considerations based on the very simple condition of support being necessary for risk-sharing, would also be renegotiation proof in their setting. This provides further motivation for the simple approach to enforcement we take.

We now consider the stability of the efficient risk-sharing networks. Since the need for groups of at least three to support risk-sharing, here we require stability with respect to a simple form of coalitional deviations for groups of three agents.[44] In particular, we call a network triple-wise stable with respect to expected utilities $\{u_i(L)\}_{i\in\mathbf{N}}$ if and only if it is pairwise stable, and for all $i, j, k \in \mathbf{N}$, if two or more of l_{ij}, l_{ik}, l_{kj} are not in L and $\hat{L}$ is the union of network L with these three links, then if $u_i(\hat{L}) \geq u_i(L)$ and $u_j(\hat{L}) \geq u_j(L)$ with at least one inequality strict, then $u_k(\hat{L}) < u_k(L)$. In words, triplet-wise stability requires a network to be pairwise stable and for no set of three players to be able to benefit by forming the links among themselves (thereby facilitating direct risk sharing among themselves).

Proposition 17.

(i) *If there exists an efficient triplet-wise stable network then all friendship networks are triplet-wise stable, and for a non-empty range of parameter specifications only friendship networks are triplet-wise stable.*

(ii) *For all inequality measures in the Atkinson class, among the set of efficient triplet-wise stable networks, friendship networks and only friendship networks maximize inequality.*

This result is analogous to results in Proposition 6 in Section 4. There a star network was the most efficient stable network, but also the most unequal. Proposition 17 shows that this result generalizes to the case in which links must be supported to facilitate risk sharing, but with friendship networks (tree networks of triangles, with one agent in the center being part of every triangle) taking the place of star networks.

The basic intuition for the result mirrors the intuition for Proposition 6. Groups of three agents have stronger incentives to deviate and form links among themselves to facilitate risk sharing when they are further apart. Among the set of efficient networks the relevant distances are minimized by the friendship network. In terms of inequality, it can be shown that agents' net payoffs are again proportional to their degrees, and the total number of links is constant for all tree unions of triangles connecting n agents. Further, in any tree union of triangles all agents must have at least degree 2. The friendship network therefore minimizes the possible degree for all but one agent, while maximizing the possible degree for

[44]In the Supplementary Appendix, Section B, we also provide results for pairwise stability.

the remaining agent. Analogous arguments to those in Section 4 establishing that the star network maximizes inequality among all efficient networks for all inequality measures in the Atkinson class can then be made to show that in the current setting the friendship network maximizes inequality among all efficient networks for the same class of inequality measures.

6.3. **Imperfect risk sharing.** The benchmark model assumes that risk-sharing is perfect on a connected component of the network. In reality, risk-sharing can be imperfect for various reasons. In this section we briefly explore two of those reasons: because state-dependent transfers are costly and because relationships sometimes fail to function. We demonstrate that this can actually provide further motives for agents to form highly centralized networks. When state-dependent transfers are costly, short paths are efficient and that requires central agents. When relationships fail alternative paths are required and providing alternative paths efficiently (with relatively few links) again demands central agents.

6.3.1. *Costly transfers.* One possible reason for imperfectness of risk-sharing is that state-dependent bilateral transfers are costly to make (for example, income realizations might need to be verified and this could be costly). Here we consider a very simple environment with this feature. Assume there are n agents, with quadratic preferences[45]. Income realizations are such that exactly one randomly selected agent i gets hit by a bad shock ($e_i = -1$), one randomly selected agent j receives a good shock ($e_j = -1$), and all other agents are not hit by any shock. We assume that an ex ante meeting has to take place between neighboring agents, in order to establish risk-sharing arrangements, and that state-independent transfers can be arranged at this point. Up to this point only network formation costs have been incurred. Given these state independent transfers let $c_i(L)$ denote agent i's baseline consumption level. However, any subsequent state-dependent (post income realization) transfer requires an extra meeting or transaction (and possibly state verification) that incurs a cost $k > 0$. If the cost of forming a link κ is small enough, and k is small enough relative to κ, then any efficient risk-sharing network has to be a tree (small κ implies that an efficient network has to be connected, and small k relative to κ implies that duplicate links are inefficient). In this environment it is easy to show that the constrained Pareto efficient risk-sharing arrangement implies that for any income realization, there is a single chain of transfers corresponding to the unique path from the agent who received a good shock, to the agent hit by a bad shock, and everyone along this chain ends up with consumption $c_i - \frac{l}{l+1}k$, where l is the length of the path (i.e., agents along the chain equally divide the costs of the l bilateral transfers required to reach the agent with the bad shock). Given such risk-sharing agreements, the tree network maximizing social efficiency is the one that minimizes the average distance (path length) between two agents. The average distance is proportional to the Wiener index (the sum of distances between different pairs of agents), and it is well-known (see for example Dobrynyn, Entringer and Gutman (2001) p213) that the n-node star is the unique network minimizing

[45]This keeps expected utilities transferable given that incomes will no longer be normally distributed.

the Wiener index among n-node trees. So when transfers are costly, there is an extra reason to expect more centralized networks, as with a limited number of links such networks minimize average distance between agents, and thus reduce the costs of risk-sharing. As opposed to the benchmark model, here the star is not only the most stable efficient network structure, but in fact the only efficient network structure.[46]

6.3.2. *Link failures.* Another potential reason for imperfect risk-sharing can be that links might fail with some probability, making it impossible to send transfers through them, representing a fallout between neighbors or the absence or unavailability of a neighbor for other reasons.[47] If such link failures are not uncommon, instead of tree networks efficiency requires denser networks, with multiple paths existing between the same two agents, providing alternative routes to connect them in case link failures cut some of the paths between them. Characterizing efficient networks in such an environment analytically is a hard problem. But Peixoto and Bornholdt (2012), in a related model, using a combination of analytical and numerical techniques, find that for a large enough number of agents efficient networks have a core-periphery structure, where a small core of nodes with high degree is responsible for most of the connectivity, serving as a central backbone to the system. Such networks are close to multi-center generalizations of the star network, in which centers are connected to every agent, while periphery agents are only connected to the center.[48]

6.4. **Permitting some free links.** In practice relationships are formed for many reasons, and there will be some relationships that exist for reasons unrelated to risk sharing, but nevertheless permit risk sharing. These links might, for example, represent family relationships or close friendships formed in childhood. In effect, these are relationships it is free to form for the purpose of risk-sharing, providing another explanation for why real world risk-sharing networks are coarser than tree networks. We extend our baseline model to permit this possibility.

Let $\widehat{L}$ denote the exogenously given set of links that can be formed for free. As, by the Myerson Value calculation, a link strictly increases the expected utility an agent receives from the risk sharing arrangement, we assume all such links are always formed. The network $\widehat{L}$ will consist of a set of components. For each such component C, we identify an agent $i^*(C) \in \operatorname{argmin}_i \max_j md_{ij}(C)$. This is agent who has the lowest maximum Myerson Distance to any other agent in the component C. We will refer to agent $i^*(C)$ as the Myerson distance central agent in component C and let C_i denote the component to which i belongs. Considering all

[46]Moreover, if villagers whom are more important to implementing the risk-sharing transfers (i.e., those that are involved in more of the transfers) extract more of the surplus from risk-sharing, the star will again be associated with the most inequality.

[47]For models of network formation in environments outside the risk-sharing framework with the possibility of probabilistically failing links, see for example Bala and Goyal (2000) and Haller and Sarangi (2005).

[48]The efficiency of such network structures requires that link failures are exogenous and independent. If there is a strategic adversary selecting which links fail, more decentralized networks become optimal (see Dobrynin et al (2001) and Haller and Sarangi (2005). However, this scenario is less relevant in the risk-sharing context.

components, we then have a set of Myerson distance central agents $I^* = (i^*(C))_C$. Finally, we identify a Myerson distance central agent associated with the largest distance, $i^{**} \in \text{argmax}_{i^* \in I^*} \max_{j \in C_{i^*}} md_{i^*j}$.

We dub a network generated by forming all free links, and the links $l_{i^*i^{**}}$ for all $i^* \in I^* \backslash \{i^{**}\}$ a *central connections network*. Central connections networks are always efficient.[49] They are also most stable within the class of efficient networks.

Proposition 18. *Suppose there is one group. If any efficient network is stable, then all central connections networks are also stable.*

Proposition 18 shows that when some links are formed for free, the most stable efficient network forms all additional links required for risk-sharing with a single agent. As payoffs are proportional to degree, this again pushes villages towards inequitable outcomes.

6.5. **General tensions between stability, efficiency and inequality.** In this subsection we point out a general fundamental tension between equality and efficient stable networks. We begin by relating different graph theoretic concepts to stability, efficiency and inequality.

6.5.1. *Equality.* We would like to say something general about inequality for all inequality measures in the Atkinson class on formed networks for *any* symmetric payoff function $u : L \to \mathbb{R}$. Unfortunately, without further restrictions on how network positions translate into payoffs, it is impossible to compare two network in general. However, it is possible to pose and answer in general the question of when payoffs will be guaranteed to be perfectly equitable.

We proceed under the assumption that only agents' network positions matter for their payoffs—specifically, we require agents in identical network positions to receive the same payoffs. Intuitively, then, if all agents are in identical positions, they must receive equal payoffs. The set of networks for which this holds, thereby guaranteeing perfectly equitable outcomes, will be a useful benchmark that helps identify a general tension between equality and efficiency/stability.

In order to formalize the idea that agents are in identical network positions, we need to introduce some graph theory notations and terminology. We limit attention to connected networks. Every network is implicitly labelled, and we identify the set of labels with the set of nodes $\mathbf{N}$. Two networks L_1 and L_2 are called *isomorphic*, written $L_1 \sim_I L_2$, if they coincide up to labelling, i.e. a permutation of $\mathbf{N}$. They are also *automorphic* if for the permutation of nodes associated with the isomorphism, every node has the same set of neighbors. More formally, the network L_1 and L_2 are *automorphic*, written $L_1 \sim_A L_2$, if they are isomorphic, and for any $i, j \in \mathbf{N}$, $i \in \mathbf{N}(j; L)$ if and only if $f(i) \in \mathbf{N}(f(j); L)$, where f is the relevant isomorphism mapping $\mathbf{N}_1$ to $\mathbf{N}_2$, called an *automorphism.* A simple undirected binary graph $L \in \mathcal{L}$ is *vertex transitive* if for every pair of vertices i and j in $\mathbf{N}$, there exists

[49]As before, the same set of risk sharing arrangements can be implemented on any given component, and as expected utility is transferable, given that formation costs have been minimized, any point on the Pareto frontier can be obtained.

an automorphism $f_{ij} : \mathbf{N} \to \mathbf{N}$ such that $f(i) = j$. Thus, when a network is vertex transitive, we can take a node i and map it to the position of any other node j, by changing the label of j to i, and there exists a way of relabelling the other nodes such that all nodes have exactly the same neighbors as before and the structure of the graph is preserved. Thus the positions of any two nodes i and j in a vertex transitive network are equivalent in a certain sense, and it is intuitive that the agents should receive the same payoffs.

Indeed, in the Supplementary Appendix, Section D, we show that allowing for a general payoff function, vertex transitivity is sufficient for all nodes to receive the same payoffs, and generically, it is also necessary.

Vertex transitivity is a strong condition to place on the network structure. All vertex transitive networks are regular, but not all regular networks are vertex transitive. So the symmetry condition required for perfectly equitable outcomes in general is stronger than the symmetry condition needed in our baseline model.

6.5.2. *Efficiency.* A network $L = (n, L)$ is *Pareto efficient* if there is no network L' such that the payoffs of the agents on the network $L' = (n, L')$ Pareto dominate those on L (i.e., all agents receive weakly higher net payoffs on L' than L and at least one agent receives a strictly higher payoff).

To get a handle on the set of *Pareto efficient* networks, we assume that shorter path lengths facilitate weakly better risk-sharing. Specifically, we assume that all *Pareto efficient* networks $L = (n, L)$ have one component, and there is no alternative network $L' = (n, L')$ such that $|L'| \leq |L|$ and the path length distribution of L' first order stochastically dominates the path length distribution of L. This enables us to eliminate some configurations as being Pareto efficient.

6.5.3. *Stability.* Finally, we turn to stability. Since we want to make a point at a high level of generality, without a concrete model specification, below we propose a weak notion of stability that can be interpreted as a necessary condition. Given the assumption we have already made that shorter path lengths enable better risk-sharing, it is natural to also suppose that there are stronger incentives for agents further way in the network to have a profitable deviation in which they form a new link. This also preserves a key ingredient from our benchmark model in terms of agents' incentives to deviate.

We say two nodes i and j are closer on a network L than L' if every path between i and j on L' can be matched to a weakly shorter path between i and j on L. We assume that if two agents are further apart in this weak, partial ordering sense, then they have stronger incentives to deviate and form a new link.

6.5.4. *A general tension.* The next result formalizes the general tension among efficiency, stability and equality, by showing that for realistic numbers of agents a network cannot be both efficient and regular, which as argued above is in general necessary but not sufficient for perfectly equitable outcomes.

Proposition 19. *If in all Pareto efficient networks connecting n agents there are fewer than $n\sqrt{n-1}/2$ links, then there does not exist a constrained Pareto efficient and regular network.*

To aid interpretation of Proposition 19 it is helpful to consider some values of n that are of the same order of magnitude as village sizes. For $n = 100$, the optimal regular degree r that minimizes the overall number of links is $r = 10$. So there are at least 500 risk-sharing links present in a regular separation efficient network on 100 nodes (and the networks that achieves this are 10-regular). For $n = 500$, the optimal regular degree r that minimizes the overall number of links required is $r = 22$. So there are at least 5670 risk-sharing links present in a Pareto efficient network on 500 nodes (and the network that achieves this is 22-regular). In both cases this is considerably more risk-sharing links than empirical research typically documented in villages, suggesting that the minimum number of links that would be necessary to maintain both equality and separation efficiency is inefficiently high.

To help understanding of Proposition 19 we provide an outline of the proof (see Section D of the Supplementary Appendix for the full proof). Observe that Pareto efficient networks must have at least $n-1$ links (as we require for all nodes to be in the same component) and a diameter of 2. This is because if there are $k \geq n-1$ links then k pairs of the nodes are directly connected, and so the best possible path length distribution is for the other pairs of nodes to have path lengths of two. This bound is achievable—consider any network that includes the star on n nodes as a subnetwork. Given this, we can apply the Moore bound from graph theory. The Moore bound says that any network component with diameter d and maximum degree no more than ψ, the number of nodes n in the network satisfies $n \leq 1 + \psi \sum_{i=0}^{d-1} (\psi - 1)^i$. Thus, for $d = 2$, in any regular network with degree r, there must be $n \leq 1 + r^2$ nodes, or equivalently, $r \geq \sqrt{n+1}$. Hence, the total number of links in the network is $rn/2 \geq \sqrt{n-1}n/2$.

7. Conclusion

Our paper provides a relatively tractable model of endogenously formed networks and surplus division in a context of risk sharing that allows for heterogeneity in correlations between the incomes of pairs of agents. Such correlations have a sizeable impact on the potential of informal risk sharing to smooth incomes. We investigate the incentives for relationships that enable risk sharing to be formed both within a group (caste or village) and across groups, giving access to less correlated income streams. We find a novel trade-off between equality and efficiency. Thus we identify new downsides to informal risk sharing arrangements that can have important policy implications. This trade-off remains present in various generalizations and extensions of our baseline model.

Although we focus our analysis on risk sharing, our conclusions regarding network formation could apply in other social contexts too, as long as the economic benefits created by the social network are distributed similarly to the way they are in our model—a question that requires further empirical investigation.

Within the context of risk sharing, a natural next step would be to provide a dynamic extension of the analysis that allows for autocorrelation between income realizations. Another important direction in which this research agenda could be advanced is by studying dynamic network formation problem. It is realistic to let links form sequentially, and to consider how forward looking agents will try to manipulate this process so that they occupy advantageous positions in the network. Interestingly, this will give rise to new forces in which agents compete to become central, although we would also expect chance to play an important role.

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Appendix I. Omitted Proofs

Proof of Proposition 1. To prove the first statement, consider villagers' certainty-equivalent consumption. Let $\hat{K}$ be some constant, and consider the certain transfer K' (made in all states of the world) that i requires to compensate him for keeping a stochastic consumption stream $c_i + \hat{K}$ instead of another stochastic consumption stream $c_i' + \hat{K}$:

$$\begin{aligned}\mathbf{E}[v(c_i + \hat{K} + K')] &= \mathbf{E}[v(c_i' + \hat{K})]\\ -\frac{1}{\lambda}e^{-\lambda\hat{K}}e^{-\lambda K'}\mathbf{E}[e^{-\lambda c_i}] &= -\frac{1}{\lambda}e^{-\lambda\hat{K}}\mathbf{E}[e^{-\lambda c_i'}]\\ e^{\lambda K'} &= \frac{\mathbf{E}[e^{-\lambda c_i}]}{\mathbf{E}[e^{-\lambda c_i'}]}\\ K' &= \frac{1}{\lambda}\left(\ln\left(\mathbf{E}[e^{-\lambda c_i}]\right) - \ln\left(\mathbf{E}[e^{-\lambda c_i'}]\right)\right) \qquad (16)\end{aligned}$$

This shows that the amount K' needed to compensate i for taking the stochastic consumption stream $c_i + \hat{K}$ instead of $c_i' + \hat{K}$ is independent of $\hat{K}$. As a villager's certainty-equivalent consumption for a lottery is independent of his consumption level, certainty-equivalent units can be transferred among the villagers without affecting their risk preferences, and expected utility is transferable.

Next, we characterize the set of Pareto efficient risk sharing agreements. Borch (1962) and Wilson (1968) showed that a necessary and sufficient condition for a risk-sharing arrangement between i and j to be Pareto efficient is that in almost all states of the world $\omega \in \Omega := \mathbf{R}^{|S|}$,

$$\left(\frac{\partial v_i(c_i(\omega))}{\partial c_i(\omega)}\right)\Big/\left(\frac{\partial v_j(c_j(\omega))}{\partial c_j(\omega)}\right) = \alpha_{ij} \qquad (17)$$

where α_{ij} is a constant. Substituting in the CARA utility functions, this implies that

$$\begin{aligned}\frac{e^{-\lambda c_i(\omega)}}{e^{-\lambda c_j(\omega)}} &= \alpha_{ij}\\ c_i(\omega) - c_j(\omega) &= -\frac{\ln(\alpha_{ij})}{\lambda}\\ \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] &= -\frac{\ln(\alpha_{ij})}{\lambda}\\ c_i(\omega) - c_j(\omega) &= \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)] \qquad (18)\end{aligned}$$

Letting i and j be neighbors such that $j \in \mathbf{N}(i)$, equation 18 means that when i and j reach any Pareto-efficient risk-sharing arrangement their consumptions will differ by the same constant in all states of the world. Moreover, by induction the same must be true for all pairs of path-connected villagers.

Consider now the problem of splitting the incomes of a set of villagers $\mathbf{S}$ in each state of the world to minimize the sum of their consumption variances:

$$\min_{\mathbf{c}} \sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) \qquad \text{subject to} \quad \sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega) \quad \text{for all } \omega. \tag{19}$$

If we denote a CDF of income probability distribution on $\Omega = \mathbf{R}^{|S|}$ by $F(\cdot)$,

$$\sum_{i\in\mathbf{S}} \mathrm{Var}(c_i) = \int_{\Omega} \sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 dF(\omega). \tag{20}$$

Since $\mathrm{Var}(c_i(\omega) + a_i) = \mathrm{Var}(c_i(\omega))$, the sum of variances is invariant to state-independent changes in a consumption profile, and the variance-minimizing consumption profile exists for any profile of expected consumptions $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$: $\sum_{i\in\mathbf{S}} \mathbf{E}[c_i] = \sum_{i\in\mathbf{S}} \mathbf{E}[y_i]$. Fix any such profile of expected consumptions, $\{\mathbf{E}[c_i]\}_{i\in\mathbf{S}}$. Similarly to Wilson (1968), we apply Theorem 1 from Zahl (1963) to our minimization problem. We denote a Lagrange multiplier attached to constraint $\sum_{i\in\mathbf{S}} y_i(\omega) = \sum_{i\in\mathbf{S}} c_i(\omega)$ by $\gamma(\omega)$. Then, the corresponding Lagrangian of the problem is

$$\int_{\Omega} \left[\sum_{i\in\mathbf{S}} (c_i(\omega) - \mathbf{E}[c_i])^2 - \gamma(\omega) \sum_{i\in\mathbf{S}} c_i(\omega) \right] dF(\omega).$$

By pointwise minimization with respect to $c_i(\omega)$ we obtain that for each $i \in \mathbf{S}$ and almost every $\omega \in \Omega$, $2(c_i^*(\omega) - \mathbf{E}[c_i]) = \gamma(\omega)$. Thus, $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for all $i, j \in \mathbf{S}$. Note that this equality as well implies that $\mathbf{E}[c_i^*(\omega)] = \mathbf{E}[c_i]$, and $\{c_i^*(\omega)\}$ indeed solves the minimization problem. Thus, the condition $c_i^*(\omega) - c_j^*(\omega) = \mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]$ for almost all ω is exactly the same as the necessary and sufficient condition for an ex-ante Pareto efficiency. Hence, a risk-sharing agreement is Pareto efficient if and only if the sum of the consumption variances for all path-connected villagers is minimized.

Using the necessary and sufficient condition for efficient risk sharing, we obtain

$$\sum_{k\in\mathbf{S}} y_k(\omega) = \sum_{k\in\mathbf{S}} c_k(\omega) = |\mathbf{S}| c_i(\omega) - \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_k(\omega)]\right), \tag{21}$$

which implies that

$$c_i(\omega) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} \left(\mathbf{E}[c_i(\omega)] - \mathbf{E}[c_j(\omega)]\right) = \frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega) + \tau_i, \tag{22}$$

where $\tau_i = \mathbf{E}[c_i(\omega)] - \mathbf{E}[\frac{1}{|\mathbf{S}|} \sum_{k\in\mathbf{S}} y_k(\omega)]$. □

Proof of Lemma 4. Agent i's net benefit from forming link l_{ij} is $(MV_i(L) - MV_i(L \setminus \{l_{ij}\}) - \kappa_w)$. We need to show that

$$MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) = md(i, j, L)V. \tag{23}$$

Some additional notation will be helpful. Suppose agents arrive in a random order, with a uniform distribution on all possible arrival orders. The random variable $\widehat{\mathbf{S}}_i \subseteq \mathbf{N}$ identifies the set of agents, including i, who arrive weakly before i. For each arrival order, we then have an associate network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ that describes the network formed upon i's arrival (the subnetwork of L induced by agents $\widehat{\mathbf{S}}_i$). Let $q(i,j,L)$ be the probability that i and j are path-connected on network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$.

The certainty-equivalent value of the reduction in variance due to a link l_{ij} in a network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ is V if the link is essential and 0 otherwise. The change in i's Myerson value, $MV_i(L) - MV_i(L \setminus \{l_{ij}\})$, is then $(q(i,j,L) - q(i,j,L \setminus \{l_{ij}\}))\,V$. However, $q(i,j,L) = 1/2$. To see this, note that $l_{ij} \in L$ and therefore in every order of arrival in which i arrives after j (which happens with probability 1/2), i and j are path-connected on the network $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$, while i and j are never path-connected on $\mathcal{L}_L(\widehat{\mathbf{S}}_i)$ when j arrives after i.

Probability $q(i,j,L \setminus \{l_{ij}\})$ can be computed by the inclusion-exclusion principle, using the fact that the probability of a path connecting i and j existing on network $\mathcal{L}_{L\setminus\{l_{ij}\}}(\widehat{\mathbf{S}}_i)$ is equal to the probability that for some path connecting i and j on $L \setminus \{l_{ij}\}$ all agents on the path are present in $\widehat{\mathbf{S}}_i$. Thus

$$q(i,j,L \setminus l_{ij}) = \sum_{k=1}^{|\mathbf{P}(i,j,L\setminus l_{ij})|} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le |\mathbf{P}(i,j,L)|} \left(\frac{1}{|P_{i1} \cup \cdots \cup P_{ik}|} \right) \right). \tag{24}$$

We therefore have that

$$MV_i(L) - MV_i(L \setminus l_{ij}) = (1/2 - q(i,j,L \setminus l_{ij}))V = md(i,j,L)V, \tag{25}$$

where the last equality follows from the definition of Myerson distance. □

Proof of Proposition 6. **Part (i):** By remark 3 and under our regularity condition, all efficient networks are tree networks. By definition, in all tree networks any pair of agents i and j have a unique path between them. Thus, for a tree network L with diameter $d(L)$, there exist agents i and j with a unique path between them of length $d(L)$ and all other pairs of agents have a weakly shorter path between them. Thus by equation 11:

$$md(i,j,L) = \frac{1}{2} - \frac{1}{d(L)} \ge md(k,k',L) \qquad \text{for all } k,k' \in \mathbf{N}. \tag{26}$$

By Proposition 5 there is no underinvestment in any stable network. Lemma 4 therefore implies that the efficient network L is stable if and only if $md(k,k',L) \le \kappa_w/V$ for all k,k' such that $l_{kk'} \notin L$. As $md(i,j,L) \ge md(k,k',L)$ and $md(i,j,L) = 1/2 - 1/d(L)$ (see equation 26), this condition simplifies and the efficient network L is stable if and only if

$$\frac{V - 2\kappa_w}{V} \le \left(\frac{2}{d(L)} \right). \tag{27}$$

As $d(L)$ gets large, the right-hand side converges from above to 0 and so in the limit, the condition for stability becomes $V \leq 2\kappa_w$, which is violated by our regularity condition. Thus, there exists a finite $\overline{d}(L)$ such that the efficient network L is stable if and only if $d(L) \leq \overline{d}(L)$.

Rearranging equation 27, L is stable if and only if

$$d(L) \leq 2\left(\frac{V}{V - 2\kappa_w}\right). \tag{28}$$

So the key threshold is $\overline{d}(\kappa_w) = \lfloor 2V/(V - 2\kappa_w) \rfloor$.

Fixing the number of agents $|\mathbf{N}|$ in an efficient (tree) network L, the star network is the unique (tree) network (up to a relabeling of players) that minimizes the diameter $d(L)$ while the line network is the unique (tree) network (up to a relabeling of players) that maximizes the diameter $d(L)$. The result now follows immediately.

Part (ii): On any efficient networks all links are essential and generate a net surplus of $V - 2\kappa_w > 0$, where the inequality follows from our regularity condition. As i and j must benefit equally at the margin from the link l_{ij} (see condition (ii) in the definition of agreements that are robust to split-the-difference renegotiation), agent i's expected payoff on an efficient network L is

$$u_i(L) = |\mathbf{N}(i; L)|(V/2 - \kappa_w) > 0. \tag{29}$$

Thus i's net payoff is proportional to his degree.

For any tree network L other than the star network let agent k be one of the agents with the highest degree. Consider a link $l_{ij} \in L$ such that $i, j \neq k$. As L is a tree there is a unique path from i to k and a unique path length from j to k. As we are on a tree network, either the path from j to k passes through i, or else the path from i to k passes through j. Hence either i or j is closer to k and without loss of generality we let i have a longer path to k than j. We now delete the link l_{ij} and replace it with the link l_{ik}. This operation generates a new tree network. Moreover, repeating this operations until there are no links l_{ij} such that $i, j \neq k$, defines an algorithm.

This algorithm terminates at star networks as the operation cannot be applied to this network; There are no links of l_{ij} such that $i, j \neq k$. Moreover the operation can be applied to any other tree network because on all other tree networks there exists an l_{ij} such that $i, j \neq k$. Finally, in each step of the algorithm the degree of k increases and so the algorithm must terminate in a finite number of steps. Moreover, the algorithm must terminates at the star network with k at the center.

By construction, at each step of the above algorithm we decrease the degree of some agent $j \neq k$ and increase the degree of k. Suppose we start with a network L and consider a step of this rewiring where the link l_{ij} is deleted and replaced by the link l_{ik}. Only the expected payoff of agents j and k on L and $L \cup l_{ik} \setminus l_{ij}$ change; The degrees of all other agents remain constant and thus by equation 29 so do their payoffs. Letting $\alpha = (V/2 - \kappa_w)$, we have $u_j(L) = \alpha d_j(L)$, $u_k(L) = \alpha d_k(L)$, $u_j(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_j(L) - 1)$ and $u_k(L \cup l_{ik} \setminus l_{ij}) = \alpha(d_k(L) + 1)$.

It follows that welfare $W(u) = \sum_i f(u_i)$ (see equation 9) decreases through the rewiring in this step if and only if

$$f(\alpha(d_j - 1)) + f(\alpha(d_k + 1)) - f(\alpha d_j) - f(\alpha d_k) < 0, \tag{30}$$

which is equivalent to:

$$f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1)) \tag{31}$$

As $f(\cdot)$ is increasing, strictly concave and differentiable $f'(\alpha d_j)\alpha < f(\alpha d_j) - f(\alpha(d_j - 1))$ and $f'(\alpha d_k)\alpha > f(\alpha(d_k + 1)) - f(\alpha d_k)$. Moreover, by concavity $f'(\alpha d_j) \geq f'(\alpha d_k)$ (as $d_k \geq d_j$). Combining these inequalities establishes the claim that $f(\alpha(d_k + 1)) - f(\alpha d_k) < f(\alpha d_j) - f(\alpha(d_j - 1))$.

Thus at each step of the rewiring welfare $W(u)$ decreases. For each network L' reached during the algorithm we can consider the average expected utility $u'(L')$ which if distributed equally would generate the same level of welfare as obtained on L. As aggregate welfare is decreasing at each step of the rewiring $u'(L)$ must be decreasing too. However, the total surplus generated by risk sharing remains constant and so average expected utility $\overline{u}$ remains constant. Recall that Atkinson's inequality measure / index is given by $I(L) = (1-(u'(L)/\overline{u})$. Thus at each step of the rewiring the inequality measure $I(L)$ increases. As this rewiring can be used to move from any tree network to the star network, stars network and only star networks maximize inequality among the set of tree networks, which correspond to the set of efficient networks under our regularity condition. As this argument holds for any strictly increasing and differentiable, concave function f it holds for all inequality measures in the Atkinson class.

Consider now an alternative rewiring of a tree network L. Let k be one of the agents with highest degree on L and let j be one of the agents with degree 1 on L. As tree networks contain no cycles, there always exists agents with degree 1 (leaf agents). Pick one of k's neighbors $i \in \mathbf{N}(k; L)$, remove the link l_{ik} from L and add the link l_{ij} to L. This operation generates a new tree network. Repeating this operation until the highest degree agent has degree 2 defines an algorithm. As the unique tree network with a highest degree of 2 is the line network, the algorithm terminates at line networks and only line networks. At each stage of the rewiring we either reduce the degree of the highest degree agent k or reduce the number of agents who have the highest degree. Thus the algorithm must terminate in a finite number of steps at a line network. Moreover, reversing the argument above, inequality is reduced at each step of the rewiring for any inequality measure in the Atkinson class. $\square$

Proof of Proposition 7. By definition, underinvestment within group for a network L requires that there exists an $l_{ij} \notin L$ such that $G(i) = G(j)$ and for which $TS(L \cup l_{ij}) - TS(L) > 2\kappa_w$. As $TS(L \cup l_{ij}) - TS(L) = 0$ for all non-essential links, l_{ij} must be essential on $L \cup \{l_{ij}\}$. Thus l_{ij} is also essential on $\widehat{L} \cup \{l_{ij}\}$ for any $\widehat{L} \subseteq L$. Equation 13 then implies that $TS(\widehat{L} \cup l_{ij}) - TS(\widehat{L}) \geq V$ for any $\widehat{L} \subseteq L$.

Consider any arrival order in which i arrives after j and let $\mathbf{S}_i$ be the agents that arrive (strictly) before i. Agent i's marginal contribution to total surplus without l_{ij} when i arrives is then $TS(L(\mathbf{S}_i \cup \{i\})) - TS(L(\mathbf{S}_i))$ while with l_{ij} it is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i))$. So i's additional marginal contribution to total surplus when l_{ij} has been formed is $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\}))$. As $L(\mathbf{S}_i \cup \{i\}) \subseteq L$, by the above argument $TS(L(\mathbf{S}_i \cup \{i\}) \cup \{l_{ij}\}) - TS(L(\mathbf{S}_i \cup \{i\})) \geq V$. As i arrives after j in half the arrival orders, i's average additional incremental contribution to total surplus when l_{ij} has been formed is at least $V/2$. Thus $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \geq V/2$. An equivalent argument establishes that $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \geq V/2$. Under our regularity condition $V/2 > \kappa_w$ and so i and j have a profitable deviation to form l_{ij} and the network L is not stable. As L was an arbitrary network within underinvestment within group, there is no stable network with underinvestment within group. □

Proof of Proposition 8. The proof of the first part of the statement has four steps.

Step 1: Consider any efficient network L that is robust to overinvestment inefficiency within group. This implies that for all path-connected agents i, j such that $G(i) = G(j)$ and $l_{ij} \notin L$, either $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ or $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$. However, by condition (i) in the definition of agreements that are robust to split-the-difference renegotiation, $MV_i(L \cup \{l_{ij}\}) - MV_i(L) = MV_j(L \cup \{l_{ij}\}) - MV_j(L)$ and so both $MV_i(L \cup \{l_{ij}\}) - MV_i(L) \leq \kappa_w$ and $MV_j(L \cup \{l_{ij}\}) - MV_j(L) \leq \kappa_w$.

Step 2: Let a network $\widehat{L} := \{l_{ij} : G(i) = G(j), l_{ij} \in L\}$ be a network formed from L by deleting all across-group links. Consider any subset of agents $\mathbf{S} \subseteq \mathbf{N}$ such that $i, j \in \mathbf{S}$. As the network L is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between i and j cannot contain an across-group link. So, i is path-connected to j on the induced subnetwork $L(\mathbf{S})$ if and only if i is path-connected to j on the induced subnetwork $\widehat{L}(\mathbf{S})$. Thus, by equation 13, the additional variance reduction that i and j can now achieve by forming a superfluous across-group link on $\widehat{L}(\mathbf{S})$ is weakly lower than on $L(\mathbf{S})$. So, by the Myerson value definition (equation 6), $MV_i(\widehat{L} \cup \{l_{ij}\}) - MV_i(\widehat{L}) \leq MV_i(L \cup \{l_{ij}\}) - MV_i(L)$ and $MV_j(\widehat{L} \cup \{l_{ij}\}) - MV_j(\widehat{L}) \leq MV_i(L \cup \{l_{ij}\}) - MV_i(L)$. This implies that $\widehat{L}$ is robust to overinvestment within group.

Step 3: Let a network $\widehat{L}'$ be a network formed from $\widehat{L}$ by rewiring (alternately deleting then adding a link) each within-group network into a star (for an algorithm that does this, see the part (ii) of the proof of Proposition 6). Consider any two agents i', j' such that $G(i') = G(j')$, $l_{i'j'} \notin \widehat{L}'$. By part (i) of Proposition 6, $MV_{i'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{i'}(\widehat{L}') \leq MV_i(\widehat{L} \cup \{l_{ij}\}) - MV_i(\widehat{L})$ and $MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}') \leq MV_j(\widehat{L} \cup \{l_{ij}\}) - MV_j(\widehat{L})$. Thus $\widehat{L}'$ is robust to overinvestment within group.

Step 4: Finally, consider any network $L' \in \mathcal{L}^{CSS}$. This network can be formed by adding a set of across-group links to a network $\widehat{L}'$ such that $\widehat{L}' \subseteq L'$ and if $l_{kk'} \in L' \setminus \widehat{L}'$ then $G(k) \neq G(k')$. Consider any subset of agents $\mathbf{S}' \subseteq \mathbf{N}$ such that $i', j' \in \mathbf{S}'$. Recall that $G(i') = G(j')$ and note that by the construction of L', $l_{i'j'} \notin L'$. On the induced subnetwork $L'(\mathbf{S}')$, either i' is path-connected to j', in which case $l_{i'j'}$ would be superfluous if added, or else i' and j' are isolated nodes. This is because the within-group network structure for group $G(i')$ is a star. Thus, whenever $l_{i'j'}$ would not be superfluous, the change in i' and j''s Myerson value if it were added is independent of the across-group links that are present: $MV_{i'}(L' \cup \{l_{i'j'}\}) - MV_{i'}(L') = MV_{i'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{i'}(\widehat{L}')$ and $MV_{j'}(L' \cup \{l_{i'j'}\}) - MV_{j'}(L') \leq MV_{j'}(\widehat{L}' \cup \{l_{i'j'}\}) - MV_{j'}(\widehat{L}')$. Thus L' is robust to overinvestment within group.

We turn now to the second part of the result. If $L \notin \mathcal{L}^{CCS}$, then there will be agents i, j such that $G(i) = G(j)$ and $l_{ij} \notin L$ such that either the within-group network structure for $G(i)$ is not a star, or else it is a star but there are across-group links being held by an agent who is not the center agent. In the first case, the inequality in step 3 will be strict by Proposition 6. In the second case, we can without loss of generality let agent i be the non-center agent holding the across-group link. Then, by equation 13, the inequality in step 2 will be strict. Thus for some parameter values L will not be robust to overinvestment within group, but L' will be. $\square$

Proof of Lemma 9. Denote the set of all possible arrival orders for the set of agents $\mathbf{N}$, by $\mathcal{A}(\mathbf{N})$. Order this set of $|\mathbf{N}|!$ arrival orders in any way, denoting the kth arrival order by $\widehat{A}_k \in \mathcal{A}(\mathbf{N})$. We will then construct an alternative ordering, in which we denote the kth arrival order by $\widetilde{A}_k \in \mathcal{A}(\mathbf{N})$, such that for arrival order $\widetilde{A}_k$,

(i) i arrives at the same time as agent i' does for the arrival order $\widehat{A}_k$;
(ii) when i arrives he connects to exactly the same set of agents from $\mathbf{N} \setminus \mathbf{S}_0$ that i' connects to upon his arrival for the arrival order $\widehat{A}_k$;
(iii) when i arrives he connects to weakly more agents from $\mathbf{S}_0$ that i' connects to upon his arrival for the arrival order $\widehat{A}_k$.

Equation 15 shows that the risk reduction, and hence the marginal contribution made by an agent $k \in \mathbf{S}_0$ from providing the across-group link l_{kj}, is an increasing function of the component size of k's groups. It then follows that

$$MV(i; L \cup l_{ij}) - MV(i; L) > MV(i'; L \cup l_{i'j}) - MV(i'; L). \tag{32}$$

To construct the alternative ordering of the set $\mathcal{A}(N)$ as claimed we will directly adjust individual arrival orders, but in a way that preserves the set $\mathcal{A}(N)$. First, for each arrival order, we switch the arrival positions of i' and i. This alone is enough to ensure that conditions (i) and (ii) are satisfied. There are $|\mathbf{S}_0|!$ possible arrival orders for the set of agents $\mathbf{S}_0$. Ignoring for now the other agents, we label these arrival orders lexicographically. First we order them, in ascending order, by when i arrives. Next, we order them in ascending order by the number of agents i is connected to upon his arrival. Breaking remaining ties in any way, we have labels $1_i, 2_i, \ldots, |\mathbf{S}_0|!_i$. We then let every element of $\mathcal{A}(N)$ inherit these labels, so that two arrival orders receive the same label if and only if the agents $\mathbf{S}_0$ arrive in the same order. We now construct a second set of labels by doing the same exercise for i', and denote these labels by $1_{i'}, 2_{i'}, \ldots, |\mathbf{S}_0|!_{i'}$. We are now ready to make our final adjustment to the arrival orders. For each original arrival order $\widehat{A}_k$ we find the associated (second) label. Suppose this is $x_{i'}$. We then take the current kth arrival order (given the previous adjustment), and reorder (only) the agents in $\mathbf{S}_0$, so that the newly constructed arrival order now has (first) label x_i. Because of the lexicographic construction of the labels, the arrival position of agent i will not change as a result of this reordering of the arrival positions of agents in $\mathbf{S}_0$, so conditions (i) and (ii) are still satisfied. In addition, condition (iii) will now be satisfied from the definition of i being more central than i'. The only remaining thing to verify is that the set of arrival orders we are considering has not changed (i.e. that we have, as claimed, constructed an alternative ordering of the set $\mathcal{A}(N)$) and this also holds by construction. □

Proof of Proposition 10. Let L be an efficient network that is robust to underinvestment across group. This implies that for any across-group link $l_{ij} \in L$ between groups $g = G(i)$ and $\hat{g} = G(j) \neq g$, $MV_i(L) - MV_i(L \setminus \{l_{ij}\}) = MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \geq \kappa_a$, where the inequality follows from condition (i) in the robustness to split-the-difference renegotiations definition.

We now rewrite L. As the network L is efficient, it is a tree network that minimizes the number of across-group links conditional on a given set of agents being in a component. This implies that the unique path between any two agents from the same group cannot contain an across-group link. We can therefore rewrite the within-group network structures of L to obtain a star by sequentially deleting and then adding within-group links (an algorithm that does this is presented in the proof of part (ii) of Proposition 6). Do this rewiring so that agent i is the agent at the center of the within-group network for group $G(i)$ and let j be the agent at the center of the within-group network for group $G(j)$. Finally, we rewire across-group links so that the same groups remain directly connected, but all across-group links are held by the center agents. Let the network obtained be L'. By construction, $L' \in \mathcal{L}^{CCS}$.

Under our definition of Myerson centrality, it is staightforward to verify that both i and j are weakly more Myerson central within their respective groups on network L' than on network L. An argument almost identical to that in the proof of Lemma 9 then implies that i' and j' have better incentives to keep the link $l_{i'j'}$ than i and j have to keep the link l_{ij} (because the argument is more or less identical we skip it). Hence,

$$MV_{i'}(L) - MV_{i'}(L \setminus \{l_{i'j'}\}) \geq MV_i(L) - MV_i(L \setminus \{l_{ij}\}) \tag{33}$$

$$MV_{j'}(L) - MV_{j'}(L \setminus \{l_{i'j'}\}) \geq MV_j(L) - MV_j(L \setminus \{l_{ij}\}) \tag{34}$$

Network L' is therefore robust to underinvestment. Moreover, whenever the within-group networks of i and j on network L are not both stars with i and j at the centers, the inequality is strict because both i and j are strictly more Myerson central within-group on L' than on L. There then exists a range of parameter specifications for which any center-connected star network $L' \in \mathcal{L}^{CCS}$ is robust to underinvestment across group but L is not. □

Supplementary Appendix: For Online Publication Only

In this supplementary appendix we provide an extended analysis that supports our main paper "Investments in social ties, risk sharing and inequality," henceforth referred to as the main paper.

A. More general environments and surplus sharing rules

This section provides a slightly more general and comprehensive treatment of the issues studied in the corresponding section of the main paper—Section 6.1. The main difference is that here we allow for multiple groups, while in Section 6.1 attention was restricted to the one-group case. Due to the generalization we present a complete and self-standing analysis, even though there is much overlap with Section 6.1. We number replicated assumptions and results so that they correspond to those in Section 6.1, and new results with the prefix SA.

The purpose of this section is to examine under what conditions our main conclusions extend to more general utility functions, income distributions and surplus division rules. The environment with CARA utilities and jointly normally distributed incomes facilitates a convenient transferrable (expected) utilities environment that is particularly tractable to analyze when social surplus is divided in accordance with the Myerson value. While analytical tractability requires a series of strong assumptions, below we show that some of the main qualitative insights of the model extend to much more general specifications.

For general specifications of the model expected utilities are nontransferable and the simple, costless means of redistributing surplus via state-independent transfers we used before is no longer available, hence we need to take a more general approach towards risk sharing. Let $v_i(c_i)$ be the utility function for agent i, mapping second period consumption into utility. We assume that $v_i = v_j$ for all i and j in the same group, and that v_i is strictly increasing and strictly concave for all $i \in \mathbf{N}$.[1] Let $\mathcal{P}_k$ be the distribution the incomes of agents in group $k \in \mathbf{M}$ are drawn from.

Let $\mathcal{L}$ be the set of all possible networks for agents in $\mathbf{N}$. We assume there is a unique risk-sharing arrangement that will be implemented for any possible network $L \in \mathcal{L}$, and that agents correctly anticipate the risk-sharing agreement that will obtain. These risk-sharing arrangements, which depend on the social network, might be dictated by social conventions, or they can be outcomes of negotiation processes for transfer arrangements once the network is formed. Let $\tau(L)$ be the transfer arrangement and $u_i^\tau(L)$ be the expected second period consumption utility of agent i implied by $\tau(L)$.[2]

We continue to assume that for every $L \in \mathcal{L}$, $\tau(L)$ specifies a pairwise-efficient risk-sharing arrangement $\tau_{ij}(L)$ for every pair of agents i, j linked in L. As shown earlier, this is equivalent

[1]These properties imply that for any number of agents more than one, and for any point of the Pareto frontier of feasible consumption plans that can be reached via risk-sharing arrangements, there is a direction along the Pareto frontier in which a given agent's expected utility is strictly increasing.

[2]More precisely, utility function v_i, the distribution of income realizations and transfer arrangement $\tau(L)$ jointly determine $u_i^\tau(L)$.

to $\tau(L)$ being Pareto efficient at a component level. Agent i maximizes the difference between expected utility from the second period risk sharing (given by u_i^τ) and her costs of establishing links.

Let $C_i(L)$ be the set of agents on the same component as i given L, and recall that G is a function mapping agents in $\mathbf{N}$ to groups in $\mathbf{M}$.

Next we impose a series of assumptions on $\tau(\cdot)$. We do not claim that the above assumptions hold universally when informal risk-sharing takes place, but they are relatively weak requirements that are natural in many settings. Our main objective is to demonstrate that our qualitative results hold for a much broader class of models than the CARA-normal setting with surplus division governed by the Myerson value.

The first assumption requires that establishing a link always strictly increases the connecting agents' expected consumption utilities.

Assumption 11(a). *$u_i^\tau(L \cup \{l_{ij}\}) > u_i^\tau(L)$ for every $L \in \mathcal{L}$, $i, j \in \mathbf{N}$ and $l_{ij} \notin L$.*

The next assumption requires that establishing an essential link does not impose a negative externality on other agents. This implies that while both i and j privately benefit from essential link l_{ij}, in terms of second period expected utility, they do not benefit over and beyond the enhancement of risk-sharing opportunities that the link facilitates.

Assumption 11(b). *$u_k^\tau(L \cup \{l_{ij}\}) \geq u_k^\tau(L)$ for every $L \in \mathcal{L}$, $i, j, k \in \mathbf{N}$ and $C_i(L) \neq C_j(L)$.*

Next we extend the idea that the private benefit that two agents receive from establishing a link should be increasing in the distance between them in the absence of the link. In the previous analysis these private benefits depended specifically on the Myerson distance between the two agents, while here we allow for a general class of distance measures. Before defining the class of distance measures we allow for, some additional notation is required. For two sets S and S' we define $\mathcal{M}(S, S')$ as the set of matching functions $\mu : S \to S' \cup \{\emptyset\}$, such that for $s \in S$ if $\mu(s) \neq \emptyset$ then $\mu(s) \neq \mu(t)$ for all $t \in S \setminus \{s\}$. Thus every $\mu \in \mathcal{M}(S, S')$ maps each element of S into a different element of S', or else the empty set.

Let $\overline{\mathbf{N}}^2 = \{(i,j) | i, j \in \mathbf{N}, i \neq j\}$.

Definition (Distance measure): A distance measure is a mapping $d : \overline{\mathbf{N}}^2 \times \mathcal{L} \to \mathbb{R}_{++}$ satisfying the following properties:

Assumption 11(c) (i)-(iii).

(i) *If i and j are in different components on L, then $d(i,j,L) = \overline{d}$, with $\overline{d}$ strictly greater than the maximum possible distance between any two path-connected agents.*

(ii) *The distance measure depends only on paths (thus ignoring walks with cycles).*

(iii) *Let S_{ij} be the set of paths between i and j and S_{kl} be the set of paths between k and l. We assume $d(i,j;L) > d(k,l;L)$ if there exists a matching function $\mu \in \mathcal{M}(S,S')$ such that each path between i and j is matched to a shorter path between k and l, and all such paths between k and l are independent (do not pass through any of the same nodes as each other).*

Assumption 11(c) (i)-(iii) places only weak restrictions on the distance measure. In particular, part (iii) in general only provides a very weak partial ordering of the distances between agents. However, there is a special case in which the ordering is complete. On a tree network, there is a unique path between any two agents, so this determines the ordering of distances between pairs of agents. In what follows, let $d(\cdot)$ be any distance measure satisfying the above requirements.

While we will use the concept of distance between agents in the general case of multiple groups, first we focus on extending our earlier results for the case of homogeneous agents. Next we make assumptions on how distance in the absence of a link influences the private benefits of two agents within the same group establishing that link.

The next assumption requires that if all agents are from the same group then the private benefit two agents receive when establishing a link only depends (positively) on their distance in the absence of the link, and on the sizes of the components they are on. Recall that in our benchmark model in the CARA-normal setting these private benefits only depended on the Myerson-distance between the agents. The requirement below allows the private benefit to depend on different distance measures, and also on the sizes of the agents' components (which for general utilities influences the difference between the Pareto frontiers of feasible consumption plans with and without the link).

Assumption 11(c) (Only Distance and Size Matter)**.** *If $G(i) = G(j)$ for all $i, j \in \mathbf{N}$ and $l_{ij} \notin L$, then*

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|),$$

Moreover, $g(d(i,j,L), |C_i(L)|, |C_i(L \cup \{l_{ij}\})|)$ is increasing in $d(i,j,L)$.

Note that Assumption 11(c) differs slightly from the corresponding assumption in the main paper—now that we are permitting there to be multiple groups a qualification is made for this assumption to only apply when all agents are in the same group. In the multiple group case the composition of each component, in terms of the groups the constituent agents come from, and their network positions, can matter.

The last assumption we need for recreating the results of the benchmark model for homogeneous agents is that the cost of link formation within a group is sufficiently small relative to the private benefits from establishing an essential link. In the CARA-normal framework with the surplus allocated according to the Myerson value and all agents being homogeneous, a pair forming an essential link received the full social surplus created by the link. This implies that the social and private benefits coincide in the benchmark model for essential links, and therefore there is no within group underinvestment for any cost of link formation. For general utility functions and surplus allocation rules such equivalence does not hold, therefore no within group underinvestment cannot be expected to hold for all possible costs of link formation. However, for any specification of the general model that satisfies the assumptions above (in particular that the private benefit of establishing any link is always strictly positive), there is no within group underinvestment *if* the cost of establishing a link between agents from the same group is small enough. While this is a nontrivial assumption, it is realistic in many settings. Indeed, in the data we consider, within group underinvestment does not appear to be a problem.

Assumption 11(d) (Within Group Cost of Link-formation Small)**.** *For all networks L,*

$$c_w/2 < \min_{L,i,j \ st. \ C_i(L)\neq C_j(L)} u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L).$$

Assumption 11(d) immediately implies that if all agents are from the same group then in all stable networks there is a single component. The next proposition shows that the same holds for all efficient networks. For the rest of the section, the above assumptions are maintained.

A network is Pareto efficient if there is a feasible transfer agreement that could be reached on that network such that there is no other network, feasible transfer agreement pair in which all agents are weakly better off and some agents are strictly better off.

Proposition 12. *If all agents are from the same group then a network is Pareto efficient if and only if it is a tree connecting all agents.*

Proof. First, we consider the "only if" direction. In any Pareto efficient network, every component has to be a tree. This is because if any component was not a tree then a link could be deleted and the same risk-sharing arrangement can be achieved as before, but the costs of establishing the link saved. Now suppose there are two components of a Pareto efficient network L that are not connected. Let agents i and j be on different components. By Assumption 11(d), total expected utilities (that is, taking into account the costs of network formation, too) of both i and j are strictly higher for network $L \cup \{l_{ij}\}$ than for network L, while by Assumption 11(b) all other agents' total expected utilities are weakly higher for $L \cup \{l_{ij}\}$ than for L. This contradicts that L is Pareto efficient.

We now consider the "if" direction. Consider a tree network and suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all i and j and all states ω. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse off. Suppose, towards a contradiction, that we can redistribute the link formation costs by forming a different tree network, and find new feasible consumptions that together constitute a Pareto improvement. Holding consumptions fixed, the change in the network will make some agents worse off if any agents are made better off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As all agents in the same group have the same utility function $v_i(c_i) = v(c_i)$ and as the utility function $v(\cdot)$ is strictly concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c_i'(\omega)) < v\left(\frac{1}{n}\sum_i c_i'(\omega)\right) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all ω. Thus the average expected utility from consumption will decrease. As total link formation costs have remained constant, this implies that at least one agent must be worse off. This is a contradiction. □

Corollary SA1. *When all agents are from the same group, there is no underinvestment.*

Given Proposition 12, Corollary SA1 follows immediately from Assumption 11(d) and we omit a proof.

Note that for any non-essential link $|C_i(L)| = |C_i(L) \cup \{l_{ij}\})|$. Thus the marginal benefits i and j receive from forming a superfluous link depends only on the distance between i and j on L and the number of agents in their component. The latter is n for any efficient network, by Proposition 12. Thus the marginal benefit i and j receive from forming a superfluous link depends only on the distance between i and j, and is increasing in this distance. Thus an efficient network will be stable if and only if the maximum distance between any two agents is sufficiently low. The next Corollary formally states this result.

Corollary 14. *If all agents are from the same group then an efficient network is stable if and only if its diameter is sufficiently small.*

Proof. Consider an efficient network L. As L is efficient there exists a unique path between i and j for all i and all $j \neq i$. Consider two such agents i and $j \neq i$. Assumptions 11(c) (i)-(iii) imply that $d(i, j, L)$ is strictly increasing in the path length between i and j, and that $d(i, j, L) = d(j, i, L)$. Further, as $|C_i(L)| = |C_i(L \cup \{l_{ij}\})| = n$, by Assumption 11(c)

$$u_i^\tau(L \cup \{l_{ij}\}) - u_i^\tau(L) = g(d(i, j, L), n, n) = g(d(j, i, L), n, n) = u_j^\tau(L \cup \{l_{ij}\}) - u_j^\tau(L).$$

Moreover, by Assumption 11(c), $g(d(i, j, L), n, n)$ is strictly increasing in $d(i, j, L)$. Thus for all i and $j \neq i$, there exists a threshold $\hat{d}$ such that i and j benefit from forming a superfluous link if and only if $d(i, j, L) > \hat{d}$.

In the absence of any underinvestment (by Corollary SA1), a network L is stable if and only if no two agents can benefit from forming a superfluous link. As the agents' furthest away from each other have the strongest incentives to form a superfluous link, L is stable if and only if $\max_{i,j} d(i,j,L) \leq \hat{d}$. As $d(i,j,L)$ is strictly increasing in the (unique) path length between i and j, this is equivalent to the diameter of L being sufficiently small. □

A network is *least* stable within a class of networks, when its stability implies the stability of any other network in that class. A network is *most* stable within a class of networks, when its instability implies the instability of any other network in that class.

Proposition 14. *If all agents are from the same group then*

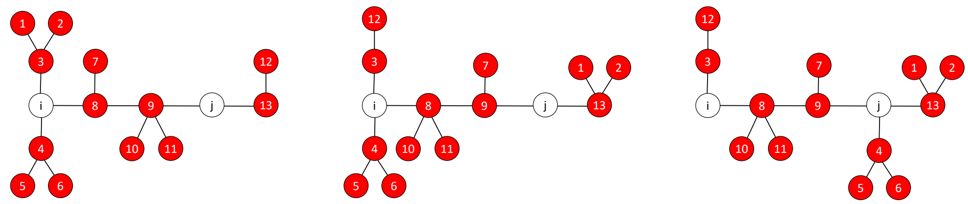
(i) *the most stable efficient network is the star,*
(ii) *the least stable efficient network is the line.*

Proof. By Corollary 14 an efficient network is stable if and only if its diameter is sufficiently low. It follows that if a network with diameter d is stable, all efficient networks with weakly lower diameter will also be stable. As the line network maximizes diameter among efficient networks, its stability implies the stability of all other efficient networks and it is least stable. Similarly, if a network with diameter d is unstable, Corollary 14 implies that all network with a weakly higher diameter are unstable. As the star network minimizes the diameter within the class of efficient networks, its instability implies the instability of all other efficient networks, and it is most stable within the class of efficient networks. □

Inequality measures within the Atkinson class will often rank utility vectors differently. In the simpler setting with CARA utilities, normally distributed incomes and the Myerson value allocation rule we were able to identify the star as the least equitable networks for any inequality measure in the Atkinson class. This was achieved by showing that any efficient network could be transformed into a star by rewiring it in a way such that, at each step of the rewiring, the utility of the center agent increased, the utility of one other agent decreased and the utility of the remaining agents remained constant. Specifically, the act of removing a link l_{ij} and adding a link l_{jk}, increased the utility of agent k, decreased the utility of agent i and held constant the utility of all other agents.

In the more general setting, this rewiring need not hold constant the utility of the other agents. This creates problems. Consider the four agent line network and suppose utilities, after link formation costs, are $(10, 25, 25, 10)$. Now suppose we remove link l_{34} and add link l_{24} to create a star network. In the more general model, utilities after this rewiring might be $(11, 35, 11, 11)$. These two vectors will be ranked differently by different inequality measures within the Atkinson class. However, if we make an additional assumption that this kind of rewiring only affects those agents who gain or lose a link, then we can relate inequality to network structure in the more general setting.

Proposition 15. *Suppose there is one group, and for all pairs of efficient networks L and L' such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then for all inequality measures in the Atkinson class, among the set of efficient networks,*



(A) Initial Network L

(B) L'

(C) Rewired network L''

FIGURE 1. An example of the rewiring used to find a contradiction in the proof of Lemma SA2 is shown. Panel (i) shows the initial network, Panel (ii) the interim network and Panel (iii) the final network after the rewiring is complete.

star networks and only star networks maximize inequality, while line networks and only line networks minimize inequality.

Proof. We begin with a Lemma:

Lemma SA2. *Suppose there is one group, and for all pairs of efficient networks L and L' such that $L' = \{L \setminus l_{ij}\} \cup l_{jk}$, the transfer arrangements satisfy $\tau_l(L) = \tau_l(L')$ for all $l \neq i, k$. Then agents with a higher degree in L have a higher utility.*

Proof. Consider an efficient network L and suppose agent i has higher degree than j. We will show that we can rewire a network in a way that weakly reduces i's utility and increases j's utility, but swaps the positions of i and j in the network such that on this new network i should have the same utility j had on the initial network. This will imply that i must have had a higher utility on the initial network.

Consider the following rewiring, an example of which is illustrated in Figure 1. As L is efficient it is a tree by Proposition 12 and there is a unique path between i and j. If i is directly connected to j we do not need to do any rewiring along this path. Otherwise, let there be $l \geq 1$ agents on this path, other than i and j, and create the following two labelings of these agents: $i, i1, \ldots, il, j$ and $i, jl, \ldots, j1, j$. Thus $i1 = jl$, $i2 = j(l-1)$, and so on. Now, if agent $i1$ has a link to an agent k on L, and k is not on the path between i and j, we remove the link $l_{i1,k}$ and add the link $l_{j1,k}$. Repeat until all of $i1$'s links to agents not on the shortest path between i and j have been rewired. We now repeat for ik, with $k = 2, \ldots l$. Note that at each step of this rewiring we reach a connected tree network.

Consider now the neighbors of j not on the path between i and j. Match each of these neighbors to a different neighbor of i's who is also not on this path. As i has a higher degree than j, such a matching exists. For each such pair we start with j's neighbor. Letting this neighbor of j be k, one by one, we rewire each of k's links on L, except l_{ij}, to the neighbor of i agent k was matched to. Let this agent be l. We then rewire each of l's links on L, except

l_{il}, to agent k. Repeat for all of j's neighbors on L not on the path between i and j. Note again that at each step of this rewiring we reach a tree network. After all this rewiring, let the network that has been reached be denoted L'.

As in all the rewiring so far i and j have kept the same links, and as at each step an efficient network has been reached, by the premise of the Proposition, $\tau_i(L) = \tau_i(L')$ and $\tau_j(L) = \tau_j(L')$, so $u_i^\tau(L) = u_i^\tau(L')$ and $u_j^\tau(L) = u_j^\tau(L')$.

Finally, we consider the neighbors of i who were not on the shortest path to j, and were not matched to one of j's neighbors. As i's degree is higher than j's there exists at least one such agent. For all agents in this set, we remove their link to i and add a link to j. Let the network reached after this be denoted L''.

By Assumption 11(a), this increases j's utility and decreases i's utility, so $u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L') < u_j^\tau(L'')$. However, by construction, after this rewiring is complete i's position in L'' is identical to j's position in L (up to a relabeling of agents), while j's position in L'' is identical to i's position in L. Thus by Assumption 11(c) $u_i^\tau(L) = u_i^\tau(L'')$ and $u_j^\tau(L) = u_j^\tau(L'')$. We then have that

$$u_i^\tau(L) = u_i^\tau(L') > u_i^\tau(L'') = u_j^\tau(L).$$

□

We can now prove the Proposition. As shown in the proof of Proposition 6(ii), the star network can be reached from any efficient network L by rewiring links to the highest degree agent in L. By Lemma SA2 the agent with the highest utility on L is the agent with the highest degree, and by Assumption 11(d), the net expected utility of this agent increases at each such step of the rewiring, while the net expected utility of all other agents weakly decreases. The argument from the proof of Proposition 6(ii) can then be applied again, and utilities become more unequal for any inequality measure in the Atkinson class.

The argument for the line network is equivalent. From any efficient network L, there is a rewiring to the line network that decreases the utility of the highest degree agent at each step, which by Lemma SA2 is also the highest utility agent, and increases the utility of all other agents. Thus, utilities become more equal for any inequality measure in the Atkinson class. □

We will now consider the multiple group case. With one group it was efficient for a network to form in which all agents are path-connected to each other. We now make an assumption to ensure this remains the case with multiple groups.

Assumption SA3 (Efficient Risk Sharing Across Group)**.** *For any network L with at least two components there exists a risk sharing agreement τ, and a pair of agents i and $j \notin C_i$, such that all agents are weakly better off on $L \cup \{l_{ij}\}$ and some agents are strictly better off.*

Relative to the single group case, agents from different groups provide each other with access to less correlated income streams. This increases the total surplus generated by risk sharing conditional on a given network being formed. Moreover, the presence of across group links provides positive externalities to others insofar as it increases the marginal value of within group links. This raises the question of how the additional surplus generated by across group risk sharing should be split among the agents. We take a parsimonious approach to this issue by making two assumptions. The first assumption builds on the single group analysis. It requires that agents receive at least the same marginal benefits they would receive were all agents from the same group. The additional surplus generated must be split in a way such that each agent receives a weakly positive share.

Assumption SA4 (Lower Bound)**.** *Consider a network L, such that l_{ij} is essential on $L \cup \{l_{ij}\}$, and two allocations of the agents to groups G, G'. If all agents are from the same group under G, such that $G(i) = G(j)$ for all $i \neq j$, then*

$$u_i^\tau(L \cup \{l_{ij}\}, G') - u_i^\tau(L, G') \geq u_i^\tau(L \cup \{l_{ij}\}, G) - u_i^\tau(L, G).$$

This assumption requires that the additional benefits an agent i gets from risk sharing, in terms of the second period agreement reached relative to the payoff i would have got were everyone from the same group, strictly increase if a link l_{jk} is removed form the network and replaced by a link l_{ij} without changing the set of agents in each component.

Assumption SA5 (Link Increasing Additional Benefits)**.** *Consider two networks L and L' connecting the same sets of agents, and two allocations of the agents to groups G, G'. If L' can be reached from L by rewiring a link to i such that, $L' = \{L \setminus l_{jk}\} \cup l_{ij}$, $i \neq j \neq k$, $l_{ij} \notin L$, $l_{jk} \in L$, G' contains agents from different groups and under G all agents are from the same group, then*

$$u_i^\tau(L', G') - u_i^\tau(L, G') > u_i^\tau(L', G) - u_i^\tau(L, G).$$

Assumption SA5 is only a coarse partial ordering on utilities. While it implies that an agent's share of the additional surplus generated by across group risk sharing increases as links are rewired to that agent, it makes no comparison between networks that cannot be reached by rewiring links to a single agent. In particular, following a rewiring to i, it does not pin down how the payoffs of other agents changes.

Proposition SA6. *Suppose all groups have the same utility functions, such that $v_i = v_j$ for all i, j. With k different groups, there exist a $\bar{\kappa}_W > 0$ such that for all $\kappa_W < \bar{\kappa}_W$ a network is Pareto efficient if and only if it is a tree with $k-1$ across group links.*

Proof. We begin by showing the "only if" direction. All Pareto efficient networks are trees. First, by Assumption SA3, risk sharing among all agents is efficient so L must connect all agents. Second, a Pareto improvement can be achieved on any connected non-tree network by implementing the same risk sharing arrangement and deleting a superfluous link, thereby saving these costs.

We now show that efficient networks must also have exactly $k-1$ across group links. We will show, by construction, that for any tree network with strictly more than $k-1$ across group links, there exists a Pareto improvement.

If there are more than $k-1$ across group links in a tree network, we claim that there must exist an across-group link l_{ij} which, upon its removal, will result in a network $L' = L \setminus \{l_{ij}\}$ such that there exists two agents (k, l), with $G(k) = G(l)$ and $C_k(L') \neq C_l(L')$.

Towards a contradiction, let there be $k' > k-1$ across group links and suppose this is not true. As L is a tree network, removing all across group links must then result in there being $k'+1$ components. If there are no agents from the same group in different components, this implies that there must be at least $k'+1 > k$ different groups, which would be a contradiction. Thus there exist two components each containing an agent from the same group. Denote these agents k, l. As L is a tree there exists a unique path between k and l on L, and as k and l are in different components following the removal of across group links there exists at least one across group link on this path. Letting this link be l_{ij} proves the claim.

As k, l are in different components on L', but from the same group, the network $L'' = L' \cup l_{kl}$ will be a connected tree network with one less across-group link, and one more within-group link than L.

On the network L'' we implement the same risk-sharing arrangement as before, with one exception. First we identify the vector of consumptions for agents i and j that make them just as well off as on the original network, and continue to satisfy the Borch rule:

$$\frac{\partial v_i(c_i(\omega))/\partial c_i(\omega)}{\partial v_i(c_i(\omega'))/\partial c_i(\omega')} = \frac{\partial v_j(c_j(\omega))/\partial c_j(\omega)}{\partial v_j(c_j(\omega'))/\partial c_j(\omega')} = \frac{\partial v_{i'}(c_{i'}(\omega))/\partial c_{i'}(\omega)}{\partial v_{i'}(c_{i'}(\omega'))/\partial c_{i'}(\omega')},$$

for all states ω, ω' and all $i' \neq k, l$.

As i and j save the cost of an across group link, and utility is strictly increasing and concave in consumption, this implies that $c_i(\omega)$ and $c_j(\omega)$ must strictly decreases in all states ω. This additional consumption is passed onto agents k and l. As there is a strictly positive amount of remaining consumption in all states of the world, and utilities are strictly increasing in consumption, there exist feasible consumption vectors for agents k and l that strictly increase $\mathrm{E}(v(c_k))$ and $\mathrm{E}(v(c_l))$. Thus, for all κ_w sufficiently small, we have $\mathrm{E}(v(c_k)) > \kappa_w$ and $\mathrm{E}(v(c_l)) > \kappa_w$. We have therefore constructed a Pareto improvement.

We now show the "if" direction. Consider a tree network with $k-1$ across group links. Suppose we implement a risk sharing agreement in which $c_i(\omega) = c_j(\omega)$, for all i and j. As all agents' consumptions are equalized in all states, there is then no way in which link formation costs can be redistributed and the risk sharing arrangement changed, without making someone worse. Suppose towards a contradictions that we can redistribute the link formation costs, by forming a different tree network with $k-1$ across group links, to generate a Pareto improvement. Holding consumption fixed, on the new network if some agents are better off, then some will be worse off. Thus, to achieve a Pareto improvement, consumptions will have to be changed. Let $c'(\omega)$ be the new consumption vector. As the utility function $v(\cdot)$ is concave, Jensen's inequality implies that

$$\frac{1}{n}\sum_i v(c_i'(\omega)) < v(\frac{1}{n}\sum_i c_i'(\omega)) = \frac{1}{n}\sum_i v(c_i(\omega)),$$

for all ω. Thus the average expected utility from consumption will decrease, and total link formation costs have remained constant, so at least one agent must be worse off. This is a contradiction.

□

In our simple CARA utility, normally distributed incomes, Myerson value allocation rule model, underinvestment across group is possible but there is no underinvestment within group. The same example establishes the possibility of underinvestment across group in our more general setting. There is also never any underinvestment within group in our more general setting as we now show.

Proposition SA7. *There is never any underinvestment within group.*

Proof. Consider any stable network L' and allocation to groups G'. Suppose, towards a contradiction, there is underinvestment within a group in L'. There must then be an essential link l_{ij} the planner could form to achieve a Pareto improvement. Stability of L' implies that either $u_i^\tau(L'\cup\{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$ or else $u_j^\tau(L'\cup\{l_{ij}\}, G') - u_j^\tau(L', G') < c_w$. Without loss of generality suppose $u_i^\tau(L'\cup\{l_{ij}\}, G') - u_i^\tau(L', G') < c_w$. Consider now the alternative grouping G in which all agents are from the same group. In this case, by Assumption 11(d) and as l_{ij} is essential, $u_i^\tau(L'\cup\{l_{ij}\}, G) - u_i^\tau(L', G) \geq c_w$. Thus, combining inequalities, $u_i^\tau(L'\cup\{l_{ij}\}, G) - u_i^\tau(L', G) > u_i^\tau(L'\cup\{l_{ij}\}, G') - u_i^\tau(L', G')$. This contradicts Assumption SA4. □

Consider the partial ordering in which an agent i is more central in a network L' than in network L if and only if L' can be reached from L by rewiring links only to i. The following result generalizes the result in the benchmark model that more centrally located agents within a group have higher incentive to create across group links.

Proposition SA8. *Suppose that*

(i) *when there is one group, for all efficient networks $L\cup\{l_{ij}\}$, $g(\overline{d}, |C_i(L)|, |C_i(L\cup\{l_{ij}\})|) = g(\overline{d}, |C_j(L)|, |C_j(L\cup\{l_{ij}\})|)$; and*

(ii) *there are two groups.*

Then, for any efficient network L with across group link l_{ij}, if it is profitable for an agent i to form l_{ij}, and the alternative efficient network L' can be reached from L by rewiring within group links to i, then it is also profitable for i to form the link $l_{ij}\in L'$.

Proof. Let G' be the grouping of agents. Agent i is weakly better incentivized to invest in the across group link l_{ij} on the network L' than the network L if and only if

$$u_i^\tau(L, G') - u_i^\tau(L \setminus \{l_{ij}\}, G') \leq u_i^\tau(L', G') - u_i^\tau(L' \setminus \{l_{ij}\}, G'). \tag{1}$$

As L and L' are efficient, and l_{ij} is an across group link on both L and L', all agents who are path-connected to i on $L \setminus \{l_{ij}\}$ are from the same group as i, as are all agents path connected to i on $L' \setminus \{l_{ij}\}$. Thus, on the networks $L' \setminus \{l_{ij}\}$ and $L \setminus \{l_{ij}\}$, by Assumption SA4 agent i must then get exactly the same payoffs as he would do in the one group case: $u_i^\tau(L \setminus \{l_{ij}\}, G') = u_i^\tau(L \setminus \{l_{ij}\}, G)$ and $u_i^\tau(L' \setminus \{l_{ij}\}, G') = u_i^\tau(L' \setminus \{l_{ij}\}, G)$, where G is the grouping in which all agents are from the same group. We can therefore rewrite equation 1 as

$$\begin{aligned} u_i^\tau(L, G') - u_i^\tau(L, G) + u_i^\tau(L, G) - u_i^\tau(L \setminus \{l_{ij}\}, G) \quad \leq \quad & u_i^\tau(L', G') - u_i^\tau(L', G) \\ & + u_i^\tau(L', G) - u_i^\tau(L' \setminus \{l_{ij}\}, G). \end{aligned} \tag{2}$$

Repeatedly applying Assumption SA5, $u_i^\tau(L, G') - u_i^\tau(L, G) < u_i^\tau(L', G') - u_i^\tau(L', G)$. Thus a sufficient condition for equation 2 to hold is that:

$$u_i^\tau(L, G) - u_i^\tau(L \setminus \{l_{ij}\}, G) \leq u_i^\tau(L', G) - u_i^\tau(L' \setminus \{l_{ij}\}, G).$$

As we are in the one group case and l_{ij} is essential on both L and L', $u_i^\tau(L, G) - u_i^\tau(L \setminus \{l_{ij}\}, G) = u_i^\tau(L', G) - u_i^\tau(L' \setminus \{l_{ij}\}, G) = g(\overline{d})$. This completes the proof.

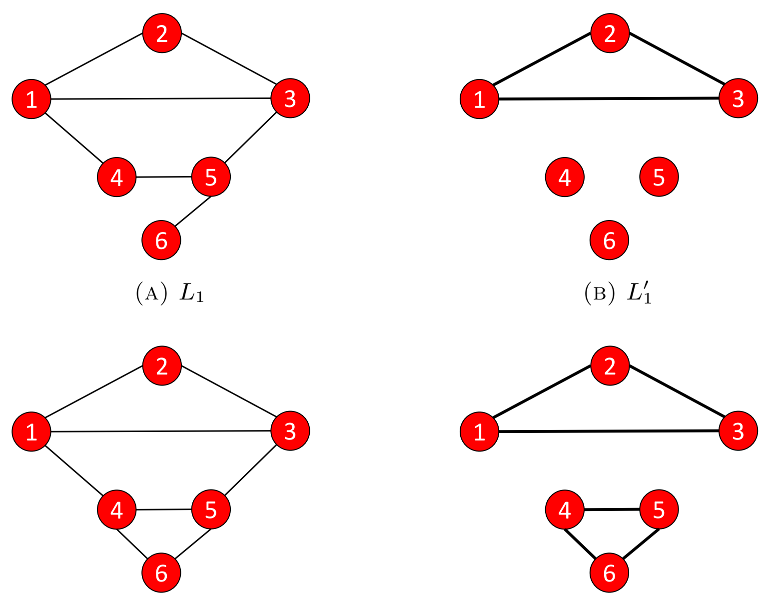
□

B. Supported risk sharing

As with the previous section, this section provides a slightly more general and comprehensive treatment of analysis in the main paper. This time the corresponding section of the main paper is Section 6.2. Again we number replicated assumptions and results so that they correspond to those in Section 6.2 of the main paper, while new results are labeled with the prefix SA.

In this section we extend the model to capture the idea that having friends in common can reduce an agent's incentives to renege on an agreement. This might be because the friend in common is able to monitor actions and identify the guilty party in a dispute, or because reneging on the agreement will lead to a damaging reputation loss with the friend in common. While it is beyond the scope of this paper to fully explore these issues, and there is a vibrant literature that focuses on network based enforcement of agreements (see, for example, Jackson et al. (2012), Wolitzky (2012), Ali and Miller (2013, 16), Ambrus et al. (2014), Nava and Piccione (2014), Ambrus et al. (2016)), in this section, motivated by this literature, we model the value of friends in common for enforcement by assuming that risk sharing between two agents is possible if, and only if, those two agents have a friend in common. This is known as closure (Coleman, 1988) and has long been thought important for cooperation because it enables collective sanctions to imposed on a deviating agent—if an agent cheats on one of their neighbors, there are friends in common that can also punish the deviating agent.

A link in L is supported and can be used for risk-sharing if, and only if, it is part of a triangle (i.e., the complete network among three agents). Let L' be the spanning subgraph of L which contains only supported links. An illustrations of this is provided in Figure 2. Risk-sharing agreements, and rent distribution, are as in Section 2 of the main paper. The only difference is that now risk-sharing takes place on the network $L'(L)$ instead of L (but agents continue to pay to form links in L).



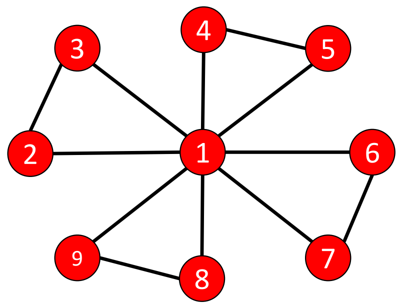
(A) L_1 (B) L_1'

(C) L_2 (D) L_2'

Figure 2. Panel (A) provides an example of a network among six villagers, while panel (B) shows which of these links support risk sharing. Panel (C) provides a different example of network among six villagers, while panel (D) shows which of these links support risk sharing.

Before we can state our main result for this section we need some new terminology.

A network L is a *tree-union of triangles* if it can be expressed as the union of m (non-node-disjoint) subnetworks ordered as $\{L(N_1), \ldots, L(N_m)\}$, such that $\cup_{i=1}^{k} N_i \cap N_{k+1} = 1$ and each subnetwork $L(N_i)$ is a triangle. Thus each subnetwork in the sequence is a triangle that has exactly one node in common with the union of all the nodes in the subnetworks preceding it in the sequence. Two different tree unions of triangles are illustrated in Figure 3 and panel (A) of Figure 4, respectively. The tree union of triangles illustrated in Figure 3 is known the *Friendship graph*, or Windmill network. This is the tree-unions of triangles in which all triangles have the same node in common.



(A)

FIGURE 3. The Friendship graph on 9 vertices.

We will focus on risk sharing within a village. We denote the cost of forming a link by $\kappa = \kappa_w$. As before, we continue to focus on the parameter range for which risk sharing among all agents is efficient. As before, the surplus obtained from enabling risk-sharing among two groups of agents is V. Proposition 16 shows it is efficient for all agents to risk-share if and only if $V \geq 3\kappa$, and that the efficient networks are then tree-unions of triangles. Thus, in comparison to Section 2 of the main paper where agreements didn't need to be supported to be enforceable, tree unions of triangles play the role of tree networks.

Proposition 16. *Suppose the number of villagers $n \geq 3$ is odd.*

(i) *If risk-sharing among all n agents is efficient, then the efficient risk-sharing networks are tree-unions of triangles.*

(ii) *Risk sharing among all n agents is efficient for all n if and only if $V \geq 3\kappa$.*

The proof of Proposition 16 is fairly long and deferred until Section B.1. Here we offer some intuition. First observe that any link that is not supported is costly to form but cannot be used for risk-sharing. While in principle such a link might still be valuable as a means for supporting an agreement on another link, this requires a triangle to be formed with the link which would make it supported. Thus in an efficient network all links must be supported, and part of a triangle. Given this, the most efficient way to organize links (among an odd number of agents) is to form a tree union of triangles. This creates distinct triangles in which no link is shared by two triangles. This might seem inefficient, but it is not because it economizes on the number of triangles required. As a comparison consider the tree union of triangles shown in panel (A) of Figure 4 and the alternative network, in which villagers 1 and 2 are connected to all villagers and there are no other links, shown in panel (B) of Figure 4. In the alternative network there are $n - 2 = 7$ triangles, while there are just $(n-1)/2 = 4$ triangles in the tree union of triangles. Thus although the triangles in the alternative network all share the link l_{12}, meaning that for $n - 3 = 6$ of the triangles only two additional links are required, there are more links in the alternative network than the tree union of triangles $(3(n-2) - (n-3) = 15$ links in comparison to $3(n-1)/2 = 12$ links).

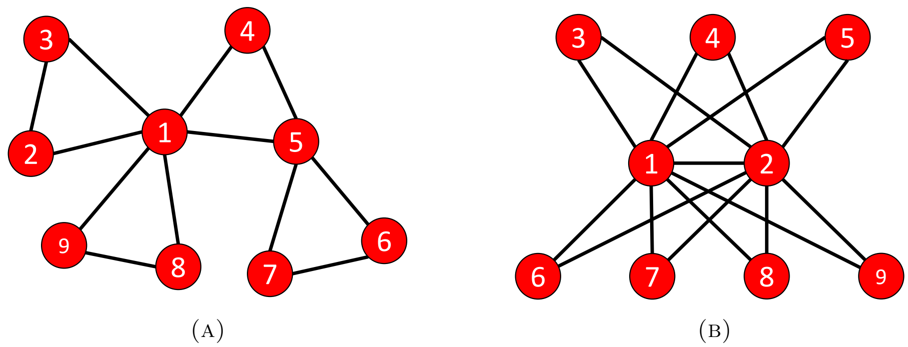


FIGURE 4. Panel (A) shows a tree union of triangles connecting nine villagers. Panel (B) shows an alternative network connecting 9 villagers in which all villagers are able to risk share and all triangles share a common link.

Jackson et al. (2012) find a class of networks they call social quilts to be those that can supporting risk-sharing agreements based on renegotiation proofness. Interestingly, tree unions of triangles are social quilts. The networks we identify through efficiency considerations based on the very simple condition of support for risk-sharing to be possible would also be renegotiation proof in their setting. This provides further motivation for the simple approach to enforcement we take.

We now consider the stability of the efficient risk-sharing networks. Unlike the corresponding result in Section 4 of the main paper, all tree unions of triangles are equally pairwise stable and the empty network is now always pairwise stable. As risk sharing now requires three agents, for an agent to extricate themselves from an agreement while not leaving unsupported links, they must delete two links at once. Thus, in addition to the pairwise stable networks, we consider networks that are pairwise stable and also stable to multiple link deletions. Such a network L must be pairwise stable and, for all agents i, $u_i(L) \geq u_i(L')$ for all L' that can be obtained by removing any of i's links in L.

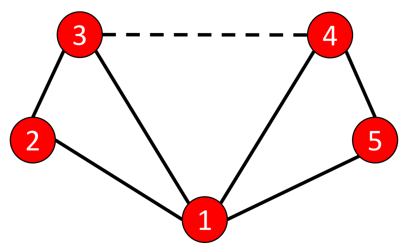
As before, we let V be the constant value of reducing the number of risk-sharing groups by 1.

Proposition SA9. *In a tree-union of triangles, an agent i receives a net payoff $|N(i;L)|(V/3-\kappa)$. A tree-union of triangles L is pairwise stable if and only if $3V/5 \leq 3\kappa \leq 2V$. A tree-union of triangles L is pairwise stable and also stable to multiple link deletions if and only if $3V/5 \leq 3\kappa \leq V$. The empty network is always pairwise stable.*

The full proof is in Section B.1. Analogously to before, on efficient networks, all links are essential and make the same expected contribution to total surplus for a random arrival order of the agents (as can be used to calculate the Myerson value). Moreover, these benefits are shared equally among two agents when they have a link. Collectively, a triangle of links contributes an amount $2V$ to total surplus. In a tree unions of triangles each link is part of only one triangle, and thus each link contributes on average $2V/3$. As these benefits are split evenly among the agents forming the link, they each get $V/3$ while it costs each agent κ to form a link. Hence each agent receives a net payoff of $|N(i;L)|(V/3-\kappa)$, which is again proportional to their degree.

If an agent deletes a link, exactly one other of its links becomes unsupported. Thus the agent's payoff decreases by $2V/3$, but they only save κ in costs. Thus a network is stable to individual link deletions if and only if $2V \geq 3\kappa$, while it is stable to multiple link deletions if and only if $V \geq 3\kappa$ (which holds by the maintained assumption that it is efficient for all agents to risk-share with each other).

Consider an agent's incentives to form an additional superfluous link. In any network, agents can only benefit from forming links that would be supported so that it can be used for risk-sharing. The key to the proof is showing that on a tree union of triangles, for any superfluous link that would be supported upon its formation, there are the same incentives to deviate form it. Thus there is a profitable deviation to form any superfluous link in any tree union of triangles if and only if it is profitable to form the link shown in Figure 5. As it is profitable to form this additional link if and only if $V/2 \geq 3\kappa$, a tree union of triangles is robust to the pairwise addition of a link if and only if $V/2 \leq 3\kappa$.



(A)

FIGURE 5. The Friendship graph on 5 vertices with a possible deviation shown by the dashed line.

Finally, to see that the empty network is always stable, just note that on this network an additional link will not be supported and so not facilitate any risk sharing; thus there are no incentives to form any link. The stability of the empty the network, and the need for groups of at least three agents to support risk sharing, suggests that it might be reasonable to permit coalitions of three agents to form links among themselves. We do so with the minimal possible extension to pairwise stability that facilitates such deviations.

A network is triple-wise stable with respect to expected utilities $\{u_i(L)\}_{i\in\mathbf{N}}$ if and only if it is pairwise stable and for all $i,j,k \in \mathbf{N}$, if two or more of l_{ij}, l_{ik}, l_{kj} are not in L and $\hat{L}$ is the union of network L with these three links, then if $u_i(\hat{L}) \geq u_i(L)$ and $u_j(\hat{L}) \geq u_j(L)$ with at least one inequality strict, then $u_k(\hat{L}) < u_k(L)$. In words, triplet-wise stability requires a network to be pairwise stable and for no set of three players to be able to benefit by forming the links among themselves (thereby facilitating direct risk sharing among themselves).

Proposition 17.

(i) *If there exists an efficient triplet-wise stable network then all friendship networks are stable, and for a non-empty range of parameter specifications only friendship networks are stable.*

(ii) *For all inequality measures in the Atkinson class, among the set of triplet-wise efficient networks, friendship networks and only friendship networks maximize inequality.*

This result is analogous to results in Proposition 6 in Section 4 of the main paper. There a star network was the most efficient stable network, but also the most unequal. Proposition 17 shows that this result generalizes to the case in which links must be supported to facilitate risk sharing, but with friendship networks taking the place of star networks.

The proof of Proposition 17 is in section B.1. The basic intuition for the result mirrors the intuition for the corresponding result in the main paper (Proposition 6). Groups of three agents have stronger incentives to deviate and form links among themselves to facilitate risk sharing when they are further apart. Among the set of efficient networks the relevant distances are minimized by the friendship network. In terms of inequality, agents' net payoffs are again proportional to their degree, and the total number of links is constant in any tree union of triangles connecting n agents. Further, in any tree union of triangles all agents must have at least degree 2. The friendship network therefore minimizes the possible degree for all but one agent, while maximizing the possible degree for the remaining agent. The star network did the equivalent thing in Section 4 of the main paper, and this was the key property of the star network that led it to generate the most inequality for any inequality measure in the Atkinson class. The argument establishing that the friendship network now generates the most inequality for any inequality measure in the Atkinson class is the same.

B.1. **Proofs.**

B.1.1. *Proof of Proposition 16.*

Proof. **Part (i)**: Consider an efficient network L. As the network is efficient all agents are then in the same risk-sharing component so $L'(L)$ is connected. Further, as the network is efficient every link must be supported, so $L'(L) = L$. This means that the network can be decomposed into a set of triangles (where the triangles can share nodes and links with each other and every node is part of at least one such triangle). There may be more than one such decomposition for L. Moreover, as $L'(L) = L$ is connected, these triangles must be connected to each other so that there is path from every triangle to every other triangle. It is therefore possible to order the triangles in the decomposition, so that as the triangle are added to the network in this sequence there is always a unique component.

Figure 6 gives an example of this triangle decomposition. In this example there is a redundant triangle such that the original network can be constructed from a set of triangles that excludes it. It doesn't matter which decomposition is selected and whether the redundant triangle is included or not.

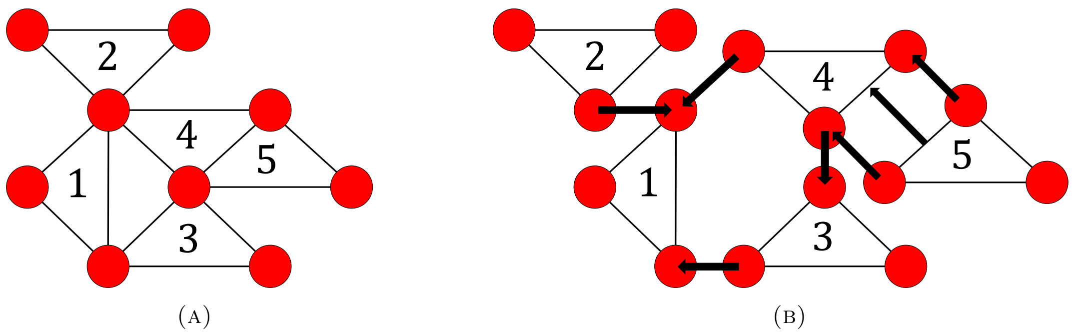


FIGURE 6. Panel (A) illustrates a network in which every link is supported (i.e., $L'(L) = L$). Panel (B) shows how this network can be represented a sequence of triangles. By combining the triangles 1, 2, 3, 4 and 5 the original network is obtained. The arrows in panel B indicate which links and nodes are combined in this construction.

Consider an efficient network and an associated triangle decomposition. Suppose we create the network associated with the decomposition. So, if there are k triangles in the decomposition, we are then left with a network consisting of k disjoint triangles (this will require creating duplicate nodes and links). This network has k components, $3k$ nodes and $3k$ links. We then order these triangles, and recombine them to create the efficient network. We start with triangle 1, add triangle 2 so that 1 and 2 now from a network component, add 3 so that triangles 1, 2 and 3 form a component, and so on. Thus, after each step in the sequence the number of components is reduced by one.[3] We consider how the number of links and nodes in the network must evolve along such a sequence.

When we connect an unconnected triangle to an existing set of connected triangles (which we term the component) the ways in which this might be done can be partitioned as follows: The new triangle can share 3 nodes with existing nodes, 2 nodes with existing nodes, or 1 node with existing nodes. In the case of sharing 3 nodes, no new nodes are being added to the network, but new links might be. As, by construction, all nodes in the component are already supported, it is without loss of generality to ignore such operations when searching for minimally connected networks that enable risk sharing among all agents (i.e., efficient networks).[4] Figure 7 shows two examples of this. The addition of the triangle as shown in

[3]For the example given in Figure 6 the sequence 1, 2, 3, 4, 5 results in a reduction in the number of components of one at each step, while the sequence 2, 5, 3, 1, 4 would not.

[4]For example, the redundant triangle in Panel A of Figure 6 could be added last in which case it would share three nodes and three links with the component and its addition would add no new links or nodes to the component.

panel (A) has no effect on the number of links or nodes in the network (see panel (B)), while the addition of the triangle as shown in panel (C) increases the number of links but not the number of nodes in the network (see panel (D)).

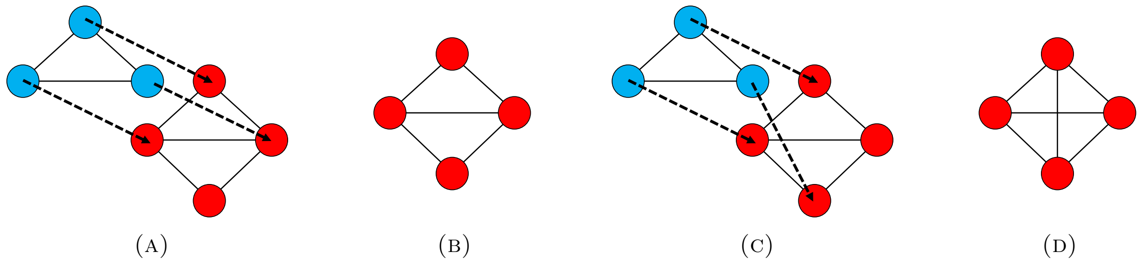


FIGURE 7. Panels (A) and (B) illustrates the addition of a triangle to a risk sharing component in which all nodes are shared by a single existing triangle. Panels (C) and (D) illustrates the addition of a triangle to a risk sharing component in which all nodes are shared by existing triangles.

When a triangle is added that shares two nodes, it can either share one link as well, or share no links. When a triangle is added that shares just one node it cannot share any links. These three possibilities are enumerated below and illustrated in Figure 8.

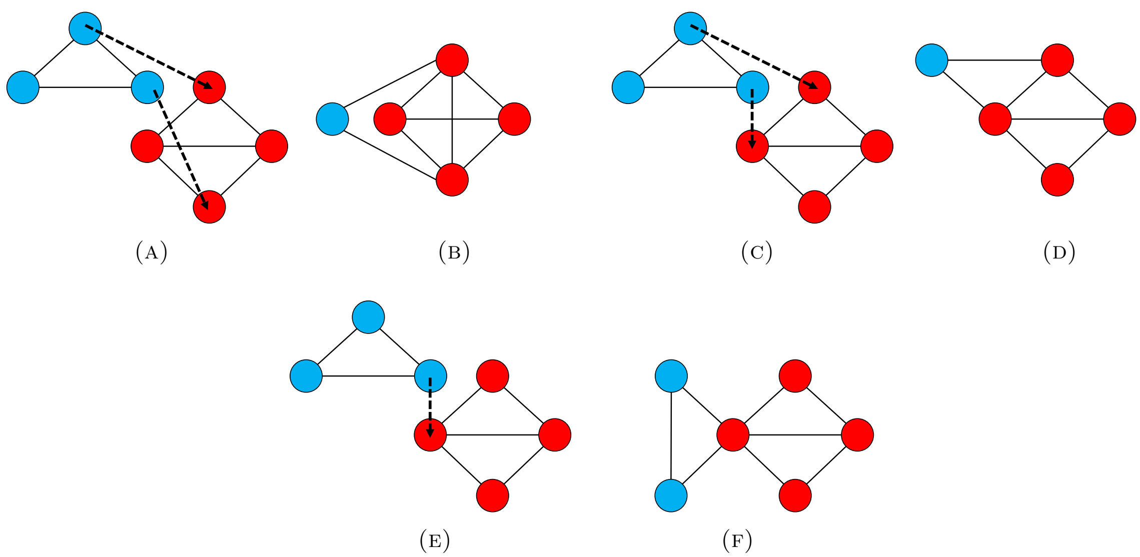


FIGURE 8. Panels (A)-(D) illustrates the possible ways in which triangles that share two nodes can be added, while panels (E)-(F) illustrate the possible ways in which triangles that share one node can be added.

(a) The triangle shares two nodes, one node with each of two different triangles. In this case, we increase the number of links in the component by 3 and increase the number of nodes in the component by 1.

(b) The triangle shares two nodes, both nodes with the same other triangle. In this case, we increase the number of links in the component by 2 and increase the number of nodes in the component by 1.

(c) The triangle shares one node. In this case, we increase the number of links in the component by 3 and increase the number of nodes in the component by 2.

Following the decomposition, along the sequence of recombining the triangles we do one of the above three operations at each of the $k-1$ steps. There are n nodes, where (by assumption) n is an odd integer. Suppose it is feasible to do any combination of the operations $(a)-(c)$, in any order to arrive at n nodes. We always start with the component being a triangle, with three nodes and three links. This means that the number of nodes in the original network is $n = 3 + a + b + 2c$, where a is the number of (a) operations, b the number of (b) operations and (c) the number of c operations. As the initial network is efficient, this sequence of operations must minimize the number of links in the resulting network, conditional on enabling all n agents to risk-share. Assuming that any sequence of operations is feasible, the sequence of operations must minimize $3+3a+2b+3c$ subject to $3+a+b+2c = n$. As n is odd, this is uniquely achieved by setting $a = b = 0$ and $c = (n-3)/2$. (Incidentally, when n is an even number greater than 3, it can be seen that this is instead achieved by setting $a = 0$, $b = 1$ and $c = (n-4)/2$—thus when n is even the structure of the efficient networks is similar to the structure of efficient networks when n is odd). Note that the efficient network is constructed through sequentially adding triangles such that at each step in the sequence the added triangle shares exactly one node with the triangles already added. But this is just the definition of a tree union of triangles. This implies that this sequence of operations is feasible and that the efficient networks are tree unions—when there are n nodes, with n odd, any tree union of triangle is efficient and no other network is efficient.

Part (ii): We have established that an efficient network is a tree union of triangles, so the number of links under full risk sharing is $3(n-1)/2$. Since every link incurs the cost κ for both agents, full risk sharing is therefore efficient if and only if $V(n-1) \geq 3(n-1)\kappa$.

□

B.1.2. *Proof of Proposition SA9.*

Proof. Let L be a tree-union of triangles and consider one such triangle τ. Without loss of generality label the agents in this triangle $1, 2$ and 3. Consider adding the agents to the network in an arbitrary permutation. For any such permutation, the last agent to be added from the set $\{1, 2, 3\}$ completes the triangle τ. As L is a tree union of triangles, prior to completion of this triangle, agents 1, 2 and 3 cannot risk share with each other and must be in different risk-sharing components of the network $L'(L)$. Thus, the completion of the triangle τ reduces the number of risk sharing components by 2, generating additional value $2V$. So in the Myerson value calculation, the presence of the triangle τ generates an additional expected payoff for each of the agents $\{1, 2, 3\}$ equal to $2V/3$ (as each is last to arrive in 1/3 of the permutations, and so each completes τ, thereby generating risk sharing benefits of $2V$, in $1/3$ of the permutations). Thus, in a tree union of triangles an agent's payoff before link formation costs is $|N(i;L)|V/3$.

As after a link is deleted one sharing triangle is lost (as no triangles share links in a tree union of triangles) deleting a link causes that agent to lose benefits $2V/3$. Thus an agent does not want to delete any one of their links in a tree union of triangles if and only if $2V/3 > \kappa$.

Consider now the incentives of two unconnected agents i and j to form an additional link l_{ij}. As i and j are unconnected they are in different risk sharing triangles. If the link l_{ij} does not create a new triangle with some agent k, then it does not facilitate any additional risk-sharing on any subnetwork that can be reached by adding the agents in sequentially. Hence, agents i's and agent j's Myerson value is unaffected, but they pay a cost κ each to form the link. As such deviations are unprofitable, we can restrict attention to link l_{ij} that would be part of a triangle once added. Let τ be the triangle on $L \cup \{l_{ij}\}$ between agents i, j and some other agent k. Thus $l_{ik} \in L$ and $l_{jk} \in L$. Upon its completion (i.e., when the last of i, j or k is added for a given arrival order) the triangle τ facilitates new risk sharing between agents i, j and k thereby reducing the number of risk-sharing components by 2, if and only if both i and k and j and k were not able to risk-share with each other before. As i and k are connected on L, and L is a tree union of triangles, they must be part of a risk sharing triangle on L with another agent k'. Hence they are already risk-share with each other if and only if k' has already been added (i.e., k' is not the last agent to be added in the permutation among the four agents i, j, k, k'). This happens in $3/4$ of the permutations. Similarly, agents j and k must also already be part of a risk-sharing triangle with another agent $k'' \neq k'$ (were $k'' = k'$ this would imply that two risk-sharing triangles in L share a link $l_{k'k}$, but then L would not be a tree union of triangles). So risk sharing among agents j and k is also already possible if k'' is not last in the permutation among the four agents i, j, k, k'' (see Figure 9).

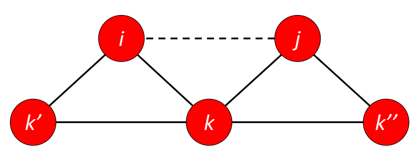


FIGURE 9. Adding a new link that is supported. The new link is the dashed link.

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The probability that the new triangle τ generates benefits $2V$ upon being added is $2(3!)/5!$. There are $5!$ permutations of i, j, k, k', k''. There are $3!$ permutations of i, j, k. For each of these permutations, there are two permutations in which k'' and k' are the last two elements for a permutation of i, j, k, k', k''. Hence the probability that k'' and k' are both after all of i, j and k in a random permutation is $2(3!)/5! = 1/10$. The probability that τ generates benefits V is the probability that either k' is after all of i, j and k or k'' is after all of i, j and

k, but k' and k'' are not both after all of i, j and k. The probability that k' is after all of i, j and k is $1/4$. The probability that k'' is after all of i, j and k is $1/4$. Thus the probability that τ generates benefits V is $1/2 - 1/10 = 2/5$. Thus the expected increase in surplus generated by the link l_{ij} is $2V/5 + 2V/10 = 3V/5$. These benefits accrue to agent i with probability $1/3$, to agent j with probability $1/3$ and to agent k with probability $1/3$. Thus agent i and j have a profitable pairwise deviation to form the link if and only if $V/5 > \kappa$. Thus a tree union of triangles is pairwise stable if and only if $3V/5 \leq 3\kappa \leq 2V$ as claimed.

When it is possible to delete multiple links at once, a lower bound on the benefit lost per link deleted in a tree union of triangles is $V/3$. Recall that in a tree union of triangles an agent's payoff before link formation costs is $|N(i; L)|V/3$. Thus, if after a deletion all remaining links still facilitate risk-sharing, only $V/3$ will be lost per link deleted. If after the deletion some of remaining links are not able to facilitate risk-sharing, the loss per link will be greater. The bound of $V/3$ is tight. For example, if an agent simultaneously deletes all their links this bound will be achieved. As the amount saved in link formation costs from deleting a link is κ, it then follows that a network is pairwise stable and also stable to multiple link deletions if and only if $3V/5 \leq 3\kappa \leq V$.

Finally, note that in the empty network the incremental benefits of forming a link l_{ij} are 0 as it does not permit any risk-sharing. Hence, the empty network is pairwise stable. $\square$

B.1.3. *Proof of Proposition 17.*

Proof. **Part (i)**: By Proposition 16, efficient networks are tree-unions of triangles and by Proposition SA9 all these networks are equally pairwise stable. Thus any difference in stability between the efficient networks in terms of stability must be due to triplet wise deviations that form at least two links among the three agents. Thus, there are two cases to consider—when a triplet deviates by adding two links and when a triplet deviates by adding three links.

We consider these cases shortly. Before that, it is helpful to define a new distance measure for tree-unions of triangles. By the definition of a tree union of triangles, any L tree union of triangles can be decomposed into a sequence of triangles such that each triangle in the sequence shares a single node with triangles earlier in the sequence. Thus, for any two nodes i and j on a tree union of triangles L, there is a minimal subset of these triangles that must be added for i and j to be path connected. We define the *triangle distance* between i and $j \neq i$ on a tree union of triangles L to be the cardinality of this set of triangles and denote the distance by $\Delta(i, j; L)$. For example, in Figure 10 we have $\Delta(i, j; L) = 4$, $\Delta(i, k; L) = 5$ and $\Delta(k, j; L) = 1$.

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Case A (two links): For the additional links to be valuable they must create a triangle. Thus, when the triplet adds two links, the other link must already be present. Without loss, label this triplet i, j, k and suppose that $l_{ij} \in L$ is the link in this triangle that is already present. We let τ denote this triangle between i, j and k. As L is a tree union of triangles,

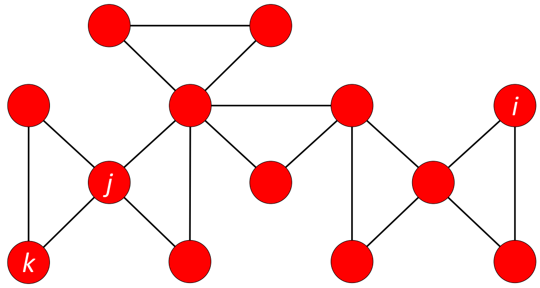

FIGURE 10. A tree union of triangles L

l_{ij} must be supported and there must be an agent k' such that $l_{ik'} \in L$ and $l_{jk'} \in L$. Figure 11 shows the subnetwork of L among agents i, j, k and k', including the links that would be formed by the deviation.

If agents i, j and k deviate to form τ, the probability that agents i and j could risk-share without the links l_{ik} and l_{jk} at the time τ is completed, for a random arrival order, is the probability that agent k' has already been added—i.e., $3/4$ (the probability that k' is not last to arrive out of i, j, k, k'). Figure 11 shows the subnetwork of L among agents i, j, k and k', and the links that would be formed by the deviation which are dashed.

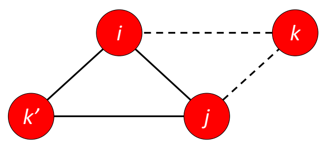

FIGURE 11. Subnetwork of a tree union of triangles L induced by agents i, j, k and k' is shown by the solid links. A possible triplet-wise deviation among agents i, j and k through the creation of the links l_{ik} and l_{jk} is shown by the dashed links.

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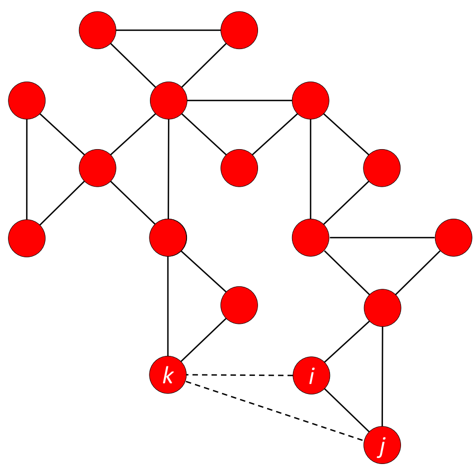
The probability that agents i and k can already risk share depends on whether there would be a supported path between them when the triangle τ is completed. Recall that $\Delta(i, k; L)$ is the triangle distance between i and k. Without loss, suppose that $\Delta(i, k; L) \geq \Delta(j, k; L)$. A supported path between i and k will exist upon the completion of τ if and only if all agents in the triangles counted in the triangle distance between i and k are already present. This requires $1+2\Delta(i, k; L)$ agents, including i, j and k to be present when τ is completed. Letting $x = 1 + 2\Delta(i, k; L)$, the probabilty of this is the probability that i, j or k arrive last in the arrival order among these x agents, i.e., $3(x-1)!/x! = 3/x$.

There are two possibilities to consider (given that $\Delta(i, k; L) \geq \Delta(j, k; L)$) when calculating the probability that agents j and k can already risk-share upon the completion of τ. First, we could have $\Delta(i, k; L) = \Delta(j, k; L)$, in which case agents j and k will be path connected upon the completion of τ if and only if i and k are path connected upon the completion of τ. Moreover, in this case, i and k (and thus also j and k) are path connected upon the completion of τ only if the triangle (i, j, k') is present upon the completion of τ. An example of this case is shown in panel (A) of Figure 12. Thus the probability that the triangle τ generates benefits $2V$ upon its completion is $1/4$ (i.e., the probabilty k' is last to arrive of i, j, k and k'), and the probability it generates benefits of exactly V upon its completion is $1 - 1/4 - 3/x$. So the expected benefits τ generates are $V(5/4 - 3/x)$.



(A) (B)

FIGURE 12. Panel (A) illustrates a triplet deviation for agents i, j and k in which the triangle distance between both agents i and k and agents j and k is 6. Panel (B) illustrates a triplet deviation for agents i, j and k in which the triangle distance between agents i and k is 6 and between agents j and k is 5.

The second possibility is that $\Delta(j, k; L) = \Delta(i, k; L) - 1$. An example of this case is shown in panel (B) of Figure 12. Consider a labeling of agents consistent with l_{ik} and l_{jk} being the new links and l_{ij} being already present. If $\Delta(i, k; L)$ is the same for both possibilities, then the incentives to deviate in this case are always weaker. This is because we can match permutations such that permutation by permutation the risk-sharing value attributable to the new links, upon completion of τ, is weakly lower now than under the first possibility. For example, in Figure 12(a), consider any permutation in which k' is the last agent to be added and j is the second to last agent to be added. In this case, τ generates value $2V$ as upon the addition of j none of i, j or k would be able to risk-share with each other without the new links. Now consider the same sequence of agents for the example shown in Figure 12(b) (where, in this figure, agents j and k' have swapped position in comparison to before). Now, when j is added, agents k and j would be able to risk-share without the new links because they will be still be path connected. Hence the new links only generate additional risk-sharing benefits of V.

On any tree union of triangles L there are at least two leaf triangles (such that two of the agents in the triangle have degree 2). Thus, if the maximal triangle distance between any two nodes on L is z, there is a pair of connected nodes i, j whom are both triangle distance z from some other node k. Hence, for the triplet of agents with the strongest incentives to deviate by forming two links on any tree union of triangles L, the triangle distance between the agents without links will be equal to the maximum triangle distance in the network. Thus, the maximum incentives over all triplets in a tree union of triangles L, for them deviate by forming two links, is increasing in the maximum triangle distance on the network which we call the triangle diameter. For example, for the tree unions of triangles shown in Figure 12 the triangle diameter is 6 and for the deviation shown in panel (A) the triangle distance between agent i and k and between agents j and k are equal to 6. The friendship network has a triangle diameter of 2, which is strictly lower than for any other tree union of triangles that is not a friendship network. The incentives for some triplet to deviate on a tree union of triangles is therefore strictly lower on a friendship network than any other tree union of triangles.

Case B (three links): Again the additional links must create a triangle and facilitate risk sharing among the agents. Without loss, label these agents i, j and k and the triangle they create from their deviation τ. For the three links to be added, these agents must all initially be in different risk-sharing triangles. Moreover, as L is a tree union of triangles, there is a unique set of risk-sharing triangles among any two of them that connects them. Let X be the set of agents in the risk-sharing triangles connecting i and k, let Y be the set of agents in the risk-sharing triangles connecting j and k, and let Z be the set of agents in the risk-sharing triangles connecting i and j.

The triangle τ, upon its completion for a random arrival order, permits new risk sharing among the triplet generating value $2V$ if and only if none of the following conditions hold: (i) agent i or k is the last to arrive among the agents in the set X; (ii) agent j or k is the last to arrive among the agents in the set Y; (iii) agent i or j is the last to arrive among the agents in the set Z. This is a complex (although tractable) combinatorial calculation to write down. However, for our purposes, what matters are the following two facts: (a) this probability increases as additional agents are added to any of the sets X, Y or Z (whether these agents are present in the other sets or not); (b) this probability increases as the sets X, Y and Z become less overlapping holding their individual cardinalities fixed. For example, holding the sets Y and Z fixed, and the cardinality of X fixed, if $|X \cup Y|$ or $|X \cup Z|$ increases the probability increases.

The triangle τ, upon its completion, permits new risk sharing among the triplet generating value V or $2V$ if and only if at most one of the following conditions hold: (i) agent i or k is the last to arrive among the agents in the set X; (ii) agent j or k is the last to arrive among the agents in the set Y; (iii) agent i or j is the last to arrive among the agents in the set Z. Again, for our purposes, what matters is the following two facts: (a) this probability increases as additional agents are added to any of the sets X, Y or Z (whether these agents are present in the other sets or not); (b) this probability increases as the sets X, Y and Z become less overlapping holding their individual cardinalities fixed.

As i, j and k are in different risk sharing triangles on L (and no triangles in a tree union of triangles share a link), we have the following inequalities on cardinalities:

(1) $|X|, |Y|, |Z| \geq 5$,
(2) $|X \cup Y|, X \cup Z|, |Y \cup Z| \geq 7$,
(3) $|X \cup Y \cup Z| \geq 7$.

For any given tree union of triangles, when considering the stability of it with respect to these deviations, we are interested in the triplet of agents that has the strongest incentives to deviate. These incentives are again minimized in the friendship graph. The friendship graph achieves the aforementioned bounds for any triplet of agents that can deviate in this way. Moreover, it is straight-forward to see that for any other tree-union of triangles, the bounds are not achieved—there must exist two agents with a tree distance greater than 2, and without loss these agents can be labeled i and k such that $|X| \geq 7$.

Part (ii): By Proposition SA9 the payoff of each agent is proportional to its degree. Among tree unions of triangles the friendship graph maximizes the degree of the highest degree agent and set the degree of all remaining agents to 2. As all agents in all tree unions of triangles must have degree of at least 2 the argument used in the proof of Proposition 6(ii) in the main paper goes through unchanged.

□

C. Permitting some free links

This section replicates and then extends Section 6.4 in the main paper.

In practice relationships are formed for many reasons, and there will be some relationships that exist for reasons unrelated to risk sharing, but nevertheless permit risk sharing. These links might, for example, represent family relationships or close friendships formed in childhood. In effect, these are relationships it is free to form for the purpose of risk-sharing, providing another explanation for why real world risk-sharing networks are coarser than tree networks. We extend our baseline model to permit this possibility.

Let $\widehat{L}$ denote the exogenously given set of links that can be formed for free. As, by the Myerson Value calculation, a link strictly increases the expected utility an agent receives in a risk sharing arrangement, we assume all such links are always formed. The network $\widehat{L}$ will consist of a set of components, each of which contains agents from the same group. For each such component C, we identify an agent $i^*(C) \in \text{argmin}_i \max_j md_{ij}(C)$. This is agent who has the lowest maximum Myerson Distance to any other agent in the component C. We will refer to agent $i^*(C)$ as the Myerson distance central agent in component C and let C_i denote the component to which i belongs. Considering all components, we then have a set of Myerson distance central agents $I^* = (i^*(C))_C$. Finally, we identify a Myerson distance central agent associated with the largest distance, $i^{**} \in \text{argmax}_{i^* \in I^*} \max_{j \in C_{i^*}} md_{i^*j}$.

We dub a network generated by forming all free links, and the links $l_{i^*i^{**}}$ for all $i^* \neq i^{**}$ a *central connections network*. Suppose there are k different groups and $k' \geq k$ initial components. The set of efficient network then comprises of the set of networks in which all free links are formed and $k' - k$ within group links are formed (i.e., the minimal number of costly links that must be formed for there to be a single component).[5] Central connections networks are always efficient. They are also most stable within the class of efficient networks.

Proposition 18. *Suppose there is one group. If any efficient network is stable, then all central connections networks are also stable.*

Proposition 18 shows that when some within-group are formed for free, the most stable efficient network forms all additional links required for risk-sharing with a single agent. As payoffs are proportional to degree, this again pushes villages towards inequitable outcomes.

We now prove Proposition 18.

Proof. Consider two components C and C'. For two agents i, j in component C, recall that $md(i, j, C)$ equals 1/2 less the probability that a path exists between i and j on C upon the arrival of i. Suppose now we take two components C and C'. Let agents i, k be in component C and agents j, k' be in component C', and form the bridging link $l_{kk'}$. The probability a path exists between i and j upon i's arrival is now is equal to the probability that a path exists between i and k on C multiplied by the probability that a path exists between k' and j on C'. This is because these events are independent, and when both path exist agents k and k' must have arrived before i and so the link $l_{kk'}$ must be present. It follows that

$$\operatorname*{argmax}_{i,j} md_{ij}(C \cup C' \cup \{l_{kk'}\}) = \{i, j : i \in \operatorname*{argmax}_{l} md_{lk}(C), j \in \operatorname*{argmax}_{l} md_{lk'}(C')\}.$$

Thus the network generated by forming all free links, and the links $l_{i^* i^{**}}$ for all $i^* \neq i^{**}$ minimizes the maximum Myerson distance on an efficient network and, by Lemma 4, is stable if any other efficient network is stable. □

When there are multiple groups, central connections networks within group with the agent i^{**} providing the across group link(s) continue to work well. With multiple groups, agents' incentives to form superfluous within-group links depend on two things. First, as before, whether the link will be essential for a random arrival order, and second, unlike before, how many agents from other groups the link provides access to upon i's arrival when it is essential. Incentives to form a superfluous within-group links are increasing in the number of agents from other groups the link provides access to, and decreasing in the number of agents within-group the link provides access to. These considerations make superfluous links to the agent providing the across group link(s) particularly valuable. However, by construction the network generated by forming a central connections network within-group, with the agent i^{**} providing the across group link(s), minimizes the maximum probability that a superfluous link to the agent providing the across group link(s) will be essential for a random arrival

[5]As before, the same set of risk sharing arrangements can be implemented on any given component, and as expected utility is transferable, given that formation costs have been minimized, any point on the Pareto frontier can be obtained.

order. It thus minimizes the maximum incentives for an agent to form a superfluous link within-group to the agent providing the across group link(s).

Considering the incentives within a group to efficiently form an across-group essential link, a central connections networks within-group is also likely to do well. By Lemma 9 more Myerson central agents have better incentives to form across group links. While central connections networks maximize a slightly different notion of the centrality of the most central agent, in this case agent i^{**}, these measures of centrality are likely to be highly correlated. We therefore expect central connections networks within-group to provide relatively good incentives for across group links to be formed.

D. General tensions between stability, efficiency and inequality

Like earlier sections, here we provide a more detailed treatment of a corresponding Section in the main paper. The corresponding section this time is Section 6.5.

The purpose of this section is to document a general fundamental tension between equality and efficient stable networks. We begin by relating different graph theoretic concepts to stability, efficiency and inequality.

D.0.1. *Equality.* We would like to say something general about inequality for all inequality measures in the Atkinson class on formed networks for *any* symmetric payoff function $u : L \to \mathbb{R}$. Unfortunately, without further restrictions on how network positions translate into payoffs, it is impossible to compare two network in general. However, it is possible to pose and answer in general the question of when payoffs will be guaranteed to be perfectly equitable.

We proceed under the assumption that only agents' network positions matter for their payoffs—specifically, we require agents in identical network positions to receive the same payoffs. Intuitively, then, if all agents are in identical positions, they must receive equal payoffs. The set of networks for which this holds, thereby guaranteeing perfectly equitable outcomes, will be a useful benchmark that helps identify a general tension between equality and efficiency/stability.

In order to formalize the idea that agents are in identical network positions, we need to introduce some graph theory notations and terminology. We limit attention to connected networks. Every network is implicitly labelled, and we identify the set of labels with the set of nodes $\mathbf{N}$. Two networks L_1 and L_2 are called *isomorphic*, written $L_1 \sim_I L_2$, if they coincide up to labelling, i.e. a permutation of $\mathbf{N}$. They are also *automorphic* if for the permutation of nodes associated with the isomorphism, every node has the same set of neighbors. More formally, the network L_1 and L_2 are *automorphic*, written $L_1 \sim_A L_2$, if they are isomorphic, and for any $i, j \in \mathbf{N}$, $i \in \mathbf{N}(j; L)$ if and only if $f(i) \in \mathbf{N}(f(j); L)$, where f is the relevant isomorphism mapping $\mathbf{N}_1$ to $\mathbf{N}_2$, called an *automorphism.* A simple undirected binary graph $L \in \mathcal{L}$ is *vertex transitive* if for every pair of vertices i and j in $\mathbf{N}$, there exists an automorphism $f_{ij} : \mathbf{N} \to \mathbf{N}$ such that $f(i) = j$. Thus, when a network is vertex transitive, we can take a node i and map it to the position of any other node j, by changing the label of

j to i, and there exists a way of relabelling the other nodes such that all nodes have exactly the same neighbors as before and the structure of the graph is preserved. Thus the positions of any two nodes i and j in a vertex transitive network are equivalent in a certain sense, and it is intuitive that the agents should receive the same payoffs.

To formalize the idea that vertex transitivity is the key network symmetry condition for equal payoffs we show that for a large class of payoff functions mapping network positions into payoffs, payoffs are identical if and only if the network is vertex transitive. In principle, an agent's payoff can depend not only on their position in a network L, but also their position in subnetworks of L. Moreover, we might want to assign different subnetwork values to agents in the same subnetwork that vary with different orderings of the agents, and in particular, some notion of the marginal effect an agent has on the subnetwork. This gives us a rich basis for considering network payoffs.

Define $\mathcal{T}$ as the *ordered* set of permutations over the nodes $\mathbf{N}$. Note that with regards to any node $i \in \mathbf{N}$, every permutation $\tau \in \mathcal{T}$ maps one-to-one onto two specific induced subgraphs: one, the subgraph supported by vertices up to and *excluding* i, and two, the subgraph supported by vertices up to and *including* i. Let $\nu : \mathcal{T} \times \mathcal{V} \times \mathcal{L} \to \mathbb{R}$ be the function which assigns to every pair $\{\tau, i\}$ a "marginal value" with regards to such implied pairs of subgraphs in L. Let $S^i_\tau \subseteq L$ denote the induced subgraph supported by those vertices up to and including i in τ, while $S^{-i}_\tau = S^i_\tau \setminus \{i\}$ is the node-deleted subgraph of S^i_τ with regards to i. We require $\nu_i(\tau_k) = \nu_j(\tau_\ell)$ if the respective subgraphs including, respectively, i and j are isomorphic ($S^i_{\tau_k} \sim_I S^j_{\tau_\ell}$) *and* the respective subgraphs excluding, respectively, i and j are isomorphic ($S^{-i}_{\tau_k} \sim_I S^{-j}_{\tau_l}$). This confines node identity only to matter in so far as it corresponds to a position in a subnetwork. Let $V_i : \mathbb{R}^{n!} \to \mathbb{R}$ be the function which maps node i's multi-set of $n!$ marginal values $\{\nu_i(\tau)\}_T$ in L onto a "graph value", where $n!$ is simply the cardinality of $\mathcal{T}$. So the overall payoff we assign to an agent depends on all their possible marginal values. We restrict how these marginal values are mapped into payoffs by requiring only anonymity, i.e. any pre-image under V_i is closed under permutation. Hereafter, we let ν_i and V_i indicate the conditioning on some node i when convenient.

Let $\mathcal{V}$ denote the space of admissible functions V_i, and $\mathcal{F}$ denote the space of admissible functions ν_i. We will say that a result applies generically if it applies to all but a zero measure set of admissible functions ν_i and all but a zero measure set of admissible functions V_i. Of course, many non-generic mappings (V_i, ν_i) may be of interest. That non-withstanding it is of interest to study what network symmetry is needed in general for agents to receive equal payoffs.

We proceed to pin down the simple graphs for which expected payoffs V_i must be uniform for any payoff mapping (including the Myerson value).

Proposition SA10. *$V_i = V_j$ for all $i, j \in \mathbf{N}$ if L is vertex transitive and, generically, $V_i = V_j$ for all $i, j \in \mathbf{N}$ only if L is vertex transitive.*

Proof. **Sufficiency.** Consider any two vertices i and j as well as some permutation τ. Let $f(\tau) \in \mathcal{T}$ be the image of τ under the automorphism mapping i to j, which exists by the vertex transitivity of L. As automorphisms by definition preserve adjacency relations, $S_\tau^j \sim_I S_{f(\tau)}^{f(i)}$ and $S_\tau^{-j} \sim_I S_{f(\tau)}^{-f(i)}$ for all $i, j \in \mathcal{V}$. Hence, $\nu_j(\tau) = \nu_i(f(\tau))$ by the earlier requirement of identity in ν under isomorphism of the implied graph arguments, for any $\tau \in \mathcal{T}$. Fix the set of all of i's marginal values in L, written $\{\nu_i\}_\mathcal{T}$, in arbitrary order. By the foregoing argument, there exists a bijection between $\{\nu_i\}_\mathcal{T}$ and $\{\nu_j\}_\mathcal{T}$ through f. By anonymity of V, hence $V(\{\nu_j\}_\mathcal{T}) = V(\{\nu_i\}_{f \circ \mathcal{T}}) = V(\{\nu_i\}_\mathcal{T})$.

Necessity. We start with a well known result from graph theory. A simple undirected binary graph of finite order is vertex transitive if, and only if, its one-node deleted subgraphs are isomorphic (Thomassen, 1985). Thus if we have a graph that is not vertex transitive there exist nodes i and j such that $L \setminus \{i\} \not\sim_I L \setminus \{j\}$. Hence generically, for any permutation τ in which i is last, and any permutation τ' in which j is last, $\nu_i(\tau) \neq \nu_j(\tau)$. So generically, $V(\{\nu_i\}_\mathcal{T}) \neq V(\{\nu_j\}_\mathcal{T})$. □

Proposition SA10 shows that for a large space of payoff functions for which all that matters is agents' network positions, vertex transitivity guarantees equal payoffs and is also required for equal payoffs, generically. A non-generic payoff function in this space, that can be applied in the special case of transferable utilities (which is not assumed for the above result) is the Myerson value. As we have seen, vertex transitivity is sufficient but not necessary for equal payoffs under the Myerson value. With the Myerson value the weaker symmetry requirement of regular networks (so that each node has the same number of neighbors) is sufficient (see Section 4 of the main paper).

Proposition SA10 takes its informational basis for determining payoffs to be similar to that used by the Myerson value. However, this informational basis is very broad and any way of determining payoffs based on coarser information is covered by the result. For example, the result covers any payoff function that depends only on each agent's set of friends (neighbors), set of friends of friends, set of friends of friends of friends, and so on, for every possible subnetwork of L. As a more specific example, if payoffs were proportional to each agent's eigenvector centrality they would depend only on the structure of the network L, and so by Proposition SA10 agents would receive identical payoffs on a vertex transitive network.[6] Similarly, if payoffs were proportional to agents' marginal contributions to the spectral radius of the network L, then they would depend only on the structure of the network L and the subnetworks $L \setminus \{i\}$ for all $i \in N$, and so, by Proposition SA10 agents' payoffs would be identical on a vertex transitive network.

D.0.2. *Efficiency.* A network L is *Pareto efficient* if there is no network L' such that the payoffs of the agents on the network L' Pareto dominate those on L (i.e., all agents receive

[6]As one of many ways in which this can be implemented, set $\nu_i(\tau)$ equal to i's eigenvector centrality on L for all τ in which i is last to arrive, and to 0 otherwise, and let V_i equal $\max_{\tau \in \mathcal{T}} \nu_i(\tau)$.

weakly higher net payoffs on L' than L and at least one agent receives a strictly higher payoff).

To get a handle on the set of *Pareto efficient* networks, we assume that shorter path lengths facilitate weakly better risk-sharing. Specifically, we assume that all *Pareto efficient* networks $G = (n, L)$ have one component, and there is no alternative network $G' = (n, L')$ such that $|L'| \leq |L|$ and the path length distribution of L' first order stochastically dominates the path length distribution of L. This enables us to eliminate some configurations from the set of networks that might be Pareto efficient.

D.0.3. *Stability.* Finally, we turn to stability. Since we want to make a point at a high level of generality, without a concrete model specification, below we propose a weak notion of stability that can be interpreted as a necessary condition. Given the assumption we have already made that shorter path lengths enable better risk-sharing, it is natural to also suppose that there are stronger incentives for agents further way in the network to have a profitable deviation in which they form a new link. This also preserves a key ingredient from our benchmark model in terms of agents' incentives to deviate.

We say two nodes i and j are closer on a network L than L' if every path between i and j on L' can be matched to a weakly shorter path between i and j on L. We assume that if two agents are further apart in this weak, partial ordering sense, then they have stronger incentives to deviate and form a new link.

D.0.4. *A general tension.* The next result formalizes the general tension among efficiency, stability and equality, by showing that for realistic numbers of agents a network cannot be both efficient and regular, which as argued above is in general necessary but not sufficient for perfectly equitable outcomes.

Proposition 19. *If in all Pareto efficient networks connecting n agents there are fewer than $n\sqrt{n-1}/2$ links, then there does not exist a constrained Pareto efficient and regular network.*

To aid interpretation of Proposition 19 it is helpful to consider some values of n that are of the same order of magnitude as village sizes. For $n = 100$, the optimal regular degree r that minimizes the overall number of links is $r = 10$. So there are at least 500 risk-sharing links present in a regular separation efficient network on 100 nodes (and the networks that achieves this are 10-regular). For $n = 500$, the optimal regular degree r that minimizes the overall number of links required is $r = 22$. So there are at least 5670 risk-sharing links present in a Pareto efficient network on 500 nodes (and the network that achieves this is 22-regular). In both cases this is considerably more risk-sharing links than empirical research typically documented in villages, suggesting that the minimum number of links that would be necessary to maintain both equality and separation efficiency is inefficiently high.

We now prove Proposition 19

Proof. Consider a network $L = (n, L)$. Nodes i and j have a path length of 1 if and only if $l_{ij} \in L$. Thus, the number of pairs of nodes with path length 1 in network L is $|L|$. In a network with n nodes there are $n(n-1)/2$ potential links. Thus a lower bound on the distribution of path lengths a network $L = (n, L)$ can achieve (in terms of first order stochastic dominance) is for $|L|$ pairs of nodes to have path length 1 and the remaining $n(n-1)/2 - |L|$ pairs of nodes to have path length 2. Moreover, this bound is tight. Any network which contains the star network as a subnetwork trivially achieves the bound.

By assumption, on a Pareto efficient network L, all nodes are path-connected and there must be no alternative network $L' = (N, L')$ that contains weakly fewer links and generates a path length distribution that first order-stochastically dominates L. Thus all efficient networks must have $|L|$ pairs of nodes with path length 1 and $n(n-1) - |L|$ pairs of nodes with path length 2 (otherwise the star network would contain weakly fewer links and generate a path length distribution that first order-stochastically dominates L). Hence all Pareto efficient networks on L must have diameter 2.

In a regular network of order r, each node has r neighbors. A given node i then has r neighbors, and each of these have $r-1$ neighbors other than i. Were there any other nodes in the graph, the diameter would be more than 2. Hence, an upper bound on the number of nodes in the network is $\bar{n} := 1 + r + r(r-1) = r^2 + 1$. Indeed, this is exactly the upper bound on the number of nodes in a regular network given by the Moore upper bound for diameter 2 networks. As $n \leq \bar{n}$, we can rearrange the inequality to conclude that $r \geq \sqrt{n-1}$ (as $r > 0$). The number of links in an r regular network is $nr/2$—there are n nodes each with r links, and each link is shared by two nodes. We therefore have that $|L| = nr/2 \geq n\sqrt{n-1}/2$.

□

E. Overinvestment and Underinvestment Examples

In this Section we provide an example of over-investment within group in the unique stable network and a related example of underinvestment across group in the unique stable network.

We begin by assuming there is one group with s members connected by a network L. Equation 11 in the main paper implies that Myerson distance of two agents i, j such that $l_{ij} \notin L$ is greater than $1/2$, while the Myerson distanace between i and j if they form the link l_{ij} would be $1/2$. Thus i and j's gross payoff strictly increases if the link l_{ij} is added. So, for κ_w sufficiently close to 0, in all stable networks for any pair of agents i, j the link l_{ij} must be formed; The unique stable network is the complete network and there is overinvestment.

Suppose now there are two groups, g, g' both with s members and keep the same parameter values from the previous example. By equation 13 in the main paper, the incentives to form within group links are weakly increased by the presence of any across group links. Thus in all stable networks the network structure within-group must be complete networks; All possible within-group links must be formed. Suppose these are the only links formed so that no across-group links are formed. Denote this network L. From equation 13 the change in total variance achieved by connecting an agent i from group g to an agent j from group g' is strictly increasing in s (the size of both groups). Given the Myerson value calculation, this means that the marginal contribution of the link l_{ij} to total surplus (the certainty equivalent value of the variance reduction) is strictly greater on $L \cup \{l_{ij}\}$ than it is on any strict subgraph, including all those formed when the later of i and j arrives in the Myerson calculation. This implies that $(MV(i; L \cup l_{ij}) - MV(i; L)) + (MV(j; L \cup l_{ij}) - MV(j; L)) < TS(L \cup l_{ij}) - TS(L)$ for all $l_{ij} : i \in \mathbf{S}_g, j \in \mathbf{S}_{g'}$. So, setting κ_a such that

$$MV(i; L \cup l_{ij}) - MV(i; L) + MV(j; L \cup l_{ij}) - MV(j; L) < 2\kappa_a < TS(L \cup l_{ij}) - TS(L),$$

the network L is the unique stable network and there is underinvestment (in across-group links) in all stable networks.

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